Heat transfer in buildings

Video n°3

Thermal mass

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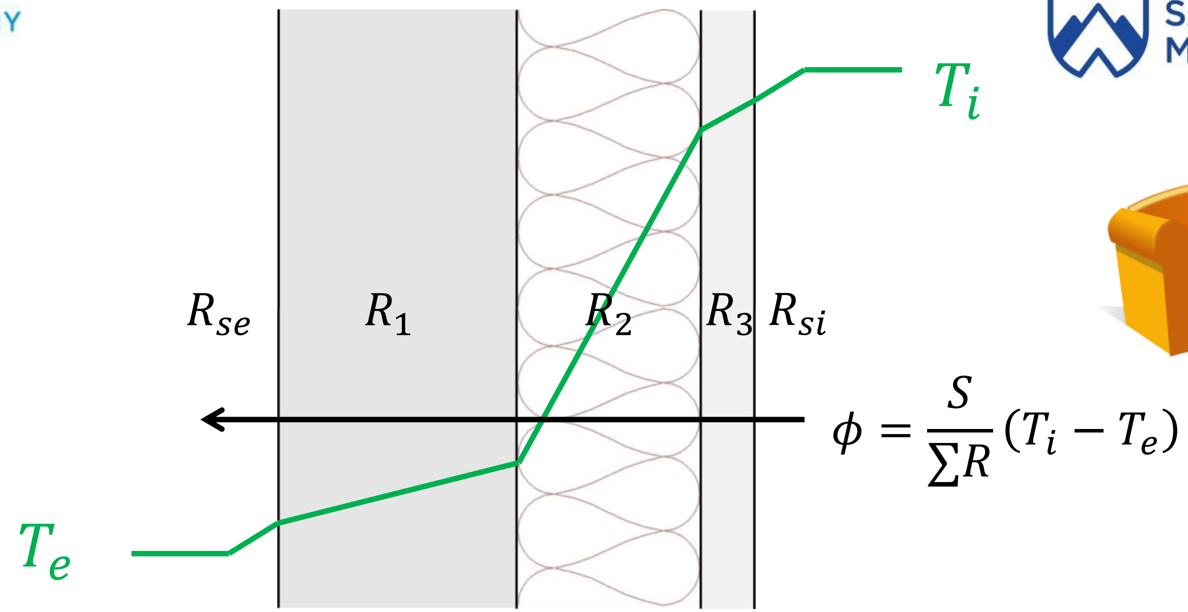
