Internet Appendix:

Excess Persistence in Return Expectations*

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A Proofs

Proof of Proposition 1. Given that $M_{t,t+n}R_{t,t+n}=1$ by assumption, $C_t^{(n)}=0$ in (4). The stated results then follow immediately.

Proof of Proposition 2. First consider n = m = 1, and write

$$\varepsilon_{t+1}^{(1)} = \mu_{t+1}^{(1)} - f_t^{(1)} = \mathcal{L}_{t+1}^{(1)} - \mathcal{L}_t^{(2)} + \mathcal{L}_t^{(1)} + \operatorname{cov}_t(MR_{t,t+2}, r_{t,t+2}) - \operatorname{cov}_t(MR_{t,t+1}, r_{t,t+1}) - \operatorname{cov}_{t+1}(MR_{t+1,t+2}, r_{t+1,t+2}). \tag{A1}$$

Consider the first covariance term. Given the joint log-normality of the SDF and returns (and the normality of $r_{t,t+n}$), Stein's lemma gives that

$$cov_{t}(MR_{t,t+2}, r_{t,t+2}) = cov_{t}(MR_{t,t+1}MR_{t+1,t+2}, r_{t,t+2})$$

$$= cov_{t}(mr_{t,t+1} + mr_{t+1,t+2}, r_{t,t+2}) \mathbb{E}_{t}[MR_{t,t+2}]$$

$$= cov_{t}(mr_{t,t+1} + mr_{t+1,t+2}, r_{t,t+1} + r_{t+1,t+2}),$$

where $mr_{t,t+n} = \ln(MR_{t,t+n})$, and where the last line uses that $\mathbb{E}_t[MR_{t,t+2}] = 1$. Having separated the two MR terms, apply Stein's lemma again to obtain

$$cov_t(MR_{t,t+2}, r_{t,t+2}) = cov_t(mr_{t,t+2}, r_{t,t+1}) + cov_t(mr_{t,t+1}, r_{t+1,t+2}) + cov_t(mr_{t+1,t+2}, r_{t,t+2})
= cov_t(MR_{t,t+2}, r_{t,t+1}) + cov_t(MR_{t,t+1}, r_{t+1,t+2})
+ cov_t(MR_{t+1,t+2}, r_{t+1,t+2}).$$
(A2)

For the first two terms in (A2), by the law of total covariance and using that $\mathbb{E}_{t+1}[MR_{t+1,t+2}] = 1$,

$$cov_{t}(MR_{t,t+2}, r_{t,t+1}) = \mathbb{E}_{t} \left[MR_{t,t+1}r_{t,t+1} cov_{t+1}(MR_{t+1,t+2}, 1) \right]
+ cov_{t}(MR_{t,t+1} \mathbb{E}_{t+1}[MR_{t+1,t+2}], r_{t,t+1})
= cov_{t}(MR_{t,t+1}, r_{t,t+1}),$$
(A3)
$$cov_{t}(MR_{t,t+1}, r_{t+1,t+2}) = \mathbb{E}_{t}[MR_{t,t+1} cov_{t+1}(1, r_{t+1,t+2})] + cov_{t}(MR_{t,t+1}, \mathbb{E}_{t+1}[r_{t+1,t+2}])
= cov_{t}(MR_{t,t+1}, \mathbb{E}_{t+1}[r_{t+1,t+2}]).$$
(A4)

Turning now to the last term in (A1), the law of total covariance can similarly be applied to obtain that as of time t,

$$\mathbb{E}_t[\text{cov}_{t+1}(MR_{t+1,t+2}, r_{t+1,t+2})] = \text{cov}_t(MR_{t+1,t+2}, r_{t+1,t+2}). \tag{A5}$$

Taking expectations in (A1), substituting in results (A2)–(A5), and applying the definition of $\hat{\varepsilon}_{t+1}^{(1)}$, we obtain:

$$\mathbb{E}_t\left[\varepsilon_{t+1}^{(1)}\right] = \mathbb{E}_t\left[\widehat{\varepsilon}_{t+1}^{(1)}\right] + \operatorname{cov}_t(MR_{t,t+1}, \mathbb{E}_{t+1}[r_{t+1,t+2}]). \tag{A6}$$

Rearranging to solve for $\mathbb{E}_t[\widehat{\varepsilon}_{t+1}^{(1)}]$ yields the stated result for the n=m=1 case. While this case is convenient for straightforward derivations, note that all the above steps apply when using t+n in place of t+1 and using t+n+m in place of t+2, so the stated result holds for general n,m. \square

Proof of Proposition 3. Starting again with (A1) and expanding the first covariance term,

$$cov_t(MR_{t,t+2}, r_{t,t+2}) = cov_t(MR_{t,t+1}MR_{t+1,t+2}, r_{t,t+1}) + cov_t(MR_{t,t+1}MR_{t+1,t+2}, r_{t+1,t+2}).$$
(A7)

We consider each of the two terms on the right side of (A7) in turn, and in both cases apply the law of total covariance. For the first term, as in (A3),

$$cov_t(MR_{t,t+1}MR_{t+1,t+2}, r_{t,t+1}) = cov_t(MR_{t,t+1}, r_{t,t+1}).$$
(A8)

For the second term,

$$cov_{t}(MR_{t,t+1}MR_{t+1,t+2}, r_{t,t+2}) = \mathbb{E}_{t}[MR_{t,t+1} cov_{t+1}(MR_{t+1,t+2}, r_{t+1,t+2})] + cov_{t}(MR_{t,t+1}, \mathbb{E}_{t+1}[r_{t+1,t+2}]).$$
(A9)

Using (A8) and (A9) in (A1), applying the definition of $\hat{\varepsilon}_{t+1}^{(1)}$, and taking expectations,

$$\mathbb{E}_{t}\left[\varepsilon_{t+1}^{(1)}\right] = \mathbb{E}_{t}\left[\widehat{\varepsilon}_{t+1}^{(1)}\right] + \mathbb{E}_{t}\left[\left(MR_{t,t+1} - 1\right)\operatorname{cov}_{t+1}\left(MR_{t+1,t+2}, r_{t+1,t+2}\right)\right]
+ \operatorname{cov}_{t}\left(MR_{t,t+1}, \mathbb{E}_{t+1}\left[r_{t+1,t+2}\right]\right)
= \mathbb{E}_{t}\left[\widehat{\varepsilon}_{t+1}^{(1)}\right] + \operatorname{cov}_{t}\left(MR_{t,t+1}, \mathbb{E}_{t+1}\left[r_{t+1,t+2}\right] + \operatorname{cov}_{t+1}\left(MR_{t+1,t+2}, r_{t+1,t+2}\right)\right).$$
(A10)

Note from (4) that $\mathcal{L}_{t+1}^{(1)} = \mathbb{E}_{t+1}[r_{t+1,t+2}] + \text{cov}_{t+1}(MR_{t+1,t+2}, r_{t+1,t+2})$. Using this in (A10),

$$\mathbb{E}_t \left[\widehat{\varepsilon}_{t+1}^{(1)} \right] = \mathbb{E}_t \left[\varepsilon_{t+1}^{(1)} \right] - \text{cov}_t \left(M R_{t,t+1}, \mathcal{L}_{t+1}^{(1)} \right).$$

The above steps again apply when using t+n in place of t+1 and using t+n+m in place of t+2, completing the proof.

Proof of Lemma A1. To compute the risk-neutral expectation of $H[P_T] = R^{\alpha} (\ln R)^{\beta}$, we apply standard spanning theorems (Bakshi and Madan 2000, Carr and Madan 2001). We have

$$\frac{1}{R_f} \mathbb{E}_t^* \left[R^{\alpha} \left(\ln R \right)^{\beta} \right] = \left(H \left[\bar{P} \right] - \bar{P} H_P \left[\bar{P} \right] \right) \frac{1}{R_f} + H_P \left[\bar{P} \right] P_t
+ \int_0^{\bar{P}} H_{PP} \left[K \right] \operatorname{put}_t^{(n)}(K) dK + \int_{\bar{P}}^{\infty} H_{PP} \left[K \right] \operatorname{call}_t^{(n)}(K) dK.$$

The result follows by setting $\bar{P} = F_t^{(n)}$ and simplifying.

Proof of Proposition A1. (A13) is immediate from Martin (2017) Result 8. (A14) and (A15) follow from Lemma A1 by setting the appropriate α and β and simplifying.

B Measurement Details

B.1 Data

United States Data. For the 1996 to 2021 period, we obtain end-of-day option prices, index prices, projected dividend yields, and risk-free rates from OptionMetrics. To maximize the sample size, we use options with both AM and PM settlement. We use the bid/ask midpoint as the option price in

the main analysis. We linearly interpolate the risk-free rate curve to match option maturities. If either the dividend yield or risk-free rate is missing, we use the last non-missing observation.

For the 1990 to 1995 period, we obtain intraday option quotes from CBOE Market Data Express, as in Kelly, Pástor, and Veronesi (2016) and Culp, Nozawa, and Veronesi (2018). We obtain end-of-day index prices/returns from CRSP and estimate dividend yields from lagged one-year cum/ex-dividend index returns. We obtain Treasury bill rates and constant maturity Treasury yields from FRED to construct risk-free rates, as in Culp, Nozawa, and Veronesi.

