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## Step 1.

### a) Definition of terms

#### (i) Market Risk Premium (MRP)

The Market Risk Premium (MRP) is an additional return that investors seek for maintaining a market portfolio rather than having a risk-free investment. It is a fundamental element in the Capital Asset Pricing Model (CAPM), and can be defined as an extra return that an investor expects to earn for holding a combination of risky assets instead of government bonds because the latter carries no or low risk.

In the CAPM formula, the expected return of an asset is expressed as:

$$E(R)_i = R_f + \beta_i(E(R_m) - R_f)$$

where:

- $R_f$  is the risk-free rate.
- $\beta_i$  is the sensitivity of the asset to the market.
- $E(R_m) - R_f$  is the market risk premium.

This premium is also the price investors place on taking a position in the stock market which carries greater risk compared to a virtually risk-free investment in a treasury bond. The MRP is variable and has linkages with other macroeconomic variables such as the level of inflation, interest rates, and global economic conditions.

#### (ii) Small minus Big (SMB)

The Small Minus Big (SMB) factor reflects the return spread between small-cap and large-cap stocks. It captures the "size effect," which shows that historically, smaller companies tend to outperform larger ones over long periods. SMB is calculated by taking the difference between the returns of a portfolio of small-cap stocks and a portfolio of large-cap stocks:

$$SMB = \text{Average return of small cap stocks} - \text{Average return of large cap stocks}$$

Consequently, small companies usually have more growth opportunities and therefore enjoy higher expected returns but with greater risk as well. Small-cap equities are more sensitive to economic downturns, are less liquid and have limited fundraising ability, which are all factors to their volatility.

#### (iii) High Minus Low (HML)

The High Minus Low (HML) factor captures the return differential between value stocks and growth stocks. Value stocks are characterized by high book-to-market ratios, meaning they are perceived as undervalued compared to their fundamental metrics, such as book value. Growth stocks, on the other hand, have low book-to-market ratios, indicating high expectations for future earnings growth but may be overpriced relative to their current fundamentals. HML is constructed by taking the difference between the returns of high book-to-market (value) stocks and low book-to-market (growth) stocks:

$$HML = \text{Average return of value stocks} - \text{Average return of growth stocks}$$

Fama and French's HML can be viewed as an example of the value premium, which suggests that value stocks, in the end, outperform growth stocks. Value stocks are perceived as riskier, and these are firms that are distressed or firms in sunset industries. Such stocks carry with them a risk premium which investors typically expect to earn.

#### **(iv) Conservative Minus Aggressive (CMA)**

The Conservative Minus Aggressive (CMA) factor distinguishes companies with conservative investment policies and those with aggressive investment strategies. Conservative firms tend to carry out less reinvestment of their earnings and such firms also tend to have low levels of capital investment as opposed to aggressive firms that are characterized by high levels of mergers, expansions or new projects that are aimed at reinvesting earnings.

CMA is calculated by taking the difference between the returns of companies with conservative investment policies and those with aggressive policies:

$$CMA = \text{Average return of conservative firms} - \text{Average return of aggressive firms}$$

The CMA is one of the factors presented by Fama and French in their Five-Factor Model in 2015. The intuition behind this factor is that such firms which have more conservative investment policies are expected to better the aggressive firms as the latter will be taking risk which does not always yield return as expected.

#### **(v) Robust Minus Weak (RMW)**

The RMW factor measures the spread of earnings between stronger earnings and weaker earnings and hence provides the difference in the returns between firms with high earnings and those with weak earnings. In most cases, profitability is determined in terms of the gross profit-to-asset ratio or the operating income-to-asset ratio. Companies with robust bid-to-cover ratios and bid premiums tend to have consistent income from their operations. On the other hand, weaker firms can hardly earn steady earnings.