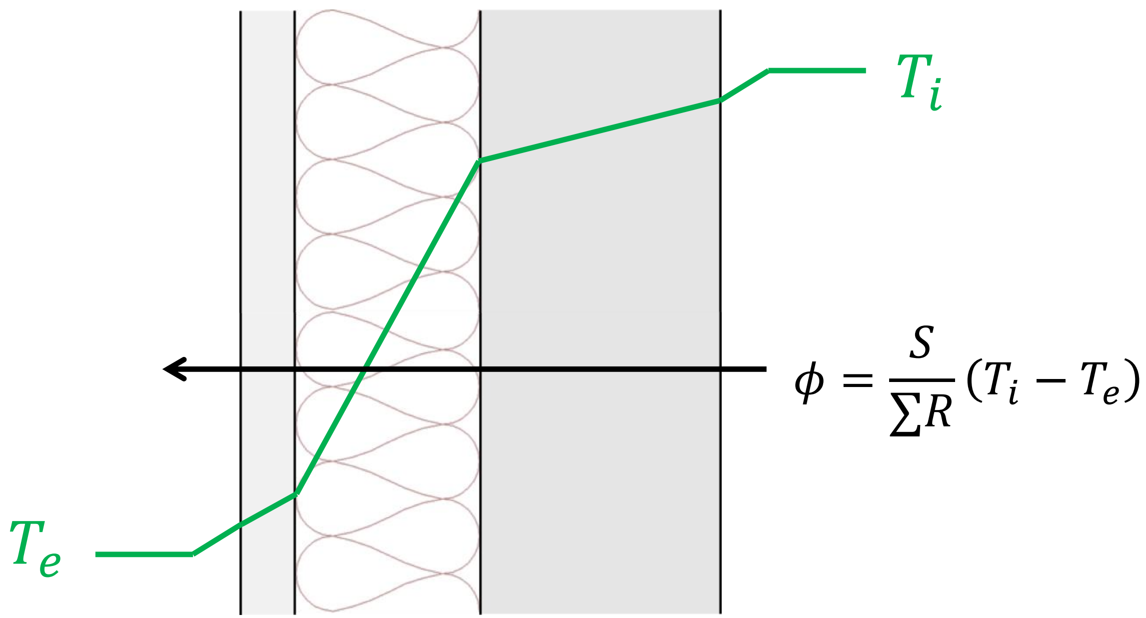
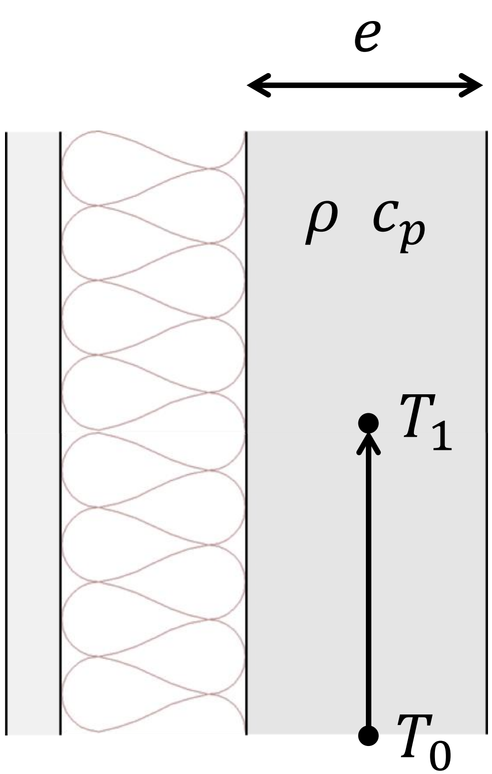
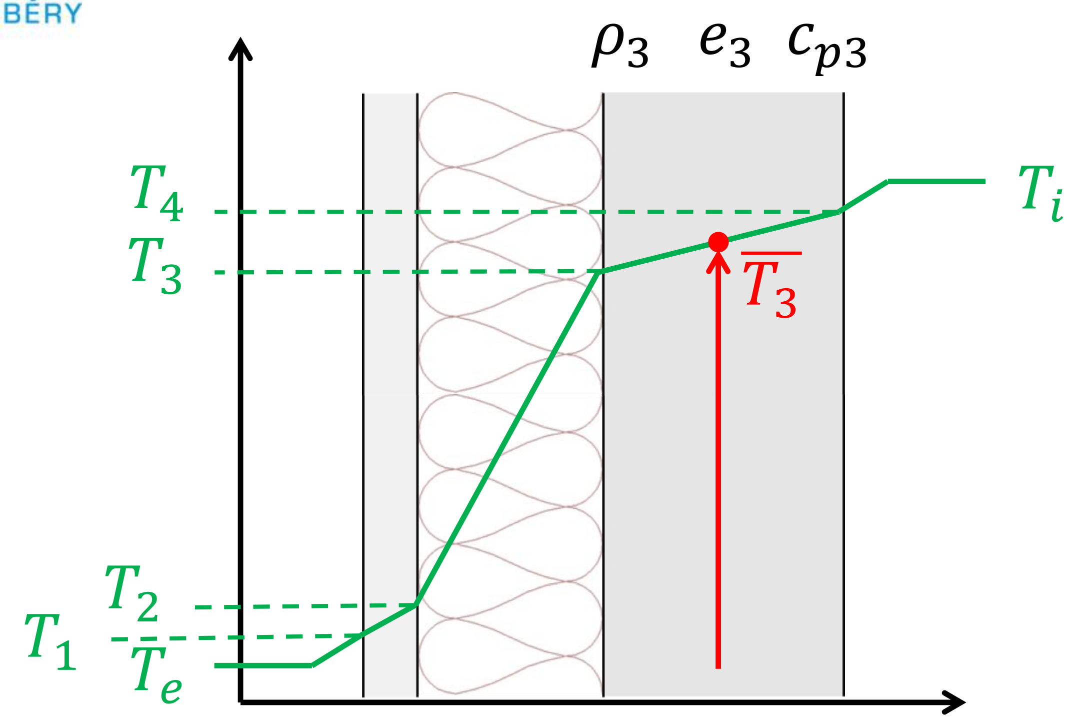

