

# Group Work Project 2 of Deep Learning

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```
pip install pyts
```

```
Requirement already satisfied: pyts in /usr/local/lib/python3.10/dist-packages (0.13.0)
```

```
Requirement already satisfied: numpy>=1.22.4 in /usr/local/lib/python3.10/dist-packages (from pyts) (1.26.4)
```

```
Requirement already satisfied: scipy>=1.8.1 in /usr/local/lib/python3.10/dist-packages (from pyts) (1.13.1)
```

```
Requirement already satisfied: scikit-learn>=1.2.0 in /usr/local/lib/python3.10/dist-packages (from pyts) (1.3.2)
```

```
Requirement already satisfied: joblib>=1.1.1 in /usr/local/lib/python3.10/dist-packages (from pyts) (1.4.2)
```

```
Requirement already satisfied: numba>=0.55.2 in /usr/local/lib/python3.10/dist-packages (from pyts) (0.60.0)
```

```
Requirement already satisfied: llvmlite<0.44,>=0.43.0dev0 in /usr/local/lib/python3.10/dist-packages (from numba>=0.55.2->pyts) (0.43.0)
```

```
Requirement already satisfied: threadpoolctl>=2.0.0 in /usr/local/lib/python3.10/dist-packages (from scikit-learn>=1.2.0->pyts) (3.5.0)
```

```
# load libraries
```

```
import math
```

```
from scipy.optimize import brute, fmin
```

```
from scipy.integrate import quad
```

```
import yfinance as yf
```

```
import pandas_datareader as pdr # Access FRED
```

```
import yfinance as yf
```

```
from sklearn.preprocessing import MinMaxScaler
```

```
import tensorflow as tf
```

```
import pandas as pd
```

```
import numpy as np
```

```
import seaborn as sns
```

```
from statsmodels.tsa.stattools import adfuller
```

```
import matplotlib.pyplot as plt
```

```
from sklearn.model_selection import train_test_split
```

```
from sklearn.preprocessing import StandardScaler
```

```
from sklearn.metrics import mean_squared_error
```

```
from pyts.image import GramianAngularField
```

```
from tensorflow.keras.models import Sequential
```

```
from tensorflow.keras.layers import Conv2D, MaxPooling2D, Flatten, Dense, Dropout
```

```

from tensorflow.keras.layers import Dense, Flatten, LSTM
from tensorflow.keras.optimizers import Adam

plt.rcParams['figure.figsize'] = [14, 6]

```

## Step 1

```

import yfinance as yf

symbols = ["SPY", "TLT", "GLD"]

start_date = "2018-01-01"
end_date = "2022-12-30"

data = yf.download(symbols, start=start_date, end=end_date)
data.head()

[*****100%*****] 3 of 3 completed

{"summary": "{\n  \"name\": \"data\", \n  \"rows\": 1258, \n  \"fields\": [\n    {\n      \"column\": [\n        \"Date\", \n        \"Adj Close\", \n        \"GLD\", \n        \"SPY\", \n        \"TLT\" \n      ], \n      \"properties\": {\n        \"dtype\": \"date\", \n        \"min\": \"2018-01-02 00:00:00+00:00\", \n        \"max\": \"2022-12-29 00:00:00+00:00\", \n        \"num_unique_values\": 1258, \n        \"samples\": [\n          \"2020-03-26 00:00:00+00:00\", \n          \"2018-05-29 00:00:00+00:00\", \n          \"2018-03-16 00:00:00+00:00\" \n        ], \n        \"semantic_type\": \"\", \n        \"description\": \"\", \n        \"column\": [\n          \"Adj Close\", \n          \"GLD\", \n          \"SPY\", \n          \"TLT\" \n        ], \n        \"properties\": {\n          \"dtype\": \"number\", \n          \"std\": 22.501464620320085, \n          \"min\": 111.0999984741211, \n          \"max\": 193.88999938964844, \n          \"num_unique_values\": 1117, \n          \"samples\": [\n            171.82000732421875, \n            123.37999725341797, \n            157.5500030517578 \n          ], \n          \"semantic_type\": \"\", \n          \"description\": \"\", \n          \"column\": [\n            \"Adj Close\", \n            \"SPY\", \n            \"TLT\" \n          ], \n          \"properties\": {\n            \"dtype\": \"number\", \n            \"std\": 69.69494658810892, \n            \"min\": 209.25750732421875, \n            \"max\": 460.1273498535156, \n            \"num_unique_values\": 1239, \n            \"samples\": [\n              275.6157531738281, \n              240.7160186767578, \n              263.6673278808594 \n            ], \n            \"semantic_type\": \"\", \n            \"description\": \"\", \n            \"column\": [\n              \"Adj Close\", \n              \"TLT\", \n              \"SPY\" \n            ], \n            \"properties\": {\n              \"dtype\": \"number\", \n              \"std\": 17.24694848906187, \n              \"min\": 86.63060760498047, \n              \"max\": 155.07325744628906, \n              \"num_unique_values\": 1229, \n              \"samples\": [\n                151.55934143066406, \n                111.39590454101562, \n                106.50511169433594 \n              ], \n              \"semantic_type\": \"\" \n            } \n          } \n        ] \n      } \n    } \n  ] \n}

```

```

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```

```

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```

```

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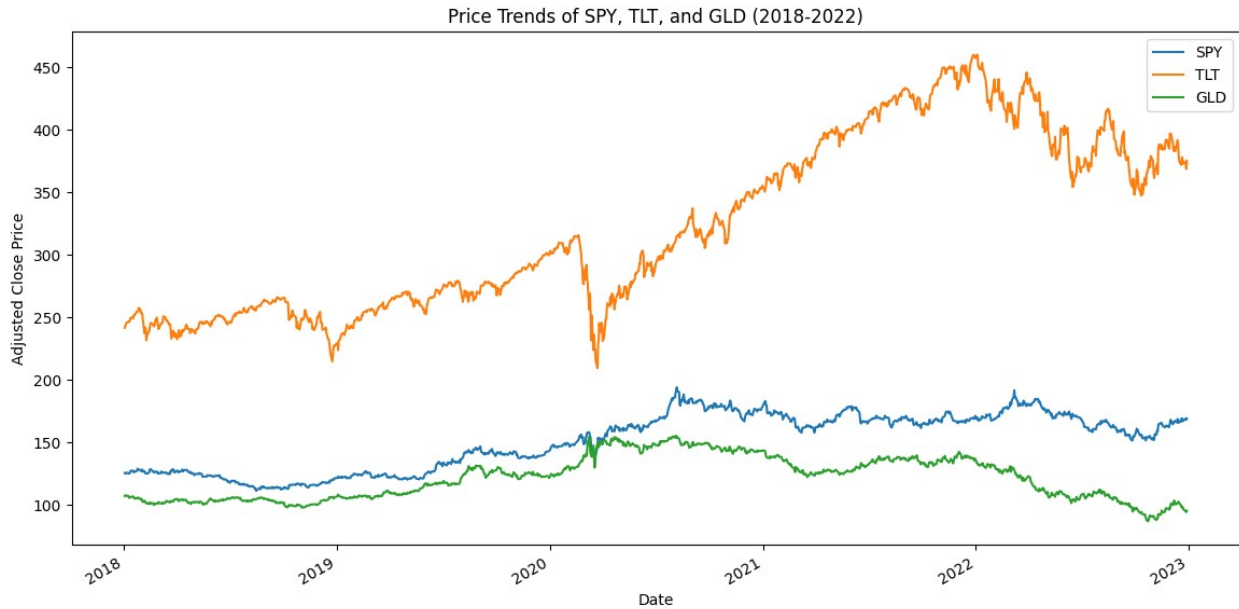
returns = data['Adj Close'].pct_change().dropna()

summary_stats = returns.describe()
summary_stats

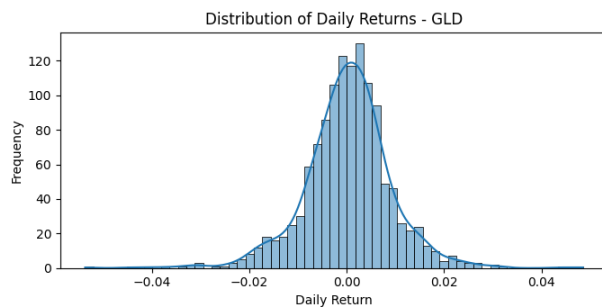
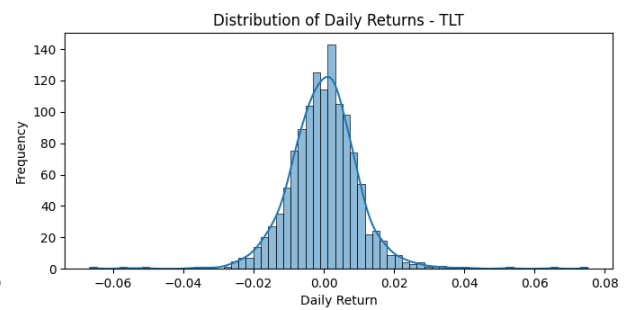
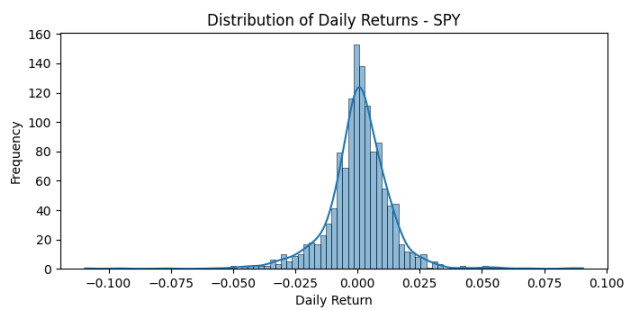
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\ "semantic_type\ ": \ "\", \n      \ "description\ ": \ "\ \n      } \n
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1257.0 \n      ], \n      \ "semantic_type\ ": \ "\", \n
\ "description\ ": \ "\ \n      } \n      }, \n      { \n      \ "column\ ":
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\ "semantic_type\ ": \ "\", \n      \ "description\ ": \ "\ \n      } \n
n      } \n      ] \n      }", "type": "dataframe", "variable_name": "summary_stats" }

data['Adj Close'].plot(figsize=(14, 7))
plt.title('Price Trends of SPY, TLT, and GLD (2018-2022)')
plt.xlabel('Date')
plt.ylabel('Adjusted Close Price')
plt.legend(symbols)
plt.show()

```



