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Motion of an Orbiter under Tidal Perturbation

We consider the motion of an orbiter about a central body which is orbiting a much more massive third body that effects a tidal perturbation on the orbiter. The effect of the tidal perturbation is shown to lead to instability in the eccentricity for high inclination orbits which would lead to impact against the central body with no control of the vehicle. This work derives a first order analytical theory for the motion of the satellite and determines stability conditions. These conditions are then compared against higher fidelity models.

I. Introduction

The exploration of the outer Solar System has led to many exciting discoveries and a unique opportunity to develop new technologies. Past and present missions have helped us to understand the composition and behavior of these planetary system, but have also raised new questions. The outcome of the Galileo mission to Jupiter and Cassini to Saturn suggest that the icy moons Europa and Enceladus, one on each system, may have liquid oceans beneath the surface that may be capable of sustaining life.

NASA defined the exploration of Europa as the second priority in space exploration [1] and an orbiter mission was conceived in partnership with ESA, Jupiter Europa Orbiter - Laplace, but it was canceled and due to budgetary constraints. While NASA still pursued the exploration of Europa through a fly-by mission, ESA shifted its approach to JUICE: a fly-by mission that would study Europa and other Jovian moons finishing with an injection into Ganymede [2] and thus being the first spacecraft to orbit a planetary satellite other than Earth's Moon.

This work will focus on the analysis of the orbital dynamics for a scenario common to all these missions: the motion of the orbiter about a satellite for low altitude orbits. For this, we will develop a first-order analytical theory to gain insight on the general behavior of the orbit which can be used in the preliminary design of the orbits drawn from Scheeres [3] [4]. The novelty of this scenario is that we will have to consider the effect of tidal force of the planet on the orbit besides the oblateness of the satellite. Unlike the perturbation from distant third bodies, the gravitational perturbation due to the central body cannot be considered constant, and this differential force will give rise to new characteristics in the motion.

We will study high inclination orbits since these are desirable for the scientific purposes of such a mission. We will show that the main effect is the effect on the eccentricity and the argument of periapsis. Depending on the inclination of the orbit, we can obtain secular effects on the eccentricity that will lead to instability and eventually to an impact against the surface. However, this behavior can also be leveraged to circularize the orbit as JUICE intends.

This work is split into the following sections. After the introduction, the definition of the model is presented: Hill's problem will be introduced ans used to define the perturbation function. In the analysis section, we will make use of averaging techniques to obtain the secular effect of the perturbations and derive from it the main characteristics of the motion. In the results we will study the motion for some specific cases in the environment of Europa and we will obtain patterns similar to those used in JUICE, and compare it with more especial perturbations solutions. Finally we will present our conclusions.

II. Definition of the Model

This section states the assumptions used for defining the problem and works through the derivation of the perturbation functions used to analyze the motion. For this purpose we expand the concepts in Scheeres [3]. The perturbing function is split and found based on its different components, and we use the linearity of the disturbing function to find their combined effect.

A. Tidal perturbation

The tidal force of the planet is modeled following Hill's approximation to the Three Body problem. This is a simplification of the Circular Restricted 3 Body Problem (CR3BP). It uses the assumptions for the CR3BP: the body of interest has negligible mass and the satellite orbits the planet circularly and the choice of a synchronous rotating reference frame, but we also assume that the mass of the satellite is negligible with respect to that of the primary [5]. In this scenario, depicted in figure 1, we consider the motion close the satellite:

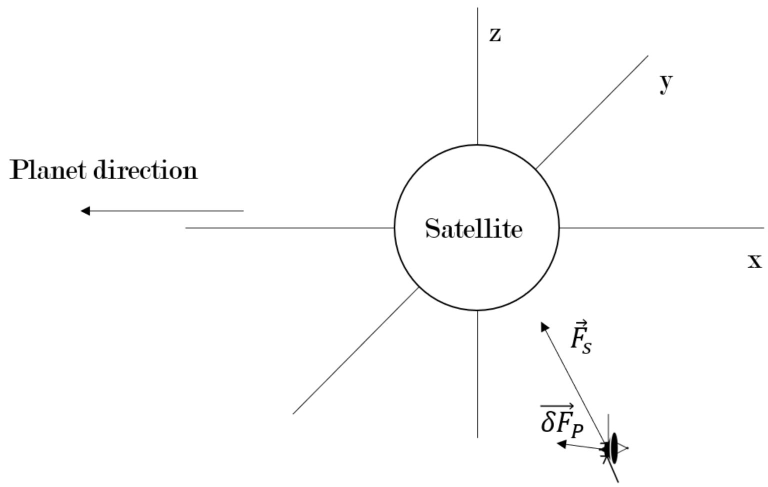

Fig. 1 Hill problem: the orbiter is under the effect of a satellite and a distant planet.

To simplify and generalize the outcome of the analysis we non-dimensionalize the problem. For this we choose a characteristic time and distance as the inverse of the angular velocity of the satellite and the radius that would achieve the unperturbed resonant orbit around the satellite respectively:

$$n_s = \sqrt{\frac{\mu_p}{r_s^3}} = \sqrt{\frac{\mu_s}{r_{res}^3}} \qquad \text{non-dimensional inverse time} \tag{1}$$

$$r_{res} = \left(\frac{\mu_s}{n_s^2}\right)^{1/3} \qquad \text{non-dimensional distance} \tag{2}$$

This implies that $\mu_s = 1$. Since we are considering perturbed orbits about the satellite, due to the difference of mass between the two massive bodies we must consider orbits in the immediate vicinity of the satellite. This means that we can obtain the perturbations from the planet by expanding the gravitational influence about the position of the satellite. We can do this by expanding using Legendre polynomials, same as in the class notes [6].

