

# Policy Evaluation with Staggered Adoption

Eli Ben-Michael

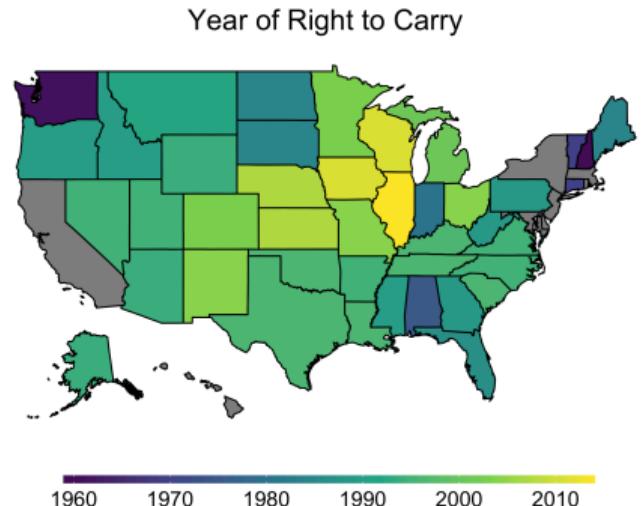
Harvard University

(Based on joint work with Avi Feller, Jesse Rothstein, and Elizabeth Stuart)



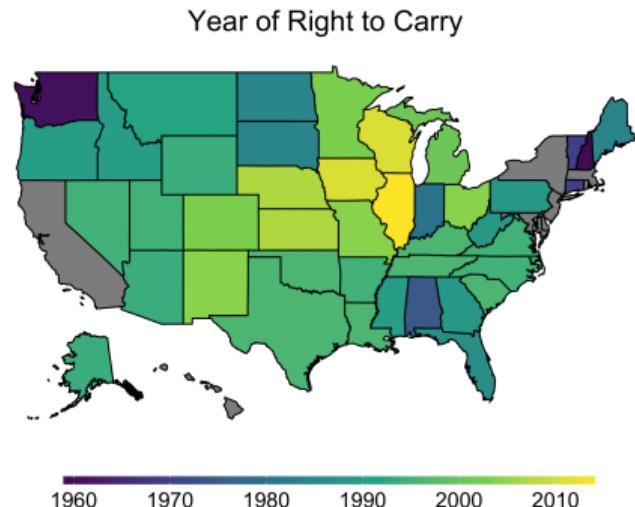
# What is the impact of right-to-carry laws on violent crime?

- 1959 - 2014: 42 states enact right-to-carry



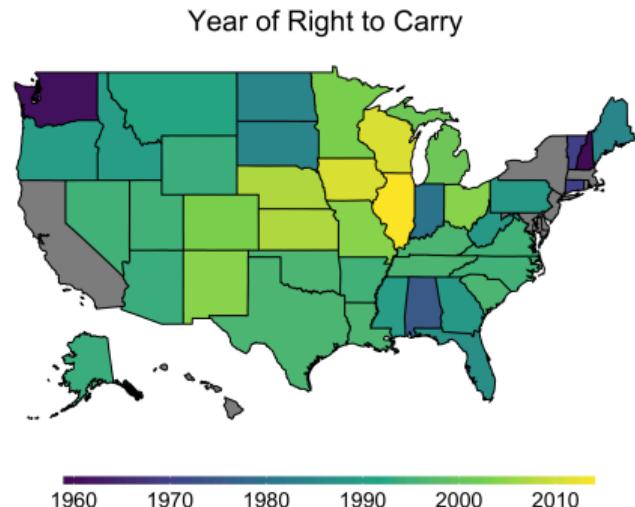
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[Lott and Mustard, 1997]



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- 1959 - 2014: 42 states enact right-to-carry
- "More guns, less crime"?  
[Lott and Mustard, 1997]
- New research says no  
[Donohue et al., 2019]



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A more design-based approach → **Policy Trial Emulation**

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A more design-based approach → **Policy Trial Emulation**

Applied to SCM → **Partially Pooled SCM**

- Modify optimization problem to target overall and state-specific fit
- Account for level differences with Intercept-Shifted SCM

# Combining ideas from Epidemiology and Econometrics

## **Target Trial Emulation**

Design an obs. study like a RCT

[Danaei et al., 2018;

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## **Panel Data Methods**

Beyond two-way fixed effects

[Abraham and Sun, 2018; Call-

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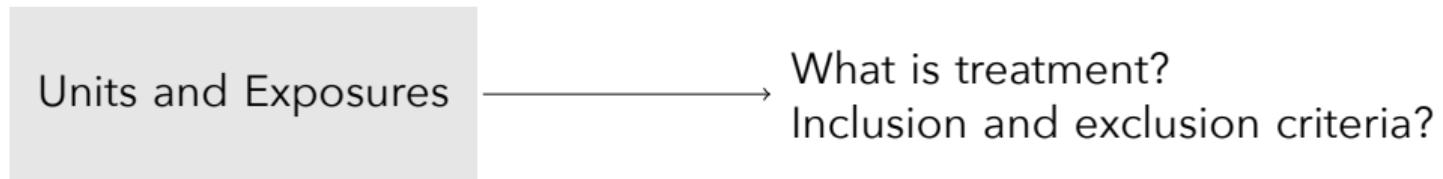
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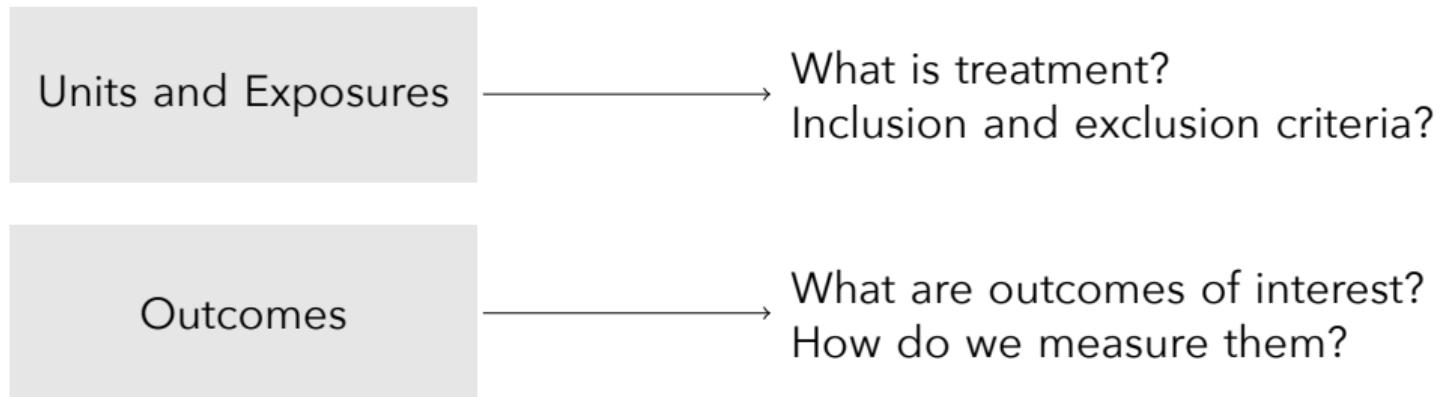
## Policy Trial Emulation

## The Elements of **Policy Trial Emulation**

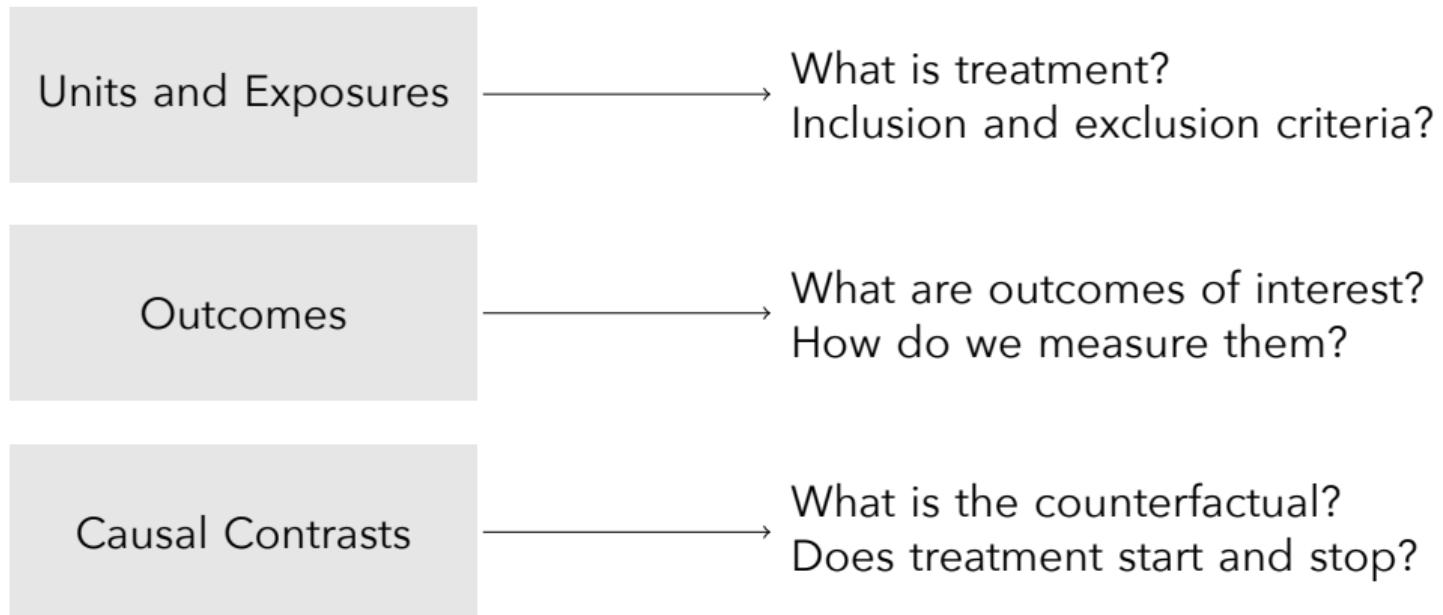
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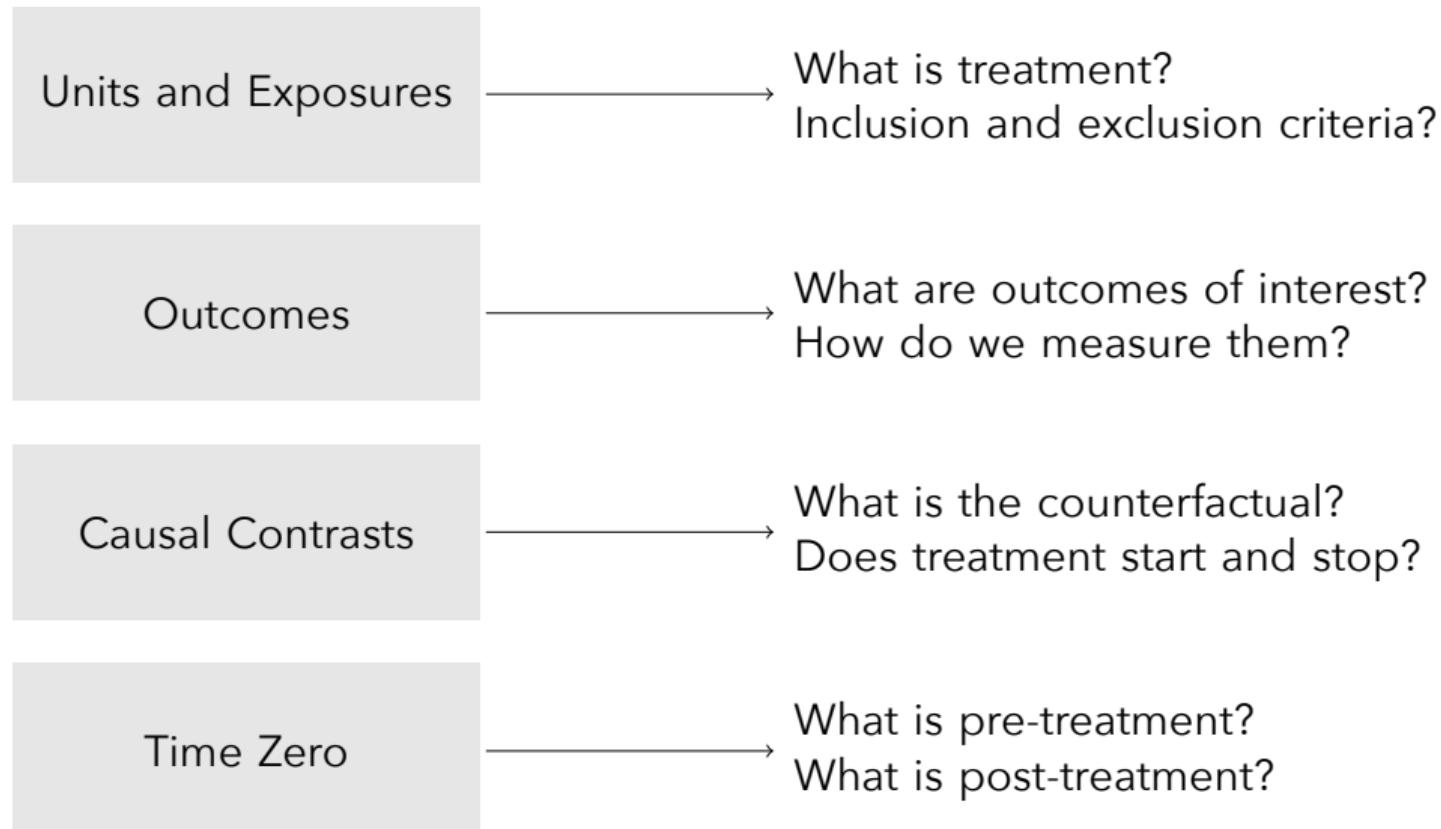
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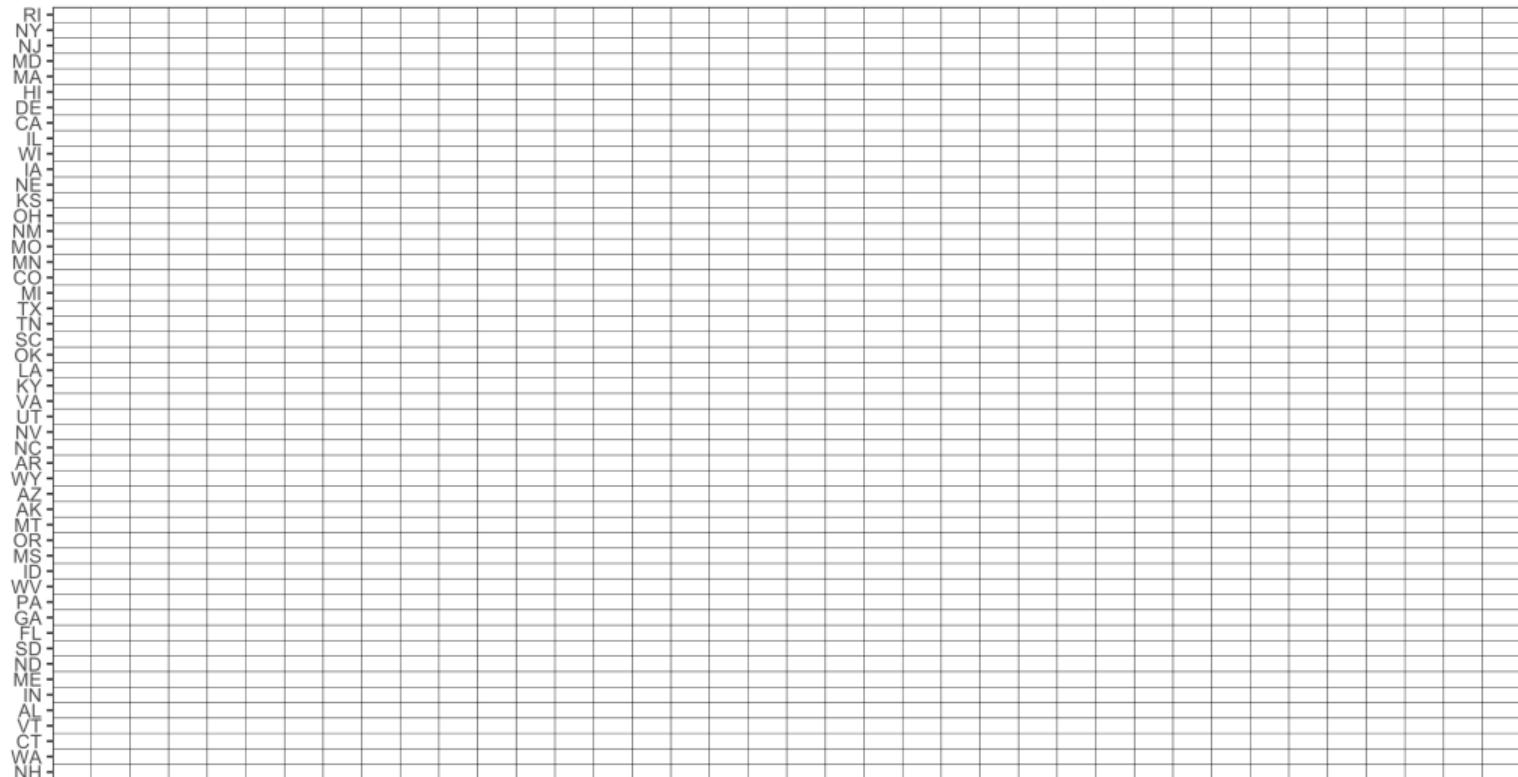


