

Double Regression with Post-stratification (DRP)

for high-dimensional survey data

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AsianPolmeth VIII & ASQPS IX

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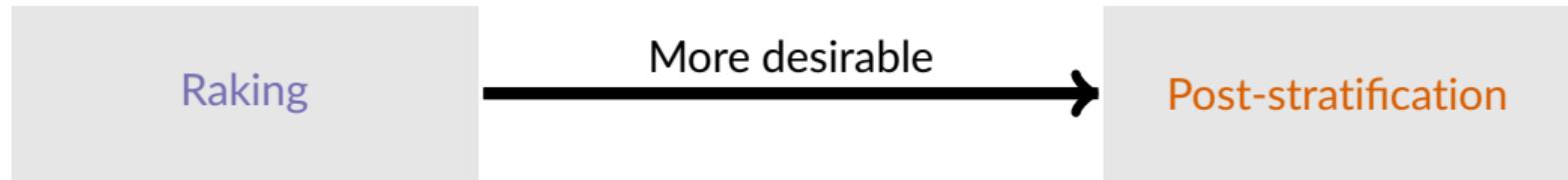
Raking

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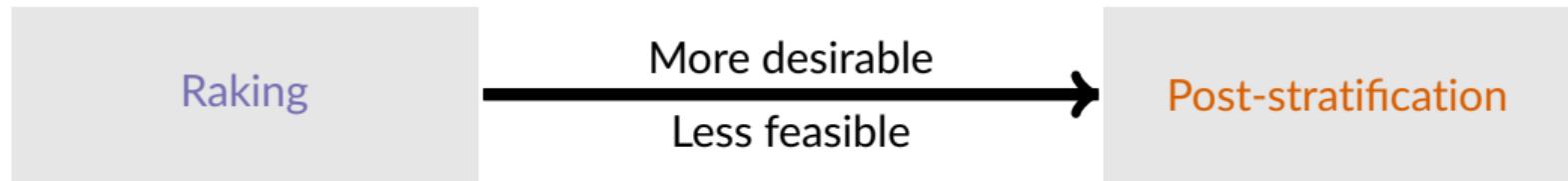
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Post-stratification

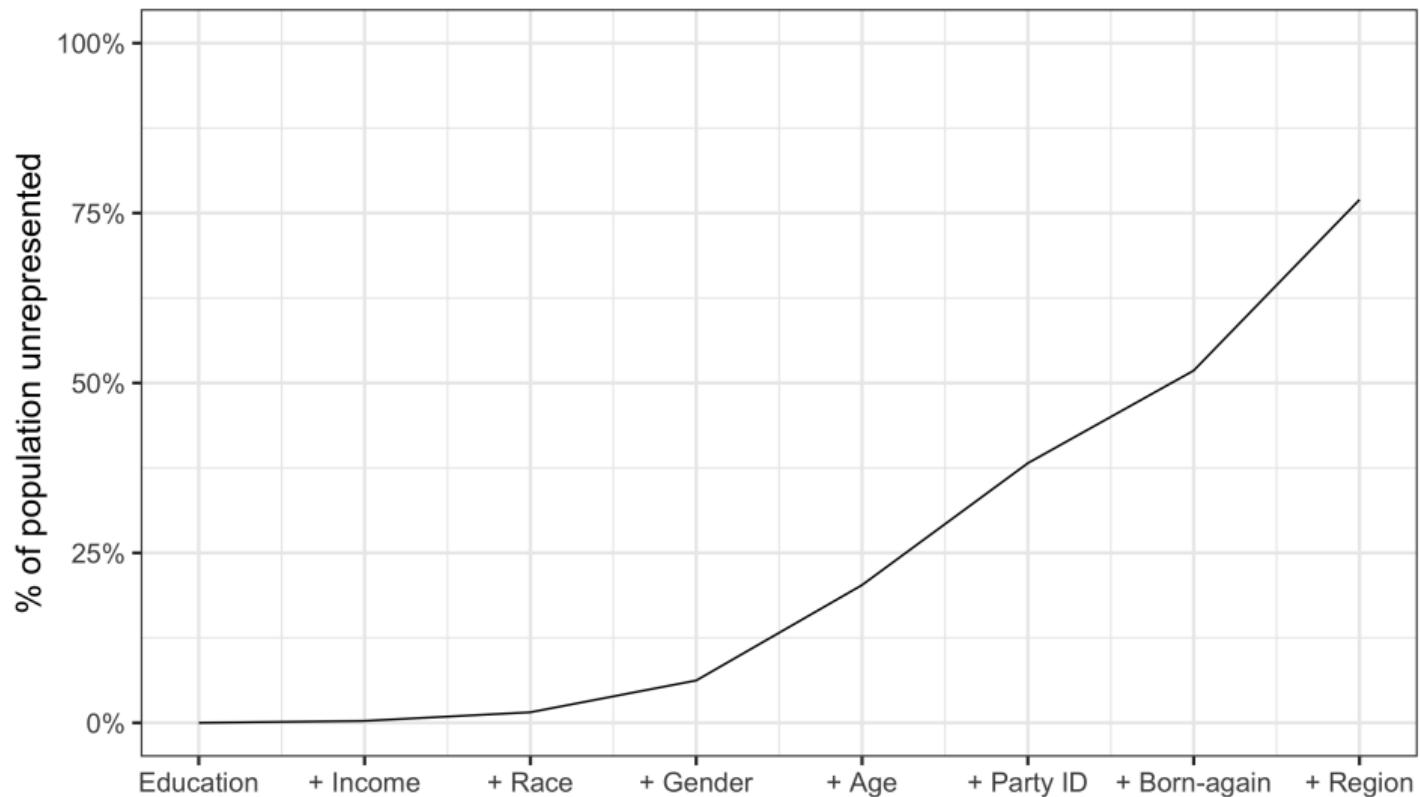
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Quickly run into empty cells



How can we account for interactions in a principled way?

Adjusting for interactions in non-response is important

- Example: 2016 presidential election polling [Kennedy et al., 2018]

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Ideally we'd **post-stratify**, but we can't

This paper: **Approximately post-stratify** while **at least raking** on margins

- Leverage the value of interactions in a parsimonious way
- Dual representation as multilevel model of non-response

Combine with outcome model → Double Regression with Post-stratification (**DRP**)

- *Explicitly* adjusting for interactions via weighting and *implicitly* via machine learning

Approximately post-stratifying
while at least raking

Notation and setup

$i = 1, \dots, N$ individuals

- Outcome Y_i , Response R_i with prob $P(R_i = 1) = \pi_i$
- d categorical covariates w/levels J_1, \dots, J_d

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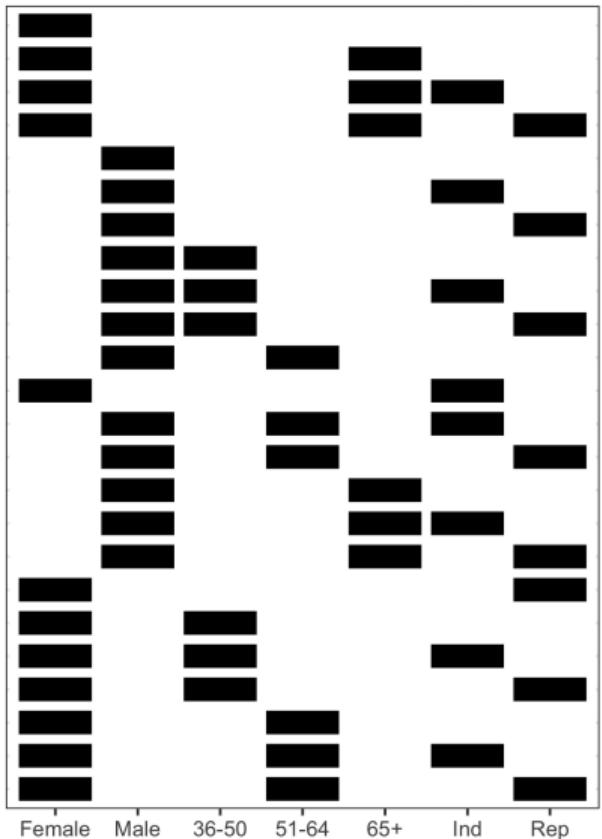
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Combine into cells $S_i \in \{1, \dots, J_1 \times \dots \times J_d \equiv J\}$

