

hw_3

July 1, 2022

1 Home #3.3

```
[1]: %matplotlib inline
import warnings
warnings.filterwarnings('ignore')
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from sklearn.linear_model import LinearRegression, Lasso, Ridge, RidgeCV, LassoCV

pd.set_option('display.max_rows', 500)
pd.set_option('display.max_columns', 500)

[2]: # read data
rets_mkt = pd.read_excel('data/hw_3_3_data.xlsx', sheet_name='benchmark rets')
rets_spx = pd.read_excel('data/hw_3_3_data.xlsx', sheet_name="s&p500 rets")
```

1.1 1. Penalized Regression

```
[3]: X = rets_spx.drop(columns='date')
y = rets_mkt.GLD
```

1.1.1 1) OLS

```
[4]: model_ols = LinearRegression().fit(X, y)
results_ols = pd.DataFrame({'regressor': X.columns, 'intercept': model_ols.
    ↪intercept_, 'beta': model_ols.coef_})
```

a) intercept and betas

```
[5]: display(results_ols)
```

	regressor	intercept	beta
0	A	0.000613	0.065328
1	AAP	0.000613	-0.030014
2	AAPL	0.000613	-0.089143

3	ABC	0.000613	-0.042302
4	ABT	0.000613	0.075277
5	ACN	0.000613	-0.022478
6	ADBE	0.000613	-0.041769
7	ADI	0.000613	0.025656
8	ADM	0.000613	-0.006127
9	ADP	0.000613	0.086004
10	ADSK	0.000613	-0.020923
11	AEE	0.000613	0.027172
12	AEP	0.000613	-0.084359
13	AES	0.000613	-0.041263
14	AFL	0.000613	-0.018153
15	AIG	0.000613	-0.011030
16	AIZ	0.000613	-0.016694
17	AJG	0.000613	0.061370
18	AKAM	0.000613	0.069468
19	ALB	0.000613	-0.020335
20	ALGN	0.000613	-0.003458
21	ALK	0.000613	0.061027
22	ALL	0.000613	-0.006670
23	AMAT	0.000613	-0.030667
24	AMD	0.000613	0.000289
25	AME	0.000613	-0.096122
26	AMGN	0.000613	-0.073429
27	AMP	0.000613	0.068424
28	AMT	0.000613	-0.028573
29	AMZN	0.000613	-0.008876
30	ANSS	0.000613	0.027064
31	ANTM	0.000613	0.007065
32	AON	0.000613	-0.032766
33	AOS	0.000613	-0.007962
34	APA	0.000613	0.036380
35	APD	0.000613	-0.106897
36	APH	0.000613	-0.095966
37	APTV	0.000613	0.061230
38	ARE	0.000613	-0.055925
39	ATO	0.000613	0.039071
40	ATVI	0.000613	0.000198
41	AVB	0.000613	-0.005838
42	AVGO	0.000613	-0.057006
43	AVY	0.000613	0.030683
44	AWK	0.000613	0.028946
45	AXP	0.000613	-0.012275
46	AZO	0.000613	-0.025685
47	BA	0.000613	-0.028732
48	BAC	0.000613	0.004094
49	BALL	0.000613	0.026438
50	BAX	0.000613	0.117546

51	BBWI	0.000613	-0.035678
52	BBY	0.000613	0.009760
53	BDX	0.000613	0.029199
54	BEN	0.000613	0.069249
55	BF/B	0.000613	0.017963
56	BIO	0.000613	-0.090708
57	BK	0.000613	-0.022655
58	BKNG	0.000613	0.037882
59	BKR	0.000613	0.017498
60	BLK	0.000613	0.009845
61	BMY	0.000613	-0.046744
62	BR	0.000613	0.123529
63	BRK/B	0.000613	-0.003556
64	BRO	0.000613	0.039116
65	BSX	0.000613	0.020688
66	BWA	0.000613	0.021731
67	BXP	0.000613	-0.025521
68	C	0.000613	-0.015276
69	CAG	0.000613	0.011301
70	CAH	0.000613	-0.028629
71	CAT	0.000613	-0.016422
72	CB	0.000613	-0.023083
73	CBOE	0.000613	0.071112
74	CBRE	0.000613	-0.071218
75	CCI	0.000613	0.014345
76	CCL	0.000613	-0.043849
77	CDNS	0.000613	0.024411
78	CE	0.000613	-0.047771
79	CF	0.000613	-0.002057
80	CHD	0.000613	0.068601
81	CHRW	0.000613	0.029579
82	CHTR	0.000613	0.052737
83	CI	0.000613	0.022323
84	CINF	0.000613	0.009664
85	CL	0.000613	-0.045731
86	CLX	0.000613	-0.006737
87	CMA	0.000613	0.001709
88	CMCSA	0.000613	-0.025949
89	CME	0.000613	-0.007473
90	CMG	0.000613	0.007149
91	CMI	0.000613	0.084812
92	CMS	0.000613	-0.051335
93	CNC	0.000613	-0.005758
94	CNP	0.000613	-0.004549
95	COF	0.000613	-0.091583
96	COO	0.000613	-0.054791
97	COP	0.000613	-0.022594
98	COST	0.000613	0.057493

