# Interest Rates and Equity Valuations*

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## Abstract

A large body of work has sought to measure the effect of interest rates on equity valuations. The challenge in doing so is that both are endogenous, and their comovement depends on the forces driving interest-rate changes. To address this problem, we develop and estimate a decomposition that splits movements in real rates into three structural drivers: changes in expected growth, risk, or "pure discounting." We show that only pure discount-rate shocks transmit one-for-one to equity valuations, with little or negative transmission of growth and risk shocks. Implementing our decomposition with a global panel of growth expectations and asset prices, we find a weak unconditional relation between valuations and real rates but a strong relation with the pure discounting component, which explains 80% of cross-country valuation changes since 1990. In the U.S., we find that 35% of the interest-rate decline is attributable to pure discounting, implying that only a fraction of the change in rates has passed through directly to equities. We use the decomposition to revisit evidence on the role of interest rates in explaining price variation, and to study higher-frequency returns, cross-sectional rate exposures, duration-matched equity premia, and reactions to monetary policy.


Keywords: Stock prices, interest rates, duration, long-term growth

JEL Codes: G10, G12, E44, F30

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# 1. Introduction

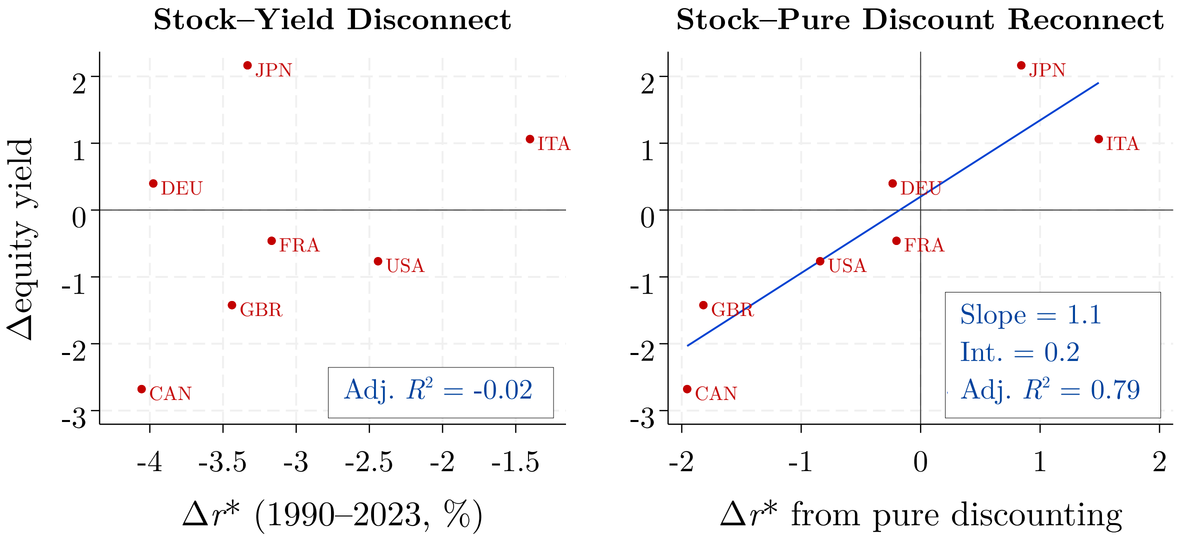
Advanced economies' long-term interest rates have declined significantly in recent decades. How do such changes in rates transmit to equity valuations? A potentially tempting line of reasoning is to assume that equity discount rates move one-for-one with interest rates; since equity is a long-term asset whose value is highly sensitive to discount rates, this assumption implies that stock valuations should have increased substantially as a result of the secular decline in interest rates. High-level evidence might appear consistent with this view: in the U.S., for example, the market's equity yield has declined substantially in recent decades, corresponding to a large increase in equity valuations over this period.

Empirically, however, there is no clear relationship between long-term changes in equity valuations and interest rates. The left panel of Figure 1 presents one view of this stock–yield disconnect: across G7 economies, the change in a country's equity yield since 1990 is effectively completely unrelated to that country's change in the trend long-term real rate $r^*$, described further below. In addition to the example in Figure 1, it is well known that the correlation between stock and bond returns is weak and often negative (e.g., Campbell, Sunderam, and Viceira 2017) — another example of the stock–yield disconnect.

The apparent stock–yield disconnect arises because the interest-rate sensitivity of stock prices is more complicated than alluded to in our opening paragraph. Interest rates are determined endogenously and may decline for multiple possible structural reasons, each of which may affect equity differently. Interest rates may, for instance, decline because of a decrease in expected growth rates in the economy, which — keeping all else constant — will decrease equity prices and thus mute the effects of the decline in interest rates on equity.

In this paper, we provide a framework and measurement approach to control for the underlying drivers of interest-rate movements and estimate the interest-rate sensitivity of equity prices. We start with a simple but general theoretical decomposition under which any change in trend real rates can be split into three mutually exclusive shocks: a change in expected growth, a change in uncertainty, and a "pure discounting" shock akin to a change in the rate of time preference. We then characterize how these shocks transmit to equity yields, the inverse of equity valuation ratios. The pure discounting shock transmits one-for-one from rates to equity yields, inducing perfect comovement between stocks and duration-matched bonds. But the remaining terms induce a weak and ambiguous relationship between stocks and bonds: a growth-rate shock affects both equity discount rates and cash-flow growth, while an uncertainty shock causes interest rates and equity risk premia to move in opposing directions. Isolating the pure discounting component of real rates is therefore key for understanding how much any given change in interest rates has passed through to equities.

**Figure 1: Preview of Main Results: Long-Term Decomposition**




*Notes:* This figure plots the country-level changes in equity yields against changes in trend interest rates (left panel) and in the estimated change in the pure discounting component of interest rates (right panel), for G7 economies. The sample is 1990–2023, or the longest available span for the given country. Details on the measurement of the equity yield, trend long-term interest rate $r^*$, and the pure discounting component of the change in $r^*$ are provided in Section 3.2.

We next implement the decomposition empirically using a combination of survey data and option prices to estimate the underlying structural drivers. The punchline of our empirical implementation is that most of the secular changes in stock valuations over the past 35 years can be explained by movements in the pure discounting part of interest rates. This result is illustrated in the right panel of Figure 1, which shows that 80% of the changes in valuation ratios of equities in G7 countries can be explained by changes in the pure discounting part of interest rates. This pure discounting component is estimated purely from our decomposition for interest rates, so there is nothing mechanical about the tight fit in explaining almost the entirety of the country-by-country change in equity valuations over recent decades.

The pure discounting part of interest rates also explains a sizable share of fluctuations in stock prices at higher frequencies, consistent with the long cash-flow duration of the stock market, and it can be used to understand the pricing of the cross-section of equities. Our framework allows us to revisit outstanding questions on why stock prices move, the size of the ex ante equity premium, the impact of interest rates on wealth inequality, and the channels through which monetary policy influences stock prices, all of which we elaborate on below.

The key input for our measurement is an international panel of long-term professional forecasts for interest rates, inflation, and growth rates, which we obtain from Consensus Economics. We back out forecast-implied series for trend real rates $r^*$ and trend growth rates by country, and we augment these with option-based measures of uncertainty to estimate our interest-rate decomposition. After stripping out growth-rate and uncertainty changes, the remaining interest-rate change is our estimate of the pure discounting shock.

In the U.S., we attribute around 35% of the decline in $r^*$ since 1990 to pure discount-rate changes, and the remaining 65% to the other components. So while equities have benefited somewhat from the decline in U.S. interest rates, assuming full pass-through of $r^*$ to equity yields would overstate the effect by close to three times. And the pass-through of the decline in rates to equities has been even lower in most other G7 countries. In the U.S., the pure discounting term declined substantially in the 1990s; since 2000, it has stayed mostly flat, implying that the decline in rates since 2000 is driven by forces other than pure discounting.

Our decomposition also speaks to higher-frequency movements in interest rates and equities. Without adjusting for the endogeneity of interest rates, the raw relation between market returns and yield changes is small and imprecisely estimated. But when we implement our decomposition, pure discount-rate shocks generate strong negative comovement between $\Delta r_t^*$ and annual equity returns. The loading of stock returns on pure discounting shocks provides a theoretically well-founded measure of equity duration, and we estimate a market duration of about 20 years in the U.S. data.[1] By contrast, equity returns have a small and insignificant relation to the interest-rate change attributable to changing expected growth rates, and a positive relation with the interest-rate change attributable to uncertainty shocks. These offsetting components illustrate why equity duration is not equivalent to the price sensitivity to arbitrary changes in interest rates, and why one must isolate the pure discounting component to estimate duration.

We next use our decomposition to better understand the cross-section of stocks and their exposure to interest rates. Following Gormsen and Lazarus (2023), we sort firms by their predicted cash-flow duration and measure these duration-sorted portfolios' returns. We show that these portfolios do not differ in their exposure to raw interest-rate changes, but that the long-duration firms have significantly greater exposure to the pure discounting shock. This holds in spite of the unconditional negative alpha to long-duration relative to short-duration stocks, and it implies a sizable spread (greater than 20 years) in the duration of long- versus short-duration firms' cash flows. These results provide a further out-of-sample validation of both the duration sort and of the construction of the pure discounting term.

[1] Equity duration refers to the weighted average time to maturity of cash flows. In the fixed income literature, duration is similarly defined as the weighted average time to maturity of a bond's cash flows. This is also used as an approximate measure of bond-price sensitivity to arbitrary interest-rate changes; for equity, as our analysis emphasizes, duration does not pin down interest-rate sensitivity. We consider the 20-year duration estimate to be a likely lower bound for true duration, given measurement uncertainty in the higher-frequency estimation. This is nonetheless a large and significant estimate.

Our results are robust across a range of alternative specifications and measurement choices. This includes the use of alternative measures of $r^*$, $g^*$, and uncertainty in our interest-rate decomposition. Moreover, we provide a set of theoretical and empirical robustness results on the role of changes in the profit share of income, or the ratio of earnings to aggregate output. Using separate forecast data on earnings and dividends, we find that our main U.S. empirical results are effectively unchanged when allowing for a time-varying profit share.

Taken together, our results show that isolating the pure discounting term is essential for understanding how shocks to interest rates are reflected in stock prices. While changes to this term are equivalent to changes in the pure rate of time preference for the marginal investor, we do not view this as the only (or main) source of likely variation. To better interpret this term, we discuss how capital flows represent a candidate source of this pure discounting variation, as they may affect interest rates in a manner not fully accounted for by changes in fundamentals (growth or risk). We then show that the changes in the estimated pure discounting terms align reasonably well with cross-country capital flows empirically. We also briefly discuss other plausible contributors to changes in the pure discount rate.

## Implications

Our findings on interest-rate pass-through speak to a range of issues raised in the literature:

(i) Our results clarify analyses of the relation between interest rates and equity valuations. A prominent interpretation of classic valuation decompositions is that valuations move mainly with risk premia rather than interest rates (e.g., Campbell and Cochrane 1999; Lettau and Ludvigson 2001). In contrast to this "risk premium view," we find that changes in the pure discounting term explain most of the secular changes in equity yields across G7 countries over the past 30 years, as well as 77% of the time variation in U.S. equity yields in this sample. Because interest rates co-move with risk premia, isolating the pure discounting term is key for understanding the role of rates in equity valuation fluctuations. Doing so reveals a larger role for rates than previously recognized.

(ii) Estimating the ex ante equity premium based on realized returns can lead to biased estimates if interest rates or risk premia change over the sample (Fama and French 2002). One approach to control for declining interest rates is to subtract from stock returns the returns on duration-matched bond portfolios.[2] While this adjustment is useful for measuring the realized returns associated with dividend risk, our analysis shows that it leads to less precise equity-premium estimates in general. This arises because of the imperfect pass-through from bonds to stocks: only the pure discounting

[2] See van Binsbergen (2024) and Polk and Vuolteenaho (2025).

term passes through to equities in line with their duration, and much of the decline in rates reflects growth and risk shocks. We instead suggest subtracting the returns on a duration-matched "pure discounting" claim, which hedges only against pure discounting changes. In simulations, this produces equity-premium estimates that are 20% more precise than comparing stock returns to ex ante yields, and 50% more precise than comparing to duration-matched bond returns. Empirically, we estimate a large ex ante equity premium using our approach.

(iii) Recent work has argued that the decline in interest rates has influenced wealth inequality. If $r^*$ declines have passed through fully to equity, held disproportionately by wealthy households, then much of the increase in inequality in recent decades may be driven purely by interest-rate changes.[3] Our findings help speak to this debate. We find that only a small share of the decline in $r^*$ was transmitted directly to equity, implying that declining interest rates had a much smaller impact on wealth inequality than calculations based on full pass-through would imply. This argument is consistent with Atkeson, Heathcote, and Perri (2025a), Greenwald, Lettau, and Ludvigson (2025), and Irie (2025), who find a larger role for other forces (particularly cash-flow changes) versus discount-rate declines in explaining equity returns and changes in inequality.

(iv) Both academic research and practitioner commentary have discussed how the decline in interest rates may have affected factor returns and cross-sectional anomalies (e.g., Asness 2022). One view holds that portfolios tilted toward short-duration cash flows (such as value, or high book-to-market, portfolios) would have performed better in recent decades in a counterfactual without the rate decline.[4] We find that while value stocks have limited exposure to raw interest-rate movements in general, they are highly exposed to the pure discounting component of rates. We use this distinction to clarify the role of declining rates in value's recent poor performance: the pure discounting component explains some, though far from all, of this underperformance.

(v) As a further application, we use our decomposition to unpack the effects of surprise changes in short-term interest rates by monetary policymakers. Some papers implicitly treat the resulting changes in long-term yields as pure discounting shocks, but this need not hold: while the change in the short-term rate is indeed exogenous, the long-term yield responds to changes in both the pure discount rate *and* perceived long-run growth and uncertainty (e.g., Hanson and Stein 2015). Using our estimation results for yield changes and stock returns, along with high-frequency asset-price changes around policy announcements, we back out announcement-specific changes in both the

[3] See Fagereng et al. (2025) for a review, as well as Catherine et al. (2023) and Greenwald et al. (2023).

[4] See Maloney and Moskowitz (2021) and van Binsbergen, Ma, and Schwert (2025) for related discussion.

pure discounting term and expected growth. On average, we find that most of the change in long-term yields around policy shocks indeed stems from the pure discounting component. But we find as well that growth expectations change in a manner consistent with an information effect on average (Nakamura and Steinsson 2018; Gilchrist, Yang, and Zhao 2024), with substantial heterogeneity across announcements.

Overall, we provide a toolkit for understanding how changes in interest rates have, and have not, affected a range of risky assets and aggregate outcomes over the short and long run.

## Additional Related Literature

In addition to the connections described above, our paper relates to a long literature on the time-varying relationship between stocks and bonds; Campbell, Pflueger, and Viceira (2025) provide a recent review. Much of this work has focused on higher-frequency comovement between stocks and *nominal* bonds, emphasizing the importance of inflation risk dynamics (e.g., Song 2017; Campbell, Pflueger, and Viceira 2020). We instead focus on real yields. Chernov, Lochstoer, and Song (2025) consider a real channel in which the relative importance of permanent vs. transitory consumption shocks drives the comovement, while Laarits (2025) focuses on changing precautionary savings motives. Our framework accommodates both of these channels via the uncertainty (i.e., entropy) term in our real-rate decomposition, though we do not focus directly on time variation in the high-frequency comovement.

Our paper is closer to work studying longer-term stock–bond comovement more directly. Campbell and Ammer (1993) use VARs to relate this comovement to long-term expected returns. We instead use surveys and asset prices to map bond and equity valuations to three structural drivers of real rates, rather than relating valuations to expected returns in reduced form. Barsky (1989) provides a two-period theoretical framework to study the comovement; we allow a role for pure discounting shocks and estimate our more general decomposition empirically.[5] Farhi and Gourio (2018) use a growth model to account for secular valuation changes. While our exercise is less tightly parameterized, we find evidence consistent with their view that real rates have comoved negatively with equity premia. Bianchi, Lettau, and Ludvigson (2022) focus on the monetary policy stance in relating equity valuations to real rates; this focus is more specific than ours, but the two approaches similarly highlight the need to isolate precise sources of rate variation for understanding pass-through to stocks.

We also relate to literature studying long-term drivers of valuations. For equity, Greenwald, Lettau, and Ludvigson (2025) argue that increased profit shares are a key driver of stock-price

[5] In allowing for pure discounting shocks, we relate to macro–finance work assigning an important role to preference-related changes in discount rates (e.g., Albuquerque et al. 2016). Lettau and Wachter (2011) also consider a framework with a reduced-form stochastic discount factor and exogenous (pure) real-rate shocks.

increases in the U.S. data. While their model-based estimation differs from ours, our results are consistent with theirs: profit-share increases have affected both prices and cash flows while having a limited effect on their ratio, as we discuss further in Section 4.2. Atkeson, Heathcote, and Perri (2025a) study corporate valuations and investment, similarly highlighting the importance of pure rents for low-frequency valuation changes. Considering investment lets them speak separately to (i) expected returns (with a capital Euler equation) and (ii) expected returns net of growth (with the ratio of free cash flow to enterprise value). Our equity dividend yield is conceptually closer to (ii), and we then measure expected growth using survey data rather than considering investment directly.[6] Our focus on the bond-stock pass-through and the role of the pure discount rate also distinguish our setting from theirs. For bonds, a recent literature studies sources of asset demand, including population aging and inequality (e.g., Auclert et al. 2025; Mian, Straub, and Sufi 2021), in explaining the decline in interest rates. We view these as potential channels driving the components of the real rate, including the pure discount rate, as discussed briefly in Section 5.1.

More broadly, our joint use of bonds and stocks relates to other work that uses joint movements across assets to disentangle underlying shocks, particularly the literature on exchange rates and interest rates (e.g., Itskhoki and Mukhin 2021; Kekre and Lenel 2024). Others also consider asset valuations and capital flows (e.g., Camanho, Hau, and Rey 2022; Atkeson, Heathcote, and Perri 2025b), as we discuss in Section 5.1. Finally, our use of long-horizon survey forecasts to link fundamentals and returns also parallels recent work in other markets (e.g., Kremens, Martin, and Varela 2025).

### Organization

We begin with our theoretical decompositions in Section 2. We then turn to our data, measurement approach, and main findings in Section 3, and Section 4 provides robustness results. In Section 5, we analyze additional implications of our findings. Section 6 discusses and concludes. Derivations and additional results can be found in the Appendix.

## 2. Theoretical Decompositions

This section provides our theoretical decomposition for the trend real rate. Our goal is a decomposition of this endogenous object into interpretable fundamental components, in a manner that both (i) is empirically estimable and (ii) contains one component that induces

[6] Our valuation ratios and expected growth series appear qualitatively consistent with theirs in the overlapping (post-1990) sample period. Our survey data limit the analysis to the post-1990 period; extending to the pre-1990 period would require additional assumptions but is an interesting direction for future work.

perfect comovement of bonds and stocks. Isolating this last component will then allow us to measure the degree to which interest-rate changes transmit to equity valuations.

We begin in Section 2.1 with a general decomposition with minimal assumptions on fundamentals or the stochastic discount factor. We then specialize to a more interpretable consumption-based case in Section 2.2. We consider what each term in the real-rate decomposition means for equity prices in Section 2.3, and for equity duration in Section 2.4.

## 2.1 A General SDF-Based Version

We start with a general stochastic discount factor (SDF) $M_{t+1}$ such that $\mathbb{E}_t[M_{t+1}R_{t+1}] = 1$ for an arbitrary asset's gross return $R_{t+1}$. This implies $R^f_{t+1} = 1/\mathbb{E}_t[M_{t+1}]$, where $R^f_{t+1}$ is the real risk-free rate. Taking logs (and denoting logged variables in lowercase),

$$r^f_{t+1} = -\mathbb{E}_t[m_{t+1}] - L_t(M_{t+1}), \tag{1}$$

where $L_t(M_{t+1}) \equiv \log \mathbb{E}_t[M_{t+1}] - \mathbb{E}_t[m_{t+1}]$ is the conditional entropy of the SDF.[7]

For now, we put very little structure on the SDF. We assume that the log SDF can be additively decomposed as follows:

$$m_{t+1} \quad = \quad \underbrace{-\rho_t}_{\substack{\text{predetermined} \\ \text{trend}}} \quad - \quad \underbrace{(f(X_{t+1}) - f(X_t))}_{\substack{\text{difference for} \\ \text{Markov } X}} \quad + \quad \underbrace{\varepsilon_{t+1}}_{\substack{\text{mean 0} \\ \text{martingale diff.}}}. \tag{2}$$

This additive representation is constructed following Hansen (2012, Theorems 3.1–3.2), and it holds under a general set of primitive assumptions. See Appendix A.1 for formal details. As discussed there, the term $f(X_{t+1}) - f(X_t)$ is either stationary or difference-stationary.

In interpreting (2), the trend $-\rho_t$ shifts the intertemporal marginal rate of substitution $m_{t+1}$ in all states, so $\rho_t$ can be thought of as a time discount rate. We interpret the Markov state $X_{t+1}$ as determining aggregate cash flows (again see Appendix A.1), so $f(X_{t+1}) - f(X_t)$ can be thought of as the realized marginal utility from cash flow growth. Finally, $\varepsilon_{t+1}$ is the remaining martingale component of the log SDF. These terms' interpretation will map to their interpretation in the consumption-based framework in the next subsection.

Plugging (2) into (1), the log risk-free rate satisfies

$$r^f_{t+1} \quad = \quad \underbrace{\rho_t}_{\text{trend (discounting)}} \quad + \quad \underbrace{\mathbb{E}_t[f(X_{t+1}) - f(X_t)]}_{\text{expected growth}} \quad - \quad \underbrace{L_t(M_{t+1})}_{\text{uncertainty/prec. savings}}. \tag{3}$$

[7]This is also the starting point for studying currency puzzles in Backus, Foresi, and Telmer (2001). We note that all expectations $\mathbb{E}_t[\cdot]$ are interpretable as either objective or subjective.

The first two terms' labels align with the interpretations discussed above. For the labeling of $L_t(M_{t+1})$ as an uncertainty or precautionary savings term, note that by definition of entropy,

$$L_t(M_{t+1}) = \sum_{n=2}^{\infty} \frac{\kappa_{n,t}(m_{t+1})}{n!}, \tag{4}$$

where $\kappa_{n,t}(m_{t+1})$ is the $n^{\text{th}}$ conditional cumulant of the log SDF distribution (assumed to be finite for all $n$). Conditional entropy therefore encodes the higher ($n \geqslant 2$) moments of marginal utility, as is standard.

Our main interest will be in understanding changes in the trend risk-free rate $r_t^*$. Analogous to Bauer and Rudebusch (2020), we define this as the Beveridge–Nelson trend in the one-period real rate, $r_t^* \equiv \lim_{\tau\to\infty} \mathbb{E}_t[r^f_{t+\tau+1}]$. But unlike Bauer and Rudebusch, we are interested in longer-horizon real rates: we think of one period as being equal to the cash-flow duration of the overall equity market. We therefore do not directly consider the term premium embedded in these long-term rates. Considering a long period length is equivalent to considering a multi-period zero-coupon yield: $R^f_{t,t+\tau} = 1/\mathbb{E}_t[M_{t,t+\tau}]$ for any $\tau \geqslant 1$, where $M_{t,t+\tau} = M_{t+1} \cdots M_{t+\tau}$, so (1) applies when replacing "$t+1$" with "$t, t+\tau$." Term premia affect yields through the entropy term in (3); see Backus, Boyarchenko, and Chernov (2018).

For the trend real rate $r_t^*$, equation (3) directly implies that it satisfies

$$r_t^* = \rho_t^* + \widetilde{g}_t^* - L^*_{t,M}, \tag{5}$$

where $\rho_t^* = \rho_t$, $\widetilde{g}_t^* = \lim_{\tau\to\infty} \mathbb{E}_t[f(X_{t+\tau+1}) - f(X_{t+\tau})]$ and $L^*_{t,M} = \lim_{\tau\to\infty} \mathbb{E}_t[L_{t+\tau}(M_{t+\tau+1})]$. This is the first version of our real-rate decomposition into three terms corresponding to discounting, expected growth, and uncertainty. The analysis in this section shows that such a decomposition can be derived quite generally — up to the issue of interpretation of each of the three terms — starting from an additive decomposition of the log SDF.

## 2.2 A Consumption-Based Version

To put more structure on the decomposition in (5), we now consider a more standard consumption-based framework. We assume an endowment economy in which a representative agent has power utility over consumption,[8]

$$U_t = \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta_t^{\tau} \frac{C_{t+\tau}^{1-\gamma}}{1-\gamma}. \tag{6}$$

[8]See Appendices A.1–A.2 for extensions with Epstein–Zin preferences, time-varying risk aversion, or other departures. Decompositions of the form (5) or (8) still hold in these alternative specifications.

The time discount factor $\beta_t$ and corresponding rate of time preference $\rho_t = -\log \beta_t$ are potentially time-varying. We assume that relative risk aversion $\gamma$, or the inverse elasticity of intertemporal substitution (EIS), is constant.

Given (6), the log SDF is $m_{t+1} = -\rho_t - \gamma g_{t+1}$, where $g_{t+1} \equiv c_{t+1} - c_t$ is log consumption growth. Plugging this into (1),

$$\begin{aligned} r^f_{t+1} &= \rho_t + \gamma \mathbb{E}_t[g_{t+1}] - L_t(M_{t+1}) \\ &= \rho_t + \gamma \mathbb{E}_t[g_{t+1}] - \sum_{n=2}^{\infty} \frac{(-\gamma)^n \kappa_{n,t}(g_{t+1})}{n!}, \end{aligned} \tag{7}$$

where the final expression for $L_t(M_{t+1})$ in terms of the growth-rate cumulants $\kappa_{n,t}(g_{t+1})$ is as in Backus, Chernov, and Martin (2011) or Martin (2013); see Appendix A.2. In a lognormal setting, this simplifies to the familiar solution $r^f_{t+1} = \rho_t + \gamma \mathbb{E}_t[g_{t+1}] - \frac{\gamma^2}{2} \mathrm{Var}_t(g_{t+1})$.

