Homework #3.2

In [ ]:
```
%matplotlib inline
import warnings
warnings.filterwarnings('ignore')
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import statsmodels.api as sm
from statsmodels.stats.diagnostic import het_white
from arch import arch_model

pd.set_option('display.max_rows', 500)
```

In [ ]:
```
# read data
macro = pd.read_excel('data/hw_3_2_data.xlsx', sheet_name='macro')
sp500 = pd.read_excel('data/hw_3_2_data.xlsx', sheet_name="s&p500")
```

1. Assessing the OLS Model

1. OLS Regression CPI and Money

In [ ]:
```
X = macro.M2
y = macro.CPI
model = sm.OLS(y, sm.add_constant(X))
results = model.fit()
results.summary()
```

Out[ ]:

OLS Regression Results

Dep. Variable:	CPI	**R-squared:**	0.827
Model:	OLS	**Adj. R-squared:**	0.827
Method:	Least Squares	**F-statistic:**	3618.
Date:	Sun, 26 Jun 2022	**Prob (F-statistic):**	8.42e-291
Time:	23:02:24	**Log-Likelihood:**	-3721.7
No. Observations:	760	**AIC:**	7447.
Df Residuals:	758	**BIC:**	7457.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	62.7397	1.645	38.145	0.000	59.511	65.969
M2	0.0145	0.000	60.151	0.000	0.014	0.015

Omnibus:	42.247	**Durbin-Watson:**	0.001
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	36.033
Skew:	-0.458	**Prob(JB):**	1.50e-08
Kurtosis:	2.454	**Cond. No.**	9.54e+03

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 9.54e+03. This might indicate that there are
strong multicollinearity or other numerical problems.

a) $R^2 = 0.827$
b) $\beta = 0.0145$

2- Regression of growth rates

In [ ]:

```
inflation = macro.set_index('date')[['CPI', 'M2']].pct_change(12, freq='M').res
X = inflation.M2
y = inflation.CPI
model = sm.OLS(y, sm.add_constant(X)).fit()
model.summary()
```

Out[ ]:

OLS Regression Results

Dep. Variable:	CPI	**R-squared:**	0.008
Model:	OLS	**Adj. R-squared:**	0.007
Method:	Least Squares	**F-statistic:**	6.370
Date:	Sun, 26 Jun 2022	**Prob (F-statistic):**	0.0118
Time:	23:02:24	**Log-Likelihood:**	1610.0
No. Observations:	748	**AIC:**	-3216.
Df Residuals:	746	**BIC:**	-3207.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	0.0320	0.002	13.839	0.000	0.027	0.037
M2	0.0729	0.029	2.524	0.012	0.016	0.130

Omnibus:	211.421	**Durbin-Watson:**	0.020
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	484.782
Skew:	1.517	**Prob(JB):**	5.38e-106
Kurtosis:	5.519	**Cond. No.**	28.2