99	CPB	0.000613	-0.062898
100	CPRT	0.000613	-0.030685
101	CPT	0.000613	-0.018228
102	CRL	0.000613	-0.031367
103	CRM	0.000613	-0.022086
104	CSCO	0.000613	-0.014767
105	CSX	0.000613	-0.010653
106	CTAS	0.000613	-0.026058
107	CTRA	0.000613	0.038704
108	CTSH	0.000613	-0.008294
109	CTXS	0.000613	0.022333
110	CVS	0.000613	0.034999
111	CVX	0.000613	-0.018951
112	D	0.000613	0.060405
113	DAL	0.000613	0.044261
114	DD	0.000613	0.017216
115	DE	0.000613	0.050269
116	DFS	0.000613	-0.013506
117	DG	0.000613	-0.056541
118	DGX	0.000613	-0.049378
119	DHI	0.000613	0.009727
120	DHR	0.000613	0.103756
121	DIS	0.000613	0.011446
122	DISH	0.000613	-0.002392
123	DLR	0.000613	0.016968
124	DLTR	0.000613	-0.046649
125	DOV	0.000613	-0.012307
126	DPZ	0.000613	-0.000748
127	DRE	0.000613	0.022309
128	DRI	0.000613	0.089492
129	DTE	0.000613	-0.116357
130	DUK	0.000613	0.023791
131	DVA	0.000613	-0.002373
132	DVN	0.000613	-0.014277
133	EA	0.000613	0.027816
134	EBAY	0.000613	-0.005852
135	ECL	0.000613	-0.029710
136	ED	0.000613	-0.104110
137	EFX	0.000613	0.001330
138	EIX	0.000613	0.088421
139	EL	0.000613	-0.019641
140	EMN	0.000613	0.047718
141	EMR	0.000613	0.097785
142	ENPH	0.000613	0.018632
143	EOG	0.000613	-0.021643
144	EPAM	0.000613	0.025034
145	EQIX	0.000613	-0.050688
146	EQR	0.000613	0.003779

147	ES	0.000613	-0.022328
148	ESS	0.000613	0.105106
149	ETN	0.000613	-0.057265
150	ETR	0.000613	0.064721
151	EVRG	0.000613	-0.015022
152	EW	0.000613	0.021961
153	EXC	0.000613	-0.029924
154	EXPD	0.000613	0.024077
155	EXPE	0.000613	-0.015193
156	EXR	0.000613	0.028600
157	F	0.000613	-0.037906
158	FAST	0.000613	0.022550
159	FBHS	0.000613	-0.018690
160	FCX	0.000613	0.060270
161	FDS	0.000613	0.062694
162	FDX	0.000613	-0.050187
163	FE	0.000613	-0.057386
164	FFIV	0.000613	-0.008263
165	FIS	0.000613	-0.046016
166	FISV	0.000613	0.011487
167	FITB	0.000613	-0.009229
168	FLT	0.000613	-0.060469
169	FMC	0.000613	0.024492
170	FRC	0.000613	-0.005107
171	FRT	0.000613	-0.015965
172	FTNT	0.000613	0.003328
173	GD	0.000613	-0.012579
174	GE	0.000613	0.040548
175	GILD	0.000613	-0.019509
176	GIS	0.000613	0.070164
177	GL	0.000613	0.020605
178	GLW	0.000613	0.037477
179	GM	0.000613	-0.015920
180	GNRC	0.000613	-0.008974
181	GOOGL	0.000613	0.040942
182	GPC	0.000613	0.037439
183	GPN	0.000613	-0.020510
184	GRMN	0.000613	-0.035781
185	GS	0.000613	-0.104874
186	GW	0.000613	-0.084099
187	HAL	0.000613	0.000490
188	HAS	0.000613	-0.013895
189	HBAN	0.000613	-0.068969
190	HCA	0.000613	0.097013
191	HD	0.000613	0.009375
192	HES	0.000613	-0.008920
193	HIG	0.000613	-0.071418
194	HII	0.000613	0.033397

195	HOLX	0.000613	-0.046990
196	HON	0.000613	0.077034
197	HPQ	0.000613	0.000550
198	HRL	0.000613	-0.002587
199	HSIC	0.000613	0.036339
200	HST	0.000613	-0.007259
201	HSY	0.000613	0.072883
202	HUM	0.000613	0.031247
203	IBM	0.000613	0.000169
204	ICE	0.000613	0.038361
205	IDXX	0.000613	-0.010009
206	IEX	0.000613	-0.009434
207	IFF	0.000613	0.011792
208	ILMN	0.000613	0.009100
209	INCY	0.000613	-0.025429
210	INTC	0.000613	0.032824
211	INTU	0.000613	0.057071
212	IP	0.000613	-0.001811
213	IPG	0.000613	0.032901
214	IPGP	0.000613	0.023769
215	IRM	0.000613	-0.018988
216	ISRG	0.000613	-0.006947
217	IT	0.000613	-0.091298
218	ITW	0.000613	0.024241
219	IVZ	0.000613	-0.026533
220	J	0.000613	0.015138
221	JBHT	0.000613	-0.020834
222	JCI	0.000613	0.059562
223	JKHY	0.000613	-0.008818
224	JNJ	0.000613	0.004549
225	JNPR	0.000613	0.020167
226	JPM	0.000613	-0.052661
227	K	0.000613	-0.069516
228	KEY	0.000613	-0.019374
229	KIM	0.000613	0.031730
230	KLAC	0.000613	0.011298
231	KMB	0.000613	0.013179
232	KMI	0.000613	-0.011330
233	KMX	0.000613	-0.027601
234	KO	0.000613	0.010142
235	KR	0.000613	-0.003538
236	L	0.000613	0.034786
237	LDOS	0.000613	0.037815
238	LEN	0.000613	-0.060274
239	LH	0.000613	0.031777
240	LHX	0.000613	0.075356
241	LIN	0.000613	0.050046
242	LKQ	0.000613	0.062192