We denote position of the orbiter about the satellite as $\vec{r} = x\hat{u}_x + y\hat{u}_y + z\hat{u}_z$. Expanding the disturbing function to the second Legendre polynomial:

$$\begin{aligned}\mathcal{R}_P &= \frac{\mu_P}{r_S^3} r^2 P_2\left(\frac{x}{r}\right) = r^2\left(-\frac{1}{2} - 3\frac{x^2}{r^2}\right) \\ &= \frac{1}{2}(-r^2 + 3x^2)\end{aligned} \tag{3}$$

B. Oblateness perturbation

When considering the perturbation due to the non-sphericity of the satellite, we start by considering the motion of the satellite about the planet. We assume the satellite is tidally locked to the planet: it is static with respect to the synchronous rotating frame with the minor axis of inertia in oriented along the x axis and the major axis of inertia oriented along the z axis [7]. If we consider the satellite to be an ellipsoid, this would give place to the second order harmonics with J_{21} vanishing due to the choice of reference frame. Also, we consider the mean motion of the orbiter to be non-commensurate with the mean motion of the satellite, we can disregard the effects the J_{22} harmonic, or as shown in [7] by averaging about one revolution to the central planet.

Thus in this first approach, we only consider the J_2 an disregard higher order harmonics. The perturbation is function found in [6] in terms of the satellite latitude angle ϕ and the semimajor axis and force:

$$\mathcal{R}_2 = -\frac{\mu_s}{r^3} J_2 P_2\left(\sin\phi\right) = \frac{J_2}{2}\frac{1}{r^3}\left(1 - 3\frac{z^2}{r^2}\right) \tag{4}$$

C. Numerical model

Later in our analysis we will compare the results of the analytic theory with those of especial perturbations. For this we obtain the equations of the motion (EOMs)for our modified Hill problem. The potential function comprises the satellite spheric potential and the disturbing function:

$$U = \frac{1}{r} + \mathcal{R}_P + \mathcal{R}_2 \tag{5}$$

And we obtain the EOMs by taking the derivative of the potential and considering the rotating frame with unitary angular velocity:

$$\ddot{\vec{r}} = \nabla U - \left(\hat{u}_z \times \hat{u}_z \times \vec{r}\right) - \left(\hat{u}_z \times \dot{\vec{r}}\right) \tag{6}$$

Obtaining the EOMs for the system:

$$\ddot{x} = +2\dot{y} - \frac{1}{r^3}x + 3x - \frac{J_2}{2}\frac{1}{r^5}\left(3 - 15\frac{z^2}{r^2}\right)x \tag{7}$$

$$\ddot{y} = -2\dot{x} - \frac{1}{r^3}y - \frac{J_2}{2}\frac{1}{r^5}\left(3 - 15\frac{z^2}{r^2}\right)y \tag{8}$$

$$\ddot{z} = - \frac{1}{r^3}y - z - \frac{J_2}{2}\frac{1}{r^5}\left(9 - 15\frac{z^2}{r^2}\right)z \tag{9}$$

The Hill problem is conservative with a constant of the motion similar to the Jacobi constant for the RC3BP. Since our modified Hill problem has only a change in the potential function it is also conservative. The conserved Jacobi-like constant is:

$$C = (x^2 + y^2) + 2U = -z^2 + 3x^2 + \frac{2}{r} + \frac{J_2}{r^3}\left(1 - 3\frac{z^2}{r^2}\right) - (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) \tag{10}$$

Hill's problem has libration points at $x = \pm\left(\frac{1}{3}\right)^{1/3} \approx 0.69$. While the libration points for our problem are not the same, since we are considering a slightly perturbed problem, we still can use the libration points as reference for the semimajor axis for the orbiter. They smaller to minimize the effect of strong perturbation from the perturbing planet.

III. Analysis

In this section we obtain the analytical first order theory for orbits about the satellite and discuss the implications. For this we will double-average the disturbing function, obtain the equivalent Lagrange Planetary Equations and analyze their implication for the case of a low altitude orbiter at high inclination scientific orbits.

A. Averaging the disturbing function

In averaging the disturbing function for the tidal perturbation, we consider that the effect of the tidal perturbation for a single orbit small so it can be approximated by keeping all orbital elements constant.

$$\tilde{\mathcal{R}}_P = \frac{1}{2\pi}\int_0^{2\pi} \mathcal{R}_P dM = \frac{1}{2}(-r^2 + 3x^2) \tag{11}$$

This averaging can be done in either true or eccentric anomaly. The first option involves expanding r as Fourier cosines series for which only the constant and second harmonics doesn't vanish in the end. Eccentric anomaly is more involved but more straight forward so we will use it, we have to change the integration variable to E, and using the appropriate expressions for x and r and expanding into harmonics of E it is clear that only the constant term will survive.