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

a) $R^2 = 0.008$
b) $\beta = 0.0729$

3- Discussion

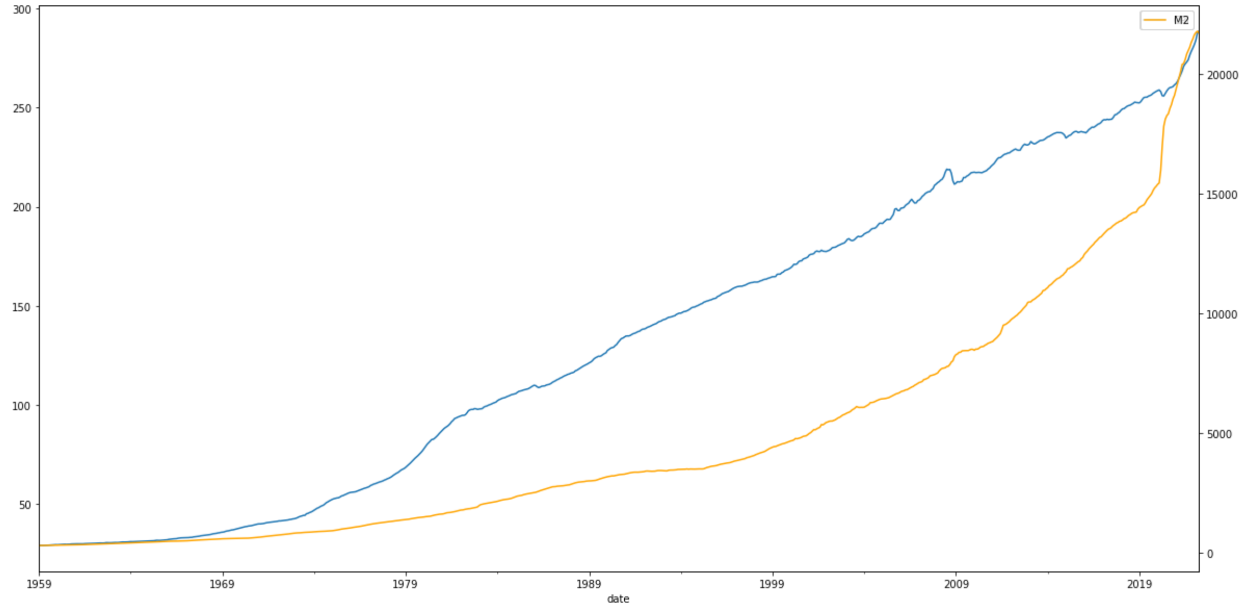
The correlation between CPI and Money seems to be stronger than the correlation between inflation and money growth.
However, CPI is non-stationary and will tend to have a strong correlation with anything that has an upward trend. Which in this case is Money.
Hence, the correlation observed here could be spurious. Regressing CPI on Money doesn't make sense.

In [ ]:

```
fig, ax1 = plt.subplots()
macro.set_index('date')[['CPI']].plot(figsize=(20, 10), ax=ax1)
ax2 = ax1.twinx()
macro.set_index('date')[['M2']].plot(figsize=(20, 10), ax=ax2, c='orange')
```

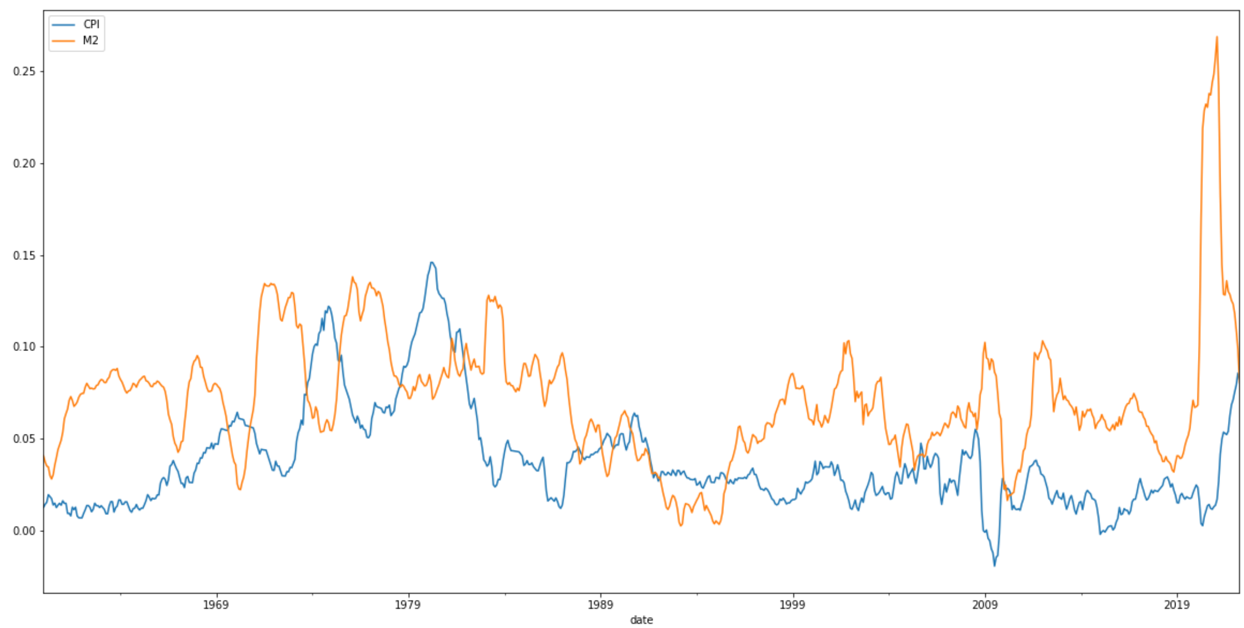
Out[ ]:

```
<AxesSubplot:xlabel='date'>
```



