Multilevel Calibration Weighting for Survey Data

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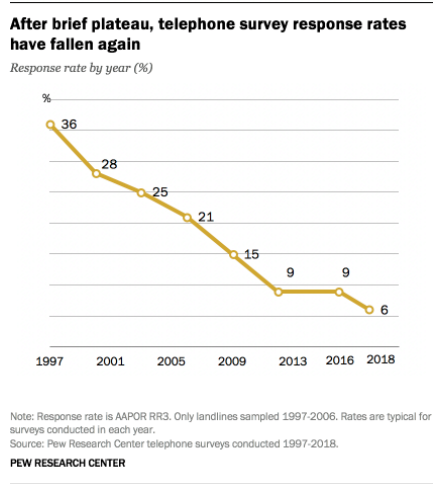
Survey non-response is a pernicious problem

- ↑ reliance on non-probability samples
- ↑ need for non-response adjustment

New and richer sources of data can help

Case study: 2016 presidential election

- Failing to adjust for **interactions** led to substantial bias
  [Kennedy et al., 2018]
- Retrospective study
  - Pre-election Pew poll
  - Post-election CCES as "ground truth"



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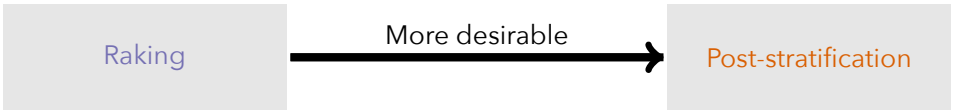

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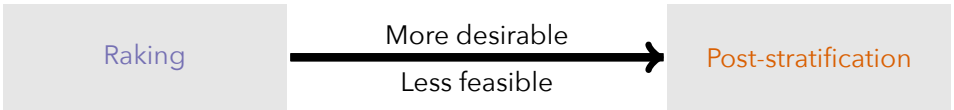
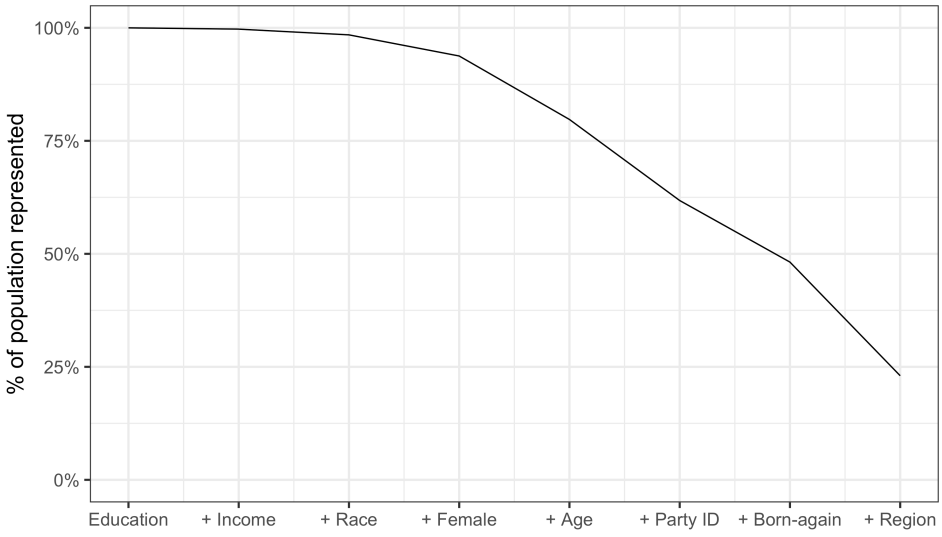


Quickly run into empty cells



How can we account for interactions in a principled way?

Adjusting for interactions in non-response is important

Ideally we'd post-stratify, but we can't

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Ideally we'd post-stratify, but we can't

This paper: Approximately post-stratify while at least raking on margins

- Leverage the value of interactions in a parsimonious way
- Dual representation as multilevel model of non-response

Combine with outcome model → Double Regression with Post-stratification (DRP)

- *Explicitly* adjusting for interactions via weighting and *implicitly* via machine learning

Approximately post-stratifying
while at least raking

Notation and setup

$i = 1, \ldots, N$ individuals

- Outcome Y_i, Response R_i with prob $P(R_i = 1) = \pi_i$
- d categorical covariates w/levels $J_1, \ldots, J_d$