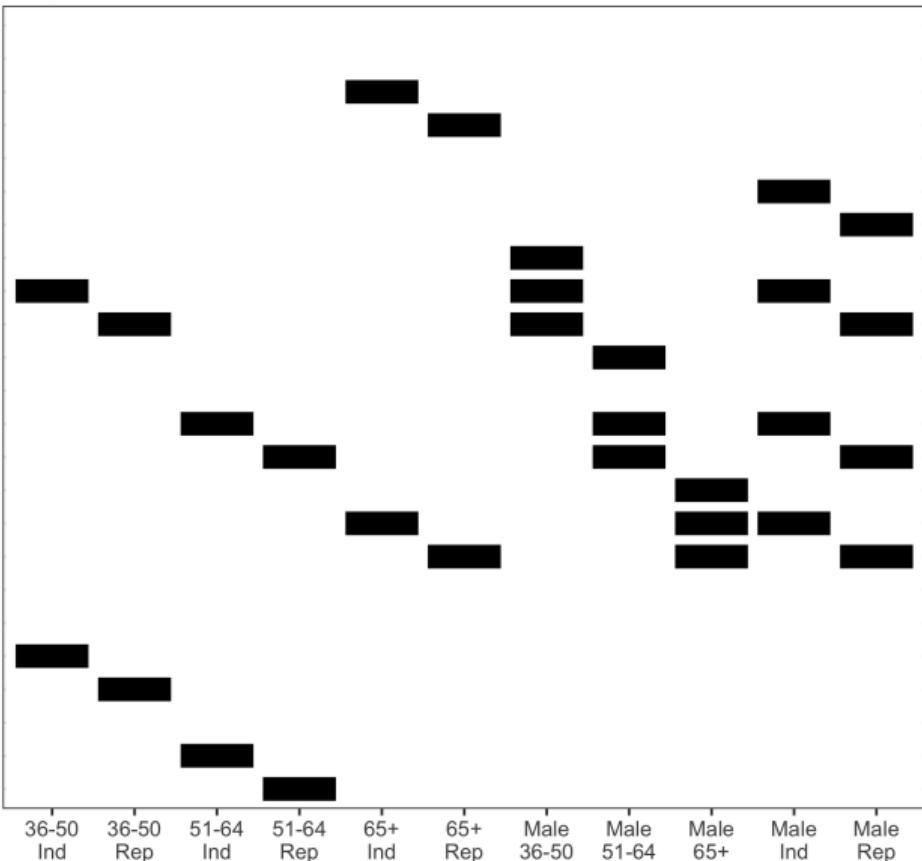
- Overall count vector $N^{\mathcal{P}} \in \mathbb{N}^J$ and response count vector $n^{\mathcal{R}} \in \mathbb{N}^J$
- Probability of responding conditional on cell s : $\pi(s) = \frac{1}{N_s^{\mathcal{R}}} \sum_{S_i=s} \pi_i$

Binary vector of k^{th} order interaction terms for cell s : D_s^k

D1: Margins



D2: 2nd order interactions



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Impute the average

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Assume responses are Missing At Random (MAR) so that within each cell

$$\mathbb{E} [\bar{Y}_s] = \mu_s$$

And positivity

$$\pi(s) > 0$$

Choosing weights: Raking and Post-stratification

Raking on margins

Exactly match the counts for margins:

$$\sum_s D_s^1 n_s^{\mathcal{R}} \hat{\gamma}(s) = \sum_s D_s^1 N_s^{\mathcal{P}}$$

- Can usually compute if d is moderate
- “Only” accounts for the linear variables

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Post-stratification

Exactly match the counts within each cell:

$$\hat{\gamma}(s) = \frac{N_s^{\mathcal{P}}}{n_s^{\mathcal{R}}}$$

- Can usually compute if d is moderate
- “Only” accounts for the linear variables

- Impossible to compute with empty cells
- Unbiased when feasible

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Is it worth it to try to **post-stratify**?

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Population regression

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If interactions are weak, approximate **post-stratification** may be enough

- Bias depends on strength of interaction \times imbalance
- Variance depends on sum of the squared weights $\|\hat{\gamma}\|_2^2$

Approximately post-stratify while at least raking on margins

Find weights via convex optimization:

$$\min_{\gamma} \sum_{k=2}^d \frac{1}{\lambda_k} \left\| \sum_s D_s^k n_s^{\mathcal{R}} \gamma(s) - D_s^k N_s^{\mathcal{P}} \right\|_2^2 + \sum_s n_s^{\mathcal{R}} \gamma(s)^2$$

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subject to $\sum_s D_s^1 n_s^{\mathcal{R}} \gamma(s) = \sum_s D_s^1 N_s, \quad \gamma(s) > 0$

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Move smoothly between two extremes

- With $\lambda_k \rightarrow \infty$, recover raking on margins
- With $\lambda_k \rightarrow 0$, recover post-stratification
- With $\lambda_k = 1$, cell weights are regularized by cell size

Based on calibration weighting and approximate balancing weights

[Deville and Särndal, 1992; Deville et al., 1993; Zubizarreta, 2015; Hirshberg et al., 2019]

Dual view: multilevel regression for response

A **regularized** model for the **inverse** probability of response:

[Zhao and Percival, 2016; Wang and Zubizarreta, 2020; Chattopadhyay et al., 2020]

$$\frac{1}{\pi_i} \sim \left[\theta_1 \cdot D_{S_i}^1 + \sum_{k=2}^K \theta_k \cdot D_{S_i}^k \right]_+$$

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Regularization makes approximate **post-stratification** feasible

$$0 \times \|\theta_1\|_2^2 + \sum_{k=2}^K \lambda_k \|\theta_k\|_2^2$$

- At least raking \rightarrow no regularization for marginal probabilities [Little and Wu, 1991]
- Approximate post-stratification \rightarrow regularizing interaction terms

Key difference with GLM: regularizing for balance

Double Regression with Post-Stratification (DRP)

MRP-style approaches

Start with an outcome model for the cells $\hat{\mu}_s$

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Post-stratify using predictions instead of outcomes

$$\hat{\mu}^{\text{mrp}} = \frac{1}{N} \sum_s \frac{N_s^{\mathcal{P}}}{n_s^{\mathcal{R}}} n_s^{\mathcal{R}} \hat{\mu}_s = \frac{1}{N} \sum_s N_s^{\mathcal{P}} \hat{\mu}_s$$

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Augmented balancing weights estimator, related to double robust and bias-corrected estimators

[Cassel et al., 1976; Robins et al., 1994; Abadie and Imbens, 2006; Hirshberg and Wager, 2019]

- Relies on **outcome model** in cells where **weighting** doesn't get it right
- Relies on **weights** to adjust cells where **model** is off
- If **post-stratifying**, collapses to weighting estimator $\hat{\mu}^{\text{drp}} = \hat{\mu}^{\text{weight}}$

Bias variance tradeoff

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Case study

Case study: 2016 presidential election

Pre-election Pew poll of vote intention with $\sim 2,000$ respondents

- Age, gender, race, region, party ID, education, income, born again Christian

Ground truth: weighted CCES, large sample $N \sim 45,000$

Interactions should be important here [Kennedy et al., 2018]

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Use this case study in 3 ways