In [ ]:
```
inflation.set_index('date')[['CPI', 'M2']].plot(figsize=(20, 10))
```

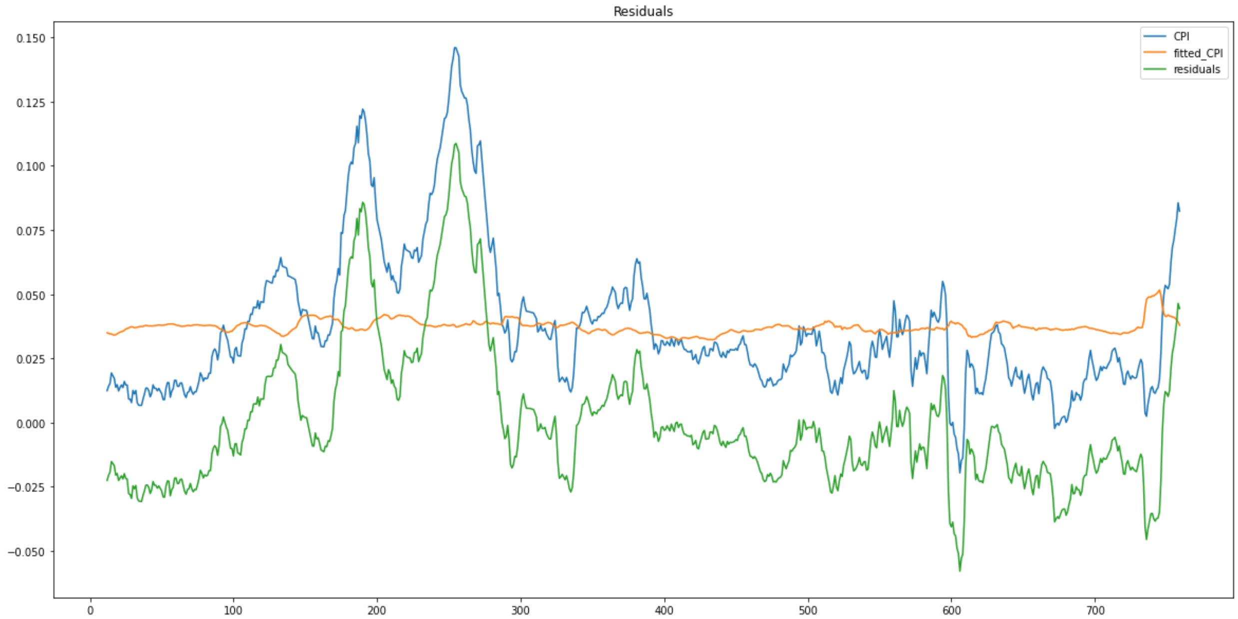
Out[ ]: <AxesSubplot:xlabel='date'>



4.

In [ ]:
```
predicted = pd.concat([y, pd.DataFrame(data=model.predict(sm.add_constant(X)),
predicted['residuals'] = predicted['CPI'] - predicted['fitted_CPI']
predicted.plot(title='Residuals', figsize=(20, 10))
```

Out[ ]: <AxesSubplot:title={'center':'Residuals'}>



a) The residuals don't seem to be drawn from a constant variance.

b) They do seem to have some serial correlation.

5.

a) Looking at the reported p-values from 2. α and β estimates seem to be statistically significant.

b) Yes, given that the homoscedasticity assumption doesn't hold the standard error is misspecified.

c) The estimated β would be more thrustworthy for larger sample sizes.