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- Outcome Y_i, Response R_i with prob $P(R_i = 1) = \pi_i$
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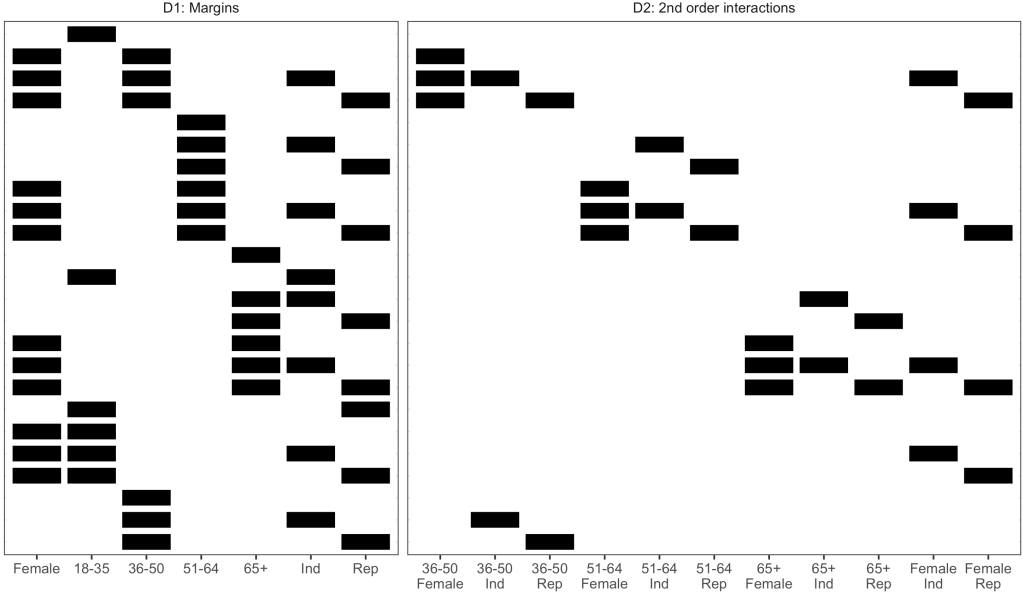
Combine into cells $S_i \in \{1, \ldots, J_1 \times \ldots \times J_d \equiv J\}$

- Overall count vector $N^{\mathcal{P}} \in \mathbb{N}^J$ and response count vector $n^{\mathcal{R}} \in \mathbb{N}^J$
- Probability of responding conditional on cell s: $\pi(s) = \frac{1}{N_s^{\mathcal{R}}} \sum_{S_i = s} \pi_i$

Binary vector of k^{th} order interaction terms for cell s: $D_s^{(k)}$


D1: Margins
D2: 2nd order interactions
Female
18-35
36-50
51-64
65+
Ind
Rep
36-50 Female
36-50 Ind
36-50 Rep
51-64 Female
51-64 Ind
51-64 Rep
65+ Female
65+ Ind
65+ Rep
Female Ind
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Goal

Impute the average

$$\mu = \frac{1}{N}\sum_{i=1}^{n} Y_i = \frac{1}{N}\sum_{s} N_s^{\mathcal{P}}\mu_s$$

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From the respondents

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Assume responses are Missing At Random (MAR) so that within each cell

$$\mathbb{E}\left[\bar{Y}_s\right] = \mu_s$$

And positivity

$$\pi(s) > 0$$

Choosing weights: Raking and Post-stratification

Raking on margins

Exactly match the counts for margins:

$$\sum_s D_s^{(1)} n_s^{\mathcal{R}} \hat{\gamma}(s) = \sum_s D_s^{(1)} N_s^{\mathcal{P}}$$

- Can usually compute if d is moderate
- "Only" accounts for the linear variables

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$$\hat{\gamma}(s) = \frac{\text{\# in Population}}{\text{\# in Sample}}$$

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- Unbiased when feasible

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"Soft" or "penalized" calibration loosens the raking constraints

[Huang and Fuller, 1978; Rao and Singh, 1997; Park and Fuller, 2009; Guggemos and Tillé, 2010]

What do we want from calibration weights?

Is it worth it to try to post-stratify?