RMW is calculated as:

$$RMW = \text{Average return of robust profitability firms} - \text{weak profitability firms}$$

The RMW factor was introduced by Fama and French in their 2015 five-factor model for explaining the profitability effect, which is ignored by previous models including the three-factor model. This means shares with higher profitability are likely to return dividends to investors since they have the capacity to pay the dividends and still support reinvestment to grow the business, and also withstand unfavorable events. Lower profitability on the other side is associated with higher likelihood of financial distress, lower growth potential, and lower survivability to shocks in the economy.

#### **b) Why does this factor help explain returns?**

### **(i) Market Risk Premium (MRP)**

The MRP is one of the most crucial factors in explaining equity returns because it directly reflects the reward investors demand for taking market risk. When the market is doing well, stocks typically outperform the risk-free rate, so a large MRP means that market returns are predicted to match risk-free assets, hence a higher return. Conversely, a low MRP might indicate a risk-averse landscape, in which investors are unwilling to invest in risky assets. This relationship is key to grasping the costs that investors weigh when choosing the level of risk to include in their portfolios

### **(ii) Small minus Big (SMB)**

SMB describes one important feature of returns in the stock market that CAPM does not capture on its own. Smaller companies are, in general, overlooked and underappreciated, shunned by institutional investors, or subject to greater inefficiencies, and their higher long-term returns are thus a reward for the increased risks of their operations. Additionally, small firms often operate in niche markets with higher growth potential compared to large established firms.

### **(iii) High Minus Low (HML)**

HML helps to understand the value-growth puzzle in stock returns. Value stocks are often undervalued by the market, and investors who buy these stocks are compensated with higher returns when the market corrects these mispricings. Growth stocks, while promising rapid future growth, tend to be more volatile and riskier in the short term. In the good times, investors have a tendency to drive the price of growth stocks ever higher, which can mean disappointing returns when the penny drops.

### **(iv) Conservative Minus Aggressive (CMA)**

CMA explains why models of stock returns become more powerful when they account for a company's investment behavior, which affects its risk and return characteristics. The companies that play it safe will appear less risky, more robust, with dependable, though potentially underwhelming, returns. Aggressive companies are usually in growth or expansion mode, and can provide huge returns if the investments work out, but are also more likely to collapse and fluctuate.

In many cases, conservative companies outperform during periods of market uncertainty or downturns, while aggressive firms may excel during economic booms. The CMA factor can be used to understand this performance gap in terms of the strategic choices that companies make about how they allocate capital.

### **(v) Robust Minus Weak (RMW)**

RMW improves the precision of stock returns forecasting by making profitability a key determinant of how firms are likely to perform in the long run. Robust firms typically offer higher returns than weak firms, as profitability indicates not only operational efficiency but also a firm's ability to allocate resources efficiently and sustain a competitive advantage. This factor helps to fill in a hole in the earlier models, which ignored the effect of profitability on stock returns.