$$r = a(1 - e\cos E) \tag{12}$$

$$dM = \frac{r}{a}dE = (1 - e\cos E)dE \tag{13}$$

x is expressed the rotating frame coordinates, which rotate with angular velocity 1. We define the longitude of the ascending node λ as a time varying constant, even in the osculating orbit, that references the ascending node in satellite coordinates (the satellite rotates synchronously with the reference frame). Then x can be obtained by a rotation from the perifocal coordinates ξ, eta, functions of the orbital parameters a, e, using the instantaneous orbital parameters λ, i, ω.

$$\begin{aligned} x =& (-\cos\lambda\cos\omega + \cos i\sin\lambda\sin\omega)\xi \\ &+ (\cos i\cos\omega\sin\lambda + \cos\lambda\sin\omega)\eta \\ =& a(-\cos\lambda\cos\omega + \cos i\sin\lambda\sin\omega)(1 - e\cos E) \\ &+ a\sqrt{1-e^2}(\cos i\cos\omega\sin\lambda + \cos\lambda\sin\omega)\sin E \end{aligned} \tag{14}$$

To obtain:

$$\tilde{\mathcal{R}}_P = \frac{1}{64}a^2 \Big[15e^2 \cos 2\omega \left(2\cos 2i + 6\cos 2\lambda + 4\sin^2 i\right) \\ -120e^2 \cos i \sin 2\lambda \sin 2\omega + 2\left(3e^2 + 2\right)\left(6\sin^2 i \cos 2\lambda + 3\cos 2i + 1\right)\Big] \tag{15}$$

We now introduce the second averaging assumption. If we consider the mean motion of the orbiter about the satellite to be $n \gg 1$, an assumption that will hold in most scenarios in the solar system, then the orbiter will complete many revolutions about the satellite while completing one about the planet. If these perturbations for a single planetary orbit are small enough, we can again average the effect of λ which as we stated, incorporates the rotation about the planet to obtain $\bar{\mathcal{R}}_P$:

$$\bar{\mathcal{R}}_P = \frac{1}{2\pi}\int_0^{2\pi} \tilde{\mathcal{R}}_P d\lambda = \frac{1}{32}a^2 \left[30e^2 \sin^2 i \cos 2\omega + \left(3e^2 + 2\right)(3\cos 2i + 1)\right] \tag{16}$$

Following the same process we obtain the disturbing function for one revolution. There's no need to consider the second averaging about the planet since it would only discard the J_{22} which has been discarded before hand. For the sake of simplicity we refer to the class notes where an equivalent expression for the averaging of the J_2 perturbation was found:

$$\bar{\mathcal{R}}_2 = \frac{n^2 J_2}{8(1-e^2)^{3/2}}(1 + 3\cos 2i) \tag{17}$$

The global averaged disturbing function $\bar{\mathcal{R}}$ is found as the sum of $\bar{\mathcal{R}}_P$ and $\bar{\mathcal{R}}_2$.

B. Derivation of the Lagrange Planetary Equations

For the derivation of the Lagrange Planetary Equations, for simplicity, we will leverage the linearity of the perturbation function. We will consider the contributions of the perturbing function due to tidal force and those due to the oblateness separately, and we will simplify them using the same assumptions. Then we will find the total rate of change as the sum of those.

The Lagrange Planetary equations considering only the tidal perturbation are:

$$\dot{a} = 0 \tag{18}$$

$$\dot{e} = \frac{15}{8}\frac{e\sqrt{1-e^2}}{n}\sin^2 i \sin 2\omega \tag{19}$$

$$\dot{\lambda} = -1 - \frac{3}{8}\frac{2+3e^2-5e^2\cos 2\omega}{\sqrt{1-e^2}n}\cos i \tag{20}$$

$$\frac{di}{dt} = -\frac{15}{16}\frac{e^2}{\sqrt{1-e^2}n}\sin 2i \sin 2\omega \tag{21}$$

$$\dot{\omega} = \frac{3}{16}\frac{3+10\cos 2i \sin^2\omega + \left(5-10e^2\right)\cos 2\omega + 2e^2}{\sqrt{1-e^2}n} \tag{22}$$

$$\dot{M} = n - \frac{3}{16}\frac{\left(e^2+1\right)\sin^2(i)\cos(2\omega) + 3\left(3e^2+7\right)\cos(2i) + 3e^2 + 7}{n} \tag{23}$$

And the equations for the J_2 perturbations are:

$$\dot{a} = 0 \tag{24}$$

$$\dot{e} = 0 \tag{25}$$

$$\dot{\lambda} = -1 + \frac{3}{2}\frac{nJ_2}{a^2\left(1-e^2\right)^2}\cos i \tag{26}$$

$$\frac{di}{dt} = 0 \tag{27}$$

$$\dot{\omega} = -\frac{3}{8}\frac{nJ_2(3+5\cos 2i)}{a^2\left(1-e^2\right)^2} \tag{28}$$

$$\dot{M} = n\left[1 + \frac{3}{8}\frac{J_2}{a^2}(1+3\cos 2i)\right] \tag{29}$$

Since we have averaged the perturbing function with respect to one revolution around the satellite and the planet both M and λ are ignorable coordinates whose averaged contribution is already included the equations above. Thus for the analysis of the shape of the orbit we can disregard the equations for both the mean motion and the node.

Instead of combining the equations above directly, we first consider the extra assumptions that we can introduce for the case of the low altitude orbiter.