# The Elements of Policy Trial Emulation



# The Elements of Policy Trial Emulation





1980

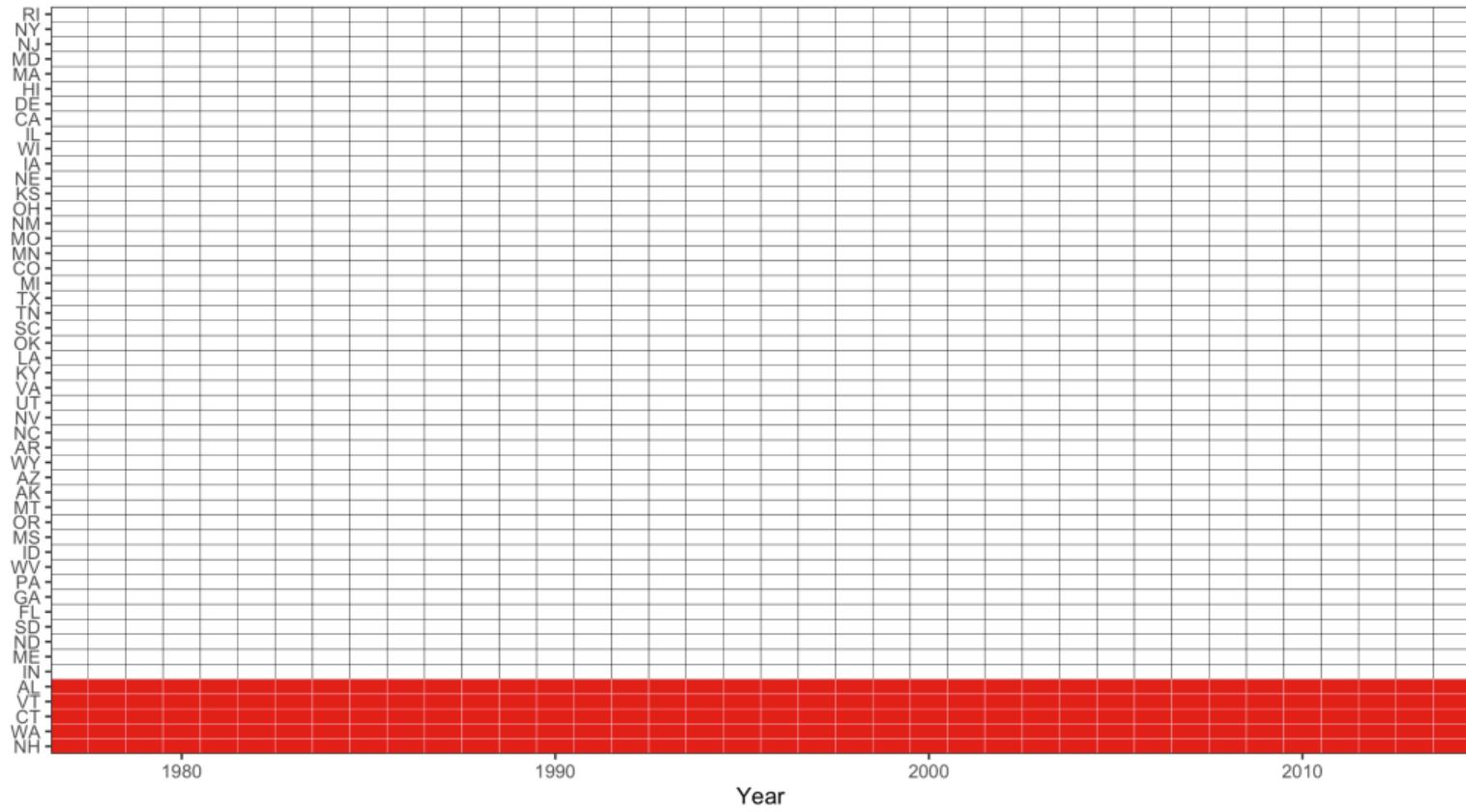
1990

2000

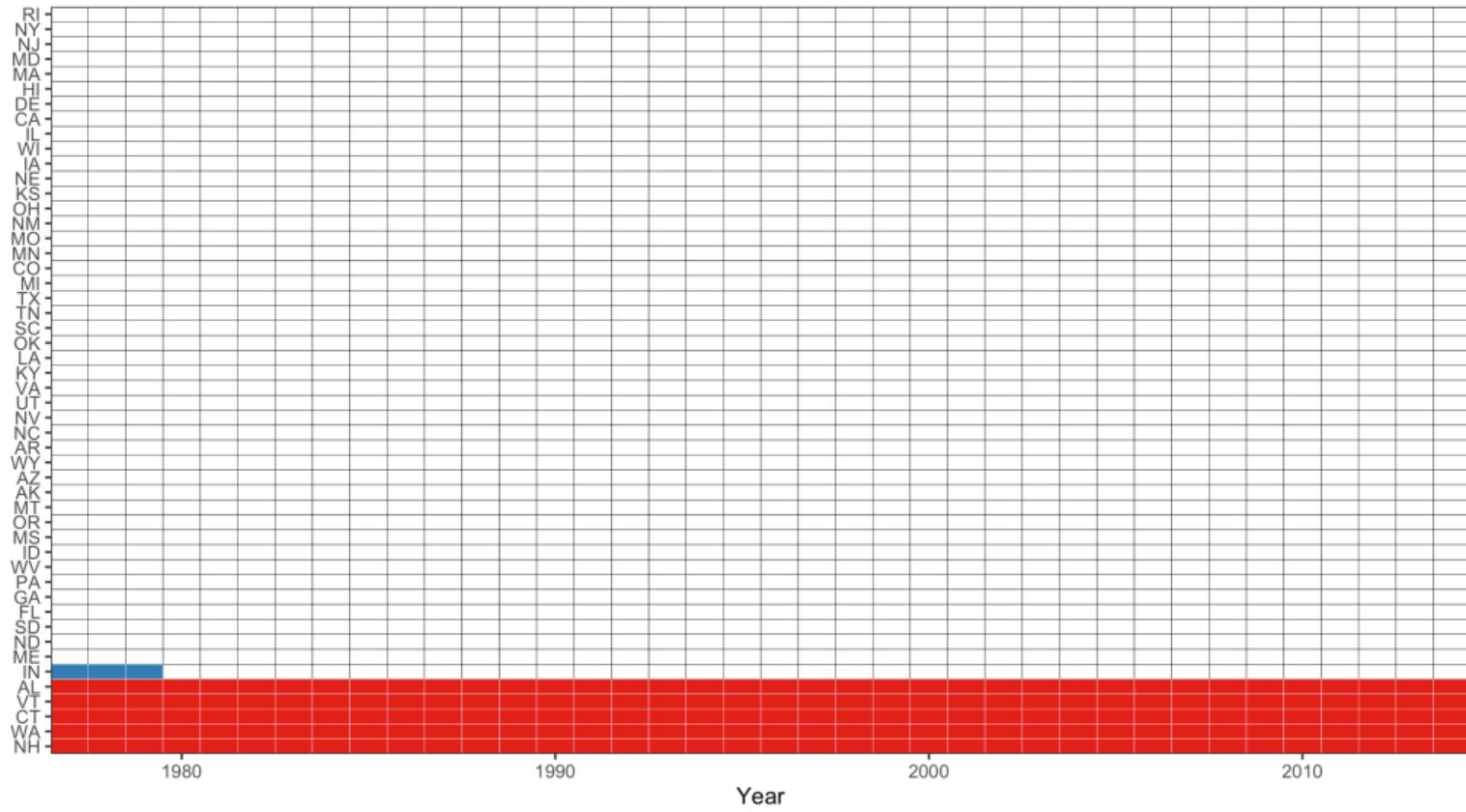
2010

Year

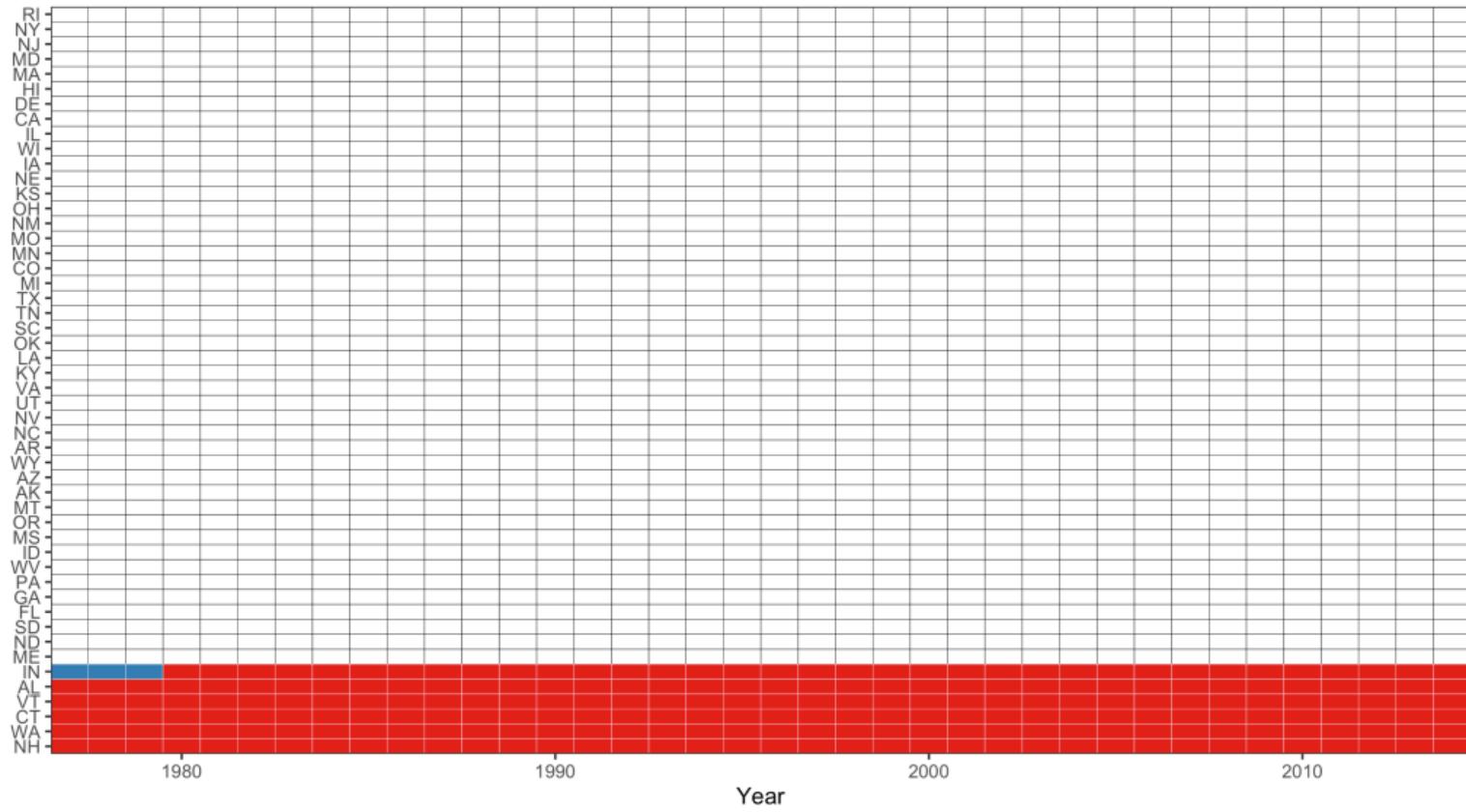
Right-to-carry  Adopted  Not Adopted



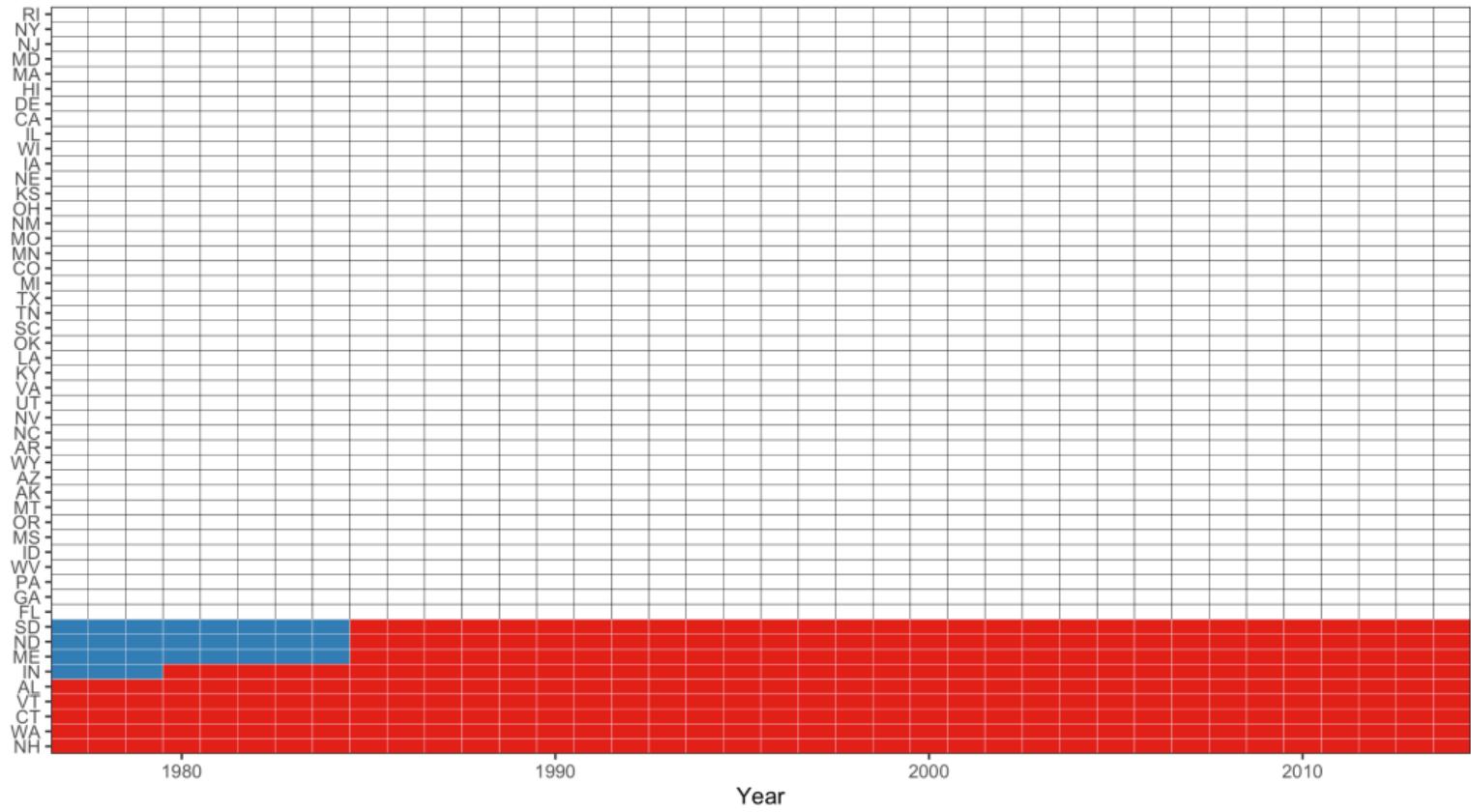
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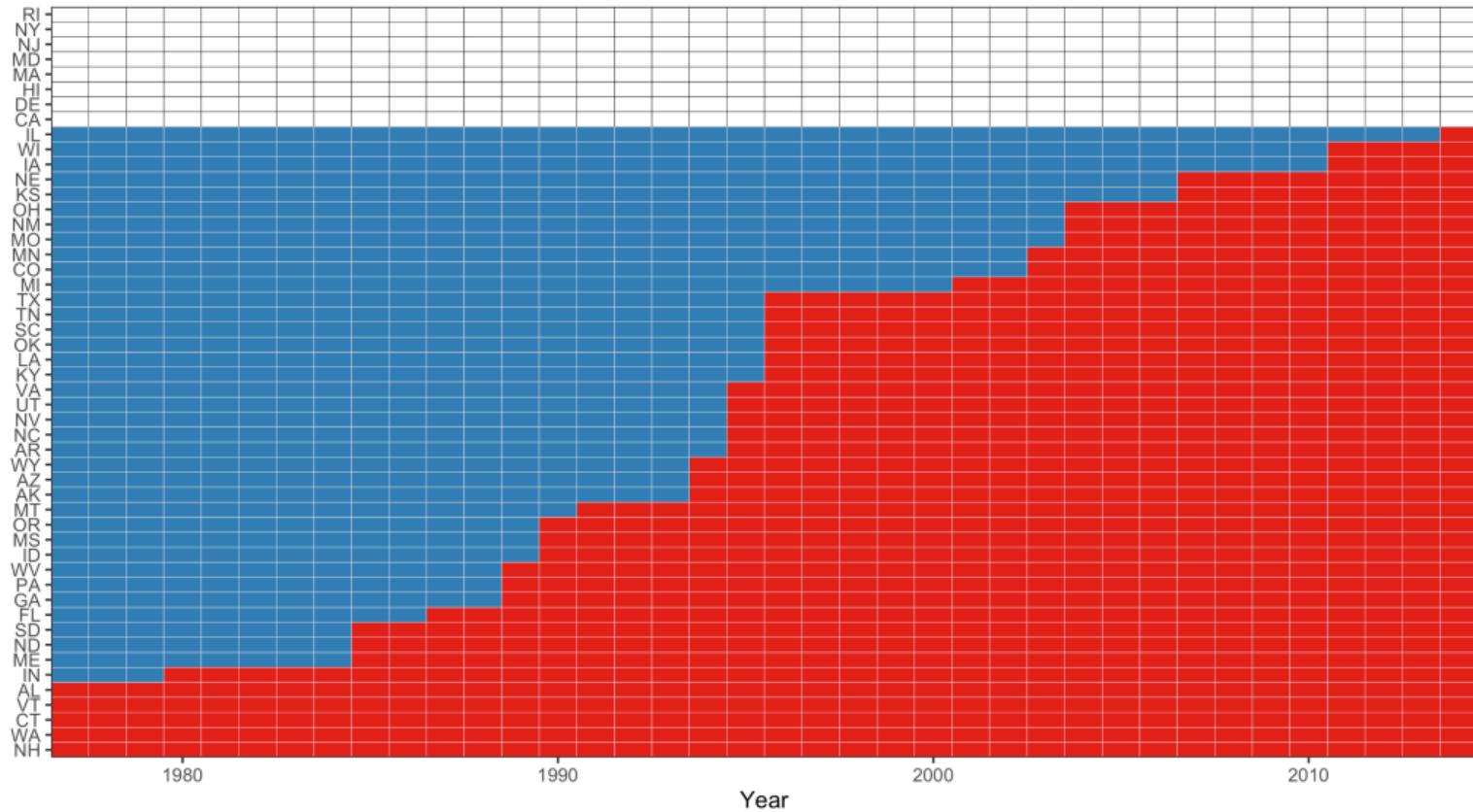
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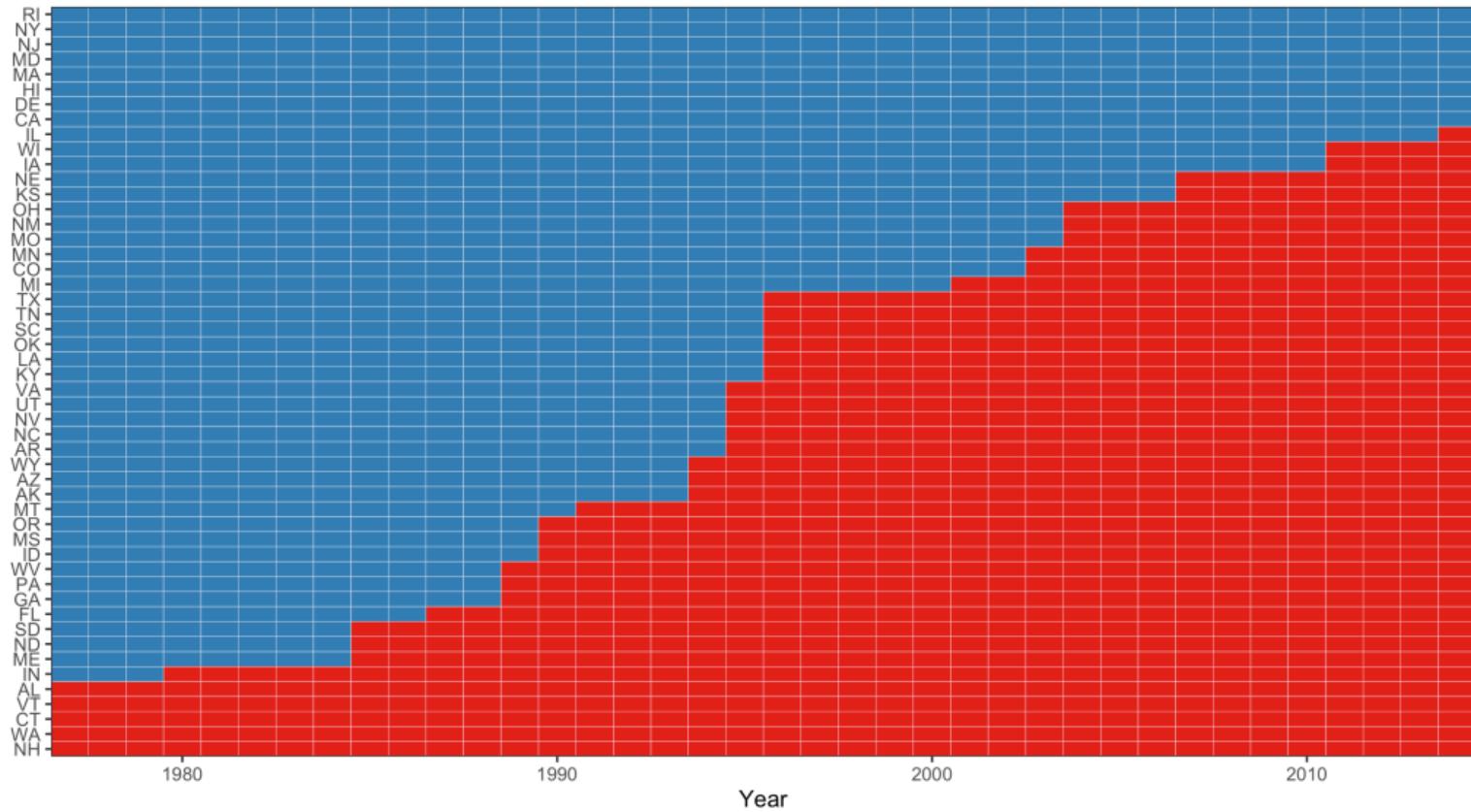
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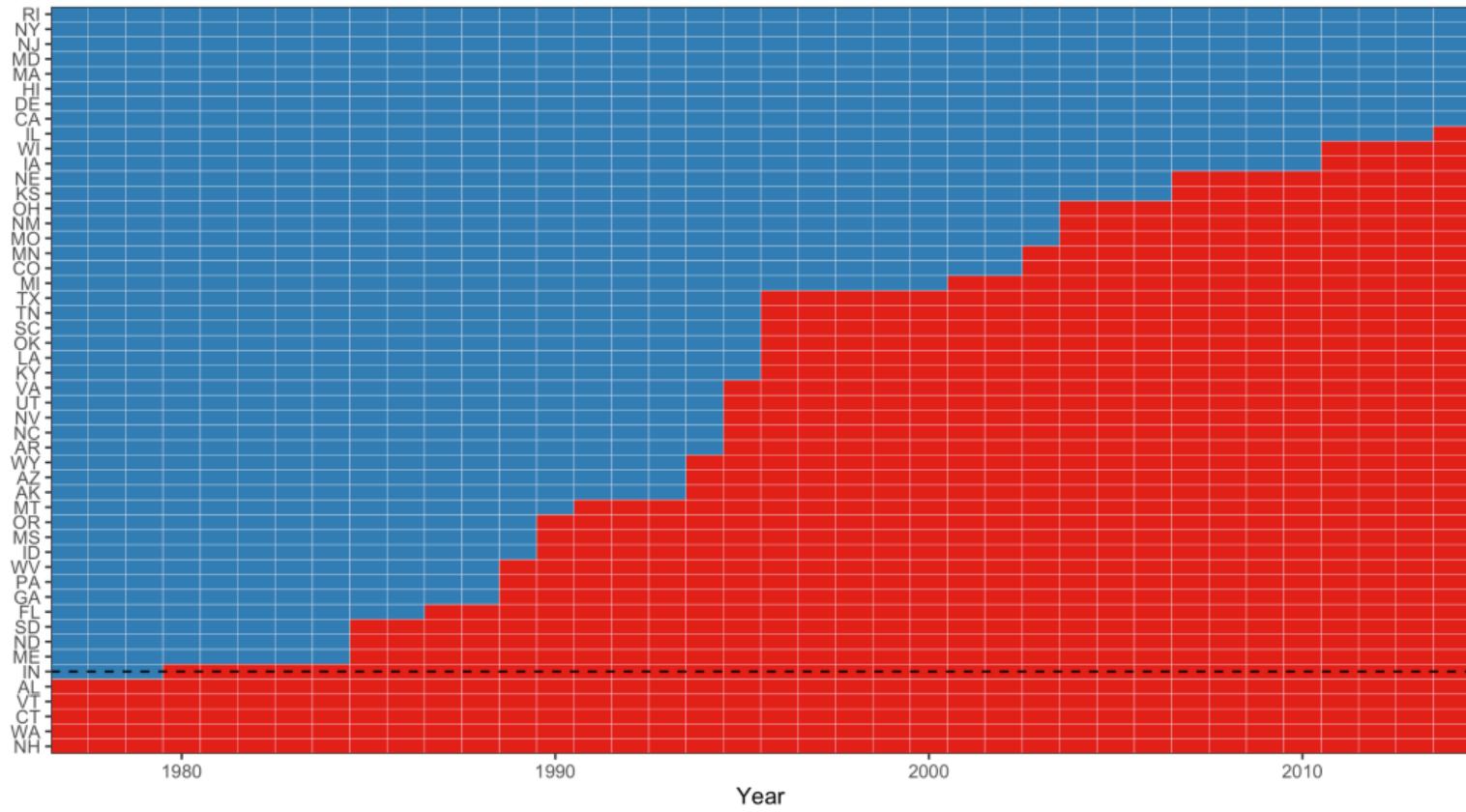
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## Causal contrasts

Units:  $i = 1, \dots, N$ ,  $J$  total treated units

Time:  $t = 1, \dots, T$ , treatment times  $T_1, \dots, T_J, \infty$

Outcome: at event time  $k$ ,  $Y_{i,T_j+k}$

- Some assumptions to write down potential outcomes  
[Athey and Imbens, 2018; Imai and Kim, 2019]

$$\text{treat} = \begin{pmatrix} \checkmark & \checkmark & \checkmark \\ & \checkmark & \checkmark \\ & & \checkmark \end{pmatrix}$$

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Basic building block:

$$\tau_{jk} = Y_{jT_j+k}(T_j) - \underbrace{Y_{jT_j+k}(\infty)}_{\sum \hat{\gamma}_{ij} Y_{iT_j+k}}$$

