Exam #3

In [ ]:
```
%matplotlib inline
import warnings
warnings.filterwarnings('ignore')
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import statsmodels.api as sm
from arch import arch_model
from statsmodels.stats.diagnostic import het_white
from sklearn.linear_model import LinearRegression, Lasso, Ridge, RidgeCV, Lassc

pd.set_option('display.max_rows', 500)
pd.set_option('display.max_columns', 500)
```

In [ ]:
```
# read data
returns = pd.read_excel('data/exam_3_data.xlsx', sheet_name='returns')
```

1. OLS

1. IYR regression on IEF

In [ ]:
```
y = returns.IYR
X = returns.IEF
```

a)

In [ ]:
```
model_1 = sm.OLS(y, sm.add_constant(X)).fit()
model_1.summary()
```

Out[ ]: OLS Regression Results

Dep. Variable:	IYR	**R-squared:**	0.003
Model:	OLS	**Adj. R-squared:**	-0.003
Method:	Least Squares	**F-statistic:**	0.5241
Date:	Thu, 07 Jul 2022	**Prob (F-statistic):**	0.470
Time:	22:35:58	**Log-Likelihood:**	241.52
No. Observations:	158	**AIC:**	-479.0
Df Residuals:	156	**BIC:**	-472.9
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	0.0134	0.004	3.156	0.002	0.005	0.022
IEF	-0.1784	0.246	-0.724	0.470	-0.665	0.308

Omnibus:	34.118	**Durbin-Watson:**	2.015
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	170.518
Skew:	0.606	**Prob(JB):**	9.39e-38
Kurtosis:	7.943	**Cond. No.**	58.7

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

$\beta^{IEF} = -0.1784\backslash\ R^2 = 0.003$

b) An R^2 of 0.003 means that only 0.3\% of IYR is explained by IEF. This is very low and negligeable.\ Also, for an univariate regression the correlation can be direclty derived from the R^2 by taking the square root.\ Hence, we have $corr(IEF, IYR) = \sqrt{0.003} = 0.055.$ We can conclude that real estate returns do not seem to be sensitive to bond returns.

2. IYR regression on IEF and SPY

In [ ]:
```
X = returns[['IEF', 'SPY']]
model_2 = sm.OLS(y, sm.add_constant(X)).fit()
model_2.summary()
```

Out[ ]: OLS Regression Results

Dep. Variable:	IYR	**R-squared:**	0.569
Model:	OLS	**Adj. R-squared:**	0.564
Method:	Least Squares	**F-statistic:**	102.4
Date:	Thu, 07 Jul 2022	**Prob (F-statistic):**	4.46e-29
Time:	22:35:58	**Log-Likelihood:**	307.80
No. Observations:	158	**AIC:**	-609.6
Df Residuals:	155	**BIC:**	-600.4
Df Model:	2		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	-0.0017	0.003	-0.583	0.561	-0.008	0.004
IEF	0.6307	0.172	3.664	0.000	0.291	0.971
SPY	1.0281	0.072	14.271	0.000	0.886	1.170

Omnibus:	72.394	**Durbin-Watson:**	1.897
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	515.901
Skew:	1.467	**Prob(JB):**	9.41e-113
Kurtosis:	11.352	**Cond. No.**	62.8

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

$\beta^{IEF} = 0.6307\backslash\ R^2 = 0.569$

3.a Correlation matrix of IYR, IEF, SPY

In [ ]:
```
returns[['IYR', 'IEF', 'SPY']].corr()
```

Out[ ]:

	IYR	IEF	SPY
IYR	1.000000	-0.057867	0.729377
IEF	-0.057867	1.000000	-0.329349
SPY	0.729377	-0.329349	1.000000

3.b Discussion

The value of β^{IEF} in 1. is smaller than the one in 2.\ Also, from the correlation matrix we can see that $corr(IYR, SPY)$ is much bigger than $corr(IYR, IEF)$. This means that most of the explanation of IYR is carried by SPY.\ Given that IEF has a slight negative correlation with SPY, β^{IEF} in 2. is greater than β^{IEF} in 1. to balance out the high β^{SPY}

4.