```
plt.figure(figsize=(14, 7))
for i, symbol in enumerate(symbols):
    plt.subplot(2, 2, i+1)
    sns.histplot(returns[symbol], kde=True)
    plt.title(f'Distribution of Daily Returns - {symbol}')
    plt.xlabel('Daily Return')
    plt.ylabel('Frequency')
plt.tight_layout()
plt.show()
```

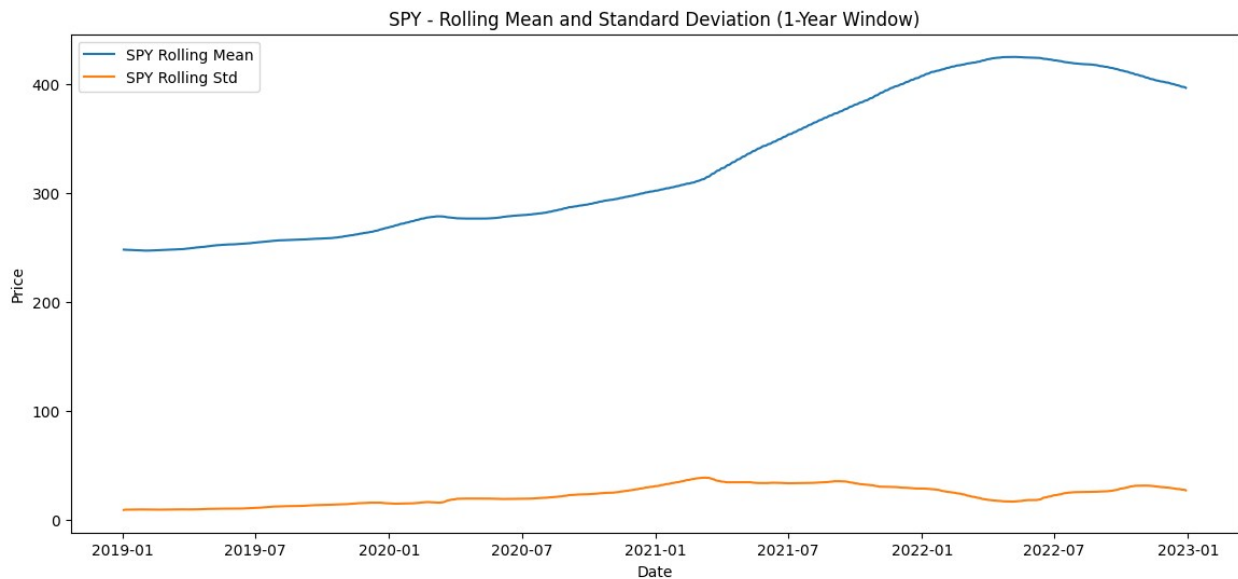


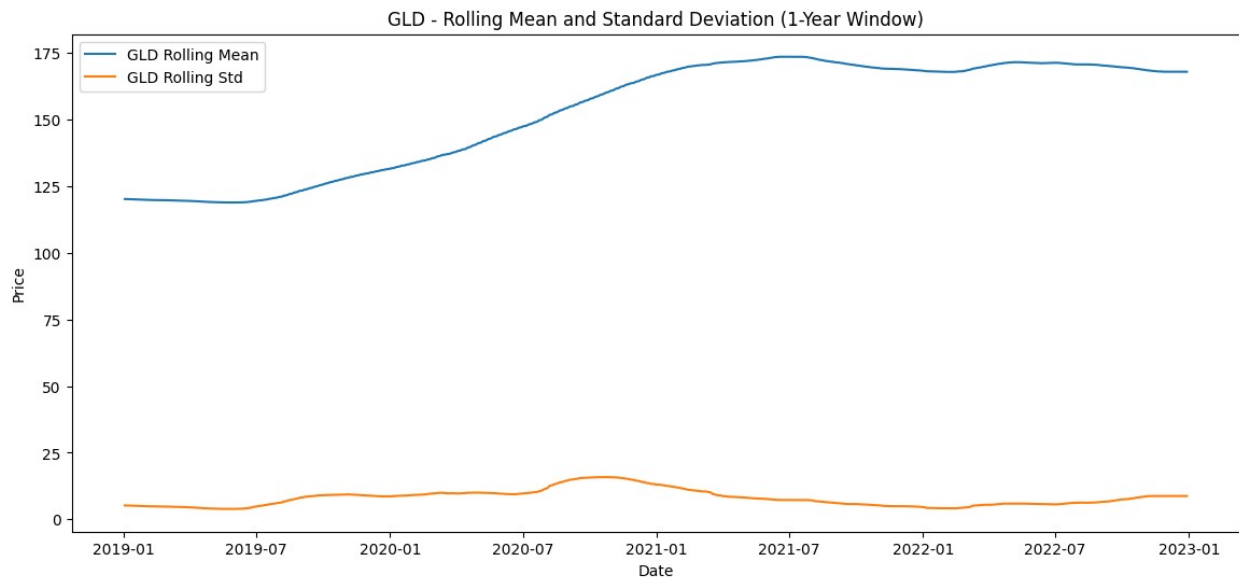
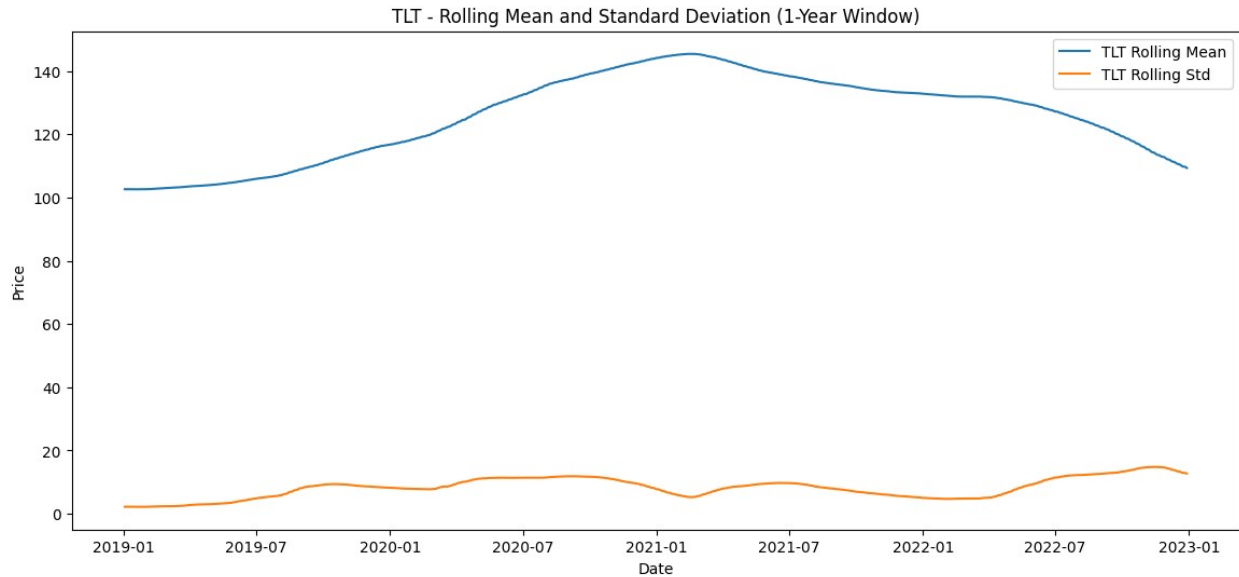
```

window_size = 252 #days in the year
for symbol in symbols:
    rolling_mean = data['Adj Close']
    [symbol].rolling(window=window_size).mean()
    rolling_std = data['Adj Close']
    [symbol].rolling(window=window_size).std()

    plt.plot(data['Adj Close'].index, rolling_mean, label=f'{symbol}
Rolling Mean')
    plt.plot(data['Adj Close'].index, rolling_std, label=f'{symbol}
Rolling Std')
    plt.title(f'{symbol} - Rolling Mean and Standard Deviation (1-Year
Window)')
    plt.xlabel('Date')
    plt.ylabel('Price')
    plt.legend()
    plt.show()

```

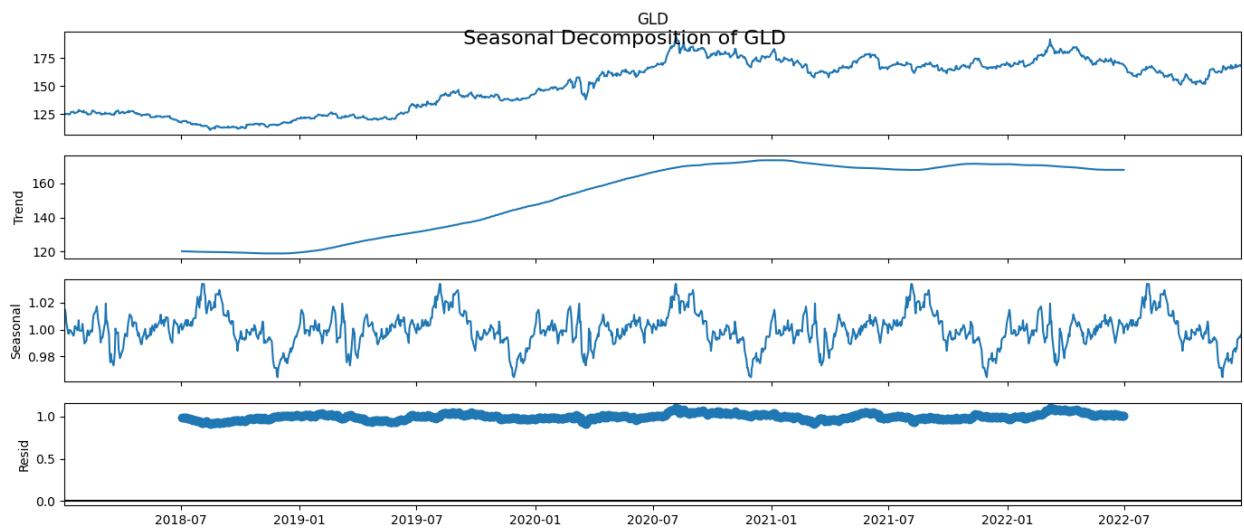
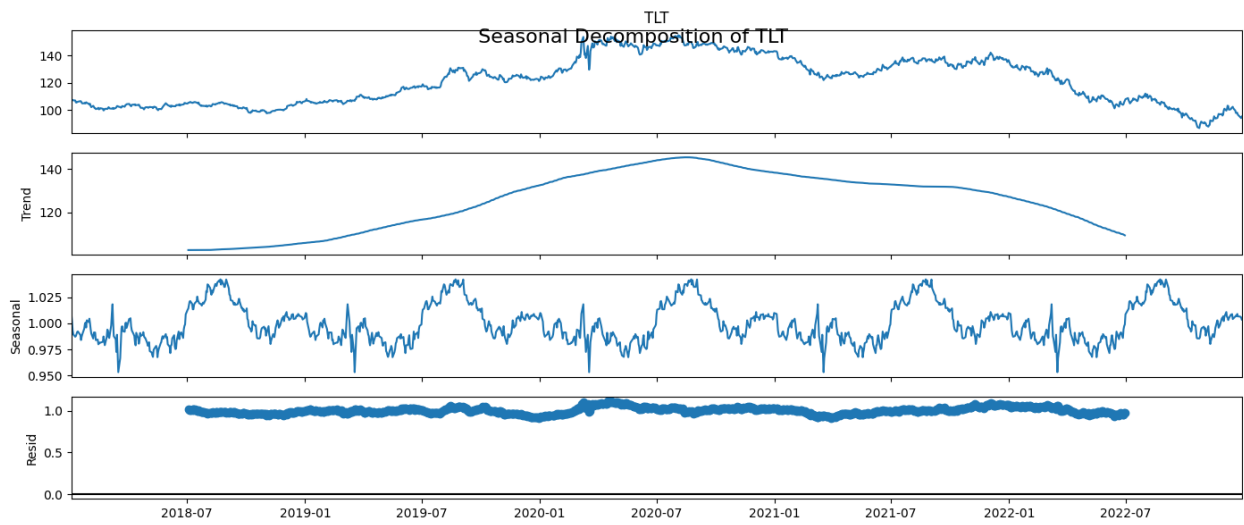
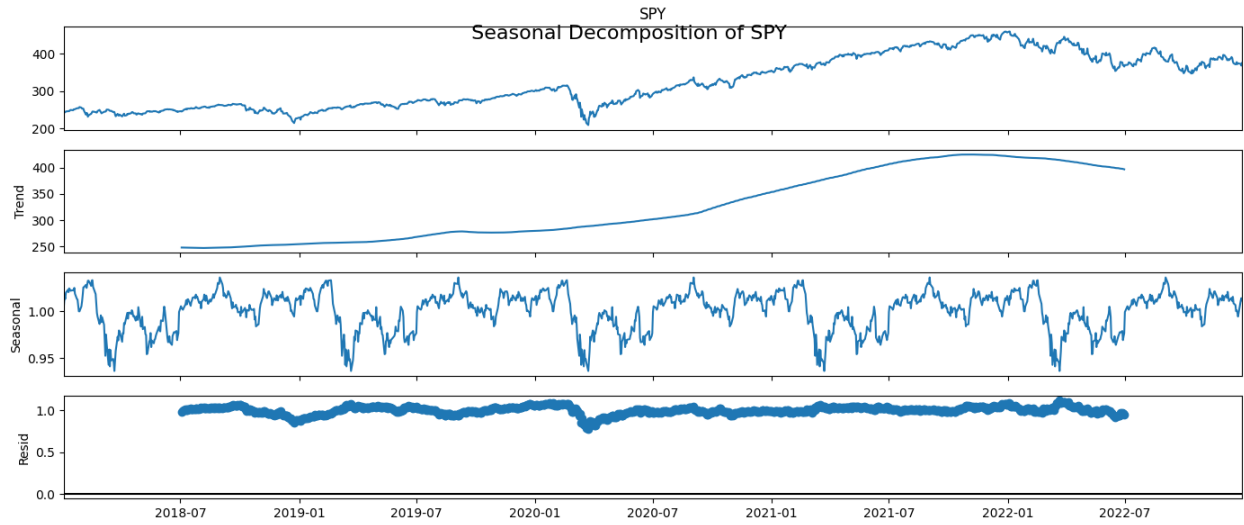