Given (7), the trend real rate can be expressed as

$$r^*_t = \rho^*_t + \gamma g^*_t - L^*_{t,M}, \tag{8}$$

where $\rho^*_t = \lim_{\tau\to\infty} \mathbb{E}_t[\rho_{t+\tau}]$, $g^*_t = \lim_{\tau\to\infty} \mathbb{E}_t[g_{t+\tau+1}]$, and $L^*_{t,M} = \lim_{\tau\to\infty} \mathbb{E}_t[L_{t+\tau}(M_{t+\tau+1})]$.[9] Thus exactly as in (5), the real rate can move due to changes in (i) time preference (a stand-in for pure discounting shocks), (ii) expected growth rates (via an intertemporal substitution channel), or (iii) risk or uncertainty (via a precautionary savings channel).

## 2.3 Implications for Equity Prices

We now move to equity and ask how each of the three channels in our $r^*$ decomposition transmit to equity valuations. Doing so requires further structure on equity cash flows and other fundamental shocks. We adopt a set of standard assumptions, with which we derive a version of a Gordon growth formula for equity dividend yields. We then apply this to show how each component of our interest-rate decomposition maps to equity valuations in a simple, intuitive way.

We start from the same consumption-based framework as in Section 2.2, though the main equity-valuation decompositions below in fact apply as well under Epstein–Zin utility (see Appendix A.3). For equity cash flows, we follow Campbell (1986) and Abel (1999) and model equity as a levered claim to consumption, paying dividends $D_t = C_t^\lambda$, with $\lambda > 0$. This imposes a tight link between dividend growth and output and consumption growth. While

[9] As discussed in Appendix A.1.3, in a stationary setting in which the limiting expectations here are constant, we redefine these terms as discounted sums of expected outcomes from date $t+1$ to $\infty$.

this works well to explain the long-term trends observed in our data, this need not always be the case (particularly at high frequencies). We accordingly extend our analysis in Section 4.2 to allow for time variation in the profit share of output, which we then discipline in the data with additional survey expectations. For now, however, we assume that $\lambda$ is constant.

Denote the gross equity return by $R^{\text{mkt}}_{t+1}$ and the log return by $r^{\text{mkt}}_{t+1}$, and define $\mu_t \equiv \mathbb{E}_t[r^{\text{mkt}}_{t+1}]$ and $rp_t \equiv \mu_t - r^f_{t+1}$. The equity yield is defined as

$$ey_t \equiv \log(1 + D_t/P_t), \tag{9}$$

where $P_t$ is the price of the equity claim. This is slightly different from the usual log dividend–price ratio ($dp_t \equiv \log(D_t/P_t)$). We define $ey_t$ as in (9) because it puts the equity yield in equivalent units as the log real rate. It also yields straightforward characterizations of steady-state $ey^*_t$ building on results from Martin (2013) and Gao and Martin (2021).

To show that our results apply across a range of standard environments, we consider three cases for the dynamics of fundamentals. All three yield nearly identical equity valuations.

**Case I (Gordon Growth):** Assume that log output growth $g_{t+1} = c_{t+1} - c_t$ is conditionally i.i.d. with arbitrary distribution, so that as of time $t$, agents expect growth to be i.i.d. with the current parameters forever. We allow for unanticipated shocks to these parameters, which then become the new steady state. Given this structure, we write $g^*_t = \mathbb{E}_t[g_{t+1}] = \mathbb{E}_t[g_{t+\tau}]$ for all $\tau \geqslant 1$, and the growth-rate cumulants satisfy $\kappa_{n,t}(g_{t+1}) = \kappa_{n,t+\tau}(g_{t+\tau+1})$ for all $\tau \geq 1$. We similarly allow for unanticipated shocks to the preference parameter $\rho^*_t = \rho_t$. We view this setting as a reasonable starting approximation given our focus on relatively long horizons.

This setting closely builds on that of Martin (2013), and we apply and extend his results; see Appendix A.3 for details and derivations. Given the constant growth rates and discount rates, a Gordon growth formula applies as follows (where we use the i.i.d. assumption to set all relevant variables equal to their conditional steady-state values):

$$ey^*_t = r^*_t + rp^*_t - \lambda g^*_t. \tag{10}$$

The log equity premium satisfies

$$\begin{aligned} rp^*_t &= \sum_{n=2}^{\infty} \frac{\kappa_{n,t}(g_{t+1})}{n!} \left((-\gamma)^n - (\lambda - \gamma)^n\right) \\ &= L_t(M_{t+1}) - L_t(M_{t+1}R^{\text{mkt}}_{t+1}) = L^*_{t,M} - L^*_{t,MR}. \end{aligned} \tag{11}$$

As discussed in the appendix, the fact that $rp_t = L_t(M_{t+1}) - L_t(M_{t+1}R^{\text{mkt}}_{t+1})$ is fully general: it holds under no arbitrage and does not require i.i.d. fundamentals or any assumptions on utility. If $\lambda = \gamma$, then $M_{t+1}R^{\text{mkt}}_{t+1} = 1$, and $rp^*_t = L^*_{t,M}$.[10] Alternatively, for arbitrary $\lambda$ and $\gamma$, if growth is lognormal so that $\kappa_{n,t}(g_{t+1}) = 0$ for $n > 2$, then $rp^*_t = \frac{1}{2}\lambda(2\gamma - \lambda)\text{Var}_t(g_{t+1}) = \frac{\lambda(2\gamma-\lambda)}{\gamma^2} L^*_{t,M}$.

Using (8), (10), and (11), we obtain a solution for equity yields summarized along with the real risk-free rate in the following result.

**Result 1.** *The steady-state real risk-free rate and equity dividend yield satisfy*

$$
\begin{aligned}
r^*_t &= \rho^*_t + \gamma g^*_t - L^*_{t,M}, \\
ey^*_t &= \rho^*_t + (\gamma - \lambda) g^*_t + (rp^*_t - L^*_{t,M}) \\
&= \rho^*_t + (\gamma - \lambda) g^*_t - L^*_{t,MR}.
\end{aligned}
$$

*Changes in the risk-free rate can arise due to (i) pure discounting shocks (changes in $\rho^*_t$), (ii) growth-rate shocks ($g^*_t$), or (iii) risk (entropy) shocks ($L^*_{t,M}$). Each of the three has different implications for equity valuations:*

*(i)* ***Pure discounting shocks:*** *Bonds and equity co-move perfectly, with $ey^*_t$ increasing by 1 basis point for each 1 basis point increase in $r^*_t$.*

*(ii)* ***Growth-rate shocks:*** *Equity yields change by $\frac{\gamma-\lambda}{\gamma}$ per unit increase in $r^*_t$. If $\gamma = \lambda$ (e.g., with log utility and an unlevered consumption claim), $ey^*_t$ is unaffected by changes in $r^*_t$ induced by growth shocks. If $\lambda > \gamma$, growth shocks induce negative comovement.*

*(iii)* ***Risk shocks:*** *Equity yields change by $-\frac{\partial rp^*_t}{\partial L^*_{t,M}} + 1$ per unit increase in $r^*_t$ if $\frac{\partial rp^*_t}{\partial L^*_{t,M}}$ is well-defined. Otherwise, equity yields change on average by $-\beta_L + 1$ per unit increase in $r^*_t$, where $\beta_L \equiv \frac{Cov(rp^*_t, L^*_{t,M})}{Var(L^*_{t,M})}$. If $\gamma = \lambda$, then $ey^*_t$ is unaffected by changes in $r^*_t$ induced by risk shocks. If $\beta_L > 1$, risk shocks induce negative comovement.*

The key implication of this result is that only the pure discounting channel generates perfect pass-through from interest rates to equity yields, and from bond prices to duration-matched stock prices (as discussed below). The range of past work assuming that the decline in rates has passed through fully to equity valuations — as discussed in Section 1 — has therefore implicitly assumed that the decline in $r^*_t$ has arisen due to such pure discounting shocks. Changes in $\rho^*_t$ can be thought of as capturing, for example, demographic changes, or something akin to a savings glut. We discuss such interpretations further in later sections.

For growth-rate changes, note that the equity yield depends on $r^*_t - \lambda g^*_t$ (our version of $r - g$), so a decline in $g^*_t$ will have roughly offsetting effects since it decreases both discount

[10] This is a restatement of the fact that the Alvarez and Jermann (2005) SDF entropy lower bound holds with equality in the growth-optimal case (i.e., with $M_{t+1}R^{\text{mkt}}_{t+1} = 1$): $L_t(M_{t+1}) = \mathbb{E}_t[r^{\text{mkt}}_{t+1}] - r^f_{t+1}$.

rates and growth rates.[11] While we do not require this to be the case, it is common to assume that $\gamma \leqslant \lambda$ — or, in the Epstein–Zin case, that $\frac{1}{\psi} \leqslant \lambda$, where $\psi$ is the EIS — so that a decline in growth rates also decreases equity valuations (corresponding to a higher $ey_t^*$). This would imply that growth-rate changes generate weakly negative comovement between bonds and stocks.

For changes in risk, there are offsetting effects on the risk-free rate and the risk premium. These changes may approximately offset or may cause stocks to move in the opposite direction of bonds. As $L_{t,M}^*$ loads on all the higher cumulants of the growth distribution as in (7), the stock-price response will depend on the specific parameter change underlying the risk shock. In Appendix A.3, we characterize the bond–stock comovement in three benchmark cases: (i) in a lognormal setting with power utility and $\gamma \neq \lambda$, $r_t^*$ and $ey_t^*$ comove positively given changes in risk, though the pass-through is less than one-for-one if $2\gamma > \lambda$; (ii) in a rare-disasters model as in Barro (2006), if $\gamma < \lambda$, then $r_t^*$ and $ey_t^*$ comove negatively given changes in the average disaster size (or, more generally, given changes in skewness or other odd moments); (iii) with Epstein–Zin utility, if $\gamma > 1$, $\psi > 1$, and $\lambda = 1$, then $r_t^*$ and $ey_t^*$ comove negatively given changes in even moments of the growth distribution (variance, kurtosis, and so on), as in Martin (2013). So while positive risk shocks robustly decrease the risk-free rate and generally induce a muted or negative stock–bond comovement, we treat their precise pass-through to stocks as an open question to be disciplined empirically.

Result 1 is our main decomposition for trend real rates and equity valuations. We now discuss how this decomposition carries through under more realistic assumptions on the evolution of fundamentals.

**Case II (Drifting Steady State):** Assume now that the distributions of growth rates and preference parameters are such that $ey_t$ follows a martingale, $ey_t = \mathbb{E}_t[ey_{t+1}]$. Campbell (2018) refers to this as a "drifting steady state" model for $ey_t^* = ey_t$, and Campbell and Thompson (2008) show that such a model has success at forecasting medium-to-long-horizon returns. To a first order for $ey_{t+1}$ around its expectation $ey_t$, we have in this case that $ey_t = \mathbb{E}_t[r_{t+1}^{\text{mkt}} - \lambda g_{t+1}]$. (This follows Gao and Martin 2021, and again see the appendix.) This implies that

$$ey_t^* = ey_t = \mathbb{E}_t[r_{t+1}^{\text{mkt}} - \lambda g_{t+1}] = \mathbb{E}_t[r_{t+2}^{\text{mkt}} - \lambda g_{t+2}] = \ldots = r_t^* + rp_t^* - \lambda g_t^*, \tag{12}$$

assuming the individual limiting values exist as $t + \tau \to \infty$. In addition, $r_t^*$ satisfies the same

[11] The $ey_t^*$ decomposition in Result 1 does not separate between discount rates and growth rates in the same manner as a Campbell–Shiller decomposition. Instead, it collects terms such that $g_t^*$ term, for example, contains both the direct cash-flow effect ($\lambda g_t^*$) and the discount-rate effect ($\gamma g_t^*$). We do so given our desire to decompose risk-free discount rates into underlying structural components rather than composite terms.

decomposition as in (8). Result 1 therefore holds exactly, and the same takeaways apply.

These steady-state facts can be equivalently stated as applying to one-period-ahead conditional expectations:

$$ey_t = \rho_t + (\gamma - \lambda)\mathbb{E}_t[g_{t+1}] + (rp_t - L_t(M_{t+1})),$$

and similarly for the risk-free rate as in equation (7). These versions are useful for interpreting higher-frequency changes in rates and prices.

**Case III (Stationarity):** Finally, we assume that $ey_t$ and all fundamental variables are stationary, with no unanticipated permanent shocks. This case does not admit permanent changes to real rates or valuations. So to non-trivially characterize the pass-through from interest-rate changes to equity valuations, we must reinterpret all the previous starred terms so as to measure persistent rather than permanent variation. Concretely, we redefine the starred terms $z_t \in \{r^f_{t+1}, rp_t, \rho_t, g_t, L_t(\cdot)\}$ as Campbell–Shiller-type discounted sums:

$$z^*_t \equiv (1-\delta)\sum_{\tau=0}^{\infty}\delta^\tau \mathbb{E}_t[z_{t+\tau+1}], \tag{13}$$

where $\delta \in (0,1)$ is a loglinearization term defined in Appendix A.3. Since $(1-\delta)\sum_{\tau=0}^{\infty}\delta^\tau = 1$, (13) defines the starred long-run terms as weighted averages of all future expected outcomes.

Given (13), we again follow Gao and Martin (2021) to obtain the loglinear approximation

$$ey^*_t \equiv ey_t = r^*_t + rp^*_t - \lambda g^*_t,$$

exactly as in (10), where $r^*_t = \rho^*_t + \gamma g^*_t - L^*_{t,M}$ and $rp^*_t = L^*_{t,M} - L^*_{t,MR}$. So given the redefinitions in (13), Result 1 again applies exactly as stated.

In all three cases, therefore, our decomposition generates effectively equivalent results, summarized in Result 1. Only shocks to the pure discounting component of real rates pass through perfectly to equity valuations (in the form of equity yields). Shocks to the other two components in our decomposition — growth rates and uncertainty — generate ambiguous and possibly negative comovement between rates and equity valuations.

## 2.4 Implications for Equity Duration

Having analyzed the relation between interest rates and equity yields using our decomposition for rates, we now consider what the decomposition implies for equity duration. Equity duration does not, as we will see, correspond to the price sensitivity of equity to an arbitrary change in interest rates. Instead, the only interest-rate change that leads to an equity price change equal to its cash-flow duration is a pure discounting shock.

To make this point, we first express equity prices in levels. We consider the constant-growth steady state from Case I and drop time subscripts to simplify:

$$\left(\frac{P}{D}\right)^* = \frac{1}{\exp(r^* + rp^* - \lambda g^*) - 1} = \frac{1}{\exp(\mu^* - \lambda g^*) - 1} \approx \frac{1}{\mu^* - \lambda g^*}. \tag{14}$$

Market-level equity duration $\mathcal{D}$ is defined as the value-weighted time to maturity of the market's expected future cash flows:

$$\mathcal{D} \equiv \sum_{n=1}^{\infty} n \frac{e^{-n(\mu^*)}\mathbb{E}_t[D_{t+n}]}{P} = \frac{1}{1 - e^{-(\mu^* - \lambda g^*)}} \approx \frac{1}{\mu^* - \lambda g^*}. \tag{15}$$

This measure is equivalent to the equity price sensitivity to the log equity discount rate,

$$-\frac{\partial \log P}{\partial \mu^*} = \frac{1}{1 - e^{-(\mu^* - \lambda g^*)}} = \mathcal{D}, \tag{16}$$

which parallels the usual result for the exposure of bond prices to a shift in the yield curve. More important here, though, is price sensitivity to interest-rate changes arising from each of the terms in our decomposition. Given (14), we have the following result describing how price sensitivity depends on the underlying structural driver of interest-rate changes.

**Result 2** (Three Interest-Rate Sensitivities)**.** *The sensitivity of stock prices with respect to each of the three terms in the interest-rate decomposition $r^* = \rho^* + \gamma g^* - L_M^*$ is as follows.*

*(i) The interest-rate sensitivity of stock prices with respect to pure discount-rate shocks is*

$$\mathcal{S}_{r(\rho)} \equiv -\frac{\partial \log P}{\partial \rho^*} = \frac{1}{1 - e^{-(\mu^* - \lambda g^*)}} = \mathcal{D}.$$

*(ii) The interest-rate sensitivity of stock prices with respect to growth shocks is*

$$\mathcal{S}_{r(g)} \equiv -\frac{\partial \log P}{\partial (\gamma g^*)} = \left(1 - \frac{\lambda}{\gamma}\right)\mathcal{D} < \mathcal{D}.$$

*(iii) Assuming that $\partial_{rp,L} \equiv \frac{\partial rp^*}{\partial L_M^*}$ is well-defined and positive, the interest-rate sensitivity of stock prices with respect to risk shocks is*

$$\mathcal{S}_{r(L)} \equiv -\frac{\partial \log P}{\partial (-L_M^*)} = (1 - \partial_{rp,L})\,\mathcal{D} < \mathcal{D}.$$

Part (i) tells us that price sensitivity to the pure discount rate pins down equity duration in a manner equivalent to price sensitivity to the equity discount rate in (16). The other interest-rate terms do not share this feature: price sensitivity to growth shocks is strictly less than duration, as is price sensitivity to risk shocks under general assumptions. To take a benchmark example, with log utility ($\gamma = 1$) and equity modeled as an unlevered consumption claim ($\lambda = 1$), the price sensitivity of equity to a change in rates due to $g^*$ or $L_m^*$ are both exactly zero. More generally, as long as equities move positively with expected growth and negatively with respect to risk (as is commonly assumed), the interest-rate sensitivities in parts (ii)–(iii) will both be negative. So only a change in rates induced by a shock to $\rho^*$ moves equities in line with their duration, and equity duration is *not* equivalent to price sensitivity to an arbitrary change in $r^*$.

The fact that equity duration is equal to price sensitivity to pure discount-rate changes also provides a novel avenue for measuring duration. Measuring duration using realized growth rates, as in (15), requires a long sample. Measurement using price sensitivity to $\mu^*$, as in (16), is challenging given the difficulty in measuring expected equity returns. So if our interest-rate decomposition provides reliable estimates of the pure discount rate term $\rho^*$ over time, then estimating the exposure of equity returns to changes in this parameter would allow for clean estimation of duration, both for the aggregate market and for individual portfolios. We pursue this approach in our empirical estimation, which we turn to now.

# 3. Empirical Implementation

We now implement our real-rate decomposition empirically and study how interest rates and their three components transmit to equities. We do so in a panel of countries, with our data and measurement approach laid out in Section 3.1. We then estimate the terms in our decomposition, first in levels to study secular trends (Section 3.2), and then in changes to study transmission to equity returns (Section 3.3) and portfolio returns in the cross-section of stocks (Section 3.4).

## 3.1 Data and Baseline Measurement Approach

Recall that our goal is to measure each of the terms in the trend real-rate decomposition from Result 1 (or equation (5)): $r_t^* = \rho_t^* + \gamma g_t^* - L_{t,M}^*$, where $\rho_t^*$ is the pure discounting term (or rate of time preference), $g_t^*$ is long-term expected output growth, and $L_{t,M}^*$ is uncertainty (entropy). Our approach will be to measure $r_t^*$ and $g_t^*$ as directly as possible; measure $L_{t,M}^*$ using a proxy from option prices; and then back out the pure discounting term as a residual.

For trend real rates and expected growth rates, our main input is a panel of long-term forecast data obtained from Consensus Economics. They collect survey expectations of country-level economic and financial outcomes from professional forecasters, with 10–30 forecasters per survey for each country. We use the long-term forecasts, which are available for the G7 countries (Canada, France, Germany, Italy, Japan, the U.K., and the U.S.) from 1990 through 2023, and for a subset of other developed economies (Netherlands, Norway, Spain, Sweden, and Switzerland) starting in 1995 or 1998. These long-term forecasts are available twice annually for 1990–2013, and quarterly since 2014.

Three key features of the Consensus data are useful here. First, data are available for a panel of countries. Second, the forecasts are provided as forward expectations: we observe separate forecasts for year $t+1$, year $t+2$, ..., as opposed to forecasts stated as averages over the next $X$ (e.g., 5 or 10) years. The long-horizon forward forecasts remove cyclical variation affecting the short-horizon or averaged forecasts; we find this is important for measuring trend variables, as we observe lower volatility and mean reversion than in other forecast data (e.g., SPF or IBES). Third, the forecasters are typically professional economists at large investment banks and firms, whose expectations are likely relevant for asset-price determination.[12]

For all series, we use consensus (mean) forecasts at the five-year forward horizon. To estimate the long-term real rate $r^*_{t,j}$ for date $t$ and country $j$, we take the consensus forecast of the 10-year nominal interest rate at the end of year $t+5$ and subtract the inflation forecast for year $t+5$. For the expected growth rate $g^*_{t,j}$, we use the forecast of real output growth for year $t+5$. One potential concern is a possible mechanical relation between this and $r^*_{t,j}$ arising from forecasters using a model tying these variables together. While this is possible, the fact that these forecasters work at institutions with a key role in trading and pricing assets means that their expectations are relevant irrespective of how they are formed. Further, our main exercises — testing if the estimated real-rate components transmit to equity in the manner predicted by theory — will provide an out-of-sample validation of the rate decomposition. We also consider alternative measures of $r^*_{t,j}$ in robustness checks in Section 4.

To proxy for the uncertainty term $L^*_{t,M,j}$, we build on results from Section 2.3 and Martin (2017). We show in Appendix A.3 that if the market is growth-optimal and the distribution of log growth is symmetric, then the entropy of the SDF is equal to that of the market, $L^*_{t,M,j} = L^*_{t,R,j}$. And as shown in Result 3 of Martin (2017), $\text{VIX}^2_{t,j}$ is proportional to the market's risk-neutral entropy. Setting $L^*_{t,M,j} \propto \text{VIX}^2_{t,j}$ is thus likely to provide a reasonable first approximation, and the constant of proportionality will be implicitly estimated in our regressions below. We will also use $\text{VIX}^2_{t,j}$ for the risk term in the equity yield decomposition,

[12]For a list of forecasters for a recent U.S. survey, for instance, see `https://web.archive.org/web/20250314034328/https://www.consensuseconomics.com/what-are-consensus-forecasts/`.

as would hold in the growth-optimal case or with lognormality. To measure $\text{VIX}^2_{t,j}$, we use a global panel of index option prices from OptionMetrics. The sample, data filters, and calculations are taken from Gandhi, Gormsen, and Lazarus (2025); see Appendix B.1 for details. We calculate the squared VIX at the six-month horizon, which is longer than the CBOE's 30-day VIX for the S&P 500 given our desire to measure long-term uncertainty.[13]

As mentioned above, we also consider a range of alternative variables to test the robustness of our decomposition results. This includes using trend short-term (rather than 10-year) interest rates for $r^*_{t,j}$; alternatives to Consensus survey data; and alternatives to $\text{VIX}^2_{t,j}$ for uncertainty. We describe these alternative approaches in more detail in Section 4.

For equities, we use data from the CRSP/XpressFeed global database to obtain market indices for each country. To measure equity yields $ey_{t,j}$, we start with the five-year earnings-to-price ratio $\overline{E}_{t-4,t,j}/P_{t,j} = [(E_{t-4,j} + \ldots + E_{t,j})/5]/P_{t,j}$, where earnings and prices are calculated on a value-weighted basis for all available stocks in the country (see Appendix B.1). All equity values are thus measured as aggregates rather than on a per-share basis. We map the earnings yield to the equity yield considered in our theory by multiplying this by 0.5, which is the full-sample average payout ratio: half of earnings (more precisely, 49.4%) are paid out to shareholders in an average year–country observation, with the remainder reinvested.[14]

For our cross-sectional analyses, we use returns on duration-sorted portfolios following Gormsen and Lazarus (2023). That paper measures duration based on analyst forecasts of long-term expected earnings growth, or LTG (with higher cash-flow growth indicating a longer duration). We also obtain data on value-sorted portfolios via Ken French's website.

## 3.2 Secular Trends

To study long-term trends, we start by estimating our decomposition for trend real rates in levels. For all available dates $t$ and countries $j$, we estimate a regression

$$r^*_{t,j} = \rho_0 + \gamma\, g^*_{t,j} + \beta\, \text{VIX}^2_{t,j} + \Gamma_j + \varepsilon_{t,j}, \tag{17}$$

with country fixed effects $\Gamma_j$. In our main specification, we also allow the $\text{VIX}^2$ loading $\beta$ to differ by country ($\beta_j$). While this is not important for our main results, it helps account for

[13]A lack of liquid long-term options preclude us from calculating something closer to a five-year VIX. Gandhi, Gormsen, and Lazarus (2025) show that implied volatility decays slowly at longer maturities, so we view our six-month proxy as a reasonable starting point for longer-term uncertainty.

[14]An alternative to this approach is to use dividend yields $dp_{t,j}$ directly in place of $ey_{t,j}$. Since $\Delta dp_{t,j}$ and $\Delta ey_{t,j}$ are highly correlated, this gives nearly identical results: the adjusted $R^2$ is 0.75 for the cross-country regression of $\Delta dp_{t,j}$ on the pure discounting change, as compared to 0.79 in the baseline in the right panel of Figure 1. Our baseline starts from an earnings yield to address the fact that payout ratios have declined in recent decades. In subsequent higher-frequency time-series plots, we generally use the dividend yield directly.

Table 1: **Regressions for Trend Real Rates** $r^*_{t,j}$

| | **(1)** U.S. | **(2)** All | **(3)** All |
|---|---|---|---|
| Expected growth $g^*_{t,j}$ | 1.8*** | 2.1*** | 2.1*** |
| | (0.2) | (0.2) | (0.2) |
| Uncertainty $\text{VIX}^2_{t,j}$ | -10.1** | -3.8 | $\beta_j$ |
| | (4.5) | (3.0) | |
| Constant | -1.9*** | -1.9*** | -2.0*** |
| | (0.5) | (0.4) | (0.4) |
| Country FEs | ✗ | ✓ | ✓ |
| Country-Specific $\text{VIX}^2_{t,j}$ Loading | ✓ | ✗ | ✓ |
| Obs. | 86 | 932 | 932 |
| $R^2$ | 0.57 | 0.65 | 0.66 |
| Within $R^2$ | — | 0.60 | 0.61 |

*Notes:* This table shows OLS coefficient estimates in the regression (17), with standard errors in parentheses. In column (1), standard errors are obtained using a block bootstrap with one-year blocks and 10,000 bootstrap draws. In columns (2)–(3), standard errors are clustered by country and date. Statistical significance at the 10% level, 5% level, and 1% level are denoted by *, **, and ***, respectively. In column (3), the country-specific loadings on the squared VIX, $\beta_j$, are statistically significant at the 1% level for 9 of the 12 countries in our sample. The sample is 1990–2023, or the longest available span for the given country.

cases in which sovereign credit risk affects a country's $r^*_t$.[15] It also allows for the possibility of country-specific measurement error in the VIX, which may be an issue particularly for countries with less-liquid option markets.

