In[1]:=
```
L1 = {{-k0 - π / T2 - I π J, k0}, {k0, -k0 - π / T2 + I π J}}
```

Out[1]=
$$\left\{\left\{-\mathrm{i}\,J\,\pi - k_0 - \frac{\pi}{T_2},\ k_0\right\},\ \left\{k_0,\ \mathrm{i}\,J\,\pi - k_0 - \frac{\pi}{T_2}\right\}\right\}$$

In[2]:=
```
L2 = {{-3 k0 - π / T2 - 3 I π J, 3 k0, 0, 0},
   {3 k0, -7 k0 - π / T2 - I π J, 4 k0, 0},
   {0, 4 k0, -7 k0 - π / T2 + I π J, 3 k0},
   {0, 0, 3 k0, -3 k0 - π / T2 + 3 I π J}}
```

Out[2]=
$$\left\{\left\{-3\,\mathrm{i}\,J\,\pi - 3\,k_0 - \frac{\pi}{T_2},\ 3\,k_0,\ 0,\ 0\right\},\ \left\{3\,k_0,\ -\mathrm{i}\,J\,\pi - 7\,k_0 - \frac{\pi}{T_2},\ 4\,k_0,\ 0\right\},\right.$$
$$\left.\left\{0,\ 4\,k_0,\ \mathrm{i}\,J\,\pi - 7\,k_0 - \frac{\pi}{T_2},\ 3\,k_0\right\},\ \left\{0,\ 0,\ 3\,k_0,\ 3\,\mathrm{i}\,J\,\pi - 3\,k_0 - \frac{\pi}{T_2}\right\}\right\}$$

In[3]:=
```
U1 = FullSimplify[MatrixExp[L1 Abs[t]], {T2 > 0, T2 ∈ Reals, k0 > 0, k0 ∈ Reals}]
```

Out[3]=
$$\left\{\left\{e^{-\mathrm{Abs}[t]\left(k_0+\frac{\pi}{T_2}\right)}\left(\mathrm{Cosh}\left[\mathrm{Abs}[t]\ \sqrt{-J^2\pi^2+k_0^2}\right] - \frac{\mathrm{i}\,J\,\pi\,\mathrm{Sinh}\left[\mathrm{Abs}[t]\ \sqrt{-J^2\pi^2+k_0^2}\right]}{\sqrt{-J^2\pi^2+k_0^2}}\right),\right.$$
$$\left.\frac{e^{-\mathrm{Abs}[t]\left(k_0+\frac{\pi}{T_2}\right)}\,\mathrm{Sinh}\left[\mathrm{Abs}[t]\ \sqrt{-J^2\pi^2+k_0^2}\right]\,k_0}{\sqrt{-J^2\pi^2+k_0^2}}\right\},$$
$$\left\{\frac{e^{-\mathrm{Abs}[t]\left(k_0+\frac{\pi}{T_2}\right)}\,\mathrm{Sinh}\left[\mathrm{Abs}[t]\ \sqrt{-J^2\pi^2+k_0^2}\right]\,k_0}{\sqrt{-J^2\pi^2+k_0^2}},\right.$$
$$\left.e^{-\mathrm{Abs}[t]\left(k_0+\frac{\pi}{T_2}\right)}\left(\mathrm{Cosh}\left[\mathrm{Abs}[t]\ \sqrt{-J^2\pi^2+k_0^2}\right] + \frac{\mathrm{i}\,J\,\pi\,\mathrm{Sinh}\left[\mathrm{Abs}[t]\ \sqrt{-J^2\pi^2+k_0^2}\right]}{\sqrt{-J^2\pi^2+k_0^2}}\right)\right\}\right\}$$

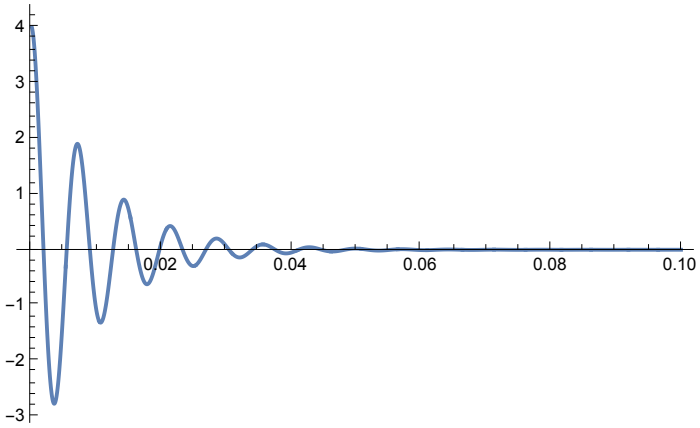
In[4]:=
```
U2 = FullSimplify[MatrixExp[L2 Abs[t]], {T2 > 0, T2 ∈ Reals, k0 > 0, k0 ∈ Reals}];
```