243	LLY	0.000613	0.055862
244	LMT	0.000613	-0.000083
245	LNC	0.000613	0.023734
246	LNT	0.000613	0.047949
247	LOW	0.000613	0.042966
248	LRCX	0.000613	-0.005932
249	LUMN	0.000613	-0.039444
250	LUV	0.000613	0.048965
251	LVS	0.000613	-0.027182
252	LYB	0.000613	0.054137
253	LYV	0.000613	-0.049070
254	MA	0.000613	0.039661
255	MAA	0.000613	-0.016273
256	MAR	0.000613	0.001836
257	MAS	0.000613	0.014023
258	MCD	0.000613	0.055834
259	MCHP	0.000613	0.009476
260	MCK	0.000613	-0.008613
261	MCO	0.000613	-0.053836
262	MDLZ	0.000613	-0.055290
263	MDT	0.000613	-0.018249
264	MET	0.000613	-0.045165
265	META	0.000613	-0.002872
266	MGM	0.000613	0.023321
267	MHK	0.000613	-0.004572
268	MKC	0.000613	-0.043545
269	MKTX	0.000613	-0.001862
270	MLM	0.000613	-0.002892
271	MMC	0.000613	-0.072019
272	MMM	0.000613	-0.041794
273	MNST	0.000613	0.025129
274	MO	0.000613	-0.034218
275	MOH	0.000613	-0.008447
276	MOS	0.000613	0.020035
277	MPC	0.000613	-0.014972
278	MPWR	0.000613	0.048571
279	MRK	0.000613	0.053791
280	MRO	0.000613	0.020902
281	MS	0.000613	0.031112
282	MSCI	0.000613	-0.001498
283	MSFT	0.000613	-0.132025
284	MSI	0.000613	-0.047584
285	MTB	0.000613	-0.000642
286	MTD	0.000613	-0.009700
287	MU	0.000613	-0.011534
288	NDAQ	0.000613	0.006418
289	NDSN	0.000613	0.020371
290	NEE	0.000613	0.054242

291	NEM	0.000613	0.159889
292	NFLX	0.000613	0.013781
293	NI	0.000613	0.007341
294	NKE	0.000613	-0.024585
295	NLOK	0.000613	-0.006802
296	NLSN	0.000613	-0.025923
297	NOC	0.000613	0.031487
298	NRG	0.000613	-0.002713
299	NSC	0.000613	-0.086940
300	NTAP	0.000613	-0.038509
301	NTRS	0.000613	0.071948
302	NUE	0.000613	0.046783
303	NVDA	0.000613	-0.005102
304	NVR	0.000613	-0.028966
305	NWL	0.000613	0.043880
306	NXPI	0.000613	0.053046
307	O	0.000613	0.080438
308	ODFL	0.000613	-0.014189
309	OKE	0.000613	-0.031967
310	OMC	0.000613	-0.032428
311	ORCL	0.000613	-0.028460
312	ORLY	0.000613	0.099030
313	OXY	0.000613	0.020136
314	PARA	0.000613	-0.025588
315	PAYX	0.000613	0.014297
316	PCAR	0.000613	-0.056029
317	PEAK	0.000613	0.052286
318	PEG	0.000613	-0.002226
319	PENN	0.000613	0.022643
320	PEP	0.000613	-0.069023
321	PFE	0.000613	-0.071840
322	PFG	0.000613	-0.007181
323	PG	0.000613	-0.089474
324	PGR	0.000613	0.001977
325	PH	0.000613	0.048665
326	PHM	0.000613	0.098021
327	PKG	0.000613	-0.043881
328	PKI	0.000613	0.098029
329	PLD	0.000613	-0.060045
330	PM	0.000613	0.134020
331	PNC	0.000613	-0.041651
332	PNR	0.000613	0.008760
333	PNW	0.000613	0.039110
334	POOL	0.000613	-0.033585
335	PPG	0.000613	0.006972
336	PPL	0.000613	-0.007179
337	PRU	0.000613	0.053835
338	PSA	0.000613	-0.073062

339	PSX	0.000613	-0.042002
340	PTC	0.000613	-0.024525
341	PVH	0.000613	-0.042802
342	PWR	0.000613	-0.074605
343	PXD	0.000613	0.002052
344	QCOM	0.000613	0.059009
345	RCL	0.000613	0.016955
346	RE	0.000613	0.008803
347	REG	0.000613	-0.055620
348	RF	0.000613	0.032118
349	RHI	0.000613	-0.069826
350	RJF	0.000613	-0.100555
351	RL	0.000613	-0.021674
352	RMD	0.000613	-0.043287
353	ROK	0.000613	-0.026967
354	ROL	0.000613	-0.079375
355	ROP	0.000613	-0.042862
356	ROST	0.000613	0.032687
357	RSG	0.000613	0.098482
358	RTX	0.000613	-0.060680
359	SBAC	0.000613	0.048636
360	SBNY	0.000613	0.027369
361	SBUX	0.000613	-0.091299
362	SCHW	0.000613	-0.099300
363	SEE	0.000613	0.026297
364	SHW	0.000613	-0.092173
365	SIVB	0.000613	0.018874
366	SJM	0.000613	0.052461
367	SLB	0.000613	0.076042
368	SNA	0.000613	0.025115
369	SNPS	0.000613	0.028688
370	SO	0.000613	-0.028068
371	SPG	0.000613	0.079783
372	SPGI	0.000613	0.039726
373	SRE	0.000613	0.039713
374	STE	0.000613	-0.040880
375	STT	0.000613	0.025059
376	STX	0.000613	0.025083
377	STZ	0.000613	0.007422
378	SWK	0.000613	0.009368
379	SWKS	0.000613	0.025971
380	SYK	0.000613	-0.052632
381	SYU	0.000613	-0.043757
382	T	0.000613	-0.100052
383	TAP	0.000613	-0.029970
384	TDG	0.000613	0.041248
385	TDY	0.000613	0.051026
386	TECH	0.000613	0.018293