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**Single Target Trial**

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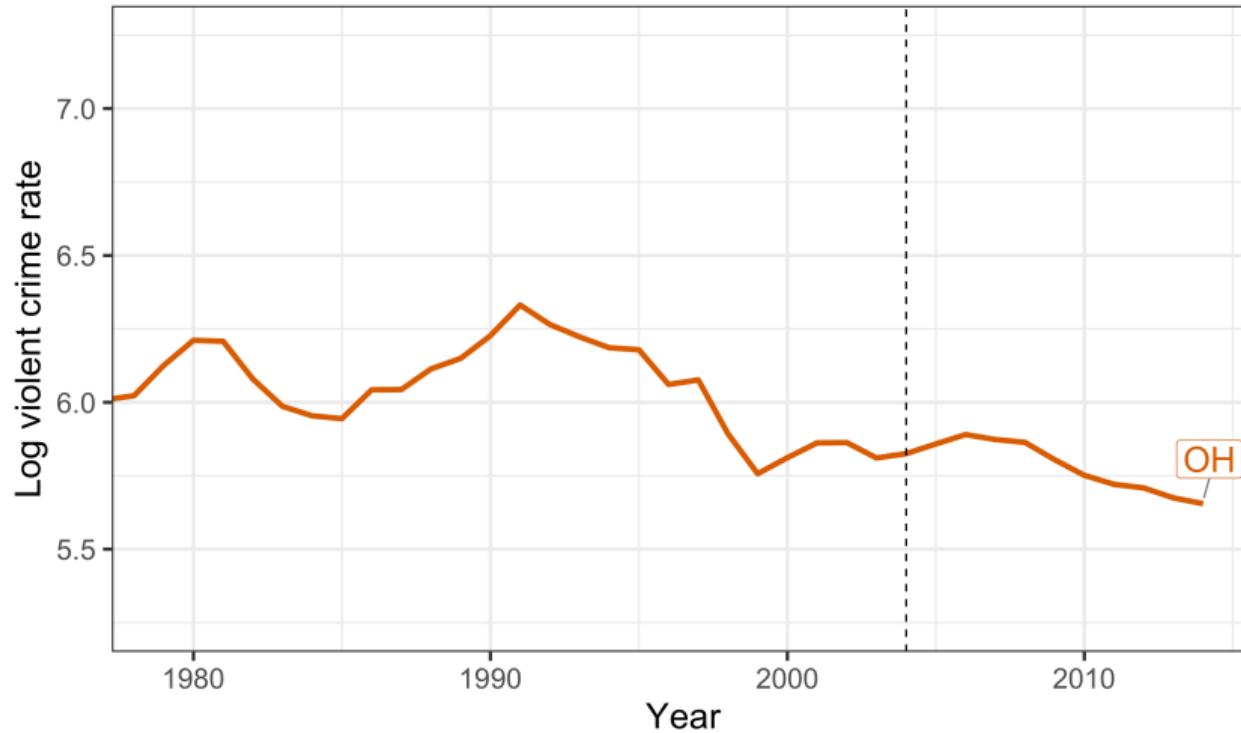
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Average at event time  $k$ :

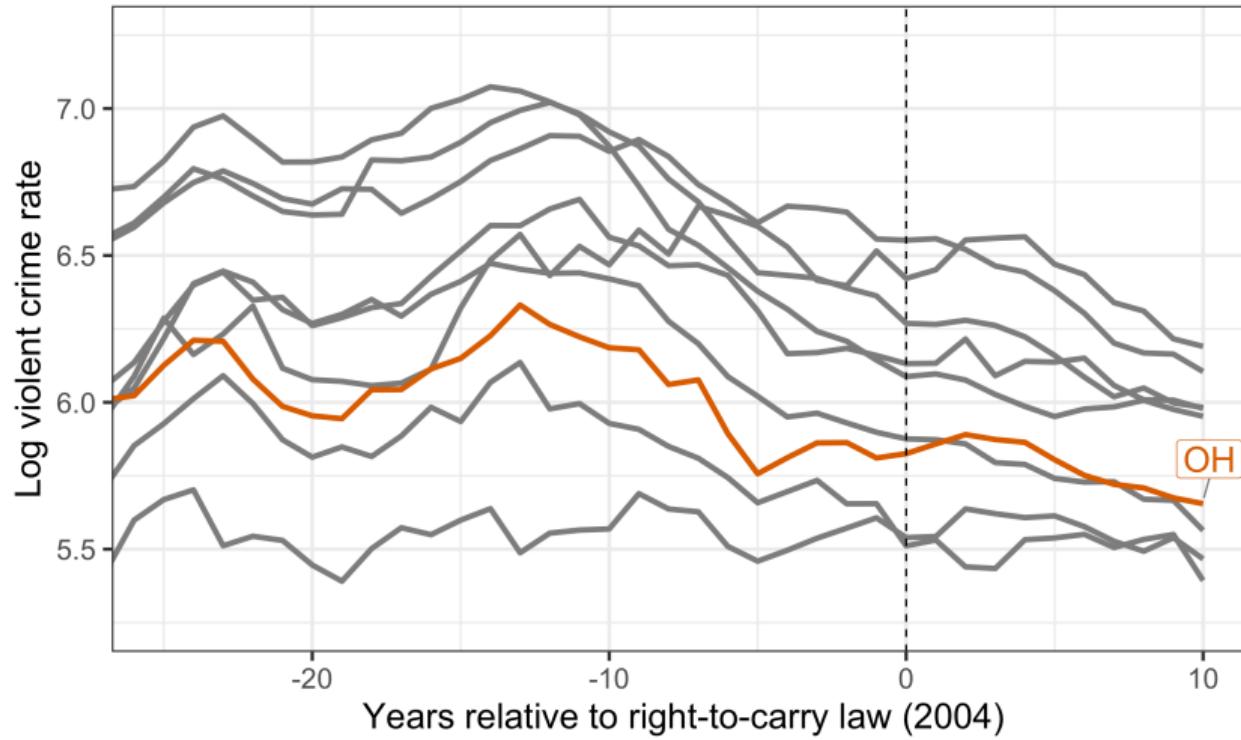
$$\text{ATT}_k = \frac{1}{J} \sum_{j=1}^J \tau_{jk}$$

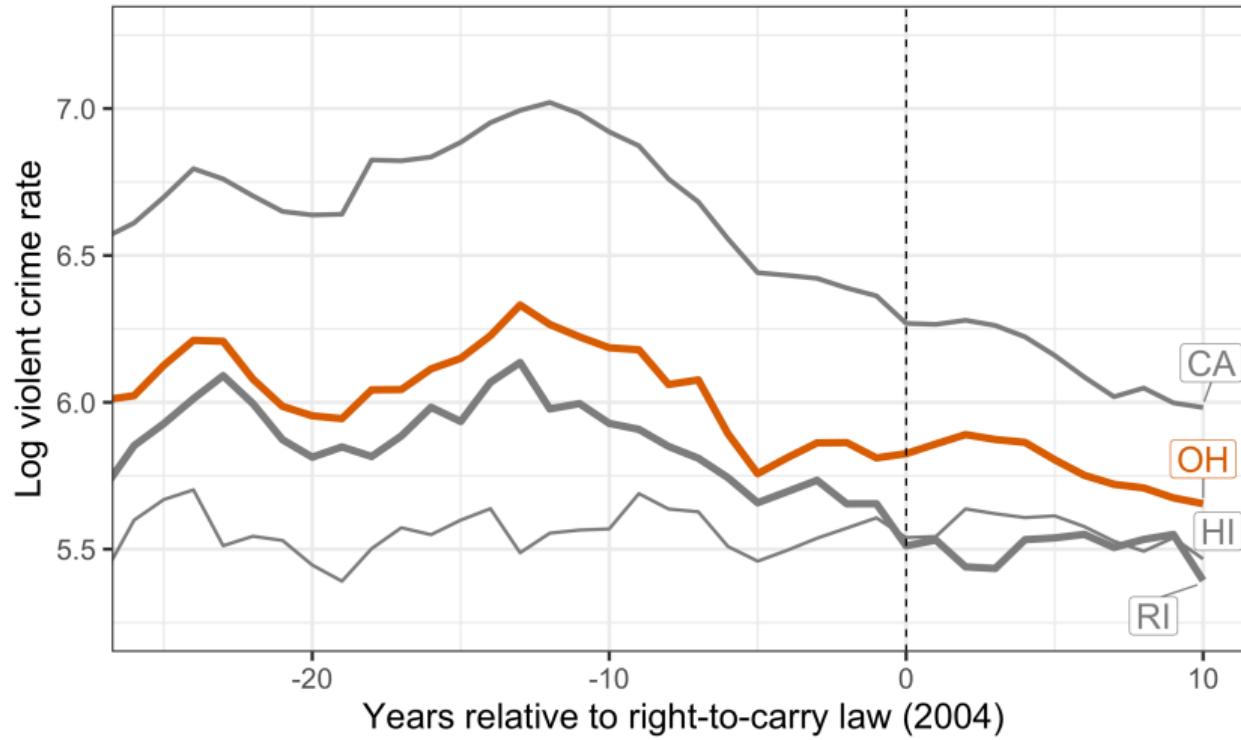
**Nested Target Trials**

# Single Target Trial Synthetic Controls

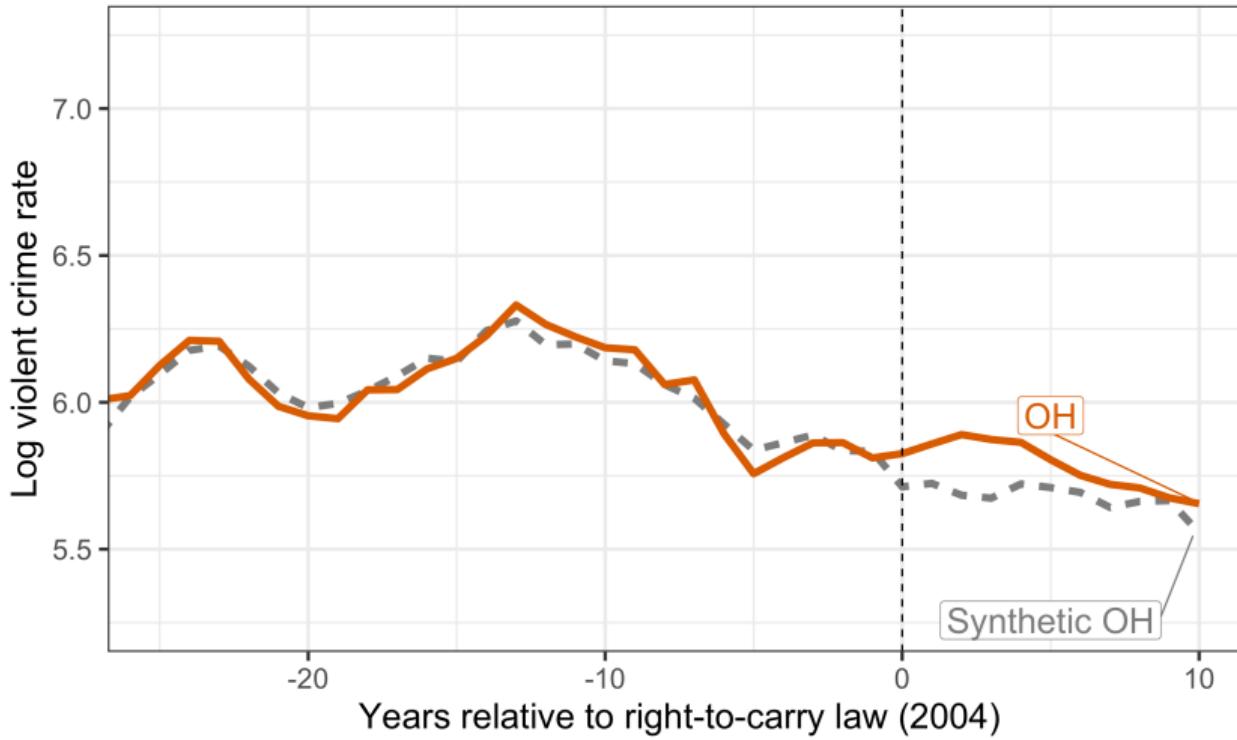




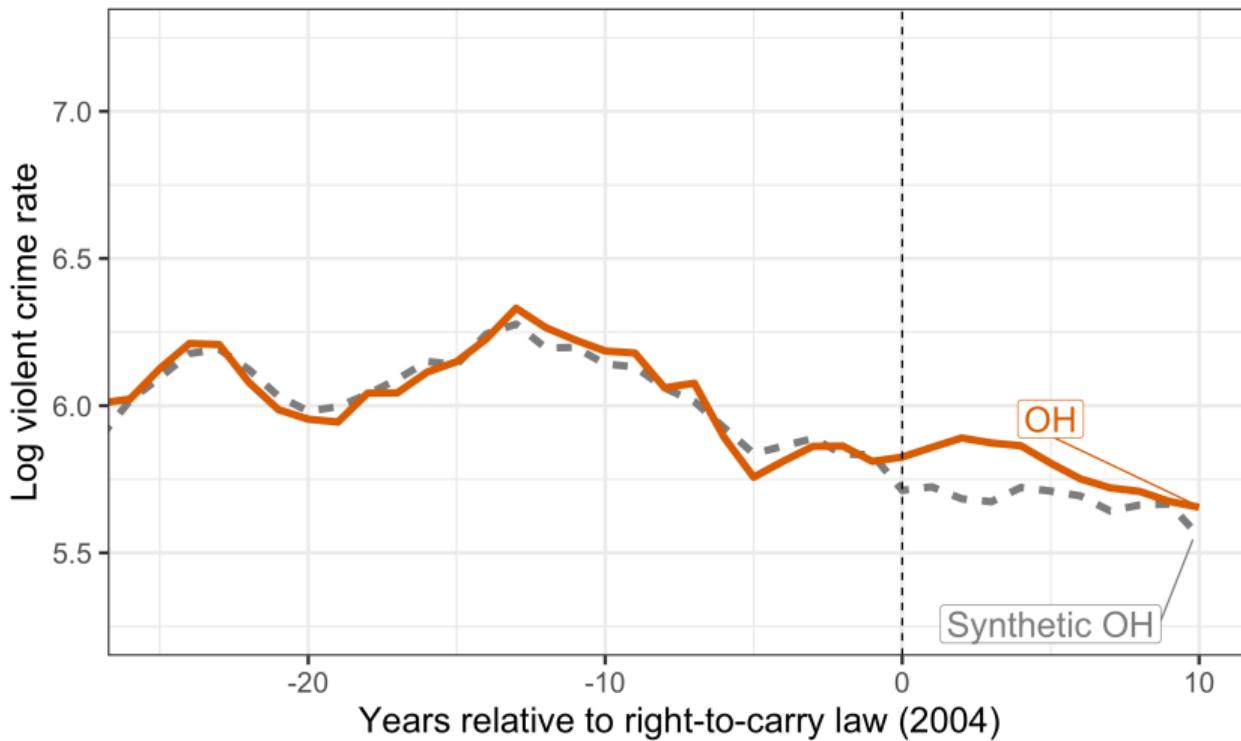




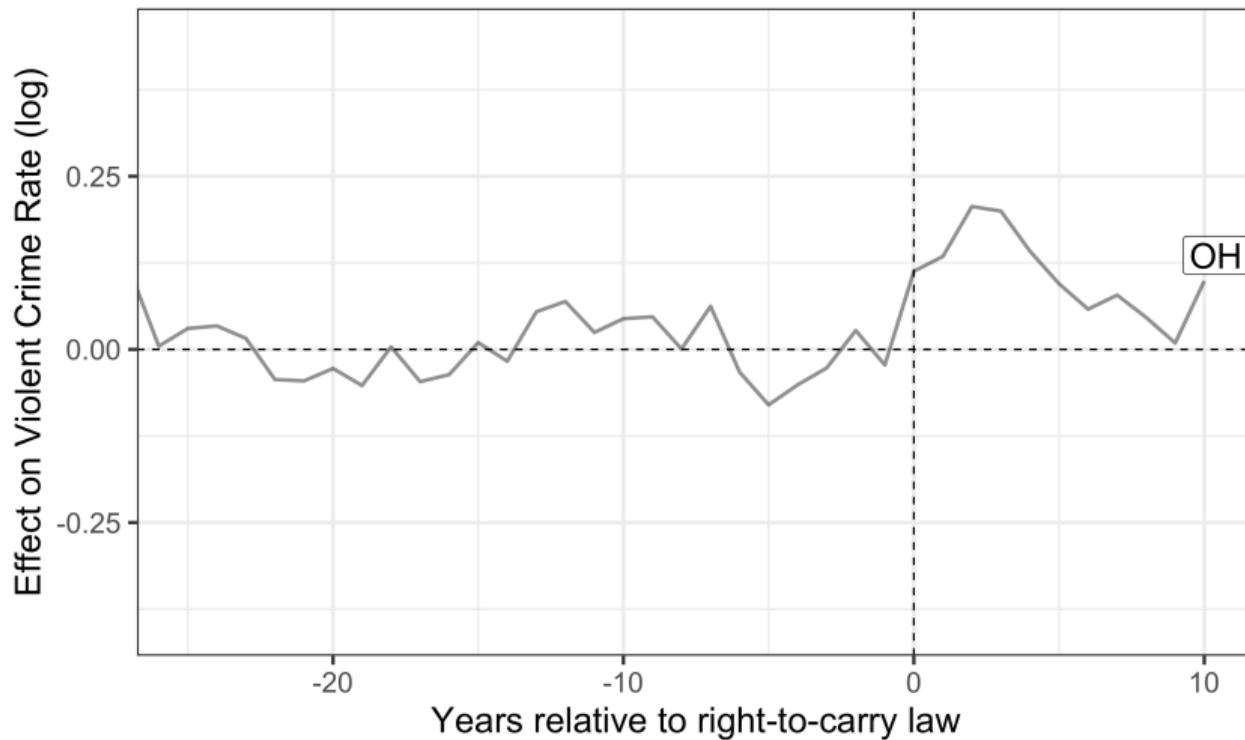
$$\min_{\gamma \in \Delta^{\text{scm}}} \left\| Y_{\text{OH}\ell} - \sum_{i \neq \text{OH}} \gamma_i Y_{i\ell} \right\|_2^2 + \text{penalty}$$



