# CMPT 210: Probability & Computing
# Practice Problems 1

**(1)** Suppose a daily diet consists of a breakfast selected from set $B$, a lunch from set $L$, and a dinner from set $D$ where:

- B = {pancakes; bacon and eggs; bagel; Doritos}
- L = {burger and fries; garden salad; Doritos}
- D = {macaroni; pizza; frozen burrito; pasta; Doritos}

How many different daily diets are possible?

**(2)** There are 75 students in this class, and they are angry with the difficult questions in the assignment.

(a) How many ways could we construct an angry lineup of 3 students?

(b) How many ways could we construct an angry mob of 3 students (the order does not matter)?

**(3)** Given a standard pack of 52 cards, a poker hand consists of 5 cards.

- How many ways can we choose a poker hand?
- A 4-of-a-kind consists of a poker hand such that 4 of the cards have the same number but different suits. How many ways can we choose a 4-of-a-kind?
- A Full House is consists of a poker hand with three cards of one number and two cards of another number. How many ways can we choose a full house?

**(4)** Prove that for any 5 points in (the interior of) a unit square (one that has side length = 1), there exist 2 points at distance less than $\frac{1}{\sqrt{2}}$.

**(5)** For an undirected graph with $n$ vertices $\{v_1, v_2, \ldots, v_n\}$,

- What is the maximum possible number of edges if i) self-loops (edges of the form $v_1 \to v_1$) are not permitted, ii) if self-loops are permitted?
- Given the answer to the previous question, what is the total number of possible graphs that can be constructed if i) self-loops (edges of the form $v_1 \to v_1$) are not permitted, ii) if self-loops are permitted?

**(6)** Give a combinatorial proof for

$$\sum_{r=0}^{n} \binom{n}{r}\binom{2n}{n-r} = \binom{3n}{n}$$

**(7)** How many positive integers not exceeding 1000 are divisible by 7 or 11?

**(8)** Express the following sum in closed form (without using a summation symbol and without an ellipsis: $\sum_{j=0}^{2n}(-1)^j\binom{2n}{j}x^j$.