387	TEL	0.000613	0.088447
388	TER	0.000613	-0.047592
389	TFC	0.000613	0.003192
390	TFX	0.000613	0.014069
391	TGT	0.000613	0.026479
392	TJX	0.000613	0.032713
393	TMO	0.000613	-0.030646
394	TMUS	0.000613	0.024611
395	TPR	0.000613	-0.036102
396	TRMB	0.000613	-0.043429
397	TROW	0.000613	0.029405
398	TRV	0.000613	0.015211
399	TSCO	0.000613	-0.076940
400	TSLA	0.000613	0.001273
401	TSN	0.000613	0.043233
402	TT	0.000613	-0.076011
403	TTWO	0.000613	0.014146
404	TXN	0.000613	-0.110645
405	TXT	0.000613	0.061742
406	TYL	0.000613	0.010014
407	UAA	0.000613	0.028357
408	UAL	0.000613	-0.087180
409	UDR	0.000613	-0.051625
410	UHS	0.000613	-0.047050
411	ULTA	0.000613	-0.065589
412	UNH	0.000613	-0.020222
413	UNP	0.000613	-0.037476
414	UPS	0.000613	0.040379
415	URI	0.000613	0.006092
416	USB	0.000613	0.132346
417	V	0.000613	0.022250
418	VFC	0.000613	-0.008681
419	VLO	0.000613	0.006484
420	VMC	0.000613	0.061214
421	VNO	0.000613	0.002257
422	VRSK	0.000613	0.061810
423	VRSN	0.000613	0.017607
424	VTR	0.000613	-0.017453
425	VTRS	0.000613	0.003724
426	VZ	0.000613	-0.070947
427	WAB	0.000613	-0.004367
428	WAT	0.000613	0.013634
429	WBA	0.000613	0.025199
430	WDC	0.000613	0.006798
431	WEC	0.000613	0.025634
432	WELL	0.000613	0.057311
433	WFC	0.000613	0.013886
434	WHR	0.000613	-0.007668

```

435      WM    0.000613 -0.060487
436      WMB    0.000613 -0.046572
437      WMT    0.000613  0.085365
438      WRB    0.000613  0.062396
439      WST    0.000613 -0.038631
440      WTW    0.000613  0.017391
441      WY     0.000613  0.021773
442      WYNN   0.000613  0.000341
443      XEL    0.000613  0.082618
444      XOM    0.000613 -0.152578
445      XRAY   0.000613 -0.027932
446      XYL    0.000613  0.028859
447      YUM    0.000613 -0.011586
448      ZBH    0.000613  0.005723
449      ZBRA   0.000613  0.054684
450      ZION   0.000613  0.008005

```

b) R-squared

```
[6]: model_ols.score(X, y)
```

```
[6]: 1.0
```

c) and d)

```
[7]: def compute_beta_sigma(spx, results):
      results['beta_sigma'] = spx.drop(columns='date').std().values * results.
      ↪beta.values
      results['abs_beta_sigma'] = results.beta_sigma.abs()
      results['abs_beta'] = results.beta.abs()
      return results

```

```
[8]: results_ols = compute_beta_sigma(rets_spx, results_ols)
      print("***** Top 10 largest betas *****")
      display(results_ols.sort_values('abs_beta', ascending=False).
      ↪head(10)[['regressor', 'beta']].\
      reset_index(drop=True))
      print("***** Top 10 largest beta-sigmas *****")
      display(results_ols.sort_values('abs_beta_sigma', ascending=False).
      ↪head(10)[['regressor', 'beta_sigma']].\
      reset_index(drop=True))

```

```
***** Top 10 largest betas *****
```

	regressor	beta
0	NEM	0.159889
1	XOM	-0.152578
2	PM	0.134020
3	USB	0.132346

```

4      MSFT -0.132025
5      BR  0.123529
6      BAX  0.117546
7      DTE -0.116357
8      TXN -0.110645
9      APD -0.106897

```

***** Top 10 largest beta-sigmas *****

```

      regressor  beta_sigma
0      NEM      0.007798
1      UAL     -0.006702
2      XOM     -0.005969
3      PHM      0.005466
4      SPG      0.005239
5      COF     -0.005195
6      USB      0.005188
7      DRI      0.005146
8      PM       0.004810
9      SCHW    -0.004725

```

e) Condition number

```

[9]: condition_number = np.linalg.cond(rets_spx.drop(columns=['date']).values);
      ↪condition_number

```

```

[9]: 212.25385331307297

```

The high condition number is indicative of presence of multicollinearity which violates OLS assumption 1.

1.1.2 2) Ridge Regression

```

[10]: model_ridge = Ridge(alpha=0.5).fit(X, y)
      results_ridge = pd.DataFrame({'regressor': X.columns, 'intercept': model_ridge.
      ↪intercept_, 'beta': model_ridge.coef_})

```

a) R-squared

```

[11]: model_ridge.score(X, y)

```

```

[11]: 0.7622443309155418

```

```

[12]: results_ridge = compute_beta_sigma(rets_spx, results_ridge)
      print("***** Top 10 largest betas *****")
      display(results_ridge.sort_values('abs_beta', ascending=False).
      ↪head(10)[['regressor', 'beta']].\
      reset_index(drop=True))
      print("***** Top 10 largest beta-sigmas *****")

```

```
display(results_ridge.sort_values('abs_beta_sigma', ascending=False).
↳head(10)[['regressor', 'beta_sigma']].\
reset_index(drop=True))
```

***** Top 10 largest betas *****

	regressor	beta
0	NEM	0.105331
1	SCHW	-0.034809
2	FCX	0.030162
3	RHI	-0.026028
4	RJF	-0.024630
5	CHTR	0.022171
6	CBRE	-0.020875
7	O	0.020077
8	SLB	0.019675
9	PKG	-0.019215

***** Top 10 largest beta-sigmas *****

	regressor	beta_sigma
0	NEM	0.005137
1	FCX	0.002314
2	ENPH	0.002027
3	SCHW	-0.001656
4	RHI	-0.001149
5	SLB	0.001125
6	RJF	-0.001085
7	CBRE	-0.000970
8	ULTA	-0.000953
9	CHTR	0.000895

1.1.3 3) LASSO

```
[13]: model_lasso = Lasso(alpha=2e-5).fit(X, y)
results_lasso = pd.DataFrame({'regressor': X.columns, 'intercept': model_lasso.
↳intercept_, 'beta': model_lasso.coef_})
```

a) intercept and beta

```
[14]: display(results_lasso)
```

	regressor	intercept	beta
0	A	0.000031	0.011062
1	AAP	0.000031	-0.001673
2	AAPL	0.000031	-0.000000
3	ABC	0.000031	-0.001622
4	ABT	0.000031	0.000000
5	ACN	0.000031	-0.000000

