Surface Mapping Feedback for Robot-Assisted Rapid Prototyping

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Abstract—A fundamental problem with many solid freeform fabrication systems is the production of large dimensional errors, due to the *freeform* construction process. The subject of this paper is surface mapping feedback (SMF), a closed-loop technique for minimizing the dimensional errors produced by rapid prototyping systems. The main steps of the technique are presented, namely part measurement, control implementation, and deposition adjustment. A test platform, the Cobra 600 rapid freeze prototyping system, is used to validate SMF. A helical gear part is constructed first with open-loop control and then with SMF control; system performance and part quality are compared for the two cases.

I. Introduction

Traditional material removal machining techniques such as milling or drilling typically produce parts with a much higher dimensional accuracy than do solid freeform fabrication (SFF) systems. This is not surprising, since any process where *formable* materials accumulate in *free* space is expected to be less controllable. The discrepancy is especially apparent in rapid prototyping (RP), where a part is constructed layer-by-layer: any error in predicting the material layer thickness is amplified with each deposited layer, thereby producing dimensional error in the vertical direction that is proportional to part height. As a result, all deposition-based RP systems will have an upper limit on the size of an object that can be built before unacceptable height error occurs.

This fundamental problem with RP has been primarily addressed at the process level, where parameters such as temperature, pressure, material flow rate, and tool position have been stabilized. Doumanidis developed a thermal feedback control model and implemented it on an RP system where an infrared camera is used to modulate the power and motion of a plasma-arc torch in laminated object manufacturing [1]. Mazumder et al. limited the maximum height of deposited material in a direct metal deposition system by using a CCD camera to measure part height and adjusting the melt pool volume [2]. Muscato et al. developed geometric feedback control for shaped metal deposition, where a CCD camera is used to measure part geometry and wire feed rate is used as the control action [3]. However, their method does not account for local part height variations: feed rate is varied once per layer, based on the average height error measured.

While implementation of closed-loop control at the process level helps to reduce the part error induced *per layer*, it

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does not maintain the nominal height over the *whole* surface of the part being built or address the error amplification problem. Recently, Cohen and Lipson developed a geometric feedback control model for solid freeform fabrication (SFF) systems that does correct local errors in part geometry [4]. Their concept is particularly suited to drop-on-demand systems, which deposit discrete droplets of material at specific locations. A claim is made that their model can be adapted to continuous-flow systems, which deposit a stream of material through a nozzle that is usually in motion. Their technique is also limited to the flow control of the deposition material.

Here, we propose a new control technique, surface mapping feedback (SMF), which also detects and corrects part geometry errors. SMF can be applied to both drop-ondemand and continuous-flow deposition; it can also be used to adjust material flow and/or deposition tool speed. In many RP systems, such as the Cobra 600 RFP system developed by the authors [5], adjusting tool speed is preferred because it is the most predictable and controllable process variable.

II. SURFACE MAPPING FEEDBACK (SMF)

Surface mapping feedback involves three main steps: part measurement, control loop iteration, and deposition adjustment. From a control perspective, it is important to distinguish between online and offline control. For RP systems, we can define an online feedback system as one where the measurement and control operate in parallel with the material deposition. For the proposed surface mapping feedback (SMF) control, online control is possible with the appropriate system hardware, including adequate computational resources and measurement and deposition systems that do not interfere with each other. However, an offline implementation involving part measurement in between periods of material deposition is much more tolerant of system hardware limitations.

A. Surface Measurement

Accurate measurement of the part surface is obviously a critical step in SMF. The measured error ϵ_m must be much smaller than the desired part error ϵ_p : a reasonable requirement is $\epsilon_m = 0.1\epsilon_p$. Additionally, to minimize part construction times, measurement should be *fast*.

