

Assignment #2

Chapter 4 Question 6

- Data on students in a statistics class
[X_1] = hours studied [X_2] = undergrad gpa Y = receive an A
↳ logistic regression
 $\beta_0 = -6, \beta_1 = .05, \beta_2 = 1$

- a) Probability that student studies 40 hr, has GPA 3.5 gets an A

↳ logistic regression equation

$$P(Y=1|X=x) = \frac{e^{-(\beta_0 + \beta_1 x_1 + \beta_2 x_2)}}{1 + e^{-(\beta_0 + \beta_1 x_1 + \beta_2 x_2)}}$$

→ with $X_1 = 40$ and $X_2 = 3.5$ probability of student getting an A is .378

- b) Same student, how many hours of study to have 50% chance?

↳ would need to study for **50 hours**

$$.5 = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_1 + \beta_2 x_2)}}$$

$$\left[.5 e^{-(\beta_0 + \beta_1 x_1 + \beta_2 x_2)} = .5 \right] \ln$$

$$-(-6 + .05x_1 + 3.5) = \ln(1) = 0$$

$$-2.5 + .05x_1 = 0$$

$$.05x_1 = 2.5$$

$$\boxed{x_1 = 50}$$

Chapter 4 Problem 7

→ Predict Yes/No on dividend based on last year's

Profit

↳ for no, $\bar{X} = 0$ for yes, $\bar{X} = 10$

σ^2 for both = 36

→ assume X follows normal dist, predict prob that a company will issue dividend given % profit was $X = 4$

→ density function for normal random variable
is $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Bayes Theorem:

$$P_r(Y=1 | X=4) = \frac{\pi_1 f_1(x)}{\sum_{i=1}^K \pi_i f_i(x)}$$

π = marginal probabilities
 σ^2 = common variance

μ = in class means

note: 80% companies issued dividend

$$\therefore \pi_0 = .2 \quad \pi_1 = .8$$

$$P_r(Y=1 | X=4) = \frac{\pi_1 f_1(4)}{\pi_1 f_1(4) + \pi_0 f_0(4)}$$

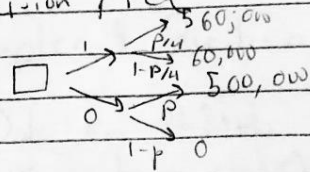
$$f_1(4) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \text{ where } \sigma^2 = 36, \mu = 10, X = 4$$
$$f_1(4) = .0403$$

$$f_0(4) = \text{"} \text{ where } \sigma^2 = 36, \mu = 0, X = 4$$

$$f_0(4) = .0532$$

$$P_r(Y=1 | X=4) = \frac{\pi_1 f_1(4)}{\pi_1 f_1(4) + \pi_0 f_0(4)} = \frac{.8(.04)}{.8(.04) + .2(.05)} = \boxed{.7518}$$

Decision Tree



To find break even point, find cost of no prescribe = cost of prescribe

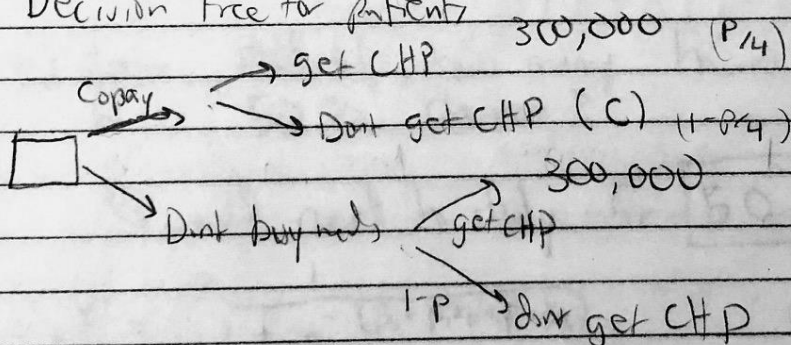
$$\therefore p(500,000) = (1 - P/4)60,000 + \frac{P}{4}(560,000)$$

$$500,000p = 60,000 - 15,000p + 140,000p$$

$$500,000p = 60,000 + 125,000p$$

$$375,000p = 60,000 \quad p = .16$$

Decision tree for patient



set the sides equal and solve for C

$$(C + 300,000) \cdot 0.04 + C \cdot 0.96 = 300,000 \cdot 0.16 + 0 \cdot 0.84$$

$$C + 300,000 \cdot 0.04 = 300,000 \cdot 0.16$$

$$C = 300,000(0.16 - 0.04) = 300,000(0.12) = 36,000$$