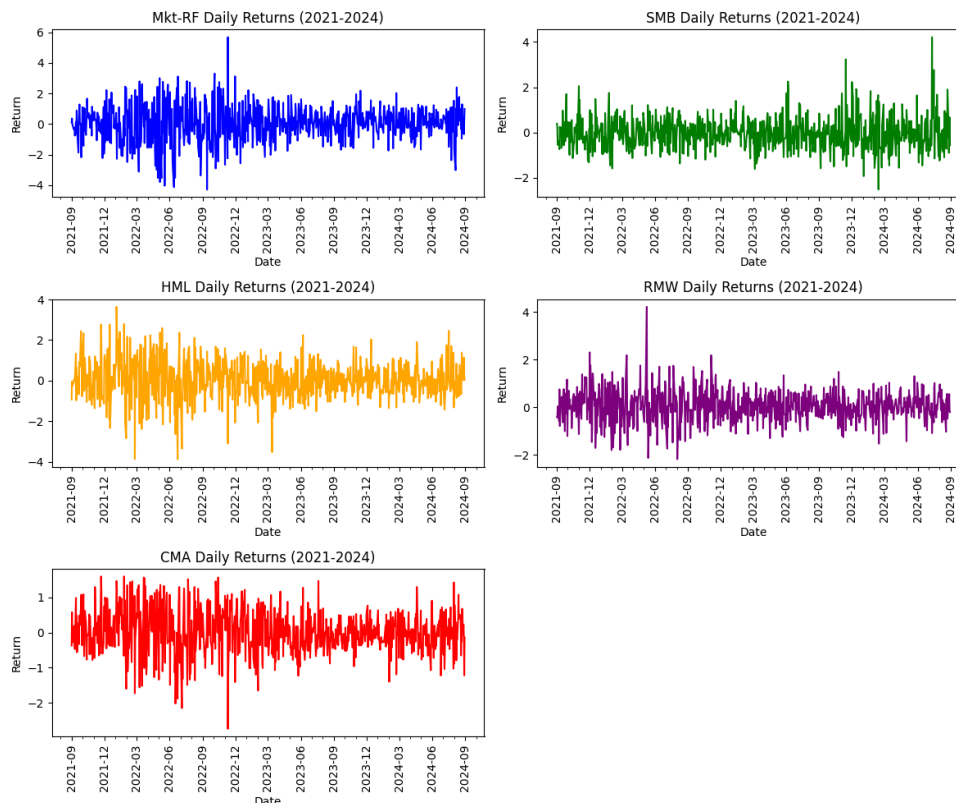
Not only does the RMW factor enhance the explanatory power of returns, but it also brings an element of quality into the analysis of investment. Investors who focus on profitability are often concerned with the quality of earnings and management's ability to maximize shareholder value. So portfolios with an overweighting of strong firms can generate both greater expected returns and less risk than portfolios filled with weak ones.

## Step 2.

### a) Importing, structuring, and graphing of the daily factor returns

Using a 3-year time period and daily data for the five factors (MRP, SMB, HML, CMA, RMW), the factor data from Professor Kenneth French's data library.

Date	Mkt-RF	SMB	HML	RMW	CMA
2021-09-01	0.11	0.39	-0.93	-0.42	-0.37
2021-09-02	0.36	0.21	-0.04	-0.43	0.58
2021-09-03	-0.01	-0.55	-0.27	0.04	-0.28
2021-09-07	-0.36	-0.37	-0.08	-0.59	0.17
2021-09-08	-0.34	-0.74	0.08	0.75	0.19



## b) Correlations of the Changes in Factor Returns

To compute the correlations of the changes in the factor returns, we will first calculate the daily returns for each factor, then compute the correlation matrix for these daily returns.

**Correlation Matrix of Daily Factor Returns:**

	Mkt-RF	SMB	HML	RMW	CMA
Mkt-RF	1.000000	-0.037117	-0.043794	0.001403	0.015672
SMB	-0.037117	1.000000	0.006820	-0.041240	-0.007376
HML	-0.043794	0.006820	1.000000	-0.025043	0.056083
RMW	0.001403	-0.041240	-0.025043	1.000000	-0.019680
CMA	0.015672	-0.007376	0.056083	-0.019680	1.000000