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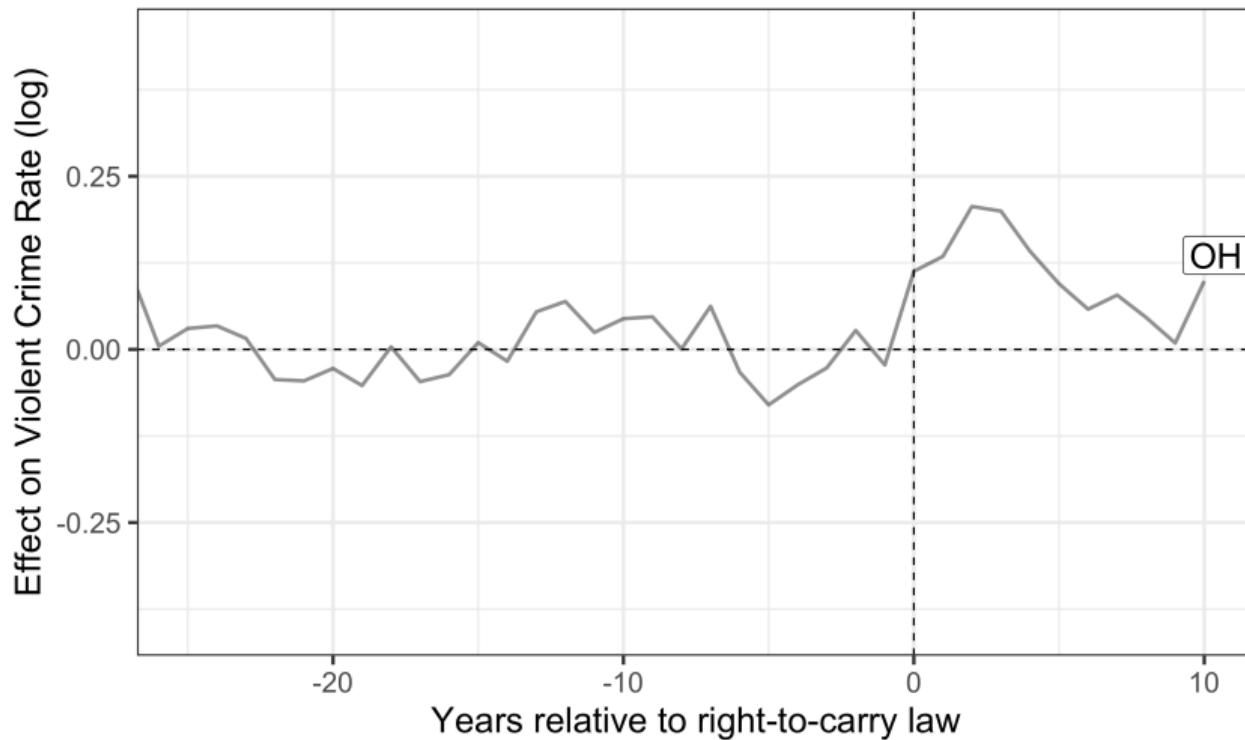
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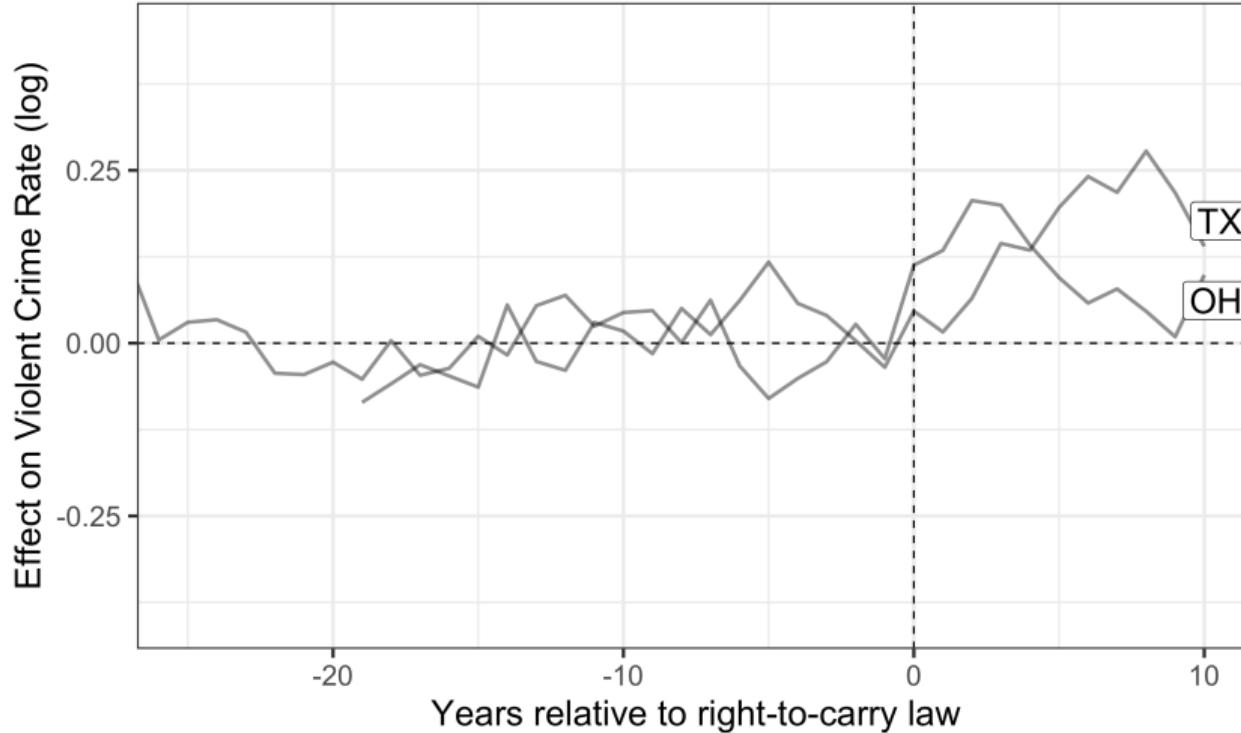
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# Towards Nested Target Trials

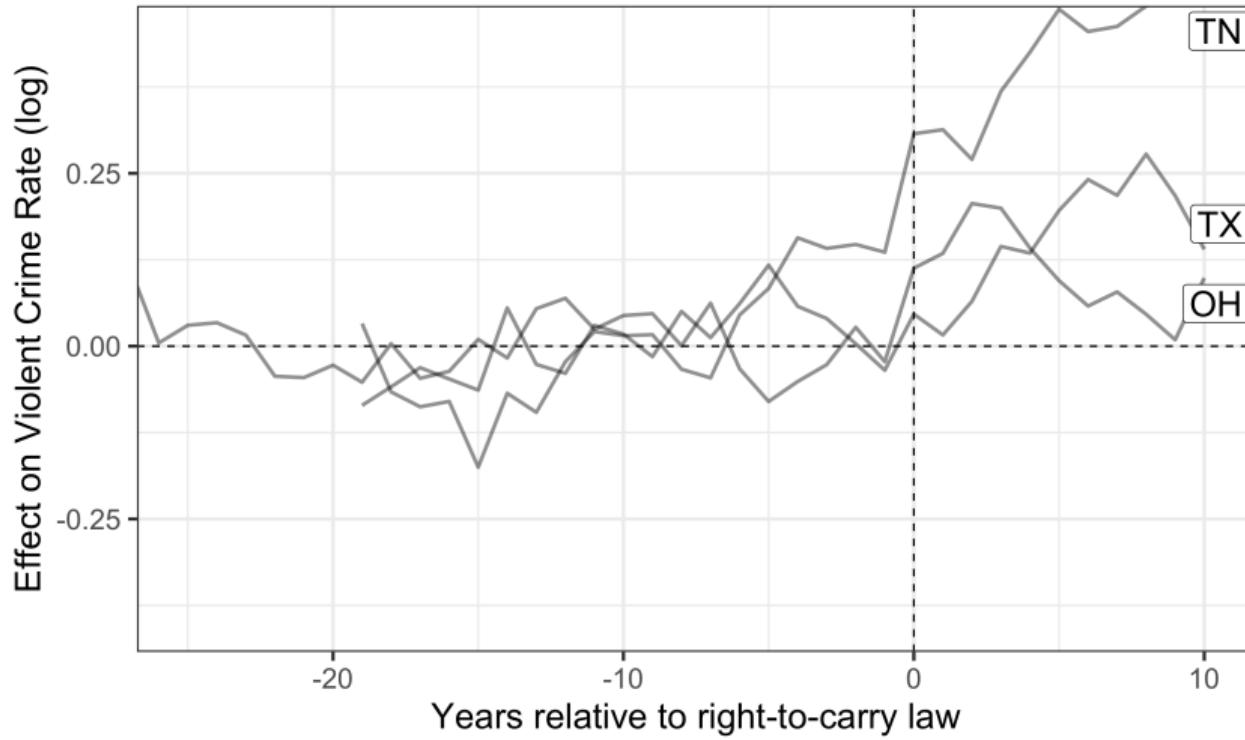
## Separate Synthetic Controls



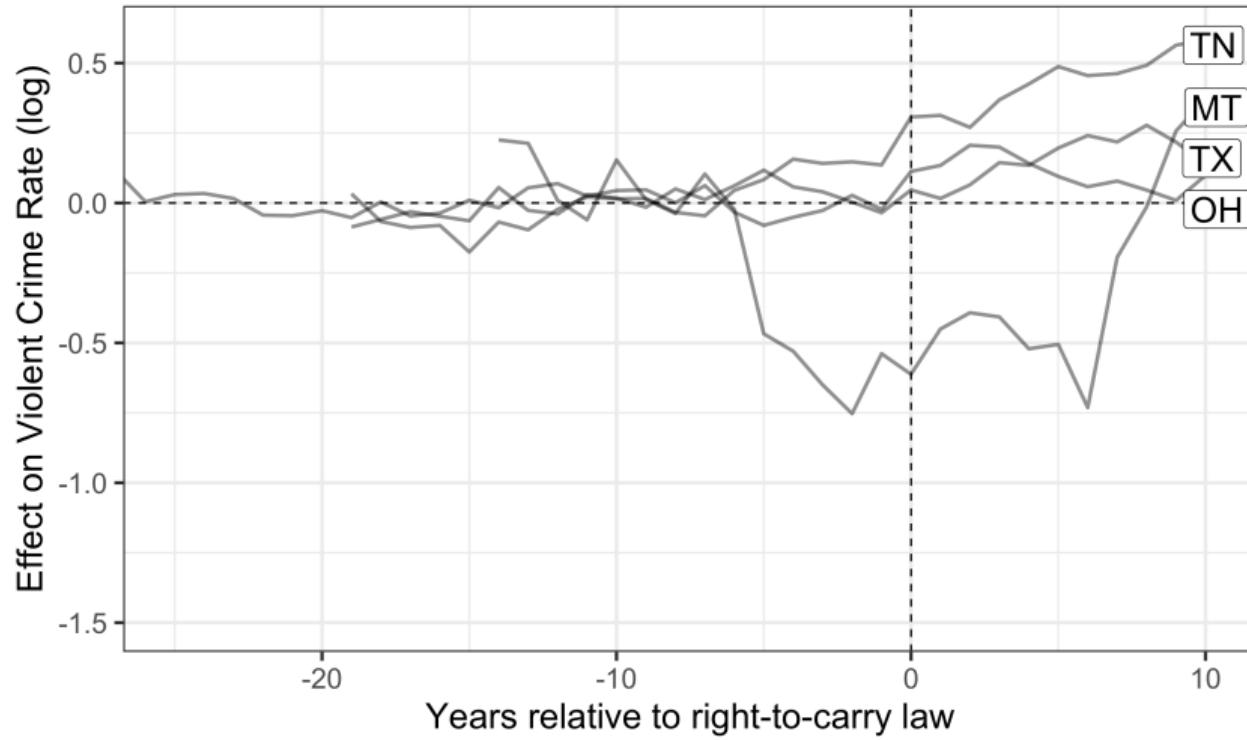
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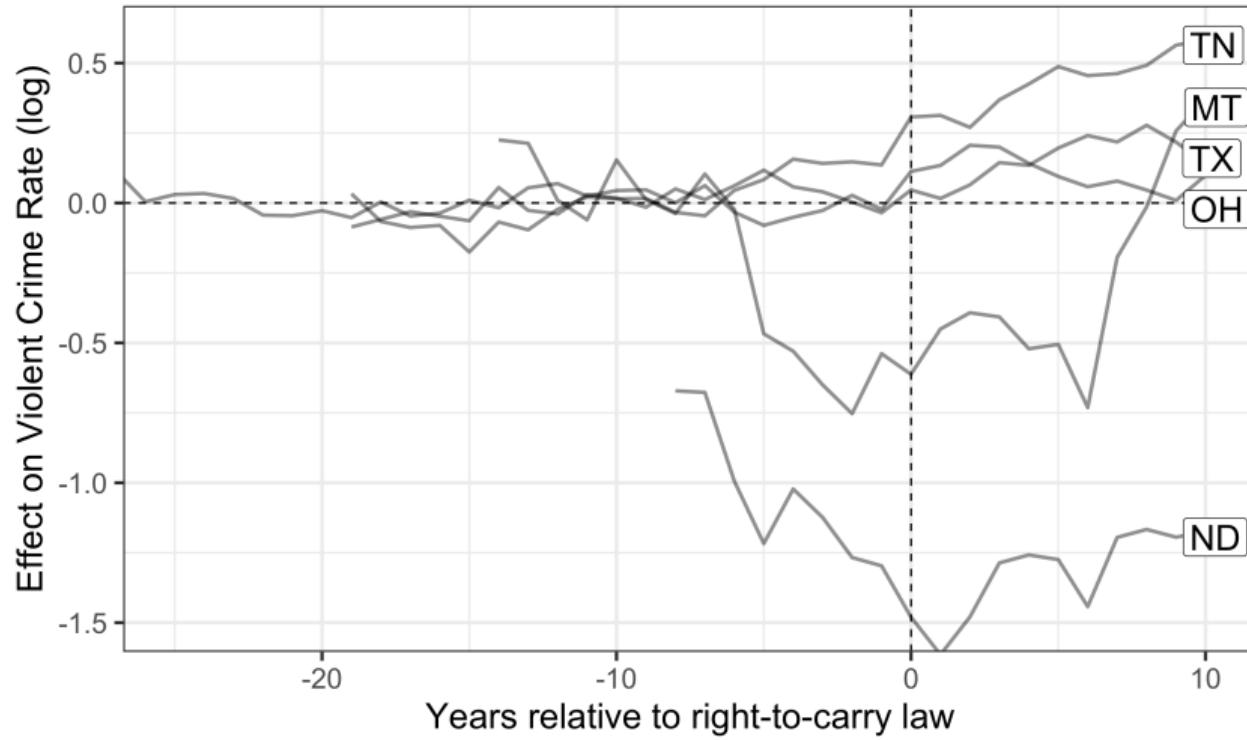
$$\min_{\Gamma \in \Delta^{\text{scm}}} \frac{1}{J} \sum_{j=1}^J \|\text{State Balance}_j\|_2^2 + \text{penalty}$$