In[5]:=
```
signal1 = Total[U1 . {2, 2}]
```

Out[5]=
$$\frac{4\,e^{-\mathrm{Abs}[t]\left(k_0+\frac{\pi}{T_2}\right)}\,\mathrm{Sinh}\left[\mathrm{Abs}[t]\ \sqrt{-J^2\pi^2+k_0^2}\right]\,k_0}{\sqrt{-J^2\pi^2+k_0^2}} +$$
$$2\,e^{-\mathrm{Abs}[t]\left(k_0+\frac{\pi}{T_2}\right)}\left(\mathrm{Cosh}\left[\mathrm{Abs}[t]\ \sqrt{-J^2\pi^2+k_0^2}\right] - \frac{\mathrm{i}\,J\,\pi\,\mathrm{Sinh}\left[\mathrm{Abs}[t]\ \sqrt{-J^2\pi^2+k_0^2}\right]}{\sqrt{-J^2\pi^2+k_0^2}}\right) +$$
$$2\,e^{-\mathrm{Abs}[t]\left(k_0+\frac{\pi}{T_2}\right)}\left(\mathrm{Cosh}\left[\mathrm{Abs}[t]\ \sqrt{-J^2\pi^2+k_0^2}\right] + \frac{\mathrm{i}\,J\,\pi\,\mathrm{Sinh}\left[\mathrm{Abs}[t]\ \sqrt{-J^2\pi^2+k_0^2}\right]}{\sqrt{-J^2\pi^2+k_0^2}}\right)$$

In[6]:=
```
Plot[signal1 //. {T2 → 3 / 100, k0 → 1 / 1000, J → 280}, {t, 0, 0.1}, PlotRange → All]
```

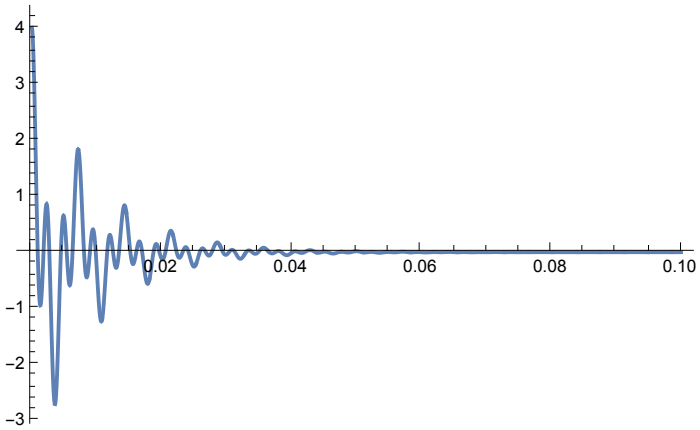
Out[6]=




In[7]:=
```
signal2 = Total[U2 . {1, 1, 1, 1}];
```

In[8]:=
```
Plot[Simplify[N[signal2 //. {T2 → 3 / 100, k0 → 1, J → 280}]],
 {t, 0, 0.1}, PlotRange → All]
```

Out[8]=




## $k_0 = 1$

In[9]:=
```
spec1 = FourierTransform[
   signal1 //. {T2 → 3 / 100, k0 → 1, J → 280}, t, ω, FourierParameters → {1, -1}]
```

Out[9]=

$$\frac{2400\,\pi\,\left(43\,536\,\pi + 715\,600\,\pi^2 + 9\,\left(4 + \omega^2\right)\right)}{858\,720\,000\,\pi^3 + 512\,083\,360\,000\,\pi^4 + 10\,800\,\pi\,\omega^2 + 81\,\omega^2\,\left(4 + \omega^2\right) - 7200\,\pi^2\,\left(-50 + 1739\,\omega^2\right)}$$

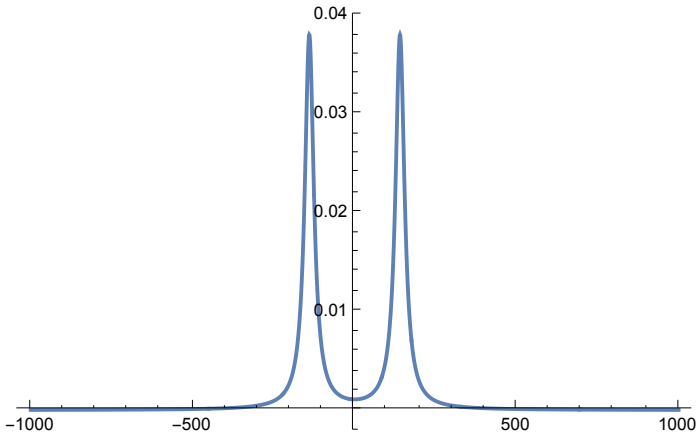
In[10]:=
```
spec2 = FourierTransform[Simplify[N[signal2 //. {T2 → 3 / 100, k0 → 1, J → 280}]],
   t, ω, FourierParameters → {1, -1}]
```