The two main classes of distance measurement devices are contact and non-contact. Since detailed mapping of the part surface would necessarily be time-consuming with contact-type devices, they can be immediately ruled out. The non-contact class includes computer vision, photographic, laser, and ultrasonic devices. It is noteworthy that the measurement ability of all non-contact displacement sensors is highly

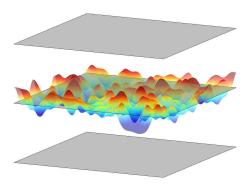


Fig. 1. Surface mapping feedback (SMF): The colored surface represents the current part height, while the gray plane intersecting it is the nominal height. All gray planes indicate nominal heights where the part surface would be mapped.

dependent on part-surface characteristics. For example, many optical sensors are unable to correctly measure distance to transparent or translucent objects.

A key characteristic of SMF is the need to map the whole top surface of the part and scaffolding, generating a surface interpolant, which is then used to adjust subsequent material deposition. A configurable interval h^* , defined as the vertical distance between measurement layers, determines the frequency of surface measurements. Figure 1 provides a graphical representation of SMF.

B. Control Technique

SMF control is similar to other feedback control techniques, except that the control loop is applied at the surface, rather than at the point level. No specific controller is required, though we will demonstrate the principles of SMF control with a PID controller.

Normally, PID control is implemented in the time domain, while for SMF, the part height domain h is used; the time step then becomes the interval h^* . The error signal e(t) of PID controllers becomes an error surface defined by the function E(x,y,h), or, upon discretization, an error matrix \mathbf{E}_k , with the dimensions of a grid which has a user-specified resolution and boundaries defined by the footprint of the part and scaffolding at height h_k , for measurement layer k. Equivalently, the controller output u(t) becomes U(x,y,h) or, upon discretization, matrix \mathbf{U}_k . The proportional, integral, and derivative gains are, as usual, K_p , K_i , and K_d .

Updating and discretizing the standard PID controller, we obtain the control law

$$\mathbf{U}_{k} = K_{p}\mathbf{E}_{k} + K_{i}\sum_{\phi=1}^{k}\mathbf{E}_{\phi} + K_{d}\left(\mathbf{E}_{k} - \mathbf{E}_{k-1}\right)$$
 (1)

where k > 1; for k = 1, the derivative term is left out.

The integral and derivative terms in (1) call for the summation of surfaces defined on different (x, y) grids. Since this summation cannot be performed directly, the matrix terms from previous measurement layers are evaluated at the points on the (x, y) grid for h_k . This transformation

is straightforward using scientific software. For example, in Matlab, a surface interpolant $\mathbb F$, is expressed as a structure where $\mathbb F.X$ is a $2\times mn$ matrix of (x,y) coordinates and $\mathbb F.V$ is a mn-dimensional vector of surface heights, where m and n are the dimensions of the (x,y) grid. $\mathbb F$ can be expressed in the new grid $\mathbb X'$ using the commands $\mathbb F.V=\mathbb F(X')$ and $\mathbb F.X=X'$. If the grid for h_k is larger than that for h_{k-1} , extrapolation could be used, or the initial integral and derivative contribution for new gridpoints could be set to zero.

C. Deposition Adjustment

For drop-on-demand systems, probably the simplest implementation of SMF would entail meshing the part and scaffolding cross-sections at each height h_k and using the U matrix to determine how many drops per gridpoint are needed to reach the next measurement plane. A more complex formulation, which could lead to enhanced accuracy, might consist of using warped pixels, defined by the part boundaries, along with interior pixels to fill in the shells. To allow for material setting or freezing time, the order of drop deposition would need to be carefully controlled.

For continuous-flow systems, a nominal deposition control data set is assumed to be available for adjustment. This data set is divided into deposition layers, each consisting of a series of deposition paths. Each path includes a mathematical description of the path itself, along with any other variables needed for controlling deposition.

Part measurement takes place every n_l deposition layers, where n_l is a user-defined parameter. If h_l is the nominal layer height for the material when used with a pre-defined, nominal set of process parameters, we have $h^* = n_l h_l$. Deposition adjustment at measurement layer k is thus achieved by applying \mathbf{U}_k to the n_l deposition layers above h_k .