Unlike OptionMetrics, CBOE provides intraday quotes. To construct end-of-day prices, we first apply filters to the intraday data and then use the last available quote. We drop quotes with the missing codes of 998 or 999. We drop quotes with negative bid-ask spreads. We correct erroneously recorded quotes – quotes with strike price less than 100 – by multiplying the strike/option price by 10. We drop end-of-day quotes that increase and then decrease fourfold (or vice versa), following similar filters in Andersen, Bondarenko, and Gonzalez-Perez (2015) and Duarte, Jones, and Wang (2022). We interpret these large reversals as probable data errors. To validate these filters, we compare data from CBOE and OptionMetrics in 1996. We match approximately 99.3% of option prices in OptionMetrics, suggesting these filters are not unreasonable.

We apply standard filters to the end-of-day data (Constantinides, Jackwerth, and Savov 2013). (1) We drop options with special settlement. (2) To eliminate duplicate quotes, we select the quote with highest open interest. (3) We drop options with fewer than seven days-to-maturity. (4) We drop options with price less than 0.01. (5) We drop options with zero bid prices or negative bid-ask spreads. (6) We drop options that violate static no-arbitrage bounds:

$$\operatorname{put}_t^{(n)}(K) \le Ke^{-r\tau} \qquad \operatorname{call}_t^{(n)}(K) \le P_t.$$

(7) We drop options for which the Black-Scholes implied volatility computation does not converge and options with implied volatility less than 5% or greater than 100%.

International Data. We again obtain end-of-day option prices, index prices, projected dividend yields, and risk-free rates from OptionMetrics. Unlike the United States, most option prices are either end-of-day settlement prices or last traded prices. Only a small fraction are from either bid/ask prices. The index price is time synchronized to the option price. If the index price is missing, we obtain the end-of-day price from Compustat Global. Risk-free rates are from currency-matched LIBOR curves. Dividend yields are from put-call parity and so are maturity-specific. As before with risk-free rates, we linearly interpolate the dividend yield curve to match option maturities. We apply the same filters to the end-of-day data as with the United States, except for filters that require bid/ask prices.

Table A1 describes the international sample. The European sample begins in January 2002 and ends in September 2021. The Asian sample begins in January 2004 and ends in April 2021. Our international sample closely follows Kelly, Pástor, and Veronesi (2016) and Dew-Becker and Giglio (2021), but we also use pan-European Stoxx indexes. These indexes represent a substantive addition to the sample. At long maturities, the Euro Stoxx 50 is arguably the most liquid options market in the world, as is the case with the dividend futures market (Binsbergen and Koijen 2017, Binsbergen et al. 2013).

Main Sample. As the international data is not equally robust across exchanges, we select the most reliable exchanges for the main analysis. We select the main sample by elimination. We drop Netherlands and Japan because they do not have sufficiently dense options, as seen in Panel A of Figure A1. We drop Finland, the Stoxx Europe 50, and the Stoxx Europe 600 because they do not

have reliable open interest data, as seen in Panel B of Figure A1. We drop Belgium, Korea, and Taiwan because they do not consistently have long-maturity options, as seen in Panel D of Figure A2. We drop China and Sweden because they do not have sufficiently deep out-of-the-money options, as seen in Figure A2. This leaves the 10 exchanges in the last column of Table A1 for the main sample. As a robustness check, we examine the full sample of 20 exchanges in Table A4 and Table A5.

B.2 Baseline Measures

Methodology. On each date and separately for puts/calls,

- 1. We convert option prices to implied volatilities via Black-Scholes. Here we follow an extensive literature on option-implied risk-neutral densities that finds interpolation more conducive in the space of implied volatilities, not option prices (Figlewski 2010, Malz 2014).
- 2. We fit a Delaunay triangulation to implied volatilities. The grid consists of strike prices between $\underline{K} = 0.10 \times P_t$ and $\overline{K} = 2.00 \times P_t$ with $\Delta K = 0.001 \times P_t$ and maturities $\tau = 30$, 60, 91, 122, 152, 182, 273, 365 days. The triangulation extrapolates as necessary with the nearest implied volatility in moneyness and time-to-maturity space.
- 3. We convert the triangulation of implied volatilities back to option prices via Black-Scholes. We then use the implied triangulation of option prices to evaluate the LVIX integral in (7) via Gaussian quadrature.
- 4. With the LVIX in hand, we can immediately compute spot rates, forward rates, and forecast errors under log utility via Proposition 1, as shown in Figure A3. Figure A4 plots contemporaneous 6-month spot rates and 6×6 -month forward rates in the full sample, analogous to Figure 2 in the United States.
- 5. We occasionally find negative forward rates. Gao and Martin (2021) argue that negative forward rates are unlikely theoretically and likely represent data errors. We follow Gao and Martin and drop such observations, but our results are not quantitatively sensitive to this choice.

Discussion. Three empirical challenges in the computation of option-implied moments – discretization, truncation, and interpolation bias – motivate our baseline methodology (Carr and Wu 2009, Jiang and Tian 2007). We discuss each in turn. First, discretization bias arises because (7) requires numerical integration. To minimize this bias, we integrate on a fine grid of interpolated option prices in step 3. Second, truncation bias arises because (7) requires integration over an infinite range of strike prices in theory. In practice, we truncate the integral. To minimize this bias, we extrapolate and integrate over strike prices well beyond the range of observable option prices in step 2. Finally, interpolation bias arises because (7) usually requires options with unavailable maturities. To address this bias, we interpolate the option surface at target maturities in step 2.

B.3 Alternative Measures

Table A3 reports robustness checks where we use alternative choices to measure spot rates, forward rates, and forecast errors. As we discuss below, the main results are largely robust to these choices.

Integration Bounds. Panel A repeats the analysis with alternative integration bounds. The first four rows consider static bounds without extrapolation. As an example, the first row evaluates the

integral in (7) between strike prices $\underline{K} = 0.65 \times P_t$ and $\overline{K} = 1.35 \times P_t$ at each maturity. The fifth row uses observable option prices between strike prices $\underline{K} = 0.10 \times P_t$ and $\overline{K} = 2.00 \times P_t$, again without extrapolation. The bounds in the first five rows naturally vary both by time and maturity with the availability of option prices. The sixth row considers static bounds with extrapolation, following a similar robustness check in Gormsen and Jensen (2022):

$$\left[\underline{K}^{(n)}, \overline{K}^{(n)}\right] = \begin{cases} [0.75, 1, 25] \times P_t & n \in \{1, 2\} \\ [0.55, 1.45] \times P_t & n \in \{3, 4, 5\} \\ [0.35, 1.65] \times P_t & n \in \{6, 9\} \\ [0.20, 1.80] \times P_t & n \in \{12\} \end{cases}.$$

These bounds vary by maturity, but not by time. The seventh row considers dynamic bounds with extrapolation, again following a similar robustness check in Gormsen and Jensen:

$$\underline{K}^{(n)} = \max\left\{0.10, 1.00 - 5\sigma_t^{(n)}\sqrt{\tau}\right\} \times P_t \qquad \overline{K}^{(n)} = \min\left\{2.00, 1.00 + 5\sigma_t^{(n)}\sqrt{\tau}\right\} \times P_t,$$

where $\sigma_t^{(n)}$, the price of the volatility contract in Bakshi, Kapadia, and Madan (2003), proxies for the risk-neutral volatility of the market return:

$$\left(\sigma_t^{(n)} \sqrt{\tau}\right)^2 = \frac{1}{R_{t,t+n}^f} \mathbb{E}_t^* \left[(\ln R_{t,t+n})^2 \right]
= \int_0^{P_t} \frac{2\left(1 + \ln\left[\frac{P_t}{K}\right]\right)}{K^2} \operatorname{put}_t^{(n)}(K) dK + \int_{P_t}^{\infty} \frac{2\left(1 + \ln\left[\frac{P_t}{K}\right]\right)}{K^2} \operatorname{call}_t^{(n)}(K) dK.$$
(A11)

These bounds vary by both time and maturity with volatility. The eighth row considers the baseline integration bounds, as discussed in the main text and Appendix B.2.

In sum, this exercise illustrates the significant effect truncation/extrapolation have on the regression estimates. With shallow bounds, forecast errors are relatively large on average but less predictable. With deep bounds, forecast errors are relatively small on average but more predictable.

Liquidity Filters. Panel B repeats the analysis with alternative liquidity filters. The first row considers an outlier filter, following similar filters in Constantinides, Jackwerth, and Savov (2013) and Beason and Schreindorfer (2022). On each date and separately for puts/calls, we first fit a quadratic function to implied volatilities in terms of moneyness K/P and time-to-maturity. To minimize the effect of deep out-of-the-money, short/long-maturity options, we only use options with maturity $14 \le \tau \le 365$ days and moneyness $0.65 \le K/P \le 1.35$. We then drop influential observations via Cook's Distance. The second row considers an open interest filter. We drop options with zero open interest. We do not have open interest data before 1996. The third row combines the outlier and open interest filters. In all, this exercise is consistent with the baseline results, suggesting that option illiquidity does not explain our findings.

