Forward Return Expectations*

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Abstract

We measure investors' short- and long-term stock-return expectations using both options and survey data. These expectations at different horizons reveal what investors think their own short-term expectations will be in the future, or forward return expectations. Across data sources, investors' forward return expectations are countercyclical. This countercyclical variation is, however, excessive, as the changes in forward return expectations are too large relative to subsequent realized short-term expectations. The excess variation in forward return expectations can account for a substantial share of the stock-price declines during the global financial crisis and the Covid-19 crisis.

Keywords: asset pricing, expected stock returns, time-varying discount rates

JEL classifications: G10, G12, G40

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1 Introduction

One of the organizing facts in asset pricing is that expected returns on equities vary over time. A key question is how investors perceive this variation ex ante. One aspect of this question, which has been studied extensively, is how investors perceive the contemporaneous level of the equity premium. Another aspect, which is the focus of this paper, is what investors expect the equity premium to be in the future, or their *forward return expectations*. These forward return expectations are important for stock price determination, as prices depend on all future return expectations and growth expectations. Our analysis reveals predictable forecast errors in investors' forward return expectations, which can account for a large share of movements in stock prices during crashes.

We study forward return expectations using both option prices and survey data. First, using a global sample of equity index options, we extract proxies for the equity premium at multiple horizons as perceived by sophisticated investors. With both long- and short-term expectations in hand, we construct forward expectations of returns based on the difference between the two. New theoretical results show that we can identify time-series changes in these expectations, which is our main focus, under relatively weak conditions. We complement the options data with return expectations from surveys of professional forecasters, CFOs, and retail investors. These surveys provide return expectations across different time horizons, enabling us to extract forward expectations in a more model-free setting.

The data reveal that forward return expectations are countercyclical among all investor types: sophisticated, professional, and retail investors. This finding aligns with estimates of the objective variation in expected returns, which are also known to be countercyclical (Campbell and Shiller 1988). The robust countercyclical variation in forward return expectations stands in contrast to the evidence on spot return expectations, for which the evidence on cyclicality is mixed (see the overview in Table 1).

The countercyclical variation is, however, excessively strong. In bad times, investors believe that expected returns will be substantially elevated in the future, but these elevated expectations are too high relative to investors' own subsequent beliefs about short-term returns. (The reverse applies in good times.) During the 2008 financial crisis and the 2020 Covid crisis, for instance, investors believed that equity premia would be substantially elevated in the future; ex post, however, their realized short-term expectations were only mildly elevated, and they reverted to their usual levels relatively quickly. This pattern applies consistently over time, and across all the settings we study, in both options and survey data.

The excess variation in forward expectations represents a new source of excess volatility in stock prices. If investors did not overestimate forward return expectations during bad times, prices would fall less in crises and would generally exhibit more modest cyclical swings. We show in particular that predictable mistakes in forward expectations can account for a significant share of the decrease in prices observed in major crisis episodes. Our lower-bound estimates explain about 20–25% of the index-level price declines in the global financial crisis and the early-2020 Covid crash, while our largest estimates imply that forward-expectation errors can account for the majority of the price drops in these periods.

Our findings on forward expectations also help explain or reconcile a number of facts documented in recent work, related to (i) disagreements in previous research related to survey expectations of returns, (ii) stylized facts about the equity term structure, and more speculatively, (iii) investors' inelastic demand for equities in response to a given change in prices. We explore these implications further below.

1.1 Framework and Stylized Facts

To better understand our analysis, we begin by defining the following notation for the expected log return on the market over n periods:

$$\mu_t^{(n)} = \mathbb{E}_t[r_{t,t+n}].$$

We refer to these expected returns as *spot rates*.[1] If at time t we observe the n-period and $(n+1)$-period spot rates, we can back out the expected returns between these two periods, which we call the *forward return expectation*, or *forward rate* for short:

$$f_t^{(n)} = \mu_t^{(n+1)} - \mu_t^{(n)} = \mathbb{E}_t\left[\mu_{t+n}^{(1)}\right].$$

After n periods, we can compare this forward rate to the realized one-period spot rate, $\mu_{t+n}^{(1)}$. Under the law of iterated expectations, the forward rate should be an unbiased predictor of the realized spot rate, which means that *forecast errors*,

$$\varepsilon_{t+n} = \mu_{t+n}^{(1)} - f_t^{(n)},$$

should have an expected value of zero and be unpredictable by any time-t information.

[1] For simplicity, this introductory discussion focuses on expected returns without explicitly netting out the risk-free rate. (The log return $r_{t,t+n}$ can without loss be taken to be a excess return, but for now we take this to be the total return unless noted.) Starting in Section 2, we also directly and separately consider expectations of the equity premium (net of the risk-free rate). To abstract from the trend and cyclical behavior of the risk-free rate, we consider forward risk premia to be our main measure of forward return expectations, and we often find somewhat stronger results for these risk premia. But we generally present estimates for both measures, finding qualitatively consistent results in both cases.

The forward rates and forecast errors can be estimated straightforwardly with survey data, as long as the given survey provides return expectations at multiple horizons. This is the case for three well-known surveys in the finance literature: the Livingston survey of professional forecasters, the Duke–Fed CFO survey of financial executives, and the Vanguard survey of retail investors. In the Livingston survey, for instance, we observe expected returns at the 6-month and 12-month horizon. These allow us to calculate a 6-month, 6-month forward rate, namely today's expectation of the 6-month spot rate, 6 months from now. By comparing this forward rate to the realized 6-month spot rate, as observed 6 months later, we can calculate forecast errors and evaluate how well forward rates predict future spot rates.

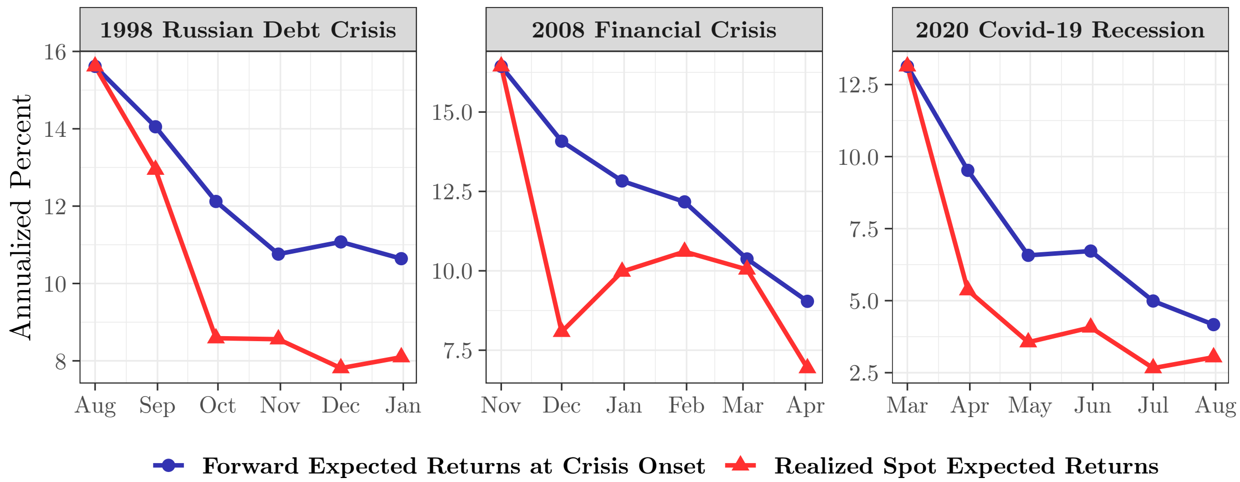
These forecast errors for return expectations can also be estimated from option prices under relatively mild conditions. We start by considering a simple special case, in which an investor has log utility over the market return. For this investor, we can directly estimate expected returns at different horizons from option prices, allowing us to estimate forward rates and forecast errors over time. But an important contribution of our paper is to show that we can extract the forecast errors under much more general conditions than log utility: for any investor, we can identify expected forecast errors up to a small risk premium term that can be managed empirically or through theory. Our forecast-error estimates are thus useful for a wide range of specifications of preferences and the data generating process. The advantage of the option-based data is that we observe forward rates and forecast errors at many different horizons, in a relatively long sample that spans many countries.

We provide a consistent set of stylized facts about the behavior of forward rates and forecast errors across investors. Forward rates are decent predictors of future realized spot rates. Across the data sources, future realized spot rates increase by around 0.7 percentage points when ex ante forward rates increase by 1 percentage point. The R^2 values vary between 0.38 and 0.71. In more detailed analysis, we find that forward rates are useful for predicting both transitory and more persistent fluctuations in spot rates. Taken together, the predictive regressions suggest that forward rates have predictive power over future spot rates, but that future spot rates move by significantly less than predicted by forward rates.

As such, investors appear to make predictable forecast errors. When forward rates are high, subsequent spot rates are lower than expected by investors ex ante. In predictive regressions of realized forecast errors on ex ante forward rates, we find that spot rates are around 0.25 percentage points lower than expected when forward rates are 1 percentage point higher. These results are statistically significant, and there is meaningful economic predictability in terms of R^2 values. This pattern reflects one of the key results of the paper, namely that there is excess cyclicality in return expectations: in bad times, investors' beliefs about future expected returns (forward rates) are elevated by significantly more than justified

Figure 1
U.S. Forward Rates and Realized Spot Rates in Crises

This figure plots forward expected returns $f_t^{(0)}$, $f_t^{(1)}$, $\cdots$, $f_t^{(5)}$ (blue) and the corresponding ex post spot expected returns $\mu_t^{(1)}$, $\mu_{t+1}^{(1)}$, $\cdots$, $\mu_{t+5}^{(1)}$ (red) in the U.S. sample. For the Russian debt crisis, forward rates are as of 08/1998 and realized spot rates are from 08/1998 to 01/1999. For the financial crisis, forward rates are as of 11/2008 and realized spot rates are from 11/2008 to 04/2009. For the Covid-19 recession, forward rates are as of 03/2020 and realized spot rates are from 03/2020 to 08/2020.



by their subsequent beliefs.[2]

To illustrate this excess cyclicality, Figure 1 shows forward rates and realized spot rates from the option-based measure during three crises: the 1998 Russian debt crisis, the 2008 global financial crisis, and the 2020 Covid-19 crisis. The blue circles show one-month forward rates maturing $n = 0$, 1, 2, 3, 4, and 5 months from the first plotted date t (which is soon after each crisis onset). The red triangles show ex post realized one-month spot rates as of months t, $t + 1$, $\ldots$, $t + 5$. In all cases, the spot equity premium (the first point) increases substantially as of the crisis onset. Forward rates also increase, but the forward curve is strongly downward sloping, suggesting that investors expect significant mean reversion of spot rates in subsequent months; the slope of the term structure of return expectations is procyclical. Going forward, the spot rate indeed decreases substantially as the crisis recedes.

In all cases, however, forward rates are too high at the peak of the crisis relative to future realized spot rates. This suggests that while investors understood that the equity premium would decrease in the future, they appear to have underestimated the speed of mean reversion. One interpretation of this finding is that the rebound following each crisis was driven by news, like a series of policy shocks, that was unexpected ex ante. Another interpretation is that investors systematically overreact to crises by thinking that future spot rates will be

[2]This cyclicality is excessive relative to their *own* subsequent beliefs: we are testing the internal consistency of expectations over time, and not just relative to an estimate of cyclicality in objective expected returns.

elevated by more and for longer than what one should rationally expect.

While Figure 1 focuses on the option-based measure, we find consistent evidence of excess cyclicality in all three measures of expectations we consider. Moreover, the forward rates share a common, countercyclical component across data sources. In all cases, forward rates comove positively with a wide range of countercyclicality indicators; leading examples include the earnings yield (measured as the excess CAPE yield, following Shiller, Black, and Jivraj 2020), volatility (measured as VIX^2), and a recession indicator. Forecast errors also share cyclical variation across the data sources: forecast errors are predictably negative after bad times. That is, in bad times, all investor groups consistently overestimate how high expected returns will be in the future. We similarly find that forward rates measured in option prices are strong predictors of future realized errors in the surveys. This cross-measure predictability alleviates measurement-error concerns. More generally, the consistency across data sources suggests patterns in expectations that are shared across a broad range of investors.

1.2 Implications and Mechanisms

Implications. Our results on forward return expectations and forecast errors have implications for four literatures in asset pricing.

The debate on the cyclicality of subjective risk premia. A recent literature studies the cyclical dynamics of return expectations. Greenwood and Shleifer (2014) document procylical variation; Nagel and Xu (2023) document acyclical variation; and Dahlquist and Ibert (2024) document countercyclical variation, all using somewhat different measures of expected returns, aggregate state variables, and investor groups. Martin (2017), meanwhile, finds countercyclicality in option-based estimates. We find that the degree of cyclicality depends on the horizon. In the surveys we study, short-run "spot" expectations of excess returns are on average acyclical, but longer-run forward rates are countercyclical. (One interpretation of this finding is that respondents understand present value logic — they understand that future long-run returns must be high during crises when prices are low — but during crises, they often believe it will take a while before prices start increasing.)

Our focus on forward rates also eliminates any apparent disagreement between option- and survey-based measures: in all cases, longer-run forward rates are robustly and excessively countercyclical. We therefore provide a unified body of evidence on the dynamics of long-run expected returns, which are the key object of interest in determining prices.

To summarize how our contribution fits in with and helps unify previous findings, Table 1 provides a selected summary of the literature on return expectations — in particular, subjective risk premia (our primary focus below) — across different horizons and investor groups. The

Table 1
Subjective Equity Premia: A Summary Across Horizons and Investors

This table summarizes findings on subjective equity return expectations from past literature and the current paper. Plus signs (**+**) indicate cases in which subjective expectations tend to co-move positively with objective measures, while minus signs (**–**) indicate cases in which subjective expectations move the wrong direction with respect to objective measures. "Excess" refers to excessive positive co-movement with objective measures (or excess cyclicality). In all cases, the "Forward" column refers to findings documented later in the paper, for forward risk premia (our main measure) and forward expected returns. All other table entries are based on past literature, for which the first column in the table provides one or more leading references containing this finding. For the past literature, expectations are for excess returns, aside from the references for the CFOs and retail investors. For the long-run entry for CFOs, De la O and Myers (2021) find that this value is close to acyclical for expected returns, while we find that it is countercyclical (**+**) when focusing on equity premia.

	Horizon of expectations				
	Short-run	+	Forward *(this paper)*	=	Long-run
Sophisticated investors					
"Mr. Market" [Martin]	**+**		**+** (excess)		**+**
Asset managers [Dahlquist–Ibert]					**+**
Other professionals					
CFOs [Greenwood–Shleifer, De La O–Myers]	**–**		**+** (excess)		**+**
Prof. forecasters [Nagel–Xu]	**+**		**+** (excess)		**+**
Retail investors					
Vanguard investors [Giglio et al.]	**–**		**+** (excess)		**+**
Gallup survey [Greenwood–Shleifer]	**–**				

second ("Forward") column illustrates that across all investor types, we find consistent evidence that forward return expectations co-move excessively with the predictable component of future objective expectations. This consistent evidence contrasts with the inconsistent evidence on the behavior of short-run expectations, which feature different signs and cyclical properties across investor groups. Put together, we find strong evidence that long-run expectations co-move positively with objective expected returns (indicating countercyclical variation).

A new source of excess volatility in prices. Our results suggest that when crises hit, prices drop by an excessive amount because investors believe future expected returns will be excessively elevated. Without such expectation errors, price fluctuations would be more modest: in our baseline option-based estimation, the expectation errors can account for most of the stock price declines during crises, and lower-bound estimates based on the survey data also account for a significant share of these crashes. Our results thus provide a new interpretation of stock-price variation, and crashes in particular, in recent decades.[3]

Stylized facts about the equity term structure. The literature on the equity term structure studies prices and expected returns for dividend claims with different maturities. The literature finds that the equity term premium — the relative return for long- versus short-maturity claims — is negative on average (Binsbergen and Koijen 2017, Binsbergen, Brandt, and Koijen 2012) and countercyclical (Golez and Jackwerth 2024, Gormsen 2021). Our results on forecast errors directly speak to these results, which are otherwise difficult to reconcile with standard models. In particular, we study a simple model where the equity term structure is flat, and show that the forecast errors uncovered in our paper result in a term structure of realized returns that is downward sloping on average and countercyclical.

Low price elasticity of demand. As a final — albeit more speculative — potential application, our results may help speak to a recent literature which documents that investors' demand for equities is relatively inelastic with respect to price changes (Gabaix and Koijen 2022). Given that a drop in prices tends to predict a significant increase in objectively estimated expected returns over short horizons, this inelasticity is puzzling. However, our results suggest that investors often mistakenly attribute a significant share of the decrease in prices to increases in expected returns at relatively long horizons and perceive short-term spot returns to increase only modestly. This structure of expectation errors lowers the elasticity of demand, as a modest increase in spot returns will not lead to a large increase in desired portfolio weights in equities. That said, we provide only a brief qualitative discussion of this issue, so the quantitative implications remain an open question.

Mechanisms. We also consider potential drivers of our results. We first address a set of alternative explanations unrelated to forecast errors in subjective expectations of log returns. For the survey data, we discuss how measurement of simple as opposed to log returns may mildly influence our results. For the options data, we investigate conditions under which our results can be explained by the behavior of the stochastic discount factor. Our option-based estimates of spot and forward rates contain a risk premium term that may be large, but

[3]We note that our results *account* for a portion of excess volatility without necessarily constituting a full *explanation* of that volatility: while we explore theoretical mechanisms that can generate our results, the analysis is to a large degree silent on the underlying sources of forecast errors. And as we also discuss, forecast errors account for somewhat less — around 1/10 — of the *unconditional* variation in valuation ratios.

importantly for our analysis, the effect on forecast errors is likely modest. For forecast errors, we show that the risk premium term on the forward rate largely cancels with the risk premium term on the spot rate, with the remaining risk correction being an order of magnitude smaller than the well-known covariance term in Martin (2017). In order for this remaining risk premium term to rationalize the behavior of forecast errors, the stochastic discount factor must feature a highly volatile and countercyclical price of risk on shocks to the equity premium. The notion that the price of risk increases in bad times is consistent with past work (Campbell and Cochrane 1999), but the pace at which this risk price must change, and the range of values it must take, appear difficult to reconcile with standard models (and with our survey evidence).

Finally, as a positive candidate explanation for our results, we study a simple model of expectation errors that matches the excess cyclicality in forward rates. In particular, one can account for observed forecast errors with a calibrated model, based on Hirshleifer, Li, and Yu (2015), in which investors overweight recent data when forecasting future fundamentals. This fundamental extrapolation then affects forecasts of the future equity premium in a manner consistent with our results. Our model also explains the cyclicality of perceived spot rates across different investor groups.

1.3 Related Literature

We build on literatures studying the behavior of investor expectations measured both from derivatives prices and in survey data, as discussed briefly above and in Table 1. For the derivatives-based measures, our paper also builds on Giglio and Kelly (2018), who document excess volatility in long-maturity relative to short-maturity claim prices across a range of term structures. They focus on the term structure of at-the-money implied variance when considering equity markets. We use a different measure of implied volatility, building on Gao and Martin (2021), and we show how this measure can be connected to the term structure of expected equity returns and forecast errors.[4] As in the table, our option-based estimates are also closely related to Martin (2017), who studies option-implied spot expected returns under a covariance condition we build on, and also Augenblick and Lazarus (2025), who derive volatility bounds for fixed-maturity index options. We differ from these papers in our focus on the term structure of expected returns.[5] We also, in contrast with much of this past work,

[4]Our paper thus connects to other work on the term-structure consistency of implied volatility, including Stein (1989) and Mixon (2007). Our result that average forecast errors are close to zero echoes the finding of Dew-Becker, Giglio, Le, and Rodriguez (2017) that it is costless on average to hedge forward variance news.

[5]This brings us slightly closer to Augenblick et al. (2025), who study how updating differs by horizon. For other recent work on measuring expectations from options, see, for example, Aït-Sahalia, Karaman, and Mancini (2020) and Chabi-Yo and Loudis (2020).

consider option-based measures jointly with survey-based measures of expectations.

Adam and Nagel (2023) provide a review of the literature on survey-based expectations. This literature often finds that investors exhibit less than fully rational expectations in their survey responses, though as above, this behavior varies across settings.[6] We consider how investors expect future expected returns to evolve, whereas past work has largely focused on contemporaneous expected returns. We also provide novel evidence of the consistency of expectations dynamics across settings when considering these forward expectations. Further, to the degree that there remain differences in the behavior of survey expectations across different groups of investors, one advantage of our option-based analysis is that we directly study the expectations expressed in equilibrium behavior.

Finally, in comparing forward and realized spot rates, our analysis appears similar in spirit to tests of the expectations hypothesis (EH) for the fixed-income term structure. Aside from focusing on equity, our methodology is geared towards estimating physical expectations (and particularly forecast errors) for future expected returns; by contrast, the spot and forward rates used in fixed-income EH tests embed risk premia by design, and violations of the EH in that setting are equivalent to bond-return predictability.[7]

2 Methodology

We begin with a theoretical analysis of the term structure of log equity risk premia. We first set up notation for spot rates, forward rates, and forecast errors. We then present our main theoretical results on forecast-error identification from option prices. We end with a discussion of identification of forecast errors from surveys.

2.1 Notation

We start by generalizing our notation slightly relative to the introduction. Continue to define the spot rate as the time-t subjective log return expectation between periods t and $t+n$:

$$\mu_t^{(n)} = \mathbb{E}_t[r_{t,t+n}], \tag{1}$$

where $r_{t,t+n} = \ln(R_{t,t+n})$ is the log return on the market.