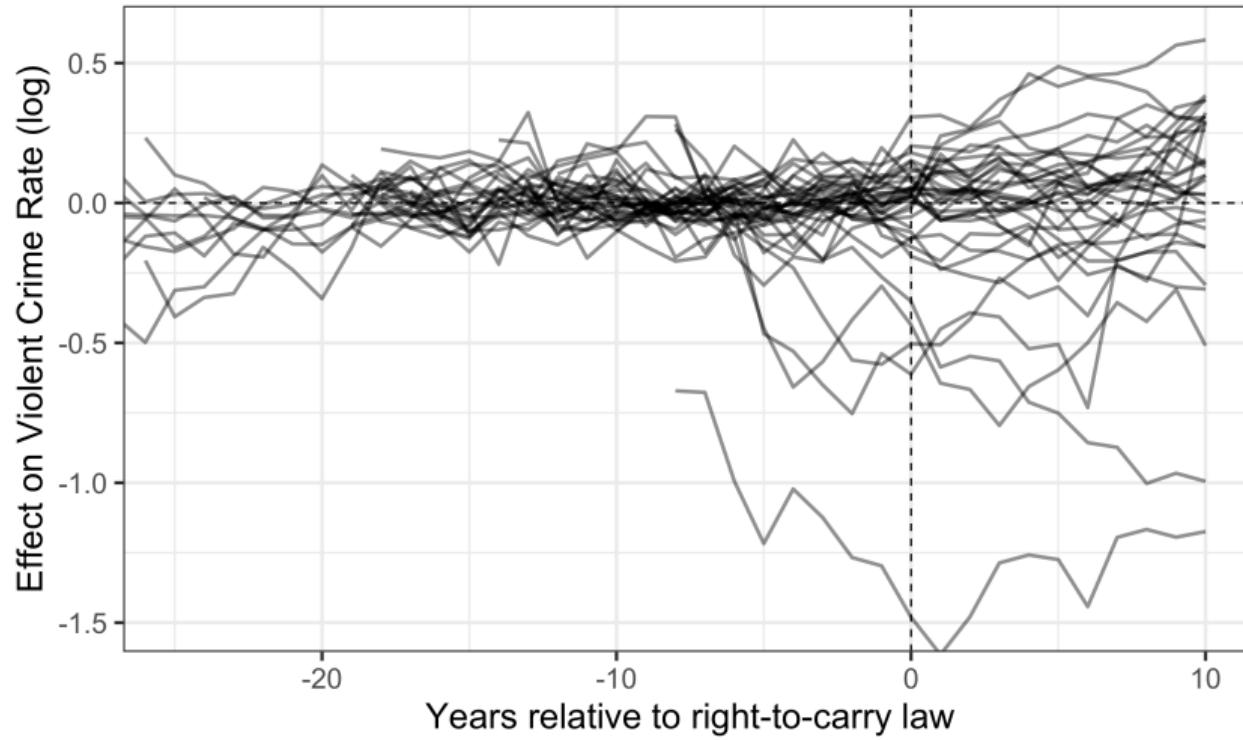
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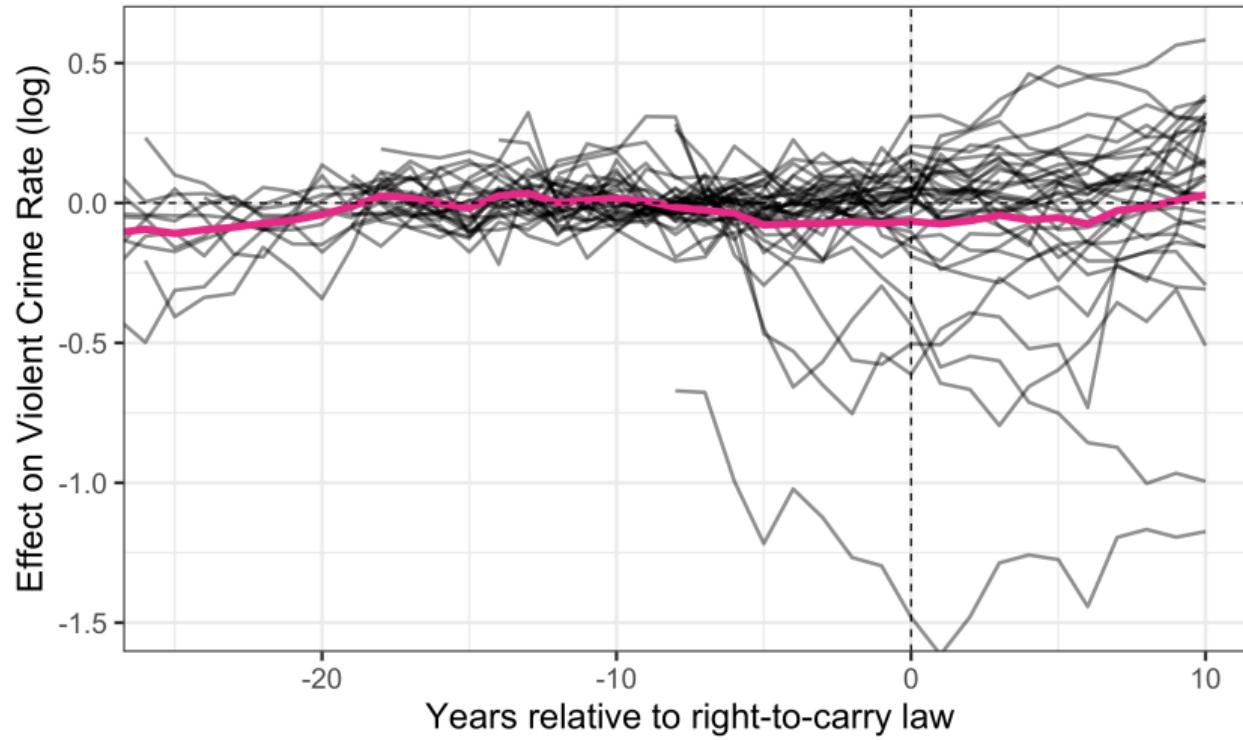
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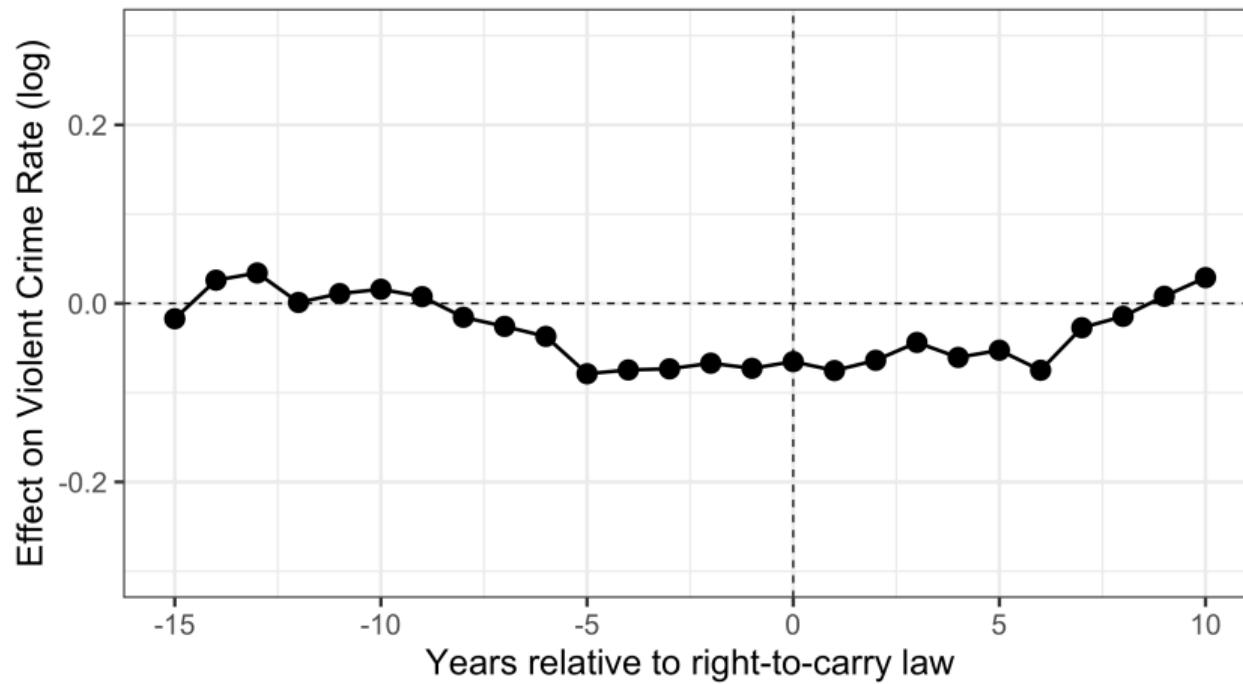


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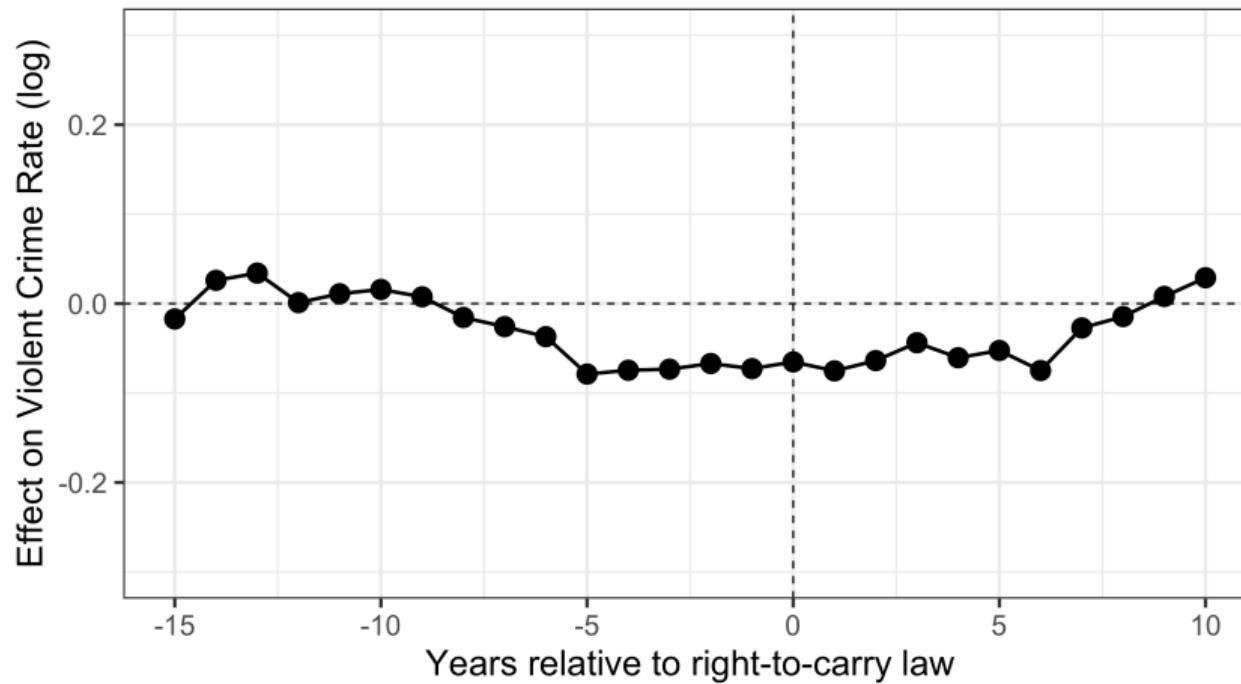
## Separate SCM



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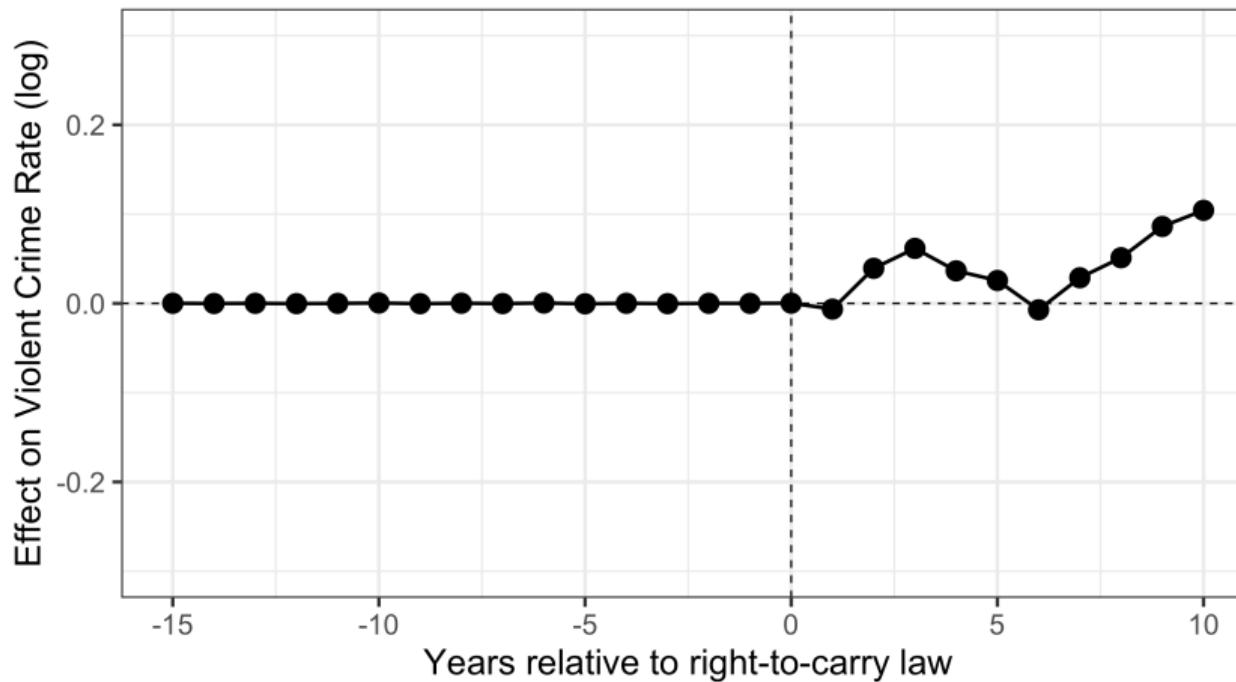
# Partially Pooled SCM