1. Calibrated simulation study
2. Look at imbalance and estimates with full weighted CCES as target

$$\frac{|N_s^P - n_s^R \hat{\gamma}(s)|}{N_s^P}$$

3. Impute Republican vote share within each state
 - This is challenging! Pew not sampled to represent states well

Calibrated simulation study

Create a combined sample from Pew and CCES, with Pew respondents as $R_i = 1$

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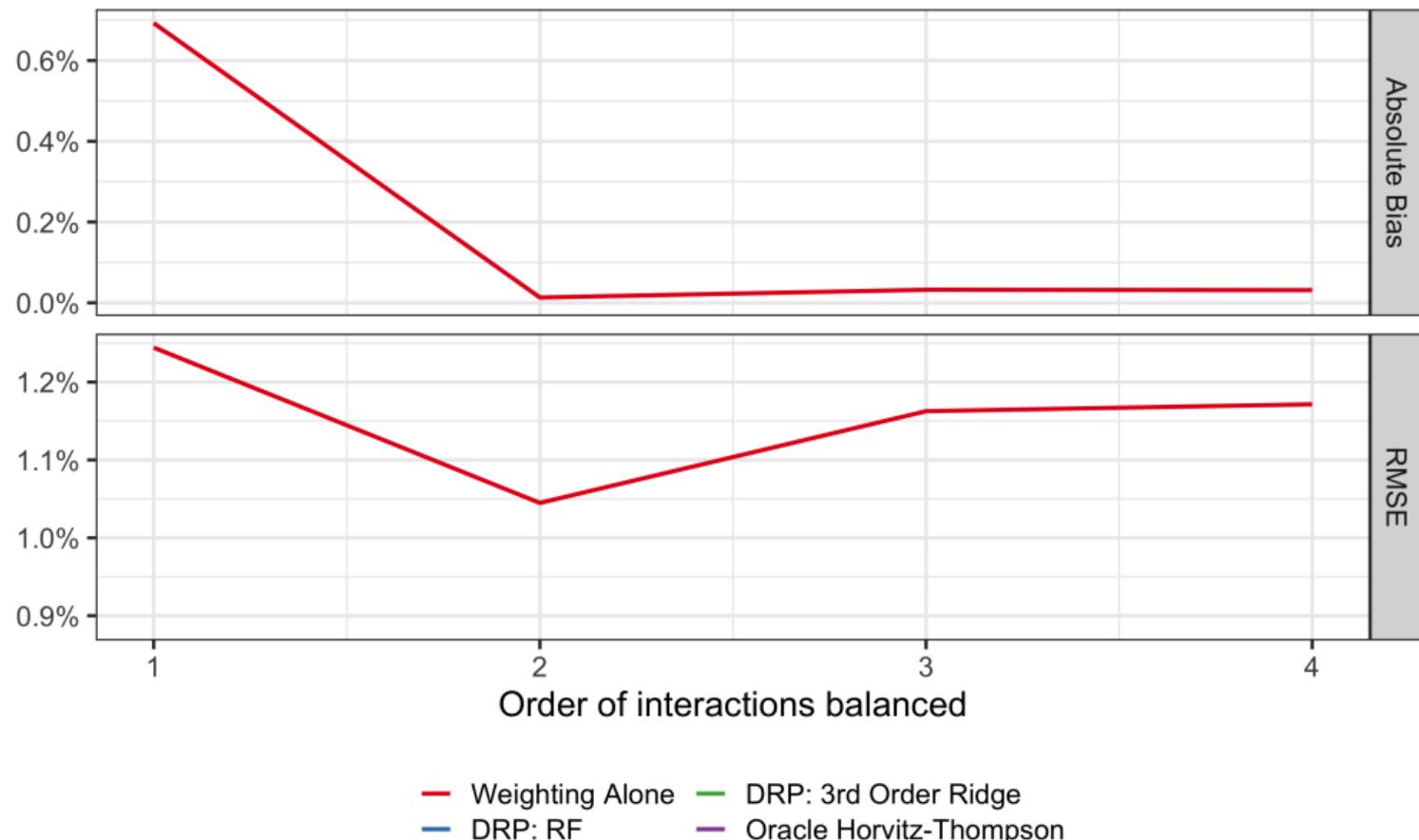
Fit two different response models:

1. Random forest
2. 4th order ridge penalized logistic regression with **low regularization**

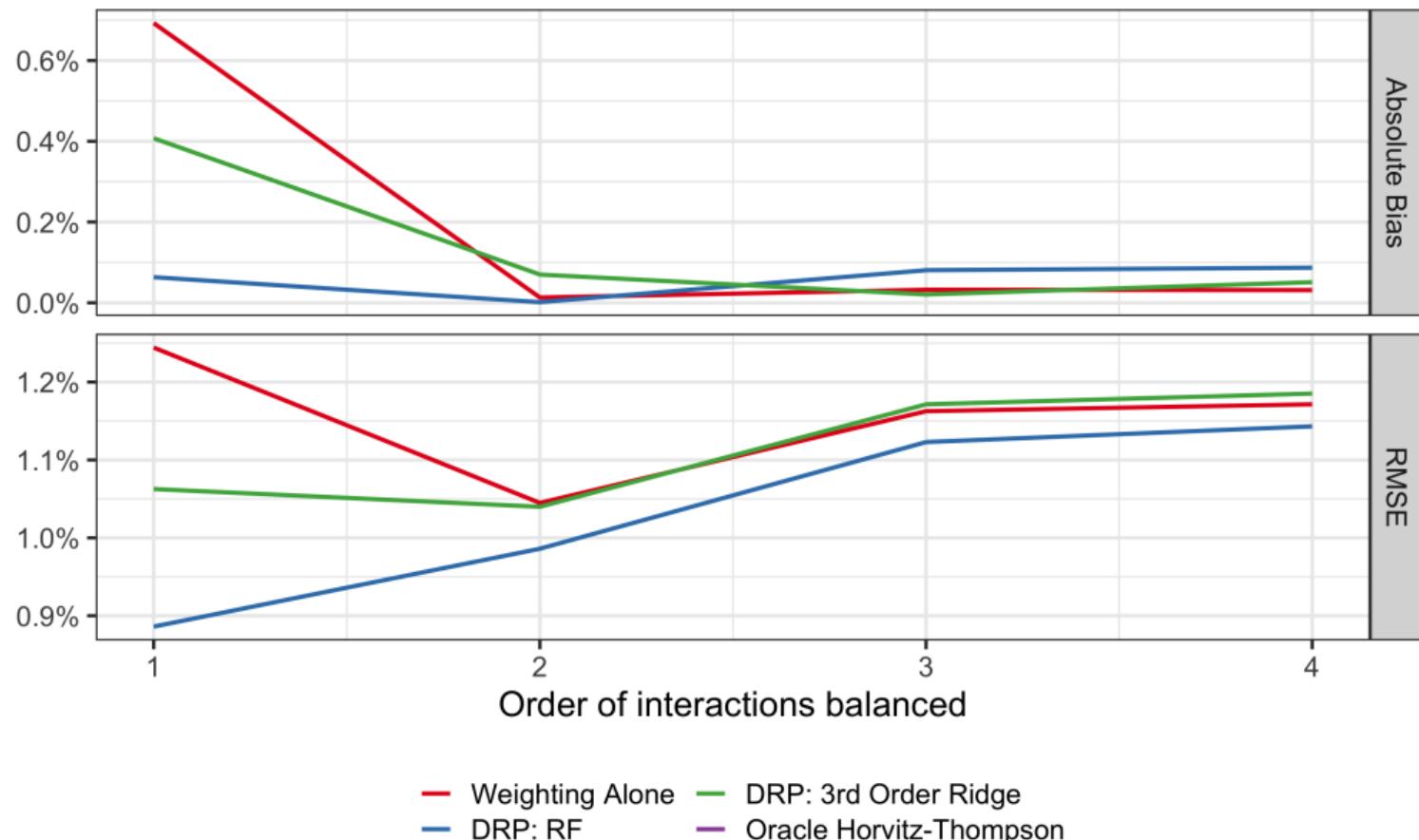
Two different outcomes:

1. Actual Republican vote
2. Sampled from 4th order ridge penalized logistic regression with **low regularization**

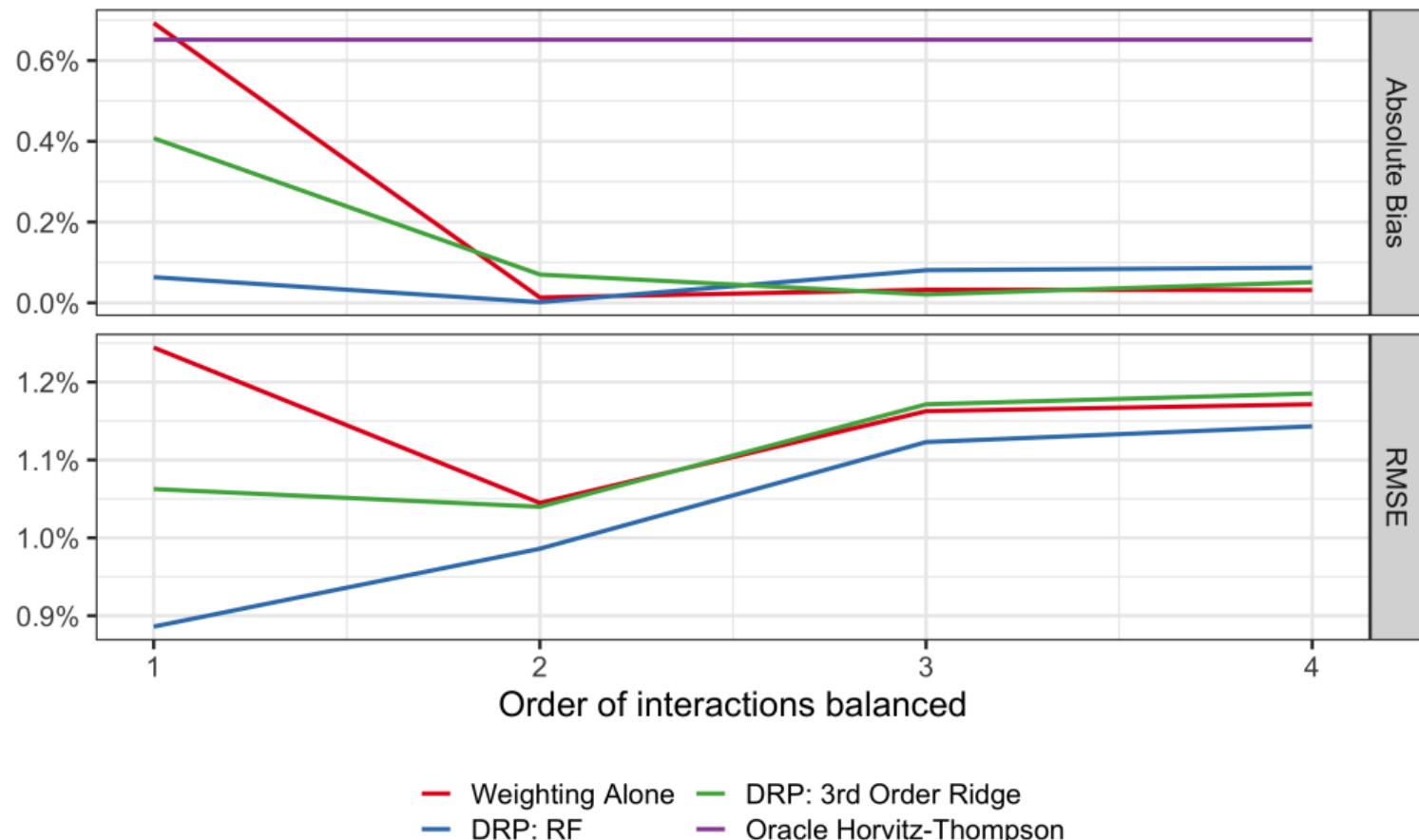
Outcome and Response: 4th order logit



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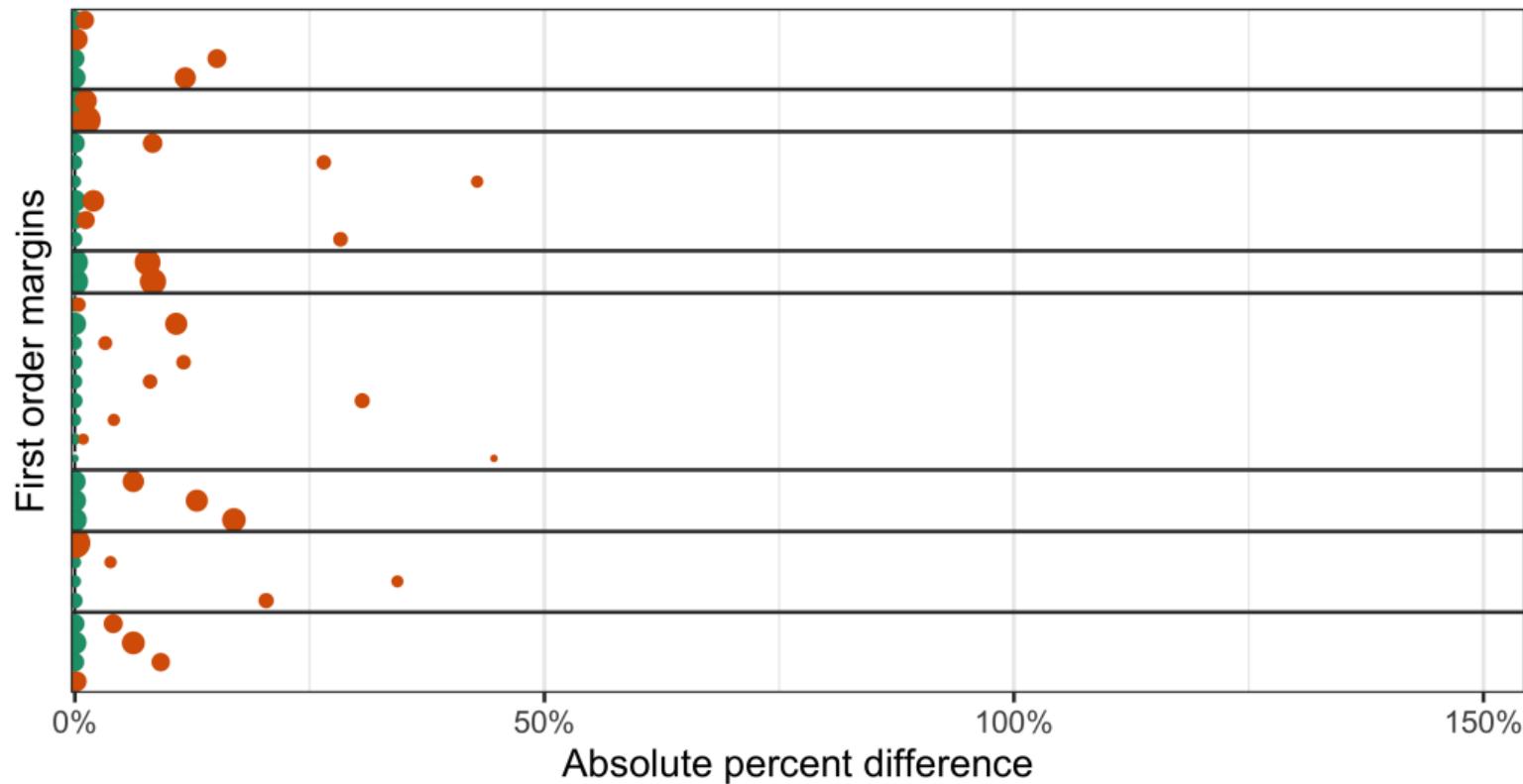