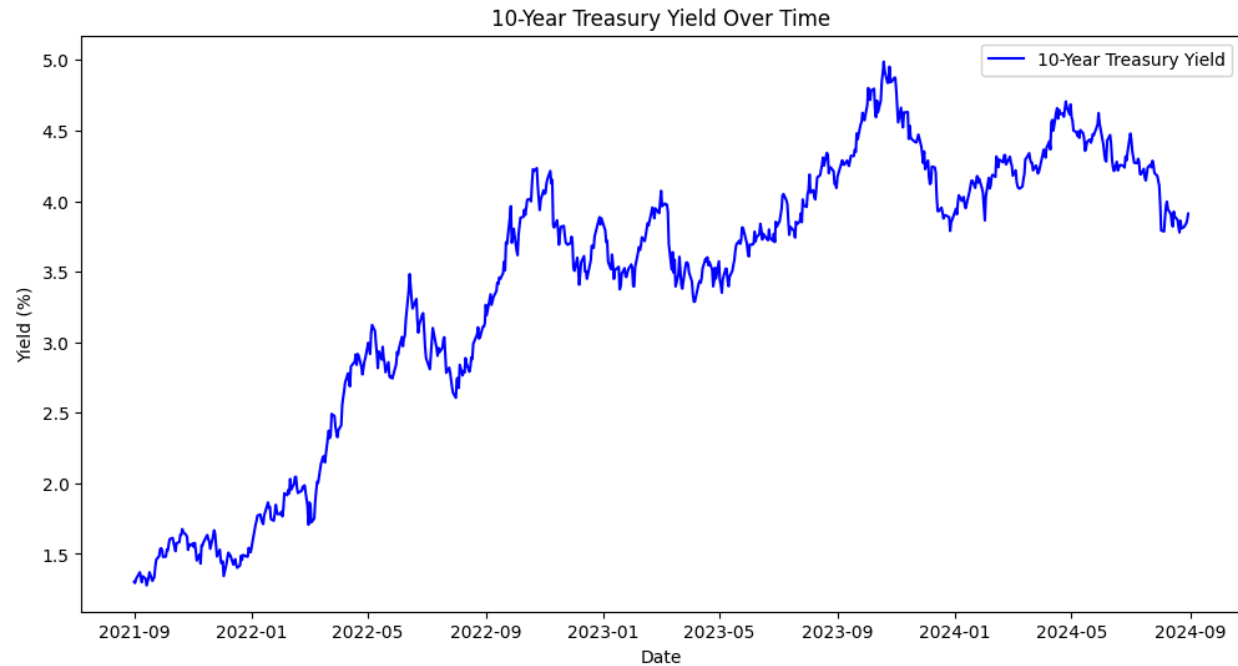
## c) 10-year Treasury Rates during the 3-year period

*10-Year Treasury Yield Data (First 5 Rows):*

Ticker	^TNX
Date	
2021-09-01 00:00:00+00:00	1.302
2021-09-02 00:00:00+00:00	1.294
2021-09-03 00:00:00+00:00	1.322
2021-09-07 00:00:00+00:00	1.370
2021-09-08 00:00:00+00:00	1.334

*10-Year Treasury Yield Data (Last 5 Rows):*

Ticker	^TNX
Date	
2024-08-26 00:00:00+00:00	3.818
2024-08-27 00:00:00+00:00	3.833
2024-08-28 00:00:00+00:00	3.841
2024-08-29 00:00:00+00:00	3.867
2024-08-30 00:00:00+00:00	3.911



### Step 3.

a) We use an OLS and a robust version with FF3 to run the regressions. Data are splitted into training and testing sets with 80/20 ratio.

b) Summary of model result:

Coefficients	OLS model	Robust model
Constant	0.0019	0.0020
Mkt-RF	0.0018	0.0016
SMB	-0.0024	-0.0028
HML	0.0064	0.0059
R-squared	0.057	
Adjusted R-squared	0.052	

Coefficients: The beta represents how sensitive the portfolio is to the respective factor. For instance, if we have a high SMB beta, that means there has been exposure to small-cap stocks.

R-squared: This explains the percentage or ratio of variation in the portfolio returns as explained by the model.

P-values: This explains the significance of each factor. Usually a low p-value of  $<0.05$  means the factor explains the returns significantly.