R_{se}
R_1
R_2
R_3
R_{si}
T_i
T_e
$\phi = \frac{S}{\sum R}(T_i - T_e)$


T_i
T_e
$\phi = \frac{S}{\sum R}(T_i - T_e)$



$$Q_{vol} = \underbrace{\rho . c_p}_{\text{Volumetric capacity [J/m}^3\text{.K]}} \; (T_1 - T_0) \qquad \text{[J/m}^3\text{]}$$

$$Q = \rho . e . c_p \; (T_1 - T_0) \qquad \text{[J/m}^2\text{]}$$



- Thermal energy of one layer

$$Q_3 = \rho_3 . e_3 . c_{p3}(\overline{T_3} - T_e) \qquad \text{where} \quad \overline{T_3} = \frac{(T_3 + T_4)}{2}$$

- Total energy

$$Q = \sum_{k=1}^{3} Q_k$$

- Time constant of the wall $\qquad \tau = \dfrac{Q}{U.(T_i - T_e)}$

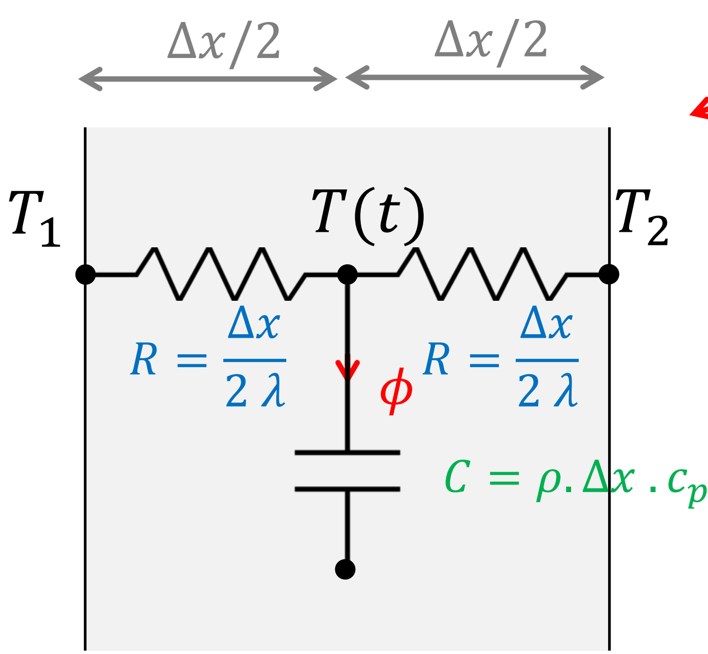
$$\rho . c_p \frac{\partial T}{\partial t} = \lambda \frac{\partial^2 T}{\partial x^2}$$

[W/m³.K]

$$\xrightarrow{\text{discretisation}}$$

$$\rho . c_p \frac{\mathrm{d}T}{\mathrm{d}t} = \frac{\lambda}{\Delta x^2/2}(T_1 - 2.T + T_2)$$

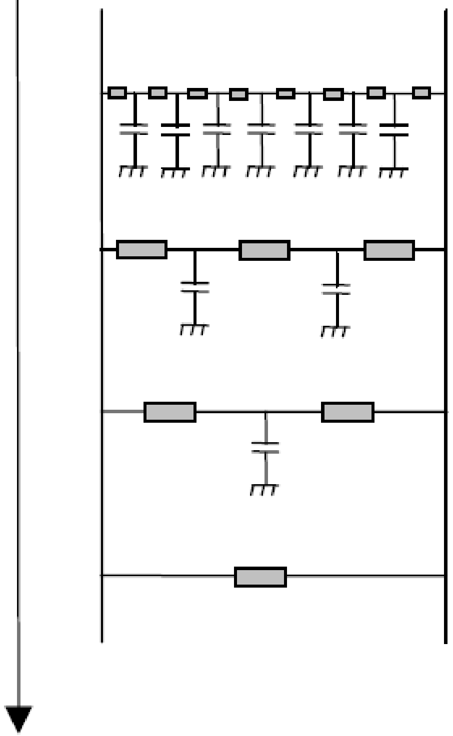
$$\underbrace{(\rho . \Delta x \,.\, c_p)}_{C} \frac{\mathrm{d}T}{\mathrm{d}t} = \frac{T_1 - T}{\underbrace{\left(\frac{\Delta x}{2\lambda}\right)}_{R}} + \frac{T_2 - T}{\left(\frac{\Delta x}{2\lambda}\right)}$$



$$I = C \frac{\mathrm{d}U}{\mathrm{d}t}$$

$$\phi = C \frac{\mathrm{d}T}{\mathrm{d}t}$$

C → [J/m².K]



Modélisation fine (LARET)

Modélisation simplifiée

Modélisation très simplifiée

Régime permanent

$$\frac{\mathrm{d}\mathbf{T}}{\mathrm{d}t} = \mathbf{A}\mathbf{T} + \mathbf{b}\,u(t)$$

$$\mathbf{T}(t) = [T_1(t)\ T_2(t)\ \ldots\ T_N(t)]$$

$$\mathbf{A} = \frac{a}{\Delta x^2}\begin{bmatrix} -2 & 2 & 0 & \ldots & 0 \\ 1 & -2 & 1 & & 0 \\ 0 & & & & \\ \ldots & & 1 & -2 & 1 \\ 0 & \ldots & 0 & 2 & -2(1+\mathrm{Bi}) \end{bmatrix}$$

$$\mathbf{b} = \frac{2}{\rho c_p \Delta x}\begin{bmatrix} 1 \\ 0 \\ \ldots \\ 0 \end{bmatrix}$$