6	ADBE	0.000031	-0.000000
7	ADI	0.000031	-0.000000
8	ADM	0.000031	0.000000
9	ADP	0.000031	-0.000000
10	ADSK	0.000031	0.000000
11	AEE	0.000031	0.000000
12	AEP	0.000031	0.000000
13	AES	0.000031	-0.000000
14	AFL	0.000031	0.000000
15	AIG	0.000031	-0.000000
16	AIZ	0.000031	-0.000000
17	AJG	0.000031	0.000000
18	AKAM	0.000031	0.014663
19	ALB	0.000031	-0.000798
20	ALGN	0.000031	-0.000498
21	ALK	0.000031	0.000000
22	ALL	0.000031	-0.000000
23	AMAT	0.000031	-0.011244
24	AMD	0.000031	-0.000000
25	AME	0.000031	-0.000000
26	AMGN	0.000031	-0.000000
27	AMP	0.000031	0.000000
28	AMT	0.000031	-0.000000
29	AMZN	0.000031	-0.000000
30	ANSS	0.000031	-0.000000
31	ANTM	0.000031	-0.000000
32	AON	0.000031	-0.000000
33	AOS	0.000031	0.000000
34	APA	0.000031	-0.000000
35	APD	0.000031	-0.009191
36	APH	0.000031	-0.000000
37	APTV	0.000031	0.000000
38	ARE	0.000031	0.000000
39	ATO	0.000031	0.000000
40	ATVI	0.000031	0.000000
41	AVB	0.000031	0.000000
42	AVGO	0.000031	-0.000000
43	AVY	0.000031	0.000000
44	AWK	0.000031	0.000000
45	AXP	0.000031	0.000000
46	AZO	0.000031	-0.000000
47	BA	0.000031	0.008285
48	BAC	0.000031	-0.000000
49	BALL	0.000031	0.019975
50	BAX	0.000031	0.000000
51	BBWI	0.000031	0.000000
52	BBY	0.000031	0.000000
53	BDX	0.000031	0.000000

54	BEN	0.000031	0.006517
55	BF/B	0.000031	-0.000000
56	BIO	0.000031	-0.000000
57	BK	0.000031	0.000000
58	BKNG	0.000031	0.012547
59	BKR	0.000031	0.006516
60	BLK	0.000031	-0.000000
61	BMV	0.000031	-0.000000
62	BR	0.000031	0.000000
63	BRK/B	0.000031	0.000000
64	BRO	0.000031	0.000000
65	BSX	0.000031	0.000000
66	BWA	0.000031	0.000000
67	BXP	0.000031	0.000000
68	C	0.000031	-0.000000
69	CAG	0.000031	0.000000
70	CAH	0.000031	-0.000000
71	CAT	0.000031	0.000000
72	CB	0.000031	0.000000
73	CBOE	0.000031	0.000000
74	CBRE	0.000031	-0.031235
75	CCI	0.000031	0.000000
76	CCL	0.000031	-0.000000
77	CDNS	0.000031	0.000000
78	CE	0.000031	-0.000000
79	CF	0.000031	-0.000000
80	CHD	0.000031	-0.000000
81	CHRW	0.000031	-0.000000
82	CHTR	0.000031	0.026566
83	CI	0.000031	0.000000
84	CINF	0.000031	0.011320
85	CL	0.000031	0.000000
86	CLX	0.000031	-0.000000
87	CMA	0.000031	0.000000
88	CMCSA	0.000031	-0.000000
89	CME	0.000031	-0.000000
90	CMG	0.000031	0.000000
91	CMI	0.000031	0.000000
92	CMS	0.000031	0.000000
93	CNC	0.000031	-0.000000
94	CNP	0.000031	0.000000
95	COF	0.000031	-0.000000
96	COO	0.000031	0.000000
97	COP	0.000031	-0.000000
98	COST	0.000031	0.000000
99	CPB	0.000031	-0.002255
100	CPRT	0.000031	0.000000
101	CPT	0.000031	0.000000

102	CRL	0.000031	0.000000
103	CRM	0.000031	0.000000
104	CSCO	0.000031	0.000000
105	CSX	0.000031	-0.000000
106	CTAS	0.000031	-0.000000
107	CTRA	0.000031	0.000000
108	CTSH	0.000031	-0.000000
109	CTXS	0.000031	-0.000000
110	CVS	0.000031	0.013270
111	CVX	0.000031	-0.000000
112	D	0.000031	-0.000000
113	DAL	0.000031	0.004996
114	DD	0.000031	0.000000
115	DE	0.000031	0.008352
116	DFS	0.000031	0.000000
117	DG	0.000031	-0.000000
118	DGX	0.000031	-0.000000
119	DHI	0.000031	0.000000
120	DHR	0.000031	-0.000000
121	DIS	0.000031	0.000000
122	DISH	0.000031	-0.000000
123	DLR	0.000031	0.000000
124	DLTR	0.000031	-0.021352
125	DOV	0.000031	0.000000
126	DPZ	0.000031	0.000000
127	DRE	0.000031	0.000000
128	DRI	0.000031	-0.000000
129	DTE	0.000031	-0.000000
130	DUK	0.000031	0.000000
131	DVA	0.000031	-0.000000
132	DVN	0.000031	0.000000
133	EA	0.000031	0.026020
134	EBAY	0.000031	-0.000000
135	ECL	0.000031	-0.000000
136	ED	0.000031	-0.000000
137	EFX	0.000031	-0.000000
138	EIX	0.000031	0.000000
139	EL	0.000031	-0.000000
140	EMN	0.000031	0.017889
141	EMR	0.000031	0.000000
142	ENPH	0.000031	0.014089
143	EOG	0.000031	-0.000000
144	EPAM	0.000031	-0.000000
145	EQIX	0.000031	-0.000000
146	EQR	0.000031	0.000000
147	ES	0.000031	-0.000000
148	ESS	0.000031	0.000000
149	ETN	0.000031	-0.000000