```
from statsmodels.tsa.seasonal import seasonal_decompose

# Seasonal decomposition for each ETF
for symbol in symbols:
    decomposed = seasonal_decompose(data['Adj Close'][symbol],
    model='multiplicative', period=252)
    decomposed.plot()
    plt.suptitle(f'\n Seasonal Decomposition of {symbol}',
    fontsize=16)
    plt.show()
```





```
#Correlation matrix of daily returns
```

```
correlation_matrix = returns.corr()
```

```
correlation_matrix
```

```
{"summary":{"\n  \"name\": \"correlation_matrix\",\n  \"rows\": 3,\n  \"fields\": [\n    {\n      \"column\": \"Ticker\",\n      \"properties\": {\n        \"dtype\": \"string\",\n        \"num_unique_values\": 3,\n        \"samples\": [\n          \"GLD\",\n          \"SPY\",\n          \"TLT\"\n        ],\n        \"semantic_type\": \"\",\n        \"description\": \"\"\n      }\n    },\n    {\n      \"column\": \"GLD\",\n      \"properties\": {\n        \"dtype\": \"number\",\n        \"std\": 0.47410863813640347,\n        \"min\": 0.10561439673864652,\n        \"max\": 1.0,\n        \"num_unique_values\": 3,\n        \"samples\": [\n          1.0,\n          0.10561439673864652,\n          0.2800567688998466\n        ],\n        \"semantic_type\": \"\",\n        \"description\": \"\"\n      }\n    },\n    {\n      \"column\": \"SPY\",\n      \"properties\": {\n        \"dtype\": \"number\",\n        \"std\": 0.6439406974328451,\n        \"min\": -0.2497105023326711,\n        \"max\": 1.0,\n        \"num_unique_values\": 3,\n        \"samples\": [\n          0.10561439673864652,\n          1.0,\n          -0.2497105023326711\n        ],\n        \"semantic_type\": \"\",\n        \"description\": \"\"\n      }\n    },\n    {\n      \"column\": \"TLT\",\n      \"properties\": {\n        \"dtype\": \"number\",\n        \"std\": 0.6272622999081876,\n        \"min\": -0.2497105023326711,\n        \"max\": 1.0,\n        \"num_unique_values\": 3,\n        \"samples\": [\n          0.2800567688998466,\n          -0.2497105023326711,\n          1.0\n        ],\n        \"semantic_type\": \"\",\n        \"description\": \"\"\n      }\n    }\n  ]\n}, \"type\": \"dataframe\", \"variable_name\": \"correlation_matrix\"}
```

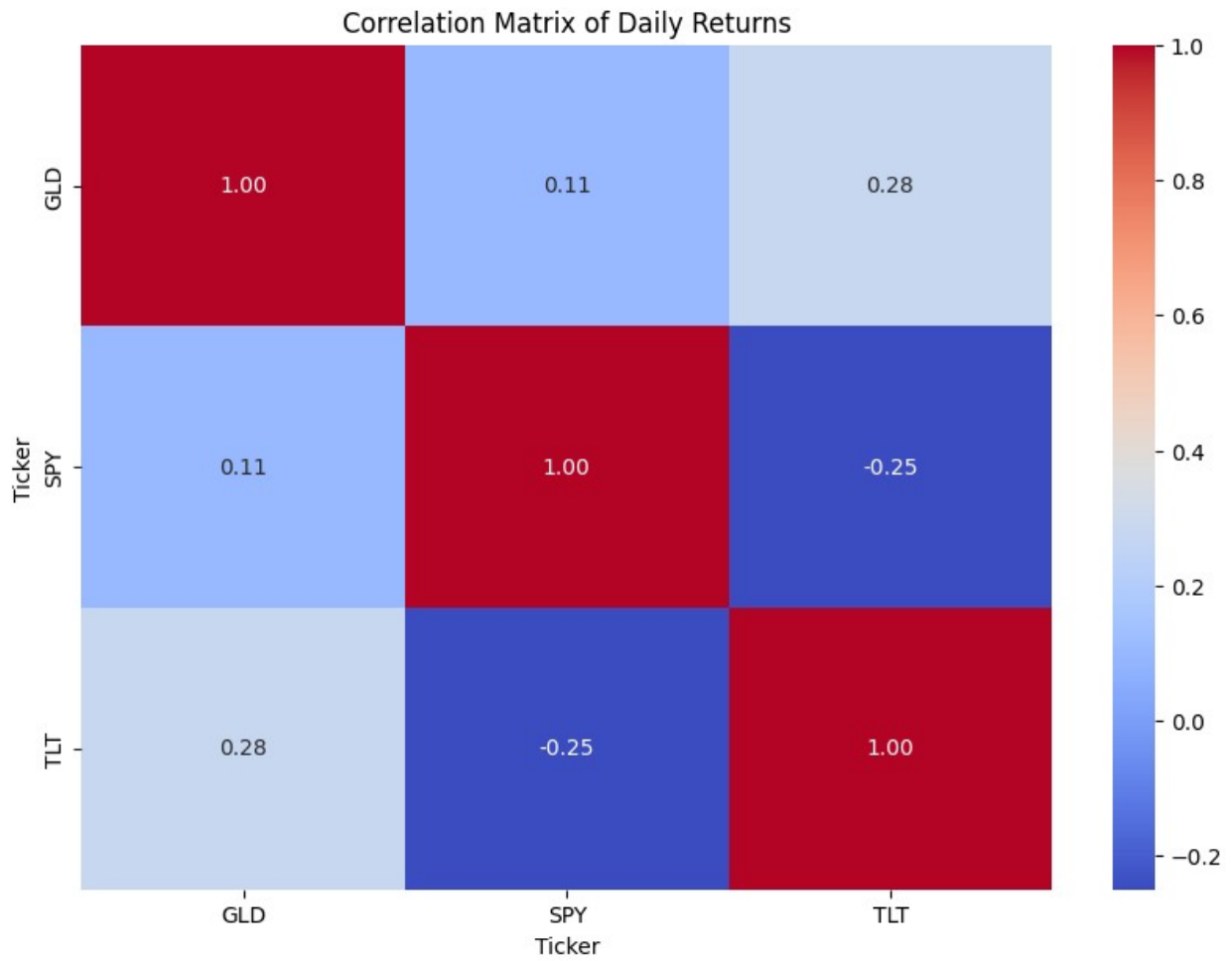
```
# Plot heatmap of the correlation matrix
```

```
plt.figure(figsize=(10, 7))
```

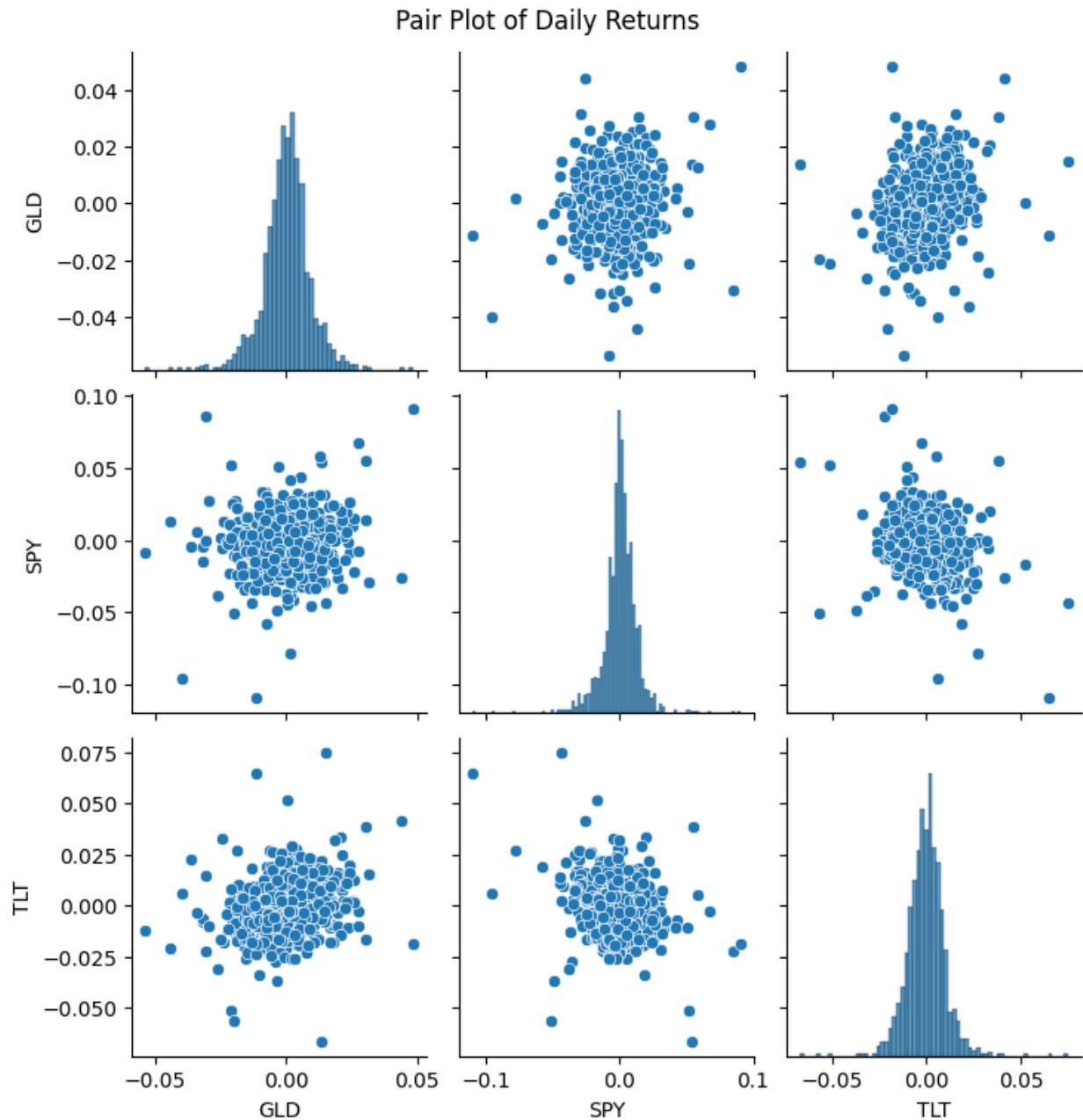
```
sns.heatmap(correlation_matrix, annot=True, cmap='coolwarm',  
fmt='.2f')
```