Population regression

$$Y_i = \sum_{k=1}^{K} \eta_k \cdot D^k_{S_i} + e_i$$

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If interactions are weak, approximate post-stratification may be enough

- Bias depends on strength of interaction $\times$ imbalance
- Variance depends on inverse of effective sample size

Approximately post-stratify while at least raking on margins

Find weights via convex optimization:

$$\min_{\gamma} \sum_{k=2}^{d} \frac{1}{\lambda_k} \|\text{Imbalance}_k\|_2^2 + \sum_s n_s^{\mathcal{R}} \gamma(s)^2$$

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Move smoothly between two extremes

- With $\lambda_k \to \infty$, recover raking on margins
- With $\lambda_k \to 0$, recover post-stratification

Based on calibration weighting and approximate balancing weights

[Deville and Särndal, 1992; Deville et al., 1993; Zubizarreta, 2015; Wong et al., 2018; Hirshberg et al., 2019]

Dual view: multilevel regression for response

A **regularized** model for the **inverse** probability of response:

[Zhao and Percival, 2016; Wang and Zubizarreta, 2020; Chattopadhyay et al., 2020]

$$\frac{1}{\pi_i} \sim \beta_1 \cdot D^1_{S_i} + \sum_{k=2}^{K} \beta_k \cdot D^k_{S_i}$$

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Regularization makes approximate post-stratification feasible

$$0 \times \|\beta_1\|_2^2 + \sum_{k=2}^{K} \lambda_k \|\beta_k\|_2^2$$

- At least raking → no regularization for marginal probabilities [Little and Wu, 1991]
- Approximate post-stratification → regularizing interaction terms

Key difference with GLM: regularizing for balance

Double Regression with Post-Stratification (DRP)

MRP-style approaches

Start with an outcome model for the cells $\hat{\mu}_s$

- Multilevel model, high dimensional regression, tree-based methods

  [Gelman and Little, 1997; Ghitza and Gelman, 2013; Gao et al., 2020; Montgomery and Olivella, 2018; Bisbee, 2019]

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Post-stratify using predictions instead of outcomes

$$\hat{\mu}^{\text{mrp}} = \frac{1}{N} \sum_s \frac{N_s^{\mathcal{P}}}{n_s^{\mathcal{R}}} n_s^{\mathcal{R}} \hat{\mu}_s = \frac{1}{N} \sum_s N_s^{\mathcal{P}} \hat{\mu}_s$$

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Augmented balancing weights estimator, related to AIPW and bias-correction

[Cassel et al., 1976; Robins et al., 1994; Abadie and Imbens, 2006; Hirshberg and Wager, 2019]

- Relies on outcome model in cells where weighting doesn't get it right
- Relies on weights to adjust cells where model is off
- If post-stratifying, collapses to weighting estimator $\hat{\mu}^{\text{drp}} = \hat{\mu}(\hat{\gamma})$

The role of the outcome model: bias correction

Bias term depends on how well the model adjusts for remaining imbalance

$$\hat{\mu}^{\text{drp}} - \mu \;=\; \frac{1}{N}\sum_{s} \text{error in cell } s \times \text{imbalance in cell } s \;+\; \text{noise}$$

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Because we can only approximately post-stratify, bias correction is key!

- Outcome model primarily performs bias correction, prioritize bias over variance
- Include higher order interactions than usual, or deeper trees
- Limits extrapolation

$\hat{\mu}^{\text{mrp}}$ relies entirely rely on outcome model

- Choose a lower variance, higher bias solution
- Fewer interaction terms, shallower trees
- Potentially unchecked extrapolation

Case study: 2016 presidential election

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Pre-election Pew poll of vote intention with $\sim$ 2,000 respondents

- Age, gender, race, region, party ID, education, income, born again Christian

Ground truth: weighted CCES, large sample $N \sim 45{,}000$

Interactions should be important here [Kennedy et al., 2018]

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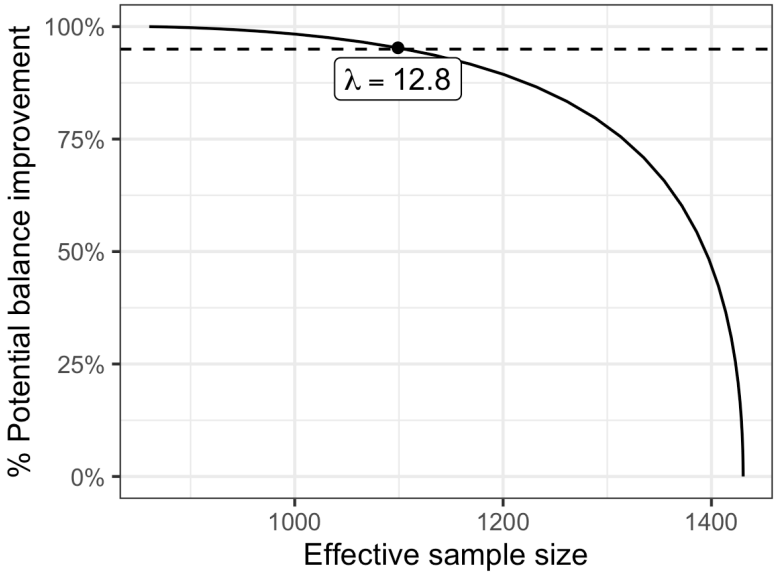
Use this case study in 3 ways

