CMPT 476

Professor office hours:
11:30 - 12:30 Friday

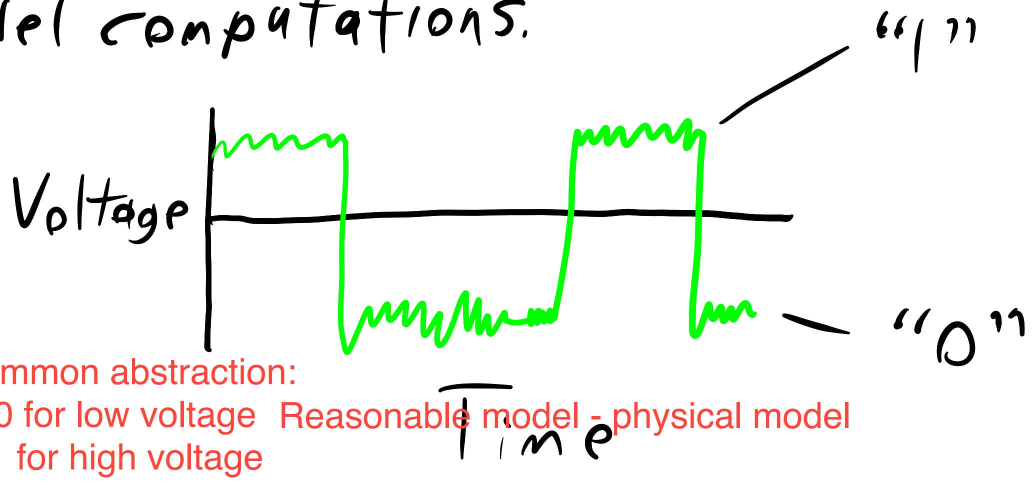
Introduction to Quantum Algorithms!

Lecture 1: What the heck is QC?

To understand what quantum computing is, we first need to understand classical computing!

Meant to change data to a form you want

Computation is a physical process of calculation (sidebar: we could instead say well defined, but how do we define well-defined?)
We use abstractions to describe and model computations.

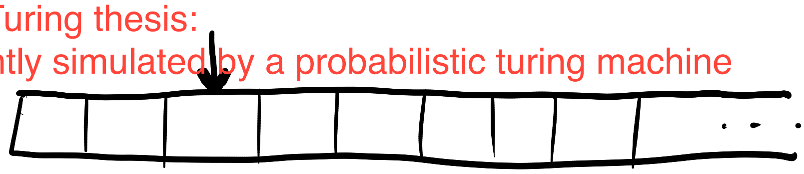


Common abstraction:
0 for low voltage
1 for high voltage

Reasonable model - physical model

A common abstraction is a Turing machine

Extended Church-Turing thesis:
utation can be efficiently simulated by a probabilistic turing machine



(Complexity)

We often want to know how difficult it is to compute something. Without (yet) saying what we mean by that, the hope is that our abstraction maps easy (physical) problems to easy abstract problems

Physical		Abstract
Easy	$\longleftarrow - - - \longrightarrow$	Easy
Hard	$\longleftarrow - - - \longrightarrow$	Hard

(Extended Church-Turing thesis)

Any reasonable (physical) model of computation can be efficiently (easy $\rightarrow$ easy) simulated by a probabilistic Turing machine

In other words, we can forget about physics because we can't do anything* physically efficiently that we can't do efficiently with a digital computer.

* $\rightarrow$ anything programmable in the input $\rightarrow$ output sense

(Feynman, 1982)

A quantum mechanical process can NOT be simulated efficiently by any classical model of computation.

$\Downarrow$

Violation of ECTT!

Ground state energy

... But can we use quantum mechanical systems to perform computations we care about? Yes!!!

Efficiently: in polynomial time

Not very important

- (Shor 1994) Integer factorization

Important

- (Lloyd 1996) Hamiltonian simulation

Research this simulation

- (Grover 1996) Unstructured search

O(n) -> O(sqrt(n))

- (HHL 2008) Sparse linear systems

In practice, cost of using quantum computer is far higher than any possible efficiency gain

And more in this course!