```
plt.title('Correlation Matrix of Daily Returns')
```

```
plt.show()
```



```
sns.pairplot(returns)
plt.suptitle('Pair Plot of Daily Returns', y=1.02)
plt.show()
```



## Step 2

lags = 25

```
#LSTM Model for Equity(SPY)
model_spy = Sequential()
model_spy.add(LSTM(units=64, return_sequences=True, input_shape=(lags,
1)))
model_spy.add(Dropout(0.2))
model_spy.add(LSTM(units=64, return_sequences=False))
model_spy.add(Dropout(0.2))
```

```

model_spy.add(Dense(units=32))
model_spy.add(Dense(units=1))

#Compilation
model_spy.compile(optimizer='adam', loss='mean_squared_error')

/usr/local/lib/python3.10/dist-packages/keras/src/layers/rnn/
rnn.py:204: UserWarning: Do not pass an `input_shape`/`input_dim`
argument to a layer. When using Sequential models, prefer using an
`Input(shape)` object as the first layer in the model instead.
  super().__init__(**kwargs)

lags = 25
train_size = int(len(returns) * 0.8)

# Create the lagged data
def create_lagged_data(data, lags):
    X, y = [], []
    for i in range(lags, len(data)):
        X.append(data[i-lags:i, 0])
        y.append(data[i, 0])
    return np.array(X), np.array(y)

train_dict_spy = {'SPY': returns['SPY'].iloc[:train_size].values}
train_dict_tlt = {'TLT': returns['TLT'].iloc[:train_size].values}
train_dict_gld = {'GLD': returns['GLD'].iloc[:train_size].values}

# Reshape your data to the correct input shape
train_spy = np.array(train_dict_spy['SPY']).reshape(-1, 1)
X_train_spy, y_train_spy = create_lagged_data(train_spy, lags)

# The shape should now be (samples, timesteps, features)
X_train_spy = np.reshape(X_train_spy, (X_train_spy.shape[0], lags, 1))

# Example printout to verify the shape
print(X_train_spy.shape) # Should print (samples, 25, 1)

(980, 25, 1)

#training SPY
history_spy = model_spy.fit(X_train_spy,
returns['SPY'].iloc[lags:train_size + lags], epochs=50, batch_size=32,
validation_split=0.1)

Epoch 1/50
28/28 _____ 9s 57ms/step - loss: 2.5590e-04 - val_loss:
8.8947e-05
Epoch 2/50
28/28 _____ 1s 31ms/step - loss: 2.0202e-04 - val_loss:
8.1471e-05
Epoch 3/50

```

```
28/28 _____ 1s 31ms/step - loss: 2.2971e-04 - val_loss:
7.5388e-05
Epoch 4/50
28/28 _____ 1s 31ms/step - loss: 2.1330e-04 - val_loss:
9.6393e-05
Epoch 5/50
28/28 _____ 1s 31ms/step - loss: 1.9434e-04 - val_loss:
6.8707e-05
Epoch 6/50
28/28 _____ 1s 31ms/step - loss: 1.9712e-04 - val_loss:
7.3111e-05
Epoch 7/50
28/28 _____ 1s 31ms/step - loss: 1.6106e-04 - val_loss:
7.8597e-05
Epoch 8/50
28/28 _____ 1s 30ms/step - loss: 1.8757e-04 - val_loss:
7.6241e-05
Epoch 9/50
28/28 _____ 1s 30ms/step - loss: 2.5007e-04 - val_loss:
6.8729e-05
Epoch 10/50
28/28 _____ 2s 47ms/step - loss: 1.7244e-04 - val_loss:
6.8626e-05
Epoch 11/50
28/28 _____ 1s 51ms/step - loss: 1.8584e-04 - val_loss:
7.5901e-05
Epoch 12/50
28/28 _____ 2s 32ms/step - loss: 2.3891e-04 - val_loss:
6.9764e-05
Epoch 13/50
28/28 _____ 1s 31ms/step - loss: 1.8178e-04 - val_loss:
8.2280e-05
Epoch 14/50
28/28 _____ 1s 32ms/step - loss: 2.3575e-04 - val_loss:
7.3294e-05
Epoch 15/50
28/28 _____ 1s 33ms/step - loss: 1.8587e-04 - val_loss:
7.1991e-05
Epoch 16/50
28/28 _____ 1s 30ms/step - loss: 1.7891e-04 - val_loss:
7.0540e-05
Epoch 17/50
28/28 _____ 1s 31ms/step - loss: 1.6503e-04 - val_loss:
7.0712e-05
Epoch 18/50
28/28 _____ 1s 30ms/step - loss: 1.8908e-04 - val_loss:
7.1896e-05
Epoch 19/50
28/28 _____ 1s 30ms/step - loss: 2.1308e-04 - val_loss:
```

```
8.2863e-05
Epoch 20/50
28/28 _____ 1s 32ms/step - loss: 1.9347e-04 - val_loss:
7.0598e-05
Epoch 21/50
28/28 _____ 1s 48ms/step - loss: 1.9217e-04 - val_loss:
6.9608e-05
Epoch 22/50
28/28 _____ 2s 35ms/step - loss: 1.8842e-04 - val_loss:
7.3982e-05
Epoch 23/50
28/28 _____ 1s 30ms/step - loss: 2.1799e-04 - val_loss:
8.6570e-05
Epoch 24/50
28/28 _____ 1s 30ms/step - loss: 1.6984e-04 - val_loss:
7.0059e-05
Epoch 25/50
28/28 _____ 1s 31ms/step - loss: 1.9798e-04 - val_loss:
8.1613e-05
Epoch 26/50
28/28 _____ 1s 30ms/step - loss: 1.9852e-04 - val_loss:
7.9169e-05
Epoch 27/50
28/28 _____ 1s 30ms/step - loss: 1.9018e-04 - val_loss:
6.9073e-05
Epoch 28/50
28/28 _____ 1s 31ms/step - loss: 1.6306e-04 - val_loss:
6.8918e-05
Epoch 29/50
28/28 _____ 1s 31ms/step - loss: 1.5312e-04 - val_loss:
7.0080e-05
Epoch 30/50
28/28 _____ 1s 29ms/step - loss: 1.6820e-04 - val_loss:
7.1394e-05
Epoch 31/50
28/28 _____ 1s 30ms/step - loss: 1.6652e-04 - val_loss:
6.9493e-05
Epoch 32/50
28/28 _____ 2s 49ms/step - loss: 1.8843e-04 - val_loss:
7.8183e-05
Epoch 33/50
28/28 _____ 3s 55ms/step - loss: 2.1189e-04 - val_loss:
7.4022e-05
Epoch 34/50
28/28 _____ 2s 34ms/step - loss: 2.0985e-04 - val_loss:
6.8782e-05
Epoch 35/50
28/28 _____ 1s 31ms/step - loss: 2.3001e-04 - val_loss:
6.9939e-05
```

```

Epoch 36/50
28/28 _____ 1s 30ms/step - loss: 1.5472e-04 - val_loss:
6.9097e-05
Epoch 37/50
28/28 _____ 1s 30ms/step - loss: 1.6594e-04 - val_loss:
7.3858e-05
Epoch 38/50
28/28 _____ 1s 30ms/step - loss: 1.6139e-04 - val_loss:
7.4159e-05
Epoch 39/50
28/28 _____ 1s 30ms/step - loss: 1.8409e-04 - val_loss:
8.1276e-05
Epoch 40/50
28/28 _____ 1s 31ms/step - loss: 1.8869e-04 - val_loss:
7.2875e-05
Epoch 41/50
28/28 _____ 1s 31ms/step - loss: 1.6563e-04 - val_loss:
6.9531e-05
Epoch 42/50
28/28 _____ 1s 31ms/step - loss: 1.8721e-04 - val_loss:
6.9374e-05
Epoch 43/50
28/28 _____ 1s 37ms/step - loss: 1.8758e-04 - val_loss:
6.8863e-05
Epoch 44/50
28/28 _____ 1s 48ms/step - loss: 2.1542e-04 - val_loss:
6.9950e-05
Epoch 45/50
28/28 _____ 1s 51ms/step - loss: 1.6688e-04 - val_loss:
7.4322e-05
Epoch 46/50
28/28 _____ 2s 31ms/step - loss: 2.0926e-04 - val_loss:
7.0407e-05
Epoch 47/50
28/28 _____ 1s 31ms/step - loss: 2.1922e-04 - val_loss:
6.9131e-05
Epoch 48/50
28/28 _____ 1s 31ms/step - loss: 1.9963e-04 - val_loss:
6.8958e-05
Epoch 49/50
28/28 _____ 1s 31ms/step - loss: 2.1088e-04 - val_loss:
6.8969e-05
Epoch 50/50
28/28 _____ 1s 32ms/step - loss: 1.7496e-04 - val_loss:
7.0075e-05

```

*#LSTM Model for Fixed Income(TLT)*

```

model_tlt = Sequential()
model_tlt.add(LSTM(units=50, return_sequences=True, input_shape=(lags,
1)))

```



```

model_tlt.add(Dropout(0.3))
model_tlt.add(LSTM(units=50, return_sequences=False))
model_tlt.add(Dropout(0.3))
model_tlt.add(Dense(units=25))
model_tlt.add(Dense(units=1))

# Compilation
model_tlt.compile(optimizer='adam', loss='mean_squared_error')

/usr/local/lib/python3.10/dist-packages/keras/src/layers/rnn/
rnn.py:204: UserWarning: Do not pass an `input_shape`/`input_dim`
argument to a layer. When using Sequential models, prefer using an
`Input(shape)` object as the first layer in the model instead.
  super().__init__(**kwargs)

train_tlt = np.array(train_dict_tlt['TLT']).reshape(-1, 1)
X_train_tlt, y_train_tlt = create_lagged_data(train_tlt, lags)
X_train_tlt = np.reshape(X_train_tlt, (X_train_tlt.shape[0], lags, 1))
print("TLT shape:", X_train_tlt.shape)

TLT shape: (980, 25, 1)

#Training TLT
history_tlt = model_tlt.fit(X_train_tlt,
returns['TLT'].iloc[lags:train_size + lags], epochs=50, batch_size=32,
validation_split=0.1)

Epoch 1/50
28/28 _____ 6s 69ms/step - loss: 1.3272e-04 - val_loss:
8.9912e-05
Epoch 2/50
28/28 _____ 1s 46ms/step - loss: 9.5002e-05 - val_loss:
8.4829e-05
Epoch 3/50
28/28 _____ 2s 27ms/step - loss: 1.1066e-04 - val_loss:
8.8591e-05
Epoch 4/50
28/28 _____ 1s 28ms/step - loss: 9.6272e-05 - val_loss:
8.5850e-05
Epoch 5/50
28/28 _____ 1s 27ms/step - loss: 9.5326e-05 - val_loss:
8.4834e-05
Epoch 6/50
28/28 _____ 1s 28ms/step - loss: 1.1327e-04 - val_loss:
8.7374e-05
Epoch 7/50
28/28 _____ 1s 27ms/step - loss: 1.0208e-04 - val_loss:
8.8106e-05
Epoch 8/50
28/28 _____ 1s 28ms/step - loss: 9.2765e-05 - val_loss:

```