1. Calibrated simulation study [see paper!]
2. Imbalance and estimates with full weighted CCES as target

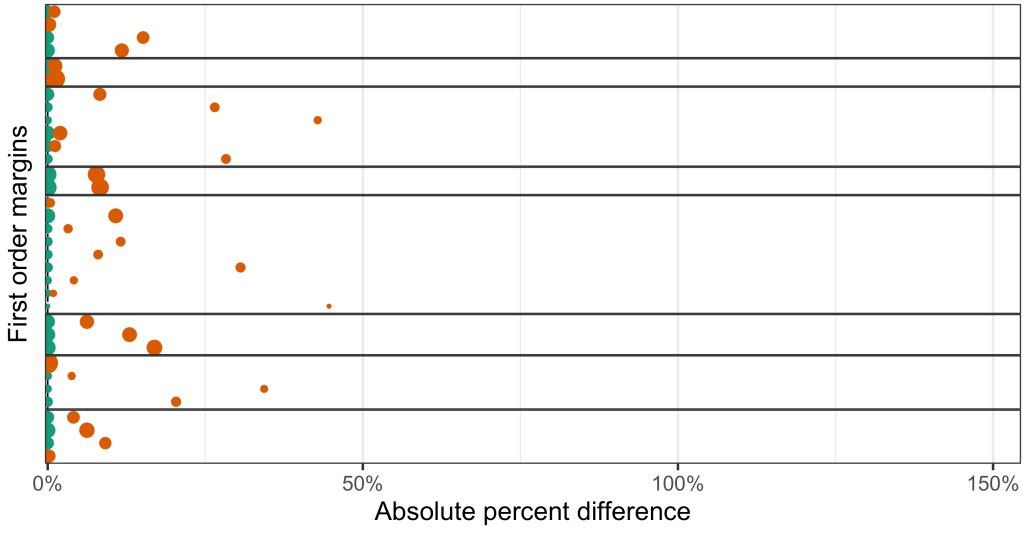
$$\frac{\left|N_s^{\mathcal{P}} - n_s^{\mathcal{R}} \hat{\gamma}(s)\right|}{N_s^{\mathcal{P}}}$$

3. Impute Republican vote share within each state
   - This is challenging! Pew not sampled to represent states well

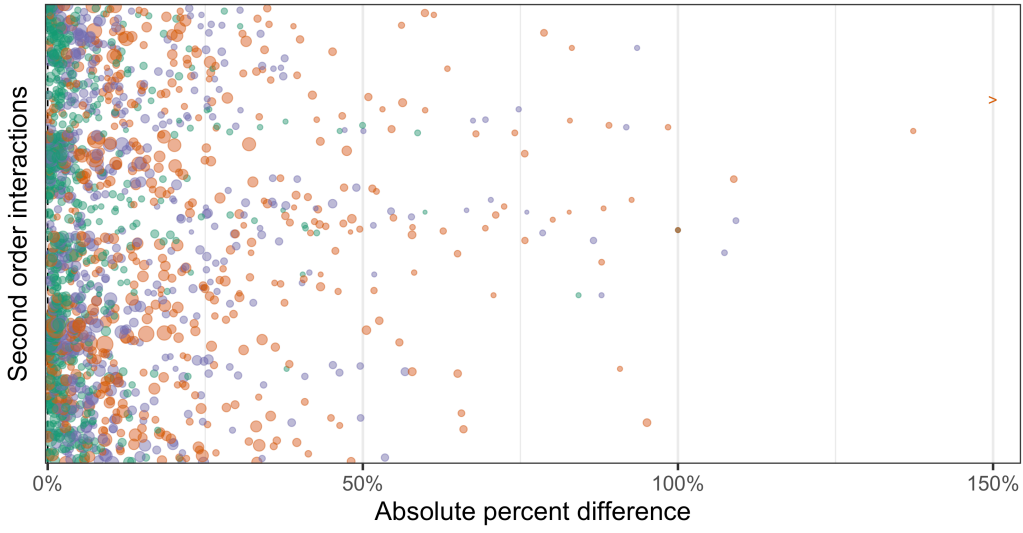
Negotiating balance vs effective sample size