[6] In addition to the references in Table 1, Couts, Gonçalves, and Loudis (2023), Boutros et al. (2025), and Gormsen and Huber (2025) provide further recent evidence. Separately, Schmidt-Engelbertz and Vasudevan (2025) study higher-order beliefs (i.e., beliefs about other investors' beliefs) about returns, finding that these may explain short- and long-horizon expected return variation in a manner consistent with our results.

[7] Bond-return predictability may, however, arise from the dynamics of physical expectations, as in Farmer, Nakamura, and Steinsson (2024) (see also d'Arienzo 2020).

For any horizon n, the expected return in (1) can be written as the one-period spot rate plus a series of one-period forward rates:

$$\mu_t^{(n)} = \mu_t^{(1)} + \sum_{i=1}^{n-1} f_t^{(i,1)},$$

where forward rates are now defined as

$$f_t^{(n,m)} = \mu_t^{(n+m)} - \mu_t^{(n)} = \mathbb{E}_t\left[\mu_{t+n}^{(m)}\right]. \tag{2}$$

We refer to $f_t^{(n,m)}$ as the $n \times m$ forward rate.

We define forecast errors as the difference between realized spot rates and ex ante forward rates,

$$\varepsilon_{t+n}^{(m)} = \mu_{t+n}^{(m)} - f_t^{(n,m)}. \tag{3}$$

Under the law of iterated expectations, the time-t conditional expectation of forecast errors is zero:

$$\mathbb{E}_t\left[\varepsilon_{t+n}^{(m)}\right] = 0.$$

In much of the analysis, we study expectations about future risk premia (net of the n-period log risk-free rate $r_{t,t+n}^f$), as opposed to expected returns (gross of the risk-free rate, as above). When considering risk premia, we use the notation $\tilde{\mu}$, $\tilde{f}$, and $\tilde{\varepsilon}$ to refer to spot rates, forward rates, and forecast errors, respectively. More specifically, in place of (1),

$$\tilde{\mu}_t^{(n)} = \mu_t^{(n)} - r_{t,t+n}^f = \mathbb{E}_t\left[r_{t,t+n} - r_{t,t+n}^f\right].$$

Then $\tilde{f}$ and $\tilde{\varepsilon}$ are defined as in (2)–(3), with $\tilde{\mu}$ in place of μ. In line with much of the past literature, we consider forward risk premia to be our main measures of forward return expectations, and we present these first in our tables and show them in the remaining figures. But we often streamline formal exposition and notation by using μ, f, and ε.

For forward risk premia, the law of iterated expectations implies $\mathbb{E}_t\left[\tilde{\varepsilon}_{t+n}^{(m)}\right] = \theta_t^{(n,m)}$, where $\theta_t^{(n,m)}$ is the term premium on the $n+m$–period bond in excess of the n-period bond. The expected value of the forecast errors for risk premia is therefore zero only when the pure expectations hypothesis for interest rates holds. Tests for forecast-error predictability, meanwhile, hold under the slightly weaker expectations hypothesis in which $\theta_t^{(n,m)}$ is constant.[8]

Finally, we write $M_{t,t+n} = M_{t,t+1}M_{t+1,t+2}\cdots M_{t+n-1,t+n}$ for the n-period stochastic dis-

[8]This assumption is typical in the literature on derivatives term structures; see Giglio and Kelly (2018) for a discussion.

count factor (SDF).

2.2 Identification from Option Prices

This section describes how to estimate spot rates, forward rates, and forecast errors (all of which depend on physical expectations) from option prices. As a starting point, using that $\mathbb{E}_t[M_{t,t+n}R_{t,t+n}] = 1$ under the law of one price, we can write expected log returns as

$$\begin{aligned}\mathbb{E}_t[r_{t,t+n}] &= \mathbb{E}_t[r_{t,t+n}]\,\mathbb{E}_t[M_{t,t+n}R_{t,t+n}] \\ &= \underbrace{\mathbb{E}_t[M_{t,t+n}R_{t,t+n}r_{t,t+n}]}_{\mathscr{L}_t^{(n)}} - \underbrace{\mathrm{cov}_t(M_{t,t+n}R_{t,t+n}, r_{t,t+n})}_{\mathcal{C}_t^{(n)}}.\end{aligned} \tag{4}$$

This follows Gao and Martin (2021), and we build on their analysis. They show that the first term on the right side of (4) is observable from option prices, and they label this term the LVIX.[9] We denote this term by $\mathscr{L}_t^{(n)}$, as in (4), and we discuss its measurement from options in Section 3. The covariance term, $\mathcal{C}_t^{(n)}$, is an unobserved risk adjustment. The size and sign of this adjustment can be controlled by theory, as discussed in detail below.

To build intuition, we first make the simplifying assumption that $M_{t,t+n} = 1/R_{t,t+n}$. This stochastic discount factor would, for instance, arise if the representative investor is fully invested in the stock market and has log utility over terminal wealth. In this case, the covariance term in (4) is equal to zero, and the LVIX directly identifies expected excess returns at different horizons. Given the identity in (4) and the definitions in Section 2.1, we can thus calculate spot rates, forward rates, and forecast errors from the data as follows.

Proposition 1 (Log Utility Identification). *Assuming that $M_{t,t+n} = 1/R_{t,t+n}$, spots, forwards, and forecast errors are given, respectively, by:*

$$\begin{aligned}\mu_t^{(n)} &= \mathscr{L}_t^{(n)} \\ f_t^{(n,m)} &= \mathscr{L}_t^{(n+m)} - \mathscr{L}_t^{(n)} \\ \varepsilon_{t+n}^{(m)} &= \mathscr{L}_{t+n}^{(m)} - \mathscr{L}_t^{(n+m)} + \mathscr{L}_t^{(n)}.\end{aligned}$$

Proofs for all theoretical results can be found in Appendix A. Proposition 1 follows straightforwardly from the fact that $M_{t,t+n}R_{t,t+n} = 1$ under log utility. Thus, given this assumption, we can directly identify forecast errors from option prices and thereby test whether

[9] We use a slightly different definition of LVIX, as theirs is in terms of excess returns and we start with total returns. Separately, one can equivalently write the LVIX in (4) as $\mathscr{L}_t^{(n)} = (R^f_{t,t+n})^{-1}\,\mathbb{E}_t^*[R_{t,t+n}r_{t,t+n}]$, where $\mathbb{E}_t^*$ denotes the risk-neutral expectation. Note that the relevant risk-free rate is the rate from t to $t+n$.

expectations are intertemporally consistent (i.e., whether $\mathbb{E}\left[\varepsilon_{t+n}^{(m)}\right]=0$ and $\mathbb{E}\left[Z_t\,\varepsilon_{t+n}^{(m)}\right]=0$ for Z_t observable as of time t). And this result does not in fact require the existence of a representative agent: the LVIX-based estimates reflect the expectations of any unconstrained log investor who is content to hold the market portfolio. These expectations can, for instance, be thought of as those of Mr. Market in the heterogeneous-agent log utility model of Martin and Papadimitriou (2022).

If we go beyond log utility, estimates of spot and forward rates are contaminated by the covariance terms in (4). When considering spot rates by themselves, it may be reasonable to assume that $\mathcal{C}_t^{(n)}=\mathrm{cov}_t(M_{t,t+n}R_{t,t+n},r_{t,t+n})\leq 0$, so that the LVIX provides a lower bound for $\mu_t^{(n)}$; this is the tack taken by Gao and Martin (2021). But when considering forward rates, $f_t^{(n,m)}=\mathscr{L}_t^{(n+m)}-\mathscr{L}_t^{(n)}-(\mathcal{C}_t^{(n+m)}-\mathcal{C}_t^{(n)})$, it is unclear whether $\mathcal{C}_t^{(n+m)}\leq\mathcal{C}_t^{(n)}$ or vice versa.

Our main innovation, however, is to consider *forecast errors*, for which we show that the unobserved covariance terms largely cancel in expectation. We present two versions of this result. First, to continue building intuition, Proposition 2 considers identification in a log-normal world given general $M_{t,t+n}$. Proposition 3 then provides a fully general identification result, which shows that the main insights from Proposition 2 carry through.

Before presenting these results, we define our empirical proxy for forecast errors:

$$\widehat{\varepsilon}_{t+n}^{(m)}=\mathscr{L}_{t+n}^{(m)}-\mathscr{L}_t^{(n+m)}+\mathscr{L}_t^{(n)},$$

which are the forecast errors one would obtain under the log-utility assumption. To streamline notation, we also define $MR_{t,t+n}=M_{t,t+n}R_{t,t+n}$. We can now show that we can study expected forecast errors up to a single covariance term.

Proposition 2 (Log-Normal Identification). *Assume a general SDF $M_{t,t+n}$, and assume that $M_{t,t+n}$ and $R_{t,t+n}$ are jointly log-normal. Then the expected value of the forecast-error proxy satisfies*

$$\mathbb{E}_t\left[\widehat{\varepsilon}_{t+n}^{(m)}\right]=\mathbb{E}_t\left[\varepsilon_{t+n}^{(m)}\right]-\mathrm{cov}_t(MR_{t,t+n},\mathbb{E}_{t+n}[r_{t+n,t+n+m}])\,, \tag{5}$$

where $\varepsilon_{t+n}^{(m)}$ is the true forecast error.

Proposition 2 shows that in a log-normal world, our forecast-error estimate is equal in expectation to the true forecast error minus an unobserved covariance term related to the pricing of shocks to expected returns (or discount-rate risk). Note that when considering spot rates, the risk adjustment term $\mathcal{C}_t^{(n)}=\mathrm{cov}_t(M_{t,t+n}R_{t,t+n},r_{t,t+n})$ depends on a covariance with *realized* returns $r_{t,t+n}$. By contrast, when considering forecast errors, the relevant risk adjustment depends on a covariance with *expected* returns $\mathbb{E}_{t+n}[r_{t+n,t+n+m}]$. By replacing (highly volatile) realized returns with (much less volatile) expected returns, the covariance in (5) is likely to be substantially smaller than the covariance in (4). We discuss how one might quantify this covariance in greater detail in Section 4.1.5.

The following proposition shows that the above intuition carries through to more general non-log-normal settings, in which case the expected return $\mathbb{E}_{t+n}[r_{t+n,t+n+m}]$ is replaced by a closely related LVIX-based proxy.

Proposition 3 (Generalized Identification). *For any SDF $M_{t,t+n}$ and any data-generating process for which the relevant expectations exist, the expected value of the forecast-error proxy satisfies*

$$\mathbb{E}_t\left[\widehat{\varepsilon}_{t+n}^{(m)}\right] = \mathbb{E}_t\left[\varepsilon_{t+n}^{(m)}\right] - \mathrm{cov}_t\left(MR_{t,t+n}, \mathscr{L}_{t+n}^{(m)}\right), \tag{6}$$

where $\varepsilon_{t+n}^{(m)}$ is the true forecast error.

The appearance of the LVIX $\mathscr{L}_{t+n}^{(m)}$ in the risk adjustment term in (6) has one benefit relative to (5): the LVIX is directly observable in the data. We can thus quantify the degree to which $\mathscr{L}_{t+n}^{(m)}$ is less volatile than the realized return $r_{t,t+n}$ in (4). In particular, under our main specification (discussed further in Section 3.1.1 below), we find that the unconditional volatility of $\mathscr{L}_{t+n}^{(m)}$ is one-tenth the volatility of the realized market return in our sample. As such, our empirical estimate of forecast errors is likely to be useful even for SDFs different from $M_{t,t+n} = 1/R_{t,t+n}$.

The above analysis pertains to expected returns. We also consider expected excess returns (risk premia) net of the n-period risk-free rate; for this case, as discussed in Section 2.1, we assume throughout that the expectations hypothesis holds, so that forward rates for risk premia correspond to expected future risk premia.

We also note in passing that expected log returns $\mathbb{E}_t[r_{t,t+n}]$ are conceptually distinct from log expected returns $\ln \mathbb{E}_t[R_{t,t+n}]$ (with the difference depending on higher moments of the return distribution). Expectations of the former are more relevant for prices, as prices depend on geometric-average expected returns (as can be seen in a Campbell-Shiller decomposition). This motivates our focus on expected log returns, but this distinction should be kept in mind in interpreting our results. We return to this issue in the subsection just below.

2.3 Identification from Surveys

We can measure spot rates, forward rates, and forecast errors using surveys that elicit return expectations at multiple horizons. While measurement of these objects in survey data is more straightforward than in options data, the exercise is still not fully model-free. First, at a basic level, the interpretation of forecast errors as stemming from intertemporally inconsistent expectations requires that the set of survey respondents stays constant over time (or that the sample is representative period by period).

Second, and more substantively, the elicited survey expectations allow for measurement of expected returns $\mathbb{E}_t[R_{t,t+n}]$ in non-log terms. As above, our interest is largely in expected log returns.[10] But as we do not observe these expected log returns directly in the survey data, we measure spot rates here as $\ln \mathbb{E}_t[R_{t,t+n}]$. This differs from the true value $\mathbb{E}_t[r_{t,t+n}]$ by

$$\ln \mathbb{E}_t[R_{t,t+n}] - \mathbb{E}_t[r_{t,t+n}] = \sum_{n=2}^{\infty} \frac{\kappa_n}{n!},$$

where κ_n is the n^{th} cumulant of the distribution for the log return $r_{t,t+n}$ (so κ_2 is the variance, κ_3 is skewness, and so on). By working with $\ln \mathbb{E}_t[R_{t,t+n}]$ rather than $\mathbb{E}_t[r_{t,t+n}]$ here, we are accordingly assuming that these higher-order cumulants are fixed over time when estimating forecast errors. If this is not the case, then our forecast errors can be thought of as being relevant for expectations of simple returns. These are of separate interest in their own right, but not exactly equivalent to the values measured using options.

3 Data and Implementation

This section describes the option and survey data and discusses implementation of the option-based and survey-based measures of expectations.

3.1 Option-Based Measures

3.1.1 Options Data

We obtain data on option prices largely from OptionMetrics. The data consists of European-exercise put and call options on major stock market indexes around the world. We have options on a total of 20 indexes from at least 15 countries, of which we select the 10 largest and most liquid indexes for the main panel analysis, as shown in Appendix Table A1. We also separately focus on the S&P 500, which is the longest available sample. For the S&P 500, the sample starts in 1990. For the global sample, the sample starts between 2002 and 2006. In both cases, the sample runs through 2021. We use end-of-month prices and apply standard filters. Details on the data, sample selection, and filters are relegated to Appendix B.

[10] This reflects both a conceptual desire to understand variation in prices, and a desire for this analysis to remain consistent and comparable with the option-based analysis.

For risk-free rates in this analysis, we measure expected returns relative to currency-matched and maturity-matched LIBOR rates, as provided by OptionMetrics.

3.1.2 Measuring the LVIX

We measure $\mathscr{L}_t^{(n)}$ using standard results from Breeden and Litzenberger (1978) and Carr and Madan (1998), following Gao and Martin (2021). We make the simplifying assumption that the ex dividend payment at time $t+n$ is known ex ante at time t.[11] We denote the relevant index price at time t as P_t and the corresponding time-t forward price at time $t+n$ as $F_t^{(n)}$. We denote the time-t prices of put and call options on P_{t+n} with strike K as $\text{put}_t^{(n)}(K)$ and $\text{call}_t^{(n)}(K)$, respectively. With these definitions,

$$\mathscr{L}_t^{(n)} = \frac{1}{P_t}\left[\int_0^{F_t^{(n)}} \frac{\text{put}_t^{(n)}(K)}{K}dK + \int_{F_t^{(n)}}^{\infty} \frac{\text{call}_t^{(n)}(K)}{K}dK\right] + r_{t,t+n}^f, \tag{7}$$

which means that the LVIX is a weighted sum of option prices on the relevant index (as well as a function of the current and forward price of the index).

3.1.3 Implementation for Option-Based Measures

We run our baseline analysis at the 6-month horizon ($n = 6$); when considering forward rates, our baseline also uses forward maturities of $m = 6$ months. Our choice of horizon reflects a tradeoff: longer horizons are more economically meaningful, but long-maturity options are generally less liquid. Longer-dated options with multi-year horizons do, however, trade on a relatively liquid basis for the Euro Stoxx 50; we make use of these options when estimating the term-structure dynamics of forecast errors. We also provide a robustness analysis at alternative horizons. The 6-month horizon also allows for useful comparison with the survey-based measures, as one of the surveys (the Livingston survey) allows us to construct spot and forward rates only for $n = m = 6$. We annualize all returns in the empirical analysis.

We do not observe options for all positive strike values, as is needed in (7). We instead truncate the integral after extrapolating well past the range of observable contracts, as is standard practice in calculating option-implied moments. Details are in Appendix B. This

[11] More generally, our option analysis can be thought of as focusing on expected capital gains, rather than expected total returns. This follows much of the literature (e.g., Adam, Marcet, and Beutel 2017, Martin 2017), and we think dividends are unlikely to meaningfully affect the analysis: while expected dividend growth is time-varying, the contribution of dividends to short-horizon expected returns depends largely on the dividend yield. The CFO and Vanguard surveys analyzed below do consider total returns.

approximation is the key issue we face in our measurement. We examine the approximation, along with a number of other assumptions with respect to measurement, in Appendix C.

3.2 Survey-Based Measures

Our analysis uses data from three surveys: the Livingston survey of professional forecasters, the Duke–Fed CFO survey, and the Vanguard retail survey.

3.2.1 Livingston Survey: Data and Implementation

We obtain data on the Livingston survey of professional forecasters from the Philadelphia Fed. Forecasts for the S&P 500 are available twice annually (in June and December), and the sample runs from June 1992 to December 2021. The survey asks respondents about the expected level of the S&P 500 in 6 months and 12 months, as well as the value at the end of the zero month.[12] We use median responses in all cases. We calculate the 6-month spot rate as the annualized log expected price change from month 0 to month 6 (without accounting for dividends), and we similarly calculate the 6-month, 6-month forward rate as the annualized log expected price change from month 6 to month 12.[13] This horizon accordingly aligns with the main horizon considered for the options data. We compare this forward rate to the realized spot rate as of the following survey to calculate forecast errors.

We also re-conduct this analysis using forecasts of equity risk premia rather than expected returns. To do so, we use the median Livingston survey forecasts of the 3-month T-bill rate, as forecasts of the 6-month rate are not elicited. For the spot rate, we subtract the annualized log T-bill rate as of month 0. For the forward rate, we subtract the annualized forecasted log T-bill rate for the 6-month forecast horizon. We then calculate forecast errors as above.

3.2.2 CFO Survey: Data and Implementation

We obtain data on the Duke–Fed CFO survey from the Federal Reserve Bank of Richmond. The sample is quarterly and runs from December 2001 to July 2025.[14] We use mean responses as median responses are not available until 2020. The survey asks respondents about expected

[12]For example, for the December 2018 survey, respondents are asked to provide their forecasts for the S&P 500 value as of the end of December 2018, the end of June 2019, and the end of December 2019. S&P forecasts are available beginning in 1990, but the zero-month responses are available only beginning in 1992.

[13]We ignore dividends in order to align this analysis with the option-based estimates. This is again unlikely to affect our conclusions given the relatively short horizon. Note that we also assume here that using the ratio of forecasted prices is valid to calculate expectations of the 6-to-12-month return.

[14]There are five quarters with missing forecast errors because the survey did not solicit return expectations once in 2019 and twice in 2020. We use the full available CFO sample in order to estimate the post-2020 effect (discussed just below) precisely. Similarly, for the Vanguard data (see Section 3.2.3), we use the full available sample given the short data span.

total annualized returns on the S&P 500 at the 1-year and 10-year horizons. With these forecasts, we calculate the 1-year, 9-year forward rate, the current expectation of the 9-year expected return in one year. We compare this forward rate to the realized 10-year spot rate in one year, both in annualized terms, to calculate forecast errors. We do not observe the realized 9-year spot rate, but it is likely close to the 10-year spot rate on an annualized basis.

We also re-conduct the analysis for risk premia. To do so, we measure spot risk premia relative to maturity-matched Treasury yields obtained from Liu and Wu (2021) via Cynthia Wu's website. We then proceed as above.

In using the CFO data, we also account explicitly for a change in the survey administration that took place in 2020. Prior to 2020, the survey was administered by Duke University; in 2020, it transitioned to being jointly administered by Duke and the Federal Reserve Banks of Richmond and Atlanta (Graham et al. 2020). The sample composition, survey design, and elicitation of return expectations changed at that date, which we discuss further in Appendix B.3. These changes generated a discrete upward level shift in the CFO forward rates, as can be seen in Appendix Figure A4. To avoid contamination from this break-induced variation in CFO forward rates and remove the level shift, all CFO survey regressions include an indicator variable equal to one for observations from 2020 onward. Figures with CFO forward rates also plot the data net of the post-2020 indicator effect.

3.2.3 Vanguard Survey: Data and Implementation

We also obtain data on the Vanguard survey of retail investors, introduced by Giglio et al. (2020) and extended in Giglio et al. (2021). The survey is conducted every two months, with the sample running from February 2017 to December 2024.[15] The survey asks roughly 2,000 retail investors about their expected total annualized returns on the U.S. stock market at the 1-year and 10-year horizons, and we observe mean forecasts. With these forecasts, we then calculate the 1-year, 9-year forward rate — the current expectation of the 9-year expected return in one year — in exactly the same way as for the CFO survey.

Forecast errors and risk-premium measures also follow an identical construction as for the CFO data. For forecast errors, we again compare the ex ante forward rate to the realized 10-year spot rate in one year, both in annualized terms. For risk premia, we subtract the relevant Treasury yields from Liu and Wu (2021) and then recalculate forward rates and forecast errors as before.