## Separate SCM



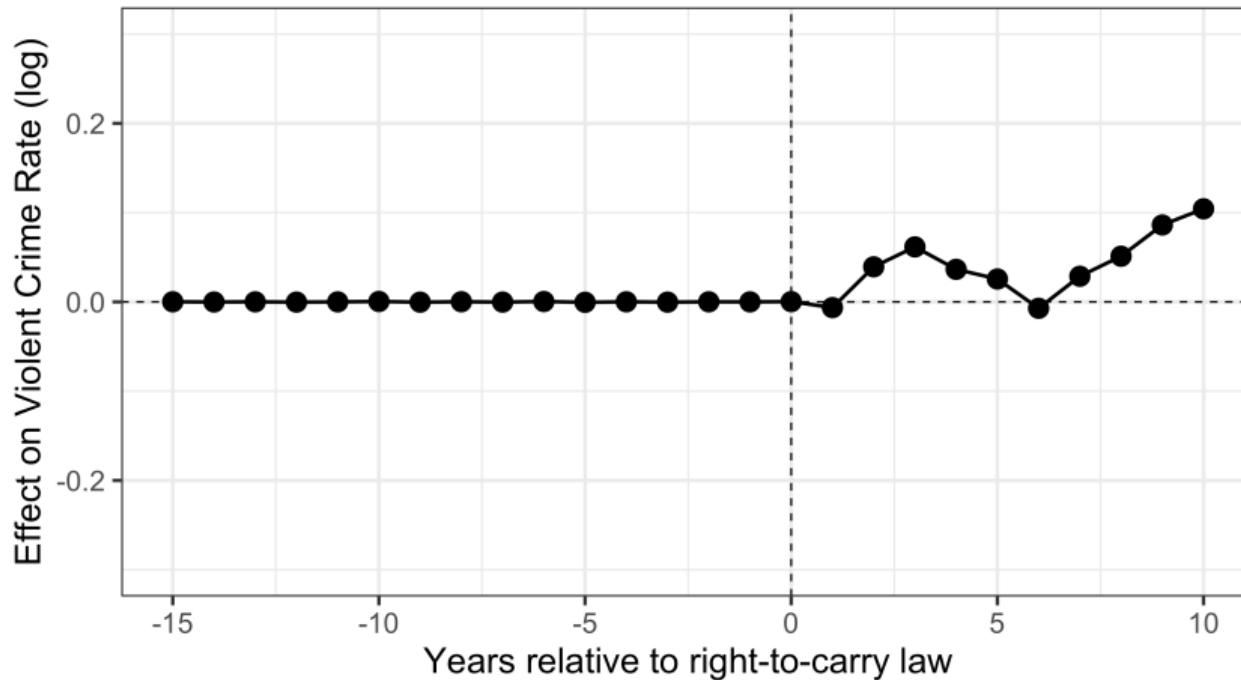
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## Pooled SCM



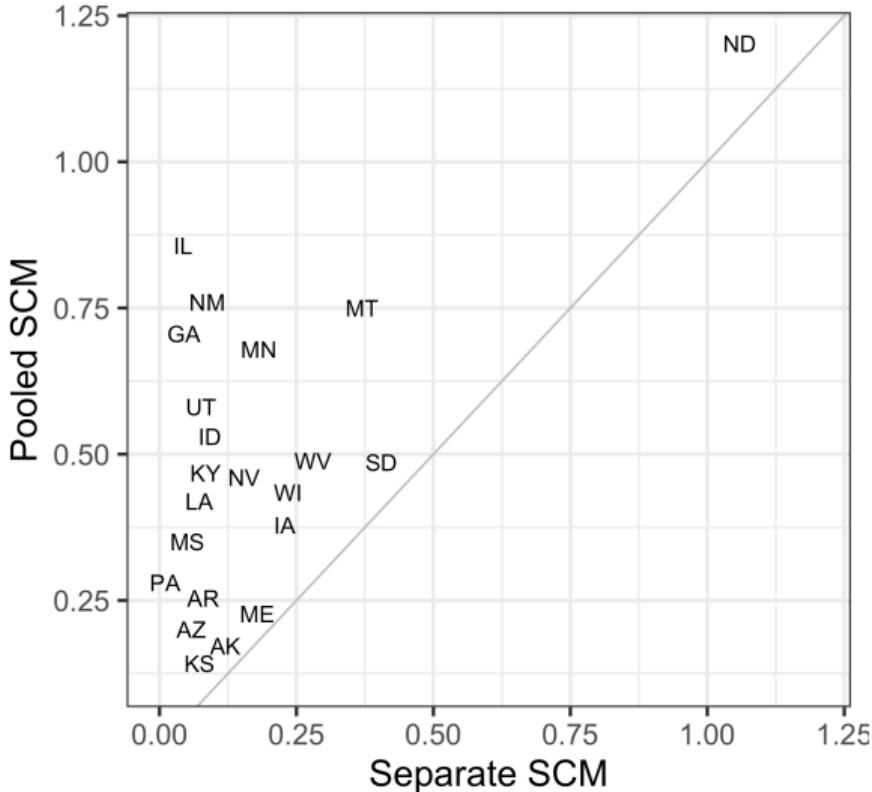
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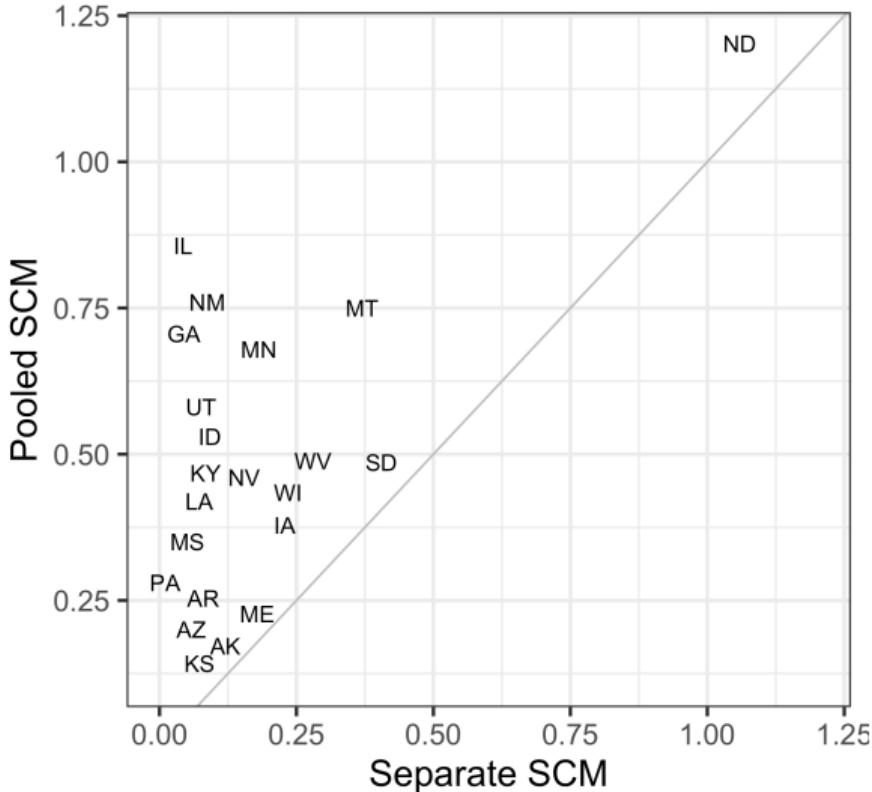
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## SCM pre-treatment imbalance



Pooled Balance is better!

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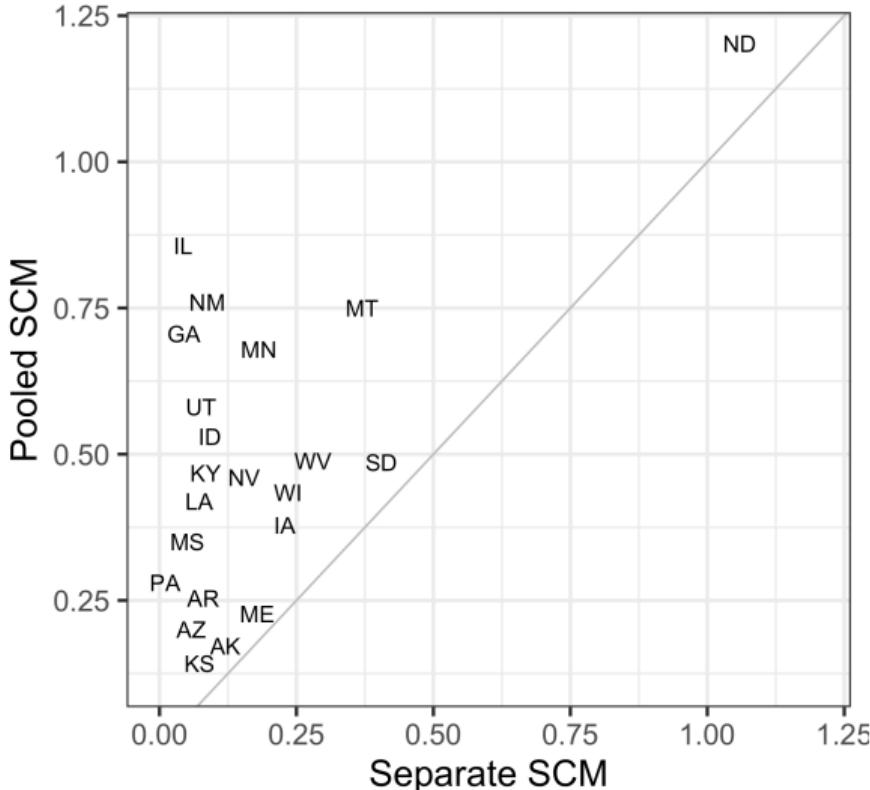


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... but State Balance is worse

- Bad for state estimates

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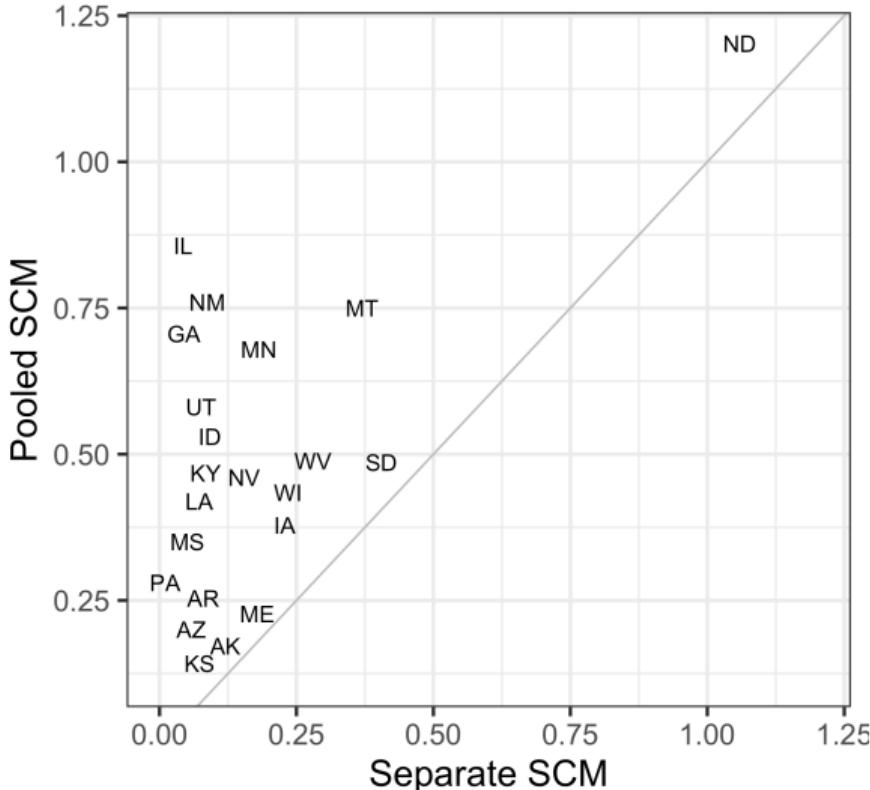
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Also bad for the **average!**