C. Simplifying the equations for a low altitude orbiter

Our interest lies in the behavior of the orbits for a low altitude orbiter. In this case the eccentricity must be small since the radius of the satellite will limit the periapsis.

Then the equations above can be simplified by keeping only the terms up to the first power of e. The equations considering the tidal and oblateness perturbations are:

$$\dot{a} = 0 \tag{30}$$

$$\dot{e} = \frac{15}{8}\frac{1}{n} e \sin^2 i \sin 2\omega \tag{31}$$

$$\frac{di}{dt} = 0 \tag{32}$$

$$\dot{\omega} = \frac{3}{8}\frac{1}{n}\left[4 - \sin^2 i + 5\sin^2 i \cos 2\omega\right] - \frac{3}{4}\frac{nJ_2}{a^2}\left(4 - 5\sin^2 i\right) \tag{33}$$

Where we have used trigonometric identities to make $\dot{e}$ and $\dot{\omega}$ similar.

For the first order approximation for small e we observe that the inclination and the semi-major axis stay constant. The behavior of the orbit can be reduced to the motion of the eccentricity in the orbital plane. Therefore we have successfully reduced the behavior of the orbit to a two-dimensional problem for which a and i are parameters.

D. The motion of the eccentricity vector

In Scheeres [4] an analytical solution is provided for the equations above. However, here we deem is more insightful to examine the properties of the behavior in terms of the phase space and the properties of conservative dynamical systems.

We define the following variable changes to further simplify the equations of motion:

$$\chi = \frac{n^2}{a^2} J_2 = \frac{\mu}{a^5} J_2 \tag{34}$$

$$\alpha = (1 + 2\chi)\frac{4 - 5\sin^2 i}{5 sin^2 i} \tag{35}$$

$$\kappa = \frac{15}{8}\frac{1}{n}\sin^2 i \tag{36}$$

And we introduce the independent variable $\tau = \kappa t$ which is just a scaling of the time that simplifies the equations. With these we obtain the equations of motion:

$$\frac{de}{d\tau} = \sin 2\omega e \tag{37}$$

$$\frac{d\omega}{d\tau} = \cos 2\omega + \alpha \tag{38}$$

This is a nonlinear system of equations. The second equation in ω is decoupled from the equation of e. The behavior of the argument of the periapsis depends on α, for $\alpha^2 > 1$ the change rate in ω will be monotonic, for $\alpha^2 < 1$ there will be several equilibria solutions (for the argument of periapsis only, not the eccentricity vector), with some being stable and others unstable. The behavior of e in instantaneously exponential with gain $\sin 2\omega$ in the eccentricity module will depend on the behavior of the argument of periapsis.

The only possible equilibrium for the this system is with $e = 0$ for any α or for the case of $\alpha^2 = 1$, but this is not a desirable operating point since the system may change properties as discussed above. While a linearized analysis of the equations may give some insight on the local equilibrium properties it does not offer insight on the global behavior. A similar problems arises for the analytic solution, it does not offer extra insight in understanding the stability properties of the system. Probably the most insightful approach is to leverage that the system is conservative and has a unique equilibrium point, then due to the symplectic properties of hamiltonian systems [8] [9], the equilibrium will be either a not asymptotic but stable point, or it will have a stable and unstable manifold. In fact, the $\alpha^2 = 1$ introduced before is a bifurcation for the system.

In figure 2 we reproduce the phase plane for the eccentricity vector in polar coordinates for the case of $\alpha^2 > 1$, the eccentricity vector exhibits a periodic motion which is stable. In figure 2 we display the case for $\alpha^2 < 1$ and we can clearly observe that the global behavior is unstable with an attractive mode and a unstable mode. Beside, the determining the stability behavior of the system, α also determines the positions of the stable and unstable manifolds since they satisfy equation 38.

While this approach is not pursued any further in this work, the existence of the attractive mode

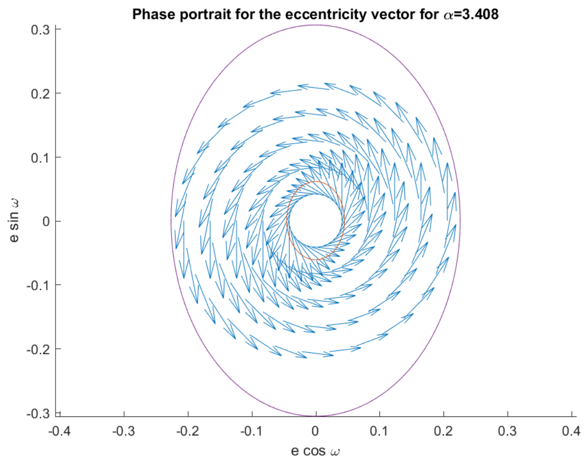

Fig. 2 Phase portrait for the eccentricity vector in a stable configuration: $\chi = .2746, \alpha = 3.4080$

could be used to circularize an orbit which is the approach followed by JUICE [2]. It is important to note that these equations are an averaged first order approximation of a more complex, this means that even if there is an attractive mode that could be exploited to circularize the orbit, there will inaccuracies that will excite the unstable modes so the control action required to follow the attractive manifold may be significant.