150	ETR	0.000031	0.000000
151	EVRG	0.000031	-0.000000
152	EW	0.000031	-0.000000
153	EXC	0.000031	-0.000000
154	EXPD	0.000031	0.000000
155	EXPE	0.000031	0.000000
156	EXR	0.000031	0.000000
157	F	0.000031	0.000000
158	FAST	0.000031	-0.012791
159	FBHS	0.000031	0.000000
160	FCX	0.000031	0.017994
161	FDS	0.000031	0.000000
162	FDX	0.000031	-0.000000
163	FE	0.000031	-0.000000
164	FFIV	0.000031	-0.000000
165	FIS	0.000031	-0.000000
166	FISV	0.000031	-0.000000
167	FITB	0.000031	0.011743
168	FLT	0.000031	-0.003762
169	FMC	0.000031	0.000000
170	FRC	0.000031	-0.000000
171	FRT	0.000031	-0.000000
172	FTNT	0.000031	-0.000000
173	GD	0.000031	0.000000
174	GE	0.000031	0.002576
175	GILD	0.000031	-0.000000
176	GIS	0.000031	0.000000
177	GL	0.000031	0.000000
178	GLW	0.000031	-0.000000
179	GM	0.000031	-0.000000
180	GNRC	0.000031	-0.000000
181	GOOGL	0.000031	-0.000000
182	GPC	0.000031	0.000000
183	GPN	0.000031	-0.000000
184	GRMN	0.000031	-0.000000
185	GS	0.000031	-0.023576
186	GWV	0.000031	-0.000000
187	HAL	0.000031	0.000000
188	HAS	0.000031	0.004908
189	HBAN	0.000031	-0.000000
190	HCA	0.000031	0.000000
191	HD	0.000031	0.000000
192	HES	0.000031	-0.021634
193	HIG	0.000031	-0.000000
194	HII	0.000031	0.010723
195	HOLX	0.000031	-0.000000
196	HON	0.000031	0.000000
197	HPQ	0.000031	0.000000

198	HRL	0.000031	-0.000000
199	HSIC	0.000031	0.000000
200	HST	0.000031	0.000000
201	HSY	0.000031	0.000000
202	HUM	0.000031	-0.000000
203	IBM	0.000031	-0.000000
204	ICE	0.000031	-0.000000
205	IDXX	0.000031	-0.000000
206	IEX	0.000031	0.000000
207	IFF	0.000031	-0.033776
208	ILMN	0.000031	0.000000
209	INCY	0.000031	-0.000000
210	INTC	0.000031	-0.009187
211	INTU	0.000031	-0.000000
212	IP	0.000031	-0.000000
213	IPG	0.000031	0.000000
214	IPGP	0.000031	-0.000000
215	IRM	0.000031	0.000000
216	ISRG	0.000031	0.000000
217	IT	0.000031	0.000000
218	ITW	0.000031	-0.000000
219	IVZ	0.000031	0.000000
220	J	0.000031	0.000000
221	JBHT	0.000031	0.000000
222	JCI	0.000031	-0.000000
223	JKHY	0.000031	-0.000000
224	JNJ	0.000031	0.000000
225	JNPR	0.000031	0.000000
226	JPM	0.000031	-0.000000
227	K	0.000031	0.000000
228	KEY	0.000031	-0.000000
229	KIM	0.000031	-0.000000
230	KLAC	0.000031	-0.000000
231	KMB	0.000031	-0.000000
232	KMI	0.000031	-0.000000
233	KMX	0.000031	-0.000000
234	KO	0.000031	0.000000
235	KR	0.000031	-0.000000
236	L	0.000031	0.000000
237	LDOS	0.000031	0.032328
238	LEN	0.000031	0.000000
239	LH	0.000031	0.000000
240	LHX	0.000031	0.003865
241	LIN	0.000031	-0.000000
242	LKQ	0.000031	0.000000
243	LLY	0.000031	0.000000
244	LMT	0.000031	0.000000
245	LNC	0.000031	-0.000000

246	LNT	0.000031	0.000000
247	LOW	0.000031	0.000000
248	LRCX	0.000031	-0.000000
249	LUMN	0.000031	-0.000000
250	LUV	0.000031	-0.000000
251	LVS	0.000031	-0.000000
252	LYB	0.000031	0.000000
253	LYV	0.000031	0.000000
254	MA	0.000031	0.000000
255	MAA	0.000031	0.000000
256	MAR	0.000031	-0.000000
257	MAS	0.000031	-0.000000
258	MCD	0.000031	0.000000
259	MCHP	0.000031	-0.000000
260	MCK	0.000031	-0.000000
261	MCO	0.000031	0.000000
262	MDLZ	0.000031	-0.000000
263	MDT	0.000031	0.000000
264	MET	0.000031	0.000000
265	META	0.000031	-0.000000
266	MGM	0.000031	0.000000
267	MHK	0.000031	0.000000
268	MKC	0.000031	-0.000000
269	MKTX	0.000031	0.000000
270	MLM	0.000031	0.000000
271	MMC	0.000031	0.000000
272	MMM	0.000031	0.000000
273	MNST	0.000031	0.000000
274	MO	0.000031	-0.000000
275	MOH	0.000031	-0.000000
276	MOS	0.000031	0.000000
277	MPC	0.000031	0.000000
278	MPWR	0.000031	0.012781
279	MRK	0.000031	0.000000
280	MRO	0.000031	-0.000000
281	MS	0.000031	-0.000000
282	MSCI	0.000031	-0.000000
283	MSFT	0.000031	-0.000000
284	MSI	0.000031	-0.023298
285	MTB	0.000031	0.000000
286	MTD	0.000031	-0.001826
287	MU	0.000031	0.000000
288	NDAQ	0.000031	-0.000000
289	NDSN	0.000031	0.000000
290	NEE	0.000031	0.000000
291	NEM	0.000031	0.232283
292	NFLX	0.000031	0.000000
293	NI	0.000031	0.000000