6.

a) From 2. the Durbin-Watson value is 0.020 which indicates presence of serial correlation.

b) Using White test (see below), we have an extremely small $p - value = 9.75 * 10^{-5}$ which means that the errors aren't homoscedastic.

In [ ]:
```
white_test = het_white(model.resid,  model.model.exog)
labels = ['LM Statistic', 'LM-Test p-value', 'F-Statistic', 'F-Test p-value']
print(dict(zip(labels, white_test)))
```

```
{'LM Statistic': 18.469958814741208, 'LM-Test p-value': 9.756620279329117e-05,
'F-Statistic': 9.43081061790568, 'F-Test p-value': 9.020683988503916e-05}
```

2. Forecasting via Regression

In [ ]:
```
# build dataset
def build_dataset(h, macro):
    inflation = macro.set_index('date')[['CPI', 'M2']].pct_change(h, freq='M').
    inflation['y'] = inflation.set_index('date')['CPI'].shift(h, freq='M').rese
```

```
    return inflation.dropna()

def fit_model(y, X):
    return sm.OLS(y, sm.add_constant(X)).fit()
```

1. Forcast using lagged infation

In [ ]:

```
h_values = [1, 12, 24, 36]
for h in h_values:
    inflation_lagged = build_dataset(h, macro)
    results = fit_model(inflation_lagged['y'], inflation_lagged['CPI'])
    print(f'\n\n\t######################## Results for h={h} #################
```

```
        ######################## Results for h=1 ########################
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.399
Model:                            OLS   Adj. R-squared:                  0.398
Method:                 Least Squares   F-statistic:                     501.8
Date:                Sun, 26 Jun 2022   Prob (F-statistic):           1.22e-85
Time:                        23:02:27   Log-Likelihood:                 3482.7
No. Observations:                 758   AIC:                            -6961.
Df Residuals:                     756   BIC:                            -6952.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0011      0.000      9.115      0.000       0.001       0.001
CPI            0.6312      0.028     22.401      0.000       0.576       0.686
==============================================================================
Omnibus:                       92.917   Durbin-Watson:                   2.143
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              798.226
Skew:                           0.113   Prob(JB):                   4.65e-174
Kurtosis:                       8.022   Cond. No.                         317.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correct
ly specified.


        ######################## Results for h=12 ########################
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.555
Model:                            OLS   Adj. R-squared:                  0.555
Method:                 Least Squares   F-statistic:                     917.1
Date:                Sun, 26 Jun 2022   Prob (F-statistic):          2.40e-131
Time:                        23:02:27   Log-Likelihood:                 1877.3
No. Observations:                 736   AIC:                            -3751.
Df Residuals:                     734   BIC:                            -3741.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0101      0.001      8.782      0.000       0.008       0.012
CPI            0.7488      0.025     30.284      0.000       0.700       0.797
==============================================================================
Omnibus:                       79.392   Durbin-Watson:                   0.089
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              159.073
Skew:                           0.650   Prob(JB):                     2.87e-35
Kurtosis:                       4.870   Cond. No.                         35.5
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correct
ly specified.


        ######################## Results for h=24 ########################
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.405
Model:                            OLS   Adj. R-squared:                  0.404
Method:                 Least Squares   F-statistic:                     483.2
Date:                Sun, 26 Jun 2022   Prob (F-statistic):           4.27e-82
Time:                        23:02:27   Log-Likelihood:                 1228.1
No. Observations:                 712   AIC:                            -2452.
Df Residuals:                     710   BIC:                            -2443.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0295      0.003     10.783      0.000       0.024       0.035
CPI            0.6309      0.029     21.983      0.000       0.575       0.687
==============================================================================
Omnibus:                      188.541   Durbin-Watson:                   0.021
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              464.720
Skew:                           1.371   Prob(JB):                    1.22e-101
Kurtosis:                       5.854   Cond. No.                         17.8
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correct
ly specified.


        ######################## Results for h=36 ########################
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.366
Model:                            OLS   Adj. R-squared:                  0.365
Method:                 Least Squares   F-statistic:                     395.3
Date:                Sun, 26 Jun 2022   Prob (F-statistic):           8.36e-70
Time:                        23:02:27   Log-Likelihood:                 886.38
No. Observations:                 688   AIC:                            -1769.
Df Residuals:                     686   BIC:                            -1760.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0507      0.004     11.476      0.000       0.042       0.059
CPI            0.5952      0.030     19.881      0.000       0.536       0.654
==============================================================================
Omnibus:                      135.194   Durbin-Watson:                   0.009
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              269.015
Skew:                           1.109   Prob(JB):                     3.84e-59
Kurtosis:                       5.112   Cond. No.                         11.9
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correct
ly specified.
```

