SIMON FRASER UNIVERSITY
School of Computing Science

CMPT 409 – Midterm Examination
June 20th, 2022

NAME:____

FAMILY NAME GIVEN NAMES

SFU-ID #:____________________________________

INSTRUCTIONS

1. Calculators are **not** permitted.
2. This exam is **closed book**.
3. Clearly print your **name** and student **ID** number on this examination (above).
4. There are **40 points** in total.
5. There are **6 pages** including this cover sheet.
6. You have **50 minutes**.

1. **[10]** The Bloch Sphere is a generalization of the representation of a complex number *z* with $|z|^2 = 1$ as a point on the unit circle in the complex plane. From this starting point, derive the equation of the Bloch sphere; namely,

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

2. **[5]** Show that the global phase factor, $e^{i\gamma}$, has no observable effect on the probabilities, $|\alpha|^2$ and $|\beta|^2$.

3. **[5]** Show that opposite states on the Bloch sphere are orthogonal.

4. **[5]** Redo the following calculation using Dirac bra-ket notation:

$$
\begin{aligned}
1 &= \langle \psi | \psi \rangle \\
&= \left(\overline{a_0} \langle 0| + \overline{a_1} \langle 1|\right) \cdot \left(a_0 |0\rangle + a_1 |1\rangle\right) \\
&= \left(\overline{a_0} \begin{bmatrix} \overline{\psi_{00}} & \overline{\psi_{01}} \end{bmatrix} + \overline{a_1} \begin{bmatrix} \overline{\psi_{10}} & \overline{\psi_{11}} \end{bmatrix}\right) \cdot \left(a_0 \begin{bmatrix} \psi_{00} \\ \psi_{01} \end{bmatrix} + a_1 \begin{bmatrix} \psi_{10} \\ \psi_{11} \end{bmatrix}\right) \\
&= \begin{bmatrix} \overline{a_0\psi_{00}} + \overline{a_1\psi_{10}} & \overline{a_0\psi_{01}} + \overline{a_1\psi_{11}} \end{bmatrix} \cdot \begin{bmatrix} a_0\psi_{00} + a_1\psi_{10} \\ a_0\psi_{01} + a_1\psi_{11} \end{bmatrix} \\
&= \overline{a_0\psi_{00}}a_0\psi_{00} + \overline{a_1\psi_{10}}a_0\psi_{00} + \overline{a_0\psi_{00}}a_1\psi_{10} + \overline{a_1\psi_{10}}a_1\psi_{10} \\
&\quad + \overline{a_0\psi_{01}}a_0\psi_{01} + \overline{a_1\psi_{11}}a_0\psi_{01} + \overline{a_0\psi_{01}}a_1\psi_{11} + \overline{a_1\psi_{11}}a_1\psi_{11} \\
&= |a_0|^2 \left(|\psi_{00}|^2 + |\psi_{01}|^2\right) + |a_1|^2 \left(|\psi_{10}|^2 + |\psi_{11}|^2\right) \\
&\quad + \overline{a_1}a_0 \left(\overline{\psi_{10}}\psi_{00} + \overline{\psi_{11}}\psi_{01}\right) + \overline{a_0}a_1 \left(\overline{\psi_{00}}\psi_{10} + \overline{\psi_{01}}\psi_{11}\right) \\
&= |a_0|^2 + |a_1|^2
\end{aligned}
$$

5. **[5]** How does one prove that two pure quantum states are entangled? Describe this in some formal detail.

6. **[5]** Describe in detail how one would make a *Hello World* program using a quantum computer. Begin by describing what such a program is intended for.

7. **[5]** Consider two widely separated entangled photons. Why do scientists call them "strongly correlated"? In other words, why can´t one say that the measured photon controls the state of the distant photon?