Two assumptions on which the classical t-stats depend are:

1. There's no multicollinearity among the regressors.
2. Errors are independant and normally distributed.

Assumpion 1 is reasonable in this case. A correlation of -0.329 between IEF and SPY is fairly small. Also, the condition number of 62.8 is ok.\ This means that the $X^T X$ matrix of regressors should be safely invertible.

For assumption 2, OLS asymptotically converges towards a normal distribution via the central limit theorem. In the eventuality that there's presence of serial correlation,\ we can safely assume that the law of large numbers will get us out of trouble.

2. Forecasting

1 a)

In [ ]:
```
y = returns.IYR.iloc[1:]
X = returns.IYR.shift(1).dropna()
```

In [ ]:
```
model_3 = sm.OLS(y, sm.add_constant(X)).fit()
model_3.summary()
```

Out[ ]:

OLS Regression Results

Dep. Variable:	IYR	**R-squared:**	0.011
Model:	OLS	**Adj. R-squared:**	0.005
Method:	Least Squares	**F-statistic:**	1.733
Date:	Thu, 07 Jul 2022	**Prob (F-statistic):**	0.190
Time:	22:35:59	**Log-Likelihood:**	256.15
No. Observations:	157	**AIC:**	-508.3
Df Residuals:	155	**BIC:**	-502.2
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	0.0124	0.004	3.171	0.002	0.005	0.020
IYR	-0.0953	0.072	-1.316	0.190	-0.238	0.048

Omnibus:	17.481	**Durbin-Watson:**	2.070
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	33.543
Skew:	-0.507	**Prob(JB):**	5.20e-08
Kurtosis:	5.025	**Cond. No.**	19.0

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

$R^2 = 0.011\backslash\ \beta^{IYR} = -0.0953$

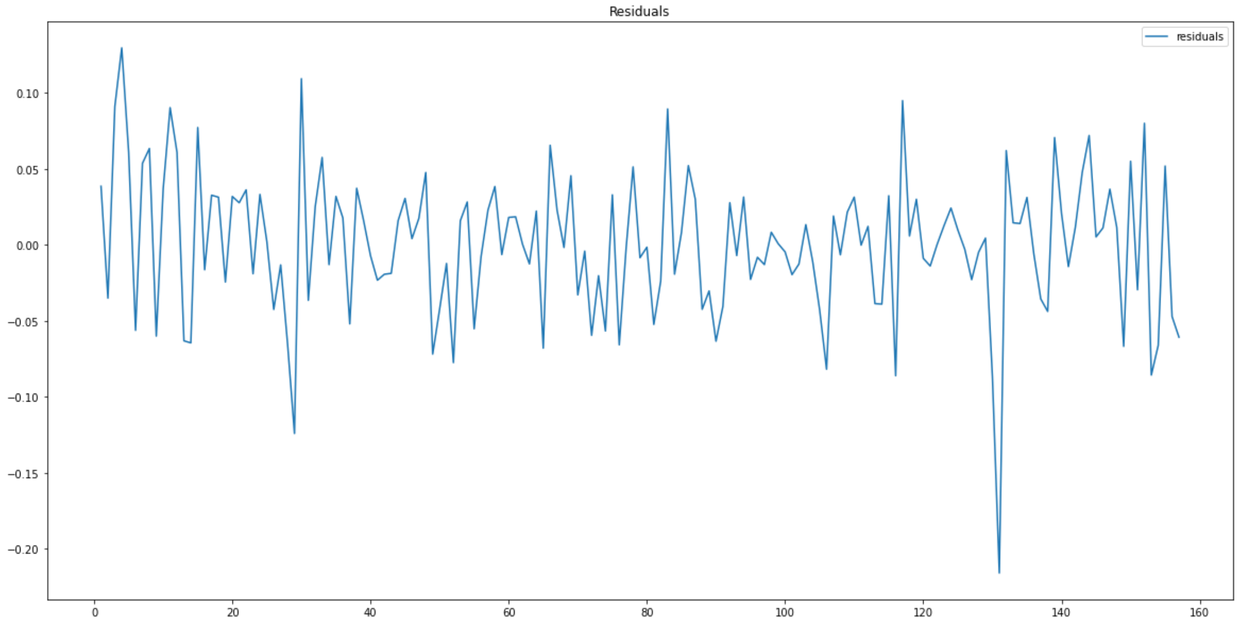
b) Residuals

In [ ]:

```
resiudals = pd.DataFrame(data=model_3.resid, index=X.index, columns=['residuals
resiudals.plot(title='Residuals', figsize=(20, 10))
```