Adjustment consists of manipulating the material volume deposited per distance traveled

$$V' = \frac{Q}{c} \tag{2}$$

where ${\cal Q}$ is the material flow rate and c is the deposition path speed.

The controller output \mathbf{U} is applied to any process parameter ρ that can be varied precisely at all points in the RP workspace and has a reasonably well-known impact on Q and/or c. This, of course, eliminates some process variables from consideration, such as extrusion temperature or deposition line pressure: neither of these can be varied *quickly* enough to achieve the needed flow rate variations along a deposition path.

One highly-responsive process parameter that directly impacts flow rate is the control signal used for a micro-solenoid valve. Pulse width modulation or frequency modulation of this signal could be used to vary the flow rate precisely and rapidly along a deposition path. The path speed c can also be varied to implement the desired control, though for some RP systems, c cannot be varied precisely along a deposition path.

The control action is implemented by using \mathbf{U} to update ρ along each path in the n_l layers to be adjusted. The exact implementation depends on how ρ is defined and the relationship between ρ and V'. For example, if $V' \propto 1/\rho$, and ρ is defined at each path point for a path discretized in terms of its coordinate points, we can form a control action matrix $\mathbf{A}_k = \mathbf{\Gamma}_k + C_u \mathbf{U}_k/n_l$, where $\mathbf{\Gamma}_k$ is a matrix of ones, with the same dimensions as \mathbf{U}_k and C_u is a constant of proportionality. The entries of the control action vector \mathbf{a} for a deposition path are defined by $a_i = A(x_i, y_i, h_k)$ for each path point i, where A(x, y, h) is the surface interpolant representation of \mathbf{A}_k . The vector $\boldsymbol{\rho}$, whose entries indicate the value of the process parameter at each point i along the path, is updated according to

$$\rho_i' = a_i \rho_i \tag{3}$$

In the above derivation, it is assumed that detected height errors are *small*, i.e., smaller than h^* . If large errors are present, large changes in ρ along a path will be called for, which might not be possible, since many process parameters that could be useful in implementing SMF will only be *partially* controllable. For example, in a system where deposition is controlled by a solenoid valve, a flow rate will exist at which there is a transition from jetting to dripping. Usually drip flow will result in poor part definition and should thus be avoided.

These problems can be dealt with by implementing another common control technique, namely, saturation. Saturation could be imposed at different steps in the algorithm. A simple but robust implementation would involve the definition of upper and lower saturation planes to bound the controller output \mathbf{U} .

III. TEST PLATFORM: THE COBRA 600 RFP SYSTEM

The SMF control technique is implemented with the Cobra 600 rapid freeze prototyping (RFP) system [5], shown in Fig. 2, consisting of an Adept Cobra 600 SCARA, which has been retrofitted for rapid prototyping using ice as the build material and hydrogenated vegetable oil methyl ester (HVOME) as the scaffolding material. Part-slicing [6] and trajectory control [7] algorithms are available to be adapted for use with SMF. This RP system is of the continuous-flow type.

The SMF software implementation consists of a robot control program written in the V+ programming language developed by Adept Technology, which runs on the Cobra controller, along with a program written in Matlab code, which runs in parallel on the control PC. Since SMF is computationally intensive and the computational resources of the controller¹ are limited, the bulk of the communication and processing load is accomplished with the Matlab code, on the control PC.



Fig. 2. The Cobra 600 RFP system

TABLE I
SPECIFICATIONS OF THE LASER DISPLACEMENT SYSTEM

Parameter	Value
Reference distance ^a	30 mm
Measurement range	$\pm 5 \text{ mm}$
Spot diameter	$30 \mu m$
Sampling rate	$20-1000 \ \mu s$
Controller Memory	65536 measurements

^aDistance from the lower face of the laser to the center of the measurement range

A. Surface Measurement

The measurement of distance to an ice part is a particularly difficult problem because of the reflective properties of ice. After consideration of several devices, a laser displacement system from Keyence, Inc., consisting of the LK-G32 laser and the LK-G3001P laser controller, was selected; relevant specifications for this system are listed in Table I.

