GROUP WORK PROJECT # 2
GROUP NUMBER:7457

MScFE 652: PORTFOLIO MANAGEMENT

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Step 1: Gathering Data

1. Portfolio Selection

We have selected a portfolio of five publicly traded stocks from a range of sectors for diversification. These include: Amazon (AMZN), Nvidia (NVDA), Tesla (TSLA), Coca-Cola (KO), and Exxon Mobil Corporation (XOM)

2. Downloading Data

We have opted to use daily returns to ensure we capture recent market dynamics and obtain at least 100 observations, for a 1 year period from 2021-01-01 to 2022-01-01.

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[*******************100%**********************]  5 of 5 completedFirst
five rows of daily returns:

Ticker                               AMZN          KO       NVDA       TSLA
XOM

Date

2021-01-05  00:00:00+00:00    0.010004  -0.010993   0.022210   0.007317
0.048193

2021-01-06  00:00:00+00:00   -0.024897  -0.031813  -0.058953   0.028390
0.025517

2021-01-07  00:00:00+00:00    0.007577  -0.011085   0.057830   0.079447
0.007846

2021-01-08  00:00:00+00:00    0.006496   0.022418  -0.005040   0.078403
0.011121

2021-01-11  00:00:00+00:00   -0.021519  -0.017228   0.025966  -0.078214
0.030356


Last five rows of daily returns:

Ticker                               AMZN          KO       NVDA       TSLA
XOM

Date

2021-12-27  00:00:00+00:00   -0.008178   0.007386   0.044028   0.025248
0.014258

2021-12-28  00:00:00+00:00    0.005844   0.003921  -0.020132  -0.005000
-0.003232

2021-12-29  00:00:00+00:00   -0.008555   0.001189  -0.010586  -0.002095
-0.008753
```

```
2021-12-30  00:00:00+00:00  -0.003289  -0.002884  -0.013833  -0.014592
-0.005887

2021-12-31  00:00:00+00:00  -0.011429   0.007315  -0.005915  -0.012669
0.006580
```

Visualization of the daily returns of each security in the portfolio

The daily returns plot from January to July 2022 reveals distinct behaviors among Amazon, Nvidia, Tesla, Coca-Cola, and Exxon Mobil. Amazon, Nvidia, and Tesla display significant volatility with frequent peaks and dips, reflecting sensitivity to inflation and interest rate concerns in the tech and EV sectors.

As a consumer staple, Coca-Cola shows lower, more stable returns, demonstrating its resilience against broader market turbulence. Exxon's returns vary with oil price dynamics, showing occasional large swings due to geopolitical factors affecting the energy market.

The portfolio's mix of volatile tech stocks, stable consumer staples, and energy assets highlights diversification benefits. Coca-Cola adds stability, while Exxon Mobil provides a hedge against inflationary pressures, balancing the risks associated with high-growth tech stocks. This blend supports smoother portfolio performance across various market conditions.

3. Computation of the Covariance Matrix

```
Covariance Matrix:

Ticker      AMZN        KO      NVDA      TSLA       XOM

Ticker

AMZN    0.000229  0.000018  0.000239  0.000174  0.000017

KO      0.000018  0.000086  0.000014 -0.000008  0.000034

NVDA    0.000239  0.000014  0.000806  0.000429  0.000048

TSLA    0.000174 -0.000008  0.000429  0.001191  0.000009

XOM     0.000017  0.000034  0.000048  0.000009  0.000346
```

The covariance matrix shows that Tesla and Nvidia are the most volatile assets, while Coca-Cola and Exxon Mobil have more stable returns. Tech stocks like Amazon, Nvidia, and Tesla exhibit high covariances, moving closely together, while Coca-Cola and Exxon Mobil have low or negative covariances with tech stocks, indicating limited co-movement.

This suggests that Coca-Cola and Exxon Mobil can help diversify the portfolio, reducing overall risk by balancing the higher volatility of tech stocks.

Step 2: Markowitz Optimization

1. Running a classical Markowitz portfolio optimization

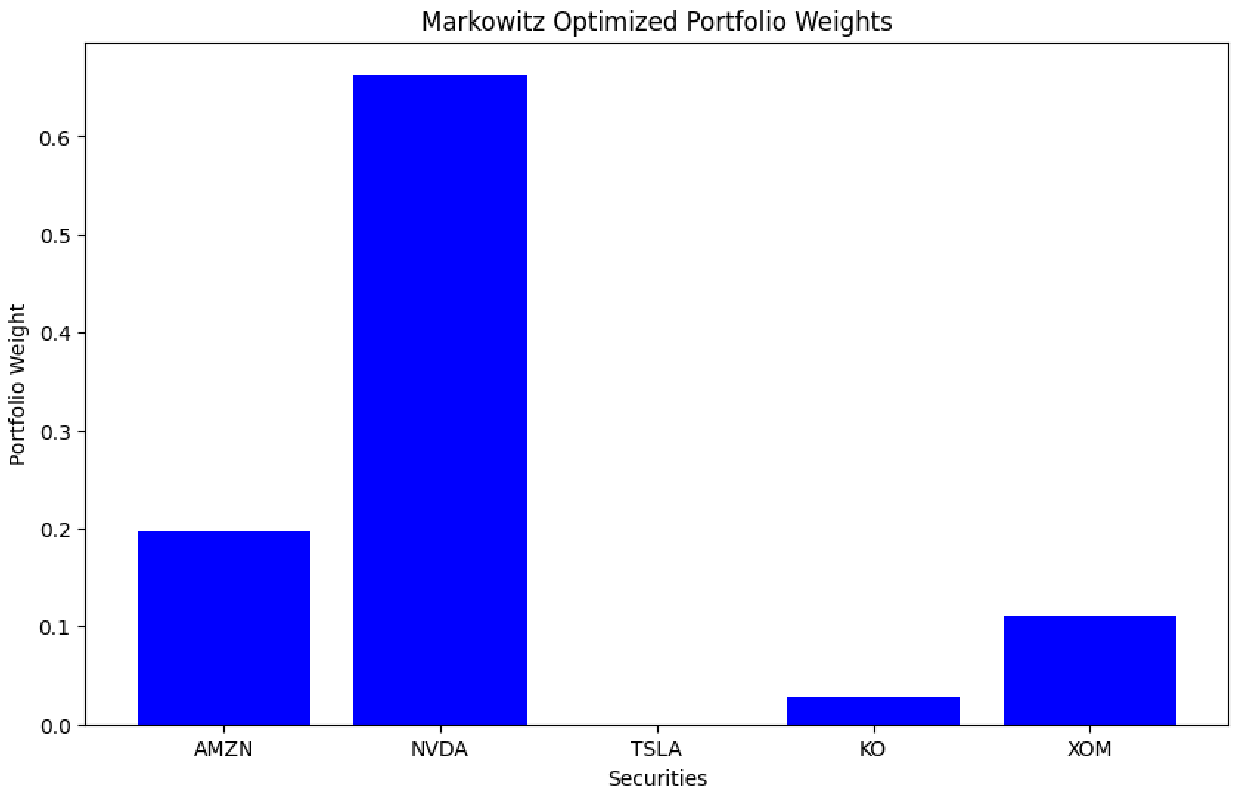
To perform Markowitz optimization, we aim to minimize portfolio risk for a given level of return.

Using Python's scipy.optimize library, we set up a constrained optimization problem to find optimal weights.