[15] We obtain the post-2020 data from Vanguard (2025), whose pre-2020 data matches that of Giglio et al. (2021). The data contain one unscheduled survey in March 2020 in addition to the usual bimonthly surveys. We drop this observation from our regression analyses, as its irregular spacing makes forecast-error calculations difficult. Given the large increase in forward rates in March 2020, this makes our results more conservative.

4 Empirical Results

This section studies spot rates, forward rates, and forecast errors for option-based and survey-based measures of expectations. Across the data sources, we find that forward rates are excessively countercyclical relative to realized spot rates.

4.1 Empirical Results from Option-Based Measures

After measuring the LVIX, we construct empirical proxies for spot rates, forward rates, and forecast errors using the expressions provided in Proposition 1. As in that proposition, these proxies are exact under log utility ($M_{t,t+n} = 1/R_{t,t+n}$), which provides a useful benchmark for interpreting the following results. We begin by considering the relationship between spot and forward rates. We then turn to forecast errors; given the results in Propositions 2–3, these forecast-error estimates are less heavily reliant on the assumption of log utility. We end with a discussion of the potential role of the risk adjustment term for our results.

Throughout the majority of this subsection, we study the behavior of both forward risk premia and forward expected returns. As discussed in Section 2, for forward risk premia to reflect expected spot premia, the expectations hypothesis must hold for the bond term structure, an assumption we do not need to make for forward expected returns. However, forward risk premia have the advantage that they are stationary in our sample, whereas the decline in risk-free rates induces a trend in forward expected returns. Many of the analyses with risk premia are accordingly somewhat better behaved. For time-series plots of spot and forward rates, we consider risk premia in order to focus on the relevant cyclical variation rather than the trend in interest rates. This focus aligns with much of the past literature on subjective expectations, though we again also consider expected returns in our main tables.

4.1.1 Spot and Forward Rates: Descriptive Statistics and Figures

We begin with an overview of our estimated spot and forward rates. Table A2 in the Appendix provides summary statistics by exchange. For a brief intuitive description of the dynamics of spots and forwards, we focus here on descriptive figures for the U.S. (S&P 500) sample. The top panel of Figure 2 shows contemporaneous time-series estimates for 6-month spot rates and 6×6-month forward rates for risk premia. Consistent with Figure 1 in the introduction, spot and forward rates increase significantly in crises. The slope of the term structure of expected risk premia is again procyclical, with forward rates increasing less than one-for-one with contemporaneous spot rates in bad times. The "contemporaneous" qualifier for spot rates is important here, as will be seen below.

The bottom panel of Figure 2 instead compares forward rates with ex post realized spot rates. (The 6×6 forward rate is a predicted value using a shorter-horizon forward as an instrument, as discussed further below.) The gap between the realized spot rate and the forward rate represents the forecast error. Again, as in Figure 1, forecast errors tend to be negative after crises, when spot rates decline from their crisis peaks. By contrast, forecast errors are frequently (though not always) positive outside of crises. So while the term structure of expected risk premia is procyclical, forecast errors are countercyclical.

To formalize this visual analysis, we move now to a set of regression analyses.

4.1.2 Forward Rates as Predictors of Future Spot Rates

We first consider Mincer-Zarnowitz (1969) regressions of realized spot rates on forward rates:

$$\mu_{i,t+6}^{(6)} = \beta_0 + \beta_1 f_{i,t}^{(6,6)} + e_{i,t+6}, \tag{8}$$

where i now indexes the exchange. We consider both panel regressions and U.S.-specific time-series regressions, and in both cases we conduct overlapping monthly regressions for $n = m = 6$ months, following Section 3.1. Standard errors are clustered by exchange and date in the panel case; in the time-series case, we use heteroskedasticity- and autocorrelation-robust Newey-West standard errors, with lags selected following Lazarus et al. (2018), and fixed-b critical values.

Table 2 presents the regression results. The first three columns consider spot and forward rates for risk premia (i.e., $\tilde{\mu}$ and $\tilde{f}$), while the last three columns consider spot and forward rates for expected returns (i.e., μ and f). The relevant null for forecast predictability features $\beta_0 = 0$, $\beta_1 = 1$; we consider these two restrictions separately as well as jointly. For risk premia, the null holds under rational expectations if $M_{t,t+n} = 1/R_{t,t+n}$ and the expectations hypothesis holds for bonds. For expected returns, it holds under rational expectations if $M_{t,t+n} = 1/R_{t,t+n}$.

Column (1) shows results for risk premia from the main panel, as defined in Section 3.1.1, without any fixed effects. The slope coefficient is around 0.64, and the intercept is statistically significant at 1.04 annualized percentage points. We can therefore reject the null of $\beta_0 = 0$, $\beta_1 = 1$ with substantial confidence for this specification. The fact that $\widehat{\beta}_1 < 1$ suggests that forward rates tend to overshoot future spot rates in the data. Thus, even though forward rates move less than one-for-one with *contemporaneous* spot rates as in Figure 2, they move *more* than one-for-one with *future* spot rates. But in spite of this overshooting of forward rates, the R^2 for this regression is around 0.2, so forward rates do predict a substantial portion (one-fifth) of the variation in the equity premium ex ante.

The next two columns consider different specifications and samples. Column (2) includes an exchange fixed effect; this is in fact our preferred specification for estimating slope coefficients in the main panel, as it cleanly identifies within-exchange (or within-country) time-series predictability. The slope coefficient in this case is slightly further below 1 than in column (1), and the within-exchange R^2 is slightly below 0.2. Column (3) reports similar results for the S&P 500 in isolation.

In columns (4)–(6) we report results of similar set of regression for expected returns, as opposed to risk premia. These regressions suggest a higher degree of predictability in spot rates, but this increased predictability is driven by the persistent variation in the risk-free rate: in the early parts of the sample, risk-free rates are substantially higher than in the later part of the sample. Since the spot and forward rates embed this variation in the risk-free rate, the slope coefficients and the R^2 go up substantially.[16] We note that we cannot reject $\beta_0 = 0$, $\beta_1 = 1$ at the 5% level, but this failure to reject does not imply that the expectations do not overreact, as will be examined in subsequent regressions.

One might be concerned with attenuation bias for the slope coefficients in Table 2 given possible measurement error in forward rates, as forward rates are estimated from approximations of the integral in (7). We now address this concern by instrumenting variation in the forward rate with another forward rate. This IV specification will account for measurement error idiosyncratic to a given forward rate, though not for measurement error in the overall forward curve. We use short-maturity forward rates as instruments, as these are likely to be better measured given that options are more liquid at shorter maturities. In particular, for regression (8), we use the 2×1-month forward rate $f_{i,t}^{(2,1)}$ as an instrument for the 6×6 rate $f_{i,t}^{(6,6)}$.

Table 3 presents the resultant two-stage least squares results.[17] For the risk-premium regressions in column (1) to (3), the estimated slope coefficients increase slightly and are now quite consistent across specifications at around 0.7–0.8. We continue to reject the null of $\beta_0 = 0$, $\beta_1 = 1$. For the expected-return regressions in column (4) to (6), the results hardly change; this is again consistent with the dominant impact of the secular trend in interest rates in the estimation of slope and intercept in these regressions.

Taken together, we find that while forward rates have substantial predictive power over future spot rates, they tend to overshoot those spot rates. This is particularly true for

[16] Appendix Table A5 examines split-sample regressions in the U.S. sample. Both the regression slope and R^2 are substantially lower in the later subsample, suggesting that the reported results are driven by persistent risk-free rate variation. While such secular variation in interest rates likely affected equity valuations (as examined by Gormsen and Lazarus 2025), our focus is instead on higher-frequency variation in forward expected returns; we accordingly prefer specifications netting out the risk-free rate.

[17] The first stage in these regressions (not shown) has an R^2 close to 70% when excluding all fixed effects, suggesting the 2×1 forward rate is a strong instrument for the 6×6 rate.

forward risk premia: when forward rates for risk premia increase by 1% relative to their within-exchange mean, this corresponds on average to an increase in future spot rates of only about 0.6%.

4.1.3 The Insignificance of Average Forecast Errors

We next turn our attention to forecast errors, which can be estimated under weaker assumptions than spot and forward rates themselves (see Propositions 2 and 3). We first consider unconditional averages of forecast errors, $\overline{\varepsilon}^{(6)}_{i,t+6}$, across different samples, in Table 4. For risk premia (columns (1) and (2)), we find average forecast errors that are statistically insignificant and effectively zero: $\overline{\varepsilon}^{(6)}_{i,t+6}$ is below 20 basis points on an annualized basis. Realized spot rates for risk premia have thus been very slightly (and insignificantly) higher than forward rates on average. This rough magnitude applies to different exchanges, though the U.S. average in column (2) is even lower, at roughly 2 basis points.

For expected returns (columns (3) and (4)), we find average forecast errors that are slightly negative and statistically significant. Realized spot rates for expected returns have thus been slightly lower than forward rates on average. This finding is likely driven by an unexpectedly large decline in interest rates over the sample. Alternatively, it could be driven by a constant term premium on interest rates that causes the pure expectations hypothesis to fail. Despite the statistical significance of these estimates, we note that the averages are still close to zero.

While forecast errors are close to zero unconditionally, this average masks substantial predictability over time, which we turn to now.

4.1.4 The Predictability of Forecast Errors

Figure 3 plots realized forecast errors along with forward rates and indications of prominent crises in the U.S. sample. The figure further illustrates the takeaway from Figure 1 in the introduction: in years following crises, forecast errors tend to be significantly negative, which is to say future realized spot rates are lower than expected ex ante. To the extent that these forecast error are systematic — that is, they occur systematically following crises — the errors represent what we refer to as excess cyclicality in forward rates: forward rates are excessively elevated during crises, relative to subsequent realized spot rates. In this section, we study whether the forecast errors are indeed systematic through a set of predictive regressions.

For a formal consideration of forecast-error predictability, we begin by testing whether forward rates predict forecast errors in regressions of the following form:

$$\tilde{\varepsilon}^{(6)}_{i,t+6} = \beta_0 + \beta_1 \tilde{f}^{(2,1)}_{i,t} + e_{i,t+6}. \tag{9}$$

We use the 2×1 forward rate $f_{i,t}^{(2,1)}$ here instead of the 6×6 rate, as we would otherwise have the same forward rate on both sides of the regression, leading to a mechanical upward bias in the slope coefficient in the presence of measurement error.[18] All other aspects of the regressions are identical to those in Table 2.

The first three columns of Table 5 report the results of equation (9). In the main panel without any fixed effects (column (1)), the slope coefficient on the forward rate is -0.13 and is significant at the 5% level.[19] In our preferred panel specification including exchange fixed effects (column (2)), the slope coefficient is now significant at the 1% level. Results are similar in the U.S.-only sample, although power is naturally slightly lower.

We next consider the forecast errors for expected returns (ε) on the left-hand side. When doing so, we continue to use the forward rate for risk premia as the predictor variable; forward rates for expected returns themselves have a strong downward trend due to the risk-free rate component, so using these forward rates as predictor variables would amount to asking if forecast errors are trending over the sample (e.g., positive in the early sample and negative in the later sample). By instead using the more transitory forward rate for risk premia ($\tilde{f}$) as the predictor variable, the predictor is more directly related to the state of the economy and better suited for capturing the type of crisis-dependent errors plotted in Figures 1 and 3. These regressions (column (4) to (6) of Table 5) thus have transitory components on both the left- and right-hand side, and they have the benefit that the theoretical predictions for the forecast errors on the left-hand side do not require assumptions about the term structure of interest rates (they require only the assumptions laid out in Propositions 2 and 3).

As can be seen in Table 5, the forecast errors for expected returns are highly predictable. In the main panel, the slope coefficients in columns (4)–(5) roughly double relative to the regressions in columns (1)–(2). The R^2 increases to around 10% with exchange fixed effects. For the U.S.-only regressions, the difference between risk premia and expected returns appears more modest. Overall, the results suggests that the patterns observed in columns (1) through (3) are not driven by the risk-free rate component (i.e., a failure of the expectations hypothesis for interest rates); if anything, considering the total expected return strengthens the results in this case. And in all cases, forecast errors are robustly predictable from the level of forward rates. This test for forecast-error predictability is evidently more powerful against a rational null than the Mincer-Zarnowitz regressions presented above.

One might be concerned that forecast-error predictability is the artifact of measurement

[18]Moreover, with the same conditioning variable, the difference between the slope in Mincer-Zarnowitz and error-predictability regressions would be one by construction.

[19]The magnitudes presented in the table are not directly comparable to those in Tables 2–3 given the use of $f_{i,t}^{(2,1)}$ as the predictor. Instead, regression (9) can be viewed as an analogue to the reduced form of the IV specification for (8) presented in Table 3.

error. As discussed in Section 3.1.3, we must approximate the integral in (7). For this approximation to have bite, we would have to *overestimate* forward rates in bad times. This upward bias would mechanically generate predictably negative forecast errors. However, our intuition is that we *underestimate* forward rates, especially in bad times, as longer-maturity options are generally less liquid.

In further tests, we find support for this intuition; see Appendix C. We find, for example, that forecast errors are less predictable in more illiquid settings, such as less-developed options markets or in the early parts of the U.S. sample. Although we of course cannot fully rule out all alternative liquidity-based explanations, it seems unlikely that forecast-error predictability is the spurious result of measurement error. We conduct further analysis in Section 4.3 below, finding similar results.

We conclude that the pattern of excess cyclicality observed in Figures 1 and 3 is systematic and statistically significant. Forward rates have exhibited excess cyclicality in this sample, in the sense that investors have believed that spot rates would be more elevated following crises than would ex post have been rational. This conclusion does require that the risk-premium adjustment in Propositions 2 and 3 is small, so that the options provide useful estimates of physical expectations of future spot rates. Our main source of confirmation for this interpretation is in our analysis of survey data, considered below. Before turning to the survey data, though, we consider the behavior of the risk-premium term in more detail.

4.1.5 Risk-Based Explanations for the Option-Based Results

We now ask what would be required in order for option-based forecast errors to be rationalized by the behavior of the stochastic discount factor alone. Our main conclusion is that we find the required behavior of the SDF difficult to reconcile with standard models. For readers more interested in corroboration of the above patterns in our survey data (which cannot be rationalized through these risk-premium terms), one can skip this self-contained theoretical discussion and resume the remainder of the paper in Section 4.2 without issue.

Overview. When going beyond log utility, our estimates of spot and forward rates partly reflect risk premium adjustments. While most of these cancel out in forecast errors, a risk premium term remains: from Propositions 2–3, expected forecast errors are

$$\mathbb{E}_t\left[\widehat{\varepsilon}^{(m)}_{t+n}\right] = \mathbb{E}_t\left[\varepsilon^{(m)}_{t+n}\right] - \varsigma_t, \quad \text{where } \varsigma_t = \begin{cases} \operatorname{cov}_t\big(MR_{t,t+n}, \mathbb{E}_{t+n}\, r_{t+n+m}\big) & (\textit{log-normal case}), \\ \operatorname{cov}_t\big(MR_{t,t+n}, \mathscr{L}^{(m)}_{t+n}\big) & (\textit{general case}). \end{cases}$$

To simplify analysis, we maintain focus on the $n = m = 1$ case (where one period is 6 months) and suppress the dependence of ς_t on n and m.

As a starting point for the potential role of risk premia, Figure 4 plots the predicted values of the risk-premium forecast errors from the regressions in Table 5. These are empirical estimates of $\mathbb{E}_t\left[\widehat{\varepsilon}^{(1)}_{t+1}\right]$, and $-\varsigma_t$ must take on these values in order to rationalize our results through risk premia alone (so that true forecast errors satisfy $\mathbb{E}_t[\varepsilon^{(1)}_{t+1}] = 0$). The main challenge is that the covariance term $-\varsigma_t$ must flip sign over time: the covariance term must be significantly negative in bad times, which means it must be positive in good times to remain close to zero on average. We show below that this requires the price of discount rate risk to be highly volatile — close to zero during good times and high during bad times.[20]

Examining the Risk Premium Term. To better understand the properties required of ς_t in order to match Figure 4, it is useful to formalize its relation to a discount-rate risk premium. As in Proposition 2, assume for now that the SDF and returns are jointly log-normal. Then by Stein's lemma and the fact that $\mathbb{E}_t[MR_{t+1}] = 1$, we have that

$$\text{cov}_t(MR_{t,t+1}, \mathbb{E}_{t+1}\, r_{t+2}) = \text{cov}_t(m_{t+1} + r_{t+1}, \mathbb{E}_{t+1}\, r_{t+2}). \tag{10}$$

For ς_t to flip sign, the covariance in (10) must thus flip sign. To see how this covariance is tied to discount-rate risk, define $P_{\mathbb{E},t}$ as the price of the claim that pays $-\mathbb{E}_{t+1}\, r_{t+2}$ next period, and define its ex ante Sharpe ratio (in log-return terms) as $SR_{\mathbb{E},t}$.[21] This Sharpe ratio captures the price of risk for exposure to increases in expected returns (or discount rates).

Rewriting (10), the sign of ς_t is given by

$$\text{Sign}\,(\varsigma_t) = \text{Sign}\Big(SR_{\mathbb{E},t} + \rho_t(r, \mathbb{E}_{t+1}\, r)\sigma_t(r_{t+1})\Big),$$

where $\rho_t(r, \mathbb{E}_{t+1}\, r) = \text{Corr}_t(r_{t+1}, \mathbb{E}_{t+1}\, r_{t+2})$. This correlation is likely negative, as low realized returns are associated with higher expected returns going forward; for simplicity, start by assuming that $\rho_t(r, \mathbb{E}_{t+1}\, r) = -1\ \forall\, t$.[22] In this case, the above expression shows that $SR_{\mathbb{E},t}$ must be higher than $\sigma_t(r)$ for ς_t to be positive, and lower than $\sigma_t(r)$ for ς_t to be negative. Since ς_t must be positive in bad times and negative in good times, $SR_{\mathbb{E},t}$ must vary more

[20] As we will show below, it is possible for standard models to generate a ς_t that is sufficiently volatile, but if ς_t does not change sign, the average of $\widehat{\varepsilon}$ will be far from zero, which is inconsistent with the average of $\widehat{\varepsilon}$ being close to zero in the data.

[21] That is, $SR_{\mathbb{E},t} = (\mathbb{E}_t[r_{\mathbb{E},t+1}] - r^f_{t,t+1} + \sigma^2_{\mathbb{E},t}/2)/\sigma_{\mathbb{E},t}$, where $r_{\mathbb{E},t+1}$ is the claim's log return and $\sigma_{\mathbb{E},t}$ is its standard deviation.

[22] This is an extreme case in which all movements in realized returns reflect variation in discount rates (Campbell and Ammer 1993).

than $\sigma_t(r)$ between good and bad times.

As a benchmark, Appendix Figure A7 plots the time variation in $\sigma_t(r)$ at the 6-month horizon from the standpoint of an investor with log utility.[23] The figure shows that volatility varies from close to 15% in good times to more than 50% in bad times. As such, the price of discount-rate risk must vary significantly and countercyclically to generate the needed variation in ς_t.

Moreover, in the case that $\rho_t(r, \mathbb{E}_{t+1} r) = -1$, then the value $-\varsigma_t$ is a scaled version of $\mathrm{cov}_t(MR_{t,t+1}, r_{t+1})$. Under the *modified negative correlation condition* (mNCC) used throughout Gao and Martin (2021), this covariance is always negative, so ς_t cannot flip sign. This illustrates that the degree of countercyclical variation in discount-rate risk prices required under $\rho_t(r, \mathbb{E}_{t+1} r) = -1$ is quite restrictive, and it is ruled out by a range of standard models considered by Gao and Martin.

In practice, however, we expect $\rho_t(r, \mathbb{E}_{t+1} r)$ to be higher than -1, as not all changes in stock returns are driven by discount rates. A $\rho_t(r, \mathbb{E}_{t+1} r) > -1$ implies a less volatile price of discount-rate risk than just discussed. However, it also implies that the price of discount-rate risk must be even lower during good times than above; for example, in the opposing extreme in which $\rho_t(r, \mathbb{E}_{t+1} r) = 0$ (i.e., with i.i.d. returns), the Sharpe ratio on the discount-rate claim must be *negative* in good times in order for ς_t to flip sign.

The discussion above can also be generalized beyond the log-normal case. For example, assume that the SDF can be written as

$$M_{t,t+1} = \beta \frac{V_W(W_{t+1}, z_{t+1})}{V_W(W_t, z_t)},$$

where V_W is an unconstrained investor's marginal utility of wealth, z_t is a state vector that includes $-\mathscr{L}^{(1)}_{t+1}$, and V_W is weakly decreasing in each entry of z_t. If this agent is fully invested in the market, has relative risk aversion $-WV_{WW}/V_W$ of at least 1, and R_{t+1} and the entries of z_{t+1} are *associated* random variables,[24] then Result 4 of Gao and Martin (2021) can be applied to obtain that ς_t again cannot change sign under the benchmark $\rho_t(r, \widehat{\mu}) = -1$.

Further Intuition Under Power Utility. The above emphasizes that the price of discount-rate risk must vary substantially over time to rationalize our results. To better understand this result, we next re-conduct our analysis under various utility functions generating a constant price of risk, and show how such functions will not be able to explain the data. This

[23] This is likely to be a conservative benchmark for the degree of variation in conditional volatility if investors are more risk-averse than implied by log utility.

[24] Formally, the elements of the vector $X_{t+1} = (R_{t+1}, z'_{t+1})'$ are associated random variables if $\mathrm{cov}_t(f(X_{t+1}, g(X_{t+1})) \geq 0$ for all nondecreasing functions f and g for which this covariance exists.

analysis will help provide intuition on the role of risk aversion in our results.