Given a set of estimated coefficients and OLS residuals, we then back out the implied pure discounting term as

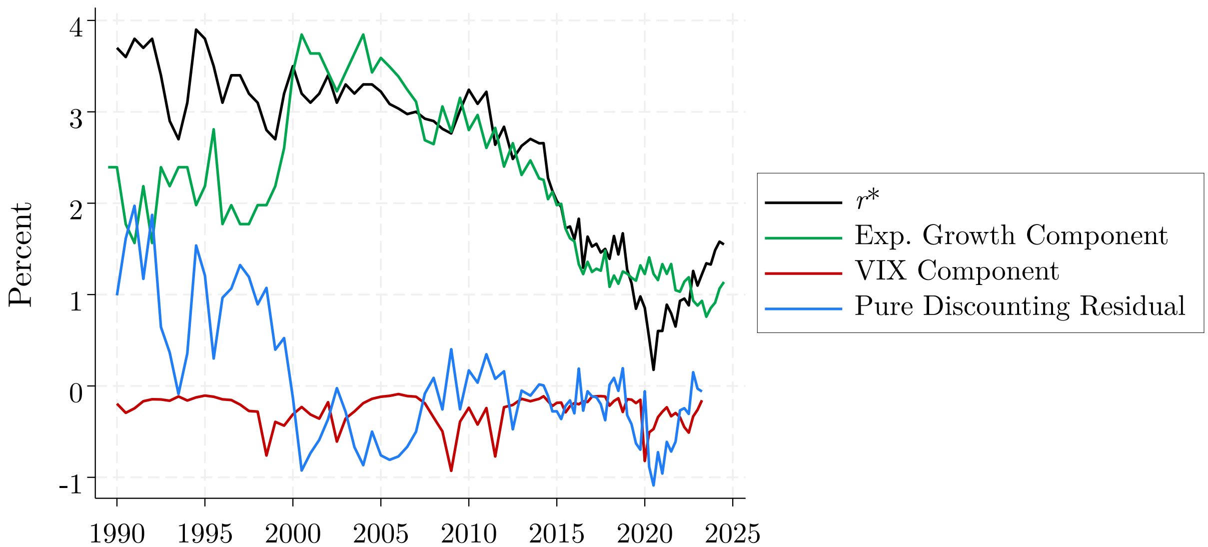
$$\widehat{\rho}^*_{t,j} = \widehat{\rho}_0 + \widehat{\Gamma}_j + \widehat{\varepsilon}_{t,j}. \tag{18}$$

We thus have, by construction, that $r^*_{t,j} = \widehat{\rho}^*_{t,j} + \widehat{\gamma} g^*_{t,j} + \widehat{\beta}\text{VIX}^2_{t,j}$, which corresponds exactly to our theoretical decomposition (with the uncertainty term $-L^*_{t,j}$ proxied by $\widehat{\beta}\text{VIX}^2_{t,j}$).

Estimates for the regression (17) are shown in Table 1, first for the U.S. only and then for the full 12-country panel. The estimates correspond well to our theory. The estimated loading on expected growth is strongly positive and consistently close to a value of 2, corresponding to implied relative risk aversion of $\gamma \approx 2$ and EIS of about 1/2. The loading on the VIX is negative and significant in the U.S. case and for most countries in the country-specific case shown in column (3). This limited set of variables explains a large share of the variation in trend real rates, with $R^2$ values of around 0.6 within-country and slightly higher overall. The

[15] Our theory suggests that the loading on uncertainty should be negative, but credit risk can induce an offsetting positive relation between risk and long-term rates. We find that this effect is small on average.

**Figure 2: U.S. Estimation Results for Decomposition of $r^*$ in Levels**



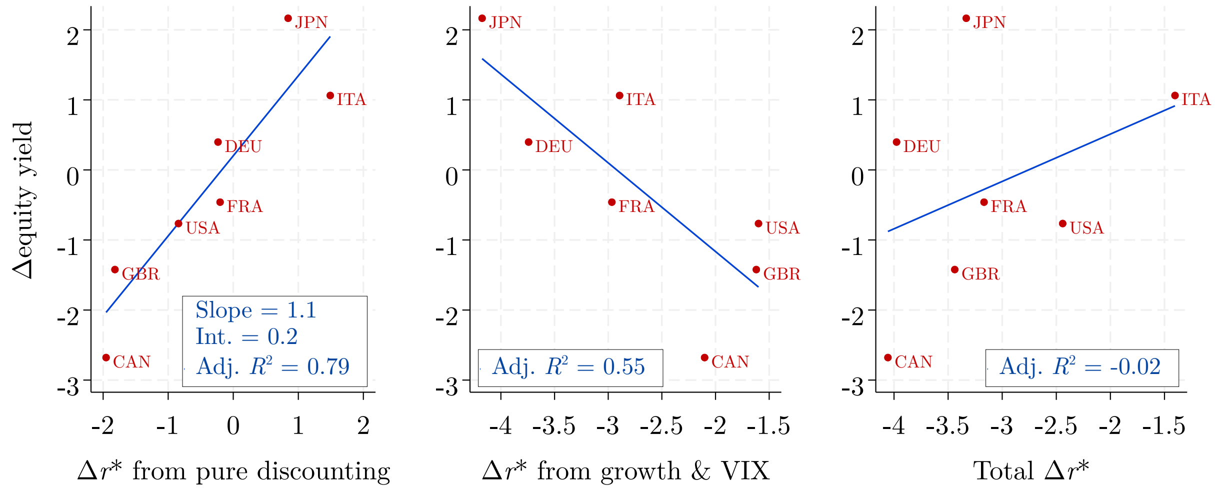

*Notes:* This figure shows the U.S. trend real rate $r^*_{t,j}$ and its components over time, estimated using (17)–(18) following the main specification in column (3) of Table 1. For readability, the expected growth component is shifted down by 3 percentage points ($\widehat{\gamma}g^*_{t,j} - 3$), and the pure discounting residual is plotted as $\widehat{\varepsilon}_{t,j}$.

remaining variation is then attributed to the pure discounting residual.

To visualize the data, Figure 2 presents the estimation results for the decomposition of $r^*_{t,j}$ over time in the U.S. data. The trend real rate has fallen by close to 2.5 percentage points (pp), or 250 basis points (bps), from the beginning to the end of the sample, starting near 4% and ending near 1.5%. As can be seen in the green line, a large share of this decline is attributed to a decline in long-term expected growth. Expected growth fell by around 0.75 pp over the sample, which when multiplied by $\gamma \approx 2$ translates to a predicted decline in yields of about 150 bps. While uncertainty affects real rates during deep recessions, it has little long-term effect over the full sample. The change in growth rates and uncertainty accordingly predicted a decline in real rates of around 150 bps overall, so the additional 100 bps of unexplained decline is attributed to the pure discounting residual. This residual was particularly important in explaining the decline in interest rates early in the sample. From 2000 onward, the decline in interest rates has been driven almost exclusively by declines in expected growth rates, implying little impact on equity valuations. As we show in Section 4, this finding is robust to numerous alternative ways of implementing our $r^*_{t,j}$ decomposition, including the use of shorter-horizon interest rates.

The remainder of this subsection studies how secular changes in equity valuations across countries relate to changes in the different components of interest rates. Our goal is to understand country-level changes in equity valuations over our sample period.

**Figure 3: Main Results: Long-Term Decomposition**




*Notes:* This figure plots the country-level changes in equity yields against changes in different components of interest rates, estimated using (17)–(18) following the main specification in column (3) of Table 1. The leftmost figure plots changes in equity yields against changes in the pure discounting term; the middle figure plots changes in equity yields against changes in the growth and VIX components; the rightmost figure plots changes in equity yields against changes in real rates themselves. The sample is 1990–2023, or the longest available span for the given country. For countries for which we can only measure equity yields starting after 1990 (see Appendix B.1), we calculate both Δequity yield and $\Delta r^*$ over the same window.

In the leftmost panel in Figure 3, we plot the change in equity yields against changes in the pure discounting term in G7 countries. This is the same figure plotted in the right panel of Figure 1 in the introduction. The figure illustrates that the large majority of the changes in equity yields over this sample can be explained by changes in the pure discounting term in interest rates. (We show that this relationship is statistically significant in Section 4.1.) We emphasize that the pure discounting term is estimated purely from the interest-rate decomposition in (17)–(18), without the use of equity valuations. As a result, there is nothing mechanical about the powerful explanatory power of pure discounting changes for stock valuations over this sample. This finding accordingly serves as a strong out-of-sample validation of the interest-rate decomposition.

The magnitude of the relation between equity yields and the pure discounting term is, in addition, almost exactly equal to that predicted by theory. The figure shows that equity yields decrease by one percentage point for every one-percentage-point decrease in the pure discounting term, as in Result 1. And the pure discounting term explains not only relative changes in equity valuations across countries, but also changes in valuations in absolute terms: the intercept for the fit is very close to zero, which means the average earnings yield has moved by as much as the pure discounting term. This does not necessarily imply that other factors influencing valuation ratios — such as growth rates and risk premia — have remained constant, but it does imply that potential movements in growth rates and risk premia have, on net, not played a significant role in changing equity valuations on average over this period. The upshot is that to understand long-run valuation changes in this sample, understanding the change in pure discount rates is nearly sufficient.

The middle panel of Figure 3 illustrates the relation between earnings yields and the change in interest rates induced by changes in expected growth rates and uncertainty, taken together. As expected, we find that valuation ratios have dropped in countries where interest rates have dropped because of declines in growth rates and increases in risk: while these changes have decreased interest rates, they have also depressed growth rates on equities and increased equity premia, with the predicted effect on equity valuations being negative. This relationship is noisier than the one plotted in the left panel, consistent with the more ambiguous theoretical predictions for equity valuations given changes in growth rates and uncertainty. But the negative relationship is nonetheless at least moderately strong in the cross-section of G7 countries.

How can the negative relation in the middle panel be squared with the fact that the pure discounting change can nearly perfectly explain the change in equity valuations over time (as documented in the left panel)? Two aspects of the results help in interpreting this. First, note that the best-fit line in the middle panel does not pass through the origin: unlike the $\Delta$equity yield—$\Delta$pure discounting relationship in the left panel (which features an intercept indistinguishable from zero), the line in the middle panel is shifted by 2.9 percentage points to the left. Enforcing an intercept of zero in this $\Delta$equity yield–$\Delta\widehat{r}^*$ relationship, we instead estimate a very small slope (close to -0.1) and an adjusted $R^2$ of -0.15. Explaining changes in valuations in absolute terms evidently requires using the pure discounting change.

Second, once we account for the pure discounting change in the left panel, the remaining terms in $\Delta r^*$ do not provide much additional explanatory power for the long-term equity valuation changes. In a regression for $\Delta$equity yield on both the pure discounting change and the remaining $\Delta\widehat{r}^*$ terms, only the coefficient on the pure discounting change is significant,[16] and the adjusted $R^2$ increases only from 0.79 (in the left panel of Figure 3) to 0.81. To visualize this marginal contribution from growth and uncertainty, Figure B.1 in Appendix B.6 shows a version of the middle panel of Figure 3 where the change in the equity yield has now been residualized against the change in the pure discounting term, $\Delta\widehat{\rho}^*_{t,j}$. The part of the equity yield change unexplained by the pure discounting change is generally small quantitatively, and it is now at most very weakly related to the change in rates from the growth and uncertainty terms, consistent with the more ambiguous effects predicted theoretically.

Moving to the right panel of Figure 3: while we observe comovement between equity

[16]The estimated loading is 0.9 ($p < 0.01$), while the estimated loading on $\Delta\widehat{r}^*$ is -0.5 ($p > 0.1$).

yields and the different components of interest rates, there is almost no relation between equity yields and rates themselves. The components of rate changes have happened to be somewhat negatively related (albeit weakly so) across countries, so adding the horizontal-axis values in the two left panels of the figure generates a muddled and weak relationship between rates and equity yields. In addition, while the point estimate of this slope is positive, note that it again does not pass through the origin: the average advanced economy had close to no equity valuation change, while nonetheless experiencing growth-rate and uncertainty shocks large enough to decrease real rates by nearly 300 bps. This emphasizes how comparing equity valuations to real rates directly can paint a misleading picture.

**Discussion and Interpretation**

Taken together, Figure 3 provides a clear view of both (i) the secular declines in real rates across countries in recent decades, and (ii) their relation to equity valuations. Taking the U.S. to begin, expected real output growth fell by around 3/4 of a percentage point over the 1990–2023 sample period, and the VIX increased slightly. Given the loadings on these terms in Table 1, those two changes together predict a decline in $r^*$ of about 1.6 percentage points. Instead, $r^*$ fell by 2.5 percentage points. We call the difference of 0.9 percentage points a pure discounting shock, akin to a decrease in the pure rate of time preference. Such a decrease predicts an increase in equity valuations (i.e., a decrease in equity yields), and this is exactly what we see in the left panel of the figure. Taking Japan as a contrasting case, its decline in $r^*$ of 3.3 percentage points is a much smaller decline than would have been expected on the basis of the large decrease in long-term expected growth, indicating a positive pure discounting shock. This positive shock similarly perfectly matches the decrease in Japanese equity valuations. The same applies for all the other countries considered.

While the pure discounting shocks provide a very good description of equity valuation changes in an accounting sense, the question of how to interpret them remains somewhat open thus far. We do not view these changes as likely representing a true aggregate preference (or patience) shock among domestic investors. Instead, a "global imbalances" view of cross-country capital flows, as described by Caballero, Farhi, and Gourinchas (2008), appears to be a reasonable candidate explanation. The main decline in the U.S.'s estimated $\rho^*$ occurred in the mid-to-late 1990s and early 2000s (see Figure 2). This period coincides with a large decrease in the U.S.'s net foreign asset position. Japan's estimated $\rho^*$, meanwhile, increased during this decade. Strong demand for U.S. assets, particularly from investors in countries experiencing large shocks to the perceived soundness of their financial system (e.g., in the wake of the Japanese stock-market crash), matches both the timing and the cross-country patterns observed in Figure 3, as we discuss in greater detail in Section 5.1 below.

## 3.3 Higher-Frequency Changes and Forecasting Regressions

Interest-rate movements influence not only secular changes in valuation ratios but also higher-frequency fluctuations. In this subsection, we study how stocks move with the different components of interest rates at a higher frequency. The higher-frequency data allows us to examine these relationships over time within a country, in contrast to considering full-sample changes across countries.

As motivating evidence, we first examine whether variation in the previously estimated pure discounting term aligns with stock-price changes over time in the U.S. data. Results are shown in Figure 4. The figure illustrates a strong comovement between the two series. Regressing the pure discounting term on the equity yield allows us to estimate the share of variation in the equity yield that can be explained by pure discounting, using the slope coefficient in this regression.[17] That number is 77%, emphasizing the important role of the pure discounting term on its own.

This strong comovement arises even though the pure discounting term is estimated without any direct information about equity yields: it uses data only on interest rates, growth expectations, and estimated entropy. One might worry that analysts back out their growth forecasts from equity yield data, and that the growth expectations we use in our decomposition thus mechanically contain information about equity yields. However, such behavior would bias us *against* finding a relation between the pure discounting term and equity yields: the pure discount rate is orthogonal to growth expectations by construction, so if analysts assign variation in equity yields to expected growth, such variation would be mechanically orthogonal to our pure discounting term.[18]

Figure 4 uses the pure discounting term estimated from a regression in levels; we now turn to estimating regressions in differences to study relationships between higher-frequency changes. This approach further sidesteps the potential for spurious comovements between slowly moving variables in levels. We balance two considerations in determining the frequency of measurement for this exercise. First, we wish to explain price and interest-rate variation for reasonably short holding periods. Second, our estimation needs to allow for inertia in forecasters' long-run growth and interest-rate forecasts, which precludes us from considering,

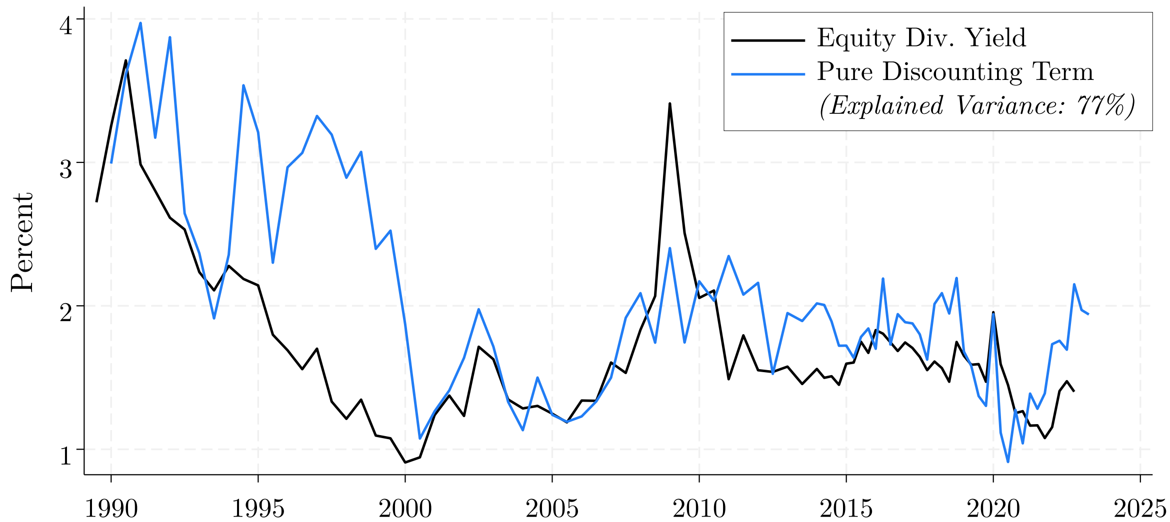
[17] Writing $ey_t = \rho_t + \text{other terms}$, equity yield variance is $\text{Var}(ey_t) = \text{Cov}(\rho_t, ey_t) + \text{Cov}(\text{other terms}, ey_t)$, yielding the decomposition $1 = \frac{\text{Cov}(\rho_t, ey_t)}{\text{Var}(ey_t)} + \frac{\text{Cov}(\text{other terms}, ey_t)}{\text{Var}(ey_t)}$. The first term is the relevant regression slope.

[18] As an extreme case, imagine that rates depend only on pure discounting and growth, and that analysts back out expected growth from the equity yield ($\mathbb{E}_t^{\text{analysts}}[g_{t+1}] = ey_t + \text{constant}$). We would then estimate

$$r_t^* = \alpha + \gamma \mathbb{E}_t^{\text{analysts}}[g_{t+1}] + \varepsilon_t = \widetilde{\alpha} + \gamma ey_t + \varepsilon_t,$$

and recover $\widehat{\rho}_t^* = \widetilde{\alpha} + \varepsilon_t$, which would be orthogonal to $ey_t$ by construction.

**Figure 4: U.S. Equity Valuations and the Pure Discount Rate Over Time**




*Notes:* This figure shows the U.S. equity dividend yield and the pure discounting term over time, estimated using (17)–(18) following the main specification in column (3) of Table 1. For readability, the pure discounting term is plotted as the pure discounting residual shifted up by 2 percentage points ($\widehat{\varepsilon}_{t,j} + 2$).

for example, monthly returns (since forecasts are collected at most once per quarter). In our baseline analysis, we consider three-year returns and estimate how these move with each of the components of interest rates.[19]

The starting point for this analysis is a regression for changes in trend real rates, analogous to equation (17) but in differences rather than levels:

$$\Delta r^*_{t,j} = \alpha_0 + \gamma\, \Delta g^*_{t,j} + \beta_j\, \Delta \mathrm{VIX}^2_{t,j} + \Gamma_j + \varepsilon_{t,j}, \tag{19}$$

where $\Delta$ denotes a three-year change and where the loading on the VIX term is again country-specific.[20] The residual term $\varepsilon_{t,j}$ is now our measure of $\Delta\rho^*_{t,j}$. Next, given this estimated pure discounting change $\widehat{\Delta\rho^*_{t,j}} = \widehat{\varepsilon}_{t,j}$, we regress three-year value-weighted net market returns on that pure discounting term, the change in expected growth, and the change in the VIX, along with a country fixed effect:

$$r^{\mathrm{mkt}}_{t,j} = \alpha_1 + \pi_\rho\, \widehat{\Delta\rho^*_{t,j}} + \pi_g\, \Delta g^*_{t,j} + \pi_V\, \Delta \mathrm{VIX}^2_{t,j} + \Lambda_j + \nu_{t,j}. \tag{20}$$

Table 2 shows the resulting estimates. Before considering our main estimates resulting

[19] In additional analysis, we find that our results are robust to the use of longer or somewhat shorter windows, though the relationships weaken for windows shorter than two years (indicating inertia or measurement error).

[20] Coefficient estimates for regression (19) are presented in Table B.1 of Appendix B.6. The estimates are similar to the level estimates in Table 1, albeit with smaller estimated coefficients. This suggests possible attenuation bias from measurement error that is amplified when estimating in differences; see Griliches and Hausman (1986) and Cochrane (2018). We thank Emi Nakamura for helpful discussions related to this point.

Table 2: Regressions for Three-Year Stock Returns

| | (1) U.S. | (2) U.S. | (3) All | (4) All |
|---|---|---|---|---|
| $\Delta$10y yield | 4.19 | | -3.39 | |
| | (3.51) | | (2.20) | |
| $\Delta$pure discount ($\widehat{\Delta\rho_t^*}$) | | -19.1** | | -9.61** |
| | | (7.64) | | (3.26) |
| $\Delta$exp. growth | | -1.49 | | 16.9* |
| | | (14.0) | | (8.82) |
| $\Delta\text{VIX}^2 \times 100$ | | -3.08** | | -5.44*** |
| | | (1.33) | | (0.90) |
| Country FEs | ✗ | ✗ | ✓ | ✓ |
| Obs. | 74 | 74 | 781 | 781 |
| $R^2$ | 0.04 | 0.20 | 0.05 | 0.27 |
| Within $R^2$ | — | — | 0.02 | 0.24 |

*Notes:* This table shows estimates from regressing three-year value-weighted market returns on changes in 10-year nominal yields (in columns (1) and (3)), and on changes in the three interest-rate components (in columns (2) and (4)). The interest-rate components are estimated from (19), and the table presents estimates from (20). Columns (1)–(2) consider the U.S. only, while (3)–(4) consider the full panel of developed countries (and include country fixed effects). In columns (1)–(2), standard errors are obtained using a block bootstrap with one-year blocks and 10,000 bootstrap draws. In columns (3)–(4), standard errors are clustered by country and date. Statistical significance at the 10% level, 5% level, and 1% level are denoted by *, **, and ***, respectively. The sample is 1990–2023, or the longest available span for the given country.

from (20), we start with a simpler exercise as a benchmark for comparison: we regress three-year stock returns on the unadjusted change in the traded 10-year nominal yield. Column (1) shows the resulting estimate for the U.S. sample. The slope coefficient is close to zero and statistically insignificant, reflecting the well-known fact that returns on stocks and bonds are close to uncorrelated.[21]

In column (2), we present coefficient estimates from (20) in the U.S. data, showing how stock returns load on each of the three drivers of interest-rate changes: changes in the pure discount term, changes in expected growth, and changes in risk. The loading on the pure discount term is -19 and statistically significant. This loading suggests that stock returns go down by 19 percentage points when trend real rates increase by 1 percentage point due to pure discounting. As in Result 2(i), this coefficient has a clear structural interpretation: it is equal to the negative of the cash-flow duration of the overall market. This market-level duration is often approximated by the dividend yield, generating an estimate on the order of 40 years (see, e.g., Gormsen and Lazarus 2023). While somewhat lower than that figure, the

[21]When regressing stock returns on the change in our $r_t^*$, the slope is also small and insignificant.

estimate of 19 years from column (2) is of the same rough order of magnitude and reinforces that the market is a long-duration claim. We view the estimate as a lower bound given the potential for attenuation bias when using the higher-frequency variation in the pure discounting term (see footnote 20 for related discussion).

The remaining estimates in column (2) show that stock returns load weakly on expected growth changes, and significantly negatively on changes to risk, again consistent with theory. Moving to columns (3) and (4) of Table 2, the results are largely similar in the global sample. The slope on the pure discounting shock is smaller in the global data than in the U.S., conceivably reflecting greater measurement error in the non-U.S. data. Growth shocks are also estimated to play a somewhat larger role in the global data.

These estimates allow for an accounting of the period-by-period contribution of different interest-rate components to stock returns. We illustrate this higher-frequency return decomposition for the U.S. data in Appendix Figure B.2, which plots the three-year annualized equity return and each of the three fitted components from the return regression. Shocks to risk appear more relevant for higher-frequency market returns than was the case in Figure 2 for lower-frequency changes. This is consistent with the importance of shocks to risk premia in explaining stock returns (Campbell 1991), and our use of the VIX as a risk proxy likely understates the return variation attributable to risk-premium shocks. Growth shocks also explain a reasonable share of returns; we include realized dividend growth in this component, since the cash-flow term in a Campbell–Shiller return decomposition includes contemporaneous dividend shocks, and returns load positively on realized growth.[22] The pure discounting term varies less dramatically than overall returns, but our estimates suggest that it has affected returns significantly. Such an exercise allows for an assessment of the contributors to stock returns, and their relation to interest rates, on an ongoing basis.

As a final out-of-sample validation test for our interest-rate decomposition in explaining aggregate market returns, we ask whether our estimated pure discounting term $\widehat{\rho}_t^*$ predicts *future* equity returns (in addition to helping account for contemporaneous realized returns). Long-horizon expected equity returns are equal to $\mu_t^* = r_t^* + rp_t^*$. Given that the uncertainty component of $r_t^*$ is likely to be negatively correlated with the equity risk premium $rp_t^*$, interest rates by themselves are unlikely to be useful for predicting future realized returns. The pure discounting component of $r_t^*$, by contrast, strips out the uncertainty component of risk-free rates, and therefore should align well with future equity returns.

We conduct such predictability tests in Table 3, which shows coefficients from regressions of annualized market returns over the subsequent three years on ex ante yield-related predictors.

[22]We omit this term in Table 2 so that the analysis aligns with the long-term decompositions in Figure 3. The loadings reported in Table 2 are nearly identical when including $\Delta d_{t,j}$, and $\Delta d_{t,j}$ has a loading of 0.62**.