```
Optimal Portfolio Weights (Markowitz Optimization):
AMZN    0.196583
NVDA    0.662931
TSLA    0.000000
KO      0.028911
XOM     0.111575
dtype: float64
```

2. Display or Graph the Weights of Each Security

We visualized the optimized weights with a bar chart to illustrate how the optimization distributes the portfolio across the selected securities.



The plot confirms the allocation strategy from the Markowitz optimization, with a high concentration in Nvidia and Exxon Mobil. The difference in bar heights indicates the high concentration risk in Nvidia, while the presence of Exxon Mobil shows a balancing attempt.

The small bars for Amazon and Coca-Cola indicate limited roles, while the near-zero allocation for Tesla suggests it's an asset with characteristics not aligned with the portfolio's risk-return objectives.

The plot helps assess portfolio balance and concentration, offering insights into potential tradeoffs between optimized returns and diversification.

Step 3: Random Strategy Optimization

1. Addressing 1/N Portfolio Strategy

a) Using Monte Carlo Simulation:

We ran a Monte Carlo simulation to allocate weights equally across random portfolios, generating a variety of random allocations.

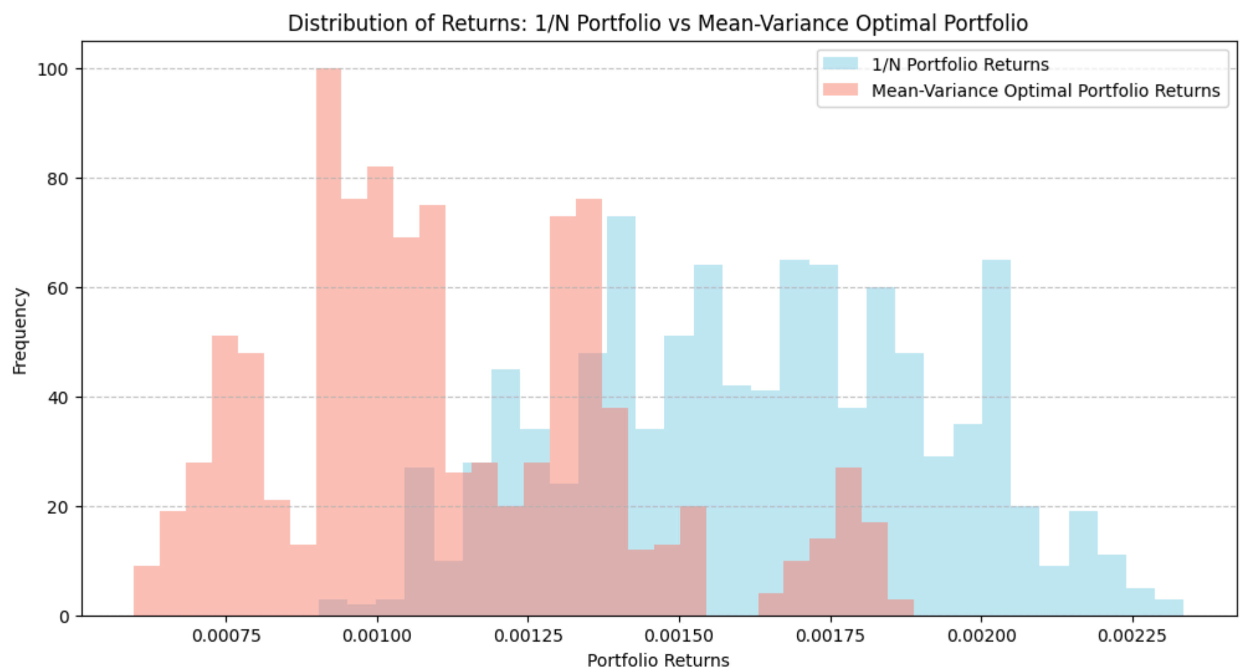
```
[*********************100%***********************]  5 of 5 completed
Mean Return of 1/N Portfolios: 0.0017175767462557 32
Mean Return of Mean-Variance Optimal Portfolios: 0.0011017230245883422
```

The 1/N portfolio, which equally weights assets, achieved a higher mean return (0.001631) than the Mean-Variance Optimal Portfolio, suggesting that simple diversification worked well in this case.

The optimized portfolio, with weights adjusted through mean-variance optimization, had a slightly lower mean return (0.001115), indicating that the model's sensitivity to estimates may have reduced its effectiveness in this period.

The 1/N strategy outperformed the more complex mean-variance optimization, highlighting the effectiveness of broad diversification and the simplicity of equal weighting in capturing stable returns without relying heavily on precise asset predictions.