294	NKE	0.000031	0.000000
295	NLOK	0.000031	0.007445
296	NLSN	0.000031	-0.001235
297	NOC	0.000031	0.009299
298	NRG	0.000031	-0.000000
299	NSC	0.000031	-0.000000
300	NTAP	0.000031	-0.000000
301	NTRS	0.000031	0.000000
302	NUE	0.000031	0.005873
303	NVDA	0.000031	-0.000000
304	NVR	0.000031	0.000000
305	NWL	0.000031	0.000000
306	NXPI	0.000031	0.000000
307	O	0.000031	0.056246
308	ODFL	0.000031	0.000000
309	OKE	0.000031	-0.001976
310	OMC	0.000031	-0.000000
311	ORCL	0.000031	-0.000000
312	ORLY	0.000031	0.000000
313	OXY	0.000031	-0.000000
314	PARA	0.000031	-0.007138
315	PAYX	0.000031	-0.000000
316	PCAR	0.000031	0.000000
317	PEAK	0.000031	0.000000
318	PEG	0.000031	-0.000000
319	PENN	0.000031	0.000000
320	PEP	0.000031	-0.000000
321	PFE	0.000031	-0.000155
322	PFG	0.000031	0.000000
323	PG	0.000031	0.000000
324	PGR	0.000031	-0.000000
325	PH	0.000031	-0.000000
326	PHM	0.000031	0.024947
327	PKG	0.000031	-0.021095
328	PKI	0.000031	0.000000
329	PLD	0.000031	0.000000
330	PM	0.000031	0.000000
331	PNC	0.000031	-0.000000
332	PNR	0.000031	0.000000
333	PNW	0.000031	0.000000
334	POOL	0.000031	-0.000000
335	PPG	0.000031	-0.000000
336	PPL	0.000031	0.000000
337	PRU	0.000031	0.000000
338	PSA	0.000031	0.000000
339	PSX	0.000031	-0.000000
340	PTC	0.000031	0.000000
341	PVH	0.000031	-0.000000

342	PWR	0.000031	0.000000
343	PXD	0.000031	-0.007346
344	QCOM	0.000031	0.015558
345	RCL	0.000031	-0.000000
346	RE	0.000031	0.000000
347	REG	0.000031	0.000000
348	RF	0.000031	0.000000
349	RHI	0.000031	-0.030106
350	RJF	0.000031	-0.046686
351	RL	0.000031	-0.000000
352	RMD	0.000031	-0.000000
353	ROK	0.000031	-0.000000
354	ROL	0.000031	-0.000000
355	ROP	0.000031	-0.000000
356	ROST	0.000031	0.013511
357	RSG	0.000031	0.000000
358	RTX	0.000031	0.000000
359	SBAC	0.000031	0.000000
360	SBNY	0.000031	0.000000
361	SBUX	0.000031	-0.000000
362	SCHW	0.000031	-0.052780
363	SEE	0.000031	0.000000
364	SHW	0.000031	-0.022257
365	SIVB	0.000031	0.000000
366	SJM	0.000031	-0.000000
367	SLB	0.000031	0.031621
368	SNA	0.000031	0.000000
369	SNPS	0.000031	-0.000000
370	SO	0.000031	0.000000
371	SPG	0.000031	0.001635
372	SPGI	0.000031	0.000000
373	SRE	0.000031	-0.000000
374	STE	0.000031	-0.000000
375	STT	0.000031	0.000000
376	STX	0.000031	-0.000000
377	STZ	0.000031	0.000000
378	SWK	0.000031	0.000000
379	SWKS	0.000031	0.000000
380	SYK	0.000031	0.000000
381	SYY	0.000031	0.005762
382	T	0.000031	-0.055258
383	TAP	0.000031	-0.000000
384	TDG	0.000031	0.000000
385	TDY	0.000031	0.000000
386	TECH	0.000031	-0.000000
387	TEL	0.000031	0.005970
388	TER	0.000031	0.000000
389	TFC	0.000031	0.000000

390	TFX	0.000031	0.000000
391	TGT	0.000031	-0.000000
392	TJX	0.000031	0.000000
393	TMO	0.000031	-0.000000
394	TMUS	0.000031	-0.000000
395	TPR	0.000031	-0.000000
396	TRMB	0.000031	-0.001159
397	TROW	0.000031	-0.000000
398	TRV	0.000031	-0.000000
399	TSCO	0.000031	-0.000000
400	TSLA	0.000031	-0.001448
401	TSN	0.000031	0.013306
402	TT	0.000031	-0.000000
403	TTWO	0.000031	0.011522
404	TXN	0.000031	-0.000000
405	TXT	0.000031	0.000000
406	TYL	0.000031	0.006167
407	UAA	0.000031	-0.000000
408	UAL	0.000031	0.000000
409	UDR	0.000031	0.000000
410	UHS	0.000031	0.012173
411	ULTA	0.000031	-0.010288
412	UNH	0.000031	0.000000
413	UNP	0.000031	-0.000000
414	UPS	0.000031	0.000000
415	URI	0.000031	0.000000
416	USB	0.000031	0.000000
417	V	0.000031	-0.000000
418	VFC	0.000031	-0.000000
419	VLO	0.000031	-0.000000
420	VMC	0.000031	0.000000
421	VNO	0.000031	0.000000
422	VRSK	0.000031	0.000000
423	VRSN	0.000031	0.000000
424	VTR	0.000031	0.000000
425	VTRS	0.000031	0.000000
426	VZ	0.000031	-0.000000
427	WAB	0.000031	0.000000
428	WAT	0.000031	0.000000
429	WBA	0.000031	0.000000
430	WDC	0.000031	0.000000
431	WEC	0.000031	0.000000
432	WELL	0.000031	0.000760
433	WFC	0.000031	0.000000
434	WHR	0.000031	0.000000
435	WM	0.000031	0.000000
436	WMB	0.000031	-0.000167
437	WMT	0.000031	0.000000

```

438      WRB    0.000031  0.000000
439      WST    0.000031 -0.000000
440      WTW    0.000031 -0.000000
441      WY     0.000031  0.000000
442      WYNN   0.000031 -0.012059
443      XEL    0.000031  0.000000
444      XOM    0.000031 -0.008749
445      XRAY   0.000031 -0.003772
446      XYL    0.000031  0.000000
447      YUM    0.000031 -0.000000
448      ZBH    0.000031 -0.000000
449      ZBRA   0.000031 -0.000000
450      ZION   0.000031  0.000000