2. Forecast using lagged money growth

In [ ]:

```
h_values = [1, 12, 24, 36]
```

```
for h in h_values:
    inflation_lagged = build_dataset(h, macro)
    results = fit_model(inflation_lagged['y'], inflation_lagged['M2'])
    print(f'\n\n\t######################## Results for h={h} #################
```

```
######################## Results for h=1 ########################
                            OLS Regression Results                            
==============================================================================
Dep. Variable:                      y   R-squared:                       0.000
Model:                            OLS   Adj. R-squared:                 -0.001
Method:                 Least Squares   F-statistic:                 3.930e-07
Date:                Sun, 26 Jun 2022   Prob (F-statistic):              0.999
Time:                        23:02:28   Log-Likelihood:                 3289.8
No. Observations:                 758   AIC:                            -6576.
Df Residuals:                     756   BIC:                            -6566.
Df Model:                           1                                         
Covariance Type:            nonrobust                                         
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0030      0.000     16.365      0.000       0.003       0.003
M2         -1.596e-05      0.025     -0.001      0.999      -0.050       0.050
==============================================================================
Omnibus:                       89.567   Durbin-Watson:                   0.737
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              579.300
Skew:                           0.280   Prob(JB):                   1.61e-126
Kurtosis:                       7.246   Cond. No.                         222.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correct
ly specified.


######################## Results for h=12 ########################
                            OLS Regression Results                            
==============================================================================
Dep. Variable:                      y   R-squared:                       0.087
Model:                            OLS   Adj. R-squared:                  0.086
Method:                 Least Squares   F-statistic:                     70.36
Date:                Sun, 26 Jun 2022   Prob (F-statistic):           2.52e-16
Time:                        23:02:28   Log-Likelihood:                 1612.6
No. Observations:                 736   AIC:                            -3221.
Df Residuals:                     734   BIC:                            -3212.
Df Model:                           1                                         
Covariance Type:            nonrobust                                         
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0208      0.002      9.274      0.000       0.016       0.025
M2             0.2373      0.028      8.388      0.000       0.182       0.293
==============================================================================
Omnibus:                      219.388   Durbin-Watson:                   0.021
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              543.560
Skew:                           1.555   Prob(JB):                   9.28e-119
Kurtosis:                       5.837   Cond. No.                         28.5
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correct
ly specified.


######################## Results for h=24 ########################
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.267
Model:                            OLS   Adj. R-squared:                  0.266
Method:                 Least Squares   F-statistic:                     258.9
Date:                Sun, 26 Jun 2022   Prob (F-statistic):           6.65e-50
Time:                        23:02:28   Log-Likelihood:                 1154.0
No. Observations:                 712   AIC:                            -2304.
Df Residuals:                     710   BIC:                            -2295.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0002      0.005     -0.037      0.970      -0.010       0.010
M2             0.5521      0.034     16.092      0.000       0.485       0.619
==============================================================================
Omnibus:                      130.247   Durbin-Watson:                   0.011
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              232.144
Skew:                           1.095   Prob(JB):                     3.89e-51
Kurtosis:                       4.740   Cond. No.                         19.5
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correct
ly specified.


        ######################## Results for h=36 ########################
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.398
Model:                            OLS   Adj. R-squared:                  0.397
Method:                 Least Squares   F-statistic:                     453.3
Date:                Sun, 26 Jun 2022   Prob (F-statistic):           1.31e-77
Time:                        23:02:28   Log-Likelihood:                 904.36
No. Observations:                 688   AIC:                            -1805.
Df Residuals:                     686   BIC:                            -1796.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0316      0.008     -4.133      0.000      -0.047      -0.017
M2             0.6930      0.033     21.290      0.000       0.629       0.757
==============================================================================
Omnibus:                       26.466   Durbin-Watson:                   0.007
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               26.889
Skew:                           0.452   Prob(JB):                     1.45e-06
Kurtosis:                       2.654   Cond. No.                         13.8
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correct
ly specified.
```

