CMPT 210: Probability & Computing
Practice Problems 4

(1) A communications system consists of n components, each of which will independently function with probability p. The total system will be able to operate effectively if at least one of its components functions. (a) What is the probability that the total system functions? (b) The total system will be able to operate effectively if at least 3 of its 5 components function. What is the probability that the total system functions?

(2) A communications system consists of n components, each of which will independently function with probability p. The total system will be able to operate effectively if at least one-half of its n components function. For what p is a 5-component system better than a 3-component system?

(3) Suppose we throw a standard die and R is the random variable corresponding to the number on the die. (a) We define a new random variable $X = 2R + 1$. What is the PDF_X? (b) Suppose $X = \max\{R - 3, 0\}$. What is the PDF_X?

(4) A lottery sells 1000000 tickets, of which 4 tickets win \$1000000, 5 tickets win \$100000, and 5000 tickets win \$1000. What is the average winning amount?

(5) Throwing Dice

- We throw a standard dice, and define a random variable R which is equal to 1 if we get an even number and 0 otherwise. What is the distribution of R? What is $\mathbb{E}[R]$?
- We throw 10 independent dice and define R to be the random variable equal to the number of dice that have an even number. What is the distribution of R? What is $\mathbb{E}[R]$?
- We repeatedly and independently throw the dice until we get an even number. We define a random variable R equal to the number of throws we need to get an even number. What is the distribution of R? What is $\mathbb{E}[R]$?

(6) A school class of 120 students are driven in 3 buses to symphonic performance. There are 36 students in the first bus, 40 in the second bus, and 44 in the third bus. When the buses arrive, one of the 120 students is randomly chosen. Let X denote the number of students on the bus of that randomly selected student. Find $\mathbb{E}[X]$.

(7) Shuffle a deck of n cards; turn them over one at a time; try to guess each card.
Case 1: We cannot even remember which card has been turned over already, and guess a card from full deck uniformly at random. Prove that the expected number of correct guesses is 1.

Case 2: We can remember which card has been turned over and guess a card uniformly at random from the cards not yet seen. Prove that the expected number of correct guesses is $O(\log(n))$.

(8) The color of one's eyes is determined by a single pair of genes, with the gene for brown eyes being dominant over the one for blue eyes. This means that an individual having two blue-eyed genes will have blue eyes, while one having either two brown-eyed genes or one brown-eyed and one blue-eyed gene will have brown eyes. An offspring receives one randomly chosen gene from each of its parents' gene pair.

- If the eldest child of a pair of brown-eyed parents has blue eyes, what is the probability that exactly 2 of the 4 **other** children (none of whom is a twin) of this couple also have blue eyes?
- If this couple has 5 **more** children (no twins), on average, how many of these children have will brown eyes?

(9) Suppose that the successive daily changes of the price of a given stock are assumed to be independent and identically distributed random variables (the daily changes are independent and have the same distribution) – for each day i, the PDF is:

$$\begin{aligned}
\Pr[\text{Daily change on day } i] &= -3 && \text{(With } p = 0.1) \\
&= -2 && \text{(With } p = 0.1) \\
&= -1 && \text{(With } p = 0.2) \\
&= 0 && \text{(With } p = 0.3) \\
&= 1 && \text{(With } p = 0.2) \\
&= 2 && \text{(With } p = 0.1)
\end{aligned}$$

If E is the event that the stocks price will increase successively by 1, 2, and 0 points in the next three days, compute $\mathbb{E}[\mathcal{I}_E]$.