Quantum computer requirements:

Two distinct measurable physical states: I0>, I1>

d ability to generate superposition of a linear combination of I0> and I1> states

Ability to generate interference

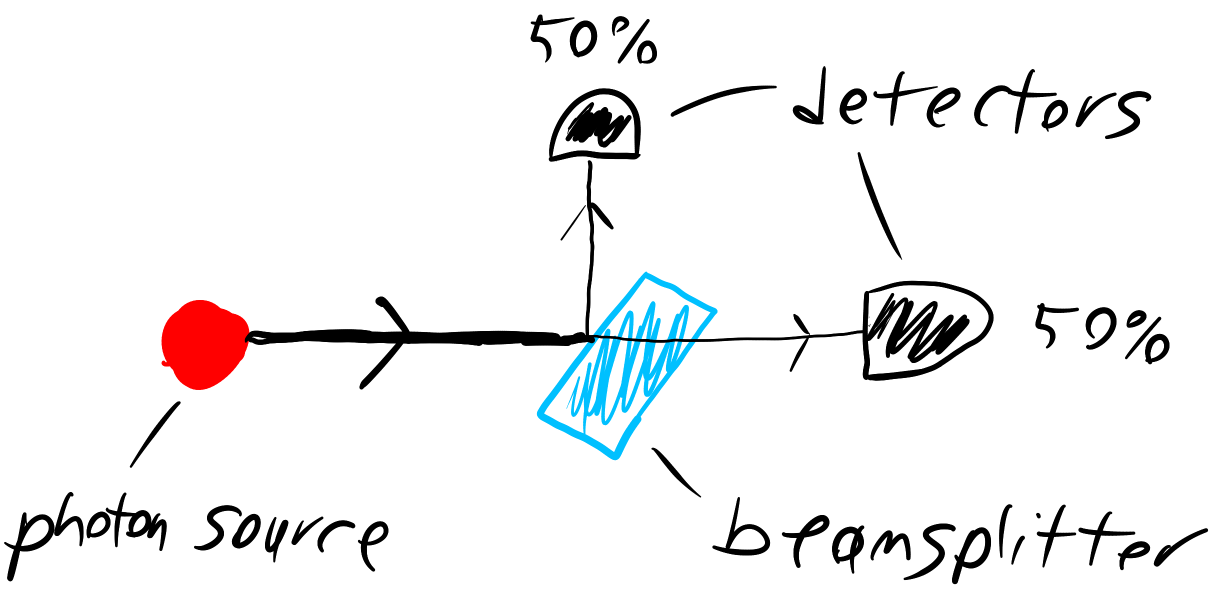
(A preview of Quantum computation)

Shopping list for QC:

1. Two (2) distinct and measurable... physical states $|0\rangle$ & $|1\rangle$
2. The ability to be in a superposition of them
3. The ability to generate interference

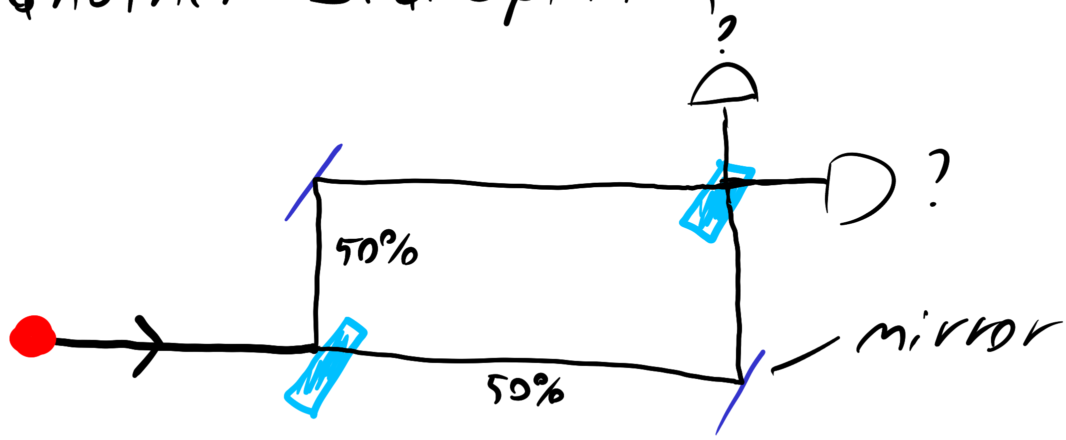
(Interferometers)

A classic example of all 3 ingredients is an interferometer. Given a photon source (light) (e.g. laser), a beam splitter reflects photons with 50% probability.



So, if we send a single photon through this set up, it should be detected in either location with 50% probability.