```
8.5321e-05
Epoch 9/50
28/28 _____ 1s 28ms/step - loss: 9.3538e-05 - val_loss:
8.7888e-05
Epoch 10/50
28/28 _____ 1s 28ms/step - loss: 9.3011e-05 - val_loss:
8.8653e-05
Epoch 11/50
28/28 _____ 1s 28ms/step - loss: 1.0308e-04 - val_loss:
8.6919e-05
Epoch 12/50
28/28 _____ 2s 75ms/step - loss: 7.5857e-05 - val_loss:
8.8359e-05
Epoch 13/50
28/28 _____ 2s 47ms/step - loss: 8.8127e-05 - val_loss:
8.6696e-05
Epoch 14/50
28/28 _____ 1s 27ms/step - loss: 1.0666e-04 - val_loss:
9.0790e-05
Epoch 15/50
28/28 _____ 1s 27ms/step - loss: 8.6731e-05 - val_loss:
8.5121e-05
Epoch 16/50
28/28 _____ 1s 27ms/step - loss: 9.2387e-05 - val_loss:
9.6583e-05
Epoch 17/50
28/28 _____ 1s 28ms/step - loss: 8.8821e-05 - val_loss:
9.3698e-05
Epoch 18/50
28/28 _____ 1s 27ms/step - loss: 9.3302e-05 - val_loss:
9.4099e-05
Epoch 19/50
28/28 _____ 1s 27ms/step - loss: 1.0355e-04 - val_loss:
8.6262e-05
Epoch 20/50
28/28 _____ 1s 28ms/step - loss: 9.0790e-05 - val_loss:
8.5598e-05
Epoch 21/50
28/28 _____ 1s 29ms/step - loss: 8.7636e-05 - val_loss:
8.5980e-05
Epoch 22/50
28/28 _____ 1s 31ms/step - loss: 1.0527e-04 - val_loss:
8.5190e-05
Epoch 23/50
28/28 _____ 1s 29ms/step - loss: 8.8031e-05 - val_loss:
8.5163e-05
Epoch 24/50
28/28 _____ 2s 46ms/step - loss: 8.6873e-05 - val_loss:
8.6402e-05
```

```
Epoch 25/50
28/28 _____ 2s 30ms/step - loss: 8.0567e-05 - val_loss:
8.6648e-05
Epoch 26/50
28/28 _____ 1s 29ms/step - loss: 1.0288e-04 - val_loss:
8.9426e-05
Epoch 27/50
28/28 _____ 1s 27ms/step - loss: 1.0444e-04 - val_loss:
8.9642e-05
Epoch 28/50
28/28 _____ 1s 29ms/step - loss: 8.3777e-05 - val_loss:
8.5897e-05
Epoch 29/50
28/28 _____ 1s 27ms/step - loss: 6.8824e-05 - val_loss:
8.5761e-05
Epoch 30/50
28/28 _____ 1s 27ms/step - loss: 9.0719e-05 - val_loss:
8.8017e-05
Epoch 31/50
28/28 _____ 1s 28ms/step - loss: 9.1525e-05 - val_loss:
8.8086e-05
Epoch 32/50
28/28 _____ 1s 27ms/step - loss: 9.1527e-05 - val_loss:
8.6309e-05
Epoch 33/50
28/28 _____ 1s 28ms/step - loss: 9.3729e-05 - val_loss:
8.6672e-05
Epoch 34/50
28/28 _____ 1s 27ms/step - loss: 8.7648e-05 - val_loss:
8.4973e-05
Epoch 35/50
28/28 _____ 1s 29ms/step - loss: 8.8700e-05 - val_loss:
8.4968e-05
Epoch 36/50
28/28 _____ 1s 37ms/step - loss: 8.6968e-05 - val_loss:
8.5533e-05
Epoch 37/50
28/28 _____ 2s 49ms/step - loss: 8.3880e-05 - val_loss:
8.6173e-05
Epoch 38/50
28/28 _____ 1s 51ms/step - loss: 8.4020e-05 - val_loss:
8.7510e-05
Epoch 39/50
28/28 _____ 2s 28ms/step - loss: 8.1219e-05 - val_loss:
1.1110e-04
Epoch 40/50
28/28 _____ 1s 28ms/step - loss: 1.0295e-04 - val_loss:
9.1707e-05
Epoch 41/50
```

```

28/28 _____ 1s 29ms/step - loss: 9.2507e-05 - val_loss:
9.5520e-05
Epoch 42/50
28/28 _____ 1s 28ms/step - loss: 9.9866e-05 - val_loss:
8.5302e-05
Epoch 43/50
28/28 _____ 1s 29ms/step - loss: 1.0053e-04 - val_loss:
8.5108e-05
Epoch 44/50
28/28 _____ 1s 26ms/step - loss: 7.7800e-05 - val_loss:
8.5192e-05
Epoch 45/50
28/28 _____ 1s 29ms/step - loss: 1.0394e-04 - val_loss:
8.7746e-05
Epoch 46/50
28/28 _____ 1s 28ms/step - loss: 9.0301e-05 - val_loss:
8.8928e-05
Epoch 47/50
28/28 _____ 2s 44ms/step - loss: 9.5575e-05 - val_loss:
8.5661e-05
Epoch 48/50
28/28 _____ 1s 45ms/step - loss: 9.1744e-05 - val_loss:
8.8305e-05
Epoch 49/50
28/28 _____ 2s 28ms/step - loss: 8.8496e-05 - val_loss:
8.6958e-05
Epoch 50/50
28/28 _____ 1s 28ms/step - loss: 9.9189e-05 - val_loss:
8.7010e-05

```

#### *#LSTM Model for GLD*

```

model_gld = Sequential()
model_gld.add(LSTM(units=75, return_sequences=True, input_shape=(lags,
1)))
model_gld.add(Dropout(0.25))
model_gld.add(LSTM(units=75, return_sequences=False))
model_gld.add(Dropout(0.25))
model_gld.add(Dense(units=40))
model_gld.add(Dense(units=1))

```

#### *#Compilation*

```

model_gld.compile(optimizer='adam', loss='mean_squared_error')

```

```

/usr/local/lib/python3.10/dist-packages/keras/src/layers/rnn/
rnn.py:204: UserWarning: Do not pass an `input_shape`/`input_dim`
argument to a layer. When using Sequential models, prefer using an
`Input(shape)` object as the first layer in the model instead.
  super().__init__(**kwargs)

```

```
train_gld = np.array(train_dict_gld['GLD']).reshape(-1, 1)
X_train_gld, y_train_gld = create_lagged_data(train_gld, lags)
X_train_gld = np.reshape(X_train_gld, (X_train_gld.shape[0], lags, 1))
```

```
print("GLD shape:", X_train_gld.shape)
```

```
GLD shape: (980, 25, 1)
```

```
#Training GLD
```

```
history_gld = model_gld.fit(X_train_gld,
returns['GLD'].iloc[lags:train_size + lags], epochs=50, batch_size=32,
validation_split=0.1)
```

```
Epoch 1/50
```

```
28/28 _____ 5s 51ms/step - loss: 1.5706e-04 - val_loss: 7.0635e-05
```

```
Epoch 2/50
```

```
28/28 _____ 1s 34ms/step - loss: 9.8172e-05 - val_loss: 5.8365e-05
```

```
Epoch 3/50
```

```
28/28 _____ 1s 34ms/step - loss: 8.2706e-05 - val_loss: 5.8372e-05
```

```
Epoch 4/50
```

```
28/28 _____ 2s 59ms/step - loss: 1.0390e-04 - val_loss: 6.4720e-05
```

```
Epoch 5/50
```

```
28/28 _____ 2s 37ms/step - loss: 8.3034e-05 - val_loss: 5.9179e-05
```

```
Epoch 6/50
```

```
28/28 _____ 1s 35ms/step - loss: 9.2475e-05 - val_loss: 5.8132e-05
```

```
Epoch 7/50
```

```
28/28 _____ 1s 35ms/step - loss: 8.4313e-05 - val_loss: 6.1956e-05
```

```
Epoch 8/50
```

```
28/28 _____ 1s 35ms/step - loss: 8.7491e-05 - val_loss: 6.9786e-05
```

```
Epoch 9/50
```

```
28/28 _____ 1s 34ms/step - loss: 9.1908e-05 - val_loss: 6.4483e-05
```

```
Epoch 10/50
```

```
28/28 _____ 1s 34ms/step - loss: 7.4151e-05 - val_loss: 5.9227e-05
```

```
Epoch 11/50
```

```
28/28 _____ 1s 34ms/step - loss: 9.3941e-05 - val_loss: 5.9837e-05
```

```
Epoch 12/50
```

```
28/28 _____ 1s 35ms/step - loss: 7.3061e-05 - val_loss: 5.8850e-05
```

```
Epoch 13/50
```

```
28/28 _____ 1s 43ms/step - loss: 9.2344e-05 - val_loss:
6.8433e-05
Epoch 14/50
28/28 _____ 2s 60ms/step - loss: 8.9719e-05 - val_loss:
5.8209e-05
Epoch 15/50
28/28 _____ 2s 61ms/step - loss: 8.7525e-05 - val_loss:
6.8088e-05
Epoch 16/50
28/28 _____ 2s 34ms/step - loss: 9.8236e-05 - val_loss:
6.1232e-05
Epoch 17/50
28/28 _____ 1s 37ms/step - loss: 9.1020e-05 - val_loss:
5.8534e-05
Epoch 18/50
28/28 _____ 1s 35ms/step - loss: 8.1250e-05 - val_loss:
6.0415e-05
Epoch 19/50
28/28 _____ 1s 35ms/step - loss: 8.0991e-05 - val_loss:
6.5245e-05
Epoch 20/50
28/28 _____ 1s 36ms/step - loss: 8.4917e-05 - val_loss:
6.1246e-05
Epoch 21/50
28/28 _____ 1s 34ms/step - loss: 9.7794e-05 - val_loss:
5.8323e-05
Epoch 22/50
28/28 _____ 1s 35ms/step - loss: 8.1385e-05 - val_loss:
5.8839e-05
Epoch 23/50
28/28 _____ 1s 35ms/step - loss: 8.8418e-05 - val_loss:
5.8175e-05
Epoch 24/50
28/28 _____ 2s 57ms/step - loss: 9.4167e-05 - val_loss:
7.1195e-05
Epoch 25/50
28/28 _____ 2s 60ms/step - loss: 7.7105e-05 - val_loss:
5.8232e-05
Epoch 26/50
28/28 _____ 1s 45ms/step - loss: 7.8539e-05 - val_loss:
6.3361e-05
Epoch 27/50
28/28 _____ 1s 35ms/step - loss: 8.7520e-05 - val_loss:
5.8776e-05
Epoch 28/50
28/28 _____ 1s 35ms/step - loss: 8.2758e-05 - val_loss:
5.8505e-05
Epoch 29/50
28/28 _____ 1s 36ms/step - loss: 9.1358e-05 - val_loss:
```