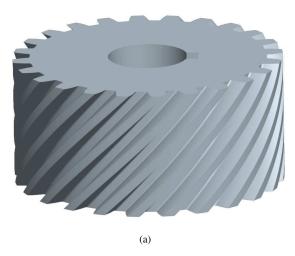
To produce the error surface \mathbf{E}_k , vertical measurements are taken with the Keyence system while the robot follows a series of paths in a reference frame defined for the laser. The part-slicing algorithm is used to produce these measurement paths, before construction begins. While the EE follows each path, measurements are accumulated at a rate of 100 Hz; after each path is followed, these data are sent to the control PC via RS-232C. The measurement paths for one layer of a helical gear part are shown in Fig. 3(b).

A measurement interval h^* of 1.2 mm is selected, based on experimental testing. Since the material layer heights are $h_{l,\mathrm{wat}}=0.2$ mm and $h_{l,\mathrm{HVOME}}=0.4$ mm, six water layers and three HVOME layers are deposited between each measurement layer.

B. Control Technique

The PID control technique introduced in Sec. II-B must be modified because of the type of deposition adjustment used for the Cobra 600 RFP system, described in Sec. III. A block diagram for the modified control system is shown in

¹An Adept C40 Compact Controller with an AWC-II 040 Processor (25 MHz), 32 MB RAM, and a 128 MB CompactFlash disk



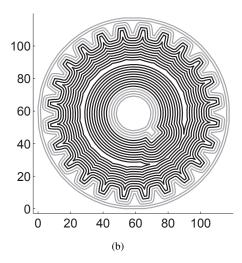


Fig. 3. (a) CAD model of a helical gear part used to compare SMF to open-loop deposition control: height is 50 mm and outside diameter is 107.6 mm; (b) Measurement paths for one layer for the part shown in (a). Gray lines indicate scaffolding paths while black lines indicate part paths. Axes indicate distance in mm

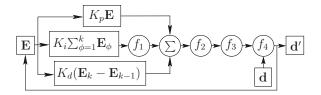


Fig. 4. The modified PID controller used for implementing SMF with the Cobra 600 RFP system. Functions f_1 - f_4 are implemented, respectively, with (4,6-8).

Fig. 4, where no setpoint is included because the Keyence laser measures deviation from the nominal height directly. Strict saturation limits on \mathbf{U} are required, which could lead to large growth of the integral term, without affecting the saturated control output. If this situation arises for a subarea of the part and the nominal height is then subsequently reached in this subarea, the integral term could contribute to a controller output that moves this subarea *away* from the setpoint. For the RFP system, this most often arises when the system shifts from depositing water to HVOME or vice versa in the subarea. To avoid this situation, saturation is also imposed on each component n_{ij} of the integral term $\mathbf{N} = K_i \sum_{\phi=1}^k \mathbf{E}_{\phi}$, according to the conditional statements

if
$$n_{ij} < \frac{h^*\psi_l}{\psi_l - 1}$$
, then $n_{ij} = \frac{h^*\psi_l}{\psi_l - 1}$ (4a)

if
$$n_{ij} > \frac{h^* \psi_h}{1 + \psi_h}$$
, then $n_{ij} = \frac{h^* \psi_h}{1 + \psi_h}$ (4b)

where ψ_l and ψ_h are the lower and upper saturation values, with $0 \le \psi \le 1$. Currently, ψ_l and ψ_h are both 0.3. These integral saturation equations are chosen to correspond with the transformation and saturation of \mathbf{U} , produced with (6-7). The gains, which have been manually tuned, are $K_p = 1$, $K_i = 0.25$, and $K_d = 0.25$.

An offline control method is implemented for the Cobra 600 RFP system, for two reasons. Firstly, the offset between the laser and the deposition nozzles would complicate path planning in an online system, since the EE would need to follow the same path in two reference frames. Secondly, online control would need to be implemented on the Cobra controller, which cannot handle the proposed, computationally intensive, control scheme.