## Step 4.

a) We use an OLS and a robust version with FF5 to run the regressions. Data are splitted into training and testing sets with 80/20 ratio.

b) Summary of model result:

Coefficients	OLS model	Robust model
Constant	0.0020	0.0021
Mkt-RF	0.0010	0.0009
SMB	-0.0050	-0.0049
HML	0.0097	0.0086
RMW	-0.0047	-0.0040
CMA	-0.0048	-0.0036
R-squared	0.072	
Adjusted R-squared	0.065	

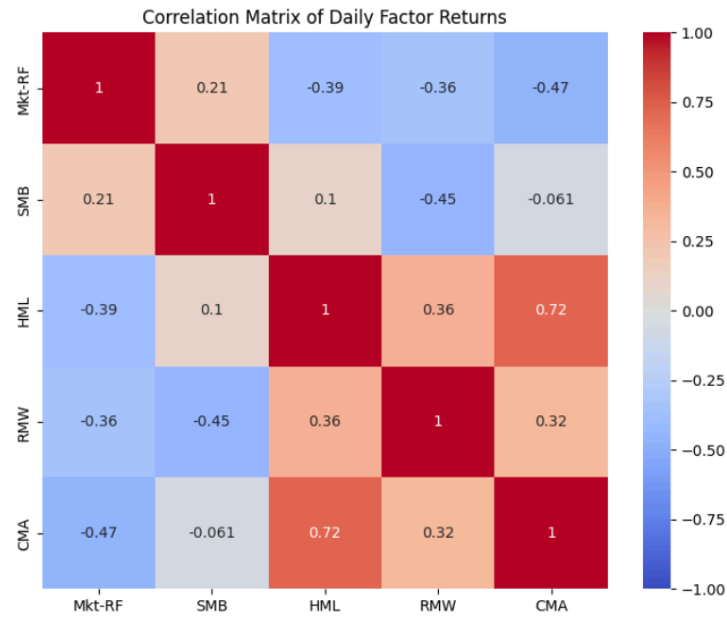
The effects of the coefficients and the R-squared are already explained above.

## Step 5

a) Correlation matrix of the daily factor returns

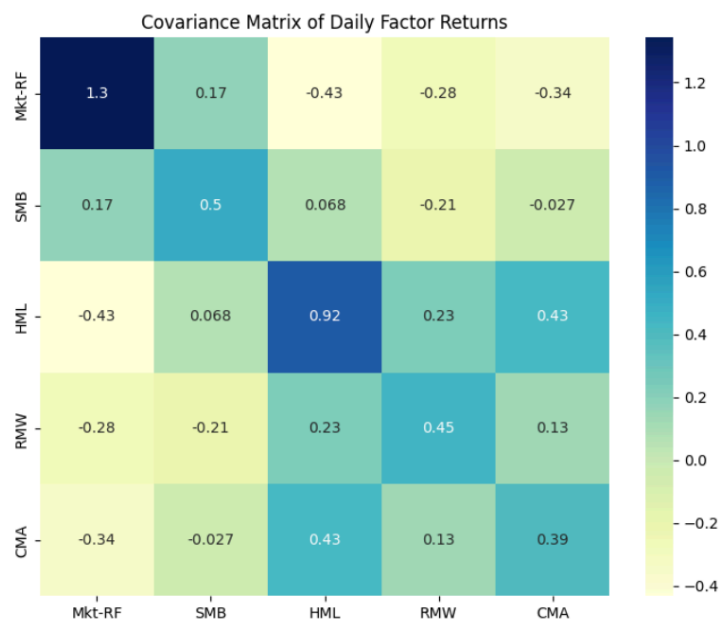
	Mkt-RF	SMB	HML	RMW	CMA
Mkt-RF	1.000000	0.208883	-0.389031	-0.359160	-0.471361
SMB	0.208883	1.000000	0.101063	-0.447998	-0.060949
HML	-0.389031	0.101063	1.000000	0.364863	0.719890
RMW	-0.359160	-0.447998	0.364863	1.000000	0.322208
CMA	-0.471361	-0.060949	0.719890	0.322208	1.000000





## b) Covariance matrix of the daily factor returns

	Mkt-RF	SMB	HML	RMW	CMA
Mkt-RF	1.342741	0.171043	-0.432316	-0.278159	-0.342076
SMB	0.171043	0.499355	0.068488	-0.211587	-0.026974
HML	-0.432316	0.068488	0.919688	0.233862	0.432374
RMW	-0.278159	-0.211587	0.233862	0.446702	0.134871
CMA	-0.342076	-0.026974	0.432374	0.134871	0.392234



### c) Compare and contrast

Correlation matrix shows the linear relationship between factors on a standardized scale from -1 to 1, while covariance matrix shows the co-movement between factor daily returns. The covariance matrix is not scaled like the correlation matrix since it depends on the variances of the two factors. The diagonal is thus not all 1 like in the correlation matrix, but represents the variance of the factor itself.