Finally, we discuss the kind of orbits that will lead to both behaviors. From the definitions of χ and α we can observe that the involved parameters are on one side the inclination of the orbit i, and on the other the semi=major axis of the orbiter a and the J_2 harmonic. The critical factor here is the inclination, low inclination orbits will lead to bigger α and therefore stability, high inclination orbits will tend to be unstable, effect that will be increased if the effect the oblateness of the planet as measured by χ increases. At the same time the semi-major axis a and inclination i are present in the time scale κ, this means that for high inclination and high altitude orbits the effect in the eccentricity will be slower.

Since the are considering the application of this theory to scientific orbiters, inclinations will typically be high and we will expect the unstable behavior.

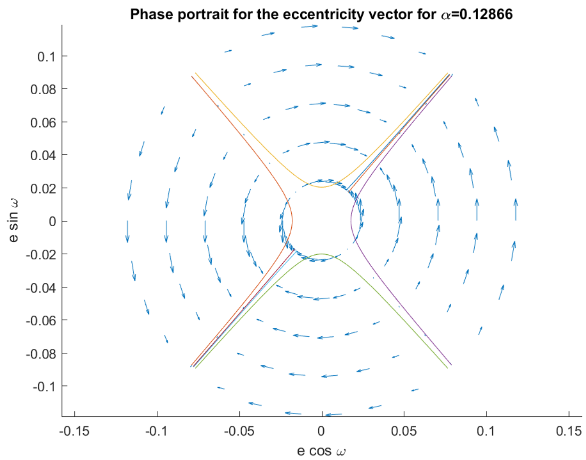

Fig. 3 Phase portrait for the eccentricity vector in a unstable configuration: $\chi = 0.4650, \alpha = 0.1287$

IV. Results

We now consider the motion of an orbiter about Europa which is orbiting Jupiter at the same time. The parameters for this environment have been obtained from JPL SSD database [10] for the parameters pertaining the 2-body motion and Schubert [11] for the radius of Europa and the J_2 coefficient. A especial perturbations approach was implemented that numerically integrates equations 7-9, details can be found on Annex A.

For reference on the non-dimensional units, the period of the orbit of Europa about Jupiter is 3.55 days, the non-dimensional distance is 19.6×10^3 km and the radius of the satellite is 1565 km. In non-dimensional units, its radius is close to .08 and an an altitude of 200km is about 0.01 units.

A. Stable case results

As a first validation of the theory, we start by comparing the motion for the stable case. We consider an orbiter around Europa with an eccentricity of 0.1 and an altitude of 400 km, ($a = 0.1$) and an null initial argument of periapsis. This scenario corresponds to $\chi = .2746, \alpha = 3.4080$ used for the phase plane in the stable case. We perform an integration for long enough for the eccentricity to complete one revolution and we compare it with the analytical results in figure 4.

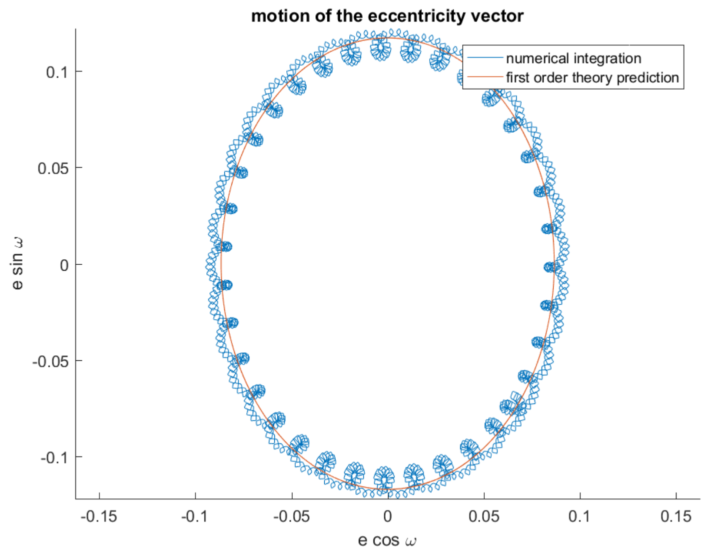

Fig. 4 Comparison of the motion of the eccentricity vector for a stable case between first order theory and especial perturbations. The theory predicts accurately the mean motion of the eccentricity.

We observe that the motion resulting from the numerical integrations fits pretty well the predictions of the analytical theory. We must have into account that analytical theory works with the mean eccentricity which will differ from the actual eccentricity due to short term perturbations which have been averaged. We also show in figure 5 that the semimajor axis and inclination stay constant in mean as predicted by theory.

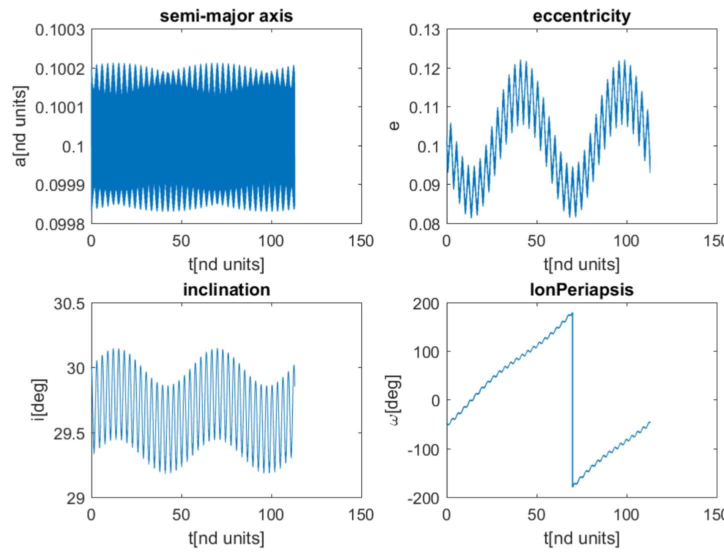

Fig. 5 Evolution of the orbital elements: the semi-major axis and inclination remain constant in average as predicted by the theory.

