CENTRAL UNIVERSITY

SCHOOL OF ENGINEERING AND TECHNOLOGY

DEPT. OF COMPUTER SCIENCE AND INFORMATION TECHNOLOGY

END OF FIRST SEMESTER EXAMINATIONS - 2020/21

COURSE TITLE:

DISCRETE MATHEMATICS 1

COURSE CODE:

COMP 107

TIME ALLOWED: TWO (2) HOURS

Student's ID: .....

INSTRUCTIONS:

1. *Answer ALL questions in SECTION A. Circle the right answer or write your answer in the provided on the question paper.*
2. *Answer any TWO questions in SECTION B in the answer booklet provided. All questions equal marks.*
3. *Fill the space indicated above with your Student ID. Fill in the details of the examination space provided on the cover of the answer booklet.*

DO NOT TURN OVER THIS PAGE UNTIL YOU HAVE BEEN T

TO DO SO BY THE INVIGILATOR

Instructor: Dr. K. Obeng Sarkodie

SECTION A [40 marks]

*Each question is followed by four lettered options **A** to **D**. Find out the correct option for each section and circle the letter that corresponds to the option you have chosen.*

Let $A = \{x, y\}$, $B = \{1, 2, 3\}$, and $C = \{a, b\}$. Use this information to answer questions 1, 2, 3, and 4

1. Find $A \times B$.
   A. $\{(x, 1), (y, 1), (x, 2), (y, b), (x, 3), (y, 3)\}$
   B. $\{(x, 1), (y, 1), (x, 2), (y, 2), (x, 3), (y, 3)\}$
   C. $\{(x, 1), (y, 1), (x, 2), (y, 2), (x, 3), (y, b)\}$
   D. $\{(x, 1), (y, 1), (a, 2), (y, 2), (x, 3), (y, 3)\}$

2. Find $B \times A$.
   A. $\{(1, x), (3, y), (2, x), (2, y), (3, x), (3, y)\}$
   B. $\{(1, x), (2, y), (2, x), (2, y), (3, x), (3, y)\}$
   C. $\{(1, x), (1, y), (2, x), (2, y), (3, x), (3, y)\}$
   D. $\{(1, x), (1, y), (1, x), (2, y), (3, x), (3, y)\}$

3. Find $A \times A$.
   A. $\{(x, x), (x, y), (y, x), (x, y)\}$
   B. $\{(x, x), (x, y), (x, x), (y, y)\}$
   C. $\{(x, x), (x, y), (y, x), (x, y)\}$
   D. $\{(x, x), (x, y), (y, x), (y, y)\}$

4. How many elements are in $A \times B \times C$?
   A. 10
   B. 11
   C. 12
   D. 13

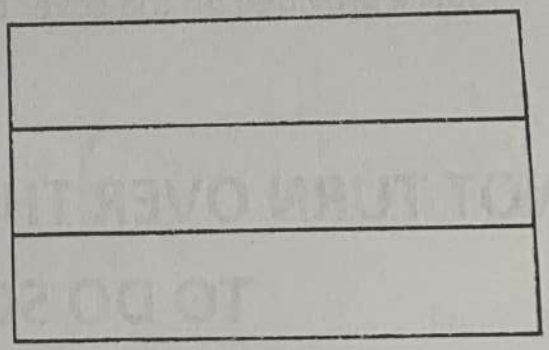
Write your answers in the boxes provided.

Assuming that *p is true*, *q is false*, and *r is true*, find the *truth value* of each proposition in questions 5, 6, 7 and 8.

5. $p \wedge q \rightarrow r$

6. $p \vee q \rightarrow \neg r$

7. $p \wedge (q \rightarrow r)$



8. $p \rightarrow (q \rightarrow r)$

In Figure 1 below, determine whether the following walks are trails, paths, closed walks, circuits, simple circuits, or just walks in questions 9, 10, 11, 12, 13 and 14.