We calculate spot and forward rates using CRRA utility with γ ranging from 0.75 to 3. Details are in Appendix D.1. As shown in Table A9, none of the power utility functions can rationalize the results. The functions with $\gamma > 1$ explain the time variation in the forecast errors, but they cannot explain the average being close to zero. To understand the intuition, consider the results derived above for log-normally distributed variables. When $\gamma > 1$, the covariance term ς_t correctly increases in bad times. However, it is always positive ($\text{cov}_t(m_{t+1} + r_{t+1}, \mathbb{E}_{t+1}\, r_{t+2}) > 0$ at all times), leading to large average forecast errors. The opposite mechanics are at play for $\gamma < 1$, for which $\text{cov}_t(m_{t+1} + r_{t+1}, \mathbb{E}_{t+1}\, r_{t+2}) < 0$ for all t.

This illustrates the challenge in explaining both the average and time variation in the covariance term, and thus in explaining the results from options through this term alone.

4.2 Empirical Results from Survey-Based Measures

We now turn our attention to spot rates, forward rates, and forecast errors estimated using survey expectations. As a preview of the results, Figure 5 plots forward rates for the three surveys, and Figure 7 shows their common variation with the option-based forward rates. As with the option-based measures, we find that (i) forward rates are countercyclical, in that they increase in bad times, and (ii) forecast errors are predictably negative following crises such as the 2008 financial crisis and the 2020 Covid-19 crisis, as will be shown in subsequent analyses. These results are again consistent with excess cyclicality in forward expectations: during bad times, investors think future spot rates returns will be more elevated than their own beliefs justify ex post. We detail this analysis below for each of the surveys in turn.

4.2.1 Spot and Forward Rates from the Livingston Survey

As explained in Section 3.2.1, we use the Livingston survey of professional forecasters to estimate the 6-month spot rate and the 6-month, 6-month forward rate. As with the option-based measures, we consider spot and forward rates for both risk premia and expected returns. Since the Livingston survey offers expectations about interest rates, we can measure forward rates for risk premia without assumptions about the expectations hypothesis for interest rates. The sample runs from 1990 to 2021.

Columns (1) and (2) of Table 6 show the results of Mincer-Zarnowitz regressions of future spot rates on past forward rates. We find slope coefficients of 0.81 for risk premia and 0.68 for expected returns. These estimates echo those from the option-based measure, suggesting that a one-percentage-point increase in forward rates is associated with spot rates that are about 0.75 percentage points higher in the future. The R^2 values are somewhat higher than those from the option-based regressions for risk premia, somewhat lower than those for expected returns; on average, around half of the variation in spot rates can be predicted by the forward rate at the 6 month horizon. Columns (3) and (4) further show that the forecast errors are close to zero on average and statistically insignificant.

Columns (5) and (6) of Table 6 report results from error-predictability regressions of future realized forecast errors based on ex ante forward rates. As for the option-based measures, we find significantly negative slope coefficients around -0.2, suggesting that high forward rates are associated with future spot rates that are lower than expected. The R^2 values are around 0.05, which are, again, higher than those for option-based risk premia, lower than those for expected returns. Given the countercyclical nature of the forward rates, as shown in Figure 7, the pattern is similar to the one obtained from the options: in bad times, investors think future expected returns will be more elevated than investors' own subsequent beliefs justify.

4.2.2 Spot and Forward Rates from the CFO Survey

As explained in Section 3.2.2, we use the Duke–Fed CFO survey to estimate the 1-year, 9-year forward rate, which is the 9-year return expectation one year from now. We again consider forecast errors for both risk premia and expected returns; since we measure risk premia relative to the yield curve, we require that expectations hypothesis holds, as with the option-based measure. The core advantage of the survey is the long horizon: longer-horizon expected returns are more important for prices. The sample runs from 2001 to 2022.

Columns (1) and (2) of Table 7 report the results of Mincer-Zarnowitz regressions of future spot rates on forward rates. Forward rates predict future spot rates with an average slope coefficient around 0.5, below that from the option-based measure and the Livingston survey. The R^2 values are comparable to that in the Livingston survey, but are somewhat different than that for the option-based measure. Columns (3) and (4) report small average forecast errors. Columns (5) and (6) of Table 7 report the results of error-predictability regressions of forecast errors on forward risk premia. For risk-premium forecast errors, we observe very strong predictability, with an R^2 of 0.48. For expected returns, this effect is statistically insignificant. The former is much stronger than that from the option-based measure or the Livingston survey, and the latter slightly weaker. The difference might arise from the longer horizon.

As a final remark on the CFO survey, there is, in this data, a clear distinction between the cyclical behavior of spot and forward rates. As shown in the top panel of Figure 6, spot rates appear at least weakly *procyclical*: short-term subjective risk premia generally decrease slightly in bad times in a manner that appears extrapolative with respect to past returns (as highlighted by Greenwood and Shleifer 2014), in spite of these being times in which risk premia are likely to be objectively high. As seen in Figure 5, however, forward rates are countercyclical, as are long-term spot rates (unplotted), so that long-term subjective risk premia increase in bad times. One interpretation of these findings is that the CFO respondents understand present value logic — they understand that future long-run returns must be high when prices are low — but during crises, they believe prices will continue to decrease for a while before increasing. This finding sheds light on the debate about whether return expectations are procyclical (Greenwood and Shleifer 2014), acyclical (Nagel and Xu 2023), or countercyclical (Dahlquist and Ibert 2024). Interestingly, there is less disagreement about forward rates: across all option-based and survey-based measures, longer-run forward rates are countercyclical, and excessively so. The apparent disagreement in previous literature is thus related to the cyclical behavior of short-term spot rates. While short-term spot rates are important for understanding the expectations-formation process, long-run expected returns and forward rates are often the key object of interest in determining prices.

4.2.3 Spot and Forward Rates from the Vanguard Retail Survey

We now consider the sample of Vanguard retail investor expectations, from which we also calculate 1-year, 9-year forward rates and associated forecast errors as in Section 3.2.3. While the 2017–2024 sample is somewhat shorter than the other data sources, it does overlap with the early-2020 downturn and contains meaningful cyclical variation before and after that. We can therefore conduct most of the same analyses for this survey as for the remaining data sources. This gives us further evidence for long-horizon errors, along the lines of the CFO analysis.

Table 8 presents the results for the Vanguard regressions, laid out identically to the previous survey results in Tables 6–7. Columns (1) and (2) show that forward rates predict future spot rates with an average slope coefficient very similar to that for the CFO survey (around 0.5), albeit with slightly lower R^2 values. In both cases, we reject the joint null of forecast efficiency: forward rates move too much relative to realized spot rates, as with the other data sources. Columns (3) and (4) show that average forecast errors are again economically and statistically insignificant. Columns (5) and (6) show that forecast errors are strongly predictable for risk premia, with a significantly negative slope coefficient of -0.57 and an R^2 of 0.21. This predictability is weaker for expected returns. Overall, the results are quite consistent with those for the other data sources.

Finally, the bottom panel of Figure 6 shows that spot rates for these retail investors appear weakly procyclical over this shorter sample, similar to the CFOs. This again contrasts with the forward rates shown in the bottom panel of Figure 5, which increase meaningfully during the Covid crisis. As noted in the previous subsection, while spot return expectations appear to behave differently across investor groups, there is much less apparent disagreement about forward return expectations: in all cases, forward rates are countercyclical. We explore this common variation, and the robustness of the countercyclicality of forward rates, in the next subsection.

4.3 Joint Results for Options and Surveys

We conclude this section by showing how the results from the different data sources align with each other and with business- and financial-cycle state variables.

4.3.1 Common Variation in Forward Rates

The analysis so far has emphasized that forward rates are countercyclical across data sources. We explore this common variation further by relating the survey-based forward rates to the forward rates obtained from option prices. Figure 7 documents a relatively strong comovement between option-based forward rates and Livingston, CFO, and Vanguard sources. The comovement is most apparent within crises periods, such as the global financial crisis and the Covid crisis, but there is also apparent comovement outside these crises.

One of our main results is that high forward rates are associated with negative forecast errors, as the high forward rates are higher than the subsequent spot rates. If this pattern is driven by a fundamental economic mechanism, we might expect it to work across data sources, such that a high forward rate in a given data source predicts negative forecasts errors in another data source. To test this hypothesis, we run cross-predictability tests in which forward rates measured from the option-based measure are used to forecast realized forecast errors from the survey-based measures. Table 9 shows that option-implied forward rates predict realized forecast errors for both Livingston and CFO surveys.[25] This is true when forward rates are measured in risk premia and in expected returns. The results are significant in all cases except for the CFO-based risk premia, for which the p-value is 12.7%. The relatively high p-value may reflect that in the particular sample relevant for the CFO survey, the predictability of the option-based errors is also diminished, as shown in column (3) of the table.

In addition to emphasizing a common driver, the cross-predictability evidence mitigates concerns about pure measurement error: if one series were merely noisy, it would not systematically forecast the other series' forecast errors. Instead, the results point to a common underlying driver of forward expectations across investor groups. Overall, the visual

[25] Given the shorter Vanguard sample, we omit it from this cross-predictability exercise to make the table more straightforward to read.

comovement in Figure 7 and the cross-predictability in Table 9 indicate that our option- and survey-based forward rates are reflective of the same cyclical component of long-horizon discount rates. We turn to this cyclical variation next.

4.3.2 Cyclical Variation in Forward Rates and Forecast Errors

To this point, our evidence on the countercyclicality of forward rates has largely come from their behavior in crisis episodes. We now turn to a more systematic statistical analysis of their comovement with standard countercyclical indicators. We then consider the cyclical behavior of forecast errors.

As an initial analysis, we consider the comovement between forward rates and three simple measures of financial market conditions that are frequently used in the literature: (i) the earnings yield,[26] (ii) the squared 1-year VIX, and (iii) an NBER recession indicator. We regress forward risk premia, as measured from the four separate data sources (including Vanguard), on each of these three countercyclical state variables separately, with results presented in Figure A5. Across the 12 regressions, the slope coefficients are all positive, indicating countercyclical variation. And 11 of the state variables have t-statistics that are above the standard 5% threshold for a one-sided normal test (t=1.645).

Since the literature has not settled definitely on a single state variable for measuring cyclicality, we conduct an additional analysis considering a wide range of 16 countercyclical state variables. These state variables include valuation ratios and different indices of financial market conditions, and they are listed in the description for Figure A6. That figure shows that across the 64 estimates for forward risk premia (16 estimates for each of the 4 data sources), 58 of the covariances are positive, indicating countercyclicality, and 39 of these estimates are statistically significant. The figure also considers forward expected returns. These results are qualitatively quite similar, although slightly weaker; only 33 of the estimates are significantly countercyclical, and one estimate is significantly procyclical. Overall, we conclude from these analyses that there is strong statistical evidence in favor of the countercyclicality of forward rates for all data sources, buttressing the evidence observed in the time-series plots and during crises.

We next turn to the cyclicality of forecast errors. Since forward rates are countercyclical and predict forecast errors, we should expect forecast errors to exhibit cyclical variation as well. To test this hypothesis, we regress future realized forecast errors on ex ante state variables capturing cyclical conditions. For simplicity, we consider only the CAPE yield

[26]We measure the earnings yield here as the excess CAPE yield, defined as the inverse of the CAPE (price to 10-year earnings ratio) less the 10-year real yield. This follows Shiller, Black, and Jivraj (2020), who advocate the use of the excess CAPE yield when considering the equity premium; it also appears more stationary than the CAPE yield. We obtain the data via Robert Shiller's website.

and the VIX (NBER recessions cannot be used in forecasting regressions, as they are dated ex post), and we standardize both forecast errors and state variables to have unit variance. Table 10 shows the results. As expected, forecast errors are consistently procyclical, with negative point estimates for all 12 combinations of forecast errors and state variables. The procyclical variation is statistically significant at the 10% level for nearly all specifications, and similar in magnitude across measures.

Overall, the results in this section confirm that forward rates are countercyclical, and excessively so: during bad times, forward rates increase more than one-for-one relative to future spot rates, yielding predictably negative forecast errors after crisis spikes. This cyclical behavior is strongly related across investor groups, and their forward rates and forecast errors all share comovement with underlying economic state variables. In the following sections, we examine the asset-pricing implications of these findings.

5 Implications for Asset Pricing

This section studies additional implications of the excess cyclicality in forward rates documented in the previous section.

5.1 A New Source of Excess Volatility

If negative asset-pricing shocks lead to predictably larger increases in forward rates than justified by subsequent realizations, then such shocks may lead to excess volatility in prices. To better understand the relevance of this source of excess volatility, we conduct two illustrative quantifications here. We first quantify the impact of option-based forecast errors on prices; we then conduct a similar exercise using survey-based forecast errors. We find that both explain a sizable share of the observed volatility during crises. Each requires different assumptions but builds off the same overarching theory, which we turn to first.

5.1.1 Defining Excess Volatility from Forecast Errors

Before we begin, we note that we focus on the component of excess volatility coming from expectations about risk premia, as opposed to expected returns.[27] Starting from a Campbell-Shiller (1988) decomposition for the log price-dividend ratio $p_t - d_t$ and using the definition

[27]This leads to smaller estimates.

of forward rates as expected equity premia:

$$\begin{aligned} p_t - d_t &= k + \sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t[\Delta d_{t+j+1}] \; - \sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t[r_{t+j,t+j+1}] \\ &= k + \underbrace{\sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t[\Delta d_{t+j+1}]}_{CF_t} \; - \underbrace{\sum_{j=0}^{\infty} \rho^j \, \tilde{f}_t^{(j,1)}}_{\mathcal{F}_t} - \underbrace{\sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t\left[r^f_{t+j,t+j+1}\right]}_{RF_t}, \end{aligned} \tag{11}$$

where k is a constant and $0 < \rho < 1$.

We further split the forward-rate term $\mathcal{F}_t$ into

$$\underbrace{\sum_{j=0}^{\infty} \rho^j \, \tilde{f}_t^{(j,1)}}_{\mathcal{F}_t} \; = \underbrace{\sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t\left[\tilde{\mu}^{(1)}_{t+j}\right]}_{\mathcal{M}_t} \; - \underbrace{\sum_{j=1}^{\infty} \rho^j \, \mathbb{E}_t\left[\tilde{\varepsilon}^{(1)}_{t+j}\right]}_{\mathcal{E}_t}. \tag{12}$$

Our interest is in the term $\mathcal{E}_t$, which quantifies the component of the log price-dividend ratio attributable to all future predicted forecast errors, or the price impact of forecast errors. Note that the predictable forecast error for $j = 0$ is zero by construction given equation (3), so we have rewritten the sum for $\mathcal{E}_t$ as being from $j = 1$ to ∞. In what follows, we often refer to this $\mathcal{E}_t$ series as *discounted forecast errors*, for short.

The true $\mathcal{E}_t$ is unobservable. The main empirical challenge is that we cannot easily estimate the entire term structure of predicted forecast errors — that is, the the entirety of the infinite sum — without additional assumptions, as we typically only observe relatively short-horizon forecast errors. Instead, we consider two proxies $\widehat{\mathcal{E}_t}$ for the true price impact.

5.1.2 Measuring Discounted Forecast Errors

We measure $\widehat{\mathcal{E}_t}$ using forecast errors from both options and surveys. For options, our approach requires more assumptions but allows us to estimate $\mathcal{E}_t$ in full; that is, our estimate captures the entire infinite sum. For surveys, our approach requires fewer assumptions but provides only a lower bound for $\mathcal{E}_t$; that is, our estimate truncates the infinite sum.

We begin with option-based measures. In the baseline U.S. sample, we estimate forecast errors for horizons only up to one year. To obtain estimates for longer horizons, we estimate the term structure of predictable forecast errors using spot and forward rates out to an 8-year horizon for the Euro Stoxx 50, which is the only exchange in our sample with index options available at such maturities.[28] For each horizon, we predict forecast errors as in (9)

[28]This is similar to the case for the dividend futures market (Binsbergen and Koijen 2017). We note that we use Euro Stoxx 50 data in this exercise only to estimate the term structure behavior of forecast

by regressing realized forecast errors on shorter-horizon forward rates. We then estimate a decay function $\phi^{(n,m)}$ for forecast errors, such that $\mathbb{E}_t[\tilde{\varepsilon}^{(m)}_{t+n}] = \phi^{(n,m)}\,\mathbb{E}_t[\tilde{\varepsilon}^{(1)}_{t+1}]$. Details on the long-horizon options data and on our estimation approach are in Appendix D.2. As reported in Table A10, the decay parameters are close to or greater than 1 at all horizons. We therefore use the assumption that predictable forecast errors have a flat term structure, $\mathbb{E}_t[\tilde{\varepsilon}^{(1)}_{t+j+1}] = \mathbb{E}_t[\tilde{\varepsilon}^{(1)}_{t+j}]$ for all j in (12), to obtain a back-of-the-envelope quantification of $\mathcal{E}_t$ for the U.S. sample:

$$\widehat{\mathcal{E}}^{\text{option}}_t = \frac{\rho}{1-\rho}\,\mathbb{E}_t\left[\tilde{\varepsilon}^{(1)}_{t+1}\right]. \tag{13}$$

The length of one period is arbitrary in (11)–(12), so for this measure we set it to 6 months to correspond to our baseline option horizon.[29] We estimate $\mathbb{E}_t[\tilde{\varepsilon}^{(1)}_{t+1}]$ in (13) using the estimates in column (3) of Table 5.

The forecast-error decay estimates in Table A10 are in many cases well above 1, suggesting that the assumption of a flat term structure of predictable forecast errors may be conservative. On the other hand, since we cannot observe very long-horizon forecast errors, assuming this flat term structure over an infinite horizon may be somewhat restrictive. It also makes $\widehat{\mathcal{E}}^{\text{option}}_t$ dependent on the value of ρ used in (13).

To relax this infinite-horizon assumption and obtain alternative price impact estimates, we turn to the survey data. We focus on the two surveys with long-horizon forward rates, the CFO and Vanguard surveys, and we quantify the price impact implied by predictable forecast errors during the 2008 financial crisis and the 2020 Covid crisis. The CFO survey is only usable for this exercise during the 2008 crisis,[30] and the Vanguard survey is only available during the 2020 crisis.

For these surveys, we quantify $\mathcal{E}_t$ with a slightly different approach. Rather than estimating the decay in predictable forward-rate errors, we exploit that the surveys directly elicit long-horizon expectations (out to 10 years) and use these to compute the contribution of the first 9 years of forecast errors. As long as predictable errors $\mathbb{E}_t[\tilde{\varepsilon}^{(1)}_{t+j}]$ at horizons $j \geq 10$ have the same sign as those for $j \leq 9$, the contribution from the first 9 years then provides a lower bound on $\mathcal{E}_t$:

$$\widehat{\mathcal{E}}^{\text{survey}}_t = \sum_{j=1}^{9} \rho^j\,\mathbb{E}_t\left[\tilde{\varepsilon}^{(1)}_{t+j}\right] \leq \mathcal{E}_t. \tag{14}$$

errors; we do not assume that longer-horizon errors are equal across exchanges on a period-by-period basis. Our assumption is therefore that the term structure dynamics are similar across exchanges on average (as documented in the yield-curve context by Diebold, Li, and Yue 2008, among others).

[29] Since forecast errors are in annualized percent, we measure ρ in annualized terms. (This normalization does not affect $\widehat{\mathcal{E}}^{\text{option}}_t$.) We estimate $\rho = (1+\exp(\overline{d-p}))^{-1}$, where $\overline{d-p}$ is the mean repurchase-adjusted log dividend yield from Nagel and Xu (2022), and obtain $\rho \approx 0.967$.

[30] As discussed in Section 3.2.2, the CFO survey underwent a redesign and break between 2019 and 2020. We therefore drop the 2020 forecast errors to avoid splicing pre-break forecasts with post-break spot rates.

For the surveys, we estimate the 1-year, 9-year predicted error $\mathbb{E}_t[\tilde{\varepsilon}^{(9)}_{t+1}]$ using column (5) of Tables 7–8. For the one-period forward rates in (14), we apply iterated expectations and set $\mathbb{E}_t[\tilde{\varepsilon}^{(9)}_{t+1}] = \mathbb{E}_t[\tilde{\varepsilon}^{(1)}_{t+j}]$, as both are in annualized terms.

5.1.3 Price Impact of Forecast Errors

We then compare variation in our estimates of $\mathcal{E}_t$ to variation in $p_t - d_t$. To account for share repurchases and ensure that $p_t - d_t$ is stationary, we use the repurchase-adjusted price-dividend series provided by Nagel and Xu (2022).

Figure 8 visualizes the results for the option-based estimates, with both discounted forecast errors $\widehat{\mathcal{E}}^{\text{option}}_t$ (red) and the log price-dividend ratio $p_t - d_t$ (blue) shown as log differences from their full-sample means. Unconditionally, the forecast errors explain a modest amount of the overall variation in prices. A regression of forecast errors on the log dividend yield gives a slope coefficient of around 0.1, suggesting that 10% of the unconditional variation in prices can be tied back to predictable forecast errors. Moreover, the two series do not always comove positively. During the Russian debt crisis in the late 1990s, for example, forward rates were quite elevated (leading to negative estimated $\mathcal{E}_t$), but the stock market's dot-com boom largely continued apace (leading to high $p_t - d_t$).

During large stock-market declines (low $p_t - d_t$), however, predictable forecast errors play a significant role in the decomposition in (11)–(12). This holds particularly during the financial crisis and the Covid crisis. Figure 9 zooms in on these episodes. From June 2007 to the February 2009 crisis trough, $\widehat{\mathcal{E}}^{\text{option}}_t$ drops by 36 log points; during the same period, the valuation ratio drops by 42 log points, suggesting that predictable forecast errors can justify a large part of the observed crash in the stock market. For the Covid crisis, $\widehat{\mathcal{E}}^{\text{option}}_t$ drops by around 22 points, which is the exact amount by which the valuation ratio drops, suggesting that the predictable forecast errors can explain effectively all of the crash.