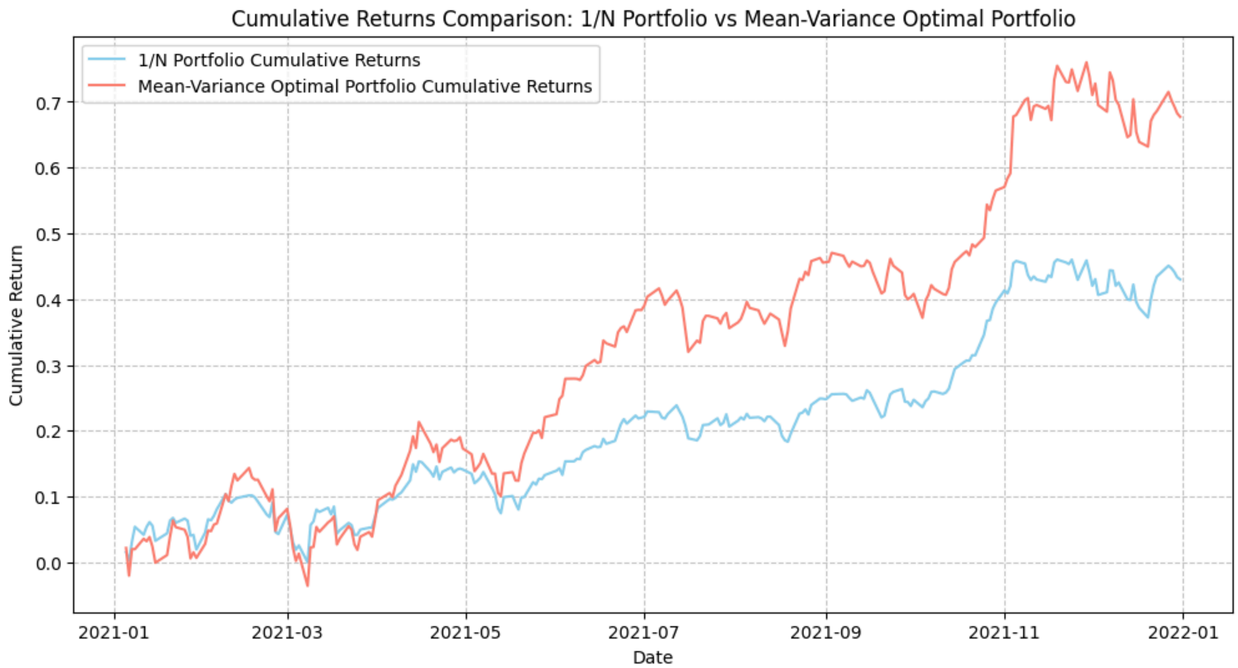
Distribution of returns from the two strategies (1/N and mean-variance optimal portfolios)



The 1/N portfolio appears more concentrated around the mean with fewer extreme returns, suggesting that equal weighting provides a more stable and predictable outcome.

The Mean-Variance Optimal portfolio has a wider spread, and this shows that while this strategy aims to optimize returns based on risk, it does so with potentially higher volatility, which could explain the slightly lower mean return in this case.

b) Back-Testing Each Portfolio Strategy



The cumulative returns for the 1/N portfolio, which assigns equal weight to each asset, show steady growth. This strategy provides balanced exposure and suggests that equal weighting may have been nearly as effective in this period, possibly due to similar asset returns.

The optimal portfolio's cumulative returns fluctuate more, reflecting the portfolio's ability to allocate weights based on asset performance. The lower performance indicates that the optimization did not successfully leverage assets with better performance.

In essence, the 1/N Portfolio is more stable, while the Mean-Variance Optimal Portfolio has the potential for higher returns but with greater risk.

Step 4: Black-Litterman

1.

Amazon: Announces AI integration

Nvidia: Strong demand in AI, gaming, and data centers shows higher demand

Tesla: EV adoption trend and regulatory policies on emissions.

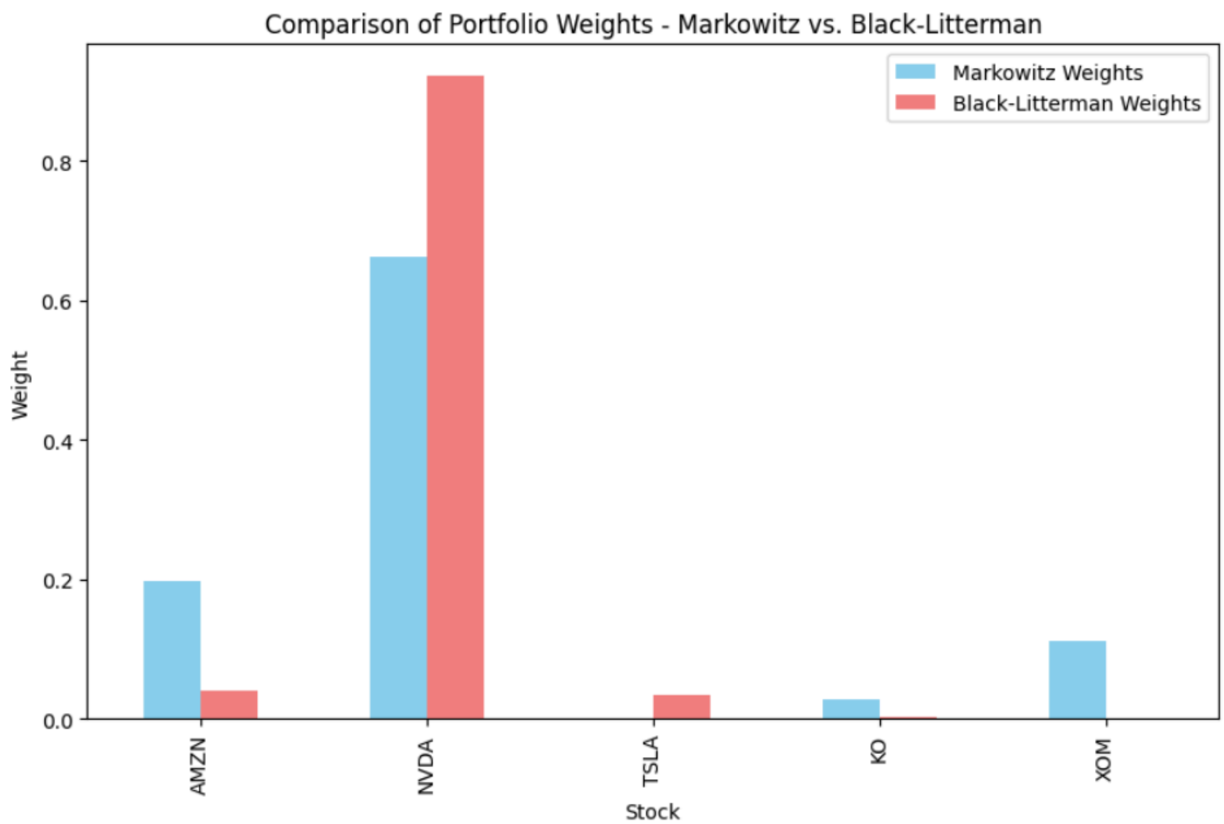
Coca-Cola: New product launches and changes of cost related to ingredients.

Exxon Mobil: Energy market fluctuations, fossil fuel demand, and oil prices.

2.

Comparison of Portfolio Weights (Markowitz vs. Black-Litterman):

	Stock	Markowitz_Weights	BL_Weights
AMZN	AMZN	0.196583	0.040354
NVDA	NVDA	0.662931	0.921292
TSLA	TSLA	0.000000	0.034855
KO	KO	0.028911	0.003499
XOM	XOM	0.111575	0.000000



Comparison of Portfolio Performance (Markowitz vs. Black-Litterman):