Out[10]=

$$\left(\left(9.0184 \times 10^{110} - 3.631 \times 10^{125}\,i\right) - \left(0. + 2.24424 \times 10^{119}\,i\right)\,\omega^2 + \left(3.41758 \times 10^{98} + 5.934 \times 10^{112}\,i\right)\,\omega^4 - \left(3.66666 \times 10^{91} + 1.57915 \times 10^{106}\,i\right)\,\omega^6\right) / \left(\left(1.92393 \times 10^{112} - 5.67004 \times 10^{128}\,i\right) + \left(7.3392 \times 10^{106} + 1.55853 \times 10^{123}\,i\right)\,\omega^2 - \left(1.7498 \times 10^{100} + 1.32792 \times 10^{117}\,i\right)\,\omega^4 + \left(5.21481 \times 10^{92} + 2.908 \times 10^{110}\,i\right)\,\omega^6 - \left(0. + 1.88497 \times 10^{103}\,i\right)\,\omega^8\right)$$

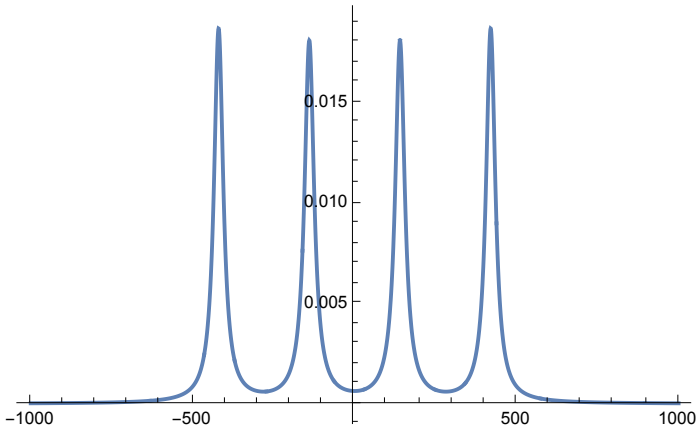
In[11]:=
```
Plot[spec1 /. ω → z * (2 π), {z, -1000, 1000}, PlotRange → All]
```

Out[11]=



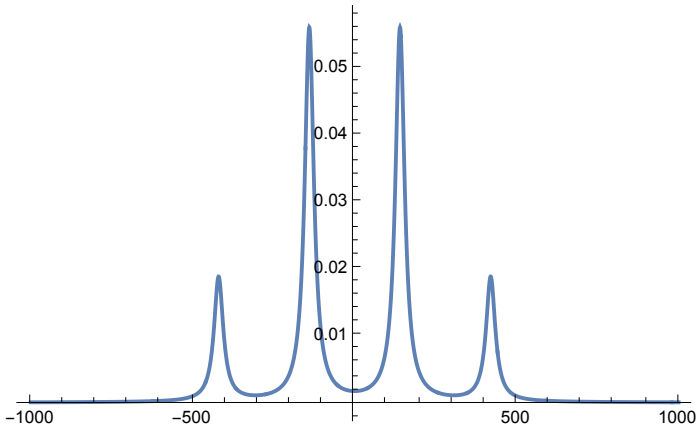

In[12]:=
```
Plot[spec2 /. ω → z * (2 π), {z, -1000, 1000}, PlotRange → All]
```

Out[12]=




In[13]:=
```
Plot[(spec1 + spec2) /. ω → z * (2 π), {z, -1000, 1000}, PlotRange → All]
```

Out[13]=



## $k_0 = 100$

In[14]:=
```
spec1a = FourierTransform[
  signal1 //. {T2 → 3 / 100, k0 → 100, J → 280}, t, ω, FourierParameters → {1, -1}]
```

Out[14]=

$$\left(2400\,\pi\,\left(4\,353\,600\,\pi + 715\,600\,\pi^2 + 9\,\left(40\,000 + \omega^2\right)\right)\right) \Big/ \left(85\,872\,000\,000\,\pi^3 + 512\,083\,360\,000\,\pi^4 + 1\,080\,000\,\pi\,\omega^2 + 81\,\omega^2\,\left(40\,000 + \omega^2\right) - 7200\,\pi^2\,\left(-500\,000 + 1739\,\omega^2\right)\right)$$

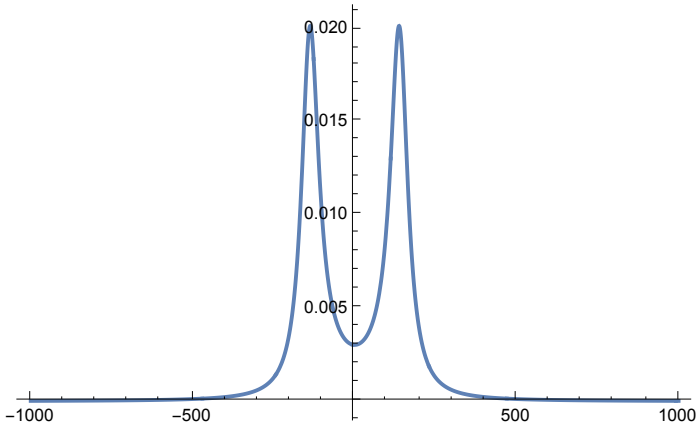
In[15]:=
```
spec2a =
 FourierTransform[Simplify[N[signal2 //. {T2 → 3 / 100, k0 → 100, J → 280}]],
  t, ω, FourierParameters → {1, -1}]
```