Both material flow and EE speed could be used to implement the SMF control action. Material flow could be varied along a path through pulse-width modulation or frequency modulation of the valve control signal, as explained in Sec. II-C. However, the relationship between the control signal and the flow rate is not *precisely* known and the stream geometry undergoes unacceptably large variations with the shape of the control signal.

Rapid prototyping systems are normally implemented with gantry-type systems, which allow for precise robot positioning but not normally precise speed control. Robot arms are much less commonly-used for rapid prototyping, though implementations do exist [8], [9] and the Cobra 600 RFP system is of this type. Since the EE speed can be precisely controlled with the Cobra 600 RFP system, this variable is selected for implementing the control action; flow rates with optimal stream geometry can thus be used.

The controller output is a height error in millimeters, to be corrected during water and SME deposition between h_k and h_{k+1} . The procedure below is used to modify a single path of an ice or HVOME layer.

The entries of the control vector \mathbf{u} for a single path are defined by the equation

$$u_i = U(x_i, y_i, h_k) \tag{5}$$

where i indicates one point along the path.

Material deposition is only programmed to occur while the EE is moving at constant speed. However, a single path can



Fig. 5. The final helical ice gear, constructed with SMF, after the scaffolding is removed

consist of several deposition *segments*; deposition is often turned on and off several times while the EE is moving at constant speed along a straight line. Since path adjustment is only needed at trajectory points that are on the deposition segments, linear interpolation among entries of **u** for these points is used to replace entries of **u** at non-depositing trajectory points. Without this adjustment, many abrupt speed variations would occur.

Normally, with PID control, the controller output will be used to move the system toward the setpoint; this scheme works well whether the system is near or far from the setpoint. Also, precise characterization of the effect the process variable has on the system state is not needed. For RFP, the system remains *near* the set-surface and part height error is normally smaller than 5% for open-loop control. Additionally, the measurement interval h^* is quite large when compared to the equivalent time step implemented for a traditional PID controller. In light of these factors, we redefine the goal of the controller to attain the setsurface within the measurement interval h^* , rather than move toward the set-surface. Since path adjustment is achieved via component-wise multiplication of a vector c, whose components are speeds at each point along a deposition path, a modified control action vector \mathbf{u}_m is defined by applying

$$u_{m,i} = \frac{h^*}{h^* - u_i} \tag{6}$$

to all components u_i of **u**.

Equation (6) can produce negative or undefined entries in \mathbf{u}_m , so saturation must be imposed. Additionally, material flow along the part surface can occur at low EE speeds because surface tension forces no longer dominate. High EE speeds are undesirable because valve timing error and EE deflection due to inertial forces can can cause significant positioning errors. For these reasons, the saturation limits ψ_h and ψ_l are applied to limit EE speed 30% above or below nominal, according to the conditional statements

if
$$u_{m,i} < 1 - \psi_l$$
, then $u_{m,i} = 1 - \psi_l$ (7a)

if
$$u_{m,i} > 1 + \psi_h$$
, then $u_{m,i} = 1 + \psi_h$ (7b)

We should note that it is not possible to directly control path speed continuously along a path in V+; point-to-point spacing or point-to-point duration must be controlled instead. Component-wise adjustment of the point-to-point duration vector d for a single path can be implemented according to

$$d_i' = d_i / u_{m,i} \tag{8}$$

producing d', the updated version of d. Unfortunately, unpredictable, jerky motion is often exhibited when the Cobra 600 is commanded to follow trajectories with highly variable duration values, even when the trajectories are theoretically smooth. Therefore, path point-spacing is adjusted instead, holding point-to-point duration constant.