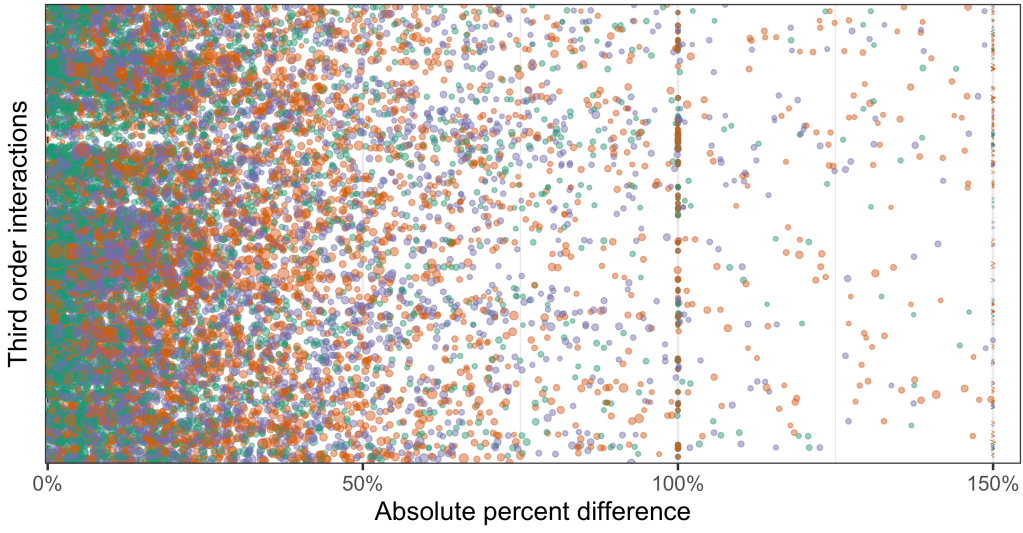
Both multilevel weighting and raking exactly match margins...



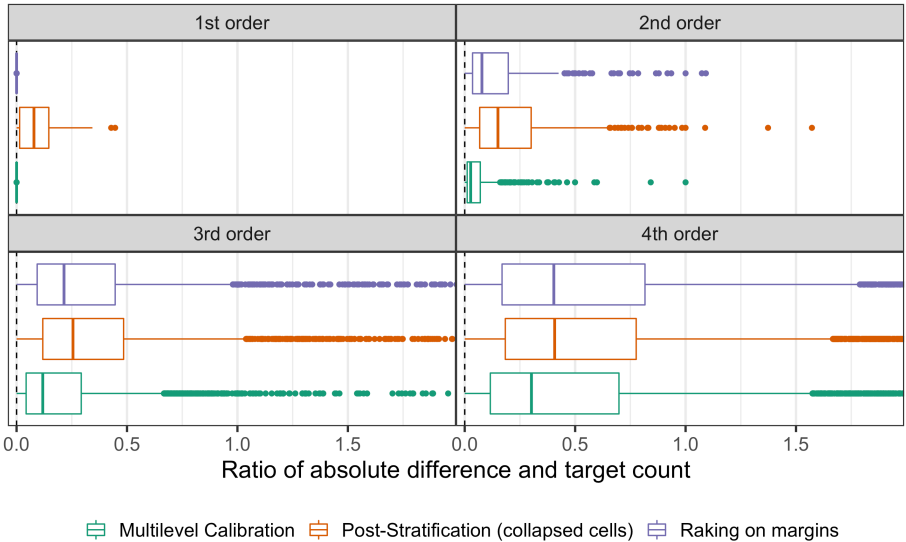
...but raking fails to balance higher order interactions