Out[15]=

$$\left(\left(5.47739 \times 10^{124} + 0.\,i\right) + \left(1.07188 \times 10^{118} - 1.28785 \times 10^{103}\,i\right)\,\omega^2 + \left(9.05264 \times 10^{110} + 7.47595 \times 10^{96}\,i\right)\,\omega^4 + \left(1.5576 \times 10^{104} - 1.14583 \times 10^{90}\,i\right)\,\omega^6\right) \Big/ \left(\left(1.62817 \times 10^{127} - 8.41718 \times 10^{111}\,i\right) - \left(5.08452 \times 10^{120} + 4.73034 \times 10^{105}\,i\right)\,\omega^2 + \left(9.26058 \times 10^{114} + 1.90957 \times 10^{99}\,i\right)\,\omega^4 - \left(2.44915 \times 10^{108} + 1.3037 \times 10^{92}\,i\right)\,\omega^6 + 1.85925 \times 10^{101}\,\omega^8\right)$$

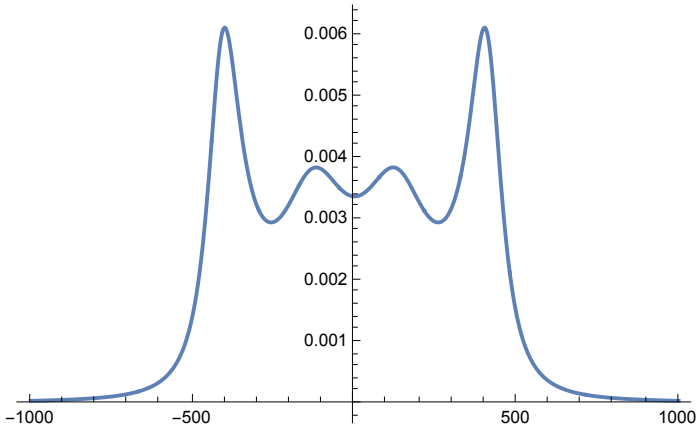
In[16]:=
```
Plot[spec1a /. ω → z * (2 π), {z, -1000, 1000}, PlotRange → All]
```

Out[16]=



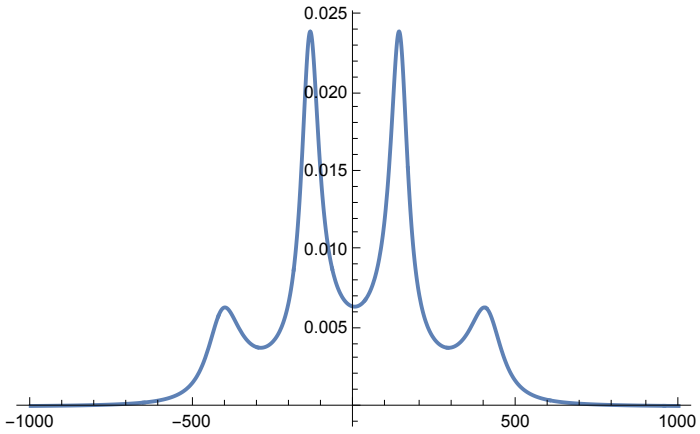

In[17]:=
```
Plot[spec2a /. ω → z * (2 π), {z, -1000, 1000}, PlotRange → All]
```

Out[17]=



In[18]:=
```
Plot[(spec1a + spec2a) /. ω → z * (2 π), {z, -1000, 1000}, PlotRange → All]
```

Out[18]=




## $k_0 = 200$

In[19]:=
```
spec1b = FourierTransform[
  signal1 //. {T2 → 3 / 100, k0 → 200, J → 280}, t, ω, FourierParameters → {1, -1}]
```

Out[19]=

$$\left(2400\,\pi\,\left(8\,707\,200\,\pi + 715\,600\,\pi^2 + 9\,\left(160\,000 + \omega^2\right)\right)\right) \Big/ \left(171\,744\,000\,000\,\pi^3 + 512\,083\,360\,000\,\pi^4 + 2\,160\,000\,\pi\,\omega^2 + 81\,\omega^2\,\left(160\,000 + \omega^2\right) - 7200\,\pi^2\,\left(-2\,000\,000 + 1739\,\omega^2\right)\right)$$

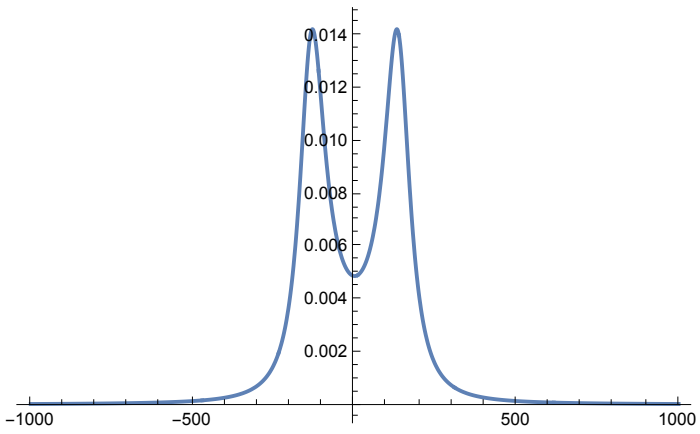
In[20]:=
```
spec2b =
 FourierTransform[Simplify[N[signal2 //. {T2 → 3 / 100, k0 → 200, J → 280}]],
  t, ω, FourierParameters → {1, -1}]
```

