CMPT 210: Probability & Computing
Assignment 2

Total marks: 200

Due: Via Coursys at 11.59 pm, Thursday, 15 February
Late Submission: 11.59 pm, Tuesday, 20 February

Instructions on how to solve, write, and submit your assignment

Solutions: Solutions to assignments must be your own. Use sample exercises from the lectures to learn methods and approaches you can use. In some cases expect that you will need to make a substantial effort to solve a problem. Discussing and collaborating with other people is okay, as long as you produce your own solution. For instance, even when two people try to solve the problem together and extensively discuss possible solutions, if they write down that solution independently, it is acceptable.

We treat any kind of academic dishonesty very seriously. For the SFU policy on academic dishonesty see the part of University Policy S 10.01 relevant for us:

> e. Cheating in assignments, projects, examinations or other forms of evaluation by:
> i. using, or attempting to use, another student's answers;
> ii. providing answers to other students;
> iii. failing to take reasonable measures to protect answers from use by other students; or
> iv. in the case of students who study together, submitting identical or virtually identical assignments for evaluation unless permitted by the course Instructor or supervisor.
>
> University Policy S 10.01
> Code of Academic Integrity and Good Conduct
> 4.1.2 Forms of Academic Dishonesty

Note that this policy treats copying and allowing to copy equally. Should anyone be caught submitting a work too similar to someone else's work, or a source found on the web, a record of the violation will permanently stay in their student file.

Writing the solution: There is no strict prescribed way to present your solution. However, make sure that your solution can be understood by another person without any help on from you. The onus to present your solution in a clear understandable way is on you. Any unclear steps or arguments will be considered incorrect. If you use some method, or result presented in this course you do not need to explain it. However, if you would like to use a result or approach from elsewhere, please, give a reference and explain what you are doing in more details.

Submission: The assignment needs to be submitted via Coursys. For some flexibility, each student is allowed 1 late-submission.

(1) **[10 marks]** Prove that the probability that exactly one of the events A or B occurs is equal to $\Pr[A] + \Pr[B] - 2\Pr[A \cap B]$.

(2) **[20 marks]** A pair of n-sided fair dice are rolled (there is equal probability of rolling each number from $\{1, 2, \ldots, n\}$).

- What is the probability that the second dice lands on a higher value than does the first? [10 marks]
- Given that the first dice is between 1 and m (for $m \leq n$), what is the probability that the second dice lands on a higher value than does the first? [10 marks]

(3) **[15 marks]** Suppose that an insurance company classifies people into one of three classes – good risks, average risks, and bad risks. Their records indicate that the probabilities that good, average, and bad risk persons will be involved in an accident over a 1-year span are, respectively, 0.05, 0.15, and 0.30. If 20% percent of the population are good risk, 50% percent are average risk and 30% percent are bad risk,

- What is the probability that a person has an accident in a fixed year? [10 marks]
- If a policy holder had no accidents in the 1-year span, what is the probability that they are good risk? [5 marks]

(4) **[10 marks]** Two integers are selected at random from $\{1, 2, \ldots, n\}$.

- What is the probability that the integers are consecutive? [5 marks]
- What is the probability that their sum is odd? [5 marks]

(5) **[20 marks]** Three cards are drawn one after the other from an ordinary 52-card deck without replacement (once a card is drawn, it is not placed back in the deck). Compute the probability that

- All of the three cards is a heart. [5 marks]
- None of the three cards is a heart. [5 marks]
- Exactly one of the three cards is a heart. [10 marks]

(6) **[15 marks]** Suppose we have n students and k tasks need to be assigned to these students. Since the professor is lazy, he assigns each task to a randomly selected student. A student gets angry if they are assigned more than one task. Calculate the probability that no student gets angry.

(7) **[15 marks]** The Canadian football team has probability 0.1 of winning against Tier 1 teams, probability 0.25 of winning against Tier 2 teams and probability 0.5 of winning against Tier 3 teams. Half the teams in the league are Tier 1 while a quarter of them are Tier 2 and a quarter of them are Tier 3. The odds of an event A is given by:

$$\text{odds[A]} = \frac{\Pr[A]}{1 - \Pr[A]}$$

- What are the odds that the Canadian football team wins against a randomly chosen team? [10 marks]
- Given that Canada wins the game, what is the probability that the opponent was a Tier 2 team? [5 marks]

(8) **[20 marks]** A class consisting of 4 men and 12 women is randomly divided into four groups (labelled Group A, Group B, Group C and Group D) each of size 4.

- What is the number of possible groups that can be formed? [5 marks]
- Suppose we impose the restriction that each group needs to have exactly 1 man, what is the number of possible groups that can be formed? [10 marks]
- Use the above results to calculate the probability that each group includes exactly man? [5 marks]

(9) **[20 marks]** A ball is in one of n boxes. It is in box i with probability p_i. If the ball is in box i, a search of that box will discover it with probability α_i.

- Prove that the conditional probability that the ball is in box j, given that a search of box i did not discover it, is

$$\frac{p_j}{1-\alpha_i p_i}, \qquad \text{if } j \neq i,$$
$$\frac{(1-\alpha_i)p_i}{1-\alpha_i p_i}, \qquad \text{if } j = i.$$

(10) **[30 marks]** Professor Plum, Mr. Green, and Miss Scarlet are all plotting to shoot Colonel Mustard. If one of these three has both an opportunity and the revolver, then that person shoots Colonel Mustard. Otherwise, Colonel Mustard escapes. Exactly one of the three has an opportunity with the following probabilities:

$$\Pr[\text{Plum has the opportunity}] = \frac{1}{6}$$
$$\Pr[\text{Green has the opportunity}] = \frac{2}{6}$$
$$\Pr[\text{Scarlet has the opportunity}] = \frac{3}{6}$$

Exactly one has the revolver with the following probabilities, regardless of who has an opportunity:

$$\Pr[\text{Plum has the revolver}] = \frac{4}{8}$$
$$\Pr[\text{Green has the revolver}] = \frac{3}{8}$$
$$\Pr[\text{Scarlet has the revolver}] = \frac{1}{8}$$

- Draw a tree diagram for this problem. Indicate edge and outcome probabilities. [15 marks]
- What is the probability that Colonel Mustard is shot? [5 marks]
- What is the probability that Colonel Mustard is shot, given that Miss Scarlet does not have the revolver? [5 marks]
- What is the probability that Mr. Green had an opportunity, given that Colonel Mustard was shot? [5 marks]

(11) **[25 marks]** Prove the following statements.

- For an event A and two mutually exclusive events E_1 and E_2, prove the following statement.

$$\Pr[E_1 \cup E_2|A] = \Pr[E_1|A] + \Pr[E_2|A]$$

  implying that the union-rule for mutually exclusive events also works if we condition on the event A. [10 marks]

- For an event A and two mutually exclusive events E_1 and E_2, prove the following statement. [10 marks]

$$\Pr[A|E_1 \cup E_2] = \frac{\Pr[A|E_1]\ \Pr[E_1] + \Pr[A|E_2]\ \Pr[E_2]}{\Pr[E_1] + \Pr[E_2]}$$

- Prove that the above rule generalizes the law of total probability, i.e. if $E_1 \cup E_2 = \mathcal{S}$ and $E_1 \cap E_2 = \emptyset$, then the above equation recovers the law of total probability, i.e. $\Pr[A] = \Pr[A|E_1]\ \Pr[E_1] + \Pr[A|E_1^c]\ \Pr[E_1^c]$, where E_1 and E_1^c are complement events. [5 marks]