Figure 9 also plots $\widehat{\mathcal{E}}^{\text{survey}}_t$ over the same two crises. The survey-based forecasts comove strongly with their option-based counterparts during these crises. During the global financial crisis, for which we rely on the CFO survey for forecast errors, $\widehat{\mathcal{E}}^{\text{survey}}_t$ drops by around 10 points over the crisis. The predictable forecast errors in the first 9 years of CFO forward rates can thus explain around one fourth of the drop in the stock market. This is again a lower bound for the full effect $\mathcal{E}_t$, under much weaker assumptions than used for the option-based measure.[31] During the Covid crisis, for which we rely on the Vanguard survey, $\widehat{\mathcal{E}}^{\text{survey}}_t$ again

[31] We note that the subjective short-horizon spot risk premium also varies over time, and this variation could exert offsetting effects on prices relative to $\mathcal{E}_t$. This is not our focus: we estimate only the predictable errors in forward rates. The spot rates in Figure 6, however, suggest that the offset is likely to be small in the two crises.

drops by around 10 points. The predictable Vanguard errors thus give a lower-bound price impact of nearly one half of the drop in the stock market during this period.

In Table 11, we provide a unified summary of the different numerical estimates of $\mathcal{E}_t$ during crises, along with $p_t - d_t$. The table shows that the option-based measure can explain more than half of the stock-price drop during the global financial crisis, and the full drop during the Covid crisis. The survey-based measures give rise to a more assumption-free lower bound, which confirms a large impact of predictable forecast errors in forward rates on valuations during crashes. In bad times, investors' overestimation of the length and magnitude of risk-premium increases can account for a significant share of observed price drops, and this holds across investor groups and measurement methods.

5.2 The Term Structure of Equity

We have argued above that a large part of the variation in stock prices during crises may come from excess cyclicality in forward rates. According to this argument, fluctuations in prices are driven by revisions in expectations that are unexpected by investors (but predictable by traditional state variables). If this mechanism helps drive realized returns on equities, we should expect it to manifest itself in the behavior of the equity term structure, i.e., the prices and expected returns of dividend claims with different maturities.

In general, equity premium forecast errors should have a larger impact on equity claims with longer maturity, because these claims have longer duration, making them more sensitive to changes in discount rates. To illustrate this effect, we calculate the impact of forecast errors on realized returns to dividend claims with different maturity under two assumptions: first, we assume that expected returns to all equity claims are the same (i.e., the equity term structure is flat); second, we assume, as in the previous section, a flat term structure of forecast errors (in this case, $\varepsilon_{t+1}^{(j)} = \varepsilon_{t+1}^{(j+1)}$ for all j). Under these assumptions, log returns on dividend claims with different maturity (ν) are given by

$$r_{t+1}^{(\nu)} = \mu_t^{(1)} + \Delta E_{t,t+1} d_{t+\nu} - (\nu - 1) \times \varepsilon_{t+1}, \tag{15}$$

where $\Delta E_{t,t+1}$ denotes the change in expectations between period t and $t+1$ (i.e., ex post forecast errors) and where we have suppressed the superscript from the forecast error because of the assumption that these errors are the same across all horizons (and claims). Equation (15) shows that mistakes about forward rates influence realized returns on equity claims, and more so for longer maturity claims.

Predictability in forecast errors thus influences the slope of the equity term structure. If forecast errors are predicted to be negative (investors have overly high expectations about future spot rates), investors will in the future revise their expectations about spot rates downwards, leading to positive realized returns. This effect will be stronger for longer-maturity claims, meaning the equity term structure will be upward sloping. Similarly, when forecast errors are predicted to be positive (investors have incorrectly low expectations about future spot rates), the equity term structure will be downward sloping.

To visualize the implications of our forecast errors for the equity term structure, we calculate the impact of forecast errors on realized returns to dividend strips with different maturities based on equation (15). We focus on forecast errors for risk premia and calculate both average returns and conditional returns in bad times (following Gormsen 2021, defined as the lowest 20% of CAPE values) and good times.

Figure 10 plots these term structures. The figure shows that the term structure of realized returns implied by the forecast errors is slightly negative on average and countercyclical. The numerical estimates happen to be close to the estimates for average returns on dividend strips in Gormsen (2021), but we interpret the results qualitatively rather than quantitatively, as our estimates rely fairly heavily on modeling assumptions (and represent log returns as opposed to arithmetic returns). Qualitatively, the figure shows how the estimated forecast errors can produce a term structure that is downward sloping on average (Binsbergen, Brandt, and Koijen 2012) and countercyclical (Golez and Jackwerth 2024, Gormsen 2021).[32] As such, our forecast errors help account for the dynamics of the equity term structure.

5.3 Impact of Forecast Errors on Demand Elasticities

Empirically, there is a strong relation between changes in prices and expected returns over the next period. Nagel and Xu (2023), for instance, show that a 1 standard deviation increase in the dividend yield increases expected returns on the market by 6 percentage points over the subsequent year. This large impact of prices on future expected returns should lead investors to allocate substantially towards equities when prices move. But our results suggest that investors often mistakenly attribute a significant share of the decrease in prices to increases in expected returns at relatively long horizons, and perceive short-term spot returns to increase only modestly. This structure of expectation errors lowers the elasticity of demand, as a modest increase in spot returns will not lead to a large increase in desired portfolio weights in equities. It can accordingly help make sense of the apparently puzzling inelasticity in investor demand for equities (Gabaix and Koijen 2022). We leave potential quantification of this effect to future work.

[32] If we considered forecast errors for expected returns instead of risk premia, the term structure would be slightly upward sloping on average but still countercyclical.

6 A Model of Expectation Errors

Finally, to provide a more positive potential explanation for our results, this section presents a simple model of expectation errors in the term structure of spot rates. The model builds on the fundamental extrapolation model proposed by Hirshleifer, Li, and Yu (2015), but it is also qualitatively similar to the natural expectations model proposed by Fuster, Laibson, and Mendel (2010) and adapted by Giglio and Kelly (2018). The model matches both the excess cyclicality in forward rates across all investors and the cyclicality of short-term spot rates across different investors.

6.1 Model Setup

We first describe the rational benchmark. The starting point is a state variable x_t. In our context, this fundamental state variable is most straightforwardly interpreted as representing market-level variance, but this labeling is not necessary for what follows. Under the objective measure, the fundamental variable follows an AR(3) process:

$$x_t = \left(1 - \sum_{j=1}^{3} \phi_j\right) \overline{x} + \phi_1 \, x_{t-1} + \phi_2 \, x_{t-2} + \phi_3 \, x_{t-3} + e_t, \tag{16}$$

where $\overline{x} = \mathbb{E}[x_t]$ and $e_t \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma_e^2)$. We assume that x_t is countercyclical. This is without loss of generality, though it applies naturally to variance. Under a rational benchmark, investors use the objective dynamics to make iterated forecasts of fundamentals:

$$\mathbb{E}_t[x_{t+n}] = \left(1 - \sum_{j=1}^{3} \phi_j\right) \overline{x} + \sum_{j=1}^{3} \phi_j \, \mathbb{E}_t[x_{t+n-j}] \;\text{ for } n > 0. \tag{17}$$

We next turn to equity premia. There is an affine mapping between contemporaneous fundamentals and short-term spot rates (henceforth *short rates* for short). Under rationality, the current short rate is:[33]

$$\mathbb{E}_t\left[\mu_t^{(3)}\right] = \alpha_0 + \alpha_1 \, x_t. \tag{18}$$

We assume that $\alpha_1 > 0$, so that the objective equity premium is countercyclical, as is standard. Investors use this mapping to translate from forecasts of fundamentals to forecasts of future

[33] We abuse notation slightly in denoting $\mu_t^{(3)}$ as $\mathbb{E}_t[\mu_t^{(3)}]$, which is somewhat redundant given that $\mu_t^{(3)} = \mathbb{E}_t[r_{t,t+3}]$. We do so to emphasize that the spot rate is itself an expected return (while maintaining focus on spot rates). This will also allow for a clearer distinction vis-à-vis subjective expectations below.

short rates:

$$\mathbb{E}_t\left[\mu^{(3)}_{t+n-3}\right] = \alpha_0 + \alpha_1\ \mathbb{E}_t[x_{t+n-3}] \text{ for } n \in \{6, 9, 12\}\,. \tag{19}$$

Under rationality, objective short rate expectations define long-term spot rates, as in (2):

$$\mathbb{E}_t\left[\mu^{(n)}_t\right] = \mathbb{E}_t\left[\mu^{(n-3)}_t\right] + \mathbb{E}_t\left[\mu^{(3)}_{t+n-3}\right] \text{ for } n \in \{6, 9, 12\}\,. \tag{20}$$

With this term structure of spot rates in hand, we can compute forward rates and forecast errors in the usual way.

We now turn to our assumptions on investors' actual subjective expectations. Investors observe current fundamentals x_t, and to limit free parameters, we also assume that they know the persistence parameters ϕ_1, ϕ_2, and ϕ_3 in (16) perfectly. We assume two deviations from rationality. The first affects how investors forecast future short rates. We refer to this as *forward rate bias* because it distorts long-term spot rates via forward rates. The second affects how investors perceive current short rates. We refer to this as *short rate bias* because it distorts long-term spot rates via current short rates. In what follows, we use $\mathbb{E}^\theta_t[\cdot]$ to differentiate subjective from objective expectations: θ_F indexes forward rate bias, θ_S short rate bias. We discuss each in turn.

We begin with forward rate bias. Investors forecast future fundamentals using (17), but to match the excess cyclicality in forward rates, we assume that they do so with a misperceived long-term mean:

$$\mathbb{E}^\theta_t[\overline{x}] = \mathbb{E}^\theta_{t-1}[\overline{x}] + \underbrace{\theta_F\left(x_t - \mathbb{E}^\theta_{t-1}[\overline{x}]\right)}_{\text{overreaction to news}}\,. \tag{21}$$

While the objective long-run mean $\overline{x}$ is constant, investors do not treat it as such, and they attach too much weight to recent fundamentals in updating about $\overline{x}$. This specification of extrapolative expectations is from Hirshleifer, Li, and Yu (2015), and we apply it to the long-term mean in the spirit of Nagel and Xu (2022) and Afrouzi et al. (2023), among others.[34] This representation nests rationality when $\theta_F = 0$. When $\theta_F > 0$, subjective expectations of the long-term mean overreact to perceived news $\hat{e}_t = x_t - \mathbb{E}^\theta_{t-1}[\overline{x}]$. In bad times, for example, overreaction leads to higher forecasts of fundamentals via (17), and thus higher forecasts of

[34] Hirshleifer, Li, and Yu (2015) assume that the one-period forecast for a fundamental growth rate $\hat{x}_{t,t+1}$ follows $\hat{x}_{t,t+1} = (1 - \rho - \tilde{\rho})\overline{x} + \rho\hat{x}_{t-1,t} + \tilde{\rho}\hat{\varepsilon}_t$, where $\hat{\varepsilon}_t$ is perceived news. They typically assume the sum $\rho + \tilde{\rho}$ is close to but just less than 1, to ensure finiteness of the value function in their general equilibrium model. We do not face this issue, so we set $\rho + \tilde{\rho} = 1$, which — setting $\tilde{\rho} = \theta_F$ — generates (21). We note that time variation in the perceived long-run mean need not be a mistake in general: investors may be learning about this parameter. But the constant-gain learning implied by (21) implies that this learning is overattentive to recent news and never converges. For further intuition, following Hirshleifer, Li, and Yu, (21) can be rewritten as $\mathbb{E}^\theta_t[\overline{x}] = (1 - \theta_F)\,\mathbb{E}^\theta_{t-1}[\overline{x}] + \theta_F\, x_t$. In this formulation, the first term reflects the persistence of extrapolation from past fundamentals, the second term extrapolation from current fundamentals.

short rates via (19) and higher forward rates via (20), replacing objective with subjective expectations $\mathbb{E}_t^\theta[\cdot]$. This generates predictably negative forecast errors, thereby allowing the model to match the excess cyclicality in forward rates.

We next turn to short rate bias. To match the cyclicality of short rates, we assume that investors incorrectly map from fundamentals to current short rates as follows:

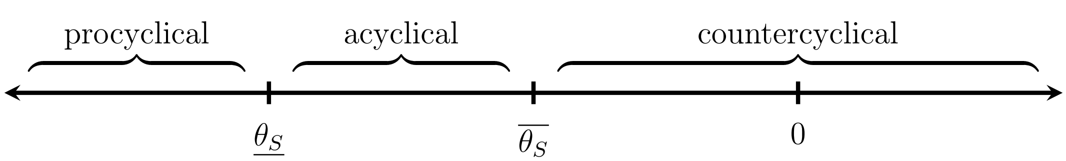
$$\mathbb{E}_t^\theta\left[\mu_t^{(3)}\right] = \mathbb{E}_t\left[\mu_t^{(3)}\right] + \alpha_1\,\theta_S\,e_t = \alpha_0 + \alpha_1\left(x_t + \theta_S\,e_t\right). \tag{22}$$

While investors understand that short rates increase in the fundamental state variable, they also include the most recent shock in this mapping.[35] This representation nests rationality when $\theta_S = 0$. When $\theta_S \neq 0$, there is a wedge between objective and subjective short rates, as shown on the left. This wedge is proportional to the most recent shock to fundamentals. Equivalently, as shown on the right, investors behave as if they misperceive fundamentals. This interpretation does not hold literally given that we assume that investors observe fundamentals correctly, but it may be more intuitive.

The covariance between fundamentals and short rates, from the standpoint of the econometrician, measures the cyclicality of short rates:

$$\underbrace{\operatorname{cov}\left(x_t, \mathbb{E}_t^\theta\left[\mu_t^{(3)}\right]\right)}_{\text{subjective}} = \underbrace{\operatorname{cov}\left(x_t, \mathbb{E}_t\left[\mu_t^{(3)}\right]\right)}_{\text{objective}} + \underbrace{\alpha_1\,\theta_S \operatorname{cov}(x_t, e_t)}_{\text{wedge}}.$$

As above, the covariance between fundamentals and subjective short rates (the subjective covariance) equals the corresponding covariance with objective short rates (the objective covariance) plus a wedge. The objective covariance is positive, but the wedge (and thus the subjective covariance) can be positive or negative. The schematic diagram below visualizes the relationship between short rate bias and the cyclicality of short rates:



Short Rate Bias θ_S and Short Rate Cyclicality $\operatorname{cov}\left(x_t, \mathbb{E}_t^\theta\left[\mu_t^{(3)}\right]\right)$

For sufficiently small $\theta_S < \underline{\theta_S}$, the subjective covariance is negative and so short rates are procyclical (Greenwood and Shleifer 2014). For $\underline{\theta_S} < \theta_S < \overline{\theta_S}$, it is approximately zero and so short rates are acyclical (Nagel and Xu 2023). For sufficiently large $\theta_S > \overline{\theta_S}$, it is positive and so short rates are countercyclical (Dahlquist and Ibert 2024). In this way, the model can match the cyclicality of short rates for multiple groups of investors.

[35] For parsimony, we set this term to e_t so that we have only a single shock.

We think of short rate bias as a stand-in for different investors' (mis)reactions to short-term data. For $\theta_S < \underline{\theta_S} < 0$, the bias can be thought of as a failure to learn from prices (Eyster, Rabin, and Vayanos 2019): after a positive shock to fundamentals (volatility) e_t, it is as if investors believe only they have observed the shock and that prices will continue to fall when others observe the shock (generating extrapolation). By contrast, $\theta_S > 0$ can be thought of as representing excessive learning from prices (Bastianello and Fontanier 2025), generating excessively high beliefs about future volatility and expected returns.

Finally, we note in passing that the short rate wedge $\alpha_1 \theta_s e_t$ does not affect forecasts of future short rates: by assumption, $\mathbb{E}_t^{\theta}[e_{t+n}] = \mathbb{E}_t[e_{t+n}] = 0$ for $n > 0$. As a result, forward rate bias is separate from short rate bias.

To summarize, our two assumed biases have distinct effects on the empirical results. Forward rate bias distorts long-term spot rates via forward rates. It affects Mincer-Zarnowitz and error-predictability regression slopes, but short rate bias does not. Short rate bias distorts long-term spot rates via short rates. It affects the cyclicality of short rates, but forward rate bias does not. This disconnect between the two biases (and thus the corresponding rates) is important. It allows the model to match both the excess cyclicality in forward rates across all investors and the cyclicality of short rates across different investors; that is, the *same* forward-rate dynamics for *different* short-rate dynamics.[36]

6.2 Calibration and Results

The discussion above suggests the model can qualitatively match the excess cyclicality in forward rates. To understand the quantitative implications, we turn to Monte Carlo simulations. To calibrate the model, we assume that the fundamental variable x_t is equal to option-implied variance (in the spirit of Table 10, and as discussed above) and estimate its dynamics in the U.S. sample. We measure spot rates using our U.S. option-implied risk premia and estimate the mapping from fundamentals to spot rates. We then consider a range of θ_F and θ_S parameters. For each combination, we solve for the 6-month, 6-month forward rate, simulating 100,000 samples of length $T = 378$ months. In the simulated data, we evaluate how regression slopes and average errors vary with deviations from rationality. We compare the simulated moments with those from the option-based measure and the Livingston survey. Details of the calibration and simulations are in Appendix E.1.

Figure 11 plots the main results from these simulations, showing regression slopes and average errors. Since short rate bias affects only regression R^2 values and not slopes or

[36] Both biases affect Mincer-Zarnowitz and error-predictability R^2 values, but neither bias affects average errors because $\mathbb{E}\left[\mathbb{E}_t^{\theta}[\overline{x}]\right] = \overline{x}$ in (21) and $\mathbb{E}[e_t] = 0$ in (22).

average errors, we set $\theta_S = 0$ for these figures and focus on how the results depend on θ_F. Overall, we find that a model with forward rate bias is consistent with the data. Starting with $\theta_F = 0$, the model recovers the rational null for both slopes and average errors. But as θ_F increases, the Mincer-Zarnowitz slope decreases toward zero and the error-predictability slope becomes more negative. Given (21), subjective expectations are on average unbiased, so the model cannot produce any non-zero average error. There is thus no relationship between θ_F and the average error. For the option-based measure, $\theta_F = 0.505$ exactly matches the slopes and closely matches the average error. This value is slightly higher than typical estimates in the literature, but those estimates are from very different settings than the one considered here; see Hirshleifer, Li, and Yu (2015) for a discussion. For the Livingston survey, the same value fits the slopes reasonably well, but misses the average error. In any case, fundamental extrapolation seems to explain the excess cyclicality in forward rates well for all investors.

6.3 Interpretation and Discussion

While the above model of expectation errors is capable of matching our empirical findings, the cyclicality of expectation errors distinguishes our setting from certain frameworks featuring return extrapolation. In the model, forward rates overreact to fundamental news: when the objective equity premium is high, forward rates increase, and forecast errors are predictably negative. This echoes our empirical findings. Those findings would not be matched by specifying a model of overreaction of forward rates to realized returns: the estimated equity premium is high in bad times, when realized returns are *low*.

If the cyclicality of our model's expectation errors were to be flipped — if forward rates were too low in bad times, generating positive errors — then these errors would not be capable of explaining any of the variation in the price-dividend ratio shown in Figure 8. Forecast errors instead would exert a *dampening* force on stock prices, making it more difficult to explain their observed volatility. We discuss and formalize this idea in more detail in Appendix E.2, which introduces a "trilemma" for expectation errors: it is difficult to simultaneously make sense of (i) volatile expectation errors, (ii) countercyclical expectation errors (e.g., extrapolation of past returns), and (iii) volatile prices. Our framework discards (ii), leaving (i) and (iii) on the table.

7 Conclusion

This paper provides novel analysis of the term structure of return expectations for equities. This term structure is a powerful laboratory for understanding how investors form expectations, and the behavior of this term structure is key for understanding variation in stock prices.

Our empirical analysis suggests that the term structure of investor expectations is well-behaved along multiple dimensions: forward rates are reasonably good predictors of future realized spot rates, they embed mean-reversion, and they exhibit countercyclical variation. However, we find strong evidence that forward rates are, in fact, excessively countercyclical: in bad times, investors' expectations about future expected returns are more elevated than their own subsequent beliefs justify.

The finding of excessively countercyclical forward rates has broad implications for asset pricing. First, mistakes about long-run risk premia have a meaningful impact on stock prices, and the excess cyclicality in forward rates therefore leads to substantial excess volatility. The excess cylicality is particularly relevant for explaining crashes in the stock market: we find that the excess cyclicality can account for a large share of the drop in prices during the global financial crisis and the Covid crisis. These crises appear to have been much more short-lived than investors expected ex ante, with meaningful price impacts.

The excess cyclicality also has implications for other areas in asset pricing. For instance, the forecast errors induced by excess cyclicality can account for the dynamics of the equity term structure studied by Binsbergen, Brandt, and Koijen (2012): the structure of the forecast errors is such that the equity term structure is slightly downward sloping on average and countercyclical. The forecast errors also tend to reduce the price-elasticity of demand.

An exciting feature of these empirical results is that they are robust across option-based and survey-based measures. Whereas the dynamics of short-run expected returns appears to vary across data sources, we find no disagreement between option-based measures and survey-based measures when considering forward rates: in all cases, longer-run forward rates are robustly and excessively countercyclical, with strikingly similar magnitudes across measures. We therefore provide a unified body of evidence on the dynamics of forward return expectations.