Volatility Surface. The first row in Panel C repeats the analysis with the interpolated volatility surface from OptionMetrics. OptionMetrics provides interpolated Black-Scholes implied volatilities on a constant moneyness/maturity grid. The literature often uses this surface for options with American exercise because OptionMetrics reports an equivalent, European exercise, implied volatility (Kelly, Lustig, and Van Nieuwerburgh 2016, Martin and Wagner 2019). We instead simply use it as a robustness check on our own Delaunay triangulation of the volatility surface. In short, this

exercise is consistent with the baseline results, although the average forecast error is somewhat smaller.

SVI Surface. The second row in Panel C repeats the analysis with the stochastic volatility inspired (SVI) surface from Jim Gatheral at Merrill Lynch (Gatheral 2011, Gatheral and Jacquier 2011, 2014). Our implementation of the SVI surface closely follows Berger, Dew-Becker, and Giglio (2020) and Beason and Schreindorfer (2022). We parameterize squared Black-Scholes implied volatilities with the function

$$\sigma_{BS}^{2}(t,\kappa,\tau) = a + b\left(\rho\left(\kappa - m\right) + \sqrt{\left(\kappa - m\right)^{2} + \sigma^{2}}\right),\tag{A12}$$

where κ is standardized forward moneyness

$$\kappa = \frac{\ln K - \ln F_t^{(n)}}{\sigma_t^{(n)} \sqrt{\tau}},$$

 $\sigma_t^{(n)}$ proxies for the risk-neutral volatility of the market return as in (A11), and each parameter is a linear function of time-to-maturity (e.g., $a=a_0+a_1\tau$). On each date, we estimate parameters $\theta=(a_0,a_1,b_0,b_1,\rho_0,\rho_1,m_0,m_1,\sigma_0,\sigma_1)$ that minimize the implied volatility RMSE between the surface (A12) and the data, subject to standard no-arbitrage constraints: option prices are nonnegative and monotonic/convex in K (Aït-Sahalia and Duarte 2003). We check these constraints on a grid with moneyness between $-20 \le \kappa \le 0.50$ for puts, between $-0.50 \le \kappa \le 10$ for calls, and maturities $\tau=30$, 60, 91, 122, 152, 182, 273, 365 days. We estimate the surface with outlier-filtered, as discussed in Appendix B.3, out-of-the-money puts/calls: puts with $\kappa \le 0$ and calls with $\kappa \ge 0$. We estimate the surface separately for puts/calls and separately for short/long-maturity options: $14 \le \tau \le 122$ days and $122 < \tau \le 365$ days, respectively.

Bid/Ask Prices. Panel D repeats the analysis with bid/ask prices, following similar robustness checks in Martin (2017) and Gao and Martin (2021). We only have bid/ask prices in the United States. The first row reports the baseline results with the bid-ask midpoint. The second row repeats the analysis with bid prices, the third ask prices. In sum, this exercise is consistent with the baseline results, although the Coibion-Gorodnichenko regression slope is somewhat smaller with ask prices.

B.4 Power Utility Measures

This section derives the power utility analogue to the LVIX. To do so, we apply results from Martin (2017) and Gao and Martin (2021). We omit time subscripts throughout to minimize clutter.

LEMMA A1 (Spanning $\mathbf{R}^{\alpha} (\ln \mathbf{R})^{\beta}$). For any α and β ,

$$\frac{1}{R_f} \mathbb{E}_t^* \left[R^{\alpha} \left(\ln R \right)^{\beta} \right] = R_f^{\alpha} \left(\ln R_f \right)^{\beta} + \int_0^{F_t^{(n)}} \omega \left(\alpha, \beta \right) \operatorname{put}_t^{(n)}(K) dK + \int_{F_t^{(n)}}^{\infty} \omega \left(\alpha, \beta \right) \operatorname{call}_t^{(n)}(K) dK,$$

where

$$\omega\left(\alpha,\beta\right) = -\frac{\alpha\left(1-\alpha\right)m^{\beta} + \beta\left(1-2\alpha\right)m^{\beta-1} + \beta\left(1-\beta\right)m^{\beta-2}}{P_{t}^{2}}\left(\frac{K}{P_{t}}\right)^{\alpha-2}$$

and $m = \ln K - \ln P_t$.

As is well-known, under certain regularity conditions, we can compute the price of any function of the index price via a replicating portfolio of bonds, stocks, and options. We simply apply this

result to the function $R^{\alpha} (\ln R)^{\beta}$, which is useful for expectations under power utility below.

PROPOSITION A1 (Expected Equity Premium with Power Utility). From the standpoint of an unconstrained power utility investor fully invested in the market,

$$\mathbb{E}_t \left[\ln R \right] - \ln R_f = \frac{\mathbb{E}_t^* \left[R^{\gamma} \ln R \right]}{\mathbb{E}_t^* \left[R^{\gamma} \right]} - \ln R_f, \tag{A13}$$

where

$$\frac{1}{R_f} \mathbb{E}_t^* [R^{\gamma} \ln R] = R_f^{\gamma} \ln R_f + \int_0^{F_t^{(n)}} \omega(\gamma, 1) \operatorname{put}_t^{(n)}(K) dK + \int_{F_t^{(n)}}^{\infty} \omega(\gamma, 1) \operatorname{call}_t^{(n)}(K) dK$$
 (A14)

and

$$\frac{1}{R_f} \mathbb{E}_t^* [R^{\gamma}] = R_f^{\gamma} + \int_0^{F_t^{(n)}} \omega(\gamma, 0) \operatorname{put}_t^{(n)}(K) dK + \int_{F_t^{(n)}}^{\infty} \omega(\gamma, 0) \operatorname{call}_t^{(n)}(K) dK$$
 (A15)

and γ is the investor's risk aversion.

The LVIX uses a special case of (A13) with $\gamma = 1$, and so the mechanics under power utility are similar, if only messier, to that under log utility. However, there is one caveat: as risk aversion γ increases, the weights $\omega(\gamma, 0)$ and $\omega(\gamma, 1)$ on deep out-of-the-money call options become untenably large. Unfortunately, these options are largely unobservable. As such, we can only realistically measure expectations for a $\gamma \leq 3$ investor in practice.

Armed with the expected equity premium from the standpoint of a power utility investor, we can compute spot rates, forward rates, and forecast errors in the usual way. Table A8 reports results analogous to those under log utility for $0.5 \le \gamma \le 3$. Figure 5 plots the conditional volatility of the market return from the standpoint of a power utility investor. The mechanics are again a direct application of Martin (2017) and standard spanning theorems, as with the expected equity premium above.

B.5 Measurement Error

Spot/Forward Rates. To better understand the role of measurement error, Figure A5 examines spot/forward rates in simulations. We first compute option prices from a parametric model. Since we know the true data generating process, we then quantify how varying integration bounds affects the integral relative to the true value. The thought experiment follows a similar exercise in Jiang and Tian (2007) for the VIX.

We first truncate the integral (7) without extrapolation, as in Table A3. In Panel A, we consider a Black-Scholes model. We find a large truncation bias in bad times. In bad times, volatility is high, deep out-of-the-money options are expensive, and so the bias is large. In contrast, in good times, volatility is low, deep out-of-the-money options are cheap, and so the bias is small. The bias is especially large for 12-month spot rates and forward rates because longer-maturity option prices have more time value. In Panel B, we consider a stochastic volatility model with jumps (SVJ). We again find an uncomfortably large truncation bias. Relative to Black-Scholes, the bias is larger when volatility is low, but smaller when volatility is high because volatility mean-reverts in SVJ.

We next truncate the integral after extrapolating beyond the range of observable strikes, as in the baseline analysis. We continue with a SVJ model in Panel C. Relative to Panel B, we find that extrapolation reduces truncation bias across the board.

We emphasize, however, that this exercise only motivates extrapolation in our baseline integration scheme and our use of shorter-maturity forward rate as instruments/predictors, as in Table 3 and Table 5. We make no claim that measurement error is unconditionally small. By construction, these simulations address only truncation bias. There is no scope for either discretization or interpolation bias, as we simulate option prices on a counterfactually dense grid. We think these biases may be non-trivial at times and especially so when options are less dense.

Coibion-Gorodnichenko Regressions. In Coibion-Gorodnichenko regressions, we use forecast revisions to predict forecast errors. Since forecast revisions/errors involve the same forward rate, measurement error may produce spurious evidence of predictability. To better understand the role of measurement error, we again turn to simulations. We quantify how much correlated measurement error would be necessary to produce the Coibion-Gorodnichenko regression slopes in the data.

We assume we observe forecast revisions and forecast errors with noise: $\tilde{x} = x\sigma_x + v\sigma_v$ and $\tilde{y} = y\sigma_y - v\sigma_v$, respectively, with $\sigma_{xy} = \sigma_{xv} = \sigma_{yv} = 0$ and $x, y, v \stackrel{\text{i.i.d.}}{\sim} \mathcal{N}(0, 1)$. We vary σ_v^2 exogenously. In each simulation draw, we set the variance of the truth $(\sigma_x^2 \text{ and } \sigma_y^2)$ such that the observed variance $(\sigma_{\tilde{x}}^2 \text{ and } \sigma_{\tilde{y}}^2)$ equals that in the data. As σ_v^2 varies, these weights ensure all variation in slopes comes from variation in noise and none from variation in observed variances. Any evidence of predictability – any non-zero slope – is spurious because $\sigma_{xy} = 0$.