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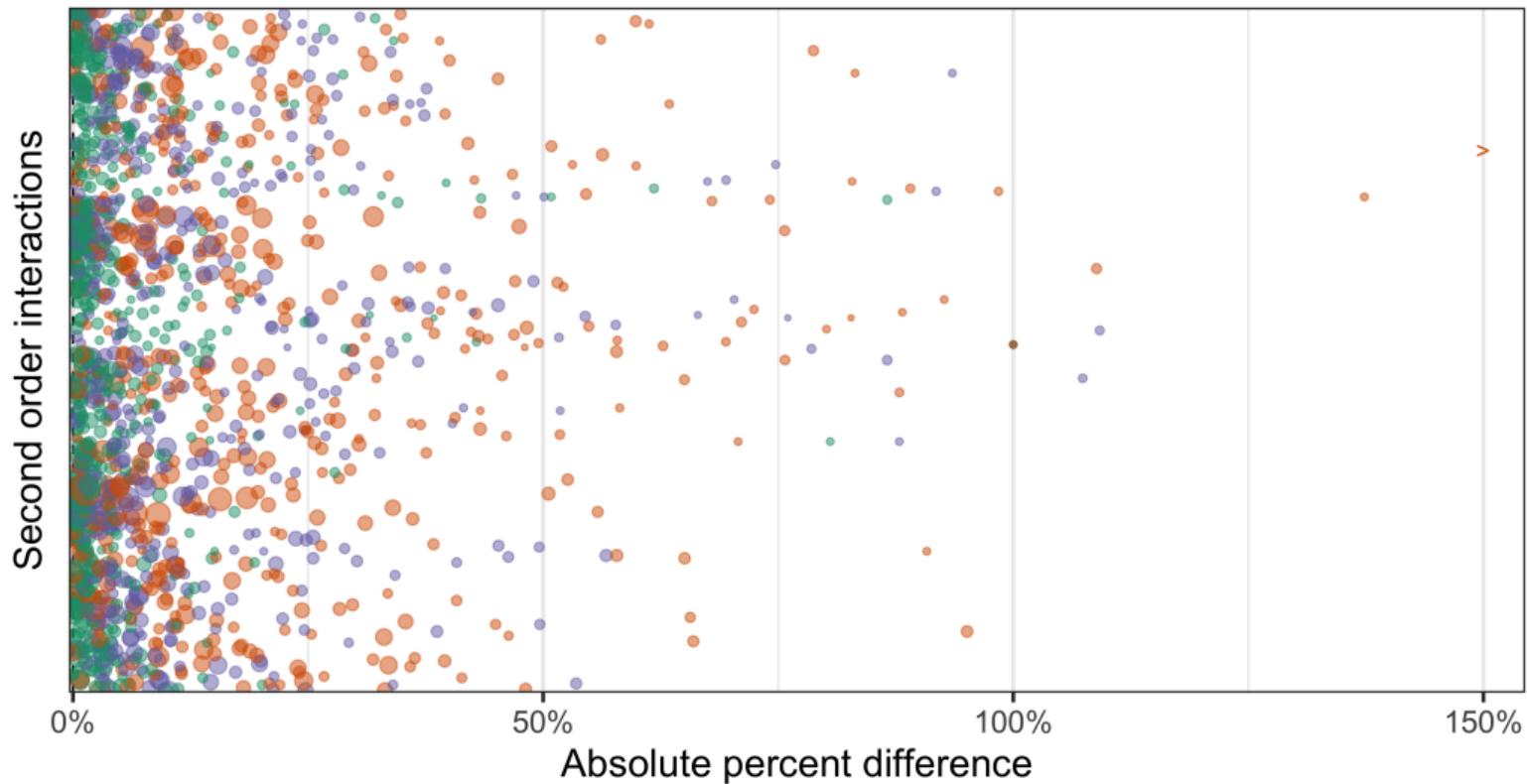
Full CCES

Both multilevel weighting and raking exactly match margins...



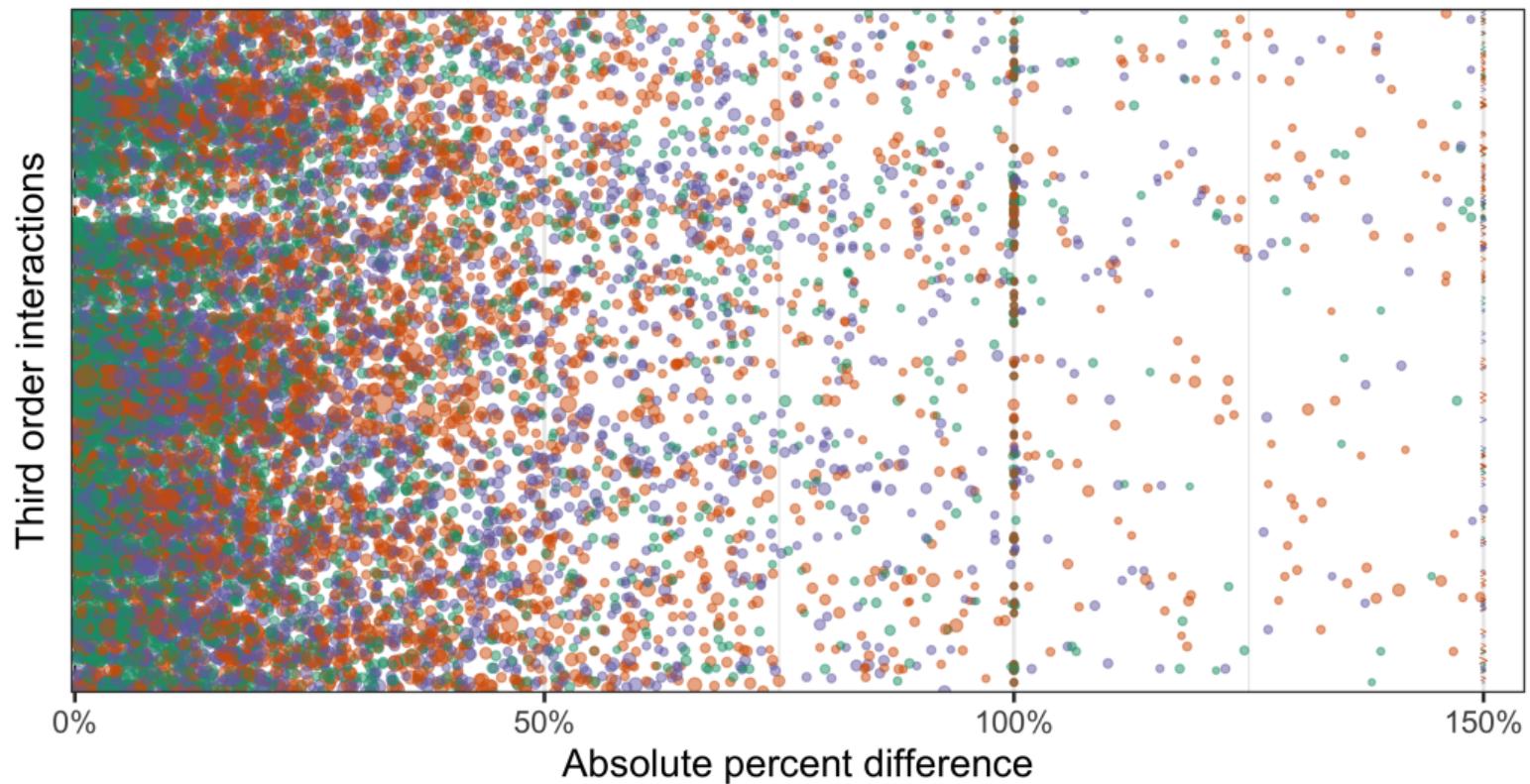
- Multilevel Weighting
- Post-Stratification (collapsed cells)
- Raking on margins