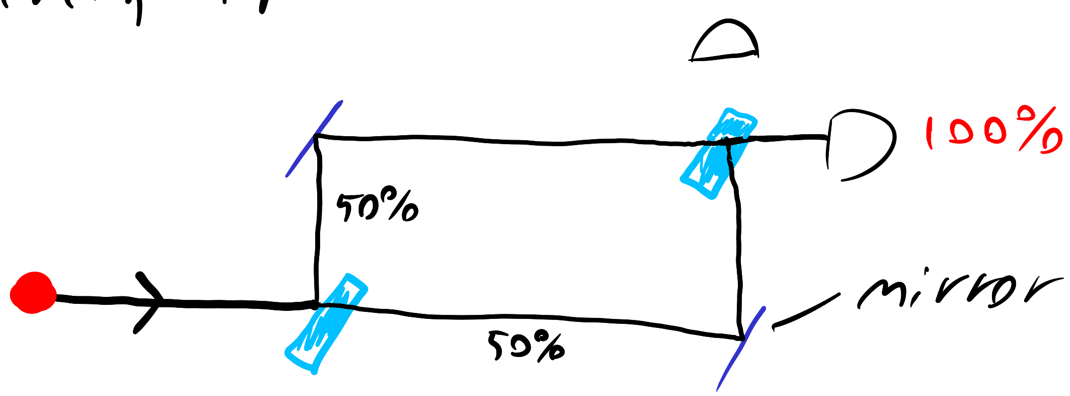
If we redirect the two beams of light back into another beamsplitter, what would happen?



Classically, we should detect at either location with 50% probability:

- 2 ways to get to the top detector, ↑ then ↑ or → then ↑
- each path has $0.5 \cdot 0.5 = 0.25$ probability
- total is $0.25 + 0.25 = 0.5$

However, in practice we find



The reason is the photon took all paths at the same time (superposition) and the paths leading to the upper detector cancelled out (interference)

(Linear algebraic model)

Mathematically, we model a superposition by a linear combination of vectors.

- $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is the initial/transmited path
- $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is the reflected path
- A photon is in the state

complex numbers

$$\alpha|0\rangle + \beta|1\rangle, \quad \alpha, \beta \in \mathbb{C}$$

a superposition of the 0 and 1 state

tate is detected, it becomes either one of the states

- If we were to measure the state by placing photon detectors along either path, we would find the photon in state $|0\rangle$ with probability $|\alpha|^2$ (similar for $|1\rangle$)

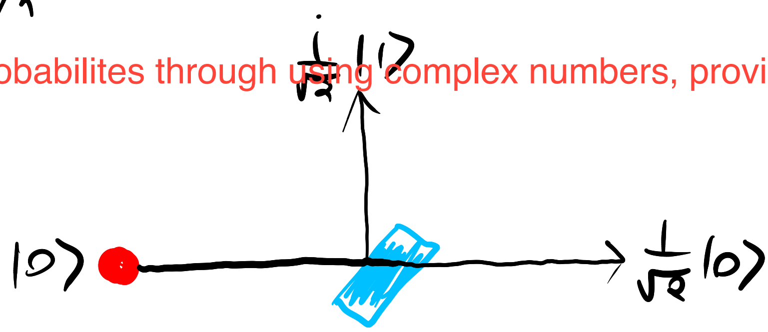
The beam splitter is modeled by applying the matrix $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$ to the state vector:

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$$

Both paths have the same weight but differ by a phase of i

Pictorally,

You have negative probabilites through using complex numbers, providing destructive interferenc



When we apply the second beam splitter, the photon is in the state $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$, so the resulting state is

We detect the photon in the zero state with a probability 0^2, and will be in the one state with a probability of |i^2|