C. The Drop-On-Demand Case

If a drop-on-demand (DOD) deposition system were used with the Cobra 600 RFP system, SMF control would be much simpler, since a nominal path data set would not be needed and saturation limits on the control output need not be imposed. A logical process parameter would be droplets per gridpoint, for a grid defined as in Sec. II-C. The number of drops required for one gridpoint would be proportional to the measured height error there. DOD is not used with the Cobra 600 RFP system because low or discrete material flow is less predictable with the currently used flow control hardware. Additionally, part construction times would be much longer with DOD.

IV. RESULTS WITH THE COBRA 600 RFP SYSTEM

A. Deposition Adjustment

The helical gear part shown in Fig. 3 was built with the Cobra RFP system to demonstrate the influence of SMF on part dimensional accuracy. Figure 5 shows the final part, constructed with SMF, after the scaffolding has been removed. Figure 6 shows surfaces measured with the laser for both the SMF and open-loop cases. The surfaces shown indicate *raw* height measurements, which are smoothed using a N-D smoothing function developed by Garcia [10], before path adjustment. An animation showing all measured layers in sequence is included in a video attached to the paper.

In both the open-loop and SMF cases, part construction time to a height of 31.2 mm was approximately 16 hours, since surface measurements are taken in both cases. Deposition time per measurement interval h^* was 30 minutes, while measurement time was 10 minutes. Therefore, construction time was about 33% higher using SMF than it would be using open-loop control. To reduce the total measurement time, h^* could be increased and/or the measurement path resolution, shown in Fig. 3(b), could be coarsened, though there would likely be an associated sacrifice in part accuracy.

The RMS deviation from the nominal height for the final layer is 0.12 mm with SMF online, and 0.84 mm for open-loop control, showing that SMF greatly enhances the part dimensional accuracy; for this case, vertical accuracy is the same or better than in the horizontal directions. As can be seen in the video, the RMS height error with SMF

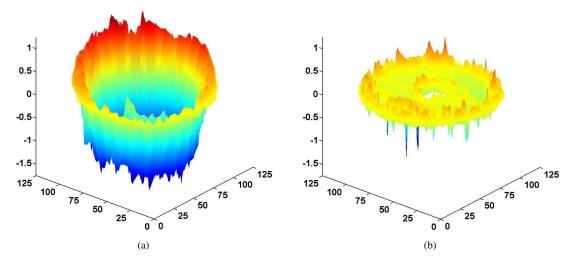


Fig. 6. The final measured surface of the helical gear part shown in Fig. 3, built 31.2 mm high (27 layers), where all axes indicate distance in mm: (a) without SMF; (b) with SMF

stays relatively constant, near 0.1 mm. We can thus conclude that SMF maintains an *absolute* vertical error, rather than an error proportional to part height. Additionally, for open-loop control, it can be seen that the height error exceeds 1 mm, which can be dangerous as the EE could eventually scrape or collide with high part sections. Also, when the EE-part clearance becomes too high, stream geometry is less predictable and part errors tend to increase at a faster rate. In these cases, the part construction must often be aborted or adjustment of the process parameters is needed during construction, rendering the system only *semi-automated*.

V. CONCLUSIONS AND FUTURE WORK

A. Conclusions

Surface mapping feedback (SMF), a novel method of geometric feedback for rapid prototyping systems, was introduced. The technique is applicable to both drop-on-demand and continuous-flow RP systems. A prerequisite for implementation of SMF is a process parameter that has a well-known impact on the part geometry. For continuous-flow systems, continuous adjustment of the parameter *along* deposition paths must also be possible. Additionally, a measurement system is needed for mapping part surface geometry. The implementation of SMF can lead to a significant increase in part accuracy, as has been shown for the Cobra 600 RFP system. Additionally, the system can help avert construction failures caused by an RP system drifting away from the set surface height.

B. Future Work

Future work with SMF will involve evaluation of system performance over a wider range of experimental conditions. For example, the system response to errors induced in the process parameters will be characterized. Additionally, the computational efficiency of the Matlab function will be improved and different settings will be tested to determine the optimal balance between part accuracy and required measurement time.

VI. ACKNOWLEDGMENTS

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