Out[ ]:

```
<AxesSubplot:title={'center':'Residuals'}>
```



There seems to be a fair amount of serial correlation

2.

a) In the case of the multivariate regression the problem that would arise the most is the misspecified signifance levels and perhaps biased beta estimates in small samples.\ In large samples however, the central limit theorem and law of large numbers gives us confidence that beta should be converge towards its true value.

b) In the case of autoregression, all the above mentioned problems can arise. Having serial correlation prevents OLS to be consistent. Which is problematic in both small and large sample estimates of beta.

3.

a) Yes, the estimated beta would change and we would have a nearly perfect r-squared.

b) The **price** is not a stationary time series. This means that the variance is undefined and this breaks most of the assumptions on which OLS relies.

4. Volatility forecast

In [ ]:

```
FREQ = 12
dbc = returns.set_index('date')[['DBC']]

### Expanding Window
var = (dbc**2).shift(1).expanding(min_periods=2*FREQ).mean().rename(columns={'I

### Rolling Window
var['Rolling Window'] = (dbc**2).shift(1).rolling(window=2*FREQ).mean()

### Garch
```

```
GARCH = arch_model(dbc, vol='Garch', p=1, o=0, q=1, dist='Normal')
GARCH_model = GARCH.fit()
GARCH_model.params
```

```
Iteration:      1,   Func. Count:      6,   Neg. LLF: 2096374.5848787487
Iteration:      2,   Func. Count:     16,   Neg. LLF: 186.1092336503408
Iteration:      3,   Func. Count:     24,   Neg. LLF: -241.9350083830496
Iteration:      4,   Func. Count:     30,   Neg. LLF: -239.9824475159594
Iteration:      5,   Func. Count:     37,   Neg. LLF: -242.30981900903456
Iteration:      6,   Func. Count:     43,   Neg. LLF: -228.4572895151698
Iteration:      7,   Func. Count:     49,   Neg. LLF: -245.04432424194397
Iteration:      8,   Func. Count:     54,   Neg. LLF: -245.0452965095418
Iteration:      9,   Func. Count:     59,   Neg. LLF: -245.0454558065374
Iteration:     10,   Func. Count:     64,   Neg. LLF: -245.0454672822969
Iteration:     11,   Func. Count:     68,   Neg. LLF: -245.0454672825398
Optimization terminated successfully    (Exit mode 0)
            Current function value: -245.0454672822969
            Iterations: 11
            Function evaluations: 68
            Gradient evaluations: 11
```

Out[ ]:

```
mu          0.001878
omega       0.000193
alpha[1]    0.111195
beta[1]     0.818933
Name: params, dtype: float64
```

In [ ]:

```
var_1 = (0.15 * (1 / (12**0.5)))**2
var[['GARCH']] = None
var.iloc[0,2:] = (dbc.iloc[:FREQ*2]**2).mean()
for i in range(1, len(var)):
    var['GARCH'].iloc[i] = GARCH_model.params['omega'] + var['GARCH'].iloc[i-1]
var = var.dropna()
```

In [ ]:

```
vol = (var * FREQ)**.5

vol.plot(figsize=(20, 10))
plt.title('Volatility Forecasts')
plt.ylabel('Volatility')

plt.show()
```

In [ ]:
```
vol.loc[['2020-04-30','2022-05-31']]
```

Out[ ]:

date	Expanding Window	Rolling Window	GARCH
2020-04-30	0.174661	0.197237	0.265092
2022-05-31	0.180528	0.211956	0.216245