**Table 3: Forecasting Regressions for Future Three-Year Market Returns**

| | (1) | (2) | (3) |
|---|---|---|---|
| 10y yield | 0.08 | | |
| | (0.38) | | |
| Survey-based $r_t^*$ | | 0.50 | |
| | | (0.68) | |
| Pure discounting term $\widehat{\rho}_t^*$ | | | 2.08*** |
| | | | (0.61) |
| Country FEs | ✓ | ✓ | ✓ |
| Obs. | 1,050 | 842 | 842 |
| $R^2$ | 0.06 | 0.03 | 0.06 |
| Within $R^2$ | 0.00 | 0.00 | 0.03 |

*Notes:* This table shows coefficient estimates from forecasting regressions $r_{t,t+3}^{\text{mkt}} = \alpha + \beta X_t + \varepsilon_{t,t+3}$, where $r_{t,t+3}^{\text{mkt}}$ is the country-level annualized three-year market return, and $X_t$ is an ex ante predictor variable. The first column uses the 10-year nominal yield as the predictor variable, using data obtained from each country's central bank. The second column uses our survey-based measure of the trend real rate $r_t^*$ as predictor. The third column uses our estimated pure discounting term $\widehat{\rho}_t^*$, estimated using (17)–(18) following the main specification in column (3) of Table 1. Each regression includes country fixed effects, and all standard errors are clustered by country and date. The sample is 1990–2023, or the longest available span for the given country.

Columns (1) and (2) show that neither nominal yields nor our measure of $r_t^*$ help predict equity returns. This provides further evidence that risk premia comove negatively with risk-free yields, as emphasized by Farhi and Gourio (2018). Meanwhile, as can be seen in column (3), the pure discounting term strongly predicts future returns. While the estimated coefficient of 2.08 is somewhat larger than the theoretical prediction of 1, the estimate is sufficiently noisy that we cannot reject a value of 1 at the 5% level. The upshot of this analysis is similar to our findings above: our interest-rate decomposition succeeds at stripping out shocks to risk-free yields with offsetting effects on equity risk premia (or growth rates), leaving us with a useful measure of the pure discounting component of long-term interest rates. This pure discounting term can be exploited as a counterfactual long-term risk-free rate to use in calculating a duration-matched equity premium; we will return to this insight in Section 5.2.

## 3.4 Cross-Sectional Portfolios

We now turn to the cross-section of stock returns, and study whether firms with different cash-flow duration have different exposure to the pure discounting term. A large literature studies the risk and return properties of firms with different cash-flow timing (see Gormsen and Lazarus 2023, and citations therein), finding that firms with shorter-duration cash flows have higher risk-adjusted returns. In this section, we use our methodology to quantify cross-sectional differences in duration, which is a key (and debated) object for this literature.

We focus here on the measure of duration used in Gormsen and Lazarus (2023), which is based on firm-level cash-flow growth estimates. We first obtain analysts' median forecasts of stock-level long-term earnings growth (LTG) from IBES (see Gormsen and Lazarus 2023 for details). For each year, we sort stocks into quintile portfolios based on their cross-sectional LTG rank; we do so separately for the U.S. and global data. These are then our five duration-sorted portfolios for which we calculate value-weighted returns. According to this exercise, firms with higher expected cash-flow growth have, all else equal, longer cash-flow duration.[23]

In Figure 5, we report slope coefficients of regressions of three-year realized returns onto three-year changes in the pure discounting term for our five portfolios of U.S. stocks with different cash-flow duration.[24] The figure shows that the portfolio of firms with the shortest cash-flow duration has a slope coefficient of around -10, while the portfolio of firms with the longest cash-flow duration has a slope of -30. At face value, these estimates suggest that the cash-flow duration of these portfolios varies from 10 to 30 years, which is significant both economically and statistically.

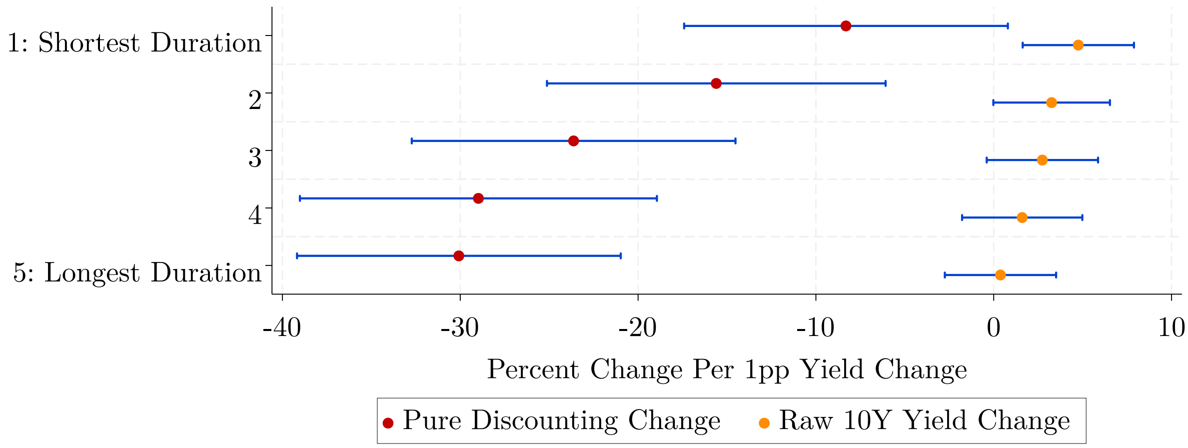
As with the previous analysis, it is possible that the slope coefficients suffer from attenuation bias. If such attenuation bias is driven by classical measurement error, it is similar (in percentage terms) for the different portfolios. In this case, it is useful to focus on the ratio of the cash-flow duration of the different portfolios, as this ratio will be unaffected by classical measurement error. We find that the portfolio of firms with the longest duration has three times as long a cash-flow duration as the portfolio of firms with the shortest duration. A lower bound on this difference appears to be 20 years, but we cannot rule out that it is longer.

By contrast, as can be seen in the coefficients plotted in orange, long-duration stocks are not substantially more exposed to raw interest-rate changes than short-duration stocks: all of them have very small estimated loadings when regressing their returns on the change in 10-year nominal yields. And the estimated coefficients go in the "wrong" direction, at least with respect to an interpretation of all interest-rate changes as being exogenous pure-discounting shocks: rather than returns *decreasing* when interest rates increase, they instead weakly increase. This further reinforces the point made in Section 2.4: equity duration does not correspond to the price sensitivity of equity to an arbitrary change in interest rates. Instead, only pure discounting shocks induce interest-rate variation that passes through to

[23] Another standard approach is to proxy for duration by valuation ratios (like book to market), since a higher valuation ratio is associated with a longer cash-flow duration (i.e., growth firms are long-duration firms). Using book-to-market ratios to measure duration does not affect this section's results.

[24] We present corresponding results for the full global sample in Figure B.3.

**Figure 5: Portfolio Exposures to Pure Discount Rates and Yields: U.S. Stocks**




*Notes:* This figure shows slope estimates from univariate regressions for three-year returns of each of five equity portfolios on the three-year change in (1) the pure discounting term ($\widehat{\Delta\rho_t^*}$), marked in red, and (2) the nominal 10-year yield, marked in orange. Each regression contains a constant. For example, the first point in the top left of the figure shows $\widehat{\beta}_1$ (and 95% confidence intervals) from $r_{t,1} = \alpha_1 + \beta_1 (\widehat{\Delta\rho_t^*}) + \varepsilon_{t,1}$, where $r_{t,1}$ is the three-year return on a value-weighted portfolio of the stocks in the bottom quintile of cash-flow duration. The pure discounting term is estimated from (19) using U.S. data. Duration-sorted portfolios and returns are calculated following Gormsen and Lazarus (2023). The sample is 1990–2023.

equity in proportion to its duration. Duration-sorted portfolios should not, and do not, vary significantly in their exposure to nominal interest rates by themselves; instead, they vary only in their exposure to pure discount-rate changes.

# 4. Robustness: Alternative Approaches and Assumptions

We now assess the robustness of our main results to alternative measurement choices. We first show in Section 4.1 that our interest-rate decomposition yields similar results under a range of approaches for measuring the trend real rate and its components. Section 4.2 then generalizes our analysis to handle the case in which dividend growth is not proportional to output growth due to time-varying profit shares, again with essentially unchanged results.

## 4.1 Alternative Measures for the Real-Rate Decomposition

Table 4 shows a series of robustness tests for our main results under alternative measurement approaches for the real-rate decomposition. The top panel repeats the main cross-country pass-through analysis from the first panel of Figure 3 (and Figure 1 in the introduction), confirming that the one-for-one relationship between equity yields and the pure discounting term holds across specifications. The bottom panel verifies the U.S. time-series properties of $\rho^*$ documented in the previous section.

**Table 4: Robustness to Alternative Measures for $r^*$ Decomposition**

| | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| | Baseline | SPF-Based | Short-Term $r^*$ | GARCH Vol. | Unc. Index |
| *Inputs changed:* | — | $r^*, g^*$ | $r^*$ | $L^*$ | $L^*$ |
| *Robustness of Figure 3: Regressions of $\Delta ey^*$ on $\Delta\rho^*$ (G7)* | | | | | |
| Slope | 1.14*** | — | — | 1.01*** | 1.15*** |
| | (0.23) | | | (0.20) | (0.24) |
| Intercept | 0.20 | — | — | 0.31 | 0.28 |
| | (0.31) | | | (0.29) | (0.30) |
| Adj. $R^2$ | 0.79 | — | — | 0.79 | 0.79 |
| *Robustness of $\rho^*$: Time-Series Properties (U.S.)* | | | | | |
| $\Delta\rho^*$ over sample | -0.84 | -1.35 | -1.29 | -0.94 | -0.91 |
| Corr($\rho^*, \rho^*_{\text{baseline}}$) | 1.00 | 0.83 | 0.71 | 0.98 | 0.99 |

*Notes:* This table shows robustness of the main results to alternative measurement approaches for the $r^*$ decomposition. All $r^*$ decompositions follow the specification in column (3) of Table 1 for measures available across countries (columns (1), (4), and (5)), and they follow column (1) of Table 1 for U.S.-only measures (columns (2) and (3)). Col. (1) uses the baseline specification as in the left panel of Figure 3. Col. (2) replaces the Consensus data with SPF forecasts for the average 10-year real yield and real GDP growth over the next decade. Col. (3) replaces the Consensus-based $r^*$ with the Bauer and Rudebusch (2020) $r^*$ estimate. Col. (4) replaces VIX$^2$ with a country-level 6-month GARCH(1,1) forecast of equity return variance. Col. (5) replaces VIX$^2$ with the country-level Baker, Bloom, and Davis (2016) uncertainty index. The top panel shows cross-country (G7) regressions of full-sample changes in equity yields on changes in $\rho^*$, for all available specifications. Standard errors in parentheses are obtained via block bootstrap by year with 10,000 draws. Statistical significance at the 10% level, 5% level, and 1% level are denoted by *, **, and ***, respectively. The bottom panel shows U.S. time-series results. The first row shows the full-sample change in the estimated $\rho^*$. The second row shows the correlation between the alternative measure of $\rho^*$ and the baseline measure from column (1). The sample is 1990–2023, or the longest available span for the given country.

Column (1) reports results from the baseline specification. For the top panel, this is the same regression as in the first panel of Figure 3, but now with standard errors and statistical significance reported. These are computed via a non-stationary block bootstrap described in Appendix B.2. This bootstrap accounts for estimation uncertainty in the generated regressor $\hat{\rho}^*$, providing statistical evidence to buttress the strong visual relationship in Figure 3. The slope coefficient of 1.14 is highly significant and statistically close to the theoretical benchmark of 1, and the intercept is not significantly different from zero.

The remaining columns vary the inputs to the real-rate decomposition, with data sources described fully in Appendix B.3. Column (2) uses forecasts from the Survey of Professional Forecasters (SPF) for the 10-year real yield and real GDP growth over the next 10 years, in place of the Consensus Economics forecasts used in our baseline. Column (3) uses the Bauer and Rudebusch (2020) short-term $r^*$ estimate, a statistical estimate of the trend short-term real rate that differs from our baseline both in maturity and in that it does not use forecast data directly.[25] Column (4) uses a country-level 6-month GARCH(1,1) forecast of equity return variance in place of VIX$^2$, to help address the possibility that VIX$^2$ may not adequately measure uncertainty or its relation to equity risk premia. Similarly, column (5) uses the country-level Baker, Bloom, and Davis (2016) economic policy uncertainty index in place of VIX$^2$. Columns (2) and (3) are U.S.-only specifications given data availability.

Across all specifications, the results are strongly consistent. For the cross-country regressions, the slope coefficients range from 1.01 to 1.15, all highly significant and close to the theoretical prediction of one-for-one pass-through. The U.S. time-series results are similarly robust: the full-sample change in $\rho^*$ ranges from -0.84 to -1.35 percentage points across specifications, in all cases less than half of the overall change in $r^*$ over the sample. The correlations between each alternative $\rho^*$ measure and the baseline are uniformly very high, indicating that the different measurement approaches identify very similar variation in the pure discounting term over time. So while we use the Consensus data as our baseline given its full cross-country panel structure, the main conclusions from Figure 3 are robust to alternative measures used to implement our decomposition.

## 4.2 Time-Varying Profit Shares

Our main analysis assumes that dividend growth is proportional to output growth. While log dividends and consumption should be cointegrated at a sufficiently long horizon, they do not comove perfectly at all dates or forecast horizons. In recent decades in the U.S., for example, equity cash flows have outpaced GDP and consumption given increases in the corporate profit share of income (Greenwald, Lettau, and Ludvigson 2025; Atkeson, Heathcote, and Perri 2025a). We consider here how time-varying profit shares affect our analysis.

We first note that our interest-rate decomposition in Result 1, $r_t^* = \rho_t^* + \gamma g_t^* - L_{t,M}^*$, is unchanged, as aggregate growth $g_{t+1}$ is the relevant outcome for the SDF. For equity cash flows, denote dividend growth by $g_{t+1,d} = d_{t+1} - d_t$. Rather than imposing $g_{t+1,d} = \lambda g_{t+1}$, we now allow for an arbitrary dividend growth process. So it may be the case that $\mathrm{Corr}(g_t^*, g_{t,d}^*) < 1$. Our equity-yield decomposition now becomes

$$\begin{aligned} ey_t^* &= \rho_t^* + \gamma g_t^* - g_{t,d}^* + (rp_t^* - L_{t,M}^*) \\ &= \rho_t^* + \gamma g_t^* - g_{t,d}^* - L_{t,MR}^*. \end{aligned}$$

[25] Our theory applies to both short- and long-term yields. As further illustration that our results hold with shorter-term yields, Appendix Figure B.4 shows $r^*$ constructed from the 5-year Consensus forecast of real bill yields (available over a shorter sample) and compares the resulting $\rho^*$ series to the baseline $\rho^*$. The two are nearly indistinguishable for the overlapping sample.

As before, only shocks to the pure discounting term $\rho_t^*$ pass through directly from rates to equity yields. And though the pass-through of growth-rate shocks may be different than in Result 1, it is still the case that such shocks induce weaker pass-through than pure-discounting shocks as long as $\text{Corr}(g_t^*, g_{t,d}^*) > 0$. While there may now be pure dividend-growth shocks (i.e., changes to $g_{t,d}^*$ without corresponding changes in $g_t^*$), these are entirely separate from the interest-rate dynamics considered in our empirical decomposition for $r_t^*$.

To reconcile this with the importance of profit-share shocks estimated in recent literature, note that such changes may raise contemporaneous cash flows without affecting expected future growth. In this case, prices and cash flows rise together, leaving equity yields unaffected. Alternatively, equity cash-flow growth expectations may have diverged meaningfully from output-growth expectations. While these changes to expected future profit-share growth would be separate from the interest-rate shocks we study, we can nonetheless estimate their magnitude and effects on valuations using two additional U.S. forecast measures: Consensus forecasts on corporate profit growth, and IBES forecasts of long-term earnings growth.

We present and discuss these analyses in Appendix B.4. In both cases, profit-share shocks appear to have materialized mainly as changes in current cash flows rather than expected future growth rates, leaving equity yields close to unaffected. Our estimated pass-through of roughly 1/3 of the decline in $r_t^*$ to equity valuations in U.S. data remains unchanged, as do our higher-frequency return regressions when we add profit-share controls. Our main results therefore continue to apply.

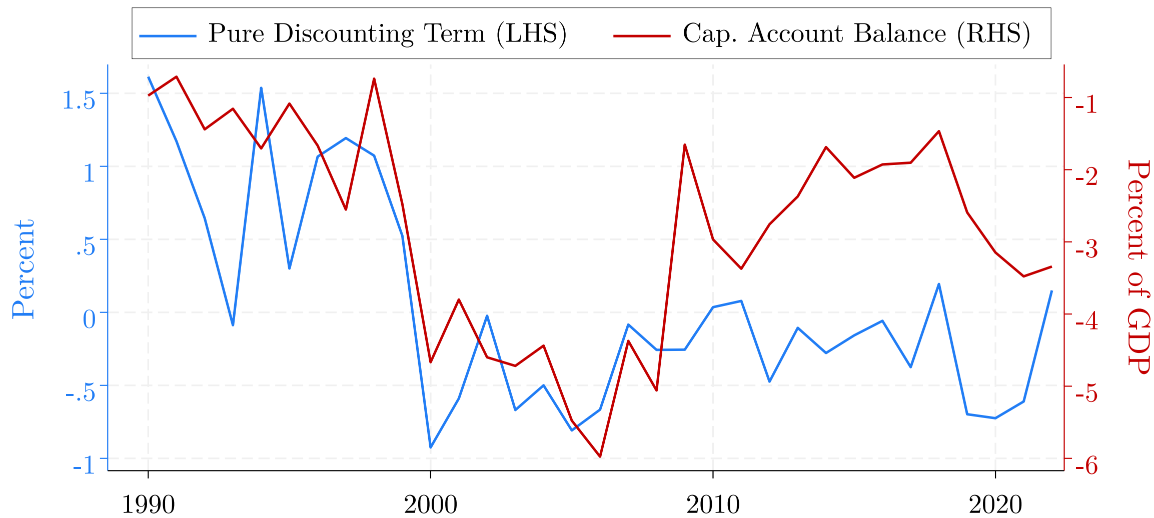
# 5. Empirical Implications and Economic Mechanisms

This section considers a series of further applications of our framework and results. In Section 5.1, we examine one potential mechanism underlying variation in the pure discount rate over time: cross-country capital flows. We then use our framework to shed light on three questions that have been debated in recent literature: (1) the "duration-matched" equity premium (Section 5.2), (2) the effect of decreasing interest rates on the value premium (Section 5.3), and (3) the potential role of an information effect in explaining stock-price responses to monetary policy news (Section 5.4).

## 5.1 Cross-Country Capital Flows

While pure discount-rate shocks align closely with equity valuation changes in our sample, how to interpret these shocks remains an open question. While $\rho^*$ is formally equivalent to the marginal investor's rate of time preference, shifts in $\rho^*$ are unlikely to reflect such

**Figure 6: Pure Discount Rates and Net Capital Flows in the U.S.**




*Notes:* The blue line shows the end-of-year pure discounting term $\widehat{\rho}^*_{t,j}$ in the U.S. data, as plotted in Figure 2. The red line shows the U.S. net capital account balance as a share of GDP, measured from the IMF's "Net Financial Account" series. The figure plots $\widehat{\rho}^*$ against annual flows, as Proposition 1 of Caballero, Farhi, and Gourinchas (2008) shows that permanent shocks lead to persistent annual flow effects.

preference changes alone: the notion that Japanese households have become significantly more impatient relative to U.S. households, for instance, seems dubious.[26] As an alternative candidate channel, we consider the potential role of cross-country capital flows in explaining changes in pure discount rates. Such capital flows can in principle affect interest rates in a manner not fully accounted for by domestic fundamentals; since $\rho^*$ absorbs any rate movements not explained by growth or risk, the flows must result in a change in this term.

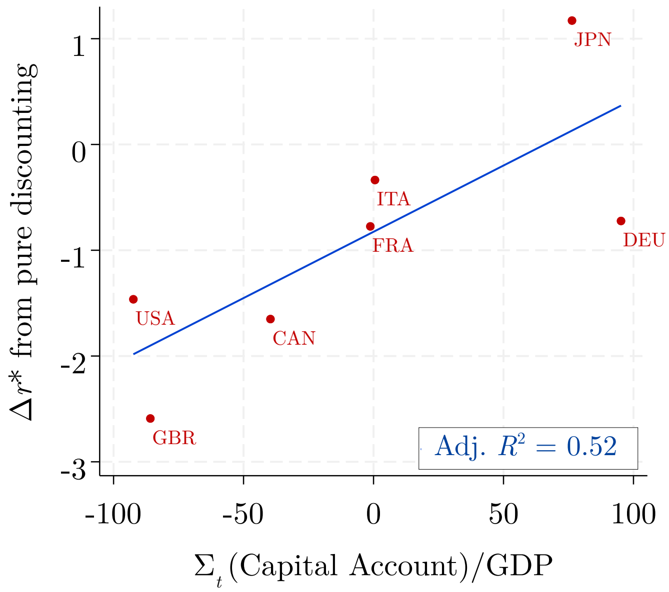
Concretely, consider a shock that generates strong foreign demand for U.S. assets. For example, following the "global imbalances" model of Caballero, Farhi, and Gourinchas (2008), such demand might arise from foreign investors experiencing a shock to the perceived soundness of their local financial system. The resulting demand for U.S. assets, if relatively inelastic, can push down U.S. interest rates without corresponding changes in U.S. growth expectations or uncertainty. In our decomposition, this would manifest as a decline in the U.S. pure discounting term $\rho^*$.[27] And as long as bond and equity markets are integrated, this change in $\rho^*$ then passes through to equity discount rates and prices following Section 2.

We therefore examine whether capital flows help explain our estimated pure discount-rate changes. We begin with time-series evidence in the U.S. data. As Figure 6 shows, the

[26]Demographic changes could in principle drive such preference shifts. However, secular-stagnation theories (e.g., Eggertsson, Mehrotra, and Robbins 2019) often predict that aging should be associated with larger declines in rates. By contrast, we find that $\Delta\rho^*$ in Figure 3 is positively related to a country's degree of aging.

[27]An alternative to the global imbalances view is that pure discount rates and interest rates may have converged toward a common global level given financial integration, with capital flows driving this convergence.

**Figure 7: Cross-Country Pure Discounting Changes vs. Cumulated Capital Flows**




*Notes:* The vertical axis shows the country-level change in the pure discounting residual, as plotted on the horizontal axis in the first panel of Figure 3. The horizontal axis shows the cumulative sum of that country's net capital account balance as a share of annual GDP, measured from the IMF's "Net Financial Account" series. The sample is 1990–2023, or the longest available span for the given country.

variation in estimated $\rho^*$ (plotted on the left axis) aligns well with annual net capital flows measured with IMF data (on the right axis, where negative numbers indicate net inflows into U.S. assets). The main decline in the U.S.'s $\rho^*$ occurred in the mid-to-late 1990s and early 2000s, a period coinciding with a sharp decrease in net capital flows.

In Figure 7, we analyze whether capital flows help account for pure discount-rate changes in the full panel of G7 countries. There is a fairly strong, positive relation between the full-sample change in a country's $\rho^*$ and the change in its net foreign asset position (calculated from cumulated net capital flows so as to remove mechanical valuation effects). Countries that received more capital inflows experienced larger declines in $\rho^*$, and such capital flows thus represent one plausible source of cross-country changes in pure discount rates. That said, capital flows are themselves determined in equilibrium, and they relate to fundamentals in ways that we do not consider here. So while these flows help account for the observed changes in pure discount rates, we leave a deeper analysis of causal drivers for future work.

## 5.2 Estimating the Equity Premium

Recent work by van Binsbergen (2024) reassesses the performance of the stock market by comparing it to the realized returns on bonds. He argues that realized returns on long-term bonds provide a natural comparison for equities, as equities have a long cash-flow duration themselves.[28] He finds that realized stock returns are similar to the realized returns on a duration-matched Treasury portfolio in recent decades, implying a realized excess return on stocks close to 0%. van Binsbergen describes this finding as a puzzle, possibly reflecting either low equilibrium compensation for dividend risk or unexpected negative shocks to the path of cash flows.

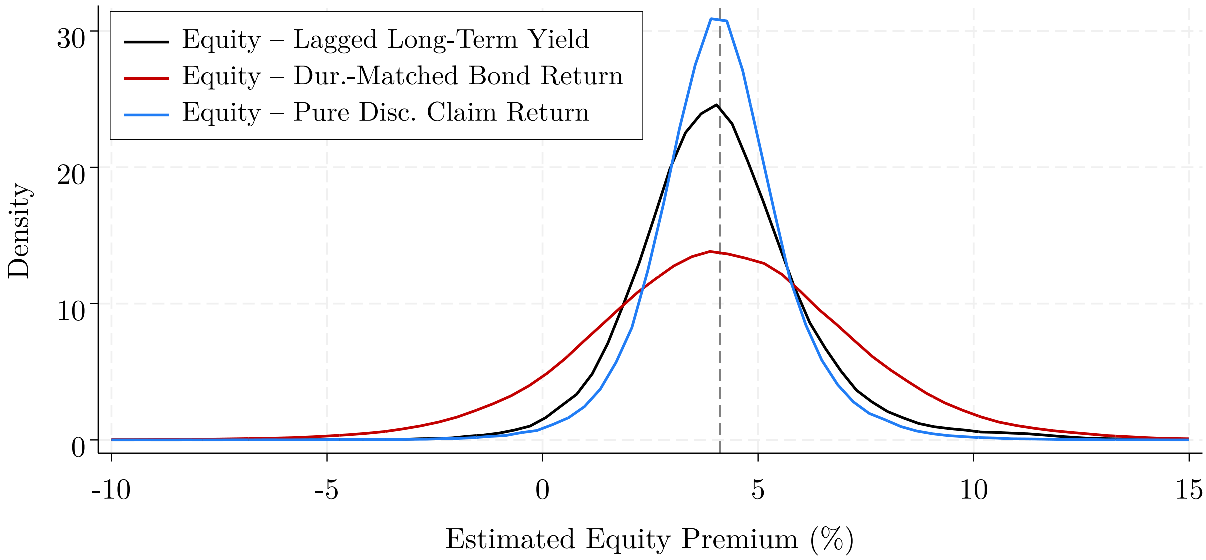
Our analysis emphasizes that realized stock returns are likely to appear low (or high) when compared to realized returns on a duration-matched portfolio: only movements in bond prices (interest rates) that are driven by pure discounting shocks are fully passed through to stock prices. Movements that are driven by growth rates are mostly unrelated to valuations, and movements driven by uncertainty have opposite effects on stocks and bonds. Since bond returns are not incorporated into stock returns in general, stock returns are likely to look high or low relative to bond returns even in relatively long samples.