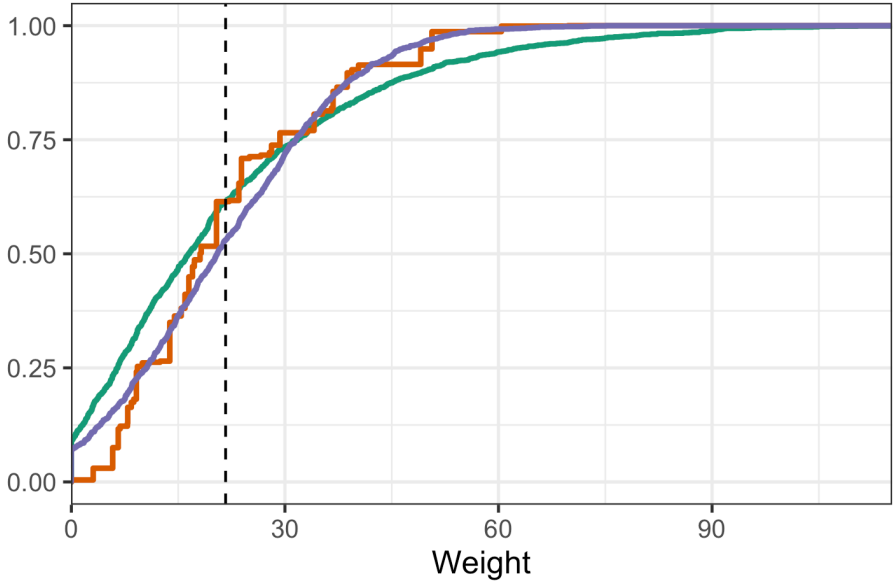
Approximate balance in 3rd order interactions



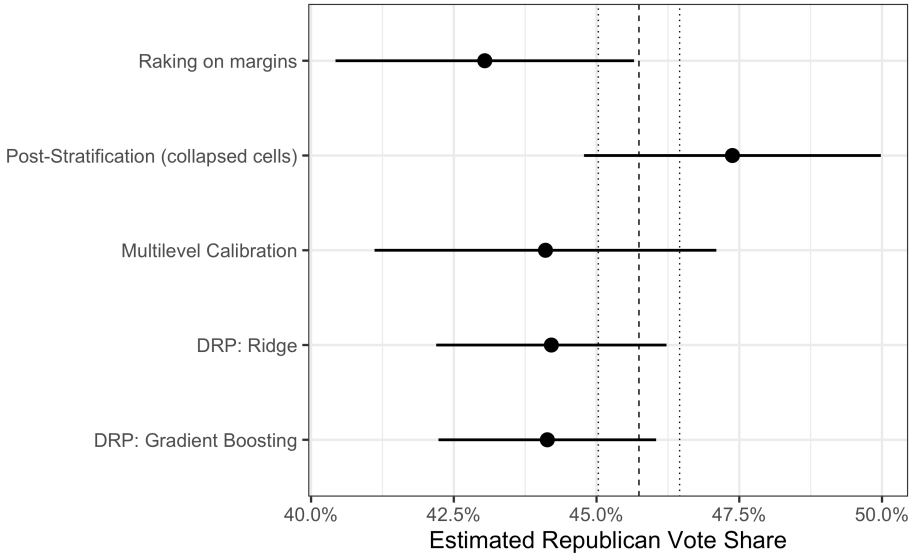
Overall multilevel calibration yields better balance



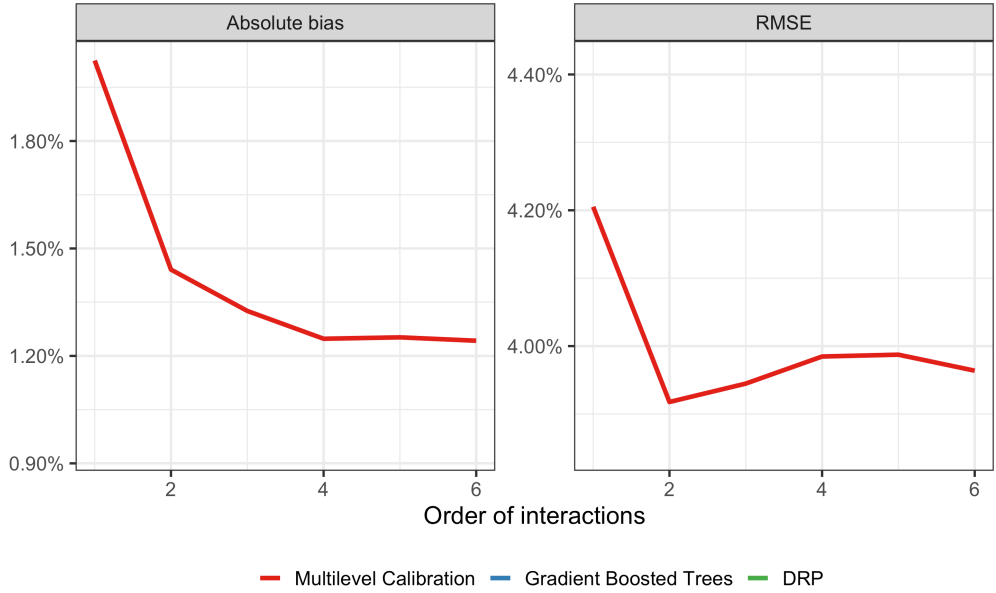
At the cost of lower effective sample sizes