3. Penalized Regression

In [ ]:
```
# split data into train and test
test_start = 2020
X_train, y_train = \
    returns.loc[returns.date.dt.year < test_start].drop(columns=['date', 'IYR']
X_test, y_test = \
    returns.loc[returns.date.dt.year >= test_start].drop(columns=['date', 'IYR'
```

1. Estimated betas for each model

In [ ]:
```
results_df ={'model': ['OLS', 'Ridge', 'Lasso']}
for regressor in X_train.columns:
    results_df[regressor] = [0]*3
results_df = pd.DataFrame(results_df)
```

In [ ]:
```
# OLS
model_ols = LinearRegression().fit(X_train, y_train)
results_df.loc[results_df.model=='OLS', 1:] = model_ols.coef_

# Ridge
model_ridge = Ridge(alpha=0.5).fit(X_train, y_train)
results_df.loc[results_df.model=='Ridge', 1:] = model_ridge.coef_

# Lasso
model_lasso = Lasso(alpha=2e-4).fit(X_train, y_train)
```

```
results_df.loc[results_df.model=='Lasso', 1:] = model_lasso.coef_

display(results_df)
```

	model	SPY	EFA	EEM	PSP	QAI	HYG	DBC	IEF
0	OLS	0.907861	0.052678	0.108224	0.043854	-1.644597	1.090110	-0.218789	0.800986
1	Ridge	0.109550	0.095692	0.114033	0.156768	0.021400	0.100272	-0.026863	0.029078
2	Lasso	0.135772	0.000000	0.051035	0.299599	0.000000	0.429314	-0.057299	0.000000

2. Regressor correlation matrix

In [ ]:

```
X_train.corr()
```

Out[ ]:

	SPY	EFA	EEM	PSP	QAI	HYG	DBC	IEF
SPY	1.000000	0.866534	0.770723	0.880032	0.789765	0.723324	0.532333	-0.430732
EFA	0.866534	1.000000	0.864663	0.903705	0.804057	0.744558	0.584627	-0.362262
EEM	0.770723	0.864663	1.000000	0.818276	0.788988	0.734362	0.596593	-0.272977
PSP	0.880032	0.903705	0.818276	1.000000	0.755128	0.811265	0.501326	-0.402218
QAI	0.789765	0.804057	0.788988	0.755128	1.000000	0.698436	0.565595	-0.100092
HYG	0.723324	0.744558	0.734362	0.811265	0.698436	1.000000	0.500861	-0.205746
DBC	0.532333	0.584627	0.596593	0.501326	0.565595	0.500861	1.000000	-0.346769
IEF	-0.430732	-0.362262	-0.272977	-0.402218	-0.100092	-0.205746	-0.346769	1.000000
BWX	0.347099	0.553046	0.601506	0.429549	0.601683	0.453230	0.443634	0.264023
TIP	-0.025633	0.069738	0.188517	0.015117	0.280105	0.129386	0.106506	0.709324

There is presence of multicollinearity among the regressors. This breaks assumption 1 of OLS related to having a full rank matrix and would make it harder for the model to precisely identiy β.

3.

Similar to OLS, Ridge uses all the regressors. However, the scale of the regressors in Ridge tend to be smaller than in OLS.\ Ridge is useful when dealing with multicollinearity because by reducing the betas it scales the diagonal of the regressors covariance matrix to break any linear dependence among them.

4.

Lasso, on the other hand, heavily penalizes values that are different from 0. This encourages "non-relevant" regressors to be zero.\ However, similar to OLS, Lasso will tend to assign high values to the non-zero regressors.\ Lasso is very useful for feature selection as it will tend to nullify regressors that are detrimental to minimizing the loss function.\ This helps greatly in reducing the model variance which is good for out-of-sample performance.

5. out-of-sample estimates

In [ ]:
```
scores_df =pd.DataFrame({'model': ['OLS', 'Ridge', 'Lasso'],
                         'r-squared':[model_ols.score(X_test, y_test),
                                      model_ridge.score(X_test, y_test),
                                      model_lasso.score(X_test, y_test)]})
display(scores_df)
```

	model	r-squared
0	OLS	0.475652
1	Ridge	0.605911
2	Lasso	0.728237