In the correlation matrix, the strongest positive linear relationships are between HML and CMA (0.7199), suggesting that these factors might have a positive trend to move together. The strongest negative relationships are between Mkt-RF and CMA (-0.471) and between SMB and RMW(-0.448), indicating that these factors might move in an opposite trend.

The covariance matrix reflects these relationships but in different magnitudes since it is affected by the variances of itself. For example, the covariance between Mkt-RF and CMA is -0.3421, corresponding to the strong negative correlation of -0.4714 between these two factors in the correlation matrix, but with units that depend on both factor variances.

## Step 6

Comparison of OLS model result between FF3 and FF5 model:

	FF3 Model	FF5 Model
Intercept	0.001916	0.002022
Mkt-RF	0.001841	0.000965
SMB	-0.002440	-0.005026
HML	0.006386	0.009714
RMW	-	-0.004661
CMA	-	-0.004761
R-squared	0.057080	0.072394
Adjusted R-squared	0.052350	0.064612

The error metrics are MSE, RMSE, and MAE:

- MSE: it helps to penalize larger errors.
- RMSE: As it is the square root of MSE, it helps interpret a balance of large errors and small errors.

- MAE: As it is less sensitive to outliers, it treats all errors equally. This means it is an average error measurement metric.

	FF3 Model	FF5 Model
OLS model MSE	0.000226	0.000224
OLS model RMSE	0.015017	0.014971
OLS model MAE	0.012169	0.012063
Robust model MSE	0.000221	0.000219
Robust model RMSE	0.014853	0.014790
Robust model MAE	0.012030	0.011922

## Step 7

5 stocks are chosen MSFT, NFLX, TSLA, NVDA, JNJ using data from the same 3-year period.

a) Using Markowitz portfolio optimization, we can find the set of optimal allocations as below:

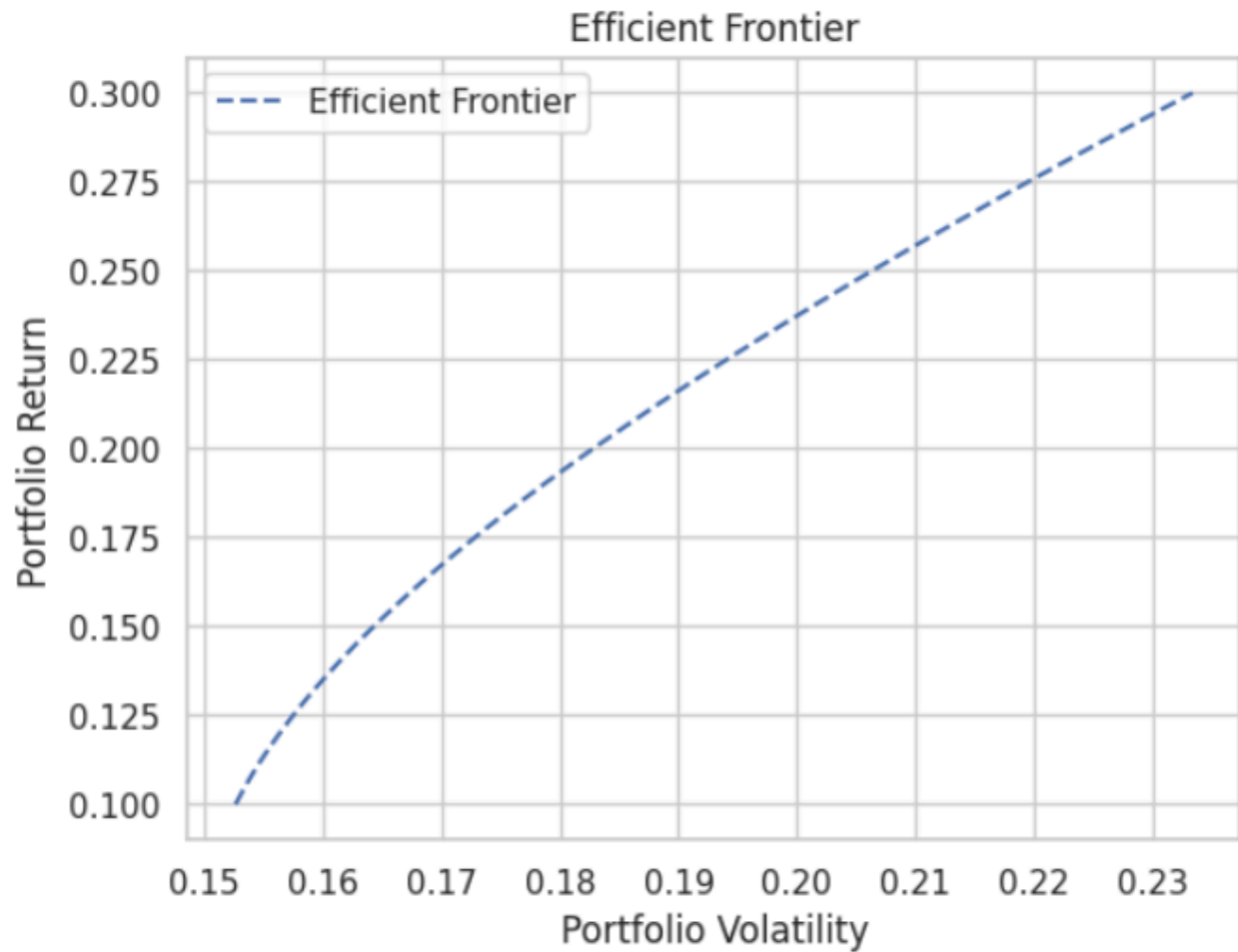
Optimal Portfolio Weights:

- 'MSFT': 0.7694341309093241
- 'NFLX': 0.18903836366175678
- 'TSLA': 0.023894151035043592
- 'NVDA': 0.0011792340965532406
- 'JNJ': 0.016454120297322287

Optimal Portfolio Return: 0.05735841388095166

Optimal Portfolio Volatility: 0.14939315279222892

The efficient frontier:



b) Fit an OLS model with FF3 factors:

Coefficients	OLS model
Constant	0.0009
Mkt-RF	0.0086
SMB	-0.0041
HML	-0.0011
R-squared	0.487
Adjusted R-squared	0.485

c) Fit an OLS model with FF5 factors:

Coefficients	OLS model
Constant	0.0009
Mkt-RF	0.0087
SMB	-0.0032
HML	-0.0017
RMW	0.0022
CMA	-0.00002
R-squared	0.493
Adjusted R-squared	0.489

## Step 8

Looking at the result with 5 factors, we can see that the constant is statistically significant ( $p=0.02$ ), which is expected if the asset pricing theory (APT) model holds. The coefficient of the market risk (Mkt-RF) is 0.0087, which shows that the returns of the portfolio move in line with the market, but with little magnitude. The coefficient of the size factor (SMB) is negative, which indicates that this portfolio of large-cap companies is less risky. The coefficient of the value factor (HML) has a negative impact, indicating that the portfolio qualifies as a growth stock rather than a value stock. The coefficient of profitability factor (RMW) is positive, indicating that the portfolio is tilted toward companies with higher profitability. The coefficient of the last factor CMA is not statistically significant ( $p=0.985$ ).

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