We formalize this point through simulation studies of an artificial calibrated economy. We consider a generic economy for which the additive decomposition of the log SDF in (2) holds and calibrate it to match the data in terms of volatility of stock prices, growth rates, pure discounting shocks, and risk shocks; see Appendix B.5 for details. We discipline the relation between interest rates, growth, and risk premia based on the empirically observed relations, but we note that using theoretically motivated parameters in the consumption-based version from Section 2.3 gives similar results. Using this model, we run 100,000 simulation of artificial 30-year samples and calculate and compare realized returns on stocks and bonds.

Figure 8 shows that realized stock returns can deviate substantially from realized returns on a duration-matched bond portfolio, and that the difference between the two is a poor estimate of the equity risk premium. The figure shows distributions of average excess returns on stocks relative to three benchmarks. For the first two benchmarks, we consider the ex ante long-term interest rate (black) and the realized return on a duration-matched bond portfolio (red). The two benchmarks deliver the same average excess returns across the simulations (the risk premium of 4%), which is mechanical because the expected return on the duration-matched bond portfolio is the long-run interest rate in our simulations. But the distribution of estimates of the excess returns is much wider when comparing stock returns to realized bond returns: while it is very unlikely to observe negative excess returns when comparing to the interest rate, there is a 10% probability of observing negative excess returns when comparing to the realized returns on the duration-matched portfolio. Seen in this light, the small duration-matched equity premium documented by van Binsbergen (2024) may be

[28]Polk and Vuolteenaho (2025) review and extend this analysis, arguing that stock returns in excess of long-term bond returns provide a "purified" measure of the equity premium.

**Figure 8: Simulated Distributions of Average Excess Equity Returns**




*Notes:* This figure shows the results of simulations of calibrated economics. We simulate changes in growth rates, pure discounting terms, and uncertainty in 30 year samples. Based on these realizations of these structural variables, we calculate realized returns on equity and bonds. The figure shows the distributions of average 30-year realized excess returns on stocks across 100,000 simulations. The excess returns are calculated relative to three benchmarks: the lagged long-term interest rate (black), the realized returns on a duration-matched bond portfolio (red), and the realized returns on a duration-matched pure-discounting claim (blue), as defined in equation (21). The true equity premium relative to the long-term yield is shown in the dashed gray line, and the mean of all three distributions is equal to the true premium. Details of the simulations can be found in Section 5.2 and Appendix B.5.

somewhat unlikely, but it is within what should be expected from such exercises.

The results suggest that controlling for duration-matched bond returns is unlikely to improve estimates of the equity risk premium. But our framework suggests that one can, in fact, improve estimates by controlling for changes in the pure discounting term. As shown in Result 2, the effect of a change in the pure discounting term $\rho$ is given by the duration of the stock market times the change in $\rho$. We can then calculate the realized excess returns net of shocks to the pure discounting term as

$$r^{\rho\text{-hedged}}_{t,t+n} = r^{\text{stocks}}_{t,t+n} - r^{f}_{t,t+n} - \mathcal{D}(\rho_t - \rho_{t+n}), \tag{21}$$

where $r^{f}_{t,t+n}$ is the ex ante long-term interest rate. The value $r^{f}_{t,t+n} + \mathcal{D}(\rho_t - \rho_{t+n})$ can be thought of as the return on a "pure discounting claim," or a bond that appreciates only when the pure discount rate decreases. The equity return net of this pure discount claim return thus provides an equity return hedged against changes in $\rho$.

As shown in the third line in Figure 8, realized excess returns net of the pure discounting claim offer a more precise estimate of the equity risk premium than the other approaches.

This distribution of excess returns (blue) has the same mean across simulations as the other approaches, so the three methods extract the same equity risk premium on average (the true equity risk premium). But the $\rho$-hedged excess returns have a tighter distribution around this true risk premium. The reason is that the realized stock returns are influenced by unexpected shocks to $\rho$, and accounting for these shocks brings the realized returns closer to the ex ante equity risk premium. Because of the market's long duration, these changes can have a meaningful impact on excess returns even though the $\rho$ shocks are modest in magnitude. Given our calibrations, the 95% confidence interval for the $\rho$-hedged excess return is 1.7 to 6.6%, which is tighter than the confidence interval for the standard approach (1.2 to 7.4%). The confidence interval for the duration-matched bond approach is twice as wide as the interval for the $\rho$-hedged approach (-1.0 to 9.2%).

Motivated by the above results, we estimate the U.S. equity risk premium over our sample period based on the three approaches above: (1) the market return in excess of the long-term nominal bond yield, (2) the market return in excess of the duration-matched nominal Treasury return, analogous to van Binsbergen (2024),[29] and (3) the market return hedged against changes in the value of the pure discounting claim.

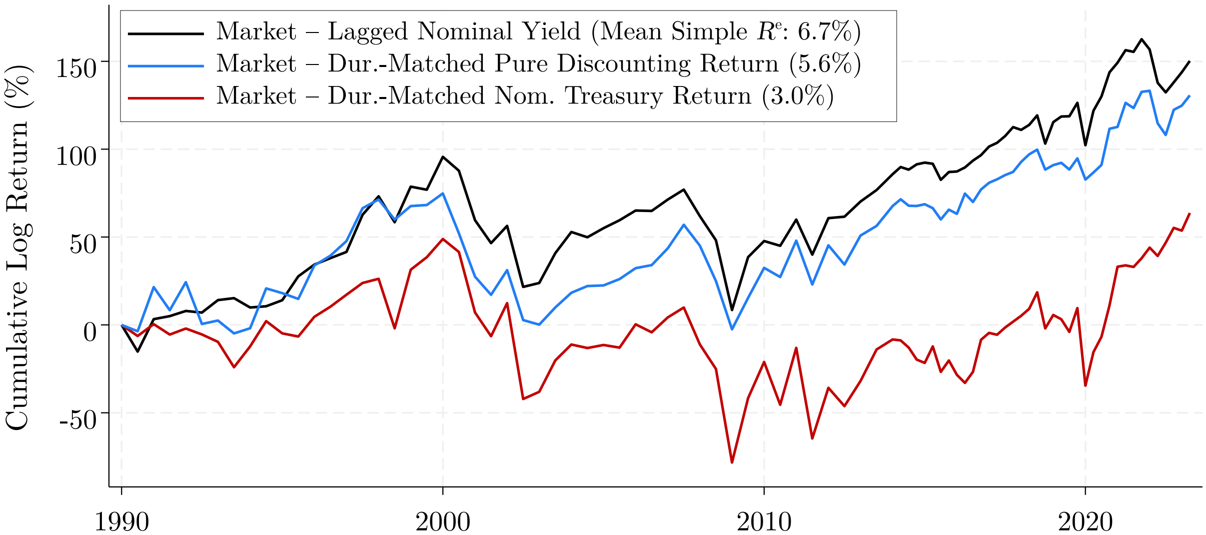
These cumulative returns are plotted in Figure 9. As can be seen in the black line, the market has had high average returns relative to the long-term risk-free rate over this period, with a realized annual arithmetic equity premium of 6.7%.[30] The red line shows a version of the finding in van Binsbergen (2024): when compared to the holding-period returns on long-term nominal Treasuries, much of this premium disappears. The full-sample average return on this nominal-Treasury-adjusted basis is 3.0%. But before considering the last three years of the sample (which featured increasing interest rates and high equity returns), there was no excess return relative to the duration-matched nominal Treasury: the red line in Figure 9 crosses zero in the fourth quarter of 2020, indicating precisely zero average excess return in the preceding thirty years of the sample.

By contrast, the stock returns in excess of shocks to the pure discount claim, shown in blue, is high and stable. On an annualized basis, this realized excess return is estimated to be 5.6% over this period, only slightly lower than the 6.7% benchmark. As a result, we can conclude that the estimated equity risk premium is close to the historical average even after accounting for the secular decline in the pure discount rate. Over the sample, the pure

[29]Our nominal bond return calculation is somewhat less sophisticated than his. He constructs a bond portfolio with multiple nominal bonds, each weighted in proportion to the value weight of the market's expected future dividend at the corresponding maturity. By contrast, our counterfactual nominal log bond return is equal to $r_{t+1,n} = y_{t,t+19} - \mathcal{D}(y_{t+1,t+20} - y_{t,t+19})$, where $\mathcal{D} = 19.1$ years from Table 2, and where we take the 19-year zero-coupon yield from Gürkaynak, Sack, and Wright (2006).

[30]We use the lagged long-term yield as our benchmark here to put all three excess returns on equal footing. The arithmetic average equity return relative to the short-term risk-free rate was 9.3% over this period.

**Figure 9: Cumulative Excess Returns for the U.S. Market**




*Notes:* This figure shows cumulative log returns on the value-weighted U.S. stock market in excess of three different counterfactual bond returns. The black line shows the return relative to the one-period-lagged 19-year nominal zero-coupon yield from Gürkaynak, Sack, and Wright (2006), where we use a 19-year duration given the results in Table 2. The blue line shows the return relative to the duration-matched pure discounting claim, calculated as in (21) using the estimated $\widehat{\rho}_t^*$ from (18). The red line shows the return relative to an unadjusted nominal Treasury security with duration $\mathcal{D} = 19.1$ years; see footnote 29 for details of construction. In all cases, we calculate cumulative log returns for equity and for bonds separately and plot the respective difference. The mean simple excess returns listed in the legend are calculated as average arithmetic (not logarithmic) excess returns for non-overlapping one-year periods.

discount rate goes down by slightly less than 1 pp, which, accounting for the duration effect, leads to a 19% reduction in the realized excess log stock return. This effect has to be averaged out over the 35 year sample; when converted to simple returns, the estimated equity risk premium decreases by roughly 1% per year.

Taken together, this section argues against comparing realized stock returns to a duration-matched bond counterfactual if the goal is to estimate the equity premium. Subtracting realized bond returns from realized stock returns adds more noise than it subtracts, leading to less precise estimates of the equity premium on average. In our sample, this approach leads to an excess stock return close to 0%. This estimate can appear puzzling, but it is explained by changes in growth rates and risk premia. Once accounting for these changes, realized stock returns are close to the historical equity premium.

## 5.3 The Value Premium and Interest Rates

We next turn to a puzzling pattern observed in the cross-section of stocks in recent decades. The value premium — measured as the average return on stocks with high book-to-market ratios minus stocks with low book-to-market ratios, or HML (Fama and French 1993) — has been substantially weaker in recent decades than implied by historical averages. One potential explanation for this underperformance could be that interest rates have dropped, which has led to an unexpected capital gain for the long-duration growth firms, leading growth firms to have performed better than expected ex ante.[31] On the surface, this effect could be meaningful. Imagine that growth firms have a 30-year longer duration than value firms. A naive calculation would imply that a roughly 3 percentage point drop in interest rates would have led to a 90 percentage point relative outperformance of growth firms. Over a 20-year span, this translates to a relative outperformance of more than 4 percent per year, which is large enough to wipe out effectively the entirety of the historical value premium.

The above calculations are, however, not the full story, as discussed in previous sections. First, while interest rates have dropped by close to 3 percentage points in the U.S., the pure discounting term has dropped by only about 1 percentage point, and it is only this component that should pass through to long-duration assets. Second, we estimate that the spread in duration for value-sorted portfolios is substantially below the 30 years assumed above. The net effect on the realized return on the value factor is therefore substantially smaller.

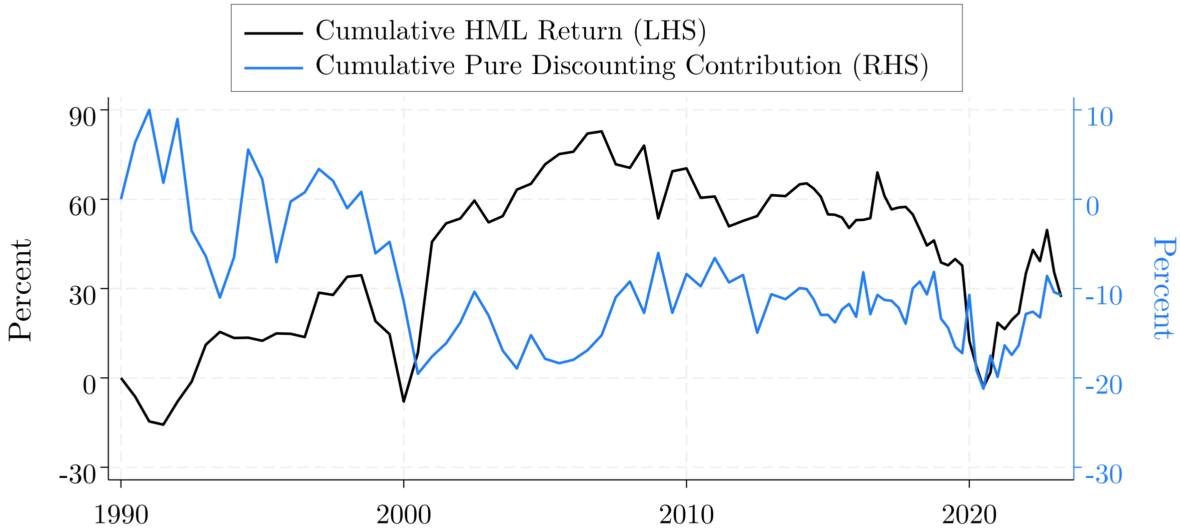
We illustrate and quantify the effect of changes in the pure discounting term for the value factor in the U.S. data in Figure 10. The figure shows both cumulative returns for the HML factor (in black, corresponding to the left axis), and the estimated contribution of changes in the pure discounting component of real rates (in blue, right axis). This pure-discounting contribution is estimated by regressing HML returns on the change in the residual $\widehat{\rho}^*_{t,j}$ from equations (17)–(18), controlling for changes in growth rates and uncertainty,[32] and then multiplying the estimated coefficient by the cumulative change in $\widehat{\rho}^*_{t,j}$ since 1990.

As the figure shows, the effect of the pure-discounting term is modest but non-trivial, reaching a cumulative effect of -20% return at the trough in 2020, but only -10% over the full sample. In addition, the crash and rebound of the value factor from 2020–2023 matches the dramatic changes to the pure discount term experienced over those years at least in timing, if not fully in magnitude: the cumulative HML return over that period is around 30% (on the left axis), while the percent attributable to the pure discounting term is around 10% (on the right axis). So while the pure discounting contribution is often important, it is clearly not the full story explaining the performance of value in recent decades in the U.S. sample.

[31] This hypothesis is discussed by Maloney and Moskowitz (2021) and Asness (2022), among others.

[32] This estimation exercise parallels the one in equation (20) for the overall market, with the exception that here we use the change in the residual $\widehat{\rho}^*_{t,j}$ estimated in levels from (17) (rather than the residual from the first-difference estimation in (19)). We do so because this allows for straightforward estimation of a cumulative effect of the change in $\widehat{\rho}^*_{t,j}$ in levels, as needed for the exercise in Figure 10. Cumulating the effects of three-year changes $\widehat{\Delta\rho^*_{t,j}}$, by contrast, would only allow for measurement of the pure discounting contribution every three years (so the blue line in Figure 10 would only show 11 equally spaced points).

**Figure 10: The Contribution of Pure Discount Changes to Value Factor Returns**




*Notes:* The black line shows the cumulative return on the Fama and French (1993) HML value factor for the U.S. sample since 1990, obtained via Ken French's website, plotted on the left axis. The blue line shows the contribution we estimate is attributable to the pure discounting component of real rates, plotted in cumulative percent terms on the right axis. For this estimated contribution, we begin with the pure discounting residual $\widehat{\rho}^{*}_{t,j}$, estimated using (17)–(18) following the main specification in column (3) of Table 1, as plotted in Figure 2. We regress three-year HML returns on the three-year change in this residual, along with the three-year change in expected growth rates and the three-year change in the squared VIX (akin to (20)). The estimated contribution from the three-year residual is then the estimated coefficient on $\Delta\widehat{\rho}^{*}_{t,j}$ multiplied by the cumulative change in $\widehat{\rho}^{*}_{t,j}$ since 1990.

In Appendix Figure B.5, we exploit our global panel to study what share of the cross-country differences in realized value returns since 1990 can be explained by cross-country differences in the evolution of the pure discounting term. The figure shows that value firms in countries that have experienced a larger decrease in the pure discount term have had lower realized premia relative to growth firms over the sample period. The cross-sectional $R^2$ demonstrates meaningful explanatory power, but it also indicates that the returns to the value factor cannot be fully summarized by changes in the pure discounting term.[33] So the pure discounting change is again important for explaining a share, but not the entirety, of the performance of value portfolios across countries.

## 5.4 Unpacking Monetary Policy Shocks

As a final exercise, we use our decomposition and estimation results to help unpack the effects of surprise changes in short-term interest rates by monetary policymakers. These surprises, when properly measured (e.g., using high-frequency changes in interest rates around policy

[33] We find that the same results hold — both within and across countries — when considering HML alpha (i.e., on a market-adjusted basis), rather than considering the raw value premium.

announcements), are by construction exogenous shocks to short-term nominal rates. But while it may seem intuitive to treat the resulting changes in long-term rates as if they represent pure discounting shocks,[34] this is not necessarily a valid assumption: while the change in the short-term rate is exogenous, the long-term yield change depends on changes to the pure discount rate *as well as* changes to perceived long-term growth and uncertainty. That is, the long-term real yield change depends not just on the short-rate shock (and its perceived persistence), but also on the market's perceived future changes to endogenous outcomes resulting from this shock, as also pointed out by Chen (2022). Further, the sensitivity of stock prices to these shocks does not identify stocks' cash-flow duration.

A benefit of our framework is that we can directly estimate the perceived effect of any given shock on the separate components of real rates. One approach to this would be to observe the change in expected growth rates around an announcement and then strip out these changes, akin to the approach taken in Section 3. But the timing of the Consensus Economics surveys makes such an approach challenging when considering high-frequency shocks like monetary policy surprises. First, the surveys are conducted infrequently (either every six months or every three months), and forecasters may exhibit inertia in changing their growth-rate forecasts after a given shock. Second, the surveys *prior* to a given monetary policy change may be stale by the time of the FOMC meeting, inducing a possibly spurious positive relation between expected-growth revisions and policy surprises; see Bauer and Swanson (2023b) for an extensive related discussion.

Instead, we can take advantage of the fact that we observe three high-frequency asset-price changes on the announcement dates themselves: we observe the change in long-term yields $\Delta y_{t,t+n}$, the return on the market $r_t^{\text{mkt}}$, and the change in uncertainty proxied by $\Delta \text{VIX}_t^2$. And our previous estimation provides a mapping from any change in $\rho_t$, expected growth $g_t$, and uncertainty $\text{VIX}_t^2$ to a change in long-term yields and stock returns. As a result, this mapping can be inverted to provide an estimate of the change in $\rho_t$ and $g_t$ implied by the observed asset-price changes. For example, a positive market return coinciding with an increase in yields implies that expected growth must have increased by enough, or uncertainty must have decreased by enough, to offset any given increase in the pure discounting term. Given that we can observe the change in uncertainty, these two reactions in fact exactly pin down the required change in both terms.[35]

To implement this idea, we start with a slightly modified version of the yield change decomposition in (19). The benchmark yield change we will use in the high-frequency data is

[34] See, for instance, Kroen et al. (2022).

[35] Our estimation approach is somewhat similar in spirit, if not in implementation, to the one used by Knox and Vissing-Jorgensen (2025) to decompose contemporaneous changes in observed returns.

a change in the traded 10-year nominal yield, whereas (19) was estimated for the survey-based $r^*$. We therefore re-estimate that equation using three-year changes in the 10-year trend real yield as our starting point. In practice, the resulting estimates are quite close to those presented in Table B.1.[36] Next, we estimate (20) using the three resulting terms from that decomposition. Estimates are again quite similar to the benchmark shown in Table 2.[37]

We then use data from Bauer and Swanson (2023a), who provide changes in 10-year nominal yields, S&P 500 futures returns, and monetary policy shocks (orthogonalized with respect to ex ante predictors) in 30-minute windows around FOMC announcements. Based on the results of Nakamura and Steinsson (2018, Table I), we assume that the change in 10-year nominal yields is equal to the change in 10-year real yields, $\Delta y_{t,t+10}$. Finally, we calculate the daily change in the $\mathrm{VIX}_t^2$ on the announcement day. Using these observed high-frequency changes and our estimated coefficients in the real-rate and stock-return regression, we invert the following two equations for the two unknowns $\Delta\rho_t$ and $\Delta g_t$:

$$\Delta y_{t,t+10} = \Delta\rho_t + \widehat{\gamma}\,\Delta g_t + \widehat{\beta}_j\,\Delta\mathrm{VIX}_t^2, \tag{22}$$

$$r_{t,j}^{\mathrm{mkt}} = \widehat{\pi}_\rho\,\Delta\rho_t + \widehat{\pi}_g\,\Delta g_t + \widehat{\pi}_V\,\Delta\mathrm{VIX}_t^2. \tag{23}$$

We then regress the recovered $\Delta\rho_t$ and $\Delta g_t$, as well as $\Delta y_{t,t+10}$ and $\Delta\mathrm{VIX}_t^2$, on the orthogonalized monetary policy shocks $\mathrm{mps}_t^\perp$ from Bauer and Swanson (2023a). Table 5 presents the results.[38] Column (1) shows that a 100-basis-point shock — that is, a shock scaled so that the one-year Eurodollar futures rate changes by +100 bps — results in a 10-year yield increase of 42 bps, nearly identical to Nakamura and Steinsson (2018, Table I). Columns (2)–(4) effectively decompose this change. The majority is attributable to an increase in the pure discount rate of 28 bps. We estimate a weak but nonetheless *positive* change in expected growth of 9 bps. There is also a significant increase in risk.

As a result, we conclude that the average monetary policy shock indeed appears reasonably close to a pure discounting shock, at least in its effect on long-term yields.[39] But there is nonetheless a small, somewhat noisily estimated *positive* expected-growth-rate change

[36]The estimated growth-rate loading is indistinguishable from the one presented in column (3) of that table, while the loading on VIX is slightly smaller (-2.3 in the U.S., rather than -4.3).

[37]In this case, they are statistically indistinguishable from the figures presented in column (2), with the very slight change being that the estimated loading on the expected-growth change is now 2.4.

[38]Results are unchanged in magnitude and significance when restricting to post-1994 announcements, or when using the Bauer and Swanson (2023a) shock that is not orthogonalized on recent macro news ($\mathrm{mps}_t$).

[39]This conclusion is similar to that of Nagel and Xu (2025), using different methods. It also aligns with Offner (2025), who finds that duration explains cross-sectional monetary policy exposures (though he finds a larger role for risk-premium changes than we do). Finally, policy shocks have on average been slightly negative since 1990 (totaling $-61$ bps), so our results suggest that easing shocks have been a modest driver for the decline in $\rho_t^*$ in the U.S. (accounting for $-17$ bps), qualitatively consistent with Hillenbrand (2025).

**Table 5: Responses of Recovered Rate Components to Monetary Policy Shocks**

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| | $\Delta$10y yield | $\Delta$pure discount ($\widehat{\Delta\rho_t^*}$) | $\Delta$exp. growth | $\Delta\text{VIX}^2 \times 100$ |
| $\text{mps}^{\perp}$ | 0.42*** | 0.28*** | 0.09* | 1.15*** |
| | (0.06) | (0.04) | (0.05) | (0.38) |
| Obs. | 292 | 292 | 292 | 292 |
| $R^2$ | 0.37 | 0.31 | 0.05 | 0.04 |

*Notes:* This table shows coefficient estimates from regressions $\Delta x_t = \alpha + \beta \text{mps}_t^{\perp} + \varepsilon_t$, where $x_t$ is the high-frequency change in outcome variable $x_t$ around an FOMC announcement, and $\text{mps}_t^{\perp}$ is the orthogonalized monetary policy shock from Bauer and Swanson (2023a) in percentage points. In column (1), $\Delta x_t$ is the 30-minute change in the 10-year nominal yield. In columns (2)–(3), $\Delta x_t$ are the recovered 30-minute changes in pure discount rates and expected growth based on the high-frequency stock-price and interest-rate changes, as obtained from (22)–(23). In column (4), the outcome variable is the daily change in the squared VIX. Standard errors are obtained using a block bootstrap with one-year blocks and 10,000 bootstrap draws. The sample is 1990–2023.

estimated as resulting from a contractionary shock. Intuitively, while stock returns decrease following contractionary shocks, they do not decrease on average by quite enough — i.e., they decrease by less than 19% (given an estimated duration of around 19 years) for every one-percentage-point change in long-term yields — to be consistent with a pure discounting shock alone.[40] As a result, we find some evidence in favor of an information effect on average, using different methods than those used by Nakamura and Steinsson (2018).

The low $R^2$ in the growth-rate regression indicates that there is meaningful announcement-specific heterogeneity in the perceived effects on growth rates (as well as the other outcome variables): there are some announcements with strong conventional policy responses,[41] and others with strong apparent information effect–type responses. To illustrate this heterogeneity, Figure B.6 in Appendix B.6 shows binned scatter plots of implied changes in the pure discount rate and expected growth rate against monetary policy shocks. The pure discount channel of monetary policy is very clean and consistent: implied $\rho_t^*$ changes increase uniformly in $\text{mps}_t^{\perp}$. The expected growth channel, meanwhile, appears non-monotonic: large easing shocks generate conventional policy responses (whereby more negative $\text{mps}_t^{\perp}$ predicts a greater increase in expected growth), while other shocks generate an apparent information effect.

[40] They would in fact need to decrease by more than 19% to be consistent with such a shock, given the small positive effect on the VIX.

[41] For example, the accommodative announcement on March 23, 2020, during the depths of the market downturn at the onset of the Covid crisis, is estimated to have increased long-term expected growth rates by 13 bps. This pattern of conventional responses to large easing shocks applies more generally (as in Figure B.6), and it is consistent with the results of Acosta (2023).

# 6. Conclusion

We provide a new framework and measurement tools to decompose any change in real interest rates into mutually exclusive underlying structural changes. According to our decomposition, only pure discounting shocks should pass through perfectly from real yields to equity valuations theoretically. When implemented empirically with long-term survey forecast data and a panel of asset prices, the decomposition works very well: pure discounting shocks pass through one-for-one to equity yields, while the other components of interest-rate changes do not.