B. Unstable case results

This is the case of high inclination orbits which is of greatest interest to the case of a sciencific mission. Since the eccentricity will be unstable for high inclinations, this will lead to decreasing altitudes in the periapsis and eventually to an impact against the satellite. In order to maximize the return of the mission, we would like to maximize the time to impact of the orbiter. The condition for impact is for the periapsis to reach surface radius, assuming the planet is a perfect sphere for this purpose. This condition can be expressed in terms of eccentricity for a given semi-major axis:

$$e_f = 1 - \frac{r_{sat}}{a} \tag{39}$$

From the analytic theory, we know this will be achieved if we start in a circular orbit or if we start in the attractive manifold. From a mission design perspective, the latter would be more interesting since we would first have to arrive to the circular orbit anyway. Thus, the initial condition can be chosen on the attractive manifold, the injection into the satellite should be made in a way that the elliptic orbit has the right argument of periapsis, this the approach followed in JUICE [2].

This behavior is showcased with an orbiter around Europa with an initial eccentricity of 0.1 and an altitude of 200 km, ($a = 0.09$) which is close to impact and an initial argument of periapsis such that the satellite is on the attractive manifold. This scenario corresponds to $\chi = 0.4650, \alpha = 0.1287$ used for the phase plane in the unstable case. We perform an integration for long enough for the eccentricity to increase and cause an impact the surface. In the case of the averaged analytical theory, if the initial conditions are chosen exactly on the manifold then the trajectory is convergent to the equilibrium, instead a perturbation equal to the observed short term oscillations in the eccentricity is introduced.

The impact condition happens when the eccentricity reaches 0.1178. The results between match closely, the non dimensional time to impact in the averaged model is 115 units, and 117 for the numeric integration, an error below 1% , also the motion of the eccentricity vector matches well the motion of the analytic model as shown in figure 6.

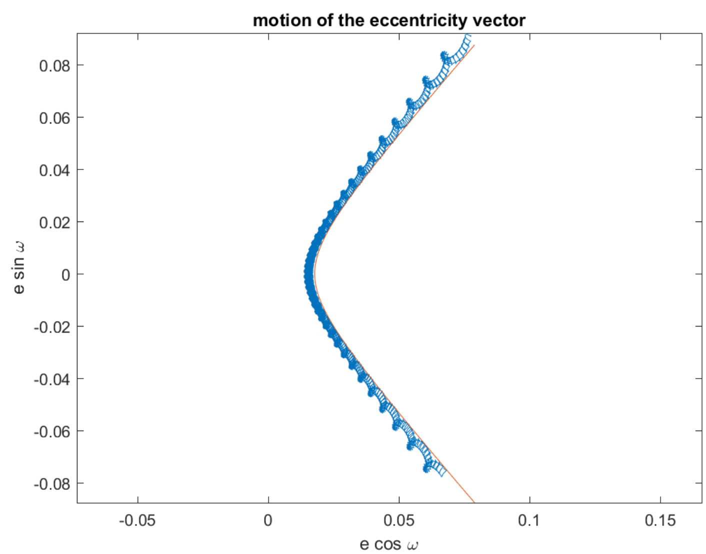

Fig. 6 Comparison of the motion of the eccentricity vector for a stable case between first order theory and especial perturbations. Short term perturbations seem to limit the effectiveness of the attractive manifold.

We also observe that the other elements behave as expected: the semi-major axis and inclination remain constant as shown in figure 7.

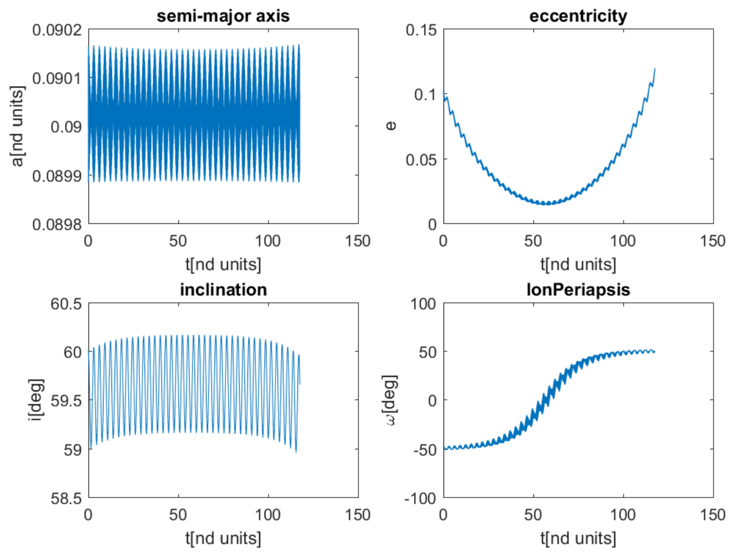

Fig. 7 Evolution of the orbital elements: the semi-major axis and inclination remain constant in average as predicted by the theory.