Tables and Figures

Table 2
Option-Based Expectations: Mincer-Zarnowitz Regressions

This table reports Mincer-Zarnowitz regressions of future realized spot rates on current forward rates for option-based risk premia (left panel) and expected returns (right panel). For risk premia, the realized spot rate

$$\tilde{\mu}^{(6)}_{t+6} = \mathbb{E}_{t+6}\left[r^{(6)}_{t+6}\right]$$

is the future expectation of the 6-month equity premium, and the forward rate

$$\tilde{f}^{(6,6)}_{t} = \mathbb{E}_{t}\left[r^{(6)}_{t+6}\right]$$

is the current expectation of the same risk premium. These expectations are analogously defined for expected returns. The units are annualized percentage points. Panel regressions, in the main sample, report standard errors clustered by exchange and date. This sample is the longest available for each exchange. Time-series regressions, in the U.S. sample, report Newey-West standard errors with $L = \lceil 1.3 \times T^{1/2} \rceil$ lags and fixed-b p-values, following Lazarus et al. (2018). This sample is from 01/1990 to 06/2021 for forward rates and from 07/1990 to 12/2021 for realized spot rates.

	Risk Premia $\tilde{\mu}^{(6)}_{t+6}$			Expected Returns $\mu^{(6)}_{t+6}$		
	(1) Main	(2) Main	(3) U.S. Only	(4) Main	(5) Main	(6) U.S. Only
$\tilde{f}^{(6,6)}_{t}$	0.64	0.56	0.67			
	(0.049)	(0.055)	(0.096)			
$f^{(6,6)}_{t}$				0.92	0.91	0.88
				(0.053)	(0.061)	(0.073)
Intercept	1.04		0.74	0.095		0.27
	(0.17)		(0.28)	(0.22)		(0.34)
p-val: $\beta_1 = 1$	<0.001	<0.001	0.003	0.157	0.181	0.114
p-val: $\beta_0 = 0$	<0.001	-	0.019	0.675	-	0.446
p-val: $\beta_1 = 1,\ \beta_0 = 0$	<0.001	-	0.012	0.067	-	0.110
Observations	2227	2227	378	2227	2227	378
Fixed Effects	None	Ex	None	None	Ex	None
Standard Errors	Cluster	Cluster	Newey-West	Cluster	Cluster	Newey-West
Clusters/Lags	Ex/Date	Ex/Date	26	Ex/Date	Ex/Date	26
Adjusted R^2	0.20	0.22	0.22	0.63	0.63	0.71
Within R^2	-	0.15	-	-	0.59	-

Table 3
Option-Based Expectations: Instrumented Mincer-Zarnowitz Regressions

This table reports instrumented Mincer-Zarnowitz regressions of future realized spot rates on current forward rates for option-based risk premia (left panel) and expected returns (right panel). For risk premia, the realized spot rate is the future expectation of the 6-month equity premium, and the forward rate is the current expectation of the same risk premium. The instrument is the current expectation of the 1-month equity premium in 2 months:

$$\tilde{f}_t^{(2,1)} = \mathbb{E}_t\left[r_{t+2}^{(1)}\right].$$

These expectations are analogously defined for expected returns. The units are annualized percentage points. Panel regressions, in the main sample, report standard errors clustered by exchange and date. This sample is the longest available for each exchange. Time-series regressions, in the U.S. sample, report Newey-West standard errors with $L = \lceil 1.3 \times T^{1/2} \rceil$ lags and fixed-b p-values, following Lazarus et al. (2018). This sample is from 01/1990 to 06/2021 for forward rates and from 07/1990 to 12/2021 for realized spot rates.

	Risk Premia $\tilde{\mu}_{t+6}^{(6)}$			Expected Returns $\mu_{t+6}^{(6)}$		
	(1) Main	(2) Main	(3) U.S. Only	(4) Main	(5) Main	(6) U.S. Only
$\tilde{f}_t^{(6,6)}$	0.77	0.70	0.73			
	(0.071)	(0.074)	(0.10)			
$f_t^{(6,6)}$				0.93	0.93	0.89
				(0.060)	(0.072)	(0.081)
Intercept	0.73		0.59	0.021		0.21
	(0.19)		(0.30)	(0.24)		(0.39)
p-val: $\beta_1 = 1$	0.001	0.003	0.018	0.278	0.331	0.187
p-val: $\beta_0 = 0$	<0.001	-	0.066	0.931	-	0.602
p-val: $\beta_1 = 1$, $\beta_0 = 0$	0.001	-	0.059	0.017	-	0.113
Observations	2227	2227	378	2227	2227	378
Fixed Effects	None	Ex	None	None	Ex	None
Standard Errors	Cluster	Cluster	Newey-West	Cluster	Cluster	Newey-West
Clusters/Lags	Ex/Date	Ex/Date	26	Ex/Date	Ex/Date	26
Adjusted R^2	0.19	0.21	0.22	0.63	0.63	0.71
Within R^2	-	0.14	-	-	0.59	-

Table 4
Option-Based Expectations: Average Forecast Errors

This table reports average forecast errors for option-based risk premia (left panel) and expected returns (right panel). For risk premia, the forecast error is the difference between the future realized spot rate and the current forward rate. The realized spot rate is the future expectation of the 6-month equity premium, and the forward rate is the current expectation of the same risk premium. These expectations are analogously defined for expected returns. The units are annualized percentage points. Panel regressions, in the main sample, report standard errors clustered by exchange and date. This sample is the longest available for each exchange. Time-series regressions, in the U.S. sample, report Newey-West standard errors with $L = \lceil 1.3 \times T^{1/2} \rceil$ lags and fixed-b p-values, following Lazarus et al. (2018). This sample is from 01/1990 to 06/2021 for forward rates and from 07/1990 to 12/2021 for realized spot rates.

	Risk Premia $\tilde{\varepsilon}_{t+6}^{(6)}$		Expected Returns $\varepsilon_{t+6}^{(6)}$	
	(1) Main	(2) U.S. Only	(3) Main	(4) U.S. Only
Average Error	0.17	0.021	-0.28	-0.40
	(0.11)	(0.15)	(0.10)	(0.17)
p-val: $\bar{\varepsilon}_t = 0$	0.136	0.891	0.026	0.035
Observations	2227	378	2227	378
Standard Errors	Cluster	Newey-West	Cluster	Newey-West
Clusters/Lags	Ex/Date	26	Ex/Date	26

Table 5
Option-Based Expectations: Predictability of Forecast Errors

This table reports predictability regressions of future realized forecast errors on current forward rates for option-based risk premia (left panel) and expected returns (right panel). For risk premia, the realized spot rate is the future expectation of the 6-month equity premium, the forward rate is the current expectation of the same risk premium, and the forecast error is the realized spot rate minus the forward rate. These expectations are analogously defined for expected returns. For both risk premia and expected returns, the predictor is the 2×1-month forward risk premium, the current expectation of the 1-month equity premium in 2 months. The units are annualized percentage points. Panel regressions, in the main sample, report standard errors clustered by exchange and date. This sample is the longest available for each exchange. Time-series regressions, in the U.S. sample, report Newey-West standard errors with $L = \lceil 1.3 \times T^{1/2} \rceil$ lags and fixed-b p-values, following Lazarus et al. (2018). This sample is from 01/1990 to 06/2021 for forward rates and from 07/1990 to 12/2021 for realized spot rates.

	Risk Premia $\tilde{\varepsilon}^{(6)}_{t+6}$			Expected Returns $\varepsilon^{(6)}_{t+6}$		
	(1) Main	(2) Main	(3) U.S. Only	(4) Main	(5) Main	(6) U.S. Only
$\tilde{f}^{(2,1)}_t$	-0.13	-0.16	-0.17	-0.25	-0.29	-0.20
	(0.047)	(0.047)	(0.067)	(0.050)	(0.047)	(0.091)
Intercept	0.51		0.39	0.39		0.039
	(0.15)		(0.23)	(0.17)		(0.29)
p-val: $\beta_1 = 0$	0.025	0.008	0.025	0.001	$<$0.001	0.050
Observations	2227	2227	378	2227	2227	378
Fixed Effects	None	Ex	None	None	Ex	None
Standard Errors	Cluster	Cluster	Newey-West	Cluster	Cluster	Newey-West
Clusters/Lags	Ex/Date	Ex/Date	26	Ex/Date	Ex/Date	26
Adjusted R^2	0.02	0.03	0.03	0.08	0.11	0.04
Within R^2	-	0.03	-	-	0.10	-

Table 6
Livingston Survey Regressions

This table reports regressions for Livingston survey expectations. Columns (1)–(2) are Mincer-Zarnowitz regressions of future realized spot rates on current forward rates and test $H_0\colon \beta_1 = 1$, $H_0\colon \beta_0 = 0$, and $H_0\colon \beta_1 = 1, \beta_0 = 0$, as in Table 2. Columns (3)–(4) are average forecast errors and test $H_0\colon \bar{\varepsilon}_t = 0$, as in Table 4. Columns (5)–(6) are error-predictability regressions of future realized forecast errors on current forward rates and test $H_0\colon \beta_1 = 0$, as in Table 5. The horizon is the 6-month spot rate, 6 months from now. The units are annualized percentage points. Each regression reports Newey-West standard errors with $L = \lceil 1.3 \times T^{1/2} \rceil$ lags and fixed-b p-values, following Lazarus et al. (2018). The sample is half-yearly from 06/1992 to 06/2021 for forward rates and from 12/1992 to 12/2021 for realized spot rates.

	Mincer-Zarnowitz		Average Error		Error Predictability	
	(1) RP $\tilde{\mu}_{t+6}^{(6)}$	(2) ER $\mu_{t+6}^{(6)}$	(3) RP $\tilde{\varepsilon}_{t+6}^{(6)}$	(4) ER $\varepsilon_{t+6}^{(6)}$	(5) RP $\tilde{\varepsilon}_{t+6}^{(6)}$	(6) ER $\varepsilon_{t+6}^{(6)}$
$\tilde{f}_t^{(6,6)}$	0.81				-0.19	-0.19
	(0.077)				(0.077)	(0.073)
$f_t^{(6,6)}$		0.68				
		(0.069)				
Intercept	0.97	1.95	0.37	0.11	0.97	0.72
	(0.64)	(0.70)	(0.34)	(0.30)	(0.64)	(0.59)
p-val: Slope	0.052	0.002	-	-	0.052	0.040
p-val: Intercept	0.205	0.032	0.347	0.733	-	-
p-val: Joint	0.125	0.005	-	-	-	-
Observations	59	59	59	59	59	59
Newey-West Lags	10	10	10	10	10	10
Adjusted R^2	0.56	0.38	-	-	0.05	0.06

Table 7
CFO Survey Regressions

This table reports regressions for CFO survey expectations. Columns (1)–(2) are Mincer-Zarnowitz regressions of future realized spot rates on current forward rates and test $H_0\colon \beta_1 = 1$, $H_0\colon \beta_0 = 0$, and $H_0\colon \beta_1 = 1, \beta_0 = 0$, as in Table 2. Columns (3)–(4) are average forecast errors and test $H_0\colon \bar{\varepsilon}_t = 0$, as in Table 4. Columns (5)–(6) are error-predictability regressions of future realized forecast errors on current forward rates and test $H_0\colon \beta_1 = 0$, as in Table 5. The horizon is the 9-year spot rate, 1 year from now. The units are annualized percentage points. Each regression reports Newey-West standard errors with $L = \lceil 1.3 \times T^{1/2} \rceil$ lags and fixed-b p-values, following Lazarus et al. (2018). The sample is quarterly from 12/2001 to 06/2024 (excluding 09/2019 and 12/2019) for forward rates and from 12/2002 to 06/2025 (excluding 09/2020 and 12/2020) for realized spot rates. Each regression includes an indicator variable that equals 1 on and after 09/2020 for forward rates and likewise 09/2021 for realized spot rates.

	Mincer-Zarnowitz		Average Error		Error Predictability	
	(1) RP $\tilde{\mu}^{(9y)}_{t+1y}$	(2) ER $\mu^{(9y)}_{t+1y}$	(3) RP $\tilde{\varepsilon}^{(9y)}_{t+1y}$	(4) ER $\varepsilon^{(9y)}_{t+1y}$	(5) RP $\tilde{\varepsilon}^{(9y)}_{t+1y}$	(6) ER $\varepsilon^{(9y)}_{t+1y}$
$\tilde{f}^{(1y,9y)}_t$	0.35				-0.65	-0.18
	(0.14)				(0.14)	(0.083)
$f^{(1y,9y)}_t$		0.64				
		(0.076)				
Intercept	2.27	2.22	0.096	-0.24	2.27	0.37
	(0.50)	(0.49)	(0.072)	(0.095)	(0.50)	(0.27)
p-val: Slope	0.001	0.001	-	-	0.001	0.069
p-val: Intercept	0.001	0.001	0.245	0.036	-	-
p-val: Joint	0.005	0.004	-	-	-	-
Observations	84	84	84	84	84	84
Newey-West Lags	12	12	12	12	12	12
Adjusted R^2	0.59	0.83	-	-	0.48	0.04

Table 8
Vanguard Survey Regressions

This table reports regressions for Vanguard survey expectations. Columns (1)–(2) are Mincer-Zarnowitz regressions of future realized spot rates on current forward rates and test $H_0\colon \beta_1 = 1$, $H_0\colon \beta_0 = 0$, and $H_0\colon \beta_1 = 1,\ \beta_0 = 0$, as in Table 2. Columns (3)–(4) are average forecast errors and test $H_0\colon \bar{\varepsilon}_t = 0$, as in Table 4. Columns (5)–(6) are error-predictability regressions of future realized forecast errors on current forward rates and test $H_0\colon \beta_1 = 0$, as in Table 5. The horizon is the 9-year spot rate, 1 year from now. The units are annualized percentage points. Each regression reports Newey-West standard errors with $L = \lceil 1.3 \times T^{1/2} \rceil$ lags and fixed-b p-values, following Lazarus et al. (2018). The sample is bimonthly from 02/2017 to 12/2023 for forward rates and from 02/2018 to 12/2024 for realized spot rates.

	Mincer-Zarnowitz		Average Error		Error Predictability	
	(1) RP $\tilde{\mu}^{(9y)}_{t+1y}$	(2) ER $\mu^{(9y)}_{t+1y}$	(3) RP $\tilde{\varepsilon}^{(9y)}_{t+1y}$	(4) ER $\varepsilon^{(9y)}_{t+1y}$	(5) RP $\tilde{\varepsilon}^{(9y)}_{t+1y}$	(6) ER $\varepsilon^{(9y)}_{t+1y}$
$\tilde{f}^{(1y,9y)}_t$	0.43				-0.57	-0.089
	(0.16)				(0.16)	(0.069)
$f^{(1y,9y)}_t$		0.54				
		(0.15)				
Intercept	2.20	3.03	-0.32	-0.10	2.20	0.29
	(0.82)	(1.08)	(0.40)	(0.070)	(0.82)	(0.34)
p-val: Slope	0.016	0.033	-	-	0.016	0.289
p-val: Intercept	0.048	0.040	0.497	0.237	-	-
p-val: Joint	0.061	0.079	-	-	-	-
Observations	42	42	42	42	42	42
Newey-West Lags	9	9	9	9	9	9
Adjusted R^2	0.13	0.26	-	-	0.21	0.03

Table 9
Predictability of Survey-Based Forecast Errors with Option-Based Forward Rates

This table reports predictability regressions of future realized forecast errors on current forward rates for risk premia (left panel) and expected returns (right panel). On the left-hand side, the option-based horizon is the 6×6-month forecast error, the Livingston horizon is the 6×6-month forecast error, and the CFO horizon is the 1×9-year forecast error. On the right-hand side, the predictor is the option-based 2×1-month forward risk premium. Forecast errors and forward rates are standardized to unit variance. The intercept is not reported. Each regression reports Newey-West standard errors with $L = \lceil 1.3 \times T^{1/2} \rceil$ lags and fixed-b p-values, following Lazarus et al. (2018). The Livingston sample is half-yearly from 06/1992 to 06/2021 for forward rates and from 12/1992 to 12/2021 for realized spot rates. The CFO sample is quarterly from 12/2001 to 06/2021 (excluding 09/2019 and 12/2019) for forward rates and from 12/2002 to 06/2022 (excluding 09/2020 and 12/2020) for realized spot rates. FedInd is an indicator variable that equals 1 on and after 09/2020 for forward rates and likewise 09/2021 for realized spot rates.

	Risk Premia				Expected Returns			
	Livingston Survey		CFO Survey		Livingston Survey		CFO Survey	
	(1) Option $\tilde{\varepsilon}^{(6)}_{t+6}$	(2) Survey $\tilde{\varepsilon}^{(6)}_{t+6}$	(3) Option $\tilde{\varepsilon}^{(6)}_{t+6}$	(4) Survey $\tilde{\varepsilon}^{(9y)}_{t+1y}$	(5) Option $\varepsilon^{(6)}_{t+6}$	(6) Survey $\varepsilon^{(6)}_{t+6}$	(7) Option $\varepsilon^{(6)}_{t+6}$	(8) Survey $\varepsilon^{(9y)}_{t+1y}$
Option $\tilde{f}^{(2,1)}_t$	-0.31	-0.21	-0.28	-0.28	-0.36	-0.24	-0.46	-0.42
	(0.075)	(0.078)	(0.10)	(0.15)	(0.12)	(0.077)	(0.092)	(0.087)
p-val: $\beta_1 = 0$	0.004	0.033	0.030	0.127	0.026	0.018	0.001	0.001
Observations	59	59	72	72	59	59	72	72
Controls	None	None	FedInd	FedInd	None	None	FedInd	FedInd
Newey-West Lags	10	10	12	12	10	10	12	12
Adjusted R^2	0.08	0.03	0.05	0.08	0.11	0.04	0.19	0.22

Table 10
Cyclical Variation in Forecast Errors

This table reports predictability regressions of future realized forecast errors on current state variables for risk premia (left panel) and expected returns (right panel). On the left-hand side, the option-based horizon is the 6×6-month forecast error, the Livingston horizon is the 6×6-month forecast error, and the CFO horizon is the 1×9-year forecast error. On the right-hand side, 1/CAPE is the cyclically-adjusted earnings yield (obtained from Robert Shiller's website), and VIX^2 is the squared 1-year CBOE Volatility Index (obtained from the CBOE's website). Forecast errors and state variables are standardized to unit variance. The intercept is not reported. Each regression reports Newey-West standard errors with $L = \lceil 1.3 \times T^{1/2} \rceil$ lags and fixed-b p-values, following Lazarus et al. (2018). The option-based sample is monthly from 01/1990 to 06/2021 for forward rates and from 07/1990 to 12/2021 for realized spot rates. The Livingston sample is half-yearly from 06/1992 to 06/2021 for forward rates and from 12/1992 to 12/2021 for realized spot rates. The CFO sample is quarterly from 12/2001 to 06/2024 (excluding 09/2019 and 12/2019) for forward rates and from 12/2002 to 06/2025 (excluding 09/2020 and 12/2020) for realized spot rates. FedInd is an indicator variable that equals 1 on and after 09/2020 for forward rates and likewise 09/2021 for realized spot rates.

	Risk Premia						Expected Returns					
	Option-Based		Livingston Survey		CFO Survey		Option-Based		Livingston Survey		CFO Survey	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	$\tilde{\varepsilon}^{(6)}_{t+6}$	$\tilde{\varepsilon}^{(6)}_{t+6}$	$\tilde{\varepsilon}^{(6)}_{t+6}$	$\tilde{\varepsilon}^{(6)}_{t+6}$	$\tilde{\varepsilon}^{(9y)}_{t+1y}$	$\tilde{\varepsilon}^{(9y)}_{t+1y}$	$\varepsilon^{(6)}_{t+6}$	$\varepsilon^{(6)}_{t+6}$	$\varepsilon^{(6)}_{t+6}$	$\varepsilon^{(6)}_{t+6}$	$\varepsilon^{(9y)}_{t+1y}$	$\varepsilon^{(9y)}_{t+1y}$
$1/\text{CAPE}_t$	-0.19		-0.20		-0.19		-0.38		-0.20		-0.24	
	(0.092)		(0.091)		(0.13)		(0.13)		(0.088)		(0.19)	
VIX^2_t		-0.25		-0.18		-0.27		-0.25		-0.20		-0.40
		(0.075)		(0.086)		(0.11)		(0.097)		(0.084)		(0.073)
p-val: $\beta_1 = 0$	0.055	0.004	0.072	0.083	0.219	0.049	0.012	0.020	0.073	0.064	0.268	<0.001
Observations	378	378	59	59	84	84	378	378	59	59	84	84
Controls	None	None	None	None	FedInd	FedInd	None	None	None	None	FedInd	FedInd
Newey-West Lags	26	26	10	10	12	12	26	26	10	10	12	12
Adjusted R^2	0.03	0.06	0.02	0.02	0.23	0.27	0.14	0.06	0.02	0.02	0.02	0.14

Table 11
Price Impact of Forecast Errors in Crises

This table reports the price impact of forecast errors in the financial crisis (top panel) and the Covid-19 recession (bottom panel) for option-based and survey-based expectations. For option-based expectations, the price impact is the discounted sum of predicted forecast errors from $j = 1$ to $j = \infty$ under the assumption of a flat term structure, as in (13). For survey-based expectations, the price impact is the discounted sum of predicted forecast errors from $j = 1$ to $j = 9$, as in (14). The predicted forecast error is from a time-series regression of future realized forecast errors on current forward rates. Forecast errors and forward rates are for risk premia. The option-based sample is monthly from 01/1990 to 06/2021 for forward rates and from 07/1990 to 12/2021 for realized spot rates. The CFO sample is quarterly from 12/2001 to 12/2018 for forward rates and from 12/2002 to 12/2019 for realized spot rates. The Vanguard sample is bimonthly from 02/2017 to 12/2023 for forward rates and from 02/2018 to 12/2024 for realized spot rates. $p_t - d_t$ is the log repurchase-adjusted price-dividend ratio in the U.S. sample and is obtained from Nagel and Xu (2022) via Zhengyang Xu's website.