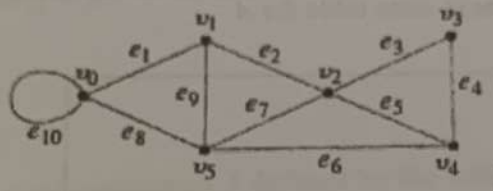


Figure 1

9. $v_1e_2v_2e_3v_3e_4v_4e_5v_2e_2v_1e_1v_0$

10. $v_2v_3v_4v_5v_2$

11. $v_4v_2v_3v_4v_5v_2v_4$

12. $v_2v_1v_5v_2v_3v_4v_2$

13. $v_0v_5v_2v_3v_4v_2v_1$

14. $v_5v_4v_2v_1$

Consider the finite-state automation A defined by the transition diagram shown in Figure 2. Use the information to answer questions 15, 16, 17, 18, 19 and 20.

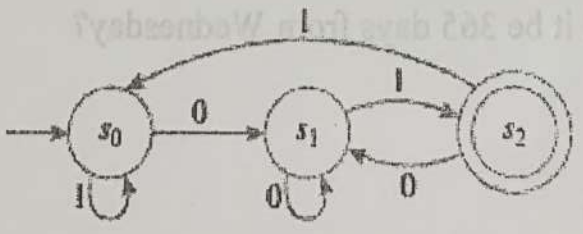


Figure 2

15. What are the states of A?

16. What are the input symbols of A?

17. What is the initial state of A?

18. What are the accepting states of A?

19. Determine $N(s1, 1)$ and $N(S2, 1)$.

20. Construct the annotated next-state table for A.

SECTION B: [60 Marks]

*Answer **ANY TWO** questions from this section. All questions carry equal marks.*

Question 1

a. Show that if $A = \{x \mid x^2 + x - 6 = 0\}$ and $B = \{2, -3\}$, then $A = B$. **[9 marks]**

b. Let $X = \{x \mid x^2 + x - 2 = 0\}$. Show that $X \subseteq \mathbf{Z}$. **[9 marks]**

c. Let $X = \{x \mid 3x^2 - x - 2 = 0\}$. Show that X is not a subset of $\mathbf{Z}$. **[9 marks]**

d. What day of the week will it be 365 days from Wednesday? **[3 marks]**

Question 2

a. Prove that the function $f(x) = 1/x^2$ from the set X of nonzero real numbers to the set Y of positive real numbers is onto Y. **[10 marks]**

b. Let $J_3 = \{0, 1, 2\}$, and define functions f and g from J_3 to J_3 as follows: For every x in J_3,

$$f(x) = (x^2 + x + 1) \ mod\ 3 \text{ and } g(x) = (x + 2)^2 \ mod\ 3.$$

Does $f = g$? **[10 marks]**

c. Let $F: \mathbf{R} \rightarrow \mathbf{R}$ and $G: \mathbf{R} \rightarrow \mathbf{R}$ be functions. Define new functions $F + G: \mathbf{R} \rightarrow \mathbf{R}$ and $G + F: \mathbf{R} \rightarrow \mathbf{R}$ as follows: For every $x \in R$,

$(F + G)(x) = F(x) + G(x)$ and $(G + F)(x) + G(x) + F(x)$.

Does $F + G = G + F$? **[10 marks]**

Question 3

Consider the finite-state automaton A defined by the following annotated next-state table:

	State	Input a	Input b	Input c
→	U	Z	Y	Y
◎	V	V	V	V
	Y	Z	V	Y
◎	Z	Z	Z	Z

a. What are the states of A? **[5 marks]**

b. What are the input symbols of A? **[5 marks]**

c. What is the initial state of A? **[5 marks]**

d. What are the accepting states of A? **[5 marks]**

e. Find $N(U, c)$ and $N(Y, a)$ and $N(Z, b)$ **[5 marks]**

f. Draw the transition diagram for A. **[5 marks]**