Overall estimate of Republican vote share



Weighting on higher order interactions reduces bias



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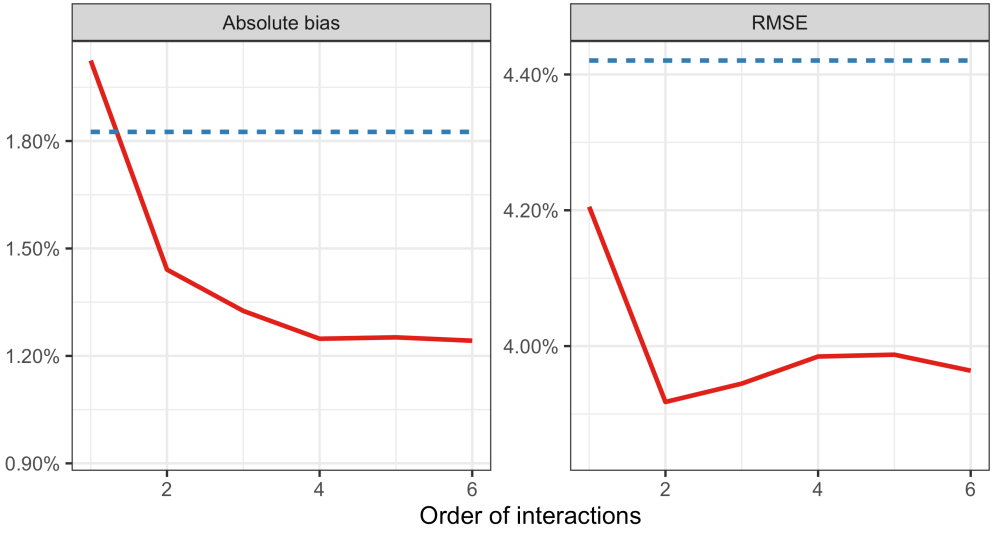
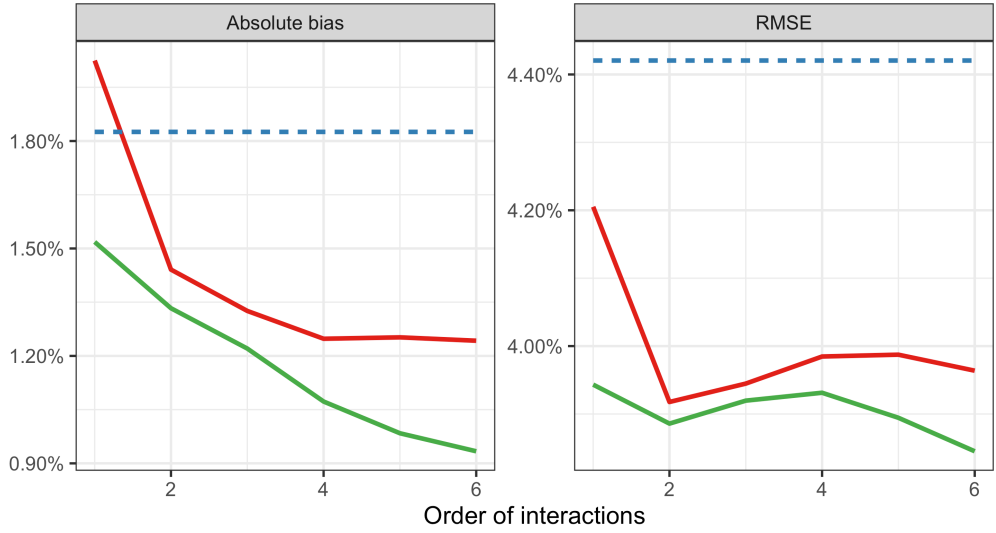