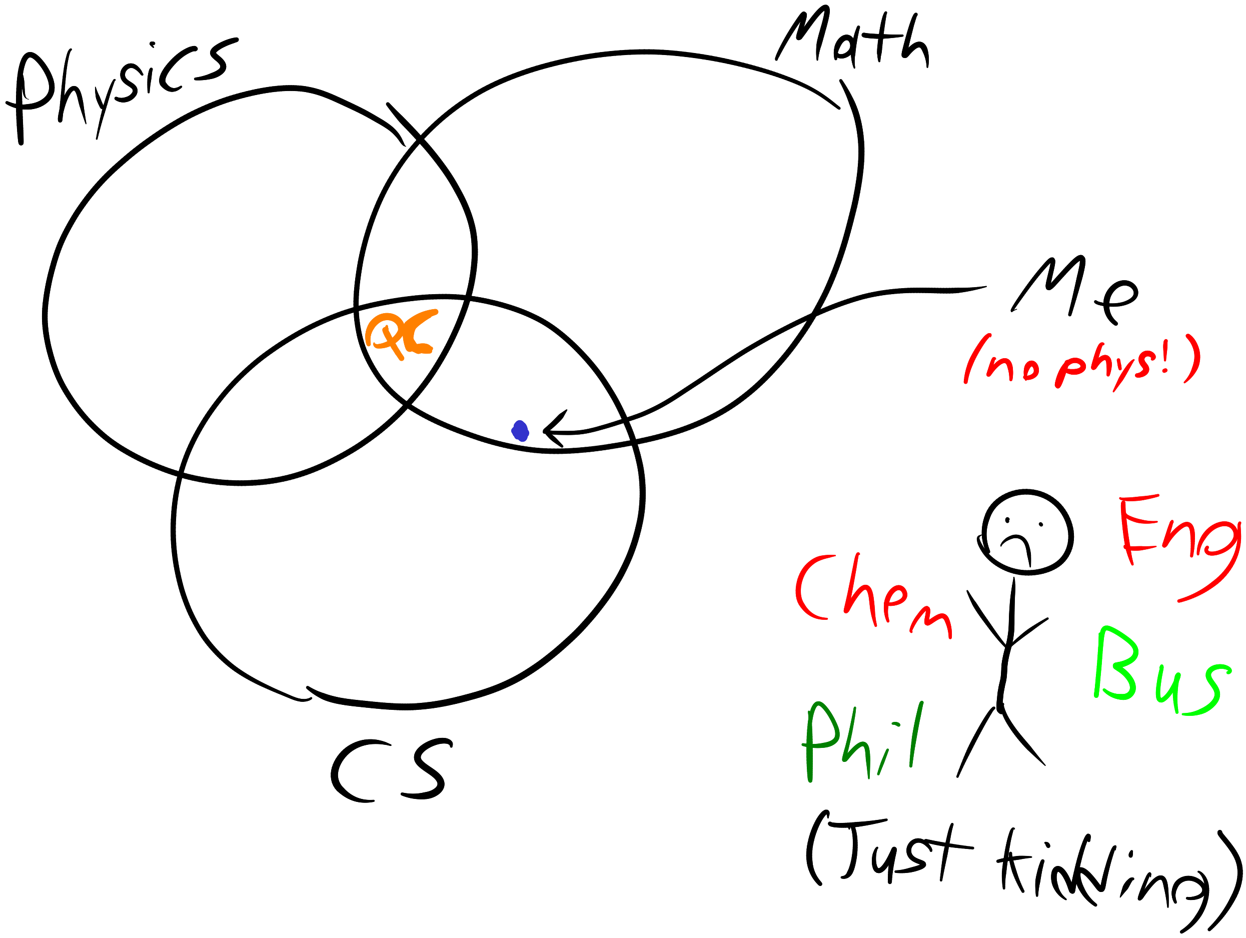
$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1+i^2 \\ i+i \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix} = i|1\rangle$$

Amplitude of i in the one path, amplitude of 0 in the 0 path

Hence we detect the photon along the transmitted path $|1\rangle$

(Does this actually correspond to reality?)

A better question is does it matter? As long as our abstract model can predict the outcome of the physical process, we can put our heads in the sand and forget about the physics ◡



If a quantum computer can solve a problem in polynomial time, it does not mean the problem cannot be solved on a classical computer in polynomial time. The method for solving the problem on a classical computer may need to change in order to solve it in polynomial time.

How do complex probabilites represent something meaningful?

(Housekeeping)

Website: (check often!)

cs.sfu.ca/~meamy/Teaching/s24/cmpt476

Evaluation:

50% assignments (approx 6)

15% mid-term exam

35% final exam

TAs:

Lucas Stinchcombe (CS)

Ming Yin (Math)

Resources:

See website!

tions by removing all the solutions that have been cancelled out by destructive
use the constructive interference to get the solutions which have what you want.

Questions:
what is computation?
what is the extended church-turing thesis?
What is reasonable?
What is efficient for a computation?

Potential application:
Nitrogen fixation

Ask professor what he said about ground energy
When is d-wave?
What assumptions can we make about the materials used?