3. Forecast using lagged inflation and money growth

In [ ]:
```
h_values = [1, 12, 24, 36]
```

```
for h in h_values:
    inflation_lagged = build_dataset(h, macro)
    results = fit_model(inflation_lagged['y'], inflation_lagged[['M2', 'CPI']])
    print(f'\n\n\t######################## Results for h={h} ##################
```

```
                        ######################## Results for h=1 ########################
                            OLS Regression Results                            
==============================================================================
Dep. Variable:                      y   R-squared:                       0.401
Model:                            OLS   Adj. R-squared:                  0.400
Method:                 Least Squares   F-statistic:                     253.2
Date:                Sun, 26 Jun 2022   Prob (F-statistic):           7.09e-85
Time:                        23:02:28   Log-Likelihood:                 3484.3
No. Observations:                 758   AIC:                            -6963.
Df Residuals:                     755   BIC:                            -6949.
Df Model:                           2                                         
Covariance Type:            nonrobust                                         
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0009      0.000      5.287      0.000       0.001       0.001
M2             0.0352      0.020      1.781      0.075      -0.004       0.074
CPI            0.6351      0.028     22.504      0.000       0.580       0.691
==============================================================================
Omnibus:                       95.393   Durbin-Watson:                   2.163
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              877.513
Skew:                           0.075   Prob(JB):                    2.82e-191
Kurtosis:                       8.269   Cond. No.                         319.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correct
ly specified.


                        ######################## Results for h=12 ########################
                            OLS Regression Results                            
==============================================================================
Dep. Variable:                      y   R-squared:                       0.614
Model:                            OLS   Adj. R-squared:                  0.612
Method:                 Least Squares   F-statistic:                     581.8
Date:                Sun, 26 Jun 2022   Prob (F-statistic):          4.81e-152
Time:                        23:02:28   Log-Likelihood:                 1928.8
No. Observations:                 736   AIC:                            -3852.
Df Residuals:                     733   BIC:                            -3838.
Df Model:                           2                                         
Covariance Type:            nonrobust                                         
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0030      0.002     -1.837      0.067      -0.006       0.000
M2             0.1939      0.018     10.494      0.000       0.158       0.230
CPI            0.7307      0.023     31.586      0.000       0.685       0.776
==============================================================================
Omnibus:                       45.483   Durbin-Watson:                   0.095
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              115.221
Skew:                           0.308   Prob(JB):                     9.55e-26
Kurtosis:                       4.838   Cond. No.                         35.9
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correct
ly specified.
```

```
            ######################## Results for h=24 ########################
                            OLS Regression Results                            
==============================================================================
Dep. Variable:                      y   R-squared:                       0.523
Model:                            OLS   Adj. R-squared:                  0.522
Method:                 Least Squares   F-statistic:                     388.6
Date:                Sun, 26 Jun 2022   Prob (F-statistic):          1.15e-114
Time:                        23:02:28   Log-Likelihood:                 1306.7
No. Observations:                 712   AIC:                            -2607.
Df Residuals:                     709   BIC:                            -2594.
Df Model:                           2                                         
Covariance Type:            nonrobust                                         
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0168      0.004     -3.927      0.000      -0.025      -0.008
M2             0.3841      0.029     13.239      0.000       0.327       0.441
CPI            0.5250      0.027     19.493      0.000       0.472       0.578
==============================================================================
Omnibus:                       79.778   Durbin-Watson:                   0.022
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              140.498
Skew:                           0.715   Prob(JB):                     3.10e-31
Kurtosis:                       4.640   Cond. No.                         22.4
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correct
ly specified.


            ######################## Results for h=36 ########################
                            OLS Regression Results                            
==============================================================================
Dep. Variable:                      y   R-squared:                       0.540
Model:                            OLS   Adj. R-squared:                  0.539
Method:                 Least Squares   F-statistic:                     402.2
Date:                Sun, 26 Jun 2022   Prob (F-statistic):          2.92e-116
Time:                        23:02:28   Log-Likelihood:                 997.05
No. Observations:                 688   AIC:                            -1988.
Df Residuals:                     685   BIC:                            -1975.
Df Model:                           2                                         
Covariance Type:            nonrobust                                         
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0389      0.007     -5.791      0.000      -0.052      -0.026
M2             0.5043      0.031     16.123      0.000       0.443       0.566
CPI            0.4079      0.028     14.555      0.000       0.353       0.463
==============================================================================
Omnibus:                       11.287   Durbin-Watson:                   0.010
Prob(Omnibus):                  0.004   Jarque-Bera (JB):               13.418
Skew:                           0.214   Prob(JB):                      0.00122
Kurtosis:                       3.534   Cond. No.                         17.0
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correct
ly specified.
```