The recovered pure discounting component of real rates helps us answer a range of questions related to asset pricing, macroeconomics, and secular economic trends observed in recent decades. In the U.S. data, we estimate that a sizable share of the decline in interest rates since 1990 — around 35% — is attributable to the pure discounting term, indicating some meaningful pass-through from declining yields to rising risky-asset valuations. But assuming perfect pass-through, as a range of literature has done, nonetheless overstates the effect of declining interest rates by roughly three times. The partial pass-through we find implies that much of the rise in household wealth (and inequality) was likely non-mechanical.[42] Our estimates also suggest that stocks have continued to exhibit a sizable equity premium relative to a duration-adjusted counterfactual. In further analysis, we use our decomposition to speak to higher-frequency equity returns, explain interest rates in the cross-section of stocks, and better understand the perceived effects of monetary policy shocks.

Unpacking the drivers of country-level changes in the pure discounting term in a structural sense, over and above the analysis of capital flows in Section 5.1, will be important for better understanding how to interpret these changes. But in spite of the work to be done on this, our paper provides a clear framework and tools to understand the relationship between stocks and bonds. This bond-stock relationship appears chaotic, both at high frequencies and over the long run, as is apparent from the stock–yield disconnect shown in the left panel of Figure 1. But our simple framework, combined with long-term survey data, works well at isolating a pure discounting component of interest rates that explains both higher-frequency stock returns and longer-term secular changes in equity valuations, as in the right panel of Figure 1.

One implication of our findings is that we can nearly perfectly explain the long-term changes in both interest rates and equities without the need for any additional convenience yield specific to Treasuries. While such market-specific shocks may be quite important for explaining shorter-term fluctuations, "standard" asset pricing evidently works reasonably well at explaining the data at a low frequency.

[42]That said, more work needs to be done to understand the pass-through of interest-rate changes to assets other than equity, which are important for many households' wealth.

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# Appendix

# A. Additional Theoretical Derivations and Discussion

This appendix provides proofs, derivations, and discussion of theoretical results as referenced in the main text.

## A.1 Additive Log SDF Decomposition

This subsection discusses the additive decomposition for the log SDF in equation (2), and its relation to the decomposition in Hansen (2012). Given our discrete-time environment, for notational simplicity we will in fact more directly build on Proposition 4.2.1 of Hansen and Sargent (2022). That result provides a discrete-time analogue to the continuous-time decomposition for additive functionals in Theorems 3.1–3.2 of Hansen (2012),[43] which we then apply to our specific setting.

We begin by defining the process $S$ such that $S_0 = 1$ and $M_{t+1} = \frac{S_{t+1}}{S_t}$, so $m_{t+1} = s_{t+1} - s_t$. We consider three separate cases for fundamental dynamics, which parallel the three cases considered in Section 2.3.[44]

### A.1.1 Stationarity with Unanticipated Breaks

Following Hansen and Sargent (2022), begin by defining $X_t$ $(t = 0, 1, 2, \ldots)$ to be a stationary Markov process of dimension $n$ with transition equation

$$X_{t+1} = \varphi(X_t, W_{t+1}), \tag{A.1}$$

where $\varphi(\cdot, \cdot)$ is a Borel-measurable function and $W_{t+1}$ is a $k$-dimensional vector of unanticipated shocks satisfying $\mathbb{E}_t[W_{t+1}] = \mathbb{E}[W_{t+1}|X_t] = 0$. The dynamics in (A.1) induce a transition distribution $\mathcal{P}$ for $X$.

Given the log SDF $m_{t+1} = s_{t+1} - s_t$, assume that the process $s$ is an additive functional, in the sense that it can be represented as

$$s_{t+1} = s_t + \kappa(X_t, W_{t+1}),$$

where $\kappa : \mathbb{R}^n \times \mathbb{R}^k \to \mathbb{R}$ is a measurable function. Define the unconditional expectation of

[43] See also Hansen (2019).

[44] The first case is a generalization of Case I in Section 2.3, allowing for arbitrary Markov dynamics (rather than the specialized version with conditionally i.i.d. dynamics in Section 2.3) along with unanticipated breaks.

the increment $s_{t+1} - s_t$ to be $\nu = \mathbb{E}[\kappa(X_t, W_{t+1})]$. Define $\overline{\kappa}(x) = \mathbb{E}[\kappa(X_t, W_{t+1})|X_t = x] - \nu$ to be the deviation of the expected increment conditional on $X_t = x$ from its unconditional mean. Using the infinite sum of all such future deviations as of $t$, define

$$H_t = \underbrace{\kappa(X_{t-1}, W_t) - \nu}_{(s_t - s_{t-1}) - \nu} + \sum_{j=0}^{\infty} \mathbb{E}_t[\overline{\kappa}(X_{t+j})], \tag{A.2}$$

and assume that the sum in (A.2) converges in mean square to a finite variable. Next, define

$$h(X_t) = \mathbb{E}_t[H_{t+1}] = \sum_{j=0}^{\infty} \mathbb{E}_t[\overline{\kappa}(X_{t+j})]. \tag{A.3}$$

Finally, define the martingale increment

$$\varepsilon_{t+1} = H_{t+1} - h(X_t), \tag{A.4}$$

so that $\mathbb{E}_t[\varepsilon_{t+1}] = 0$ by construction.

Given the above setup and the additional assumption that $\mathbb{E}[\kappa(X_t, W_{t+1})^2] < \infty$, Proposition 4.2.1 of Hansen and Sargent (2022) then gives that $m_{t+1} = s_{t+1} - s_t$ satisfies the following additive decomposition:[45]

$$m_{t+1} = \nu + h(X_{t+1}) - h(X_t) + \varepsilon_{t+1}. \tag{A.5}$$

The first term represents the linear trend in $s_t$. The second component, $h(X_{t+1}) - h(X_t)$, is a stationary difference. The last term is a mean-zero martingale increment.

We now map to our interpretation of the first term as representing discounting and the second term as depending on cash-flow growth. Denote the log cash-flow process by $c_t$, and assume that

$$\begin{aligned} c_{t+1} - c_t &= \mu_c(X_t) + \sigma_c(X_t) B_c W_{t+1}, \\ X_{t+1} &= A_x X_t + \sigma_x(X_t) B_x W_{t+1}, \end{aligned} \tag{A.6}$$

where $\mu_c(X_t)$, $\sigma_c(X_t)$, and $\sigma_x(X_t)$ are measurable functions. This nests many common specifications for fundamentals. If (i) $W_t$ is a $3 \times 1$ i.i.d. standard normal vector, where the first entry contains a shock to contemporaneous cash flows (so only the first entry of

[45] Theorem 3.2 of Hansen (2012) gives an exactly analogous decomposition in continuous time, with the added interpretation of $h$ as a finite-second-moment solution to $\lim_{t \searrow 0} \frac{1}{t} \mathbb{E}_0[h(X_t) - h(x)|X_0 = x] = \overline{\kappa}(x)$, where $\overline{\kappa}(x)$ is the deviation of the local mean of the increment in $s$ from its unconditional mean $\nu$.

the row vector $B_c$ is non-zero), (ii) $X_t$ is a $2 \times 1$ vector, where the first entry $X_{1,t}$ affects expected cash-flow growth ($\mu_c(X_t) = \overline{\mu} + X_{1,t}$), the second captures stochastic volatility ($\sigma_c(X_t) = \sigma_x(X_t) = \sqrt{\overline{\sigma}^2 + X_{2,t}}$), and $A_x$ and $B_x$ are $2 \times 2$ diagonal matrices, then this represents the long-run risks model of Bansal and Yaron (2004) with stochastic volatility. The Campbell and Cochrane (1999) habit formation model, meanwhile, applies if $W_t$ is scalar i.i.d. standard normal, $\mu_c(X_t) = \overline{\mu}$, $\sigma_c(X_t) = \sigma$, $B_c = 1$, and $X_{t+1}$ is the deviation of the log surplus consumption ratio $\widetilde{s}_t$ from its long-run mean (with $\sigma_x(X_t)$ representing Campbell and Cochrane's "sensitivity function" $\lambda(\widetilde{s}_t)$). Other settings fit similarly within the framework.

To relate the SDF dynamics to cash flows, assume that the log SDF can be written as

$$m_{t+1} = -\rho - \widetilde{\gamma}(c_{t+1} - c_t) + n_{t+1}, \tag{A.7}$$

for some constant $\widetilde{\gamma}$ and process $n_{t+1} = \mu_n(X_t) + \sigma_n(X_t) B_n W_{t+1}$. This is again quite general.[46] As in Section 2.2, it holds in a representative-agent, power-utility setting, where $\rho = -\log \beta$ is the time discount rate, $\widetilde{\gamma} = \gamma$ is relative risk aversion, and $n_{t+1} = 0$. If the representative agent has Epstein-Zin preferences with elasticity of intertemporal substitution (EIS) $\psi$, time discount rate $\rho$, and relative risk aversion $\gamma$, (A.7) holds, but now with $\widetilde{\gamma} = \frac{1}{\psi}$ and $n_{t+1} = (1/\psi - \gamma)(v_{t+1} - (1-\gamma)^{-1}(\log \mathbb{E}_t[V_{t+1}^{1-\gamma}])$, where $V_{t+1}$ is continuation utility and $v_{t+1}$ is its log.[47] In the Campbell and Cochrane (1999) habit setting, (A.7) holds, with $\widetilde{\gamma} = \gamma$ again representing risk aversion and $n_{t+1} = -\gamma(\widetilde{s}_{t+1} - \widetilde{s}_t)$, where $\widetilde{s}_t$ is the log surplus consumption ratio. It can also be mapped straightforwardly to various heterogeneous-agent models, such as that of Constantinides and Duffie (1996) or subsequent models.

To relate the above representation to the additive decomposition (A.5), we can construct each of the terms in that decomposition under our assumptions on cash flows in (A.6) and the SDF in (A.7). Define $\nu_c = \mathbb{E}[c_{t+1} - c_t] = \mathbb{E}[\mu_c(X_t)]$ and $\nu_n = \mathbb{E}[n_{t+1}] = \mathbb{E}[\mu_n(X_t)]$, and assume that $\nu_n = 0$. This will hold as long as the additional perturbation $n_{t+1}$ to the log SDF either (i) follows a martingale difference sequence or (ii) features transitory, unconditional-mean-zero disturbances, as is the case in many models.[48] The $\nu$ in (A.5) is therefore

$$\nu = -\rho - \widetilde{\gamma}\nu_c. \tag{A.8}$$

[46] It is slightly more general, for example, than the assumption in Backus, Chernov, and Zin (2014, eq. (22)).

[47] In the unit EIS case with $\psi = 1$, $N_{t+1} = \exp(n_{t+1})$ is a martingale, but $n_{t+1}$ is typically not.

[48] This holds in, for example, the Campbell and Cochrane (1999) model, and see Borovička, Hansen, and Scheinkman (2016) for further discussion. If it does not hold, then the $\rho$ in our decomposition should be understood to contain both the time discount rate and any small component arising from $\mathbb{E}[n_{t+1}]$ (e.g., from a Jensen's inequality correction for the continuation utility term in an Epstein–Zin framework). See Appendix A.2 for a discussion of how this affects the $r_t^*$ decomposition in a tractable alternative case.

Similarly, split $\overline{\kappa}(X_t)$ into two parts, $\overline{\kappa}(X_t) = \overline{\kappa}_c(X_t)+\overline{\kappa}_n(X_t)$, where $\overline{\kappa}_c(X_t) = -\widetilde{\gamma}\mu_c(X_t)+\widetilde{\gamma}\nu_c$ and $\overline{\kappa}_n(X_t) = \mu_n(X_t)$. Build up $h_c(X_t)$ and $h_n(X_t)$ accordingly from these $\overline{\kappa}$ functions as in (A.3), and $h(X_t) = h_c(X_t) + h_n(X_t)$. The $\varepsilon_{t+1}$ term inherits the remaining martingale-difference components of $-\widetilde{\gamma}(c_{t+1} - c_t)$ and $n_{t+1}$, as in (A.4).[49] Define these two processes' respective martingale increment terms as $\varepsilon_{c,t+1}$ and $\varepsilon_{n,t+1}$. Finally, define the martingale difference $\widetilde{\varepsilon}_{t+1} = \varepsilon_{t+1} - \varepsilon_{c,t+1} = \varepsilon_{n,t+1}$.

To map the above steps and results to the decomposition in (2), we construct an expanded state vector $\widetilde{X}_t = (c_t, X_t)'$, where $X_t$ is the previous state vector. This expanded state vector still follows a Markov process. With non-zero average consumption growth, $\widetilde{X}_t$ will no longer be stationary, but its differences will be. This expanded state vector will stand in for the state vector used in (2). We can now construct our alternative decomposition. Define

$$f(\widetilde{X}_{t+1}) - f(\widetilde{X}_t) = \widetilde{\gamma}(c_{t+1} - c_t) - (h_n(X_{t+1}) - h_n(X_t)). \tag{A.9}$$

Using (A.5), (A.8), and (A.9), we accordingly have our decomposition

$$m_{t+1} = -\rho - (f(\widetilde{X}_{t+1}) - f(\widetilde{X}_t)) + \widetilde{\varepsilon}_{t+1}, \tag{A.10}$$

where $f(\widetilde{X}_{t+1}) - f(\widetilde{X}_t)$ is a stationary difference and $\widetilde{\varepsilon}_{t+1}$ is a mean-zero martingale difference, as in (2). Note that while $\mathbb{E}_t[f(\widetilde{X}_{t+1}) - f(\widetilde{X}_t)]$ does not depend only on cash-flow growth $c_{t+1} - c_t$ in general, the limiting forward expectation $\lim_{\tau\to\infty} \mathbb{E}_t[f(\widetilde{X}_{t+\tau+1}) - f(\widetilde{X}_{t+\tau})]$ used in the trend real-rate decomposition (5) does:

$$\widetilde{g}_t^* = \lim_{\tau\to\infty} \mathbb{E}_t[f(\widetilde{X}_{t+\tau+1}) - f(\widetilde{X}_{t+\tau})] = \widetilde{\gamma}\mathbb{E}[c_{t+1} - c_t] = \widetilde{\gamma}\nu_c. \tag{A.11}$$

Finally, to complete the characterization of this setting's decomposition, we now allow for unanticipated changes in the economic environment. We accordingly denote the transition distribution describing the Markov process $X$ at date $t$ to be $\mathcal{P}_t$, and assume that this distribution governs $X_{t+1}$ and is expected to govern $X_{t+1+\tau}$ for all $\tau > 0$. There may then be an unanticipated change in the transition distribution to $\mathcal{P}_{t+1}$, at which point this distribution will govern $X_{t+2}$ and will be expected to govern all future $X_{t+2+\tau}$ thereafter. Again defining the expanded state vector $\widetilde{X}_t = (c_t, X_t)'$, the stationary (under $\mathcal{P}_t$) difference $f(\widetilde{X}_{t+1}) - f(\widetilde{X}_t)$ in (A.9), and the now potentially time-varying $\rho_t$ in (A.7), our decomposition (A.10) becomes

$$m_{t+1} = -\rho_t - (f(\widetilde{X}_{t+1}) - f(\widetilde{X}_t)) + \widetilde{\varepsilon}_{t+1}. \tag{A.12}$$

[49]See Borovička, Hansen, and Scheinkman (2016) for explicit characterizations of the martingale components in alternative models.

This now aligns with (2), with $\widetilde{X}_t$ and $\widetilde{\varepsilon}_{t+1}$ here in place of $X_t$ and $\varepsilon_{t+1}$ in the text. The limiting expectation $\widetilde{g}_t^*$ as defined in (A.11) may also be time-varying here, as $\widetilde{g}_t^* = \widetilde{\gamma}_t \nu_{c,t}$.

### A.1.2 Drifting Steady State

Analogous to Case II in Section 2.3, we now consider an alternative case in which fundamentals (and resulting expected returns and valuation ratios) follow a random walk, or "drifting steady state." Define $W_{t+1}$ as in Appendix A.1.1, and assume that the cash-flow process $c$ and one-dimensional process $X$ are modified from (A.6) to now follow

$$
\begin{aligned}
c_{t+1} - c_t &= \mu_c + X_t + \sigma_c(X_t) B_c W_{t+1}, \\
X_{t+1} &= X_t + \sigma_x(X_t) B_x W_{t+1}.
\end{aligned}
$$

The volatility term $\sigma_x(X_t)$ may be specified so as to ensure that $X$ remains bounded in $L^2$ in order to rule out explosive dynamics (or one could assume alternative bounded-martingale dynamics for $X$), but we do not impose this directly.[50]

In place of (A.7), the log SDF follows

$$
\begin{aligned}
m_{t+1} &= -\rho_t - \widetilde{\gamma}(c_{t+1} - c_t) + n_{t+1}, \\
\rho_{t+1} &= \rho_t + B_\rho W_{t+1}, \\
n_{t+1} &= \eta_{t+1} - \eta_t,
\end{aligned}
$$

where $\mathbb{E}_t[\eta_{t+1}] = \eta_t$. Defining $\widetilde{X}_t = (c_t, X_t)'$, we again immediately obtain a decomposition of the form (A.12) and (2):

$$
m_{t+1} = -\rho_t - (f(\widetilde{X}_{t+1}) - f(\widetilde{X}_t)) + \widetilde{\varepsilon}_{t+1},
$$

where $f(\widetilde{X}_{t+1}) - f(\widetilde{X}_t) = \widetilde{\gamma}(c_{t+1} - c_t) = \widetilde{\gamma}(\mu_c + X_t + \sigma_c(X_t) B_c W_{t+1})$ is difference-stationary, and where $\widetilde{\varepsilon}_{t+1} = n_{t+1}$ is a martingale difference.

### A.1.3 Stationarity

Finally, analogous to Case III in Section 2.3, we consider a setting building on Appendix A.1.1, without unanticipated breaks but with stationary variation in $\rho_t$. The stationarity of $X_t$ and $\rho_t$ in this setting will imply that infinite-horizon expectations of the discounting and growth

[50] Instead, the drifting steady state model is intended as a convenient tool to represent persistent fluctuations in fundamentals, rather than being a reasonable candidate model over an infinite horizon.

terms are constant. As a result, we discuss how to redefine long-run expectations in a manner incorporating persistent time variation in these processes.

We start with exactly the same setting as in Appendix A.1.1, with everything through equation (A.6) unchanged. We modify (A.7) slightly to allow for $\rho_t$ to follow stationary Markov dynamics: assume that $\rho_t$ is an element of the state vector $X_t$ in (A.1) and (A.6), and

$$m_{t+1} = -\rho_t - \widetilde{\gamma}(c_{t+1} - c_t) + n_{t+1},$$

with the assumptions on the remaining terms in $m_{t+1}$ unchanged. We also define $\overline{\rho} = \mathbb{E}[\rho_t]$. The above time variation in $\rho$ can be thought of as representing a time-varying subjective discount rate for the representative agent, but it also serves as a stand-in for many other sources of potential variation in the intertemporal marginal rate of substitution. It may arise from demographic changes, a heterogeneous-agents model with time variation in the marginal investor (and differences in investors' personal discount rates), or as discussed later in the appendix, capital flows from foreign investors.

We can then follow nearly exactly the same steps as in Appendix A.1.1 to obtain the following valid decomposition:[51]

$$m_{t+1} = -\rho_t - (f(\widetilde{X}_{t+1}) - f(\widetilde{X}_t)) + \widetilde{\varepsilon}_{t+1},$$

with $f(\widetilde{X}_{t+1}) - f(\widetilde{X}_t)$ defined as in (A.9) and $\widetilde{\varepsilon}_{t+1}$ defined as before (A.9).

In this case, all infinite-horizon expectations defined after the $r_t^*$ decomposition (5) are constant, with $\rho_t^* = \lim_{\tau\to\infty} \mathbb{E}_t[\rho_{t+\tau}] = \overline{\rho}$, $\widetilde{g}_t^* = \lim_{\tau\to\infty} \mathbb{E}_t[f(\widetilde{X}_{t+\tau+1}) - f(\widetilde{X}_{t+\tau})] = \widetilde{\gamma}\nu_c$, and $L_{t,M}^* = \lim_{\tau\to\infty} \mathbb{E}_t[L_{t+\tau}(M_{t+\tau+1})] = \sum_{n=2}^{\infty} \frac{\kappa_n(m_{t+1})}{n!}$, where $\kappa_n(m_{t+1})$ is the $n^{\text{th}}$ cumulant of the unconditional log SDF distribution. To formalize long-horizon variation in the terms in our $r_t^*$ decomposition in this context, we redefine the above terms as discounted sums:

$$z_t^* = (1-\delta)\sum_{\tau=0}^{\infty} \delta^\tau \mathbb{E}_t[z_{t+\tau+1}], \tag{A.13}$$

$$z_t \in \{\rho_t, g_t, L_t(M_{t+1}), r_{t+1}^f\},$$

[51]To spell out these steps in further detail: first, with the same definitions and assumptions on $\nu_c$ and $\nu_n$ as before, the $\nu$ in (A.5) becomes $\nu = -\overline{\rho} - \widetilde{\gamma}\nu_c$. We now split $\overline{\kappa}(X_t)$ into three parts, $\overline{\kappa}(X_t) = \overline{\kappa}_\rho(X_t) + \overline{\kappa}_c(X_t) + \overline{\kappa}_n(X_t)$, where $\overline{\kappa}_c(X_t)$ and $\overline{\kappa}_n(X_t)$ are as before, and $\overline{\kappa}_\rho = -\mu_\rho(X_t) + \overline{\rho}$. Build up $h_\rho(X_t)$, $h_c(X_t)$, and $h_n(X_t)$ accordingly from these $\overline{\kappa}$ functions as in (A.3), and $h(X_t) = h_\rho(X_t) + h_c(X_t) + h_n(X_t)$. The $\varepsilon_{t+1}$ term inherits the remaining martingale components of $c_{t+1} - c_t$ and $n_{t+1}$ as in (A.4), with no additional term for $\rho_t$ given its stationary Markov dynamics, so we define $\widetilde{\varepsilon}_{t+1}$ as before. The expanded state vector is again $\widetilde{X}_t = (c_t, X_t)'$. The decomposition thus applies as stated.

where $\delta \in (0,1)$ is a loglinearization constant and where $g_{t+\tau+1} \equiv f(\widetilde{X}_{t+\tau+1}) - f(\widetilde{X}_{t+\tau})$. We define the loglinearization constant $\delta$ in the appendix for Section 2.3 below, and we discuss how the definition (A.13) maps well to the equity yield decomposition in the stationary case. As $(1-\delta)\sum_{\tau=0}^{\infty}\delta^{\tau} = 1$, equation (A.13) defines the starred long-run terms as weighted averages of all future expected realizations of $z_t$, as in (13) in the main text.

## A.2 Interest-Rate Decomposition

### A.2.1 Benchmark Case

The general version of the decomposition for $r_t^*$ provided in equation (5) in Section 2.1 follows immediately from the SDF decomposition derived and discussed in Appendix A.1, along with equations (1) and (3). For the consumption-based version in Section 2.2, the second expression provided in (7) starts from equation (4) and then applies equation (25) of Backus, Chernov, and Martin (2011), which relates the log SDF's cumulants to the consumption-growth cumulants in a power-utility setting according to

$$\kappa_{n,t}(m_{t+1}) = (-\gamma)^n \kappa_{n,t}(g_{t+1}). \tag{A.14}$$

See also equation (8) and the preceding equation in Martin (2013), which provides the same interest-rate expression as in (7). Equation (8) then follows directly from the preceding steps. Again see the previous appendix subsection for details on the definitions of the terms in (5) and (8) given each of our three sets of assumptions on the dynamics of fundamentals.

### A.2.2 Extension with Epstein–Zin Preferences

As discussed above equation (A.8) and in detail in footnote 48, if the additional term $n_{t+1}$ in the log SDF specification (A.7) does not feature $\mathbb{E}[n_{t+1}] = 0$, then the $\rho$ (or $\rho_t$) in our decomposition should be understood to contain both the time discount rate and any small component arising from $\mathbb{E}[n_{t+1}]$. This does not pose serious issues for either the decompositions or for the paper's interpretation of $\rho_t$: as discussed in Section 3.2, we do not view shifts in $\rho_t$ in the data as likely to be arising purely from changes in aggregate patience among domestic investors. That said, given the use of Epstein–Zin preferences in Section 2.3, we briefly discuss precisely how such preferences affect the interest-rate decomposition in a tractable case.

In place of (6), we follow Epstein and Zin (1989) and set

$$U_t = \left\{(1-\beta_t)C_t^{\frac{1-\gamma}{\theta}} + \beta_t(\mathbb{E}_t[U_{t+1}^{1-\gamma}])^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\gamma}},$$

where $\theta \equiv (1-\gamma)/(1-\frac{1}{\psi})$ and where $\psi$ is the elasticity of intertemporal substitution (EIS). We again set $\rho_t = -\log \beta_t$. In a complete-markets setting in which there is a consumption claim whose value is aggregate wealth, the SDF is

$$M_{t+1} = \left(\beta_t \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\right)^{\theta} \left(R^w_{t+1}\right)^{-(1-\theta)}, \tag{A.15}$$

where $R^w_{t+1}$ is the gross return on the wealth portfolio. If we further assume jointly lognormal and homoskedastic $C_{t+1}$ and $R^w_{t+1}$, the risk-free rate is then

$$r^f_{t+1} = \rho_t + \frac{1}{\psi}\mathbb{E}_t[g_{t+1}] - \frac{\theta}{2\psi^2}\sigma_g^2 - \frac{1-\theta}{2}\sigma_w^2, \tag{A.16}$$

as in Campbell (2018), equation (6.44), where $\sigma_g^2 = \text{Var}_t(g_{t+1})$ and $\sigma_w^2 = \text{Var}_t(r^w_{t+1})$. Given (A.15), the SDF's entropy is

$$L_t(M_{t+1}) = \frac{\theta^2}{2\psi^2}\sigma_g^2 + \frac{(1-\theta)^2}{2}\sigma_w^2 + \frac{\theta(1-\theta)}{\psi}\sigma_{gw}, \tag{A.17}$$

where $\sigma_{gw} = \text{Cov}_t(g_{t+1}, r^w_{t+1})$. Combining this with (A.16), we can write

$$\begin{aligned} r^f_{t+1} &= \rho_t + \frac{1}{\psi}\mathbb{E}_t[g_{t+1}] - L_t(M_{t+1}) + \frac{\theta(\theta-1)}{2}\text{Var}_t\left(\frac{g_{t+1}}{\psi} - r^w_{t+1}\right) \\ &= \tilde{\rho}_t + \frac{1}{\psi}\mathbb{E}_t[g_{t+1}] - L_t(M_{t+1}), \end{aligned}$$

where $\tilde{\rho}_t = \rho_t + \frac{\theta(\theta-1)}{2}\text{Var}_t\left(\frac{g_{t+1}}{\psi} - r^w_{t+1}\right)$. As a result, (8) still holds, with $\tilde{\rho}_t^*$ in place of $\rho_t^*$ and with $\frac{1}{\psi}$ in place of $\gamma$.