Figure A6 reports the results from this simulations. To produce the Coibion-Gorodnichenko regression slopes in the data, we require σ_v be about 40 basis points or more than one-quarter the volatility of forecast errors in the data. This, at least to us, seems implausibly large. We conclude that correlated measurement error cannot fully explain forecast-error predictability in Coibion-Gorodnichenko regressions, although we cannot fully rule out some bias due to measurement error.

C Additional Empirical Results and Robustness Checks

Full Sample. Table A4 considers how our results extend to the full sample with all 20 available exchanges, as opposed to the 10 developed-market exchanges used in our main panel. The table presents results from a concise set of key regressions from Tables 2–5 in the main text. The main results hold here. The Mincer-Zarnowitz regression coefficients are similar to (in fact slightly below) the results in the main sample; the average forecast error is nearly identical to that in the main sample; and the error-predictability and Coibion-Gorodnichenko results are slightly stronger in the extended sample than in the main sample.

Additional Robustness Checks. Table A5 examines additional robustness checks. Panel A reports the baseline results. Panel B winsorizes spot rates, forward rates, forecast errors, and forecast revisions at the 2.5% level by exchange. Panel C is the trimming analogue to Panel B. Panel D repeats the analysis in balanced panels. Panel E repeats the analysis in subsamples. This exercise is generally consistent with the baseline results, although the Coibion-Gorodnichenko regression slopes are sensitive to winsorization/trimming and the forecast errors are somewhat less predictable in the later subsample.

Alternative Horizons. Table A6 examines alternative horizons for Mincer-Zarnowitz, average error, and error-predictability regressions. The baseline analysis in Section 4 considers the 6-month spot rate in 6 months (n = m = 6 in Panel E). This exercise illustrates the effect of the horizon on the regression estimates. Holding n + m fixed, forecast errors are relatively small on average and

less predictable with small n; forecast errors are relatively large on average and more predictable with large n.

Table A7 examines monthly forecast revisions for Coibion-Gorodnichenko regressions. The baseline analysis considers quarterly forecast revisions. The Coibion-Gorodnichenko regression slopes are generally similar between horizons, although the slope in the United States is substantially smaller with monthly revisions.

Power Utility Regressions. Table A8 presents results for option-based risk-premium forecast errors when re-estimated under the assumption of an unconstrained investor with power utility, rather than log utility, for different values of constant relative riska version. See Appendix B.4 above and Section 4.1.5 in the text for details and discussion.

Long-Horizon Predicted Forecast Errors. For the forecast-error quantification in Section 5.1, we re-estimate spot rates, forward rates, and forecast errors at longer horizons (up to m+n=8years) for the Euro Stoxx 50. The sample runs from September 2005 through September 2014 (beyond which we cannot yet observe realized forecast errors). The combinations of m and n (in months) can be seen in Table A9. For each such combination, we predict forecast errors as in (9) using a regression of realized forecast errors on shorter-horizon forward rates; we use the $n-12\times12$ forward rate (with horizons again now in months) for $n \geq 24$, and for n = 12 we use the 6×6 rate. After obtaining these predicted forecast errors, we calculate a decay parameter for each date's forecast errors, $\phi_t^{(n,m)}$, as the ratio of estimated $\mathbb{E}_t[\varepsilon_{t+n}^{(m)}]$ to $\mathbb{E}_t[\varepsilon_{t+12}^{(12)}]$ for each available m, n > 12. This decay specification builds on the one used by De la O and Myers (2021, eq. (13)). The entries of Table A9 report the median decay parameter over all t for each combination of m and n. In all cases the estimates are very close to 1. Assuming that predictable forecast errors are permanent at all horizons might be thought of as providing an estimate of their maximal possible effect. That said, when we estimate the decay parameter in the U.S. (at shorter horizons, unreported), we in fact generally obtain estimates greater than 1, suggesting that setting $\phi^{(n,m)} = 1$ may, if anything, be slightly conservative in the U.S. sample.

D Additional Model Discussion: A Trilemma for Expectation Errors

This appendix continues the discussion in Section 6.3 on how different moments of the data are tied together by the cyclicality of forecast errors. We begin with the Campbell-Shiller price-dividend decomposition in (11). Assume that the expectations $\mathbb{E}_t[\cdot]$ in that decomposition refer to agents subjective beliefs, and $p_t - d_t$ is the observed log price-dividend ratio. Now consider an alternative economy in which all agents have rational expectations. For arbitrary equilibrium variable x_t in the observed data, denote the corresponding variable in the alternative RE economy by x_t^{RE} . Define the wedge between these two variables to be $\tilde{x}_t = x_t - x_t^{RE}$. For example, $p_t - d_t$ is the wedge between the observed price-dividend ratio and the one that would be observed in the alternative economy with RE. Up to a constant, it satisfies

$$\widetilde{p_t - d_t} = \widetilde{CF_t} - \widetilde{\mathcal{F}_t} - \widetilde{RF_t}. \tag{A16}$$

Assume for simplicity that $\widetilde{RF}_t = 0$. The following variance decomposition for the price-dividend wedge therefore holds:

$$\operatorname{var}\left(\widetilde{p_t - d_t}\right) = \operatorname{var}\left(\widetilde{CF_t}\right) + \operatorname{var}\left(\widetilde{\mathcal{F}_t}\right) - 2\operatorname{cov}\left(\widetilde{CF_t}, \widetilde{\mathcal{F}_t}\right). \tag{A17}$$

Alternatively, one can also use the following decomposition given (A16):

$$\operatorname{var}\left(\widetilde{p_t - d_t}\right) = \operatorname{cov}\left(\widetilde{p_t - d_t}, \widetilde{CF_t}\right) - \operatorname{cov}\left(\widetilde{p_t - d_t}, \widetilde{\mathcal{F}_t}\right). \tag{A18}$$

The wedges \widetilde{CF}_t and $\widetilde{\mathcal{F}}_t$ can be understood as expectation errors along the lines considered in Section 6: if subjective expectations are too high relative to RE, then the wedge will be positive (and forecast errors, defined as realized – forecast, are likely to be negative). According to either of the decompositions in (A17)–(A18), therefore, one must choose from at most two of the following three features of any model of expectation errors:

- 1. Volatile expectation errors for returns (and/or fundamentals)
- 2. Volatile price-dividend ratio relative to a rational benchmark
- 3. Countercyclical return expectation errors (positive return expectation errors in bad times)

For example, if excessively positive cash-flow and return forecast revisions occur in good times (after positive news), then $\operatorname{cov}\left(\widetilde{CF}_t,\widetilde{\mathcal{F}}_t\right)>0$ in (A17). Alternatively, in the version expressed in (A18), positive comovement between price-dividend and forward-rate wedges similarly detracts from a model's ability to generate volatile $\widetilde{p_t-d_t}$. This form of overreaction to realized outcomes (cash flows and/or returns) may be intuitively appealing, but it limits a model's ability to speak to variation in the price-dividend ratio through expectation errors alone.

Our empirical results, and our model of expectation errors, instead suggest overreaction of forward rates to *spot rates*, rather than realized returns. Unlike realized returns, we find that spot and forward rates *increase* in bad times. The negative covariance between fundamental news and return expectation errors in principle allows for a volatile price-dividend ratio.

¹For example, Nagel and Xu (2022) obtain a price-dividend ratio volatility about 50% lower than that observed in the data (see their Table 5). Similarly, De la O and Myers (2021) report that in the model of Barberis et al. (2015), "movements in dividend change expectations are almost completely negated by movements in price change expectations. This leads to low variation in the price-dividend difference" (p. 1370); Campbell (2017) provides a related discussion of the Barberis et al. (2015) results.

Appendix Tables and Figures

Table A1
Option Sample

This table reports the region, the abbreviation, the underlying index, the sample period, and the sample length in months for each exchange. The last column indicates whether the exchange is in the main sample. See Appendix B.1 for more details.

Region	Abbrv	Index	Start	End	Length	Main
Panel A. North America	ca					
United States	USA	S&P~500	199001	202112	384	Y
Panel B. Europe						
Belgium	BEL	BEL 20	200201	202109	228	
Switzerland	CHE	SMI	200201	202109	237	Y
Germany	DEU	DAX	200201	202109	237	Y
Spain	ESP	IBEX 35	200610	202109	180	Y
Finland	FIN	OMXH25	200201	202109	237	
France	FRA	CAC 40	200304	202109	222	Y
United Kingdom	GBR	FTSE 100	200201	202109	237	Y
Italy	ITA	FTSE MIB	200610	202109	180	Y
Netherlands	NLD	AEX	200201	202109	219	
Sweden	SWE	OMXS30	200705	202109	173	
Panel C. Pan-Europe						
Euro Stoxx 50	SX5E	SX5E	200201	202109	237	Y
Stoxx Europe 50	SX5P	SX5P	200201	202109	237	
Stoxx Europe 600	SXXP	SXXP	200509	202109	193	
Panel D. Asia						
Australia	AUS	ASX 200	200401	202104	208	Y
China	CHN	HSCEI	200601	202104	184	
Hong Kong	HKG	Hang Seng	200601	202104	184	Y
Japan	JPN	Nikkei 225	200405	202104	204	
Korea	KOR	KOSPI 200	200407	202104	202	
Taiwan	TWN	TAIEX	200510	202104	187	

Table A2 Summary Statistics

This table reports summary statistics for the 6-month spot rate $\mu_{t+6}^{(6)}$ (left panel) and the 6×6 -month forward rate $f_t^{(6,6)}$ (right panel). The units are annualized percentage points. The sample is the longest available for each exchange in the full sample.