...but raking fails to balance higher order interactions

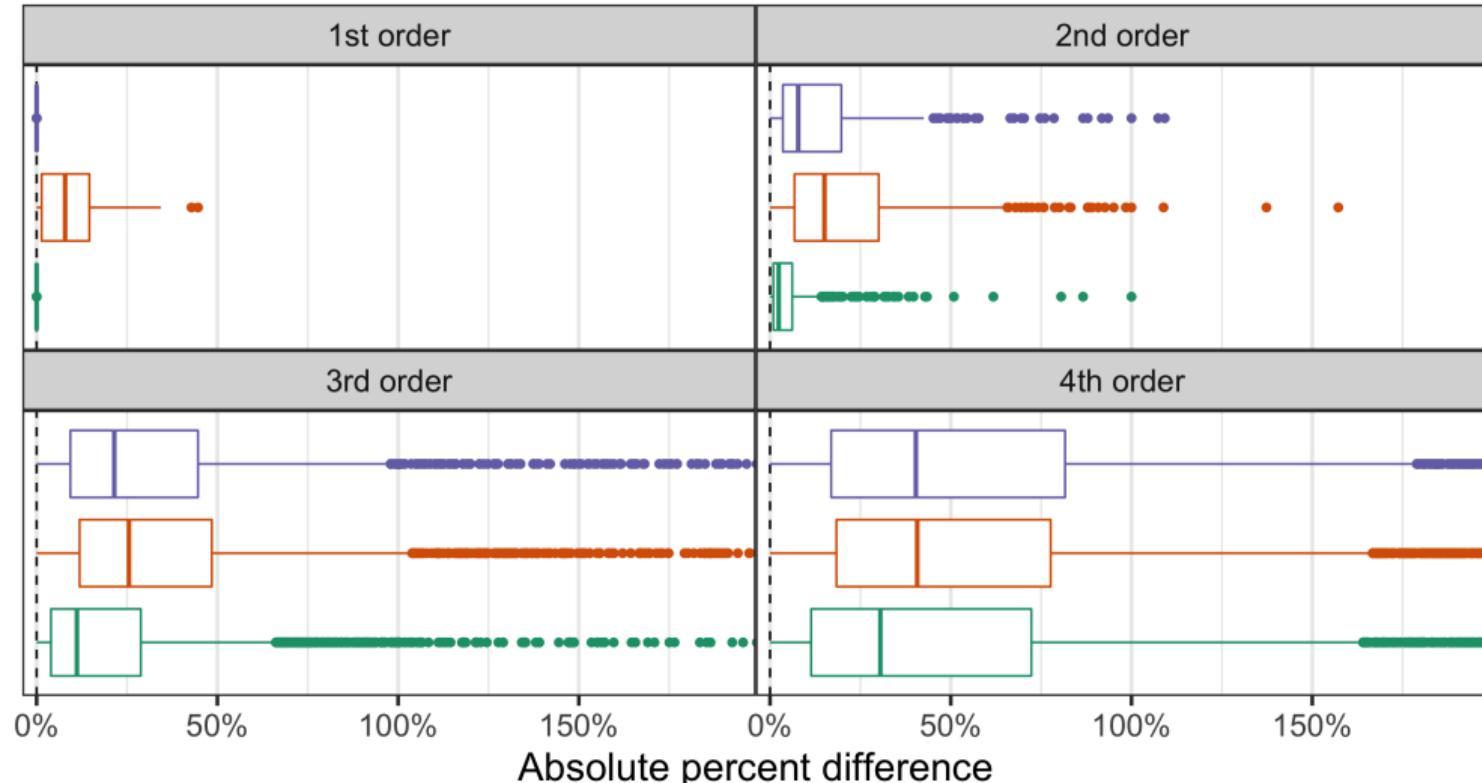


► Multilevel Weighting ► Post-Stratification (collapsed cells) ► Raking on margins

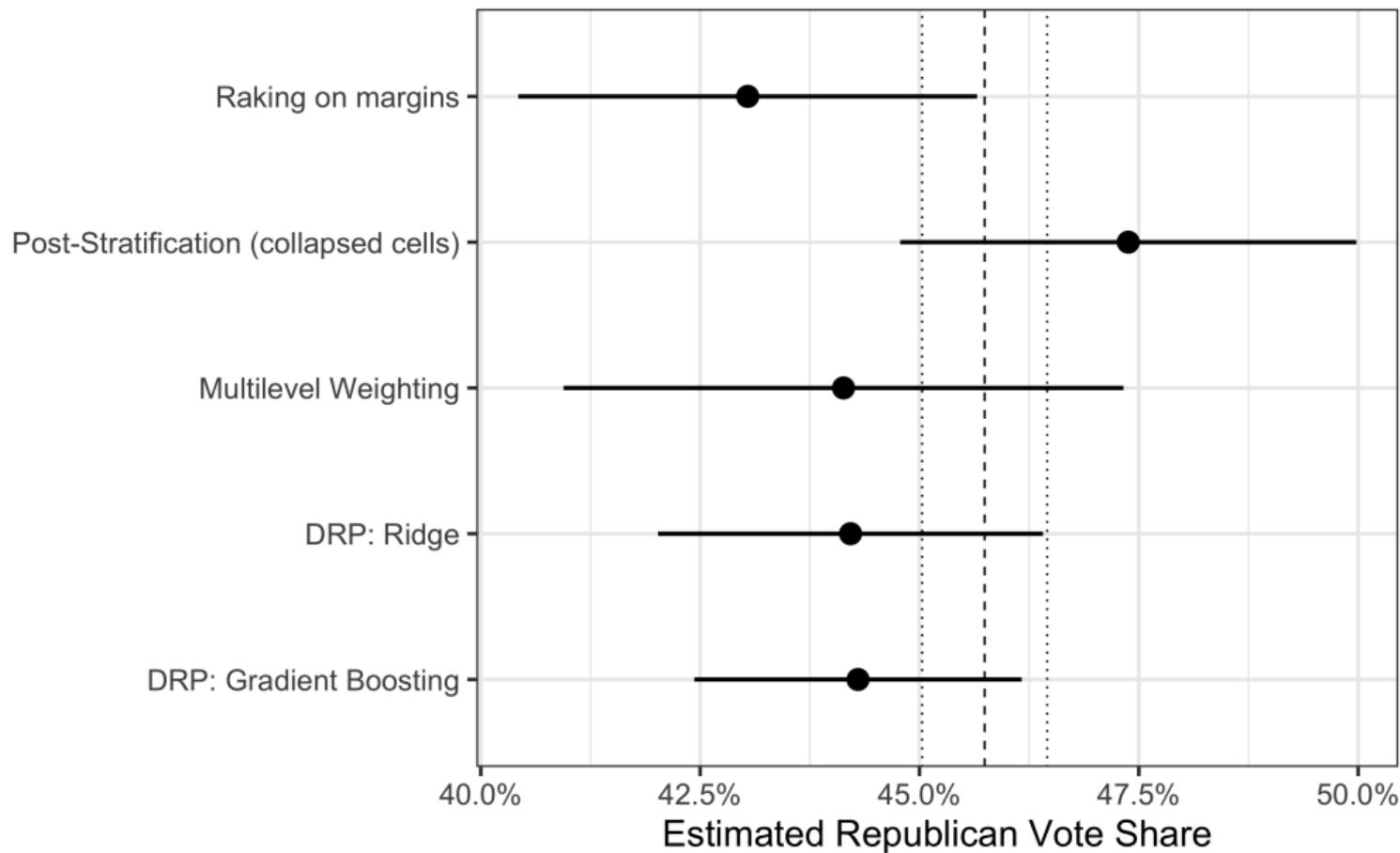
Approximate balance in 3rd order interactions



- ▶ Multilevel Weighting
- ▶ Post-Stratification (collapsed cells)
- ▶ Raking on margins

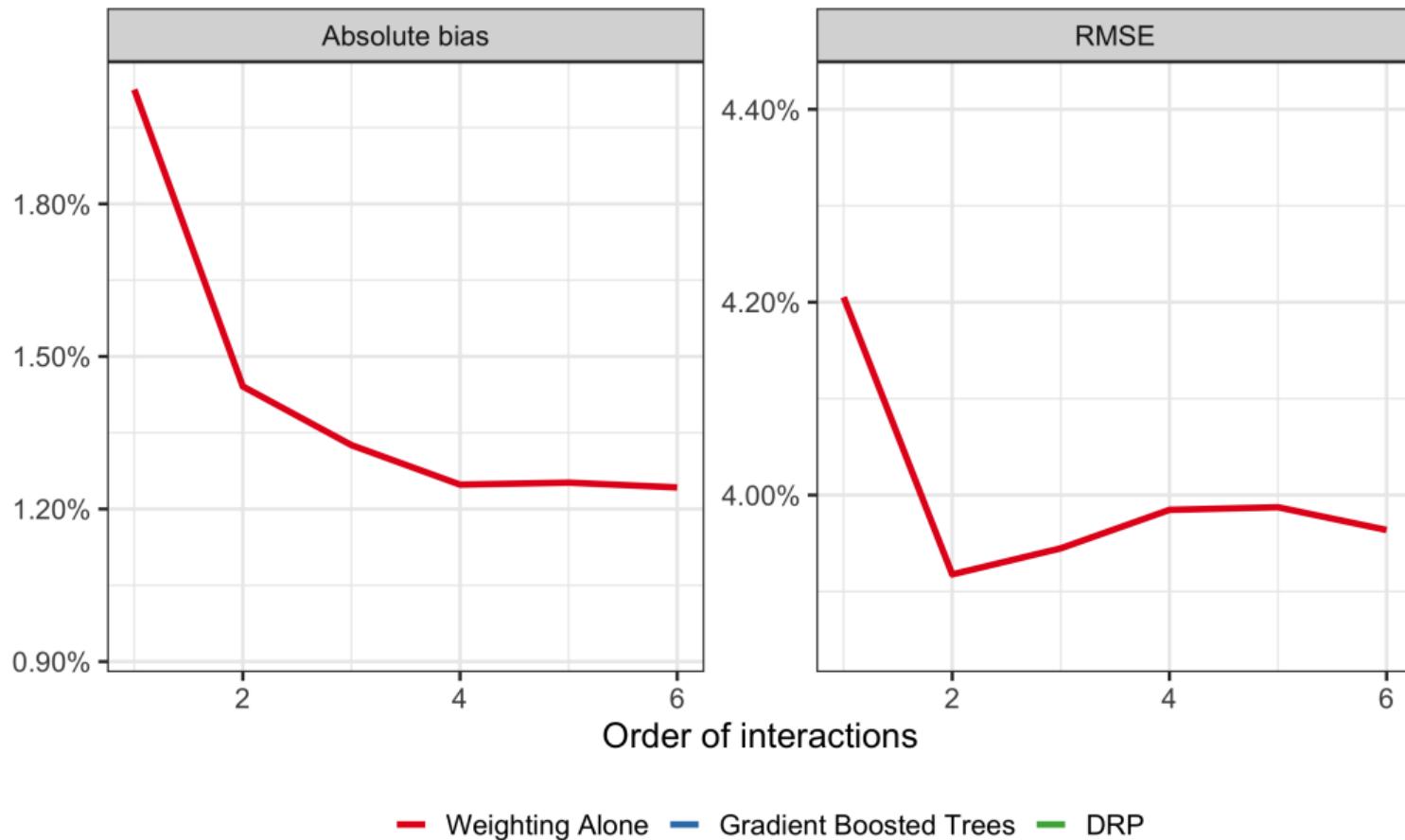


◻ Multilevel Weighting □ Post-Stratification (collapsed cells) ▨ Raking on margins