The additional term in $\tilde{\rho}_t$ can be thought of as capturing the variance of the innovation to the Epstein–Zin certainty equivalent.[52] One might thus prefer to group this additional term with the risk component of the SDF instead of $\rho_t$. To obtain this equivalent representation, note that since $L_t(M_{t+1}) = \frac{1}{2}\text{Var}_t\left(\frac{\theta g_{t+1}}{\psi} - (\theta-1)r^w_{t+1}\right)$ from (A.17), we can write

$$r^f_{t+1} = \rho_t + \frac{1}{\psi}\mathbb{E}_t[g_{t+1}] - (1+\omega)L_t(M_{t+1}),$$

$$\text{where} \quad \omega \equiv \frac{\theta(1-\theta)\text{Var}_t\left(\frac{g_{t+1}}{\psi} - r^w_{t+1}\right)}{\text{Var}_t\left(\frac{\theta g_{t+1}}{\psi} - (\theta-1)r^w_{t+1}\right)}$$

[52]We thank Mike Chernov for helpful discussion related to this point.

is a constant given the homoskedastic setting. Given that our empirical estimation allows for a flexible loading of the risk-free rate on our SDF entropy proxy, this constant of proportionality $1+\omega$ will be incorporated in the empirical versions of the decompositions in such a setting.

As an alternative to the assumption of lognormality after (A.15), assume that $g_{t+1}$ is i.i.d., in which case the log SDF becomes $m_{t+1} = -\rho_t - \gamma g_{t+1}$, exactly as in the benchmark case in Section 2.2, so the previous decomposition applies.

## A.3 Equity Yields and Duration

Following the main text (and similar to Appendix A.1), we derive our results for equity yields in three different settings. We then briefly discuss how the implications for equity duration follow directly.

### A.3.1 Case I (Gordon Growth)

**Equity Yields and Risk Premia.** Given i.i.d. consumption growth $g_{t+1} = c_{t+1} - c_t$ and $d_{t+1} - d_t = \lambda g_{t+1}$, the setting here mirrors that of Martin (2013, Section 1). He works with a risk premium defined slightly differently than ours: while we use the expected log return and set $rp_t \equiv \mathbb{E}_t[r^{\text{mkt}}_{t+1}] - r^f_{t+1}$, he instead uses the log expected return and considers what we will define as $\widetilde{rp}_t \equiv \log \mathbb{E}_t[R^{\text{mkt}}_{t+1}] - r^f_{t+1}$. By definition of entropy, these two versions of the risk premium differ by

$$\begin{aligned}\widetilde{rp}_t - rp_t &= L_t(R^{\text{mkt}}_{t+1}) \\ &= \sum_{n=2}^{\infty} \frac{\lambda^n \kappa_{n,t}(g_{t+1})}{n!},\end{aligned} \tag{A.18}$$

where the second line uses that $r^{\text{mkt}}_{t+1} = \lambda g_{t+1} + \text{constant}$ (where the constant is in fact $ey$) in this i.i.d. setting, and then applies the same relation as in (A.14).

Result 1 of Martin (2013), and in particular equation (7), gives that

$$ey^*_t = r^*_t + \widetilde{rp}^*_t - \sum_{n=1}^{\infty} \frac{\lambda^n \kappa_{n,t}(g_{t+1})}{n!}, \tag{A.19}$$

where we note that the summation in the last term starts with the first cumulant ($n=1$) rather than the second as in (A.18). This last term is therefore equal to the cumulant-generating function (CGF) for consumption growth, $\mathbf{c}(\vartheta) \equiv \sum_{n=1}^{\infty} \frac{\vartheta^n \kappa_{n,t}(g_{t+1})}{n!}$, evaluated at $\vartheta = \lambda$.

Using (A.18) and (A.19), we have that

$$ey_t^* = r_t^* + rp_t^* - \lambda g_t^*, \tag{A.20}$$

as stated in equation (10) and Result 1.

For the risk-premium expressions in (11), we start from equation (5) of Martin (2013), which gives that $\widetilde{rp}_t^* = \mathbf{c}(\lambda) + \mathbf{c}(-\gamma) - \mathbf{c}(\lambda - \gamma)$, and solve for $rp_t^*$ using (A.18):

$$\begin{aligned} rp_t^* &= \widetilde{rp}_t^* - (\mathbf{c}(\lambda) - \lambda g_t^*) \\ &= \lambda g_t^* + \mathbf{c}(-\gamma) - \mathbf{c}(\lambda - \gamma) \\ &= \sum_{n=2}^{\infty} \frac{(-\gamma)^n \kappa_{n,t}(g_{t+1})}{n!} - \sum_{n=2}^{\infty} \frac{(\lambda - \gamma)^n \kappa_{n,t}(g_{t+1})}{n!} \\ &= L_t(M_{t+1}) - L_t(M_{t+1} R_{t+1}^{\text{mkt}}). \end{aligned} \tag{A.21}$$

The first line uses the definition of the CGF in (A.18); the second substitutes in the $\widetilde{rp}_t^*$ solution above; the third expands the CGFs and uses that the first moments cancel; and the last uses that $m_{t+1} + r_{t+1}^{\text{mkt}} = \text{constant} + (\lambda - \gamma) g_{t+1}$ and applies (A.14). (See also Backus, Chernov, and Martin 2011, p. 2008, for similar steps.) Both lines of (11) follow directly.

To see that $rp_t = L_t(M_{t+1}) - L_t(M_{t+1} R_{t+1}^{\text{mkt}})$ holds in any no-arbitrage setting, one can follow Backus, Boyarchenko, and Chernov (2018, p. 12): take logs of the pricing equation $\mathbb{E}_t[M_{t+1} R_{t+1}^{\text{mkt}}] = 1$ and use the definition of entropy and equation (1) to obtain

$$\begin{aligned} 0 = \log \mathbb{E}_t[M_{t+1} R_{t+1}^{\text{mkt}}] &= \mathbb{E}_t[m_{t+1} + r_{t+1}^{\text{mkt}}] + L_t(M_{t+1} R_{t+1}^{\text{mkt}}) \\ &= \mathbb{E}_t[r_{t+1}^{\text{mkt}}] - r_{t+1}^f - L_t(M_{t+1}) + L_t(M_{t+1} R_{t+1}^{\text{mkt}}). \end{aligned}$$

Rearranging gives that $rp_t = L_t(M_{t+1}) - L_t(M_{t+1} R_{t+1}^{\text{mkt}})$, as stated.

Using (A.20) and (A.21), along with the interest-rate decomposition derived in the previous appendix sections, Result 1 then follows. We note as well that equations (5) and (7) of Martin (2017) also hold with Epstein–Zin utility, so (A.20) and (A.21) are identical in this case, as stated on page 10.

**Risk Shocks.** In part (iii) of Result 1, it is stated that equity yields change by $-\frac{\partial rp_t^*}{\partial L_{t,M}^*} + 1$ per unit increase in $r_t^*$ if $\frac{\partial rp_t^*}{\partial L_{t,M}^*}$ is well-defined. The third line of (A.21) shows the need for this qualification: $L_{t,M}^*$ and $rp_t^*$ are both functions of the consumption growth cumulants $\kappa_{n,t}(g_{t+1})$, and there are many potential changes to the different cumulants that generate identical changes in $L_t(M_{t+1})$ but different effects on $rp_t^*$.[53] In certain settings, though, this partial derivative is well-defined. One such setting is when $\gamma = \lambda$: in this case, $rp_t^* = L_{t,M}^*$, so $\frac{\partial rp_t^*}{\partial L_{t,M}^*} = 1$.

Case (i) of the three bond–stock comovement cases described on page 13 is another such setting. This case assumes lognormal growth, so that $\kappa_{n,t}(g_{t+1}) = 0$ for $n > 2$. We therefore have that $rp_t^* = \frac{1}{2}\lambda(2\gamma - \lambda)\mathrm{Var}_t(g_{t+1}) = \frac{\lambda(2\gamma-\lambda)}{\gamma^2}L_{t,M}^*$, so

$$-\frac{\partial rp_t^*}{\partial L_{t,M}^*} + 1 = -\frac{\lambda(2\gamma - \lambda)}{\gamma^2} + 1 = \frac{(\lambda - \gamma)^2}{\gamma^2},$$

which is strictly positive if $\lambda \neq \gamma$. In a lognormal, power-utility setting, the discount-rate effect of an increase in volatility always dominates, so equity prices increase given an increase in volatility as considered here. But the pass-through of interest rates to equity yields (i.e., $-\frac{\partial rp_t^*}{\partial L_{t,M}^*} + 1$ above) is nonetheless strictly below one as long as $(\lambda - \gamma)^2 < \gamma^2$, or equivalently $2\gamma > \lambda$, as stated in the text.

We now consider the other two cases introduced on page 13. Case (ii) is based on the rare-disasters model of Barro (2006). Consumption growth is modeled as $g_{t+1} = g_{1,t+1} + g_{2,t+1}$, where the two components $g_{1,t+1}$ and $g_{2,t+1}$ are independent of each other and independent over time. The first component is normal with $\mathbb{E}_t[g_{1,t+1}] = g_1^*$ and variance $\sigma^2$. The second component is a jump component, and we follow Backus, Chernov, and Martin (2011) and assume that it follows a Poisson-normal mixture: a given period has $j \in \mathbb{N}$ jumps with probability $e^{-\omega}\omega^j/j!$ (so $\omega$ is the effective jump intensity), and conditional on $j$, the jump size is normal with mean $-jm$ (with $m > 0$) and variance $js^2$. This implies (see p. 2002 of Backus, Chernov, and Martin) that the consumption-growth CGF is

$$\mathbf{c}(\vartheta) = \vartheta g_1^* + \frac{1}{2}\vartheta^2\sigma^2 + \omega\left(e^{-\vartheta m + \frac{1}{2}\vartheta^2 s^2} - 1\right).$$

Given that $L_t(\exp(\vartheta g_{t+1})) = \mathbf{c}(\vartheta) - \vartheta\mathbb{E}_t[g_{t+1}]$ and that in this setting $\mathbb{E}_t[g_{t+1}] = g_1^* - \omega m$, we have that

$$\begin{aligned} L_{t,MR}^* = L_t(\exp((\lambda - \gamma)g_{t+1})) &= \mathbf{c}(\lambda - \gamma) - (\lambda - \gamma)(g_1^* - \omega m) \\ &= (\lambda - \gamma)\omega m + \frac{1}{2}(\lambda - \gamma)^2\sigma^2 + \omega\left(e^{-(\lambda-\gamma)m + \frac{1}{2}(\lambda-\gamma)^2 s^2} - 1\right). \end{aligned}$$

[53] This motivates our consideration of the average change using the beta of the risk premium with $L_{t,M}^*$, where we use that $\mathrm{Cov}(rp_t^* - L_{t,M}^*, L_{t,M}^*) = \mathrm{Cov}(rp_t^*, L_{t,M}^*) - \mathrm{Var}(L_{t,M}^*)$. When the partial derivative does not exist, one can take $\frac{\partial rp_t^*}{\partial L_{t,M}^*}$ to represent a stand-in for $\beta_L$.

Using this,

$$\begin{aligned} ey_t^* &= \rho_t^* + (\gamma - \lambda) g_t^* - L_{t,MR}^* \\ &= \rho_t^* + (\gamma - \lambda) g_1^* - \frac{1}{2}(\lambda - \gamma)^2 \sigma^2 - \omega \Big( e^{-(\lambda-\gamma)m + \frac{1}{2}(\lambda-\gamma)^2 s^2} - 1 \Big) . \end{aligned}$$

Given a change in the average disaster size $m$, we therefore have

$$\frac{\partial e y_t^*}{\partial m} = (\lambda - \gamma)\, \omega\, e^{-(\lambda-\gamma)m + \frac{1}{2}(\lambda-\gamma)^2 s^2}$$

This is strictly positive (so valuations go down given an increase in mean disaster size) iff $\lambda > \gamma$.

Meanwhile, following similar steps for the SDF's entropy and plugging into the risk-free rate decomposition,

$$r_t^* = \rho_t^* + \gamma g_1^* - \frac{1}{2}\gamma^2 \sigma^2 - \omega \Big( e^{\gamma m + \frac{1}{2}\gamma^2 s^2} - 1 \Big) .$$

As a result,

$$\frac{\partial r_t^*}{\partial m} = -\gamma\, \omega\, e^{\gamma m + \frac{1}{2}\gamma^2 s^2} < 0,$$

so $r_t^*$ strictly decreases given an increase in mean disaster size. We conclude that such changes induce negative comovement between equity yields and real rates as long as $\gamma < \lambda$.

More generally, for any change in higher ($n \geqslant 2$) moments $\kappa_{n,t}(g_{t+1})$ for $n$ odd, if $\lambda > \gamma$,

$$\begin{aligned} \frac{\partial L_{t,M}^*}{\partial \kappa_{n,t}(g_{t+1})} &= \frac{(-\gamma)^n}{n!} < 0, \\ \frac{\partial L_{t,MR}^*}{\partial \kappa_{n,t}(g_{t+1})} &= \frac{(\lambda - \gamma)^n}{n!} > 0. \end{aligned}$$

Thus, greater negative skewness (i.e., a decrease in $\kappa_{3,t}$) will increase the SDF's entropy $L_{t,M}^*$ but decrease the entropy of the discounted return $L_{t,MR}^*$, and similarly for other higher odd cumulants. Since the risk-free rate decreases in $L_{t,M}^*$ and the equity yield decreases in $L_{t,MR}^*$, these odd-higher-moment shocks will induce negative comovement in the $\gamma < \lambda$ case.

For case (iii) on page 13, with Epstein–Zin utility (and parameters as defined in Appendix A.2.2), using equations (4)–(5) in Martin (2013) and considering a consumption claim

($\lambda = 1$), we can write

$$r_t^* = \rho_t^* + \sum_{n=1}^{\infty} \frac{\kappa_{n,t}(g_{t+1})}{n!} \left((1 - 1/\theta)(1-\gamma)^n - (-\gamma)^n\right)$$
$$ey_t^* = \rho_t^* + \sum_{n=1}^{\infty} \frac{\kappa_{n,t}(g_{t+1})}{n!} \left(-(1-\gamma)^n/\theta\right).$$

So considering any change in higher ($n \geqslant 2$) moments $\kappa_{n,t}(g_{t+1})$ for $n$ even, we can use the assumptions $\psi > 1$, $\gamma > 1$, and plug in for $\theta$ (and use $\theta < 0$ with those assumptions) to get

$$\frac{\partial r_t^*}{\partial \kappa_{n,t}(g_{t+1})} = \frac{1}{n!}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)^{n-1} - (-\gamma)^n < 0,$$
$$\frac{\partial ey_t^*}{\partial \kappa_{n,t}(g_{t+1})} = \frac{1}{n!}\left(-(1-\gamma)^n/\theta\right) > 0,$$

so equity yields and risk-free rates move in opposite directions, as stated.

### A.3.2 Case II (Drifting Steady State)

If $\mathbb{E}_t[ey_{t+1}] = ey_t \equiv ey_t^*$ (and all underlying fundamentals similarly are martingales), we can follow Gao and Martin (2021, eq (12)–(14)): since $R_{t+1}^{\text{mkt}} = \frac{D_{t+1}+P_{t+1}}{P_t}$, taking logs and expectations yields

$$ey_t^* = \mathbb{E}_t[r_{t+1}^{\text{mkt}}] - \lambda \mathbb{E}_t[g_{t+1}] - \log\left(1 - e^{-ey_t}\right) + \mathbb{E}_t\left[\log\left(1 - e^{-ey_{t+1}}\right)\right].$$

To a first order for $ey_{t+1}$ around its expectation $ey_t$, the last two terms cancel, giving equation (12) as stated. Given that $r_t^*$ satisfies the same decomposition (as shown in Appendix A.1.2), Result 1 therefore holds as stated.

### A.3.3 Case III (Stationarity)

In this case, we apply Result 2 of Gao and Martin (2021) directly to obtain that to a first order around the unconditional expectation $\overline{ey} \equiv \mathbb{E}[ey_t]$,

$$ey_t^* \equiv ey_t = (1-\delta)\sum_{\tau=1}^{\infty} \delta^\tau \mathbb{E}_t[r_{t+\tau+1}^{\text{mkt}} - \lambda g_{t+\tau+1}],$$

where $\delta \equiv e^{-\overline{ey}}$. Therefore, defining $rp_t^*$ (and its underlying components) and $g_t^*$ as in (13), and using the definition for $r_t^*$ and its underlying components from (A.13), the stated decomposition follows. So given the decomposition for $r_t^*$ as shown in Appendix A.1.3, Result 1 again holds as stated.

#### A.3.4 Equity Duration

The equity duration implications in Section 2.4 for the most part follow from direct application of the previous expressions. For equation (15), we use that $\mathbb{E}_t[D_{t+n}] = D_t e^{n(\lambda g)}$ and then evaluate the resulting series. The remaining derivatives in (16) and Result 2 then follow directly from (14)–(15).

#### A.3.5 Empirical Entropy Proxy

As stated in Section 3 (see page 17), if the market is growth-optimal and the distribution of log growth is symmetric, then $L^*_{t,M,j} = L^*_{t,R,j}$. To see this, note from (A.14) and (A.18) that

$$\kappa_{n,t}(m_{t+1}) = (-\gamma)^n \kappa_{n,t}(g_{t+1}),$$
$$\kappa_{n,t}(r^{\text{mkt}}_{t+1}) = \lambda^n \kappa_{n,t}(g_{t+1}).$$

Growth optimality requires $\lambda = \gamma$, and a symmetric log growth distribution implies that $\kappa_{n,t}(g_{t+1}) = 0$ for all odd $n$ (aside from $n = 1$, which does not enter into the entropy sum). And for even $n$, given $\gamma = \lambda$, we have $(-\gamma)^n = \lambda^n$. As a result, $\kappa_{n,t}(m_{t+1}) = \kappa_{n,t}(r^{\text{mkt}}_{t+1})$ for all $n$, and so $L^*_{t,M,j} = L^*_{t,R,j}$, as stated. This (along with Martin 2017, Result 3) motivates our use of the squared VIX to proxy for the SDF entropy term $L^*_{t,M}$ in estimating our $r_t^*$ decomposition.[54]

# B. Additional Empirical Details and Results

This appendix provides additional measurement details and empirical results as referenced in the main text.

## B.1 Baseline Measurement Details

Section 3.1 explains most of our data sources and variable definitions for the baseline analysis. Here, we provide additional details on two aspects of the data mentioned in the text.

[54] Martin's result is for risk-neutral entropy, so this proxy additionally requires an assumption of a constant entropy risk premium (or, in the log-normal case, variance risk premium) over the sample. This is likely a reasonable approximation for the full-sample differences considered in the secular trends analysis, though it likely induces further measurement error for the higher-frequency analyses.

**VIX Measurement.** The squared VIX is defined for horizon $T - t$ as

$$\text{VIX}^2_{t,T} = \frac{2R^f_{t,T}}{T-t}\Big(\int_0^{F_{t,T}} \frac{\text{put}_{t,T}(K)}{K^2} dK + \int_{F_{t,T}}^{\infty} \frac{\text{call}_{t,T}(K)}{K^2} dK\Big),$$

where $F_{t,T}$ is the forward price and $\text{put}_{t,T}(K)$ and $\text{call}_{t,T}(K)$ are prices of European put and call options with strike $K$ expiring at $T$. To implement this formula, we use a cleaned version of a global panel of index option prices from OptionMetrics. As in the main text, the sample, data filters, and implementation approach are taken from Gandhi, Gormsen, and Lazarus (2025); see that paper for full details.

Our options data are available starting in 1990 in the U.S. sample, but the samples for other countries start between 2002 and 2006. To obtain a full sample corresponding to the forecast data, we project $\text{VIX}^2_{t,j}$ in the available sample onto realized volatility in the country $j$ index return, and then obtain predicted values $\widehat{\text{VIX}}^2_{t,j}$ using the observed volatility for any dates in the sample for which we cannot calculate VIX directly.

**Equity Yields and Data Availability.** As in the text, the equity yield $ey_{t,j}$ is measured by starting with the five-year earnings-to-price ratio $\overline{E}_{t-4,t,j}/P_{t,j} = [(E_{t-4,j} + \ldots + E_{t,j})/5]/P_{t,j}$, where earnings and prices are calculated on a value-weighted basis for all available traded stocks in the country. Earnings $E_{t,j}$ are defined as net income for the full calendar year corresponding to date $t$, while prices $P_{t,j}$ are end-of-period aggregate market capitalizations. We then scale this $\overline{E}_{t-4,t,j}/P_{t,j}$ by 0.5, very close to the unconditional average payout ratio of 0.494 in our post-1990 sample, to obtain our final measure of equity yields $ey_{t,j}$. Algebraically, $ey \equiv \log(1 + D/P) \approx D/P = (D/E) \times (E/P)$. Our unconditional average payout ratio is calculated as the average ratio of five-year-average common dividends to five-year-average net income, to put the dividend and earnings figures in common terms, and the average is across all years and G7 countries starting in 1990.

In some countries, the share of publicly traded companies with available earnings data is low in the early part of our sample. We drop any country–year equity yield observations with such coverage issues. The resulting samples start in 1990 for the U.S. and Canada; 1992 for the U.K.; 1993 for Japan; 1994 for France and Germany; and 1998 for Italy. Any estimated relationship between changes in earnings yields and changes in interest rates uses a country-specific start date consistent with the beginning of this equity yield sample; for example, the $r^*$ difference for Italy in Figure 3 is the difference starting from 1998, consistent with the difference calculated for its equity yields. While all our analyses use data through 2023, the need for full-year net income data to calculate $ey_{t,j}$, along with the fact that our data is only available through part of 2023, means that we measure equity yields through the end of 2022. In all cases where we calculate full-sample differences in the real rate (and its components), we again match this full-sample difference with the actual available sample for our equity data. (Any analysis focusing solely on the real-rate data — for example, the first-stage regression in Table 1 — uses all the data through 2023.)

## B.2 Details on Block Bootstrap Inference

For the top panel of Table 4 in Section 4.1, we compute standard errors and confidence intervals using a block bootstrap that clusters at the year level. We draw years with replacement from the original sample, and for each bootstrap replication, we then construct a resampled panel by including all country-quarter observations from each drawn year. We then run the first-stage regression (17) using the specification in column (3) of Table 1 and obtain the estimated pure discounting observations as in (18). Given these $\widehat{\rho}^*_{t,j}$ values and the resampled equity yields, we then collapse to the first and last available observation by country to calculate full-sample differences and run the regression in the top panel of Table 4.

This last step is conducted in a manner that accounts for the pronounced trends in our sample. In particular, before collapsing to the first and last observation by country to calculate full-sample differences, we sort the resampled data so that the temporal ordering of the years is preserved from the original sample. Specifically, if the bootstrap draw selects years $\{t_1, t_2, \ldots, t_T\}$ from the original sample (where $t_1 \leqslant t_2 \leqslant \ldots \leqslant t_T$), we label these as $\{1990, 1991, \ldots, 1990 + T - 1\}$ in the resampled data (where $T = 34$). This means that early observations in the bootstrap sample are drawn from years that appeared early in the original sample, and late observations are drawn from years that appeared late in the original sample. So when collapsing the data to the first observation in the year labeled 1990 and the last observation in the year labeled $1990 + T - 1$, we preserve the nonstationarity of the original data (i.e., the pronounced change from beginning to end), while nonetheless allowing the possibility of moderate (local) changes in the start and end date.[55]

This approach implements a form of the *local block bootstrap* proposed and analyzed by Paparoditis and Politis (2002) and Dowla, Paparoditis, and Politis (2013) for inference with non-stationary time series. The classic block bootstrap assumes stationarity, which is violated in our setting.[56] The local block bootstrap instead ensures that resampled observations for

[55] For example, if the first year in the resampled data set is 1992, then the full-sample change will be calculated starting from 1992. In our setting, 99.8% of replications begin and end within the first and last five years of the original sample, respectively. The order-preserving feature of this bootstrap helps explain the strong statistical significance in the main regression despite the small apparent sample size of 7 countries: we in fact have many observations per country with which to estimate the pure discount rate, and its behavior from the start to the end of the sample is quite robust.

[56] This violation does not affect inference in cases where the ordering of the data does not matter. So for

each time period are drawn from a local window around that period in the original sample, preserving any local dependence structure and accommodating slowly varying distributions.

We use 10,000 bootstrap replications for all reported standard errors and significance tests. Statistical significance is determined by inverting basic bootstrap confidence intervals, computed as $(2\widehat{\beta}-\beta^*_{1-\alpha/2}, 2\widehat{\beta}-\beta^*_{\alpha/2})$, where $\widehat{\beta}$ is the full-sample point estimate and $\beta^*_x$ denotes the $x$ percentile of the bootstrap distribution.

## B.3 Robustness Measures

This section describes the alternative input data used in Section 4 in more detail:

- **Survey of Professional Forecasters (SPF):** We obtain mean responses for three SPF forecast variables via the Federal Reserve Bank of Philadelphia's website: the average annual 10-year nominal Treasury yield over the next 10 years, average CPI inflation over the next 10 years, and average real GDP growth over the next 10 years. These forecasts are provided once per year in the first quarter of each year. We calculate the forecasted average real yield by subtracting the CPI forecast from the Treasury yield forecast, and we use this as our alternative measure of $r^*$. We use the GDP forecast as our alternative measure of $g^*$. These forecasts are only available starting in 1992; to align the available sample with our main analysis, we use run regressions of the SPF variables on the corresponding variable from the baseline Consensus data and calculate the fitted values, which we use for the 1990 and 1991 observations.