	6-	Month Spot 1	Rate $\mu_{t+6}^{(6)}$	3	6 × 6-	Month Forwa	rd Rate	$f_t^{(6,6)}$
	Mean	St. Dev.	p10	p90	Mean	St. Dev.	p10	p90
Panel A. N	orth Ame	rica						
USA	2.17	1.40	0.97	3.81	2.15	0.99	1.11	3.45
Panel B. E	urope							
BEL	2.42	2.07	0.95	3.83	2.56	2.22	1.07	4.24
CHE	1.96	1.34	0.98	3.25	1.88	0.80	1.19	2.71
DEU	2.85	1.86	1.38	4.75	2.63	1.07	1.57	3.97
ESP	3.37	1.83	1.50	5.92	2.89	1.17	1.60	4.45
FIN	2.79	2.21	0.91	6.00	2.38	1.85	0.82	4.42
FRA	2.57	1.56	1.23	4.36	2.44	1.04	1.43	3.87
GBR	2.21	1.51	0.97	3.95	2.12	1.00	1.19	3.42
ITA	3.60	1.67	2.03	5.86	3.17	1.35	1.85	5.21
NLD	2.85	2.18	1.10	5.11	2.43	1.26	1.24	4.07
SWE	2.79	1.80	1.36	4.58	2.44	1.41	1.41	3.77
Panel C. P	an-Europe	;						
SX5E	2.89	1.83	1.31	5.15	2.65	1.08	1.52	4.12
SX5P	2.23	1.62	1.04	3.86	2.11	1.48	1.12	3.40
SXXP	2.14	1.63	0.73	4.17	2.05	1.53	0.74	4.03
Panel D. A	sia							
AUS	1.97	1.39	0.86	3.52	1.84	1.20	0.87	3.18
CHN	4.24	4.01	2.06	7.42	3.90	3.29	2.20	6.37
$_{ m HKG}$	2.98	2.56	1.33	5.63	2.85	1.93	1.63	4.53
$_{ m JPN}$	2.93	2.20	1.49	4.15	2.60	1.57	1.55	3.48
KOR	2.23	2.16	0.85	3.91	2.14	1.85	0.88	3.77
TWN	2.39	1.90	0.98	4.50	2.36	1.45	1.17	4.22

Table A3 Alternative Measures

See Appendix B.3 for more details. Panels A to C report estimates from panel regressions in the main sample. The sample is the longest available for each exchange. Panel D reports estimates from time-series regressions in the United States. The sample is from January 1990 to June 2021. Mincer-Zarnowitz regressions test $H_0: \beta_1 = 1$, as in Table 2. The average forecast error tests $H_0: \bar{\varepsilon}_t = 0$, as in Table 4. Error-predictability regressions test $H_0: \beta_1 = 0$, as in Table 5. Coibion-Gorodnichenko regressions test $H_0: \beta_1 = 0$. The risk premium is the 6-month spot rate in 6 months for Mincer-Zarnowitz, average error, and error-predictability regressions. The risk premium is the 3-month spot rate in 6 months for Coibion-Gorodnichenko regressions. The units are annualized percentage points. All regressions include exchange fixed effects and report a within R^2 . Panel regressions, in the main sample, report standard errors clustered by exchange and date. Time-series regressions, in the United States, report Newey-West standard errors with lags selected following Lazarus et al. (2018) and fixed-b p-values.

						$\mu_{t}^{(0)}$	6) +6							μ	$\iota_{t+6}^{(3)}$		
	1	Mincer-Z	arnowit	\mathbf{z}	Ave	rage Eri	or	Err	or Predi	ctabilit	у		Coib	ion-Goro	dniche	nko	
	β_1	$se(\beta_1)$	p-val	R^2	$ar{arepsilon_t}$	$se(\bar{\varepsilon}_t)$	p-val	β_1	$se(\beta_1)$	p-val	R^2	N	β_1	$se(\beta_1)$	p-val	R^2	N
Panel A. Alternative Integration Bounds																	
Truncation: 35 Moneyness	0.78	0.091	**	0.18	0.34	0.094	***	-0.019	0.039		0.00	2139	-0.16	0.16		0.00	2104
Truncation: 45 Moneyness	0.68	0.074	***	0.17	0.26	0.097	**	-0.082	0.041	*	0.01	2137	-0.22	0.15		0.01	2103
Truncation: 55 Moneyness	0.63	0.065	***	0.16	0.22	0.099	*	-0.12	0.042	**	0.02	2140	-0.24	0.14		0.01	2106
Truncation: 65 Moneyness	0.59	0.059	***	0.15	0.20	0.10	*	-0.15	0.044	***	0.03	2138	-0.24	0.13	*	0.01	2104
Truncation: Observable Moneyness	0.56	0.053	***	0.15	0.19	0.10	*	-0.17	0.043	***	0.04	2140	-0.22	0.10	*	0.01	2106
Extrapolation: Static Moneyness	0.56	0.056	***	0.15	0.14	0.10		-0.15	0.041	***	0.04	2244	-0.16	0.12		0.01	2204
Extrapolation: Dynamic Moneyness	0.56	0.055	***	0.15	0.16	0.11		-0.17	0.046	***	0.03	2241	-0.25	0.12	*	0.01	2199
Extrapolation: Baseline	0.56	0.055	***	0.15	0.17	0.11		-0.16	0.047	***	0.03	2227	-0.24	0.12	*	0.01	2186
Panel B. Alternative Liquidity Filters																	
Outliers	0.60	0.055	***	0.17	0.18	0.098		-0.20	0.050	***	0.05	2241	-0.26	0.16		0.01	2208
Open Interest: After 01/1996	0.52	0.057	***	0.14	0.16	0.11		-0.18	0.045	***	0.04	2033	-0.26	0.13	*	0.02	1966
Outliers and Open Interest: After $01/1996$	0.56	0.051	***	0.16	0.19	0.097	*	-0.21	0.050	***	0.06	2040	-0.27	0.17		0.01	1975
Panel C. Alternative Surfaces																	
Volatility Surface: After 01/1996	0.57	0.051	***	0.17	0.087	0.095		-0.21	0.053	***	0.06	2163	-0.22	0.16		0.01	2131
SVI Surface: USA and SX5E	0.59	0.047	*	0.17	0.051	0.11		-0.19	0.039		0.04	609	-0.077	0.14		0.00	597
Panel D. Alternative Prices																	
Bid-Ask Midpoint: USA	0.67	0.096	***	0.22	0.021	0.15		-0.17	0.066	**	0.03	378	-0.29	0.12	**	0.01	375
Bid Prices: USA	0.64	0.099	***	0.21	0.034	0.14		-0.15	0.077	**	0.03	378	-0.34	0.12	***	0.02	374
Ask Prices: USA	0.66	0.089	***	0.23	0.0051	0.15		-0.18	0.060	***	0.04	378	-0.15	0.12		0.00	375

Table A4 Full Sample Regressions

This table reports estimates from panel regressions in the full sample. Columns (1)–(2) are Mincer-Zarnowitz regressions of future realized spot rates on forward rates, as in Table 2. Columns (3)–(4) are instrumented Mincer-Zarnowitz regressions, as in Table 3. Column (5) is the average forecast error, as in Table 4. Columns (6)–(7) are error-predictability regressions, as in Table 5. Columns (8)–(9) are Coibion-Gorodnichenko regressions. The risk premium is the 6-month spot rate in 6 months for Mincer-Zarnowitz, average error, and error-predictability regressions. The risk premium is the 3-month spot rate in 6 months for Coibion-Gorodnichenko regressions. The units are annualized percentage points. Standard errors are clustered by exchange and date. The sample is the longest available for each exchange in the full sample.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				$\mu_{t+6}^{(6)}$				μ	$t_{t+6}^{(3)}$
	Mincer-Z	Zarnowitz	Mincer-Za	rnowitz (IV)	Average Error	Average Error Error Predictability		Coibion-Ge	orodnichenko
	$\mu_{t+6}^{(6)}$	$\mu_{t+6}^{(6)}$	$\mu_{t+6}^{(6)}$	$\mu_{t+6}^{(6)}$	$-\!$	$\varepsilon_{t+6}^{(6)}$	$\varepsilon_{t+6}^{(6)}$	$\varepsilon_{t+6}^{(3)}$	$\varepsilon_{t+6}^{(3)}$
$f_t^{(6,6)}$	0.53*** (0.058)	0.48*** (0.053)	0.63*** (0.062)	0.58*** (0.061)					
$f_t^{(2,1)}$	(0.000)	(0.000)	(0.002)	(0.001)		-0.23*** (0.049)	-0.26*** (0.049)		
$f_t^{(6,3)} - f_{t-3}^{(9,3)}$						(0.010)	(0.010)	-0.35*** (0.11)	-0.36*** (0.11)
Intercept	1.33*** (0.16)		1.09*** (0.17)		0.19 (0.12)	0.81*** (0.16)		0.28* (0.14)	(0.11)
p -value: $\beta_1 = 1$	0.0000	0.0000	0.0000	0.0000	-	_	_	-	_
Observations	4199	4199	4199	4199	4199	4199	4199	4111	4111
Fixed Effects	None	$\mathbf{E}\mathbf{x}$	None	$\mathbf{E}\mathbf{x}$	None	None	$\mathbf{E}\mathbf{x}$	None	Ex
Adjusted \mathbb{R}^2	0.17	0.19	0.17	-	-	0.06	0.08	0.05	0.05
Within R^2	-	0.14	-	0.13	-	-	0.08	-	0.05