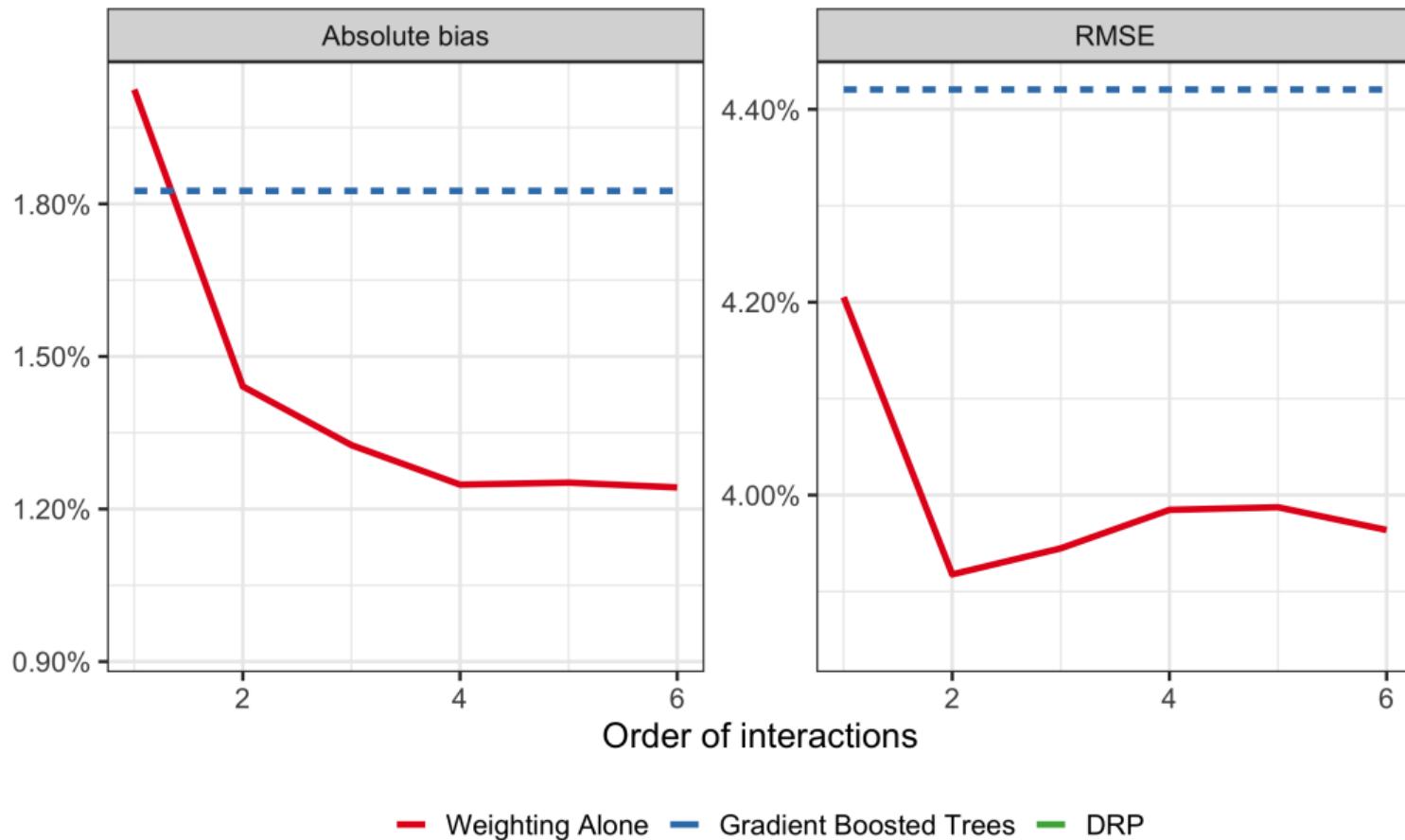


State-level results

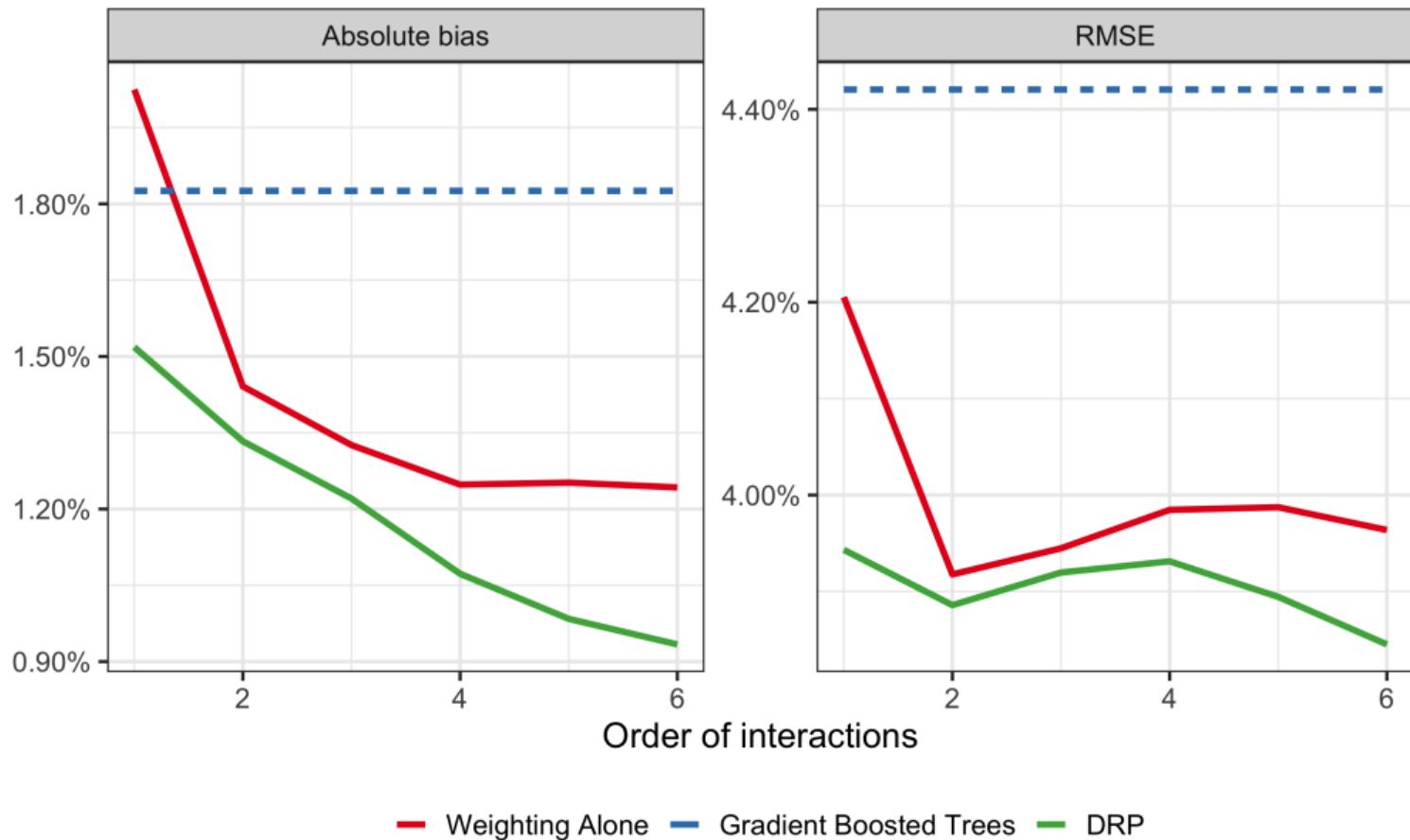
Weighting on higher order interactions reduces bias



