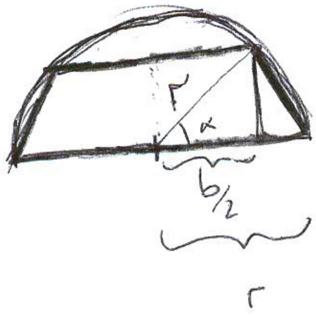
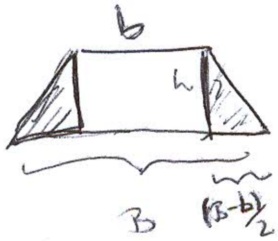


$\alpha \in [0, \pi/2]$

Área de trapecio:



$$\text{Área}_{\text{Trapecio}} = bh + h\cdot\left(\frac{B-b}{2}\right) = h\,\frac{(B+b)}{2}$$

$$\left.\begin{array}{l} b/2 = r\cos\alpha \\ B = 2r \\ h = r\,\text{sen}\,\alpha \end{array}\right] \Rightarrow A(\alpha) = (r\,\text{sen}\,\alpha)\,\frac{(2r + 2r\cos\alpha)}{2} =$$

$$= r^2\,\text{sen}\,\alpha + r^2\,\text{sen}\,\alpha\cos\alpha$$

$$= r^2(\text{sen}\,\alpha + \text{sen}\,\alpha\cos\alpha)$$

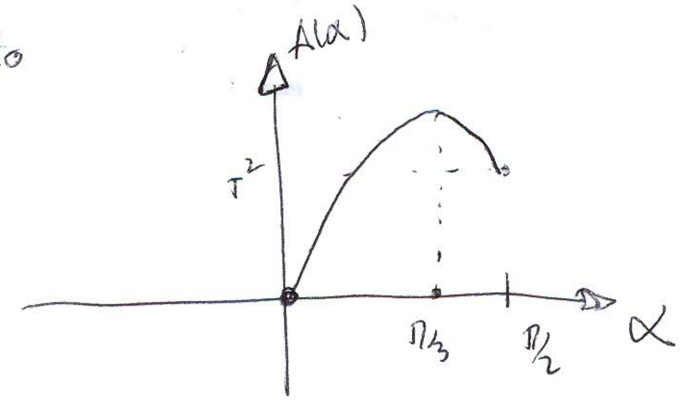
$$A'(\alpha) = r^2(\cos\alpha + \underbrace{\cos^2\alpha - \text{sen}^2\alpha}_{\cos 2\alpha}) = r^2(\cos\alpha + \cos 2\alpha)$$

$$A'(\alpha) = 0 \Leftrightarrow \cos\alpha + \cos 2\alpha = 0 \Rightarrow \cos(2\alpha) = -\cos\alpha = \cos(\pi-\alpha)$$

$$\Leftrightarrow \begin{cases} 2\alpha = (\pi-\alpha) + 2\pi k \\ 2\alpha = -(\pi-\alpha) + 2\pi k \end{cases} \Rightarrow \begin{cases} \alpha = \frac{\pi}{3} + \frac{2\pi}{3}k, & k\in\mathbb{Z} \\ \alpha = -\pi + 2\pi k, & k\in\mathbb{Z} \end{cases}$$

Como $\alpha \in [0, \pi/2]$ tenemos que $\boxed{\alpha = \pi/3}$

Por tanto



$\cancel{A(\alpha) \xrightarrow[\alpha \to 0]{} 0}$

$\cancel{A(\alpha) \to}$

$$A(0) = 0$$

$$A(\pi/2) = r^2$$

$$A(\pi/3) = r^2\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4}\right) = r^2\left(\frac{2\sqrt{3}+\sqrt{3}}{4}\right) = \frac{3\sqrt{3}}{4}r^2$$

① ¿ $w, z \in \mathbb{C}$ tales que $z \cdot w = -8$ y $\boxed{z = w^2}$?

$$z \cdot w = -8 \Rightarrow w^3 = -8$$

$z = w^2$

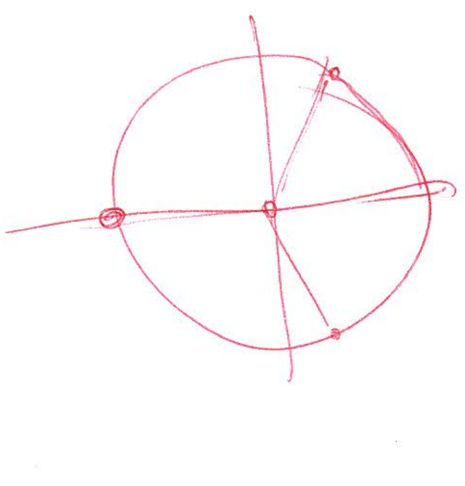
$$w = |w| e^{\alpha i} \Rightarrow w^3 = |w|^3 e^{3\alpha i} = 8 e^{\pi i} \Rightarrow \begin{cases} |w|^3 = 8 \\ 3\alpha = \pi + 2\pi k \end{cases}$$

$-8 = 8e^{\pi i}$

$$\Rightarrow \begin{cases} |w| = 2 \\ \alpha = \frac{\pi}{3} + \frac{2\pi k}{3}, \ k \in \mathbb{Z} \end{cases}$$