Table A5 Additional Robustness Checks

See Appendix C for more details. Mincer-Zarnowitz regressions test $H_0: \beta_1 = 1$, as in Table 2. The average forecast error tests $H_0: \bar{\varepsilon}_t = 0$, as in Table 4. Error-predictability regressions test $H_0: \beta_1 = 0$, as in Table 5. Coibion-Gorodnichenko regressions test $H_0: \beta_1 = 0$. The risk premium is the 6-month spot rate in 6 months for Mincer-Zarnowitz, average error, and error-predictability regressions. The risk premium is the 3-month spot rate in 6 months for Coibion-Gorodnichenko regressions. The units are annualized percentage points. All regressions include exchange fixed effects and report a within R^2 . Standard errors are clustered by exchange and date.

						$\mu_{t}^{(\epsilon)}$	5) ⊦6								$\mu_{t+6}^{(3)}$		
		Mincer-Zarnowitz			Ave	erage Er	ror	E	rror Pred	ictabili	ty		Coil	oion-Goro	odniche	nko	
	β_1	$se(\beta_1)$	p-val	R^2	$ar{arepsilon_t}$	$se(\bar{\varepsilon}_t)$	p-val	β_1	$se(\beta_1)$	p-val	R^2	N	β_1	$se(\beta_1)$	p-val	R^2	N
Panel A. Baseline																	
Main Sample: 01/1990 to 06/2021	0.56	0.055	***	0.15	0.17	0.11		-0.16	0.047	***	0.03	2227	-0.24	0.12	*	0.01	2186
Full Sample: $01/1990$ to $06/2021$	0.48	0.053	***	0.14	0.19	0.12		-0.26	0.049	***	0.08	4199	-0.36	0.11	***	0.05	4111
Panel B. Winsorization																	
Main Sample: 01/1990 to 06/2021	0.60	0.059	***	0.19	0.15	0.093		-0.15	0.048	**	0.03	2227	-0.16	0.10		0.01	2186
Full Sample: $01/1990$ to $06/2021$	0.56	0.073	***	0.20	0.16	0.095		-0.21	0.051	***	0.05	4199	-0.15	0.084	*	0.01	4111
Panel C. Trimming																	
Main Sample: 01/1990 to 06/2021	0.57	0.056	***	0.18	0.091	0.078		-0.10	0.049	*	0.01	2029	0.066	0.087		0.00	2008
Full Sample: 01/1990 to 06/2021	0.55	0.061	***	0.20	0.11	0.074		-0.15	0.047	***	0.03	3828	0.029	0.079		0.00	3791
Panel D. Balanced Panel																	
Main Sample: 10/2006 to 06/2019	0.49	0.058	***	0.12	0.22	0.13		-0.20	0.050	***	0.05	1674	-0.29	0.12	**	0.02	1658
Full Sample: 05/2007 to 06/2019	0.43	0.052	***	0.12	0.19	0.15		-0.31	0.041	***	0.10	3214	-0.39	0.11	***	0.06	3194
Panel E. Subsamples																	
Main Sample: 01/1990 to 10/2011	0.49	0.078	***	0.11	0.37	0.19	*	-0.25	0.064	***	0.07	1113	-0.41	0.14	**	0.03	1076
Main Sample: 11/2011 to 06/2021	0.30	0.12	***	0.06	-0.024	0.099		-0.11	0.081		0.01	1114	0.016	0.077		0.00	1110

Table A6 Alternative Horizons

See Appendix C for more details. Mincer-Zarnowitz regressions test $H_0: \beta_1 = 1$, as in Table 2. The average forecast error tests $H_0: \bar{\varepsilon}_t = 0$, as in Table 4. Error-predictability regressions test $H_0: \beta_1 = 0$, as in Table 5. The risk premium is the m-month spot rate in n months. The units are annualized percentage points. All regressions include exchange fixed effects and report a within R^2 . Standard errors are clustered by exchange and date. The sample is the longest available for each exchange in the main sample.

						$\mu_t^($	m) + n					
		Mincer-Z	arnowitz		Av	verage Err	or	Е	rror Predi	ctability		
	β_1	$se(\beta_1)$	p-val	R^2	$ar{arepsilon_t}$	$se(\bar{\varepsilon}_t)$	p-val	β_1	$se(\beta_1)$	p-val	R^2	N
Panel A. 4-Month Equity Premium												
1-Month Ahead: $n = 1, m = 3$	0.90	0.048	*	0.62	0.044	0.073		-0.12	0.040	**	0.03	2269
2-Months Ahead: $n = 2, m = 2$	0.77	0.058	***	0.35	0.055	0.11		-0.20	0.053	***	0.04	2268
3-Months Ahead: $n = 3, m = 1$	0.68	0.065	***	0.20	0.094	0.14		-0.24	0.055	***	0.04	2235
Panel B. 5-Month Equity Premium												
1-Month Ahead: $n = 1, m = 4$	0.94	0.043		0.67	0.067	0.064		-0.075	0.033	*	0.02	2269
2-Months Ahead: $n=2, m=3$	0.83	0.053	**	0.42	0.12	0.10		-0.13	0.043	**	0.02	2268
3-Months Ahead: $n = 3, m = 2$	0.74	0.063	***	0.25	0.16	0.13		-0.15	0.049	**	0.02	2241
4-Months Ahead: $n = 4$, $m = 1$	0.62	0.078	***	0.13	0.27	0.15		-0.19	0.053	***	0.02	2236
Panel C. 6-Month Equity Premium												
1-Month Ahead: $n=1, m=5$	0.95	0.037		0.70	0.067	0.058		-0.054	0.029	*	0.01	2269
2-Months Ahead: $n=2, m=4$	0.86	0.047	**	0.46	0.14	0.091		-0.094	0.036	**	0.01	2268
3-Months Ahead: $n = 3, m = 3$	0.78	0.058	***	0.30	0.21	0.12		-0.11	0.044	**	0.01	2249
4-Months Ahead: $n = 4, m = 2$	0.67	0.072	***	0.17	0.27	0.14	*	-0.15	0.050	**	0.02	2238
5-Months Ahead: $n = 5, m = 1$	0.57	0.076	***	0.09	0.32	0.17	*	-0.20	0.057	***	0.02	2228
Panel D. 9-Month Equity Premium												
3-Months Ahead: $n = 3, m = 6$	0.81	0.055	***	0.39	0.15	0.087		-0.065	0.036		0.01	2257
4-Months Ahead: $n = 4, m = 5$	0.73	0.066	***	0.26	0.21	0.10	*	-0.10	0.044	**	0.01	2242
5-Months Ahead: $n = 5, m = 4$	0.65	0.071	***	0.17	0.26	0.12	*	-0.14	0.049	**	0.02	2232
6-Months Ahead: $n = 6$, $m = 3$	0.55	0.071	***	0.10	0.28	0.14	*	-0.17	0.051	***	0.02	2224
Panel E. 12-Month Equity Premium	1											
3-Months Ahead: $n=3, m=9$	0.81	0.049	***	0.44	0.094	0.070		-0.059	0.036		0.01	2258
6-Months Ahead: $n = 6, m = 6$	0.56	0.055	***	0.15	0.17	0.11		-0.16	0.047	***	0.03	2227
9-Months Ahead: $n = 9, m = 3$	0.39	0.097	***	0.06	0.25	0.15		-0.24	0.052	***	0.04	2191

Table A7 Alternative Horizons: Coibion-Gorodnichenko Regressions

This table reports Coibion-Gorodnichenko regressions of future realized forecast errors on current 1-month forecast revisions:

 $\varepsilon_{i,t+4}^{(1)} = \beta_0 + \beta_1 \left(f_{i,t}^{(4,1)} - f_{i,t-1}^{(5,1)} \right) + e_{i,t+4}$

The realized spot rate is the future expectation of the 1-month equity premium, the forward rate is the current expectation of the same risk premium, and the forecast error is the realized spot rate minus the forward rate. The units are annualized percentage points. Panel regressions, in columns (1)–(4), report standard errors clustered by exchange and date. Time-series regressions, in columns (5)–(6), report Newey-West standard errors with lags selected following Lazarus et al. (2018). The sample is the longest available for each exchange in the main sample.