We note initial conditions were chosen to be on the envelope of the short terms oscillations for the analytical model and provide a good fit. This reinforces the idea that the short term perturbations excite the unstable modes and limit the lifetime of the satellite. Another source for this error could be the that the mean argument of the periapsis is not exactly on the attractive manifold due to short term perturbations. For a closer analysis of this effect, it would necessary to find the expressions for the short term perturbations in terms of the anomaly of the orbit and the latitude of the ascending node. This could lead to better choices of the initial conditions that increase the lifetime. The last test will on this.

C. Application to higher altitude orbits: JUICE mission profiles

As a final test, we change the scenario to that of JUICE orbiting Ganymede. In [2] the mission altitude profile for the beginning of the orbit is given. This consists on an elliptical injection into a 200×10000 km altitude orbit in the stable manifold that will circularize the orbit, the circular orbit is maintained by means of the AOCS system and finally the orbit evolves into a 200×10000 altitude orbit. In non-dimensional units the semi-major axis of the orbit is $a \approx .167$ and the 200 km altitude is $r \approx 0.062$. With these conditions the eccentricity is $e \approx .63$. These clearly violate the assumptions of small eccentricity introduced for the development of the theory.

We obtain the attractive and unstable globalized manifolds by integrating for negative and positive time respectively from a purely circular orbit until the maximum eccentricity is obtained. The mission profile shown in figure 8resembles that of ESA to a large extent, if we remove the periodic bits where the AOCS is assumed to be acting, the natural evolution of the orbit is about 90 days.

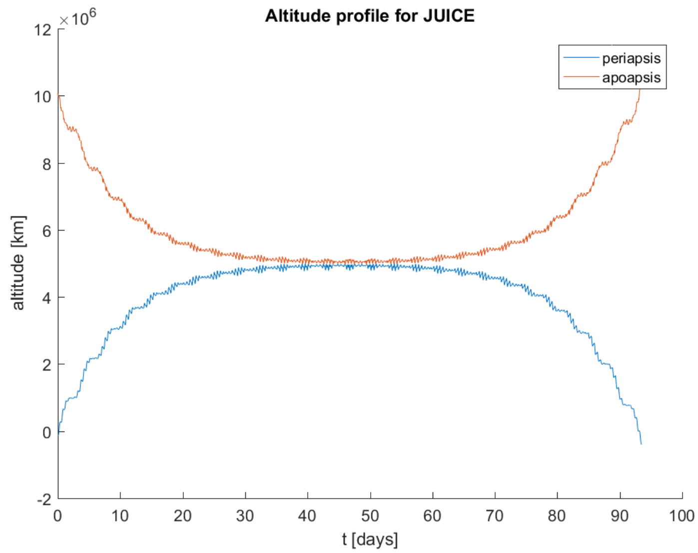

Fig. 8 Altitude profile for JUICE mission from orbit injection to first major maneuver from our model. Reproducing to ESA [2], figure 7-8

In order assess the accuracy of the theory for greater eccentricities, we compare two things: the differences in the phase plane of the eccentricity and differences in the time evolution of the system. In the phase plane we know that for small eccentricities the argument of periapsis for the manifolds is constant, but as we globalize for greater eccentricities this doesn't have to hold. In figure 9 this is done for one trajectory, we observe that the argument of periapsis for the manifolds starts to drift slowly but being sufficiently close, but the short term perturbation increase with eccentricity, as can be expected from our small eccentricity assumption.

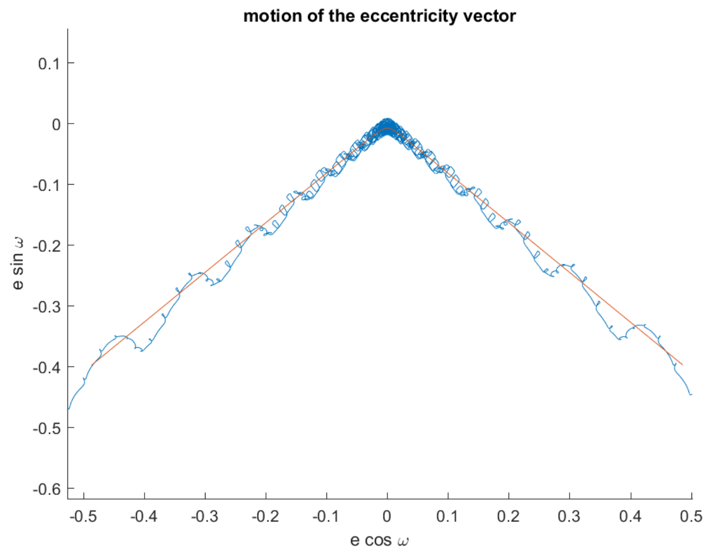

Fig. 9 Phase plane for high eccentricities. The averaged behavior resembles the behavior of the global manifolds but differences begin to show.

Even if a phase portrait is similar, there may be important differences in the rates of convergence and divergence as the manifolds are globalized. To asses this, we compare the time evolution of the altitude profile obtained from the numerical integration with the eccentricity predicted by the first-order theory. We compare the evolution of the profile for the first 40 days of the mission, when it is converging to the circular orbit. Figure 10 shows that the difference is small, meaning that the time evolution doesn't change greatly.