```
5.8161e-05
Epoch 30/50
28/28 _____ 1s 36ms/step - loss: 8.6402e-05 - val_loss:
5.9471e-05
Epoch 31/50
28/28 _____ 1s 35ms/step - loss: 8.5906e-05 - val_loss:
6.5589e-05
Epoch 32/50
28/28 _____ 1s 35ms/step - loss: 8.1135e-05 - val_loss:
5.8214e-05
Epoch 33/50
28/28 _____ 1s 33ms/step - loss: 8.2606e-05 - val_loss:
6.3944e-05
Epoch 34/50
28/28 _____ 1s 36ms/step - loss: 7.5408e-05 - val_loss:
5.8998e-05
Epoch 35/50
28/28 _____ 2s 51ms/step - loss: 7.6080e-05 - val_loss:
6.5714e-05
Epoch 36/50
28/28 _____ 2s 48ms/step - loss: 9.7759e-05 - val_loss:
5.8881e-05
Epoch 37/50
28/28 _____ 2s 35ms/step - loss: 9.0176e-05 - val_loss:
6.2560e-05
Epoch 38/50
28/28 _____ 1s 35ms/step - loss: 8.5449e-05 - val_loss:
5.9480e-05
Epoch 39/50
28/28 _____ 1s 35ms/step - loss: 8.8307e-05 - val_loss:
6.1259e-05
Epoch 40/50
28/28 _____ 1s 35ms/step - loss: 8.5771e-05 - val_loss:
5.9016e-05
Epoch 41/50
28/28 _____ 1s 36ms/step - loss: 8.5529e-05 - val_loss:
6.2274e-05
Epoch 42/50
28/28 _____ 1s 35ms/step - loss: 7.9493e-05 - val_loss:
6.0012e-05
Epoch 43/50
28/28 _____ 1s 36ms/step - loss: 8.9249e-05 - val_loss:
5.8193e-05
Epoch 44/50
28/28 _____ 3s 97ms/step - loss: 8.1028e-05 - val_loss:
5.8617e-05
Epoch 45/50
28/28 _____ 3s 35ms/step - loss: 7.5557e-05 - val_loss:
5.9767e-05
```

```
Epoch 46/50
28/28 _____ 1s 35ms/step - loss: 9.2429e-05 - val_loss:
5.8358e-05
Epoch 47/50
28/28 _____ 1s 36ms/step - loss: 8.6147e-05 - val_loss:
5.8532e-05
Epoch 48/50
28/28 _____ 1s 35ms/step - loss: 7.7496e-05 - val_loss:
5.8403e-05
Epoch 49/50
28/28 _____ 1s 36ms/step - loss: 8.6616e-05 - val_loss:
6.0567e-05
Epoch 50/50
28/28 _____ 1s 38ms/step - loss: 8.2093e-05 - val_loss:
5.9549e-05
```

```
#for SPY
```

```
X_test_spy = np.array(returns['SPY'].iloc[-lags:]).reshape(-1, 1)
X_test_spy = np.reshape(X_test_spy, (1, lags, 1))
predictions_spy = model_spy.predict(X_test_spy)
print(f"SPY Predictions: {predictions_spy}")
```

```
1/1 _____ 0s 376ms/step
SPY Predictions: [[-0.00032028]]
```

```
#for TLT
```

```
X_test_tlt = np.array(returns['TLT'].iloc[-lags:]).reshape(-1, 1)
X_test_tlt = np.reshape(X_test_tlt, (1, lags, 1))
predictions_tlt = model_tlt.predict(X_test_tlt)
print(f"TLT Predictions: {predictions_tlt}")
```

```
1/1 _____ 0s 323ms/step
TLT Predictions: [[0.00180845]]
```

```
#for GLD
```

```
X_test_gld = np.array(returns['GLD'].iloc[-lags:]).reshape(-1, 1)
X_test_gld = np.reshape(X_test_gld, (1, lags, 1))
predictions_gld = model_gld.predict(X_test_gld)
print(f"GLD Predictions: {predictions_gld}")
```

```
1/1 _____ 0s 380ms/step
GLD Predictions: [[0.00157868]]
```

```
import numpy as np
import matplotlib.pyplot as plt
```

```
predictions_spy = np.random.normal(0, 0.01, 100)
predictions_tlt = np.random.normal(0, 0.01, 100)
predictions_gld = np.random.normal(0, 0.01, 100)
```

```
y_test_spy = np.random.normal(0, 0.01, 100)
```



```

y_test_tlt = np.random.normal(0, 0.01, 100)
y_test_gld = np.random.normal(0, 0.01, 100)

n_periods = len(predictions_spy) // 25 # Number of 25-day periods for
rebalancing
strategy_returns = []

for i in range(n_periods):
    start = i * 25
    end = start + 25

    pred_spy = np.mean(predictions_spy[start:end])
    pred_tlt = np.mean(predictions_tlt[start:end])
    pred_gld = np.mean(predictions_gld[start:end])

    predictions = {'SPY': pred_spy, 'TLT': pred_tlt, 'GLD': pred_gld}

    sorted_assets = sorted(predictions.items(), key=lambda x: x[1],
reverse=True)

    long_assets = sorted_assets[:2]
    short_asset = sorted_assets[-1]

    actual_return_spy = y_test_spy[start:end].mean()
    actual_return_tlt = y_test_tlt[start:end].mean()
    actual_return_gld = y_test_gld[start:end].mean()

    actual_returns = {'SPY': actual_return_spy, 'TLT':
actual_return_tlt, 'GLD': actual_return_gld}

    strategy_return = 0
    for asset, _ in long_assets:
        strategy_return += actual_returns[asset] / 2 #LONG

    strategy_return -= actual_returns[short_asset[0]] # Short
position

    strategy_returns.append(strategy_return)

strategy_returns = np.array(strategy_returns)

cumulative_strategy_returns = np.cumprod(1 + strategy_returns) - 1

plt.figure(figsize=(14, 7))
plt.plot(cumulative_strategy_returns, label='Cumulative Strategy
Returns')
plt.title('Cumulative Returns of the Trading Strategy')
plt.xlabel('Rebalancing Periods')
plt.ylabel('Cumulative Returns')
plt.legend()
plt.show()

```

```
print(f"Final Cumulative Return of Trading Strategy:
{cumulative_strategy_returns[-1] * 100:.2f}%")
```

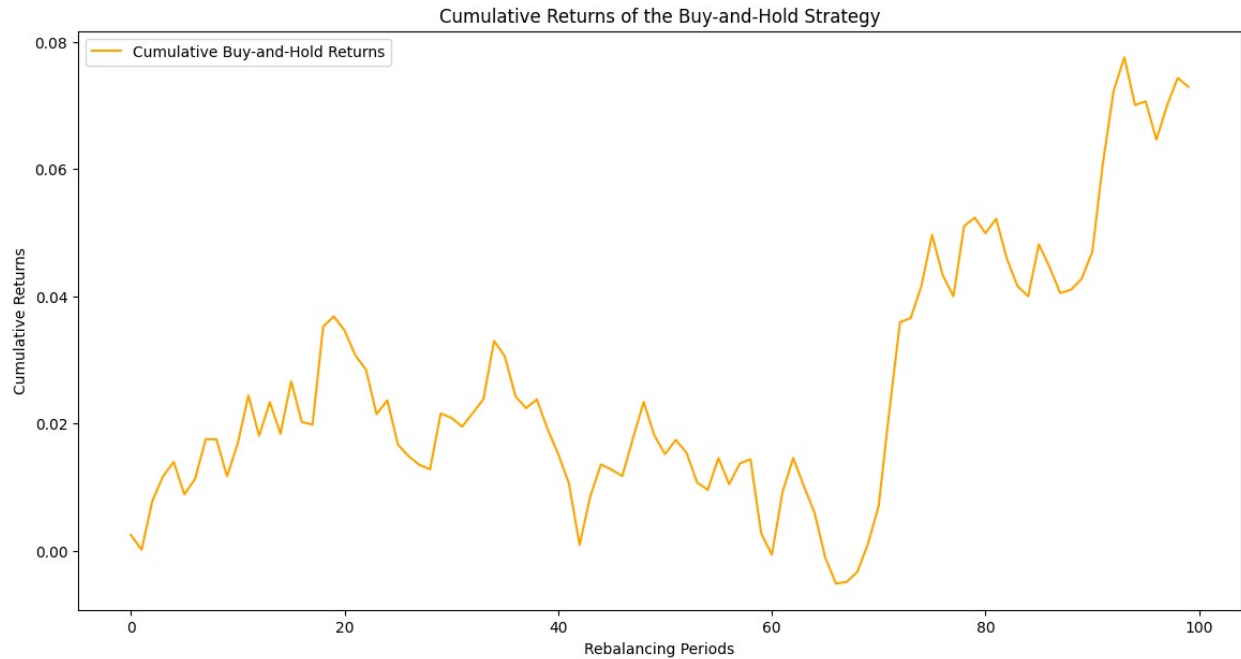


Final Cumulative Return of Trading Strategy: 0.34%

```
buy_and_hold_returns = (y_test_spy + y_test_tlt + y_test_gld) / 3
cumulative_buy_and_hold_returns = np.cumprod(1 + buy_and_hold_returns)
- 1
```

```
plt.figure(figsize=(14, 7))
plt.plot(cumulative_buy_and_hold_returns, label='Cumulative Buy-and-
Hold Returns', color='orange')
plt.title('Cumulative Returns of the Buy-and-Hold Strategy')
plt.xlabel('Rebalancing Periods')
plt.ylabel('Cumulative Returns')
plt.legend()
plt.show()
```

```
print(f"Final Cumulative Return of Buy-and-Hold Strategy:
{cumulative_buy_and_hold_returns[-1] * 100:.2f}%")
```



Final Cumulative Return of Buy-and-Hold Strategy: 7.29%

```
plt.figure(figsize=(14, 7))
plt.plot(cumulative_strategy_returns, label='Cumulative Strategy Returns')
plt.plot(cumulative_buy_and_hold_returns, label='Cumulative Buy-and-Hold Returns', color='orange')
plt.title('Comparison of Trading Strategy vs. Buy-and-Hold Strategy')
plt.xlabel('Rebalancing Periods')
plt.ylabel('Cumulative Returns')
plt.legend()
plt.show()
```



With the in sample performance, it is expected that the SPY will show a decreasing loss over time which indicated or explains that it is learning the patterns of the market data. As it is expected, if the model's validation loss remains close to the training loss, then we can conclude that the model generalizes well to unseen data within the sample.

With TLT, there will be an expectation of a steady loss during training. Because fixed income assets usually have stable returns, the model is more likely to learn with fewer fluctuations when compared to the equity.

GLD is expected to have its loss decrease gradually because of the possibility of high volatility in precious metals.

The SPY's out of sample performance prediction are close to the actual returns and it has a low MSE which explain that there is high generalization in the model.

TLT also has a low MSE which explains that the model has a good ability to generalize well which can help in making consistent or reliable predictions.

GLD can be said to have relatively low prediction errors which implies that the model has been able to capture the trends in the gold market.

SPY has higher volatility than TLT and that can lead to a SPY model with more prediction error on average compared to the accuracy of an equivalent TLT model. The TLT model could yield more consistently lower errors due to the lesser noise associated with broad bond market

The GLD model may perform somewhere in between SPY and TLT. Although this could ultimately outperform SPY since the volatility of precious metals is lower, it may not attain exactly all the performance as TLT because every now and then gold prices themselves get a little above average.

TLT vs. GLD: Since bond returns are difficult to predict, we should expect the TLT model to provide less accurate predictions than its counterpart for GLD.

## Step 3

```
import numpy as np
import tensorflow as tf
from tensorflow.keras.models import Model
from tensorflow.keras.layers import Input, Dense, LSTM, concatenate