	(1) Main	(2) Main	(3) Main	(4) Main excl U.S.	(5) U.S. Only	(6) SX5E Only
$f_t^{(4,1)} - f_{t-1}^{(5,1)}$	-0.26**	-0.27**	-0.36***	-0.30**	-0.0063	-0.24*
Intercept	(0.11) 0.28 (0.16)	(0.11)	(0.065)	(0.11)	(0.21) -0.051 (0.16)	(0.14) 0.42 (0.32)
Observations Fixed Effects	2217 None	2217 Ex	2070 Ex/Date	1838 Ex	379 None	232 None
Standard Errors	Cluster	Cluster	Cluster	Cluster	Newey-West	Newey-West
Cluster Adjusted R^2 Within R^2	Ex/Date 0.01	Ex/Date 0.01 0.01	Ex/Date 0.83 0.03	$\begin{array}{c} \mathrm{Ex/Date} \\ 0.01 \\ 0.01 \end{array}$	-0.00 -	-0.00 -

Table A8 Option-Based Power Utility Regressions

This table reports regression estimates from the standpoint of an unconstrained power utility investor fully invested in the market. See Section 4.1.5 and Appendix B.4 for more details. Panel A reports estimates from panel regressions in the main sample. Panel B reports estimates from time-series regressions in the United States. Mincer-Zarnowitz regressions test $H_0: \beta_1 = 1$, as in Table 2. The average forecast error tests $H_0: \bar{\varepsilon}_t = 0$, as in Table 4. Error-predictability regressions test $H_0: \beta_1 = 0$, as in Table 5. Coibion-Gorodnichenko regressions test $H_0: \beta_1 = 0$. The risk premium is the 6-month spot rate in 6 months for Mincer-Zarnowitz, average error, and error-predictability regressions. The risk premium is the 3-month spot rate in 6 months for Coibion-Gorodnichenko regressions. The units are annualized percentage points. All regressions include exchange fixed effects and report a within R^2 . Panel regressions, in the main sample, report standard errors clustered by exchange and date. This sample is the longest available for each exchange. Time-series regressions, in the United States, report Newey-West standard errors with lags selected following Lazarus et al. (2018) and fixed-b p-values. This sample is from January 1990 to June 2021.

						μ	(6) t+6								$\mu_{t+6}^{(3)}$		
		Mincer-Z	arnowitz		Av	erage Erro	or	E	error Predi	ctability			Co	oibion-Gord	odnichenk	.0	
	β_1	$se(\beta_1)$	$p ext{-val}$	R^2	$ar{arepsilon_t}$	$se(\bar{\varepsilon}_t)$	p-val	β_1	$se(\beta_1)$	p-val	R^2	N	β_1	$se(\beta_1)$	$p ext{-val}$	R^2	N
Panel A. Mai	n Samp	le															
$\gamma = 0.50$	0.18	0.041	***	0.05	-0.047	0.010	***	-0.40	0.10	***	0.11	2094	-0.30	0.11	**	0.08	1999
$\gamma = 0.75$	0.51	0.059	***	0.14	0.046	0.057		-0.19	0.045	***	0.04	2202	-0.29	0.11	**	0.02	2149
$\gamma \rightarrow 1.00$	0.57	0.054	***	0.15	0.17	0.11		-0.17	0.046	***	0.03	2195	-0.26	0.12	*	0.01	2140
$\gamma = 1.25$	0.60	0.056	***	0.16	0.36	0.15	**	-0.14	0.049	**	0.02	2181	-0.23	0.13		0.01	2122
$\gamma = 1.50$	0.62	0.061	***	0.16	0.58	0.19	**	-0.10	0.054	*	0.01	2164	-0.20	0.13		0.01	2097
$\gamma = 2.00$	0.64	0.075	***	0.16	1.12	0.28	***	-0.054	0.059		0.00	2114	-0.16	0.16		0.00	2016
Panel B. Uni	ted Stat	es															
$\gamma = 0.50$	0.31	0.059	***	0.17	-0.041	0.015	***	-0.63	0.12	***	0.28	378	-0.26	0.11	**	0.03	375
$\gamma = 0.75$	0.61	0.094	***	0.21	-0.022	0.083		-0.20	0.069	***	0.05	378	-0.30	0.12	***	0.01	375
$\gamma \to 1.00$	0.67	0.096	***	0.22	0.021	0.15		-0.17	0.066	**	0.03	378	-0.29	0.12	**	0.01	375
$\gamma = 1.25$	0.72	0.098	**	0.23	0.12	0.20		-0.13	0.064	**	0.02	378	-0.27	0.13	**	0.01	375
$\dot{\gamma} = 1.50$	0.76	0.100	**	0.24	0.25	0.26		-0.099	0.062		0.01	378	-0.24	0.14	*	0.00	375
$\dot{\gamma} = 2.00$	0.85	0.10		0.26	0.57	0.35		-0.048	0.059		0.00	378	-0.18	0.15		0.00	375
$\dot{\gamma}=2.50$	0.91	0.10		0.27	0.94	0.44	**	-0.0087	0.057		-0.00	378	-0.12	0.17		-0.00	375
$\dot{\gamma} = 3.00$	0.97	0.10		0.28	1.32	0.52	**	0.022	0.056		-0.00	378	-0.075	0.18		-0.00	375

Table A9 Long-Horizon Forecast Error Decay

This table reports estimates of the predicted forecast error decay for the Euro Stoxx 50. The decay $\phi_t^{(n,m)}$ parameter follows the expected decay specification from De la O and Myers (2021):

$$\mathbb{E}_{t}\left[\varepsilon_{t+n}^{(m)}\right] = \phi_{t}^{(n,m)} \,\mathbb{E}_{t}\left[\varepsilon_{t+12}^{(12)}\right]$$

The estimate is the median by horizon:

$$\phi^{(n,m)} = median\left\{ \left| \phi_t^{(n,m)} \right| \right\}$$

The predicted forecast error is from a time-series regression of future realized forecast errors on current forward rates: the predictor is the $n-12\times 12$ -month forward rate for $n\geq 24$ and the 6×6 -month forward rate for n=12. The sample is from September 2005 to September 2014.

		n-months											
m-months	12	24	36	48	60	72	84						
12 24 36 48 60 72	1.00 1.00 1.00 1.00 1.00 1.00	1.05 1.06 1.04 0.96 1.01 1.02	1.13 1.04 1.06 0.95 0.97	1.08 1.10 1.01 1.07	1.04 1.05 0.90	0.99 0.97	0.99						

Table A10 Model Calibration: Objective Parameters

This table reports AR(3) time-series regressions for 3-month spot rates:

$$\mu_t^{(3)} = \left(1 - \sum_{j=1}^3 \phi_j\right) \bar{\mu} + \phi_1 \,\mu_{t-1}^{(3)} + \phi_2 \,\mu_{t-2}^{(3)} + \phi_3 \,\mu_{t-3}^{(3)} + e_t$$

The units are annualized percentage points. Standard errors are from 10,000 Monte Carlo simulations of length T months. The sample is the longest available for each exchange in the main sample.

	AUS	CHE	DEU	ESP	FRA	GBR	HKG	ITA	SX5E	USA
ϕ_1	0.59 (0.07)	0.78 (0.07)	0.79 (0.07)	0.76 (0.08)	0.87 (0.07)	0.90 (0.07)	0.74 (0.07)	0.66 (0.08)	0.77 (0.07)	0.94 (0.05)
ϕ_2	0.02 (0.08)	-0.01 (0.08)	-0.06 (0.08)	-0.07 (0.09)	-0.14 (0.09)	-0.19 (0.08)	$0.16 \\ (0.09)$	-0.02 (0.09)	-0.03 (0.08)	-0.26 (0.07)
ϕ_3	0.22 (0.07)	$0.05 \\ (0.06)$	0.11 (0.06)	$0.09 \\ (0.08)$	0.12 (0.07)	$0.14 \\ (0.06)$	-0.05 (0.07)	$0.11 \\ (0.07)$	0.11 (0.06)	0.18 (0.05)
$ar{\mu}$	6.19 (1.36)	6.13 (1.00)	8.89 (1.62)	10.49 (1.48)	7.69 (1.23)	6.73 (1.21)	9.30 (2.44)	11.63 (1.27)	8.99 (1.59)	6.40 (0.89)
σ_e	3.23 (0.16)	2.86 (0.13)	3.82 (0.18)	4.41 (0.23)	2.89 (0.14)	2.89 (0.13)	5.04 (0.27)	4.31 (0.23)	3.69 (0.17)	2.56 (0.09)
T	208	237	237	180	222	237	184	180	237	384

Figure A1 Filtered Option Prices

Panel A plots the number of options after filters. Panel B plots the share of filtered options with positive open interest. Each bar is the annual median from daily data. The black line is the full sample median from daily data. The sample is the longest available for each exchange. Option prices have maturity $30 \le \tau \le 365$ days. See Appendix B.1 for more details.