Por tanto

$$\begin{cases} w_0 = 2e^{\frac{\pi}{3} i} = 2\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right) = 1 + \sqrt{3} i \rightsquigarrow z_0 = w_0^2 = 4e^{\frac{2\pi}{3} i} = 4\left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right) = -2 + 2\sqrt{3} i \\ w_1 = 2e^{\pi i} = -2 \rightsquigarrow z_1 = w_1^2 = (-2)^2 = 4 \\ w_2 = 2e^{\frac{5\pi}{3} i} = e^{-\frac{\pi}{3} i} = 1 - \sqrt{3} i \rightsquigarrow z_2 = w_2^2 = 4e^{-\frac{2\pi}{3} i} = 4\left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right) = -2 - 2\sqrt{3} i \end{cases}$$



a) $z = \dfrac{1+i\sqrt{3}}{1+i} = \dfrac{2e^{\pi/3\, i}}{\sqrt{2}e^{\pi/4\, i}} = \dfrac{2}{\sqrt{2}} e^{(\pi/3 - \pi/4)i} = \sqrt{2}\, e^{\frac{4\pi - 3\pi}{12} i} = \sqrt{2}\, e^{\frac{\pi}{12} i}$

$|1+i\sqrt{3}| = \sqrt{1+3} = \sqrt{4} = 2 \Rightarrow 1+i\sqrt{3} = 2(\cos\alpha + i \operatorname{sen}\alpha)$

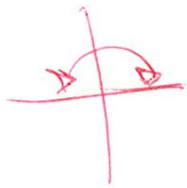
$\Rightarrow \begin{cases} \cos\alpha = 1/2 \\ \operatorname{sen}\alpha = \sqrt{3}/2 \end{cases} \Rightarrow \alpha = \pi/3$

$|1+i| = \sqrt{1+1} = \sqrt{2} \Rightarrow 1+i = \sqrt{2}(\cos\alpha + i \operatorname{sen}\alpha)$

$\Rightarrow \begin{cases} \sqrt{2}\cos\alpha = 1 \\ \sqrt{2}\operatorname{sen}\alpha = 1 \end{cases} \Rightarrow \begin{matrix} \cos\alpha = 1/\sqrt{2} = \sqrt{2}/2 \\ \operatorname{sen}\alpha = 1/\sqrt{2} = \sqrt{2}/2 \end{matrix} \Rightarrow \alpha = \pi/4$

b) $z^n = \underbrace{\sqrt{2}^n e^{\frac{\pi}{12} n i}}$

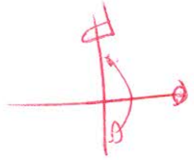
Es real si el argumento es un múltiplo de π



$\frac{\pi}{12} n = \pi k \Rightarrow n = \frac{12}{\cancel{\pi}} \cancel{\pi} k = 12k$ para algún $k \in \mathbb{Z}$

Para $k=1$, es decir, $n=12$, tenemos que

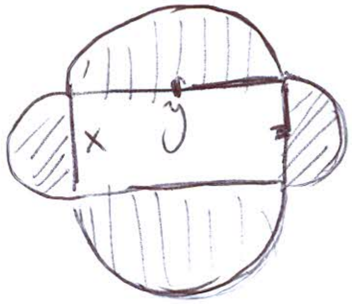
$$z^{12} = \sqrt{2}^{12} e^{\pi i} = -\sqrt{2}^{12} = -2^6 = -64$$



c) Es parecido $\rightsquigarrow$ $z^n = \underbrace{\sqrt{2}^n e^{\frac{\pi}{12} n i}}$

Es imaginario si su argumento es $\pi/2 + \pi k$ para algún $k \in \mathbb{Z}$

$\Rightarrow \frac{\pi}{12} n = \pi/2 + \pi k \Rightarrow n = 6 + 12k$ para algún $k \in \mathbb{Z}$



$$2x+2y=4$$

$$\Rightarrow y = 2-x$$

Área círculo pequeño $= \pi\left(\frac{x}{2}\right)^2$

Área círculo grande $= \pi\left(\frac{y}{2}\right)^2 = \pi\,\frac{(2-x)^2}{4}$

Área cuadrado $= xy = x(2-x)$

$A(x)=$ Área total $= x(2-x) + \pi\,\frac{x^2}{4} + \frac{\pi}{4}(2-x)^2$

$$A'(x) = 2-2x+\frac{\pi}{2}x+\frac{\pi}{2}(2-x)(-1) = \cancel{2+\pi}$$

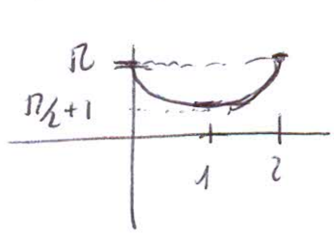
$$= 2-2x+\pi x-\pi = (\pi-2)x+(2-\pi)$$

$$A'(x)=0 \iff x = \frac{-(2-\pi)}{\pi-2} = 1.$$

$$A(x) = 2x-x^2+\frac{\pi}{4}x^2+\pi-\pi x+\frac{\pi}{4}x^2 =$$

$$= \left[\frac{\pi}{2}-1\right]x^2 + (2-\pi)x+\pi$$

$$\left.\begin{array}{l} A(0)=\pi \\ A(2)=\pi \\ A(1)=\frac{\pi}{2}-1+2-\cancel{\pi}+\cancel{\pi}=\frac{\pi}{2}+1 \end{array}\right] \longrightarrow$$



El área está entre $\pi/2+1$ y π