Out[20]=

$$\left(\left(5.17936 \times 10^{124} - 5.87136 \times 10^{108}\,i\right) + \left(7.51717 \times 10^{117} - 5.59936 \times 10^{101}\,i\right)\,\omega^2 + \left(6.41151 \times 10^{110} + 4.00498 \times 10^{95}\,i\right)\,\omega^4 + \left(3.34983 \times 10^{103} - 4.7743 \times 10^{88}\,i\right)\,\omega^6\right) \Big/ \left(\left(1.43252 \times 10^{127} + 1.20245 \times 10^{111}\,i\right) + \left(3.24386 \times 10^{120} + 0.\,i\right)\,\omega^2 + \left(1.05834 \times 10^{113} + 0.\,i\right)\,\omega^4 - \left(2.89253 \times 10^{107} + 0.\,i\right)\,\omega^6 + 3.99856 \times 10^{100}\,\omega^8\right)$$

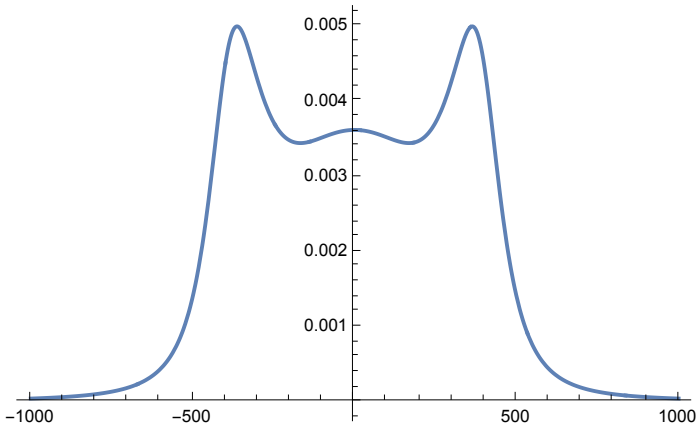
In[21]:=
```
Plot[spec1b /. ω → z * (2 π), {z, -1000, 1000}, PlotRange → All]
```

Out[21]=



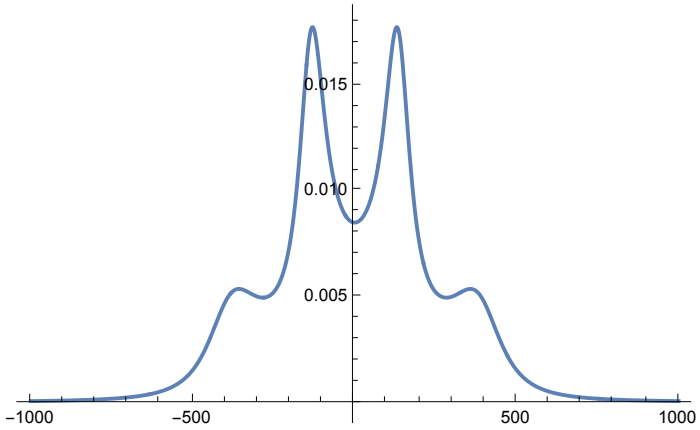
In[22]:=
```
Plot[spec2b /. ω → z * (2 π), {z, -1000, 1000}, PlotRange → All]
```

Out[22]=




In[23]:=
```
Plot[(spec1b + spec2b) /. ω → z * (2 π), {z, -1000, 1000}, PlotRange → All]
```

Out[23]=




$k_0 = 300$

In[24]:=
```
spec1c = FourierTransform[
  signal1 //. {T₂ → 3 / 100, k₀ → 300, J → 280}, t, ω, FourierParameters → {1, -1}]
```

Out[24]=

$$\left(2400\,\pi\,\left(13\,060\,800\,\pi + 715\,600\,\pi^2 + 9\,\left(360\,000 + \omega^2\right)\right)\right) \Big/ \left(257\,616\,000\,000\,\pi^3 + 512\,083\,360\,000\,\pi^4 + 3\,240\,000\,\pi\,\omega^2 + 81\,\omega^2\,\left(360\,000 + \omega^2\right) - 7200\,\pi^2\,\left(-4\,500\,000 + 1739\,\omega^2\right)\right)$$