	Metric	Markowitz	Black-Litterman
0	Return	0.679818	11.589106
1	Volatility	0.127413	0.139964
2	Sharpe Ratio	5.257047	82.729025

3. Background information

The concept of optimizing a portfolio has become relevant as it helps in seeking or ensuring a balance of risk and return in a portfolio investment. There happens to be many approaches used but Markowitz's Mean-Variance Optimization (MVO) is more popular or widely used. It uses historical data to estimate risk, return, covariances and also the optimal weight of the portfolio.

As the MVO used only historical data, the Black Litterman Model (BLM) offers or uses an approach that includes or incorporates investors' views in the optimization process. Some usually argue that the investor might not have much knowledge of the market but I believe with this approach, the investor can also share his or her thoughts on when he or she thinks the market will look like. This is even a balanced approach as investors can bring on board some categorical or qualitative data if not quantitative that can definitely help in creating a better portfolio.

To talk about the Markowitz Mean-Variance Optimization (MVO) approach, it should be noted that its aim is to maximize the Sharpe Ratio of the portfolio where the Sharpe Ratio represents or explains the return of the portfolio from a unit risk. The MVO is computed in such a way that the optimal weights will be found while minimizing the variance of the portfolio at a specified return.

$$Mini: \sigma_p^2 = w^T \Sigma w$$

$$\sum_{i=1}^{n} w_i = 1 \text{ and } E(R_p) = \sum_{i=1}^{n} w_i \mu_i$$

where:

- σ_p^2 = the portfolio variance, that is the total risk
- w = the weight of the asset allocation in the portfolio
- Σ = this is the covariance of the asset returns which explains the relationship between the returns of each asset.
- $E(R_p)$ = this is simply the expected returns of the portfolio and as seen above, it is as a result of each asset u multiplied by its corresponding weight.