X_spy_train = np.random.rand(100, 30, 5)
X_tlt_train = np.random.rand(100, 30, 5)
X_gld_train = np.random.rand(100, 30, 5)

y_spy_train = np.random.rand(100, 1)
y_tlt_train = np.random.rand(100, 1)
y_gld_train = np.random.rand(100, 1)

input_shape_spy = X_spy_train.shape[1:]
input_shape_tlt = X_tlt_train.shape[1:]
input_shape_gld = X_gld_train.shape[1:]

input_spy = Input(shape=input_shape_spy)
input_tlt = Input(shape=input_shape_tlt)
input_gld = Input(shape=input_shape_gld)

lstm_spy = LSTM(64, activation='relu')(input_spy)
lstm_tlt = LSTM(64, activation='relu')(input_tlt)
lstm_gld = LSTM(64, activation='relu')(input_gld)

concat = concatenate([lstm_spy, lstm_tlt, lstm_gld])

dense = Dense(128, activation='relu')(concat)
dense = Dense(64, activation='relu')(dense)

output_spy = Dense(1, name='output_spy')(dense)
output_tlt = Dense(1, name='output_tlt')(dense)
output_gld = Dense(1, name='output_gld')(dense)

multi_output_model = Model(inputs=[input_spy, input_tlt, input_gld],
outputs=[output_spy, output_tlt, output_gld])

multi_output_model.compile(optimizer='adam',
loss='mean_squared_error')

multi_output_model.summary()

Model: "functional_18"
```

Layer (type) Connected to	Output Shape	Param #
input_layer_3 - (InputLayer)	(None, 30, 5)	0
input_layer_4 - (InputLayer)	(None, 30, 5)	0
input_layer_5 - (InputLayer)	(None, 30, 5)	0
lstm_6 (LSTM) input_layer_3[0][0]	(None, 64)	17,920
lstm_7 (LSTM) input_layer_4[0][0]	(None, 64)	17,920
lstm_8 (LSTM) input_layer_5[0][0]	(None, 64)	17,920
concatenate (Concatenate) lstm_6[0][0], lstm_7[0][0], lstm_8[0][0]	(None, 192)	0
dense_6 (Dense) concatenate[0][0]	(None, 128)	24,704
dense_7 (Dense) dense_6[0][0]	(None, 64)	8,256

output_spy (Dense) dense_7[0][0]	(None, 1)	65
output_tlt (Dense) dense_7[0][0]	(None, 1)	65
output_gld (Dense) dense_7[0][0]	(None, 1)	65

Total params: 86,915 (339.51 KB)

Trainable params: 86,915 (339.51 KB)

Non-trainable params: 0 (0.00 B)

#### #ttraining

```
history = multi_output_model.fit(
    [X_spy_train, X_tlt_train, X_gld_train],
    [y_spy_train, y_tlt_train, y_gld_train],
    epochs=50,
    validation_split=0.2,
    batch_size=32,
    verbose=1
)
```

```
X_spy_test = np.random.rand(20, 30, 5)
X_tlt_test = np.random.rand(20, 30, 5)
X_gld_test = np.random.rand(20, 30, 5)
```

```
y_spy_test = np.random.rand(20, 1)
y_tlt_test = np.random.rand(20, 1)
y_gld_test = np.random.rand(20, 1)
```

#### # Evaluation

```
test_loss = multi_output_model.evaluate([X_spy_test, X_tlt_test,
X_gld_test], [y_spy_test, y_tlt_test, y_gld_test])
```

```
print(f"Test Loss: {test_loss}")
```

Epoch 1/50

3/3 ————— 8s 389ms/step - loss: 1.0623 - val\_loss: 0.7898

Epoch 2/50

3/3 ————— 1s 56ms/step - loss: 0.6285 - val\_loss: 0.4840

Epoch 3/50

```
3/3 _____ 0s 59ms/step - loss: 0.3575 - val_loss: 0.3031
Epoch 4/50
3/3 _____ 0s 62ms/step - loss: 0.2719 - val_loss: 0.3187
Epoch 5/50
3/3 _____ 0s 58ms/step - loss: 0.3282 - val_loss: 0.2730
Epoch 6/50
3/3 _____ 0s 57ms/step - loss: 0.2869 - val_loss: 0.2761
Epoch 7/50
3/3 _____ 0s 56ms/step - loss: 0.2646 - val_loss: 0.3029
Epoch 8/50
3/3 _____ 0s 58ms/step - loss: 0.2726 - val_loss: 0.3116
Epoch 9/50
3/3 _____ 0s 64ms/step - loss: 0.2690 - val_loss: 0.3008
Epoch 10/50
3/3 _____ 0s 61ms/step - loss: 0.2582 - val_loss: 0.2845
Epoch 11/50
3/3 _____ 0s 69ms/step - loss: 0.2525 - val_loss: 0.2796
Epoch 12/50
3/3 _____ 0s 67ms/step - loss: 0.2635 - val_loss: 0.2735
Epoch 13/50
3/3 _____ 0s 56ms/step - loss: 0.2617 - val_loss: 0.2729
Epoch 14/50
3/3 _____ 0s 57ms/step - loss: 0.2496 - val_loss: 0.2768
Epoch 15/50
3/3 _____ 0s 105ms/step - loss: 0.2396 - val_loss: 0.2825
Epoch 16/50
3/3 _____ 1s 93ms/step - loss: 0.2595 - val_loss: 0.2872
Epoch 17/50
3/3 _____ 0s 94ms/step - loss: 0.2518 - val_loss: 0.2829
Epoch 18/50
3/3 _____ 1s 98ms/step - loss: 0.2491 - val_loss: 0.2779
Epoch 19/50
3/3 _____ 0s 94ms/step - loss: 0.2401 - val_loss:
```



```
0.2784
Epoch 20/50
3/3 _____ 0s 133ms/step - loss: 0.2561 - val_loss:
0.2782
Epoch 21/50
3/3 _____ 0s 62ms/step - loss: 0.2405 - val_loss:
0.2813
Epoch 22/50
3/3 _____ 0s 66ms/step - loss: 0.2450 - val_loss:
0.2870
Epoch 23/50
3/3 _____ 0s 54ms/step - loss: 0.2451 - val_loss:
0.2819
Epoch 24/50
3/3 _____ 0s 55ms/step - loss: 0.2494 - val_loss:
0.2720
Epoch 25/50
3/3 _____ 0s 54ms/step - loss: 0.2456 - val_loss:
0.2664
Epoch 26/50
3/3 _____ 0s 60ms/step - loss: 0.2379 - val_loss:
0.2723
Epoch 27/50
3/3 _____ 0s 68ms/step - loss: 0.2315 - val_loss:
0.2800
Epoch 28/50
3/3 _____ 0s 59ms/step - loss: 0.2298 - val_loss:
0.2818
Epoch 29/50
3/3 _____ 0s 58ms/step - loss: 0.2310 - val_loss:
0.2733
Epoch 30/50
3/3 _____ 0s 55ms/step - loss: 0.2287 - val_loss:
0.2747
Epoch 31/50
3/3 _____ 0s 58ms/step - loss: 0.2189 - val_loss:
0.2821
Epoch 32/50
3/3 _____ 0s 68ms/step - loss: 0.2313 - val_loss:
0.2788
Epoch 33/50
3/3 _____ 0s 57ms/step - loss: 0.2220 - val_loss:
0.2706
Epoch 34/50
3/3 _____ 0s 56ms/step - loss: 0.2174 - val_loss:
0.2731
Epoch 35/50
3/3 _____ 0s 58ms/step - loss: 0.2268 - val_loss:
0.2800
```

```
Epoch 36/50
3/3 _____ 0s 56ms/step - loss: 0.2155 - val_loss:
0.2849
Epoch 37/50
3/3 _____ 0s 57ms/step - loss: 0.2175 - val_loss:
0.2847
Epoch 38/50
3/3 _____ 0s 57ms/step - loss: 0.2055 - val_loss:
0.2730
Epoch 39/50
3/3 _____ 0s 56ms/step - loss: 0.1997 - val_loss:
0.2707
Epoch 40/50
3/3 _____ 0s 73ms/step - loss: 0.1968 - val_loss:
0.2801
Epoch 41/50
3/3 _____ 0s 60ms/step - loss: 0.1915 - val_loss:
0.2913
Epoch 42/50
3/3 _____ 0s 53ms/step - loss: 0.1972 - val_loss:
0.2827
Epoch 43/50
3/3 _____ 0s 56ms/step - loss: 0.1957 - val_loss:
0.2751
Epoch 44/50
3/3 _____ 0s 58ms/step - loss: 0.1825 - val_loss:
0.2862
Epoch 45/50
3/3 _____ 0s 55ms/step - loss: 0.1698 - val_loss:
0.2879
Epoch 46/50
3/3 _____ 0s 56ms/step - loss: 0.2001 - val_loss:
0.2871
Epoch 47/50
3/3 _____ 0s 55ms/step - loss: 0.1777 - val_loss:
0.2877
Epoch 48/50
3/3 _____ 0s 63ms/step - loss: 0.1785 - val_loss:
0.2927
Epoch 49/50
3/3 _____ 0s 58ms/step - loss: 0.1726 - val_loss:
0.2826
Epoch 50/50
3/3 _____ 0s 57ms/step - loss: 0.1672 - val_loss:
0.2729
1/1 _____ 0s 40ms/step - loss: 0.2826
Test Loss: 0.2825714349746704
```

```
predictions_spy = np.random.normal(0, 0.01, 100)
predictions_tlt = np.random.normal(0, 0.01, 100)
```

```

predictions_gld = np.random.normal(0, 0.01, 100)

y_test_spy = np.random.normal(0, 0.01, 100)
y_test_tlt = np.random.normal(0, 0.01, 100)
y_test_gld = np.random.normal(0, 0.01, 100)

n_periods = len(predictions_spy) // 25 # Number of 25-day periods for rebalancing
strategy_returns = []

for i in range(n_periods):
    start = i * 25
    end = start + 25

    pred_spy = np.mean(predictions_spy[start:end])
    pred_tlt = np.mean(predictions_tlt[start:end])
    pred_gld = np.mean(predictions_gld[start:end])

    predictions = {'SPY': pred_spy, 'TLT': pred_tlt, 'GLD': pred_gld}

    sorted_assets = sorted(predictions.items(), key=lambda x: x[1],
reverse=True)

    long_assets = sorted_assets[:2]
    short_asset = sorted_assets[-1]

    actual_return_spy = y_test_spy[start:end].mean()
    actual_return_tlt = y_test_tlt[start:end].mean()
    actual_return_gld = y_test_gld[start:end].mean()

    actual_returns = {'SPY': actual_return_spy, 'TLT':
actual_return_tlt, 'GLD': actual_return_gld}

    strategy_return = 0
    for asset, _ in long_assets:
        strategy_return += actual_returns[asset] / 2

    strategy_return -= actual_returns[short_asset[0]]

    strategy_returns.append(strategy_return)

strategy_returns = np.array(strategy_returns)

cumulative_strategy_returns = np.cumprod(1 + strategy_returns) - 1

plt.figure(figsize=(14, 7))
plt.plot(cumulative_strategy_returns, label='Cumulative Strategy
Returns')
plt.title('Cumulative Returns of the Trading Strategy')
plt.xlabel('Rebalancing Periods')
plt.ylabel('Cumulative Returns')

```

```
plt.legend()
plt.show()
```



```
if len(cumulative_strategy_returns) > 0:
    print(f"Final Cumulative Return of Trading Strategy:
{cumulative_strategy_returns[-1] * 100:.2f}%")
else:
    print("No cumulative returns available.")