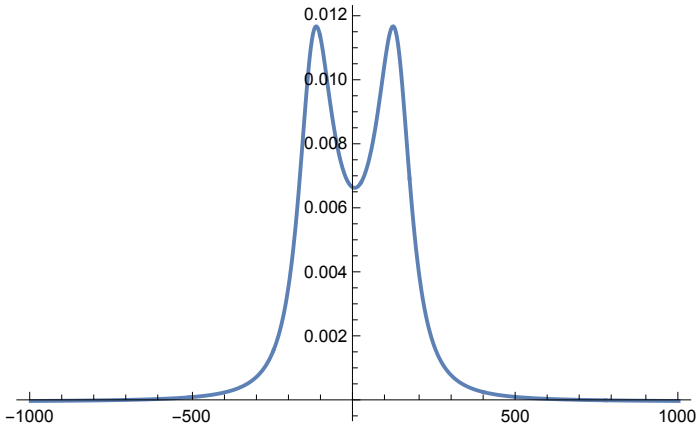
In[25]:=
```
spec2c =
 FourierTransform[Simplify[N[signal2 //. {T₂ → 3 / 100, k₀ → 300, J → 280}]],
  t, ω, FourierParameters → {1, -1}]
```

Out[25]=

$$\left(\left(9.94597 \times 10^{72} + 3.65125 \times 10^{57}\,i\right) + \left(1.17287 \times 10^{66} + 5.6999 \times 10^{50}\,i\right)\,\omega^2 + \left(7.26159 \times 10^{58} - 8.27229 \times 10^{43}\,i\right)\,\omega^4 + \left(2.00886 \times 10^{51} - 9.74499 \times 10^{36}\,i\right)\,\omega^6\right) \Big/ \left(\left(2.81673 \times 10^{75} + 0.\,i\right) + \left(3.43479 \times 10^{68} + 0.\,i\right)\,\omega^2 - \left(1.14086 \times 10^{62} + 0.\,i\right)\,\omega^4 + \left(5.71891 \times 10^{54} + 0.\,i\right)\,\omega^6 + 2.39791 \times 10^{48}\,\omega^8\right)$$

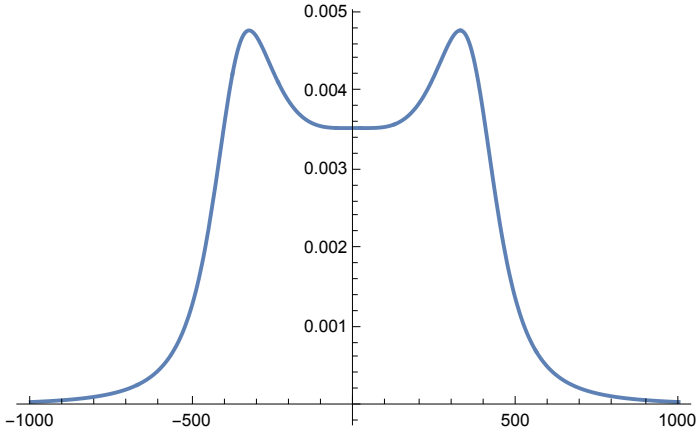
In[26]:=
```
Plot[spec1c /. ω → z * (2 π), {z, -1000, 1000}, PlotRange → All]
```

Out[26]=



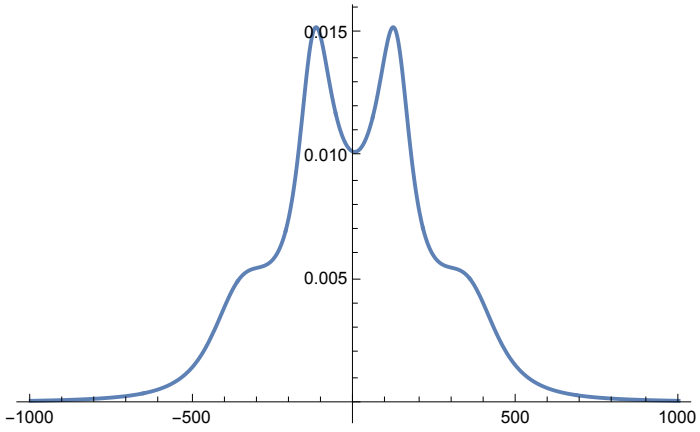

In[27]:=
```
Plot[spec2c /. ω → z * (2 π), {z, -1000, 1000}, PlotRange → All]
```

Out[27]=




In[28]:=
```
Plot[(spec1c + spec2c) /. ω → z * (2 π), {z, -1000, 1000}, PlotRange → All]
```

Out[28]=



## $k_0 = 400$

In[29]:=
```
spec1d = FourierTransform[
  signal1 //. {T2 → 3 / 100, k0 → 400, J → 280}, t, ω, FourierParameters → {1, -1}]
```

Out[29]=

$$\left(2400\,\pi\,\left(17\,414\,400\,\pi + 715\,600\,\pi^2 + 9\,\left(640\,000 + \omega^2\right)\right)\right) \Big/ \left(343\,488\,000\,000\,\pi^3 + 512\,083\,360\,000\,\pi^4 + 4\,320\,000\,\pi\,\omega^2 + 81\,\omega^2\,\left(640\,000 + \omega^2\right) - 7200\,\pi^2\,\left(-8\,000\,000 + 1739\,\omega^2\right)\right)$$