4.

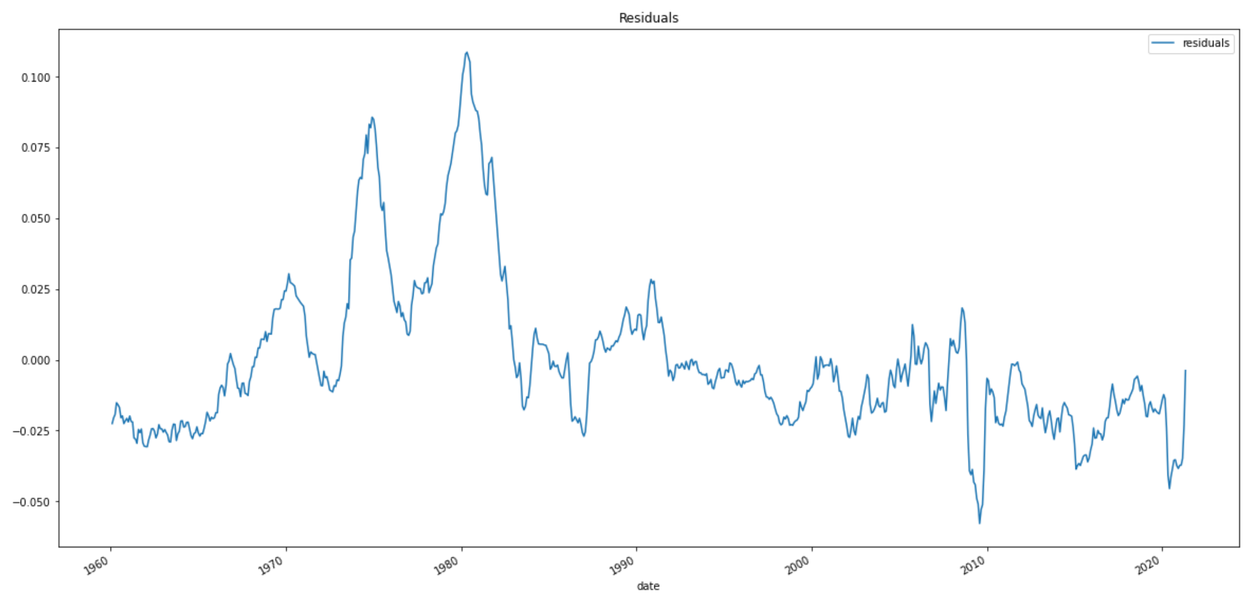
The regression with the combined regressors with $h = 12$ gives the best $R^2 = 0.612$
Combining the regressors seems to improve the forecast

5.

a) Plot residuals for $h = 12$

In [ ]:
```
h = 12
inflation_lagged = build_dataset(h, macro)
results = fit_model(inflation_lagged['y'], inflation_lagged[['M2', 'CPI']])
residuals = pd.concat([inflation_lagged['date'], pd.DataFrame(model.resid, colu
residuals.set_index('date').plot(title='Residuals', figsize=(20, 10))
```

Out[ ]:
```
<AxesSubplot:title={'center':'Residuals'}, xlabel='date'>
```



b) Yes, there seems to have some amount of serial correlation. This is also confirmed by the value of the Durbin-Watson test which is 0.095.
c) The betas as well because with serial correlation the assumption that ϵ and the regressor are uncorrelated is broken.