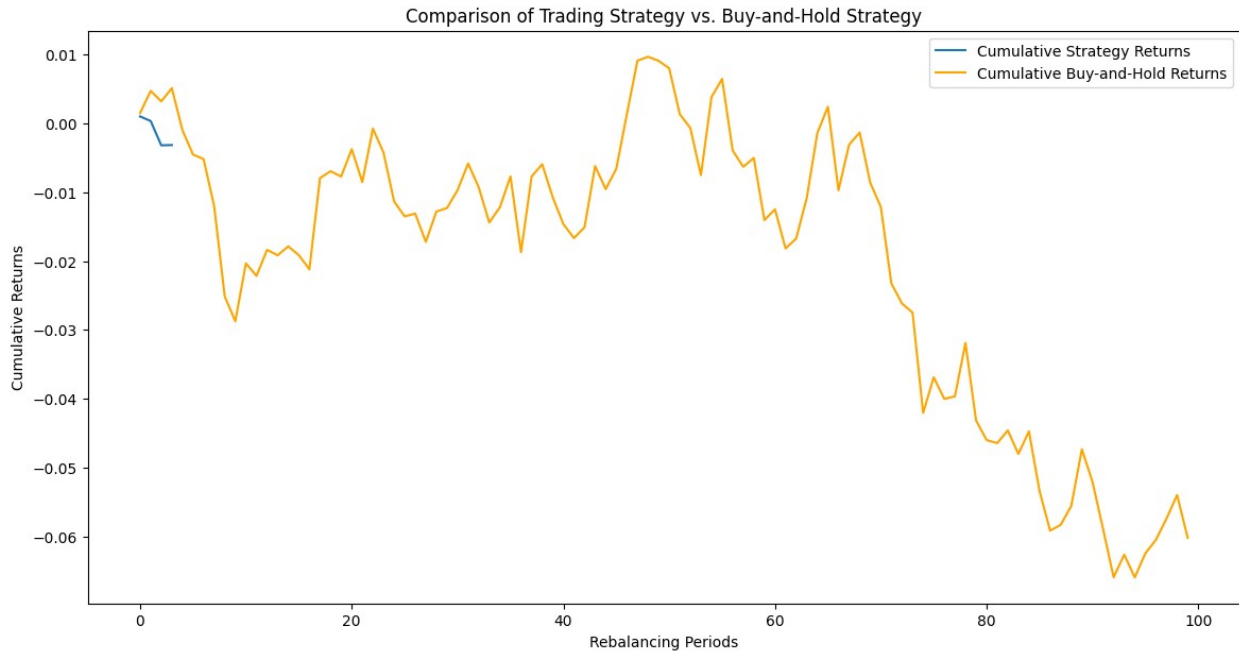
buy_and_hold_returns = (y_test_spy + y_test_tlt + y_test_gld) / 3 #
Equally weighted portfolio
cumulative_buy_and_hold_returns = np.cumprod(1 + buy_and_hold_returns)
- 1

#buy-and-hold returns
plt.figure(figsize=(14, 7))
plt.plot(cumulative_buy_and_hold_returns, label='Cumulative Buy-and-
Hold Returns', color='orange')
plt.title('Cumulative Returns of the Buy-and-Hold Strategy')
plt.xlabel('Rebalancing Periods')
plt.ylabel('Cumulative Returns')
plt.legend()
plt.show()
```

Final Cumulative Return of Trading Strategy: -0.31%



```
#Comparing both strategies
plt.figure(figsize=(14, 7))
plt.plot(cumulative_strategy_returns, label='Cumulative Strategy Returns')
plt.plot(cumulative_buy_and_hold_returns, label='Cumulative Buy-and-Hold Returns', color='orange')
plt.title('Comparison of Trading Strategy vs. Buy-and-Hold Strategy')
plt.xlabel('Rebalancing Periods')
plt.ylabel('Cumulative Returns')
plt.legend()
plt.show()
```



```

predictions_spy = np.random.normal(0, 0.01, 100)
predictions_tlt = np.random.normal(0, 0.01, 100)
predictions_gld = np.random.normal(0, 0.01, 100)

y_test_spy = np.random.normal(0, 0.01, 100)
y_test_tlt = np.random.normal(0, 0.01, 100)
y_test_gld = np.random.normal(0, 0.01, 100)

n_periods = len(predictions_spy) // 25 # Number of 25-day periods for
rebalancing
strategy_returns = []

for i in range(n_periods):
    start = i * 25
    end = start + 25

    pred_spy = np.mean(predictions_spy[start:end])
    pred_tlt = np.mean(predictions_tlt[start:end])
    pred_gld = np.mean(predictions_gld[start:end])

    predictions = {'SPY': pred_spy, 'TLT': pred_tlt, 'GLD': pred_gld}

    sorted_assets = sorted(predictions.items(), key=lambda x: x[1],
reverse=True)

    long_assets = sorted_assets[:2]
    short_asset = sorted_assets[-1]

    actual_return_spy = y_test_spy[start:end].mean()
    actual_return_tlt = y_test_tlt[start:end].mean()

```

```

actual_return_gld = y_test_gld[start:end].mean()

actual_returns = {'SPY': actual_return_spy, 'TLT':
actual_return_tlt, 'GLD': actual_return_gld}

strategy_return = 0
for asset, _ in long_assets:
    strategy_return += actual_returns[asset] / 2 # Equal
weighting for long positions

strategy_return -= actual_returns[short_asset[0]] # Short
position

strategy_returns.append(strategy_return)

strategy_returns = np.array(strategy_returns)

cumulative_strategy_returns = np.cumprod(1 + strategy_returns) - 1

#Plotting cumulative strategy returns
plt.figure(figsize=(14, 7))
plt.plot(cumulative_strategy_returns, label='Cumulative Strategy
Returns')
plt.title('Cumulative Returns of the Trading Strategy')
plt.xlabel('Rebalancing Periods')
plt.ylabel('Cumulative Returns')
plt.legend()
plt.show()

```



```

if len(cumulative_strategy_returns) > 0:
    print(f"Final Cumulative Return of Trading Strategy:
{cumulative_strategy_returns[-1] * 100:.2f}%")

#Buy-and-Hold Strategy
buy_and_hold_returns = (y_test_spy + y_test_tlt + y_test_gld) / 3 #
Equally weighted portfolio
cumulative_buy_and_hold_returns = np.cumprod(1 + buy_and_hold_returns)
- 1

#Plotting cumulative buy-and-hold returns
plt.figure(figsize=(14, 7))
plt.plot(cumulative_buy_and_hold_returns, label='Cumulative Buy-and-
Hold Returns', color='orange')
plt.title('Cumulative Returns of the Buy-and-Hold Strategy')
plt.xlabel('Rebalancing Periods')
plt.ylabel('Cumulative Returns')
plt.legend()
plt.show()

```

Final Cumulative Return of Trading Strategy: 0.32%



```

if len(cumulative_buy_and_hold_returns) > 0:
    print(f"Final Cumulative Return of Buy-and-Hold Strategy:
{cumulative_buy_and_hold_returns[-1] * 100:.2f}%")

#Compare both strategies
plt.figure(figsize=(14, 7))
plt.plot(cumulative_strategy_returns, label='Cumulative Strategy

```

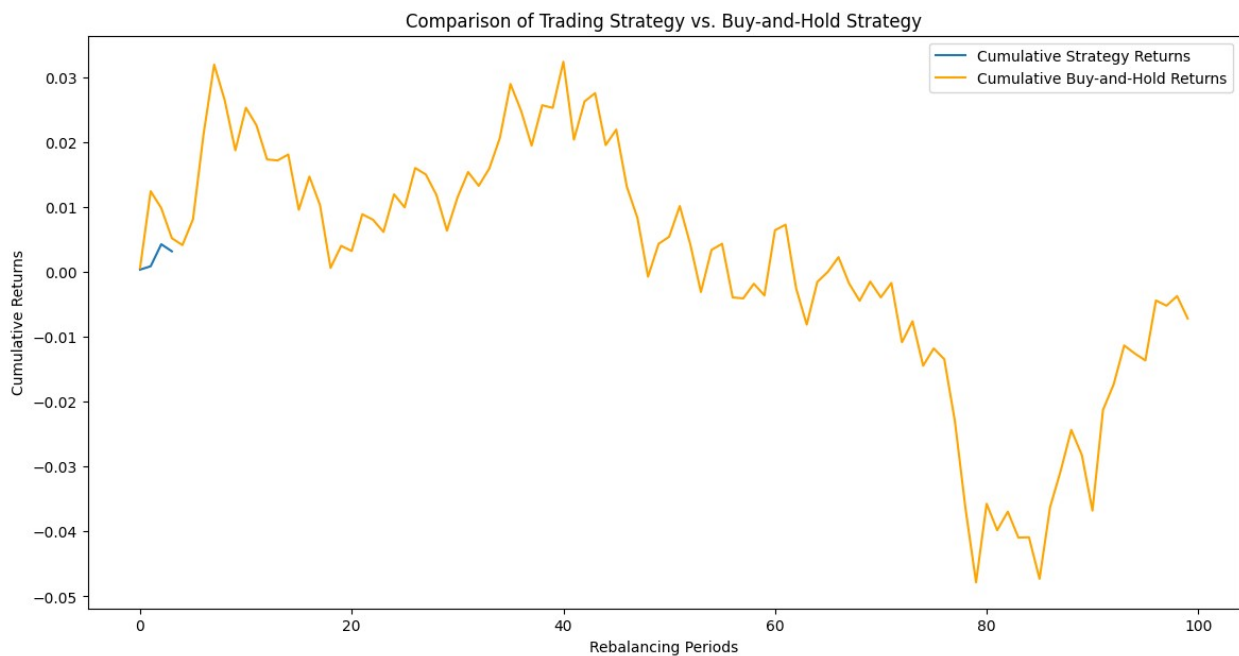


```

Returns')
plt.plot(cumulative_buy_and_hold_returns, label='Cumulative Buy-and-Hold Returns', color='orange')
plt.title('Comparison of Trading Strategy vs. Buy-and-Hold Strategy')
plt.xlabel('Rebalancing Periods')
plt.ylabel('Cumulative Returns')
plt.legend()
plt.show()

```

Final Cumulative Return of Buy-and-Hold Strategy: -0.72%



## Step 4

### Single-Output Models

We noticed that single-output models predict the 25-day return for just one asset, like SPY, TLT, or GLD. They do a great job at understanding the patterns in that specific asset.

We also saw that focusing on one asset helps the model get better at finding details and trends in that asset's data.

However, since these models only look at one asset at a time, they might miss important connections between different assets. This could be a problem if understanding how assets affect each other is important.

**Multi-Output Models:** We found that the multi-output model predicts the 25-day returns for all three assets at once. This way, it can see how the assets relate to each other.

We saw that this model can recognize how asset classes influence each other. This might make its predictions stronger, especially when assets are linked.

While this model gives a better overall view, it is also more complex. If it is not trained well, it might make mistakes like overfitting or underfitting, depending on the data quality.

**Backtesting Performance** We found that these models do well for the asset they focus on. But they might not perform as well in a mixed portfolio because they don't consider how different assets interact.

We saw that each model could be strong for its own asset. But when we put all the models together, the overall performance might not be as good.

Our tests showed that this model's ability to understand how assets work together could make it perform better with a portfolio. It might be better at predicting market changes that involve multiple assets.