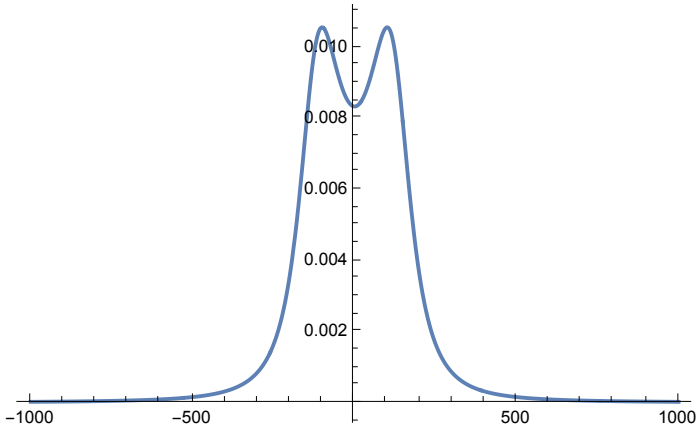
In[30]:=
```
spec2d =
 FourierTransform[Simplify[N[signal2 //. {T2 → 3 / 100, k0 → 400, J → 280}]],
  t, ω, FourierParameters → {1, -1}]
```

Out[30]=

$$\left(\left(4.07945 \times 10^{76} + 7.28026 \times 10^{60}\,i\right) + \left(3.97085 \times 10^{69} - 2.30003 \times 10^{54}\,i\right)\omega^2 + \left(1.78472 \times 10^{62} + 1.68591 \times 10^{47}\,i\right)\omega^4 + \left(3.14014 \times 10^{54} + 3.87621 \times 10^{40}\,i\right)\omega^6\right) \Big/ \left(\left(1.15395 \times 10^{79} + 0.\,i\right) + \left(9.96303 \times 10^{70} + 0.\,i\right)\omega^2 - \left(2.37146 \times 10^{65} + 0.\,i\right)\omega^4 + \left(5.87872 \times 10^{58} + 0.\,i\right)\omega^6 + 3.74827 \times 10^{51}\,\omega^8\right)$$

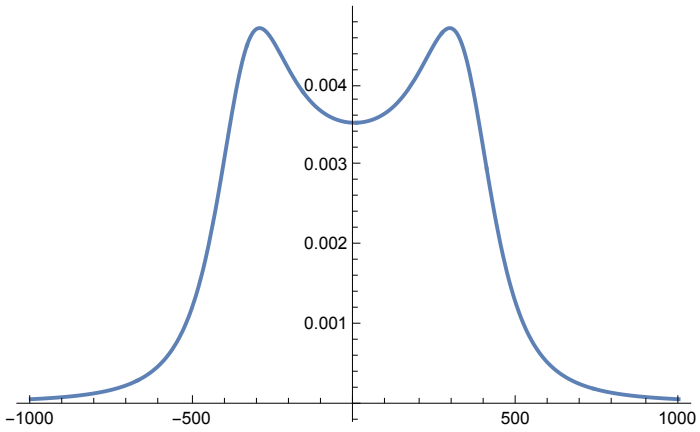
In[31]:=
```
Plot[spec1d /. ω → z * (2 π), {z, -1000, 1000}, PlotRange → All]
```

Out[31]=



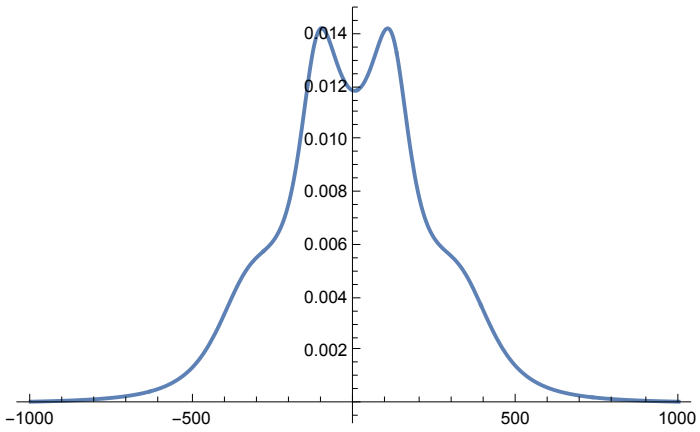

In[32]:=
```
Plot[spec2d /. ω → z * (2 π), {z, -1000, 1000}, PlotRange → All]
```

Out[32]=



In[33]:=
```
Plot[(spec1d + spec2d) /. ω → z * (2 π), {z, -1000, 1000}, PlotRange → All]
```

Out[33]=




## $k_0 = 1000$

In[34]:=
```
spec1e = FourierTransform[
  signal1 //. {T2 → 3 / 100, k0 → 1000, J → 280}, t, ω, FourierParameters → {1, -1}]
```

Out[34]=

$$\left(2400\,\pi\,\left(43\,536\,000\,\pi + 715\,600\,\pi^2 + 9\,\left(4\,000\,000 + \omega^2\right)\right)\right) \Big/ \left(858\,720\,000\,000\,\pi^3 + 512\,083\,360\,000\,\pi^4 + 10\,800\,000\,\pi\,\omega^2 + 81\,\omega^2\,\left(4\,000\,000 + \omega^2\right) - 7200\,\pi^2\,\left(-50\,000\,000 + 1739\,\omega^2\right)\right)$$