```

b) R-squared

```
[15]: model_lasso.score(X, y)
```

```
[15]: 0.7541466465557551
```

c) Most influential factors

```
[16]: results_lasso = compute_beta_sigma(rets_spx, results_lasso)
print("***** Top 10 largest betas *****")
display(results_lasso.sort_values('abs_beta', ascending=False).
    head(10)[['regressor', 'beta']].\
    reset_index(drop=True))
print("***** Top 10 largest beta-sigmas *****")
display(results_lasso.sort_values('abs_beta_sigma', ascending=False).
    head(10)[['regressor', 'beta_sigma']].\
    reset_index(drop=True))
```

```
***** Top 10 largest betas *****
```

	regressor	beta
0	NEM	0.232283
1	O	0.056246
2	T	-0.055258
3	SCHW	-0.052780
4	RJF	-0.046686
5	IFF	-0.033776
6	LDOS	0.032328
7	SLB	0.031621
8	CBRE	-0.031235
9	RHI	-0.030106

```
***** Top 10 largest beta-sigmas *****
```

	regressor	beta_sigma
0	NEM	0.011329

1	SCHW	-0.002511
2	O	0.002376
3	RJF	-0.002057
4	SLB	0.001808
5	ENPH	0.001698
6	T	-0.001688
7	CBRE	-0.001451
8	PHM	0.001391
9	FCX	0.001381

d) Number of regressors with non-zero beta

```
[17]: non_zero_regressors = results_lasso.loc[results_lasso.beta != 0]
      print(f"Number of non-zero regressors: {len(non_zero_regressors)}")
```

Number of non-zero regressors: 75

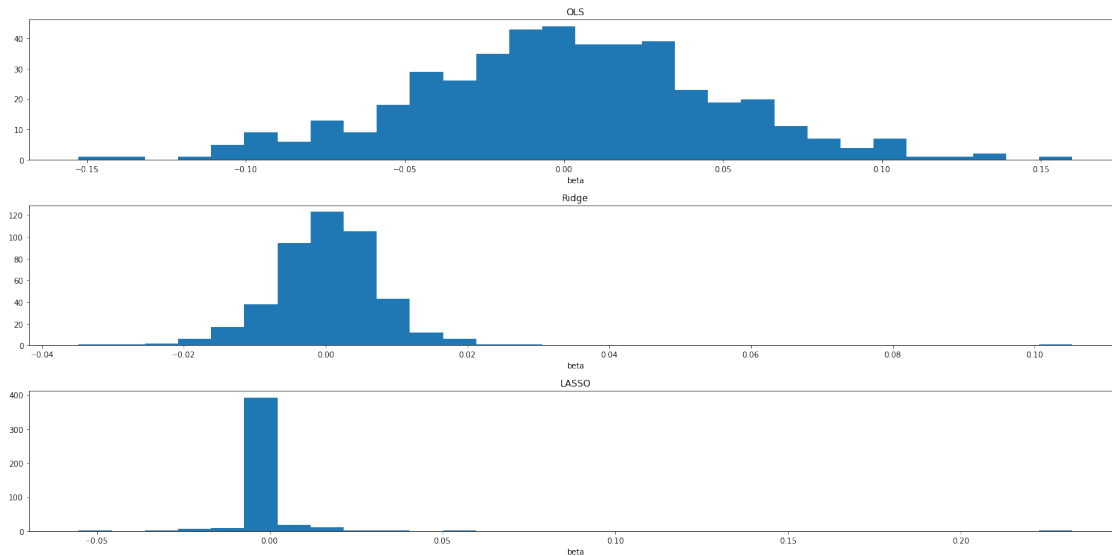
1.2 4. Comparaison

```
[18]: # histogram
fig, axis = plt.subplots(3, 1, figsize=(20, 10))
axis[0].hist(results_ols.beta, bins=30)
axis[0].set_xlabel('beta')
axis[0].set_title('OLS')

axis[1].hist(results_ridge.beta, bins=30)
axis[1].set_xlabel('beta')
axis[1].set_title('Ridge')

axis[2].hist(results_lasso.beta, bins=30)
axis[2].set_xlabel('beta')
axis[2].set_title('LASSO')

fig.tight_layout()
```

- a) OLS and Ridge have nonzero betas. LASSO as we saw previously has 75 nonzero regressors. All three have both positive and negative values. However, the range in magnitude is larger for OLS and LASSO than it is for Ridge.
- b) OLS has the largest R-squared. This is not a surprise because OLS isn't penalized for using all the regressors to perfectly fit the data.

1.3 5. Cross Validation

```
[19]: # split data into train and test
X_train, y_train = \
    rets_spx.loc[rets_spx.date.dt.year < 2021].drop(columns='date'), rets_mkt.
    ↪loc[rets_mkt.date.dt.year < 2021].drop(columns='date').GLD
X_test, y_test = \
    rets_spx.loc[rets_spx.date.dt.year >= 2021].drop(columns='date'), rets_mkt.
    ↪loc[rets_mkt.date.dt.year >= 2021].drop(columns='date').GLD
```

1.3.1 LassoCV

```
[20]: model_lasso_cv = LassoCV().fit(X_train, y_train)
model_lasso_cv.alpha_
```

```
[20]: 7.887365550464517e-05
```

1.3.2 RidgeCV

```
[21]: model_ridge_cv = RidgeCV().fit(X_train, y_train)
      model_ridge_cv.alpha_
```

```
[21]: 0.1
```

2 6. OOS predictions

```
[22]: score_oos_lasso = model_lasso_cv.score(X_test, y_test)
      score_oos_ridge = model_ridge_cv.score(X_test, y_test)
      print(f"Lasso OOS R2: {score_oos_lasso:.2f}\nRidge OOS R2: {score_oos_ridge:.2f}")
```

```
Lasso OOS R2: 0.51
```

```
Ridge OOS R2: 0.32
```