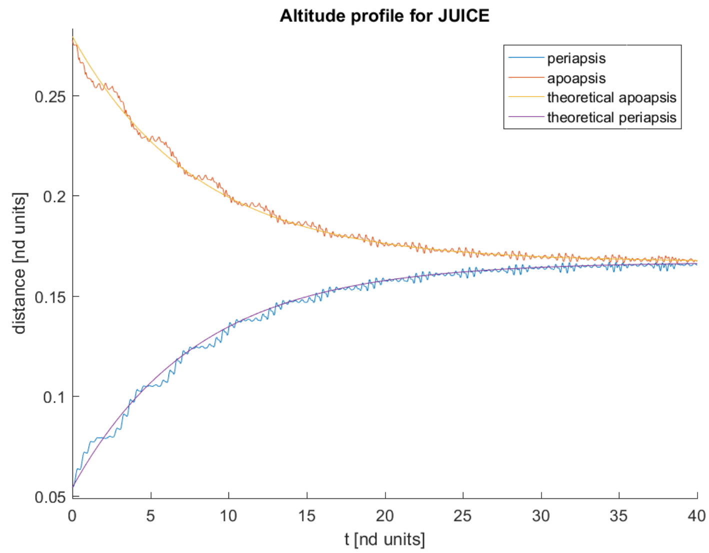

Fig. 10 Altitude profile for the mission compared with the prediction from the small eccentricity theory.

Overall, we can state that the insight the developed analytical theory still applies for higher eccentricity orbits. However, the difference in the time to impact between initial conditions chosen a priori and those obtained from globalizing the manifolds has increased significantly. Meaning that especial perturbation methods become increasingly more accurate and necessary for mission design. Figure 11 shows the orbit in a non-rotating frame where the changes in the eccentricity vector can be appreciated.

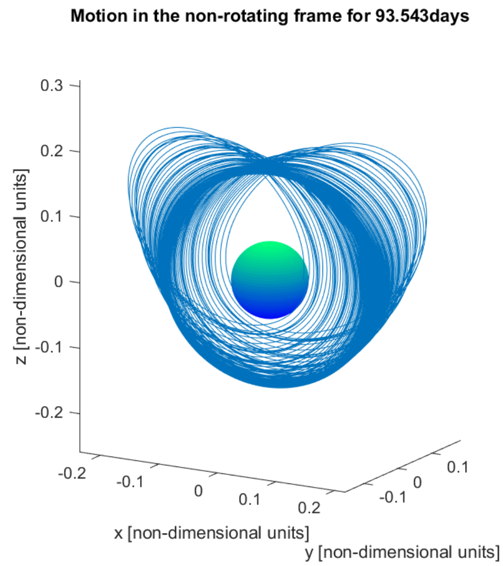

Fig. 11 Altitude profile for the mission compared with the prediction from the small eccentricity theory.

V. Conclusions

In this project a first-order analytical theory has been developed for the motion of an orbiter about a planetary satellite. The considered perturbations have been the effect of tidal force due to the planet and the J_2 harmonic due to the oblateness of the satellite which are averaged over non-commensurate revolutions about the satellite and the planet. The Lagrange planetary equations are obtained and simplified for the case of a small eccentricity orbiter.

Under these assumptions two kind of behaviors appear for the eccentricity vector, all other elements stay constant or have been averaged. For low inclination orbits the perturbations lead to bounded oscillations in modulus and argument, for high inclination of greater scientific interest a bifurcation leads to an unstable behavior. This behavior is characterized by an equilibrium for circular orbits and an attractive and unstable manifold. The unstable manifold imposes a restriction on the lifetime of the orbiter since the altitude of periapsis decreases and will eventually cause an impact against the satellite. The stable manifold offers the possibility of leveraging the natural motion of the orbiter to circularize the orbit after an elliptical injection.

The results of the theory have been validated against a especial perturbations propagator than implements the same model. Comparison between the theory and the numerical results for an orbiter in Europa show a great degree of agreement and offer valuable insight for the determination of initial conditions that can be used either to delay the impact against the surface or to leverage the attractive manifold for circularizing the orbit after injection. The effects of the short period perturbations cause the orbiter to eventually excite the unstable modes and thus a control action would necessary to maintain the orbit in the long term.

Finally, the results of the theory are compared with those of a more demanding scenario. Mission profiles obtained from this model are compared with ESA's JUICE mission showing a great degree of agreement. In this setup, the behavior of a highly eccentric orbit insertion is analyzed by comparing the results of the theory against the trajectories obtained from globalizing the manifolds. The theory still proves to be able to offer insight, especially in terms of the convergence rate, but numerical work would become increasingly more necessary for mission design purposes.

Future work should focus on obtaining insight in the short term perturbations. While their effect in the long term behavior seems almost negligible, the transformation between instantaneous and mean orbital elements limits the ability to choose initial conditions from the theory alone.

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Appendices

A. Code

All code files developed for this project and the report itself can be found on GitHub: `https://github.com/ebabio/orbiterHillProblem.git`.

The following resources are available:

1) Pdf version of the report
2) Mathematica script used to derive the analytical theory.
3) MATLAB library for numerical integration of orbits in the Hill problem.
4) MATLAB script to work with phase portraits of the analytical theory.