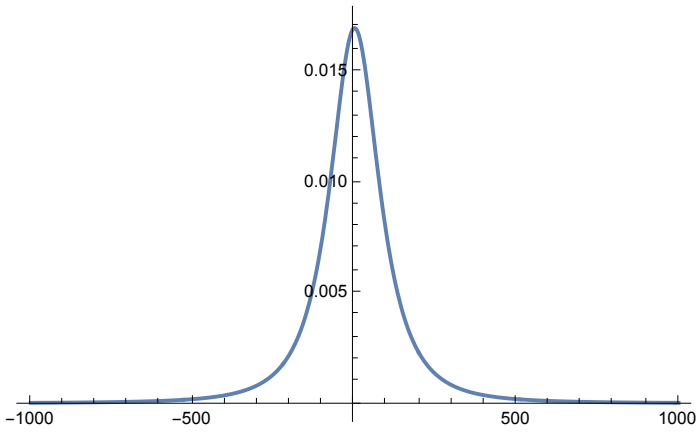
In[35]:=
```
spec2e =
 FourierTransform[Simplify[N[signal2 //. {T2 → 3 / 100, k0 → 1000, J → 280}]],
  t, ω, FourierParameters → {1, -1}]
```

Out[35]=

$$\left(\left(1.33546 \times 10^{78} + 1.5068 \times 10^{62}\,i\right) + \left(4.881 \times 10^{70} - 4.18152 \times 10^{55}\,i\right)\omega^2 + \left(6.16235 \times 10^{62} - 3.56504 \times 10^{48}\,i\right)\omega^4 + \left(2.38626 \times 10^{54} - 2.17089 \times 10^{40}\,i\right)\omega^6\right) \Big/ \left(\left(2.75528 \times 10^{80} + 0.\,i\right) - \left(7.05995 \times 10^{72} + 0.\,i\right)\omega^2 + \left(1.26995 \times 10^{67} + 0.\,i\right)\omega^4 + \left(4.92079 \times 10^{59} + 0.\,i\right)\omega^6 + 2.84839 \times 10^{51}\,\omega^8\right)$$

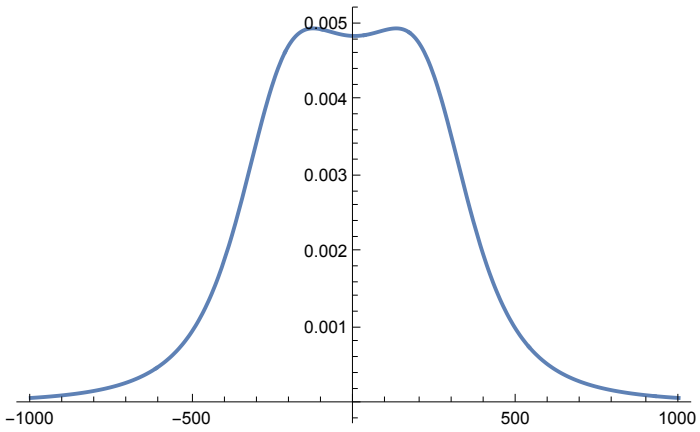
In[36]:=
```
Plot[spec1e /. ω → z * (2 π), {z, -1000, 1000}, PlotRange → All]
```

Out[36]=



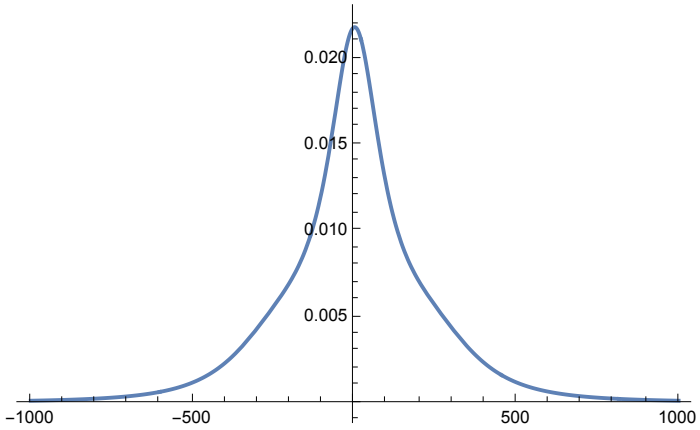
In[37]:=
```
Plot[spec2e /. ω → z * (2 π), {z, -1000, 1000}, PlotRange → All]
```

Out[37]=




In[38]:=
```
Plot[(spec1e + spec2e) /. ω → z * (2 π), {z, -1000, 1000}, PlotRange → All]
```

Out[38]=




In[39]:=
```
Plot[{(spec1 + spec2) /. ω → z * (2 π),
  (spec1a + spec2a) /. ω → z * (2 π), (spec1b + spec2b) /. ω → z * (2 π),
  (spec1c + spec2c) /. ω → z * (2 π), (spec1d + spec2d) /. ω → z * (2 π),
  (spec1e + spec2e) /. ω → z * (2 π)}, {z, -1000, 1000}, PlotRange → All]
```

Out[39]=