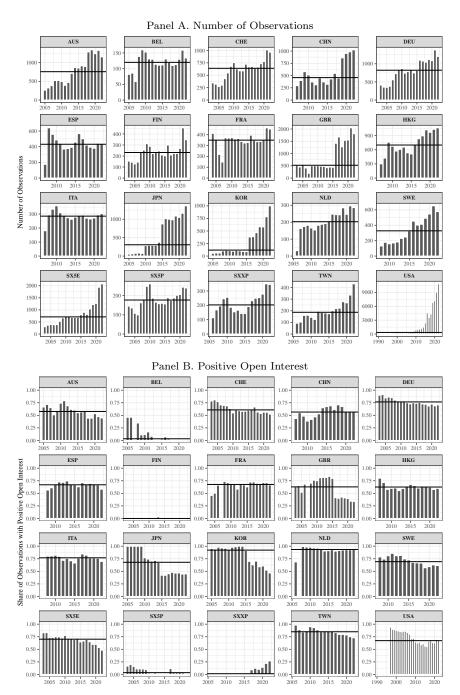
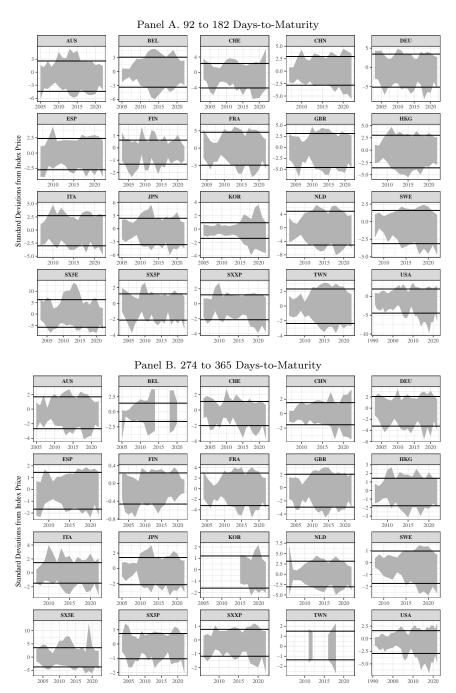


Figure A2 Minimum/Maximum Strike Price by Maturity

This figure plots the minimum/maximum strike price by maturity bin. The minimum/maximum is the annual median from daily data. The black line is the full sample minimum/maximum from daily data. The units are risk-neutral standard deviations from the index price. The sample is the longest available for each exchange. See Appendix B.1 for more details.



 ${\bf Figure~A3}$ ${\bf Timeline:~Current/Realized~Spot~Rates~and~Forward~Rates}$

See Section 3 for more details.

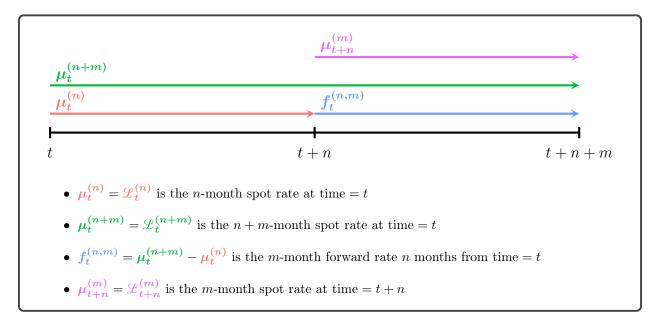


Figure A4
Current Spot and Forward Rates in the Full Sample

This figure plots the current 6-month spot rate $\mu_t^{(6)}$ (blue) and the 6×6 -month forward rate $f_t^{(6,6)}$ (red) in the full sample. The sample is the longest available for each exchange.

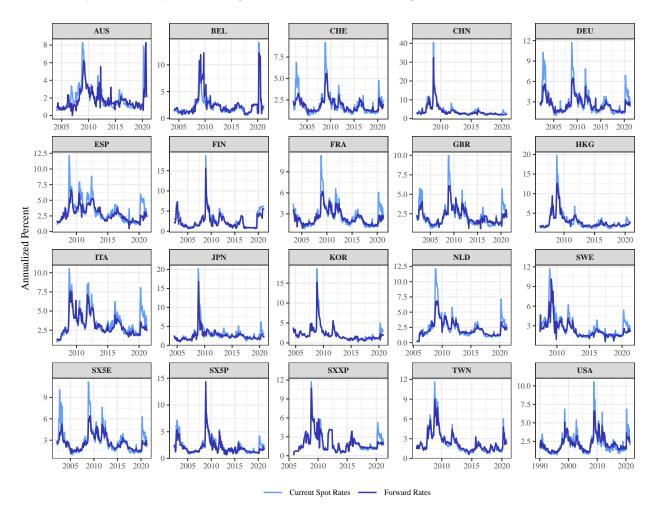


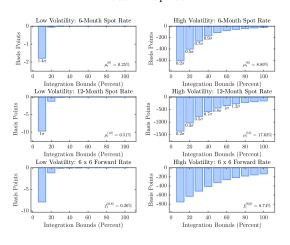
Figure A5 Measurement Error: Spot and Forward Rates

This figure describes truncation bias in the Black-Scholes model (left panel) and the stochastic volatility jump (SVJ) model (right panel). Integration bounds are in moneyness K/P units from the index price. Bar labels are in volatility standard deviations from the index price. Black-Scholes parameters: $P_t = 100$, r = 0.05, q = 0.02. SVJ parameters under the risk-neutral measure are from Bakshi, Cao, and Chen (1997):

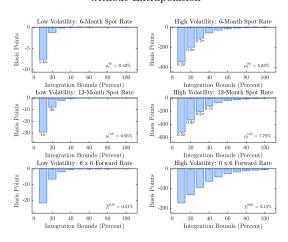
θ_v	κ_v	σ_v	ρ	μ_J	σ_J	λ
0.040	2.030	0.380	-0.570	-0.050	0.070	0.590

Under Black-Scholes (SVJ), low volatility is IV = 10% ($\sqrt{v_t} = 10\%$) and high volatility is IV = 60% ($\sqrt{v_t} = 60\%$). The units are non-annualized basis points. See Appendix B.5 for more details.

Panel A. Black-Scholes: Truncation Bias without Extrapolation



Panel B. SVJ: Truncation Bias without Extrapolation



Panel C. SVJ: Truncation Bias with Extrapolation

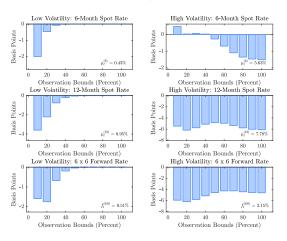
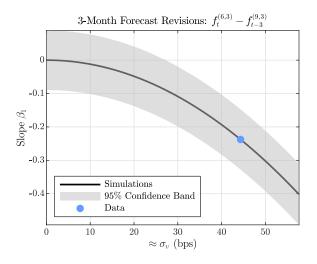
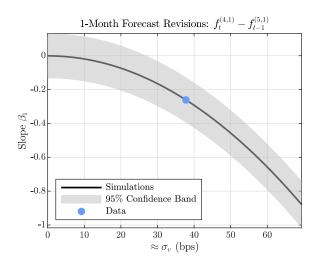


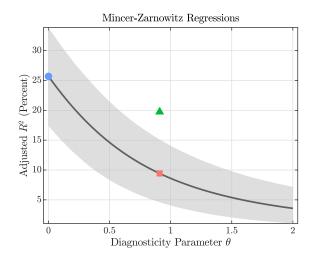
Figure A6 Measurement Error: Coibion-Gorodnichenko Regressions

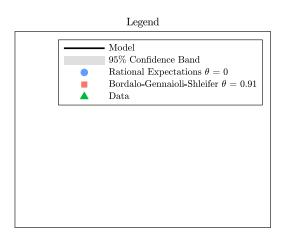
This figure quantifies how much correlated measurement error is necessary to produce the Coibion-Gorodnichenko regression slopes in the data with monthly forecast revisions (left panel) and quarterly forecast revisions (right panel). The solid lines are the slopes in simulations. The shaded regions are 95% confidence bands in 50,000 samples. The blue circles are slopes in the data. The sample is the longest available for each exchange in the main sample. See Appendix B.5 for more details.

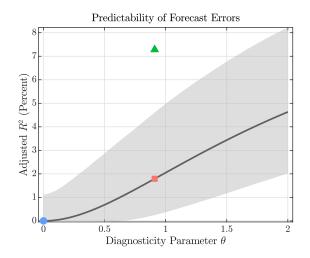


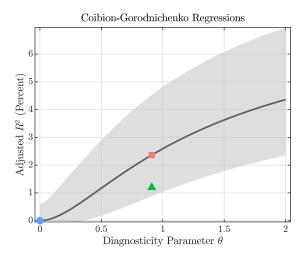


This figure reports regression R^2 s in the calibrated model of expectation errors. The model is calibrated from the standpoint of an unconstrained log utility investor fully invested in the market. Table A10 reports the objective parameters. The solid lines are model-implied population R^2 s in a single long sample. The shaded regions are model-implied 95% confidence bands in 10,000 short samples. The blue circles are model-implied R^2 s under rational expectations with $\theta = 0$. The red squares are model-implied R^2 s under diagnostic expectations with $\theta = 0.91$ from Bordalo, Gennaioli, and Shleifer (2018). The green triangles are R^2 s in the data. The sample is the longest available for each exchange in the main sample.









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