	Change in Discounted Risk Premia Forecast Errors			
		Survey-Based Expectations		
	Option-Based (Flat Term Structure)	CFO Survey (Lower Bound)	Vanguard Survey (Lower Bound)	Change in $p_t - d_t$
2008 Financial Crisis				
Peak to Trough: 06/2007 to 03/2009	-0.3576	-0.1127	-	-0.4273
2020 Covid-19 Recession				
Peak to Trough: 12/2019 to 03/2020	-0.2183	-	-0.0940	-0.2060

Figure 2
Option-Based Spot and Forward Rates

The top panel plots contemporaneous 6-month spot rates $\tilde{\mu}_t^{(6)}$ (light blue) and 6×6-month forward rates $\tilde{f}_t^{(6,6)}$ (dark blue). The bottom panel plots instrumented 6×6-month forward rates $\tilde{f}_t^{(6,6)}$ (dark blue) and the corresponding realized 6-month spot rates $\tilde{\mu}_{t+6}^{(6)}$ (red). The instrument is the 2×1-month forward rate, as in Table 3. Spot and forward rates are for risk premia. Gray bands are NBER recessions. The sample is from 01/1990 to 06/2021 in the U.S.

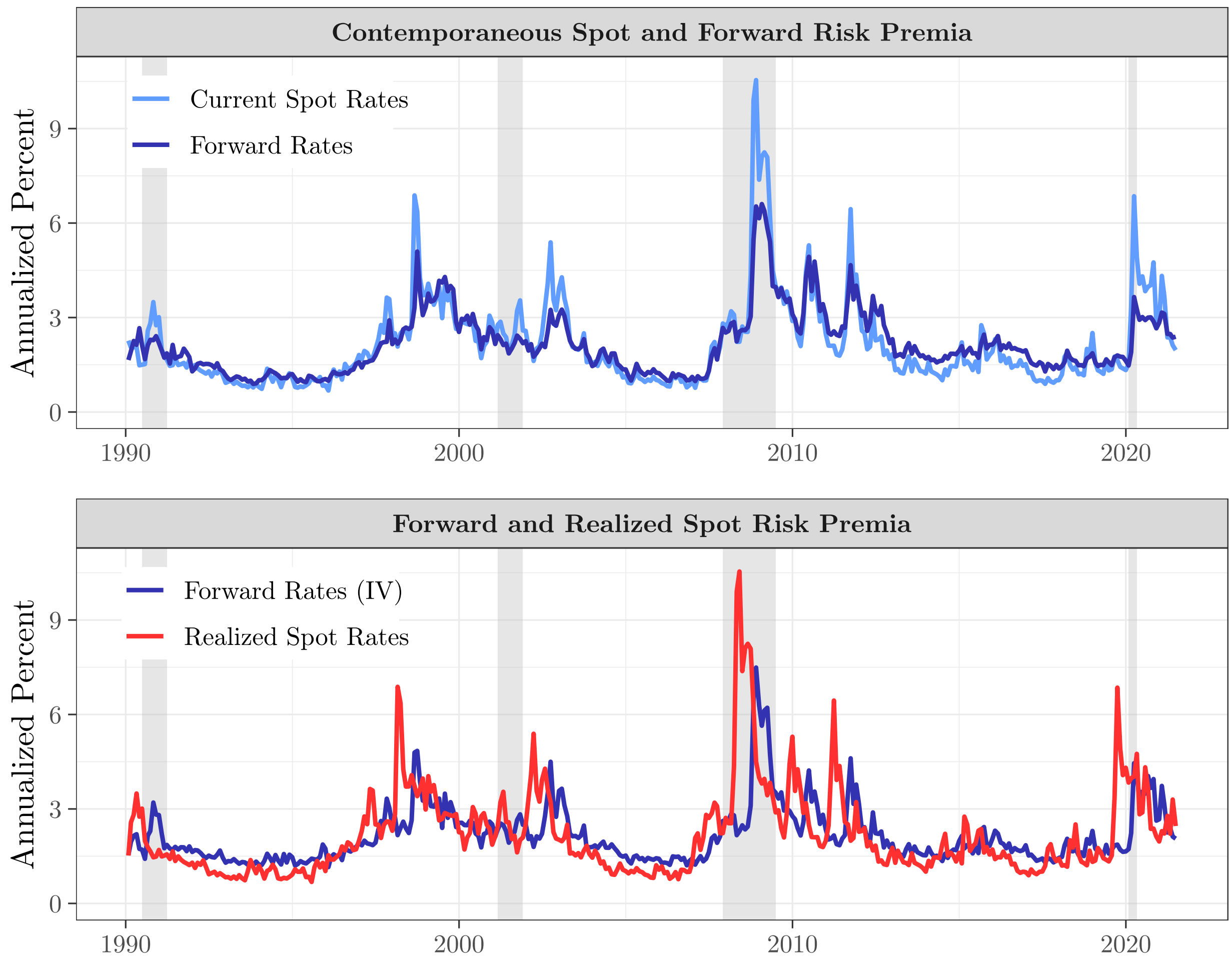


Figure 3
Option-Based Forecast Errors and Lagged Forward Rates

This figure plots 6×6-month forecast errors $\tilde{\varepsilon}_{t+6}^{(6)}$ (orange bars) and the corresponding lagged forward rates $\tilde{f}_t^{(6,6)}$ (dark blue line). Forecast errors and forward rates are for risk premia. The sample is from 01/1990 to 06/2021 in the U.S.

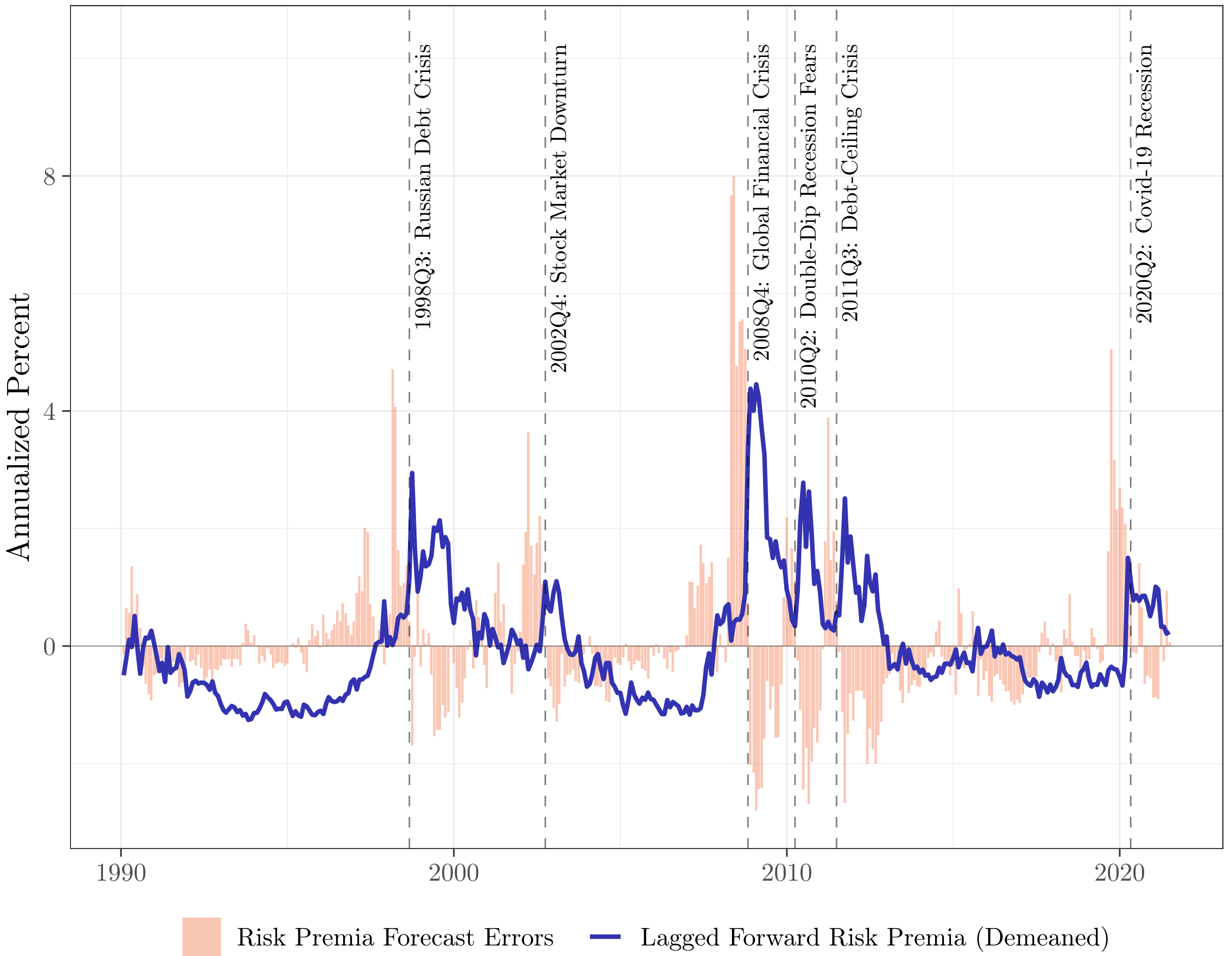


Figure 4
Option-Based Forward Rates and Predicted Forecast Errors

This figure plots instrumented 6×6-month forward rates $\tilde{f}_t^{(6,6)}$ (top panel) and the corresponding predicted forecast errors $\varepsilon_{t+6}^{(6)}$ (bottom panel). The instrument is the 2×1-month forward rate, as in Table 3. The predicted forecast error is from a time-series regression of future realized forecast errors on current forward rates, as in Table 5. Forward rates and forecast errors are for risk premia. Gray bands are NBER recessions. The sample is from 01/1990 to 06/2021 in the U.S.

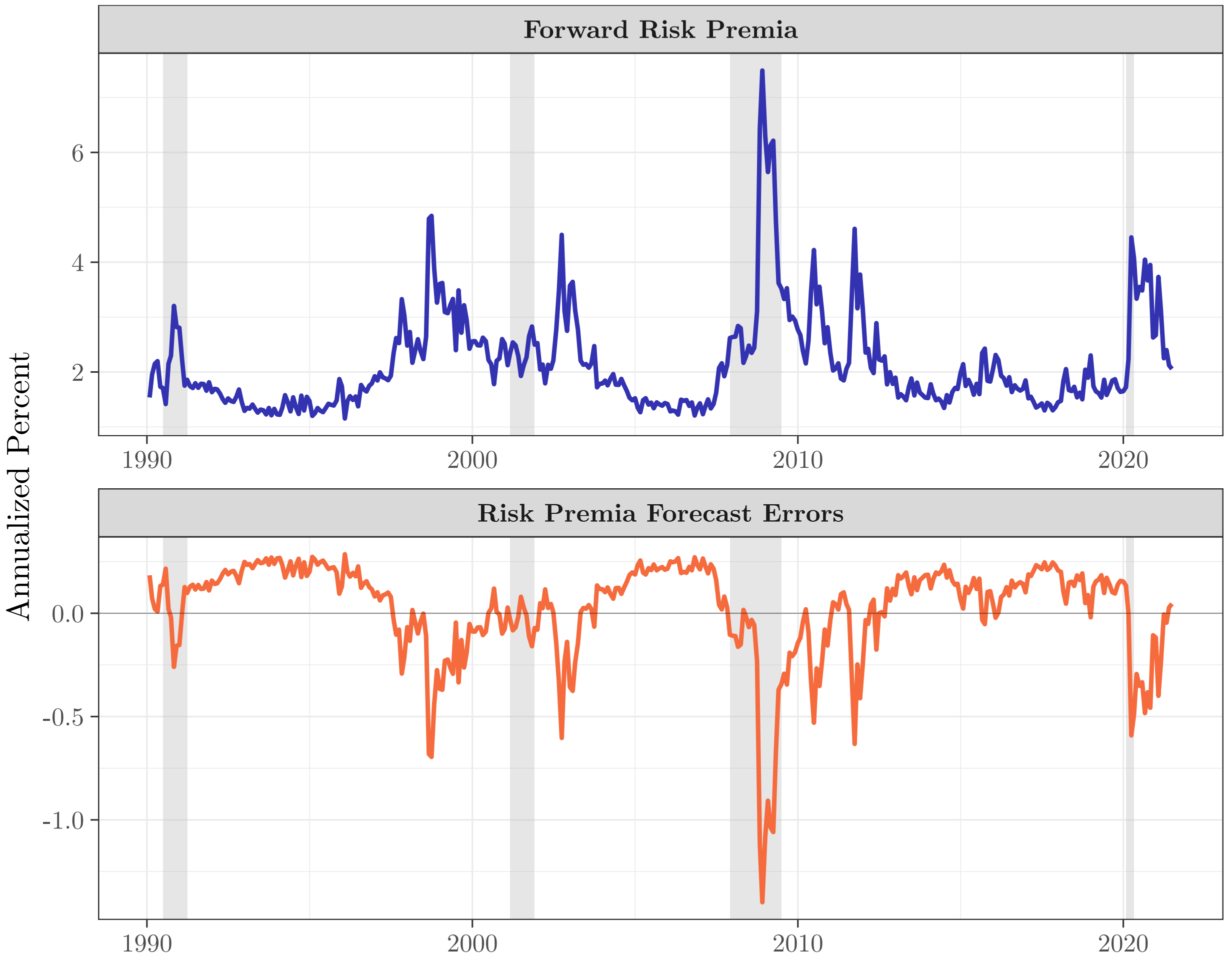


Figure 5
Survey-Based Forward Rates

This figure plots forward rates for Livingston (top panel), CFO (middle panel), and Vanguard (bottom panel) survey expectations. Forward rates are for risk premia. Gray bands are NBER recessions. The Livingston sample is half-yearly from 06/1992 to 12/2021. The CFO sample is quarterly from 12/2001 to 06/2025 and is adjusted for the structural break in 2020. The break adjustment is from a time-series regression of unadjusted forward rates on an indicator variable that equals 1 on and after 09/2020, as in Figure A4. The Vanguard sample is bimonthly from 02/2017 to 12/2024 (including 03/2020).

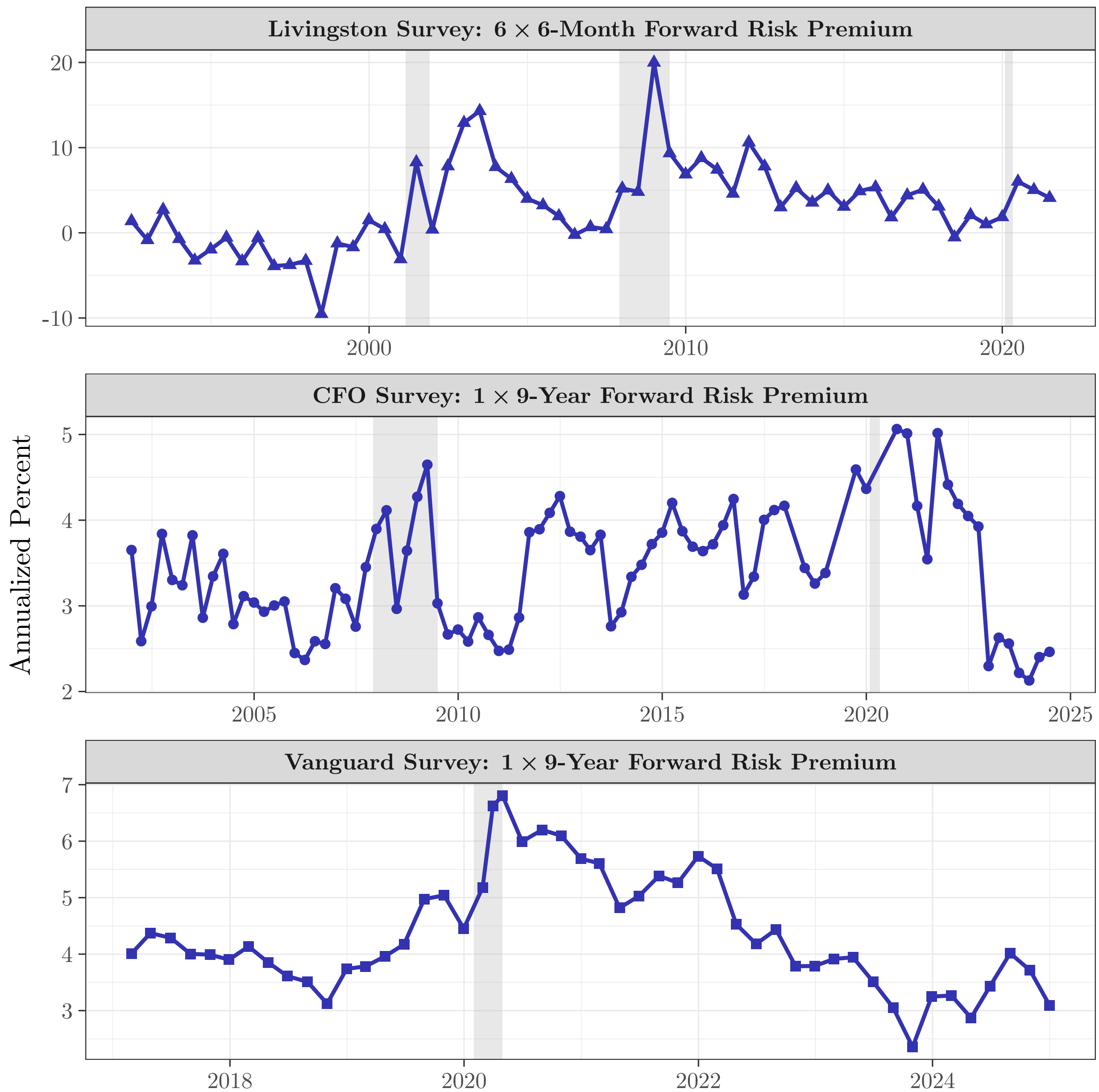


Figure 6
Cyclical Variation in Survey-Based One-Year Spot Rates

This figure plots 1-year spot rates for CFO (top panel) and Vanguard (bottom panel) survey expectations. Spot rates are for risk premia. Gray bands are NBER recessions. The CFO sample is quarterly from 12/2001 to 06/2025. The Vanguard sample is bimonthly from 02/2017 to 12/2024 (including 03/2020).

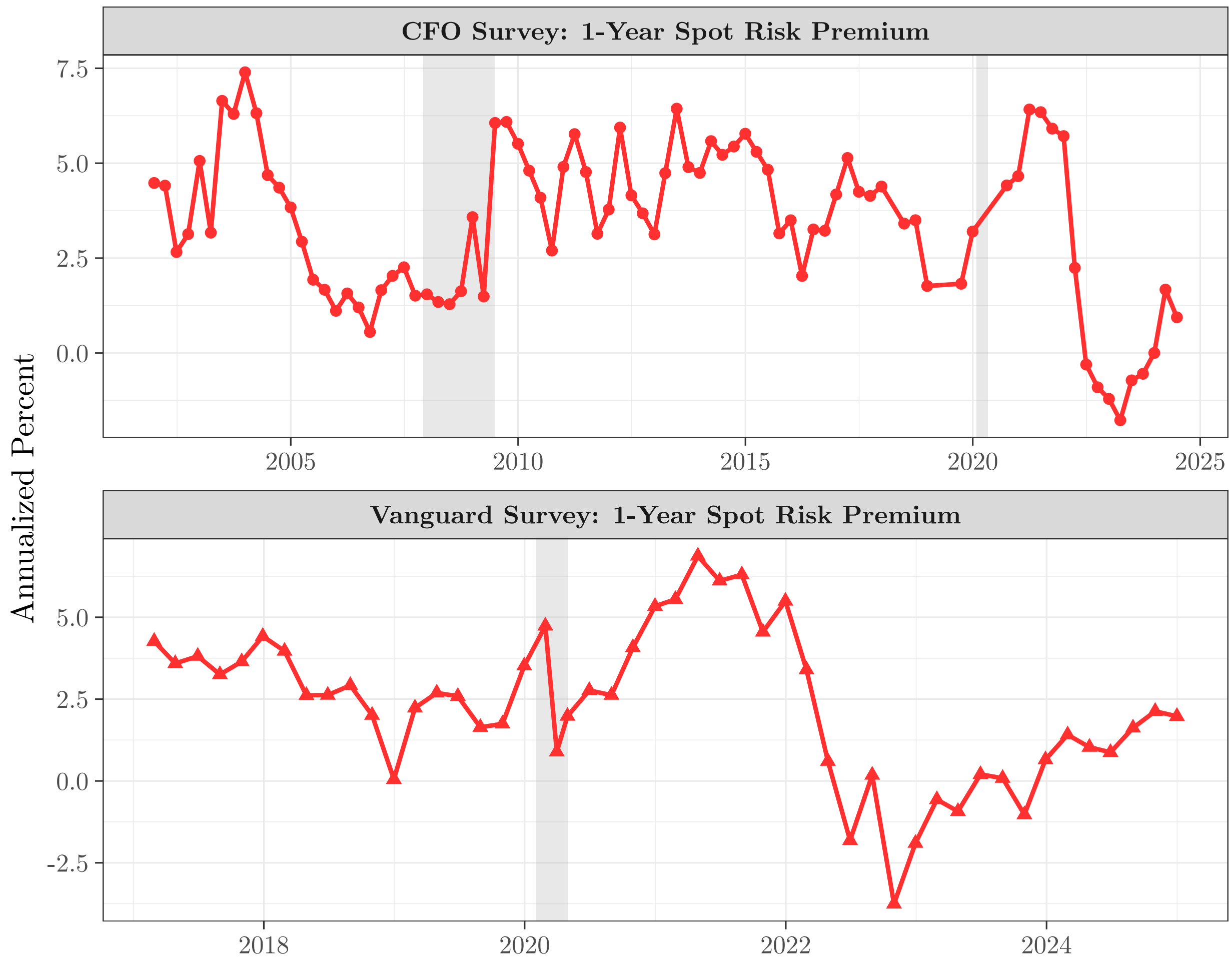


Figure 7
Common Variation in Forward Rates

The left axis plots option-based forward rates. The right axis plots Livingston survey (top panel), CFO survey (middle panel), and Vanguard survey (bottom panel) forward rates. Forward rates are for risk premia. Gray bands are NBER recessions. The Livingston sample is half-yearly from 06/1992 to 06/2021. The CFO sample is quarterly from 12/2001 to 06/2021 and is adjusted for the structural break in 2020. The break adjustment is from a time-series regression of unadjusted forward rates on an indicator variable that equals 1 on and after 09/2020, as in Figure A4. The Vanguard sample is bimonthly from 02/2017 to 06/2021 (including 03/2020).