DRP gives further improvements



Recap

Principled, parsimonious way of leveraging the value of interactions

- Multilevel weighting as middle ground between raking and post-stratification
- Dual view as multilevel model for non-response
- DRP adjusts cells where weighting misses the mark

Implemented in `multical` `R` package

Recap

Principled, parsimonious way of leveraging the value of interactions

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Thank you!

ebenmichael.github.io
arxiv.org/abs/2102.09052
multical

Appendix

Calibrated simulation study

Create a combined sample from Pew and CCES, with Pew respondents as $R_i = 1$

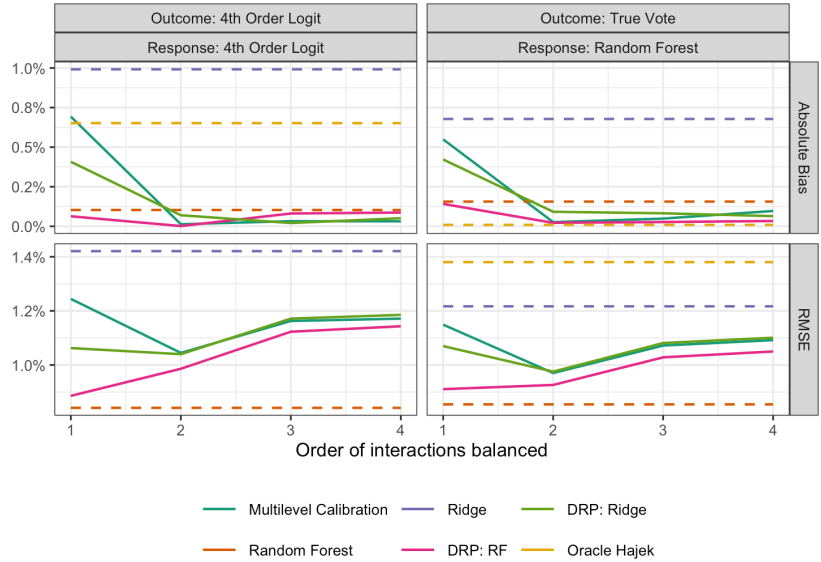
Fit two different response models:

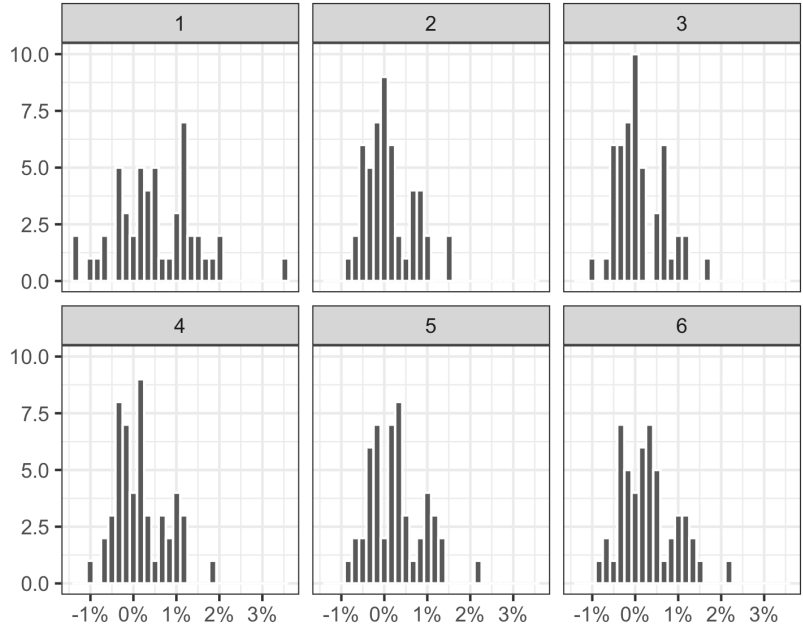
1. Random forest
2. 4^{th} order ridge penalized logistic regression with **low regularization**

Two different outcomes:

1. Actual Republican vote
2. Sampled from 4^{th} order ridge penalized logistic regression with **low regularization**

Balancing higher order interactions helps, so does including an outcome model




1
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4
5
6
10.0
7.5
5.0
2.5
0.0
-1%
0%
1%
2%
3%
Difference between DRP and weighted estimate

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