3. Time-Series Model of Volatility

In [ ]:
```
r = sp500
volatility_df = pd.DataFrame(r.Date, columns=['Date']).iloc[61:].set_index('Dat
```

Expanding series

In [ ]:
```
column_name = 'expanding_window'
def compute_var(values):
    return (values**2).sum() / len(values)
expanding_series_df = r.copy().set_index('Date').expanding(min_periods=61).appl
```

```
volatility_df[column_name] = np.sqrt(expanding_series_df.SPY.values)

vol = volatility_df[:'2008-10-31'].iloc[-1][column_name]; print(f"Volatility or
vol = volatility_df[:'2020-04-30'].iloc[-1][column_name]; print(f"Volatility or
vol = volatility_df[:'2022-05-31'].iloc[-1][column_name]; print(f"Volatility or
```

```
Volatility on October 2008:     0.040591
Volatility on April 2020:       0.042309
Volatility on May 2022:         0.043220
```

Rolling Window

In [ ]:

```
column_name = 'rolling_window'
def compute_var(values):
    return (values**2).sum() / len(values)
roling_window_df = r.set_index('Date').rolling(window=61).apply(compute_var).sh
volatility_df[column_name] = np.sqrt(roling_window_df.SPY.values)

vol = volatility_df[:'2008-10-31'].iloc[-1][column_name]; print(f"Volatility or
vol = volatility_df[:'2020-04-30'].iloc[-1][column_name]; print(f"Volatility or
vol = volatility_df[:'2022-05-31'].iloc[-1][column_name]; print(f"Volatility or
```

```
Volatility on October 2008:     0.029947
Volatility on April 2020:       0.039193
Volatility on May 2022:         0.047886
```

IGARCH

In [ ]:

```
column_name = 'IGARCH'
theta = 0.97
sigma = [0.15/np.sqrt(12)]
for i in range(1, len(r)+1):
    sigma += [np.sqrt(theta*sigma[i-1]**2 + (1-theta)*r.iloc[i-1].SPY**2)]

volatility_df[column_name] = sigma[62:]

vol = volatility_df[:'2008-10-31'].iloc[-1][column_name]; print(f"Volatility or
vol = volatility_df[:'2020-04-30'].iloc[-1][column_name]; print(f"Volatility or
vol = volatility_df[:'2022-05-31'].iloc[-1][column_name]; print(f"Volatility or
```

```
Volatility on October 2008:     0.046590
Volatility on April 2020:       0.048550
Volatility on May 2022:         0.047759
```

GARCH(1,1)

In [ ]:

```
column_name = 'GARCH'
am = arch_model(r.SPY)
forecasts = []
for i in range(61, len(r)):
    res = am.fit(first_obs=0, last_obs=i+1, disp="off")
    temp = res.forecast(horizon=1, reindex=False).variance
    fcast = temp.iloc[0]
    forecasts += [fcast.values[0]]

volatility_df[column_name] = np.sqrt(forecasts)

vol = volatility_df[:'2008-10-31'].iloc[-1][column_name]; print(f"Volatility or
vol = volatility_df[:'2020-04-30'].iloc[-1][column_name]; print(f"Volatility or
vol = volatility_df[:'2022-05-31'].iloc[-1][column_name]; print(f"Volatility or
```

```
Volatility on October 2008:     0.091378
Volatility on April 2020:       0.084973
Volatility on May 2022:         0.051495
```

In [ ]:
```python
volatility_df.plot(title='Volatility', figsize=(20, 10))
```

Out[ ]:
```
<AxesSubplot:title={'center':'Volatility'}, xlabel='Date'>
```