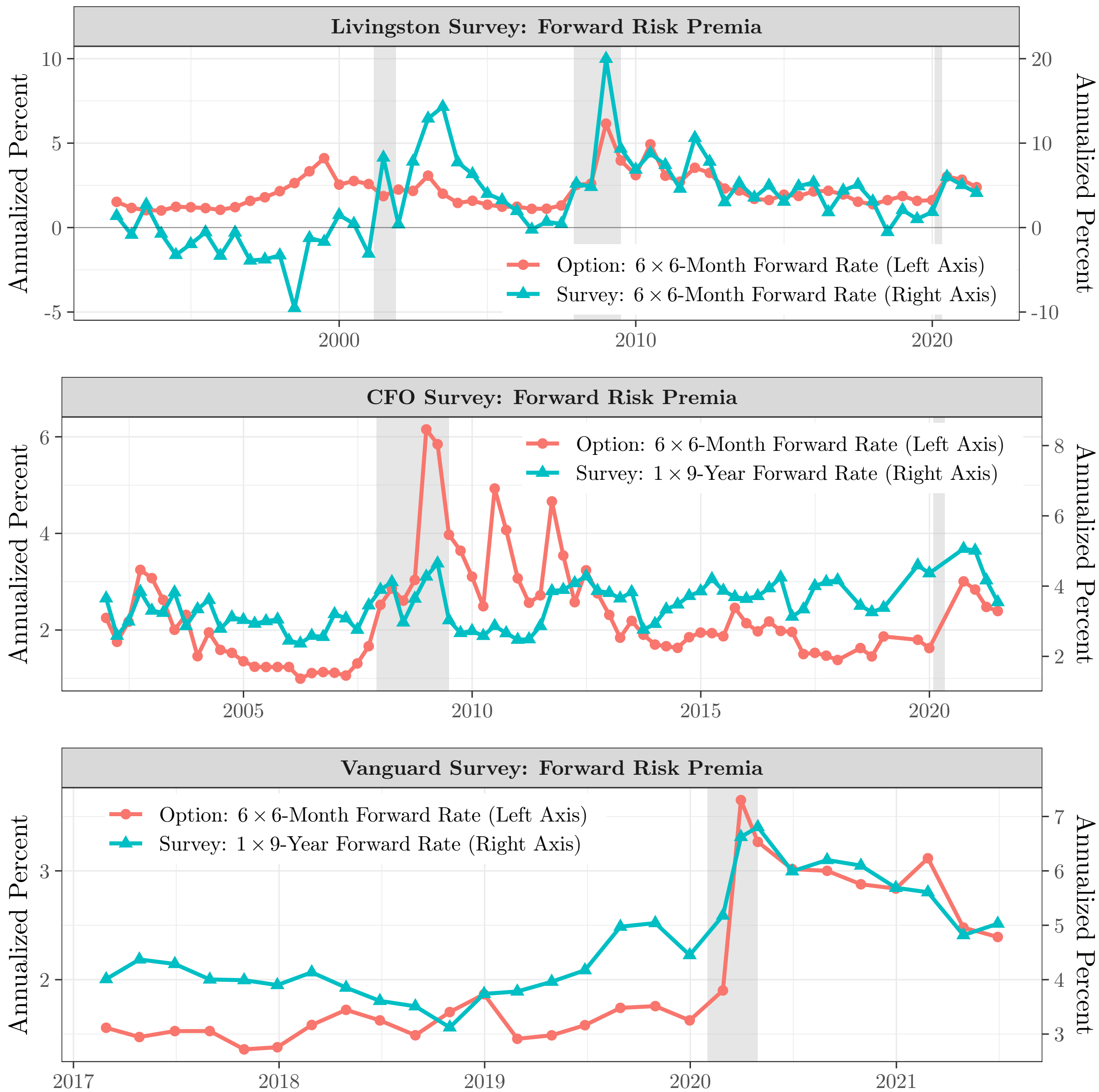


Figure 8
Price Impact of Option-Based Forecast Errors

This figure plots the option-based estimates for discounted $\mathcal{E}_t$ (red) and the log repurchase-adjusted price-dividend ratio $p_t - d_t$ (blue) in the United States. Discounted forecast errors $\mathcal{E}_t$ are estimated as in (13). The monthly repurchase-adjusted $p_t - d_t$ for the CRSP value-weighted stock index is obtained from Nagel and Xu (2022) via Zhengyang Xu's website. Both series are plotted as log differences from their full-sample means. Gray bands are NBER recessions. The sample is from 01/1990 to 06/2021.

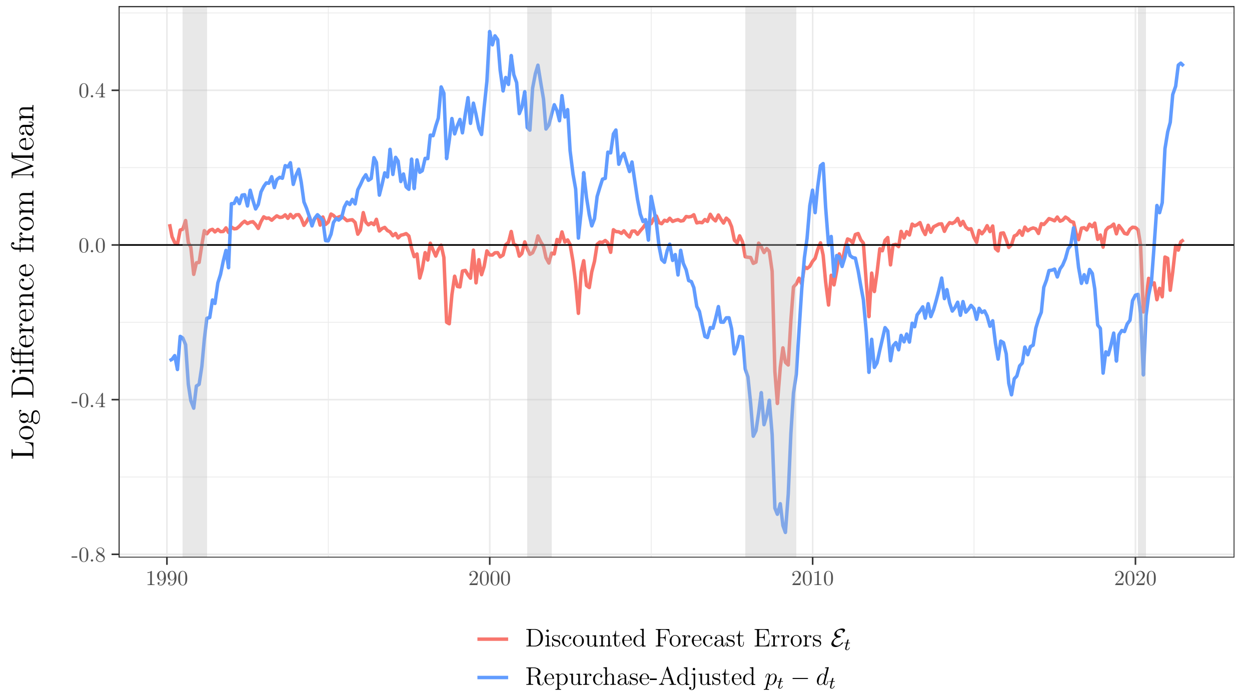


Figure 9
Price Impact of Forecast Errors in Crises

The figure plots the price impact of forecast errors in the financial crisis (top panel) and the Covid-19 recession (bottom panel) for option-based (left axis) and survey-based (right axis) expectations. For option-based expectations, the price impact is the discounted sum of predicted forecast errors from $j = 1$ to $j = \infty$ under the assumption of a flat term structure, as in (13). For survey-based expectations, the price impact is the discounted sum of predicted forecast errors from $j = 1$ to $j = 9$, as in (14). The predicted forecast error is from a time-series regression of future realized forecast errors on current forward rates. Forecast errors and forward rates are for risk premia. The option-based sample is monthly from 01/1990 to 06/2021 for forward rates and from 07/1990 to 12/2021 for realized spot rates. The CFO sample is quarterly from 12/2001 to 12/2018 for forward rates and from 12/2002 to 12/2019 for realized spot rates. The Vanguard sample is bimonthly from 02/2017 to 12/2023 for forward rates and from 02/2018 to 12/2024 for realized spot rates.

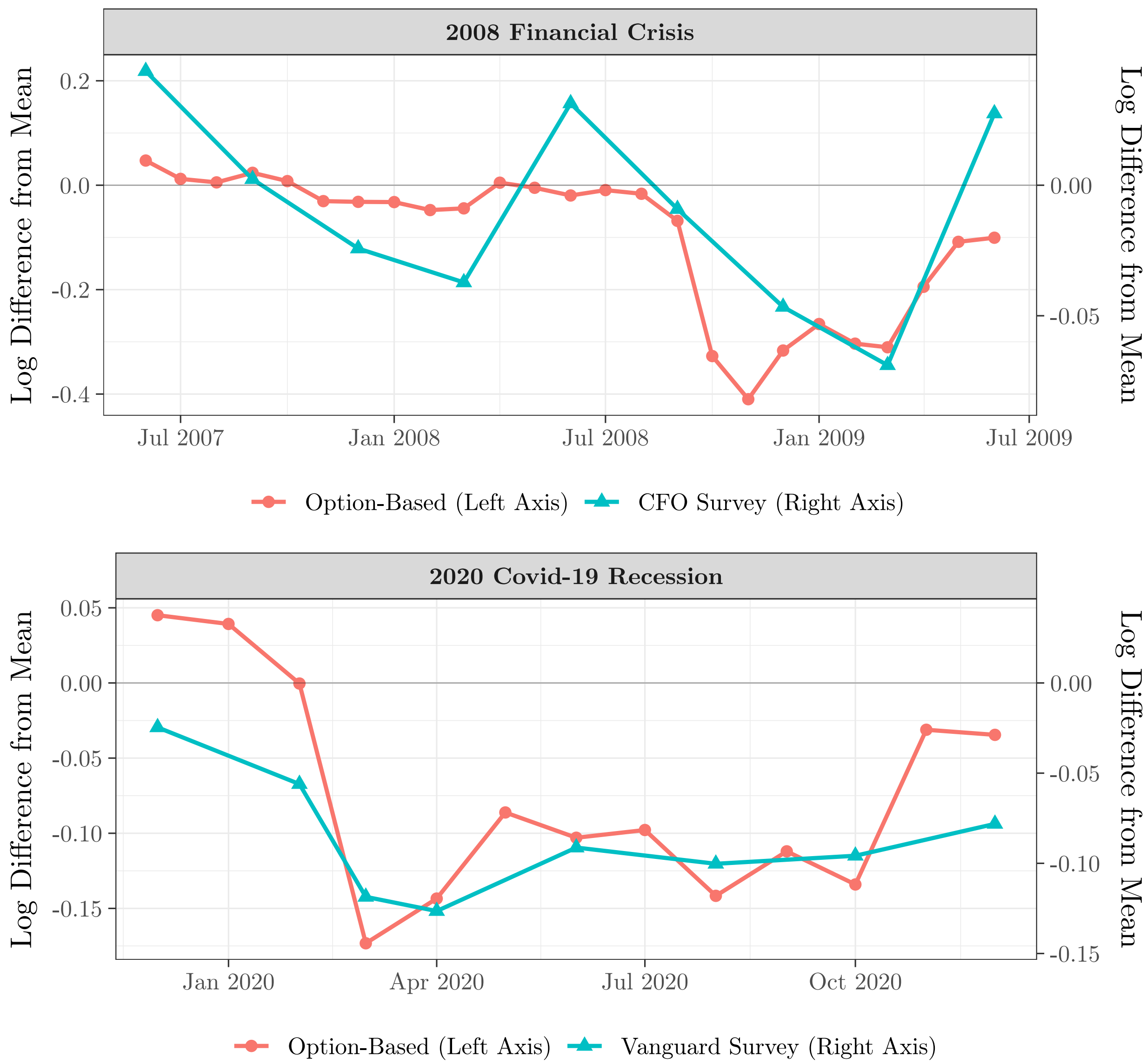


Figure 10
The Effect of Forecast Errors on Realized Returns for the Equity Term Structure

This figure shows the impact of predicable variation in forecast errors on realized returns to dividend claims with different maturity. We calculate realized returns for each equity claim based on the assumption that spot rates are the same for all dividend claims and given by the spot rates from Proposition 1. Difference in realized returns arise from differences in exposure to forecast errors (duration) across claims. We divide the sample into good and bad times based on the ex ante CAPE ratio, with good times being the 80% of the months where the CAPE ratio is highest. See Section 5.2 for more details.

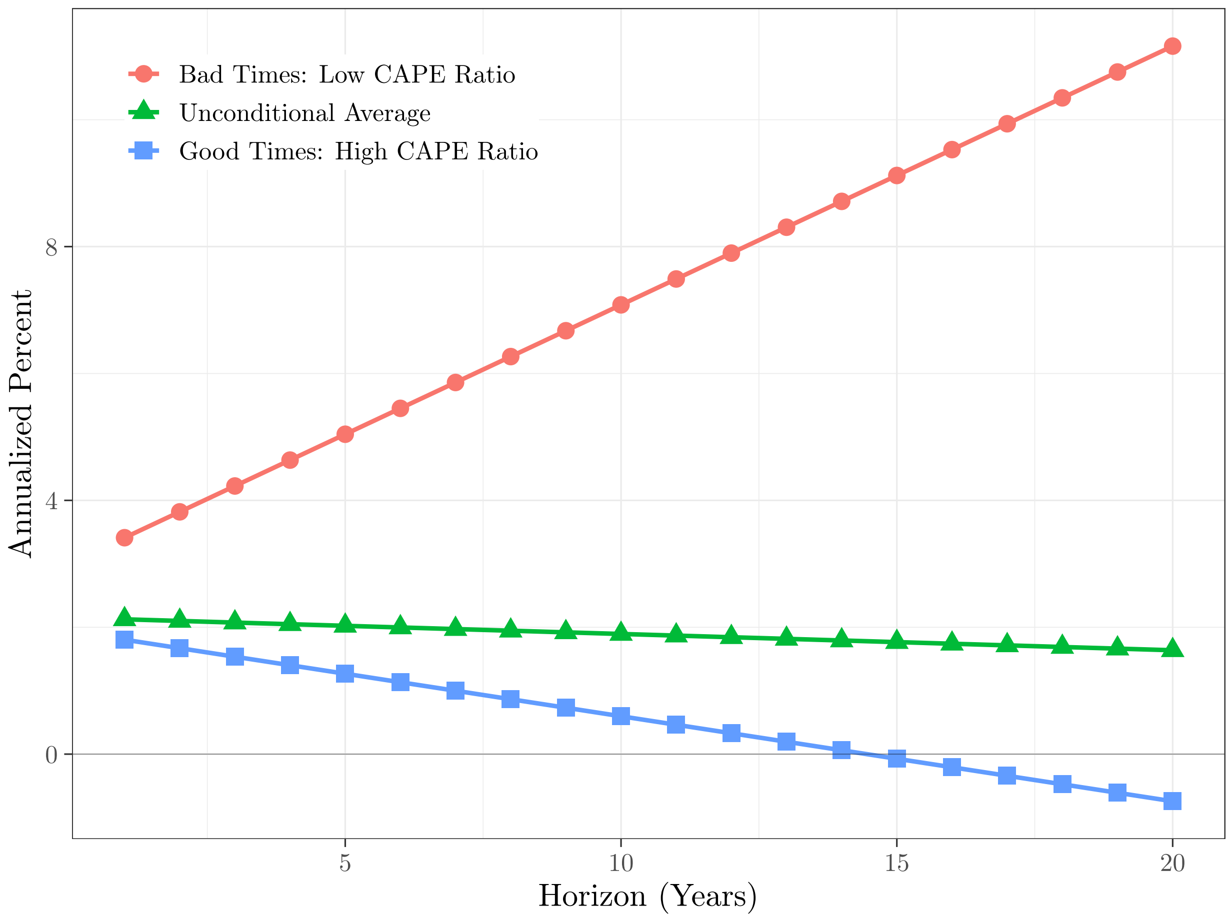
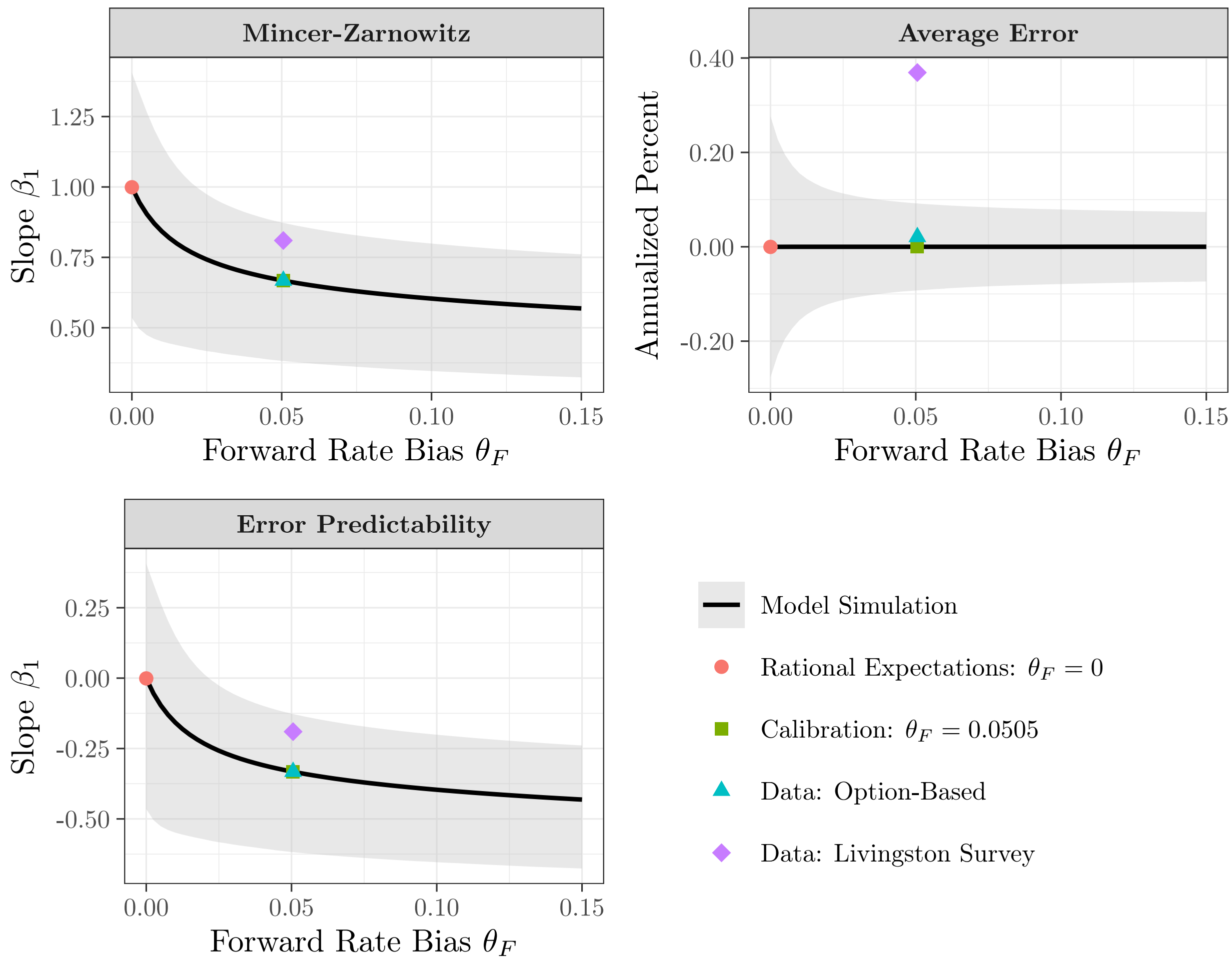


Figure 11
Model Calibration: Regression Slopes and Average Forecast Errors

This figure reports estimates in the calibrated model of expectation errors. The model is calibrated with option-based risk premia in the U.S. sample. The black line is the simulated population moment in a single long sample with $T = 37,800,000$ months. The gray ribbon is the simulated 95% confidence interval in 100,000 short samples with $T = 378$ months. The red circle is the simulated moment under rational expectations with $\theta_F = 0$. The green square is the simulated moment under forward rate bias with $\theta_F = 0.0505$. The teal triangle is the observed moment in the data with option-based expectations. The purple diamond is the observed moment in the data with Livingston survey expectations. The horizon is the 6-month spot rate, 6 months from now. The units are annualized percentage points. Short rate bias $\theta_S = 0$ for all simulated moments. See Appendix E.1 for more details.



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