An efficient frontier is the main solution from this optimization which gives the portfolio with the highest expected returns given a level of risk or the lowest level of risk as explained earlier with the Sharpe Ratio above.

The BLM addresses the little limitation of the Markowitz model which was explained not only using historical data but also including investors' views of the returns. In this two step approach, we first estimate the equilibrium returns considering the that of market capitalization weights and that of the investor's risk aversion. Thereafter, the equilibrium returns are adjusted to incorporate the subjective views.

Equilibrium returns will be $\Pi = \delta\Sigma\omega_{mkt}$, where:

- δ = the risk aversion coefficient.
- ω_{mkt} = the market capitalization weights which indicates the relative size of each of the assets on the market.
- Σ = this is the covariance of the asset returns

At this point, the investors' views are incorporated. Their views can be expressed in a form of matrices.

This is how is computed:

$$\mu\beta L = ((\tau\Sigma)^{-1} + P^T\Omega^{-1}P)^{-1}((\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q)$$

where:

- τ = the scaling parameter
- $\mu\beta L$ = the adjusted returns after incorporating the investor’s view.
- P = this is the view matrix
- Q = this is the expected returns on the views
- Ω = the covariance of the views

If the risk aversion coefficient δ has a higher value, it means there is greater aversion to risk and thereby the model will favor lower risk assets.

The scaling parameter reflects or shows the balance between historical returns and the views of the investor. If it is low, then it means more weight is given to the investor’s views

Step 5: Kelly Criterion

1. Perform back-testing using the Kelly criterion for each security in the portfolio to size the allocation to that security.

First, we calculate the mean and variance of returns for each security. Then, we calculate the Kelly criterion weights for each stock and normalize to sum to 1 for portfolio allocation.

The Kelly weights are:

```
AMZN    0.062999

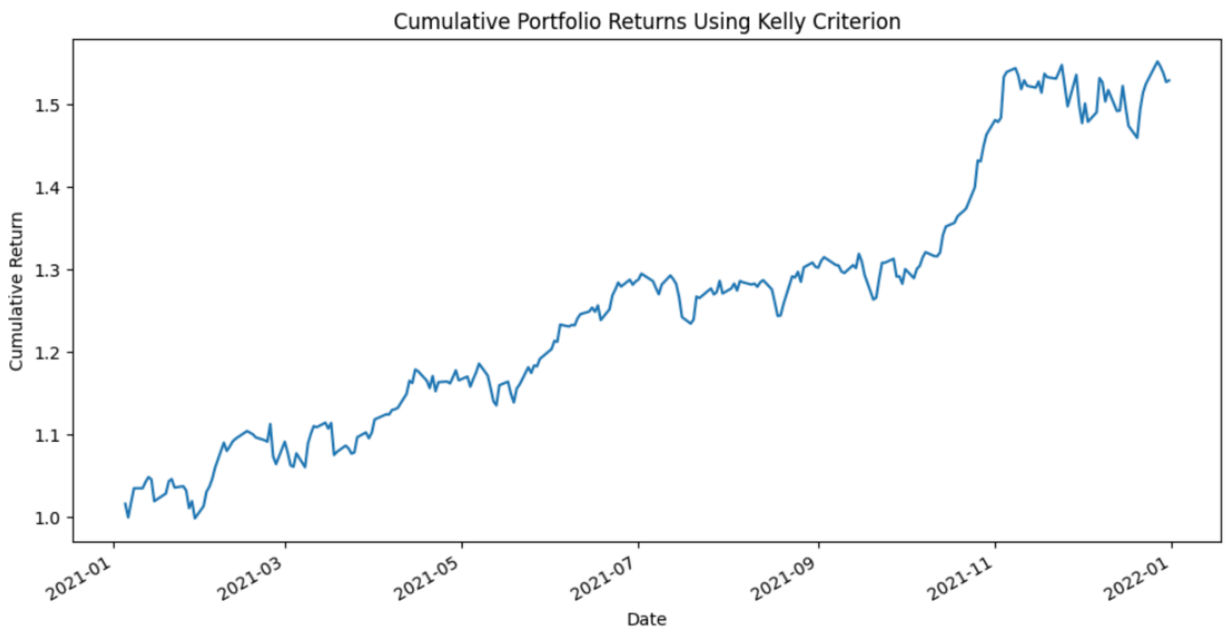
KO      0.354851
```

```
NVDA     0.220086

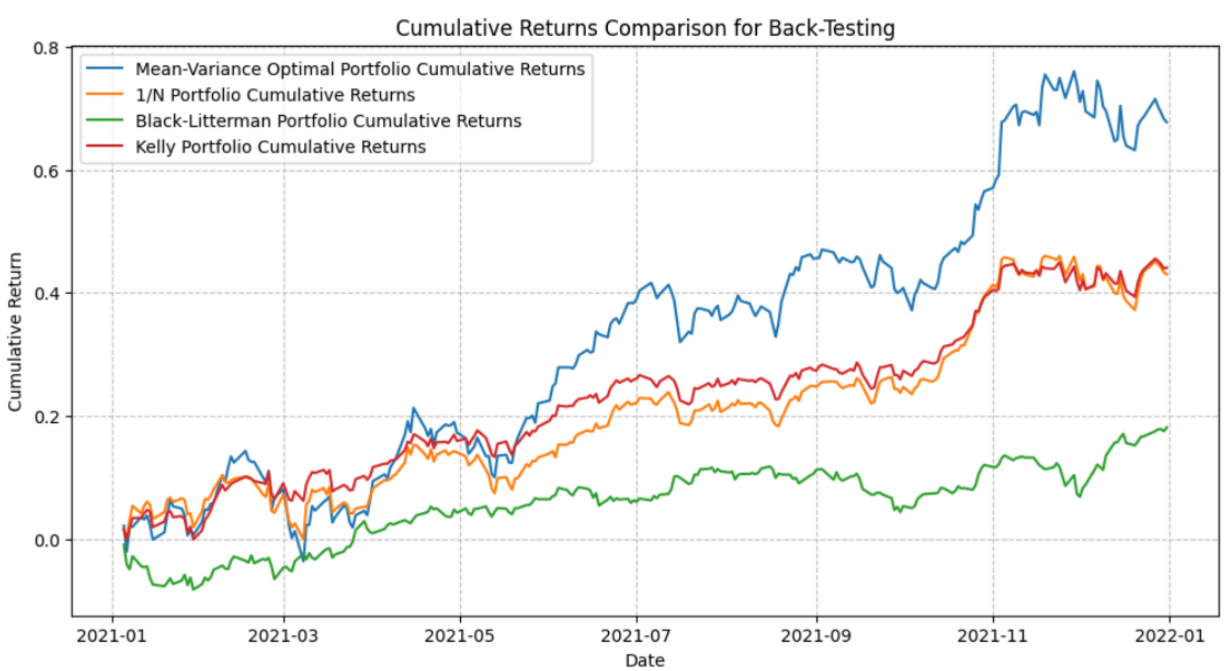
TSLA     0.084756

XOM      0.277307
```

After that, we calculate portfolio returns using the Kelly weights and calculate cumulative returns over the back-test period. The result is shown in the graph below:



2. Perform a series of historical backtests to see how the combined portfolio performs



First, we calculate the portfolio returns for different strategies: mean-variance optimal portfolio, 1/N portfolio, Black-Litterman portfolio, Kelly portfolio. Then, we calculate the cumulative returns for back-testing. The result is shown above.

3. Background information

The Kelly Criterion represents how to optimally bet for gambling and can be applied in investment. It tries to maximize the expected long-term growth rate for portfolio allocation, which means to maximize the expected logarithmic utility and the expected value of the logarithm of the total wealth. This results in a proportion of the total wealth to be allocated to the asset. Kelly Criterion reflects the trade-off between the expected reward and the risk of loss.

The Kelly Criterion formula

The Kelly fraction for an investment is calculated as:

$$f^* = \frac{\mu - r}{\sigma^2}$$

where:

- f^* is the Kelly fraction (the proportion of total capital allocating to the asset)
- μ is the expected return
- r is the risk-free rate
- σ^2 is the return variance

f^* is maximized to find the optimal fraction of the total wealth to invest in a given security.

$f^* > 1$ means to use leverage and $f^* < 1$ means to invest in both a risk-free bond and a risky asset.

The formula is similar to the Sharpe ratio, but not the same. It is actually the Sharpe ratio divided by the volatility of the asset.

For portfolios with multiple assets, the Kelly Criterion is adjusted as below:

$$f^* = (1 + r)\,\Sigma^{-1}\,(\mu - r)$$

where Σ is the variance covariance matrix for the returns of n assets.

To calculate the Kelly Criterion, the expected returns and covariance of all asset returns should be measured. The expected return, the variance and covariance can be calculated from the historical returns over a past period.

Advantages

Kelly Criterion provides an optimal way for asset allocation to maximize growth. It considers the reinvestment of returns over a multi-period horizon. This compounding of returns is a geometric calculation, in contrast with the arithmetic sum of returns that Markowitz and Sharpe are near-sighted (only sizing the current investment).

Disadvantages

Kelly might suggest wagers that are very large and thus, using the Kelly criterion can be very risky in the short term. The estimated volatility from Kelly Criterion is often high and estimation errors can result in over-allocated positions, which can lead to significant loss.

Besides, although theory and practical application of the Kelly criterion is straightforward, the underlying probability distributions are fairly known precisely.

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