- **Bauer and Rudebusch (2020):** For this alternative measure of $r^*$, we use the "Average real-time $r^*$" as presented in the right panel of Figure 2 of Bauer and Rudebusch (2020). This is an average of multiple statistical and model-based estimates, each of which is real-time in the sense that it uses only data through period $t$ to estimate $r^*_t$. To align the sample with our baseline sample, we obtain up-to-date data from the Federal Reserve Bank of New York's $r^*$ dashboard (`https://www.newyorkfed.org/research/policy/rstar`) on the real-time $r^*$ estimates from the Laubach–Williams and Holston–Laubach–Williams models, and we again calculate fitted values by regressing the Bauer and Rudebusch (2020) $r^*$ series on these two alternative $r^*$ series.

- **GARCH(1,1):** For each country, we fit a GARCH(1,1) model to their daily equity returns data, and we use this to forecast variance at the six-month horizon. We then use this in place of $\text{VIX}^2$ in the interest-rate decomposition. Results are also quantitatively

the bootstrapped standard errors reported in Section 3 (for which the regressions are invariant to the data ordering), we use this standard block bootstrap.

nearly identical when using the GARCH variance forecast *in addition to* the VIX in the decomposition, rather than as a replacement.

- **Baker, Bloom, and Davis (2016) Uncertainty Index:** We obtain the country-level economic policy uncertainty series from `https://www.policyuncertainty.com` for the set of available countries in our sample, and we use these data in place of $\text{VIX}^2$ in the interest-rate decomposition. Results are again quantitatively nearly identical when using the uncertainty index *in addition to* the VIX in the decomposition, rather than as a replacement.

## B.4 Robustness to Time Variation in Profit Shares

This section presents and discusses the robustness results introduced in Section 4.2. We use two sources of additional forecast data to estimate the contribution of profit-share changes. First, the long-term Consensus Economics forecast data provides nominal corporate profit growth forecasts since 1998 for only the U.S. sample. Using this, we construct a proxy for real $g^*_{t,d}$ by subtracting the inflation forecast from the nominal profit-growth forecast for year $t+5$. In this available post-1998 U.S. sample,[57] profit-growth forecasts have in fact fallen by meaningfully more than output-growth forecasts:

$$\Delta g^*_t = -0.50, \quad \Delta g^*_{t,d} = -1.26.$$

With a leverage parameter of $\lambda \approx 2$ — close to estimated coefficient on expected growth in the real-rate decomposition in Table 1, consistent with the insignificant equity effect estimated in (2) — we accordingly estimate that the change in the expected profit share $\pi^*_t \equiv g^*_{t,d} - \lambda g^*_t$ has been very close to 0 over this sample. As a result, our main conclusions for the U.S. data are unchanged: profit-share shocks have affected current cash flows but not expected future growth rates, leaving equity yields close to unaffected. As a result, the previous estimated effect of the pure discounting residual, and the resulting pass-through of roughly 1/3 of the decline in $r^*_t$ to equity valuations in U.S. data, remains valid.

As a secondary check on this analysis, we obtain aggregate long-term earnings growth (LTG) forecasts for U.S. equities from Nagel and Xu (2022). These are analyst forecasts for earnings growth for U.S. equities, which Nagel and Xu aggregate to the index level and convert to real terms by subtracting forecasted inflation. Using this series as our second proxy for $g^*_{t,d}$, for the full sample over which we can implement our real-rate decomposition

[57]This post-1998 sample corresponds to a period during which the majority of the decline in expected output growth took place in the U.S. forecasts.

and measure equity yields, we estimate that $\Delta g^*_t = -0.70$, $\Delta g^*_{t,d} = -0.60$.[58] If one again assumes a leverage parameter of about $\lambda = 2.5$, this implies that there has been a modest increase in the profit-share term $\Delta\pi^*_t$, but our main results are largely unaffected.

While Greenwald, Lettau, and Ludvigson (2025) differ from us in their focus on profit-share effects on prices, our results echo some of their findings. They find that "essentially all of the increase in equity values relative to output from the mid-1990s to the end of the sample" can be accounted for by assuming a fixed ratio of market equity to earnings (p. 1093), indicating that shocks to contemporaneous earnings relative to output drive the vast majority of equity price (but not equity yield) movements. This is also reflected in the analysis of Atkeson, Heathcote, and Perri (2025b), who find that increases in the ratio of cash flows to value added have affected valuation much more than the ratio of price to cash flows. While we do not have direct evidence on the importance of profit-share changes outside the U.S., Atkeson, Heathcote, and Perri (2025b) also estimate (see their Figure 7) that the increase in cash flows to value added appears fairly specific to the U.S. data.

Finally, as noted in the main text, we rerun our higher-frequency return regressions in Table 2, but now with changes in the two $g^*_{t,d}$ proxies (profit growth and LTG) as explanatory variables in addition to the change in expected output growth. Results are presented in Table B.2 below. The results are very consistent with those presented in Table 2, with the coefficient on $(\widehat{\Delta\rho^*_t})$ very close to the original estimate of -19.1. The loading on $\Delta LTG$ is also significant and positive. Overall, our main results continue to apply even when considering changes in the profit share.

## B.5 Details on Simulations for Figure 8

Figure 8 plots the distribution of average realized stock returns across 100,000 simulations of an artificial economy. We now explain how each simulation is conducted. Each simulation has two periods (0,1), and one period amounts to 30 years. We simulate expected consumption growth $g_1 = g_0 + \varepsilon^g_1$ (abusing notation slightly), pure discounting terms $\rho_1 = \rho_0 + \varepsilon^\rho_1$, and SDF entropy $L_{0,M} = L_{1,M} + \varepsilon^L_1$, as well as dividends $d_1 = d_0 + \mu^d + \varepsilon^D_1$. The equity risk premium is $rp_t = \mathbb{E}_t[r^{\text{mkt}}_{t+1}] - r^f_{t+1} = \theta_t + L_{t,M}$, where $\theta_0$ is chosen to match the average equity premium

[58] We prefer the Consensus Economics data because the LTG-based forecasts are much more volatile than the other forecast series (see Bordalo et al. 2024), so the full-sample differences are highly sensitive to the start and end date. Starting the sample in 1992 instead of 1990 gives $\Delta g^*_{t,d} = -1.88$; meanwhile, ending the sample a year later gives $\Delta g^*_{t,d} = 0.14$. Given such large cyclical LTG forecast variation ($\sigma_{\text{LTG}} = \text{SD(LTG)} = 2.0$, vs. $\sigma_{\text{profit}} = 0.8$, $\sigma_{\text{GDP}} = 0.4$), we view the baseline forecasts as better capturing low-frequency variation. For another earnings-growth series, we obtained aggregate annual data on analyst-report-based terminal growth expectations (typically at a longer horizon than the IBES LTG measure) from Décaire and Guenzel (2025), and we thank Paul Décaire and Marius Guenzel for sharing their data. In line with the other $\Delta g^*_{t,d}$ proxies, terminal growth expectations have declined in the U.S. since the early 2000's (the start of their sample).

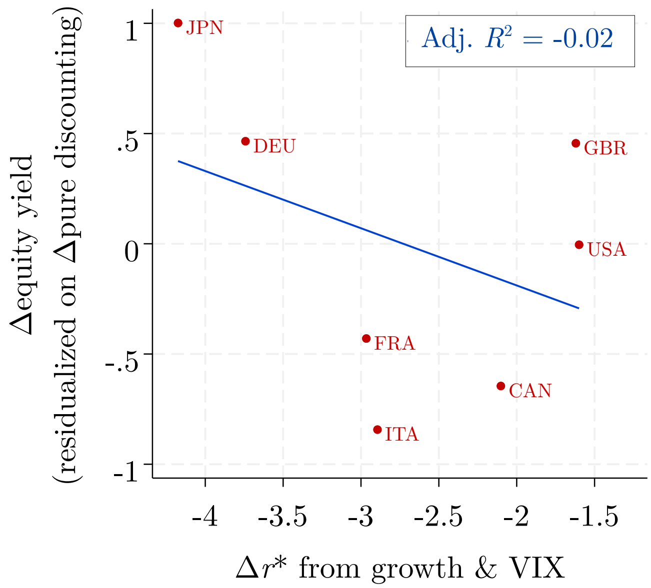
and $\theta_1$ is either equal to $\theta_0$ or adjusts to ensure that the equity yield does not drop below 0.5%. Based on these variables, we calculate the ex ante interest rate as $r_t^f = \rho_t + \gamma g_t - L_{t,M}$, where we choose $\gamma = 2$ to match the empirically observed value. We calculate equity yields as $ey_t = r_t^f + rp_t - \lambda g_t$, and set the leverage parameter $\lambda = 2$ to match the empirical observation that earnings yields are close to orthogonal to movements in growth rates. At time zero, our calibration implies $rp_t = 4\%$ (relative to long-term yields), $r_0^f = 4\%$, $g_0 = 2\%$, such that $ey_0 = 4\%$, and the duration of the stock market is 25 years (which we view as somewhat conservative).

The shocks $\varepsilon$ are all drawn from heavy-tailed $t$-distributions with 5 degrees of freedom. We choose the standard deviations of $\varepsilon^\rho$ and $\varepsilon^g$ to match the standard deviations in our global panel, and we choose the standard deviation of $\varepsilon^L$ such that the volatility of realized bond returns matches the annualized volatility of realized 30-year bond returns in the long U.S. sample; we estimate this empirical counterpart to be around 2% based on data from Robert Shiller's website. Finally, we set the volatility of $\varepsilon^D$ such that the volatility of realized stock returns matches the volatility of realized 30-year stock returns in the long U.S. sample, again based on data from Robert Shiller's website. We conduct 100,000 simulations, and we calculate and plot the distribution of realized excess returns relative to all three benchmarks described in the main text.

## B.6 Additional Results

See below for additional figures and tables referenced in the text.

**Figure B.1: Residualized Equity Yield Changes vs. Growth and Uncertainty**



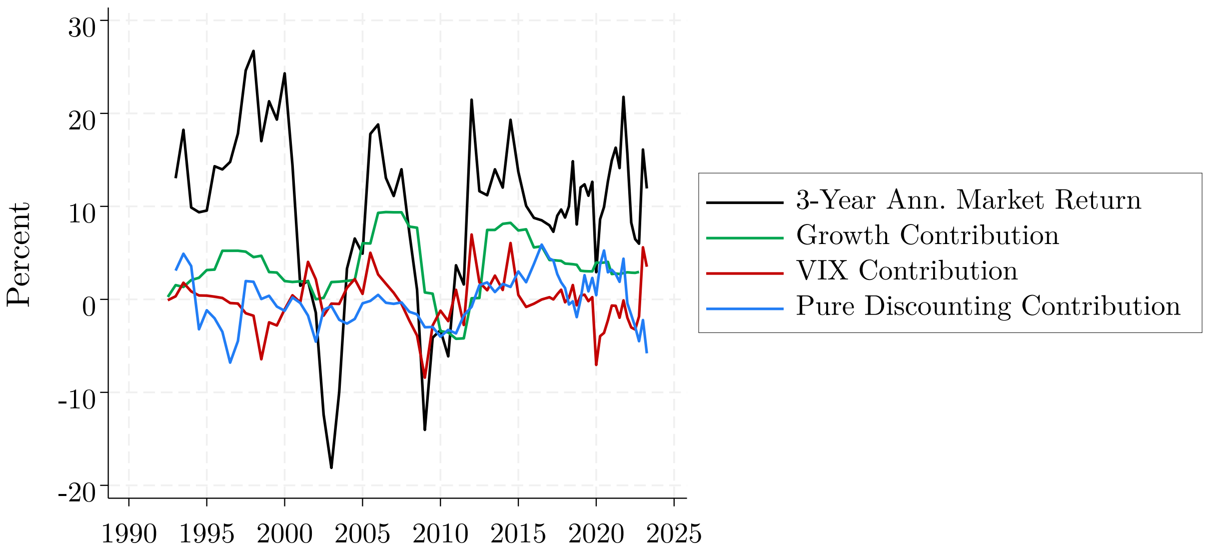

*Notes:* This figure plots the country-level change in equity yields against changes in interest rates from growth rates and uncertainty, where the equity yield change has now been residualized against the pure discounting change shown in the left panel of Figure 3. The sample is 1990–2023, or the longest available span for the given country. See Figure 3 for additional details.

**Table B.1: Regressions for Three-Year Changes in Trend Real Rates**

| | **(1)** U.S. | **(2)** All | **(3)** All |
|---|---|---|---|
| Change in expected growth $\Delta g^*_{t,j}$ | 0.5** | 0.3** | 0.3** |
| | (0.2) | (0.1) | (0.1) |
| Change in uncertainty $\Delta \mathrm{VIX}^2_{t,j}$ | -4.3** | -0.9 | $\beta_j$ |
| | (2.1) | (1.8) | |
| Constant | -0.3*** | -0.5*** | -0.5*** |
| | (0.1) | (0.1) | (0.1) |
| Country FEs | ✗ | ✓ | ✓ |
| Country-Specific $\mathrm{VIX}^2_{t,j}$ Loading | ✓ | ✗ | ✓ |
| Obs. | 74 | 784 | 784 |
| $R^2$ | 0.17 | 0.05 | 0.06 |
| Within $R^2$ | — | 0.02 | 0.04 |

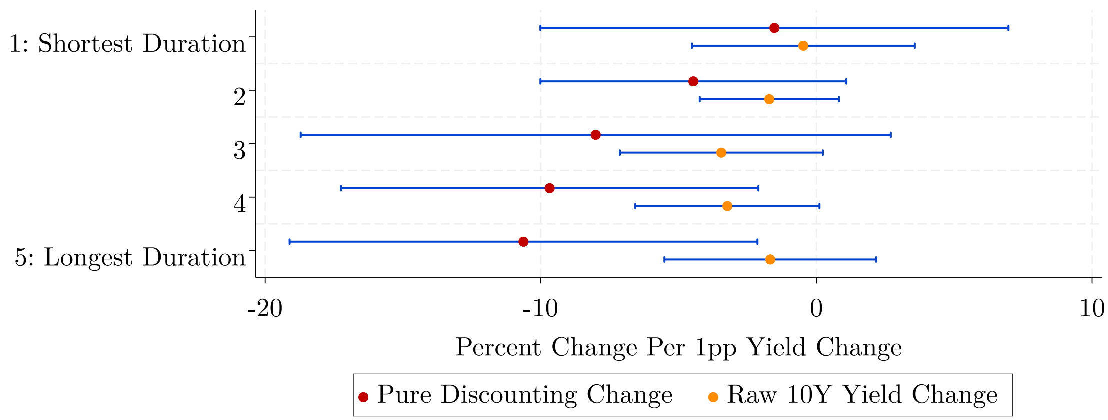
*Notes:* This table shows estimated OLS coefficients in the regression (19), along with standard errors in parentheses. In column (1), standard errors are obtained using a block bootstrap with one-year blocks and 10,000 bootstrap draws. In columns (2)–(3), standard errors are clustered by country and date. Statistical significance at the 10% level, 5% level, and 1% level are denoted by *, **, and ***, respectively. In column (3), the country-specific loadings on the squared VIX, $\beta_j$, are statistically significant at the 10% level for 6 of the 12 countries in our sample, and at the 5% level for 3 of the 12 countries (including the U.S.). The sample is 1990–2023, or the longest available span for the given country.

**Figure B.2: Decomposition of U.S. Value-Weighted Equity Returns**



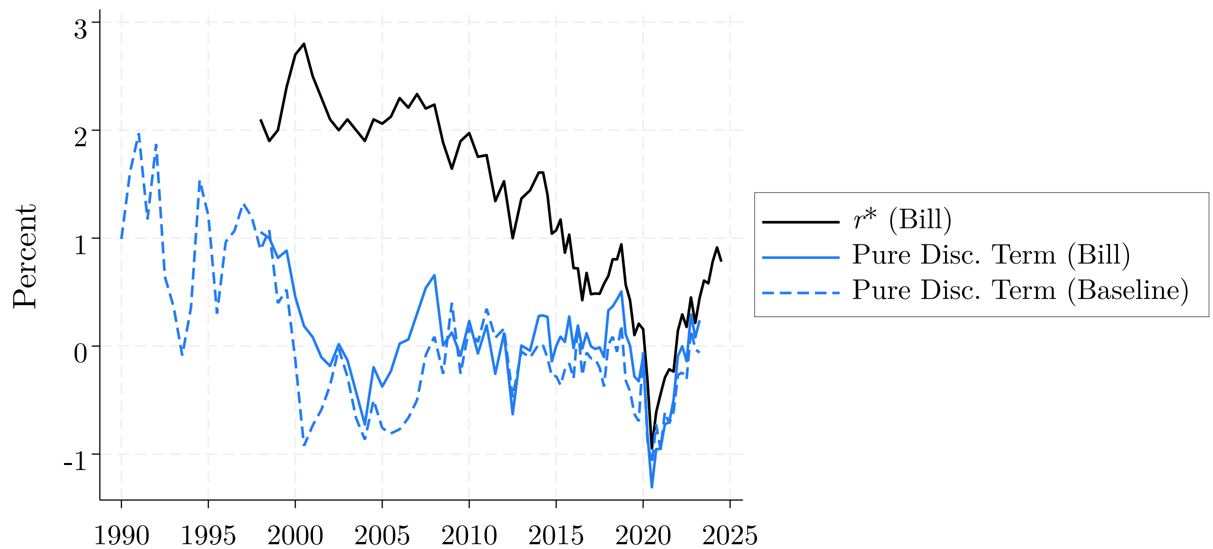

*Notes:* This figure shows three-year annualized average returns for the value-weighted U.S. stock market, along with estimated underlying contributors. The growth contribution is the estimated coefficient $\widehat{\pi}_g$ in (20) times the three-year change $\Delta g^*_{t,j}$, plus the estimated contribution of contemporaneous cash flows. For this additional contribution, we estimate a regression identical to (20) but with realized three-year log dividend growth $\Delta d_{t,j}$ as an additional predictor; the contemporaneous cash-flow contribution is equal to $\Delta d_{t,j}$ times its estimated coefficient. Each of the other two contribution terms in the figure is equal to the corresponding predictor times its estimated coefficient in (20). The coefficient estimates are taken from column (2) of Table 2, but divided by 3 (e.g., the pure discount loading is -6.3 rather than -19.1). This is to account for the use of annualized returns in this plot, whereas the outcome variable for Table 2 is cumulative non-annualized returns. The pure discounting predictor for (20) is obtained from the first-stage estimation in (19).

**Figure B.3: Portfolio Exposures to Pure Discount Rate Changes: Global Stocks**




*Notes:* This figure repeats the analysis shown in Figure 5 using the full global sample of stocks. We form duration-sorted portfolios in the international panel following Gormsen and Lazarus (2023), and then we estimate the same regressions as in Figure 5, with country-level fixed effects. The sample is 1990–2023.

**Figure B.4: U.S. Estimation Results: Alternative with Short-Rate Forecasts**



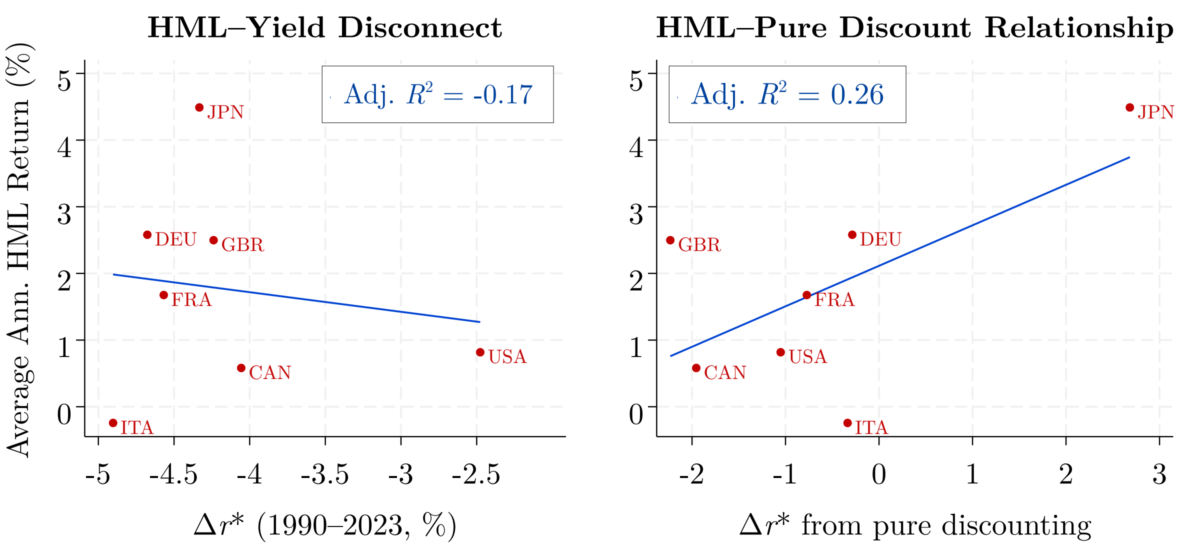

*Notes:* This figure replicates the U.S. decomposition in Figure 2, but with $r^*$ measured using the Consensus forecast of real bill yields (i.e., the forecasted nominal 3-month Treasury bill rate in year $t+5$, minus forecasted inflation at the same horizon). The black line shows this bill-based $r^*$ series, and the solid blue line shows the implied pure discounting term, estimated as in Figure 2 but with the bill-based $r^*$ as the outcome variable in the decomposition regression. The dashed blue line shows the same pure discounting residual as plotted in Figure 2. The short-rate sample is 1998–2023.

**Table B.2: Three-Year Return Regressions: Profit-Share Robustness**

| | (1) U.S. | (2) U.S. | (3) U.S. | (4) U.S. |
|---|---|---|---|---|
| $\Delta$10y yield | 4.19 | | | |
| | (3.51) | | | |
| $\Delta$pure discount ($\widehat{\Delta\rho_t^*}$) | | -19.1** | -24.6*** | -17.9*** |
| | | (7.64) | (8.95) | (6.61) |
| $\Delta$exp. growth | | -1.49 | -15.3 | -2.30 |
| | | (14.0) | (16.7) | (12.7) |
| $\Delta$exp. profit growth | | | -0.14 | |
| | | | (4.02) | |
| $\Delta$LTG | | | | 5.94*** |
| | | | | (2.03) |
| $\Delta\text{VIX}^2 \times 100$ | | -3.08** | -4.62*** | -2.88*** |
| | | (1.33) | (1.46) | (1.09) |
| Obs. | 74 | 74 | 58 | 74 |
| $R^2$ | 0.04 | 0.20 | 0.47 | 0.36 |

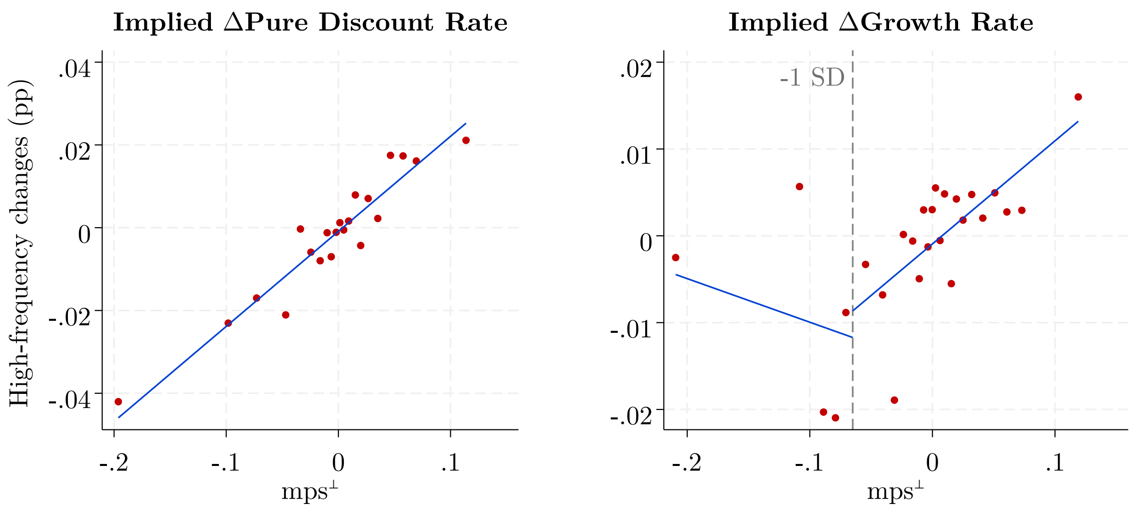
*Notes:* This table replicates the analysis in Table 2 for U.S. data, but with additional predictor variables for other measures related to equity dividend growth. See the notes for Table 2 for details on the estimation and inference, and see Section 4.2 for descriptions of the additional predictor variables.

**Figure B.5: Interest Rates and Value Returns: Long-Term Global Evidence**




*Notes:* The left panel plots the country-level average annual return on the high-minus-low (HML) book-to-market value factor versus the change in estimated trend real rate $r^*$ over the 1990–2023 sample. Following Fama and French (1993), each country's HML factor is constructed based on a 2×3 size and book-to-market double sort, with returns calculated as the average of the high-minus-low return for small and large firms. See Gormsen and Lazarus (2023) for details. The right panel plots the same average annual HML return against the change in the pure discounting term estimated using (17)–(18) following the main specification in column (3) of Table 1. The sample is 1990–2023. Note that HML returns are available starting in 1990 for all countries, whereas the equity yield samples start later than this in some cases (see Appendix B.1). As a result, the $\Delta r^*$ terms in this figure may differ relative to Figure 3, given the earlier start date for the changes calculated in this figure.

**Figure B.6: Changes in Decomposition Terms Around Monetary Policy Shocks**




*Notes:* This figure shows binned scatter plots of the change in the pure discount rate $\widehat{\Delta\rho_t^*}$ (left panel) and the expected growth rate $g_t$ (right panel) against the Bauer and Swanson (2023a) orthogonalized monetary policy shock $\text{mps}_t^{\perp}$ in percentage points (pp). The changes in pure discount rates and expected growth are recovered from (22)–(23) using the high-frequency (30-minute) stock-price and interest-rate changes. The sample is 1990–2023. In the left panel, the line of best fit is estimated from all available observations using the same regression as in Table 5, column (2). In the right panel, we show two fit lines: one for large expansionary shocks for which $\text{mps}_t^{\perp}$ is less than one standard deviation below its sample mean ($\text{mps}_t^{\perp} < -0.065$), and one for the remaining shocks, separated by the vertical dashed line. Large expansionary shocks generate conventional policy responses, whereby larger easing shocks lead to higher expected growth (as in the left part of the panel). The remaining shocks otherwise generate a positive slope consistent with an information effect (as in the right part of the panel).