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DRP gives further improvements



Recap

Principled, parsimonious way of leveraging the value of interactions

- Multilevel weighting as middle ground between **raking** and **post-stratification**
- Dual view as multilevel model for non-response
- **DRP** adjusts cells where weighting misses the mark

Recap

Principled, parsimonious way of leveraging the value of interactions

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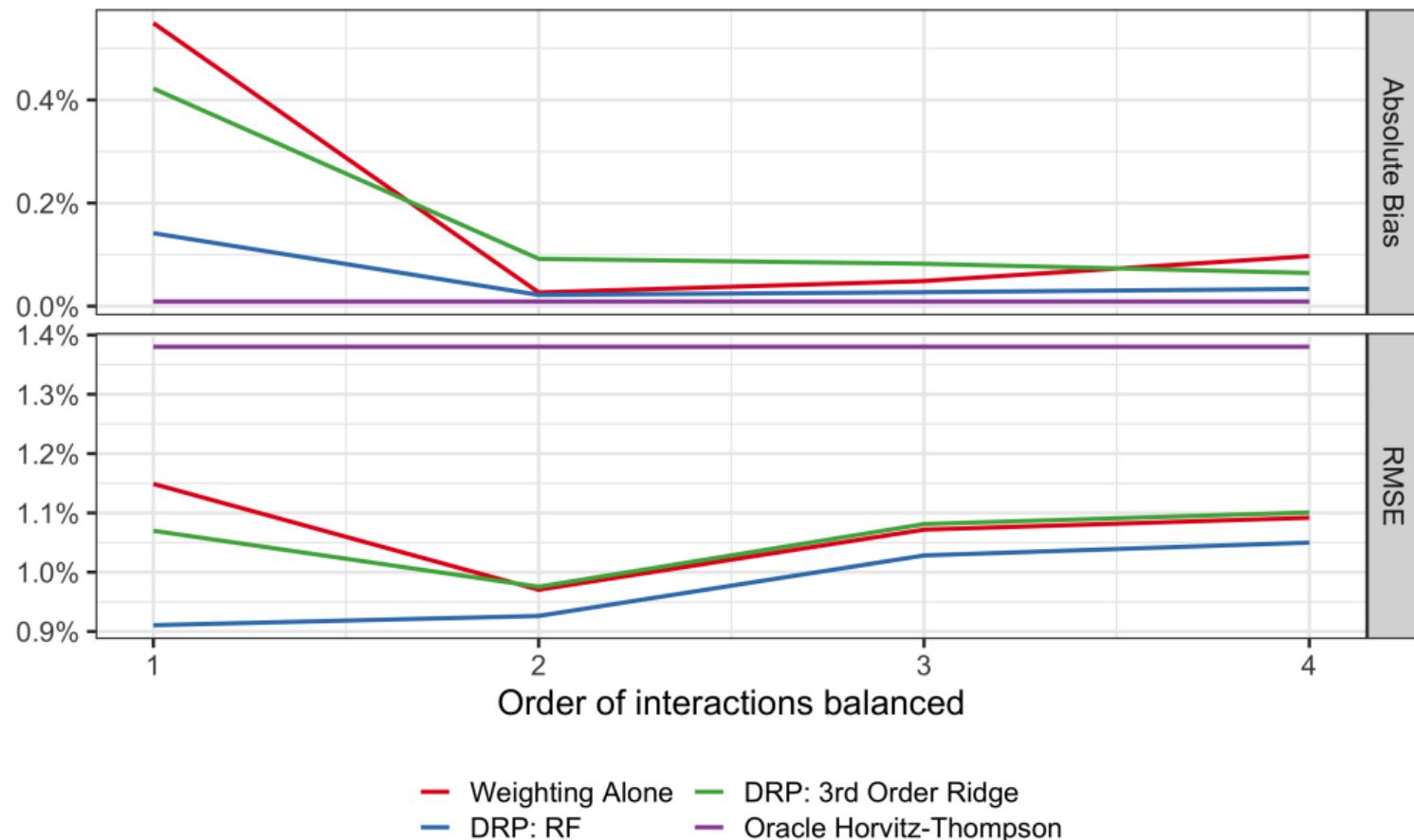
Thank you!

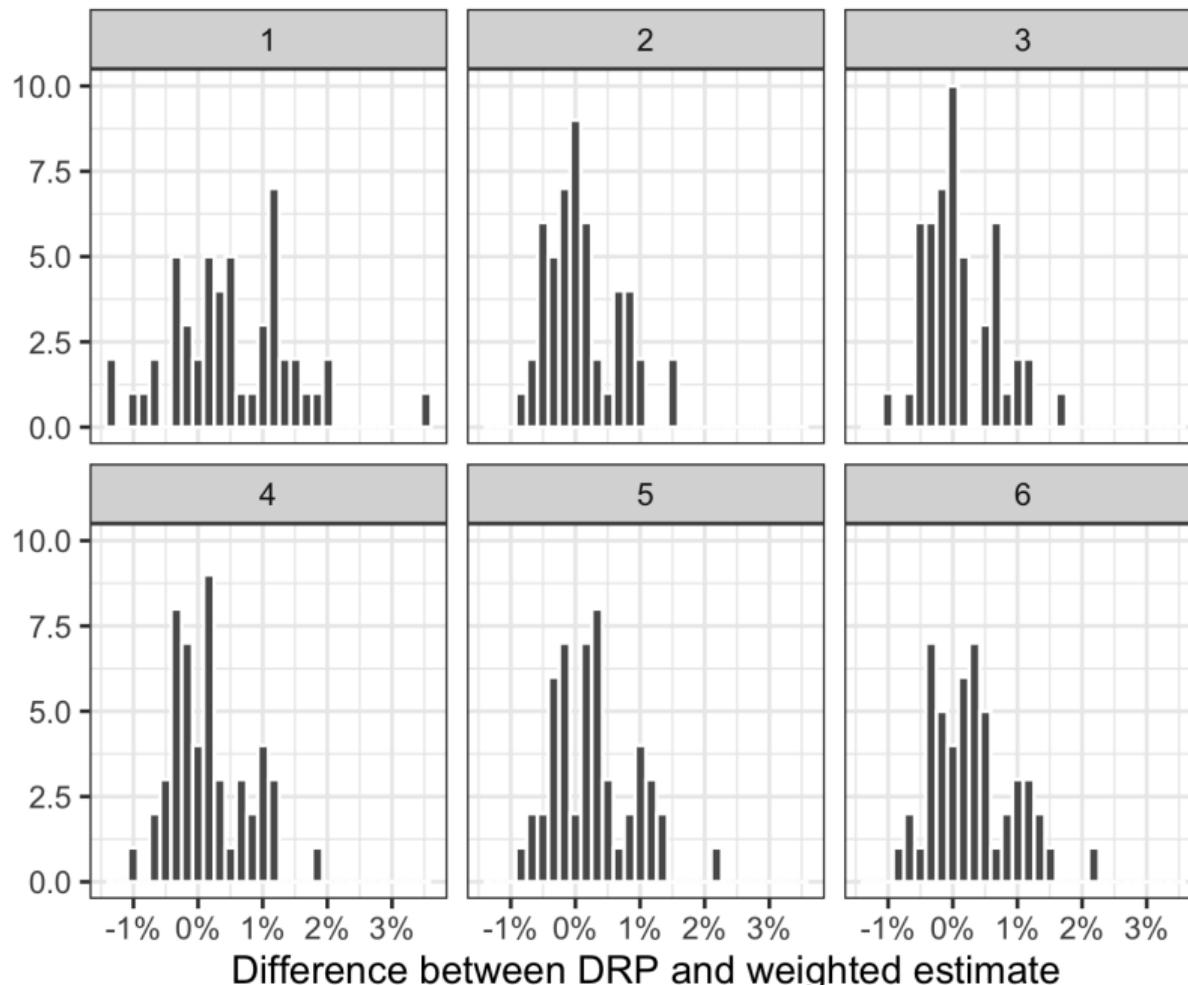
ebenmichael.github.io



Appendix

Actual Republican vote and RF response





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