- When DGP varies over time

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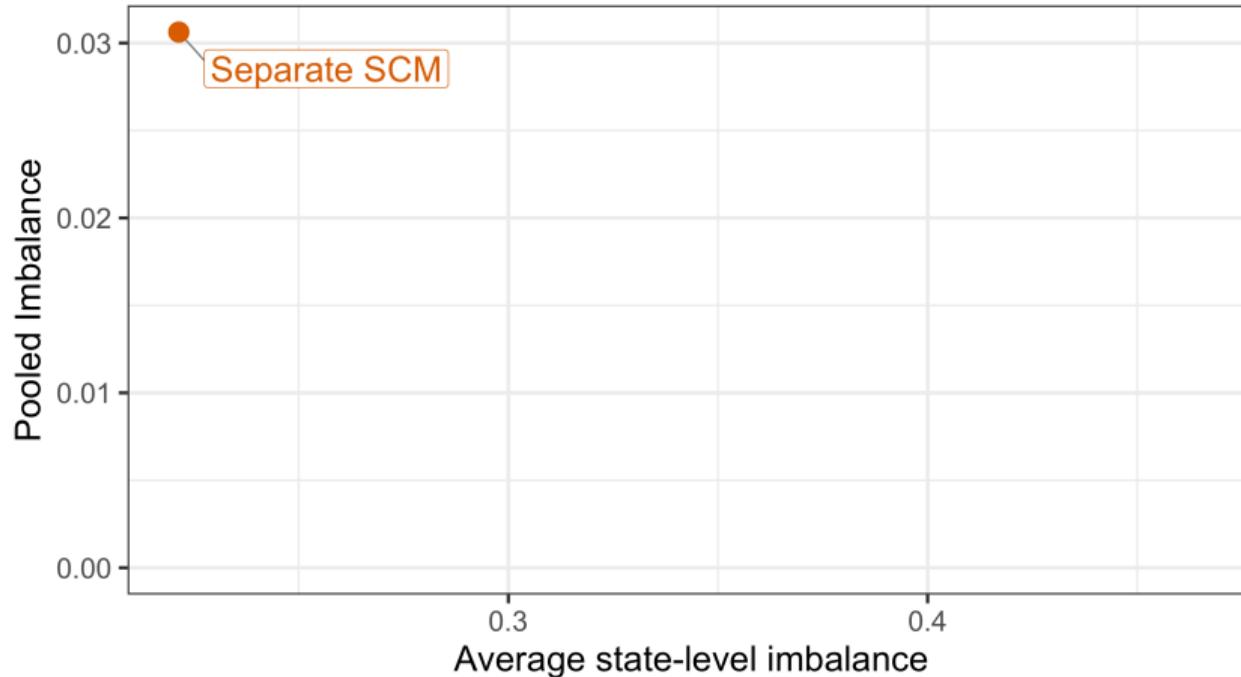
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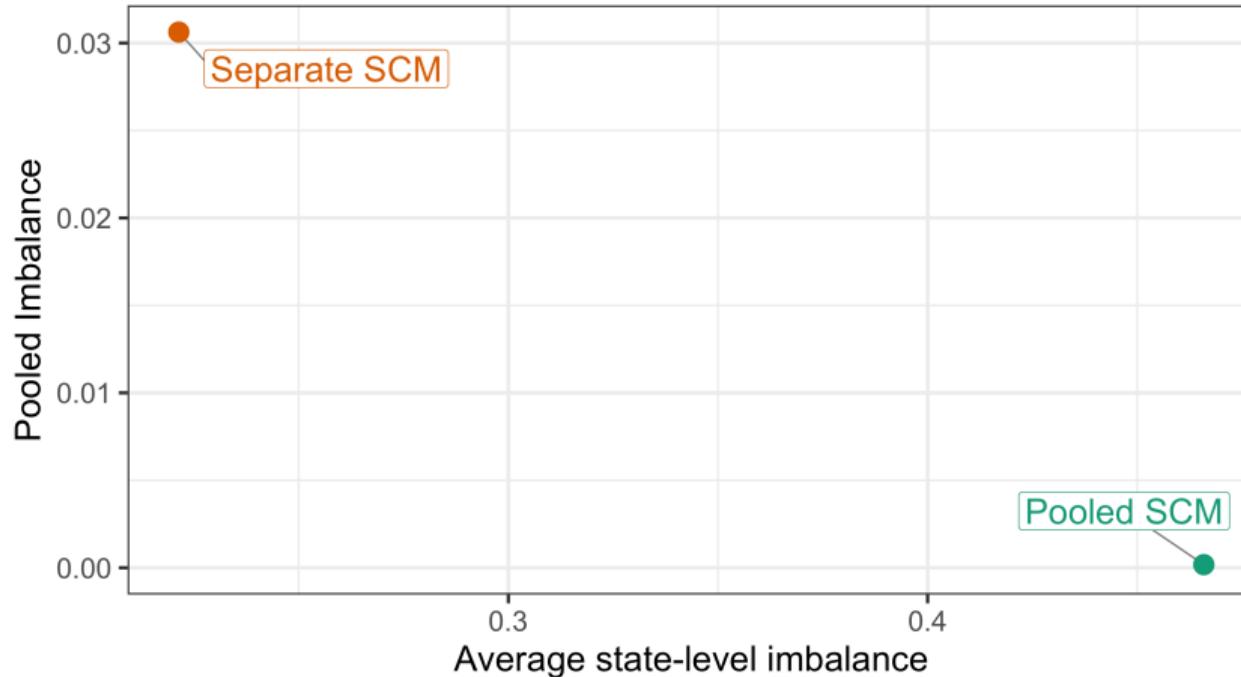
Find weights that balance both  
Pooled Balance and State Balance

## Balance possibility frontier



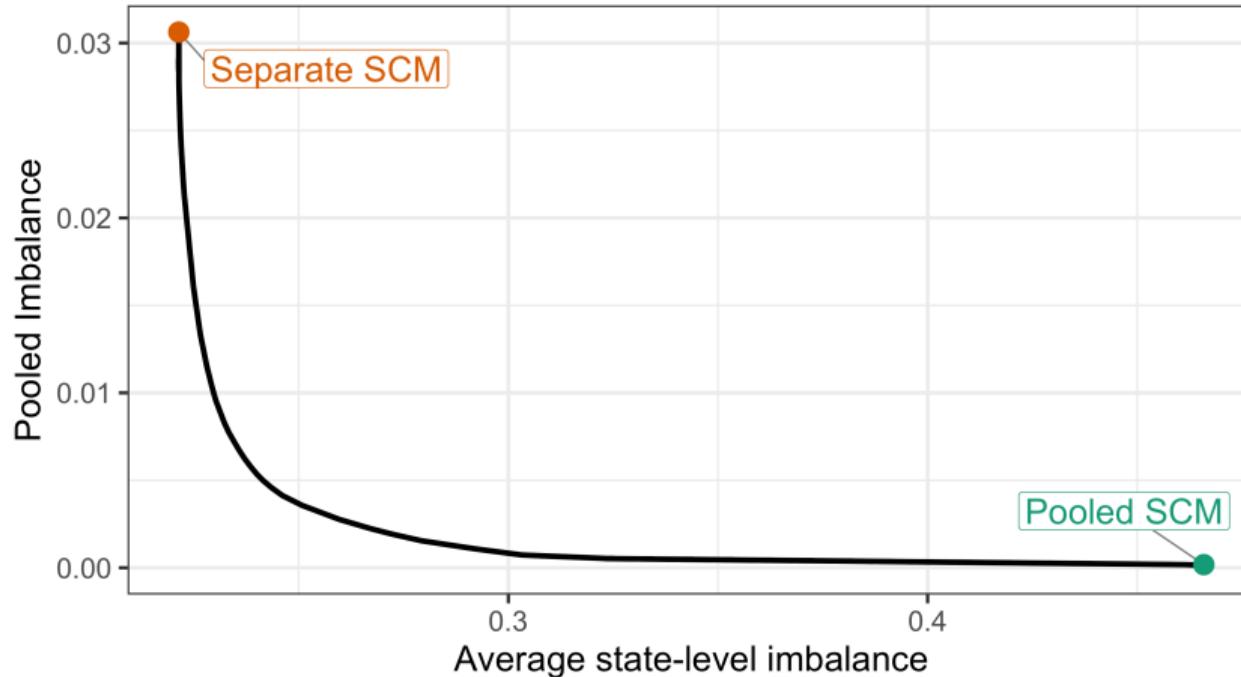
$$\min_{\Gamma \in \Delta^{\text{scm}}} \nu \|\text{Pooled Balance}\|_2^2 + \frac{1-\nu}{J} \sum_{j=1}^J \|\text{State Balance}_j\|_2^2 + \text{penalty}$$

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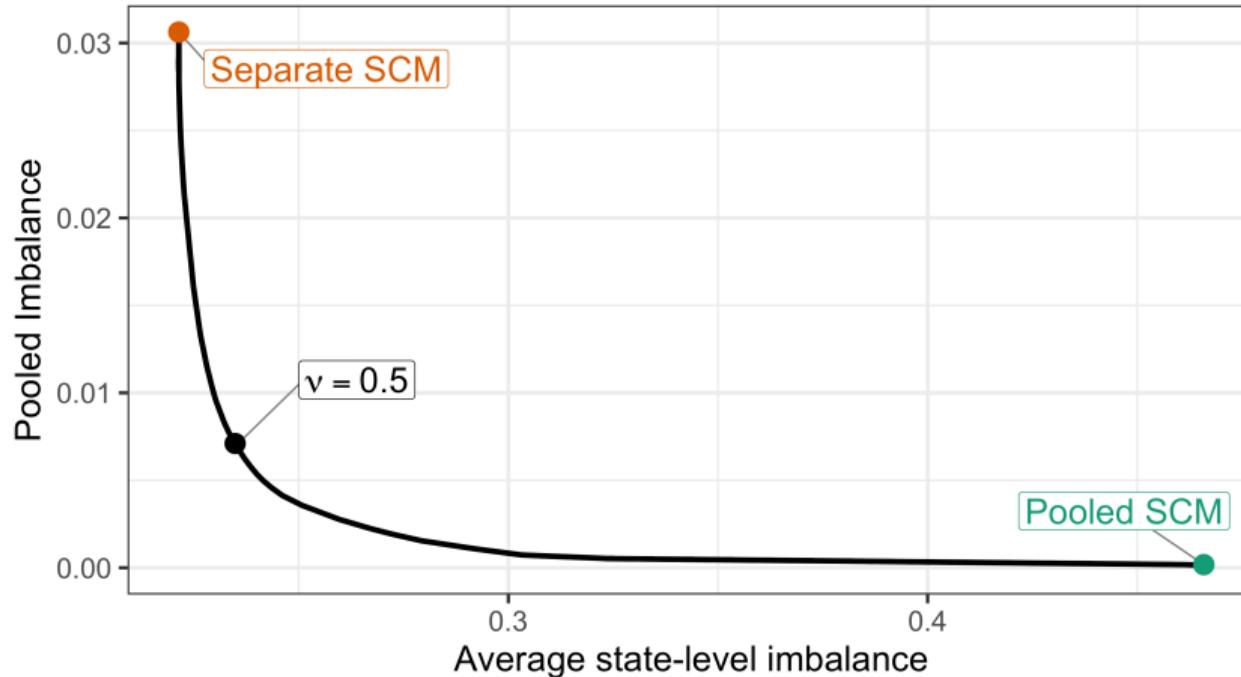
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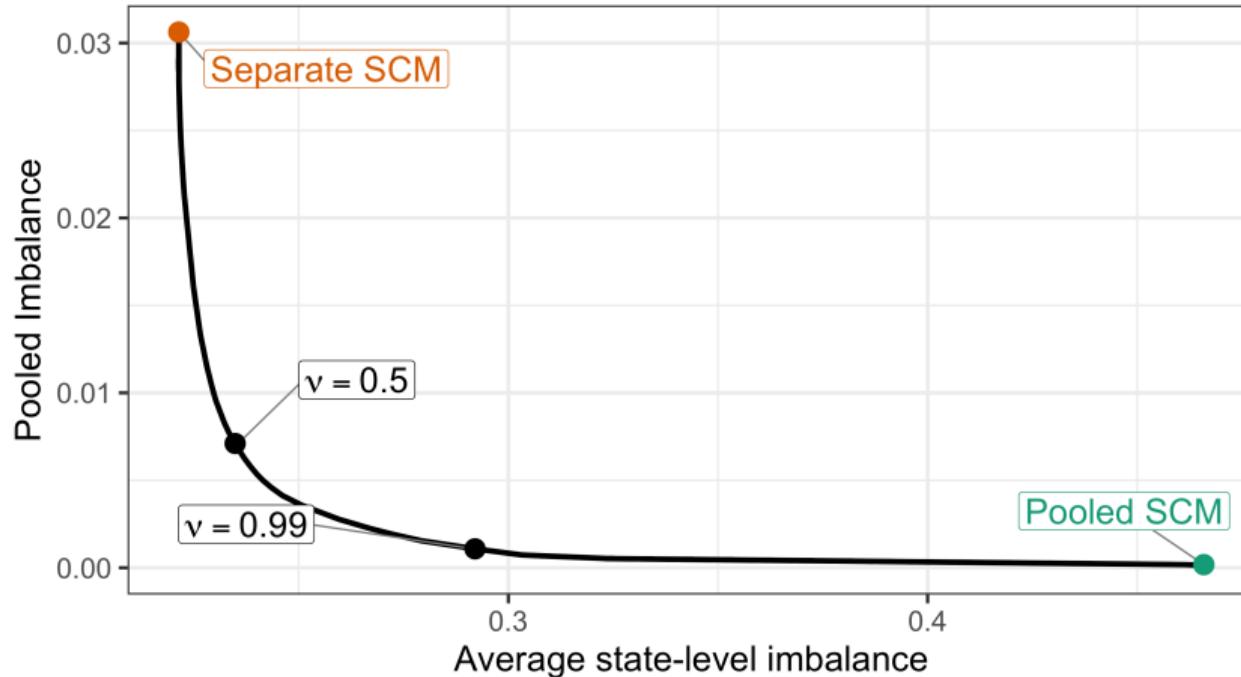
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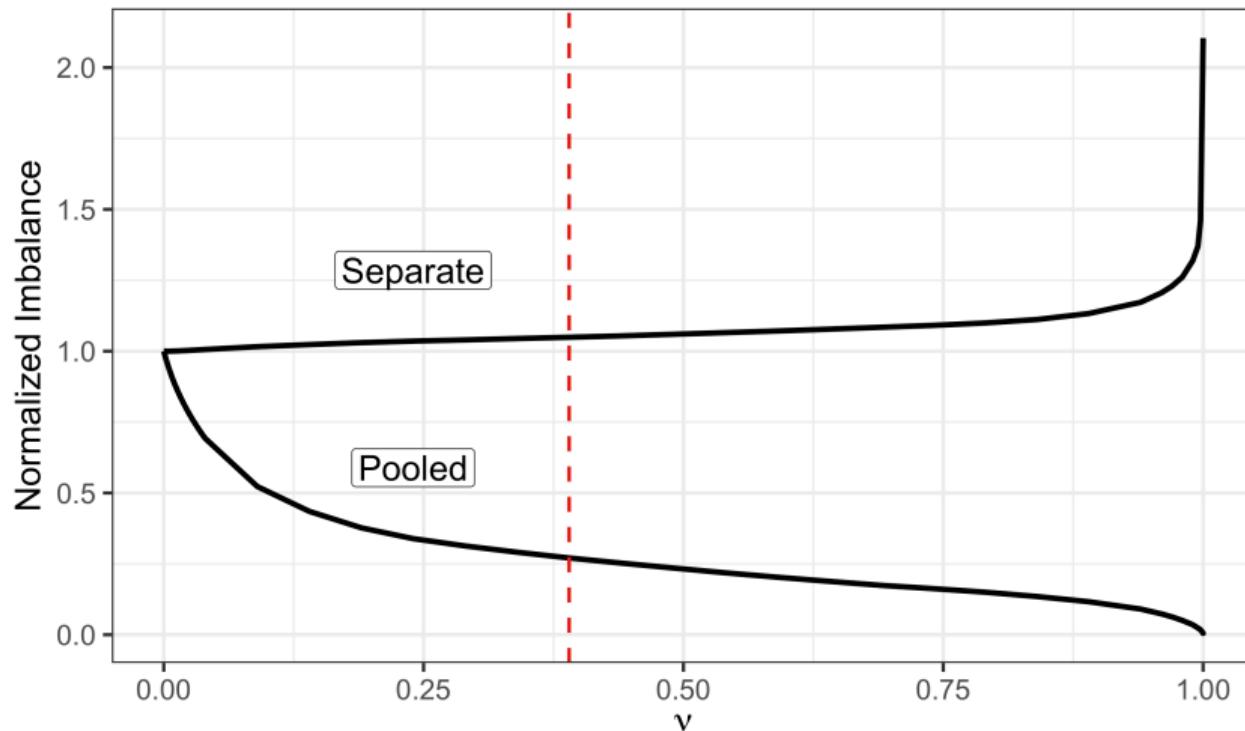


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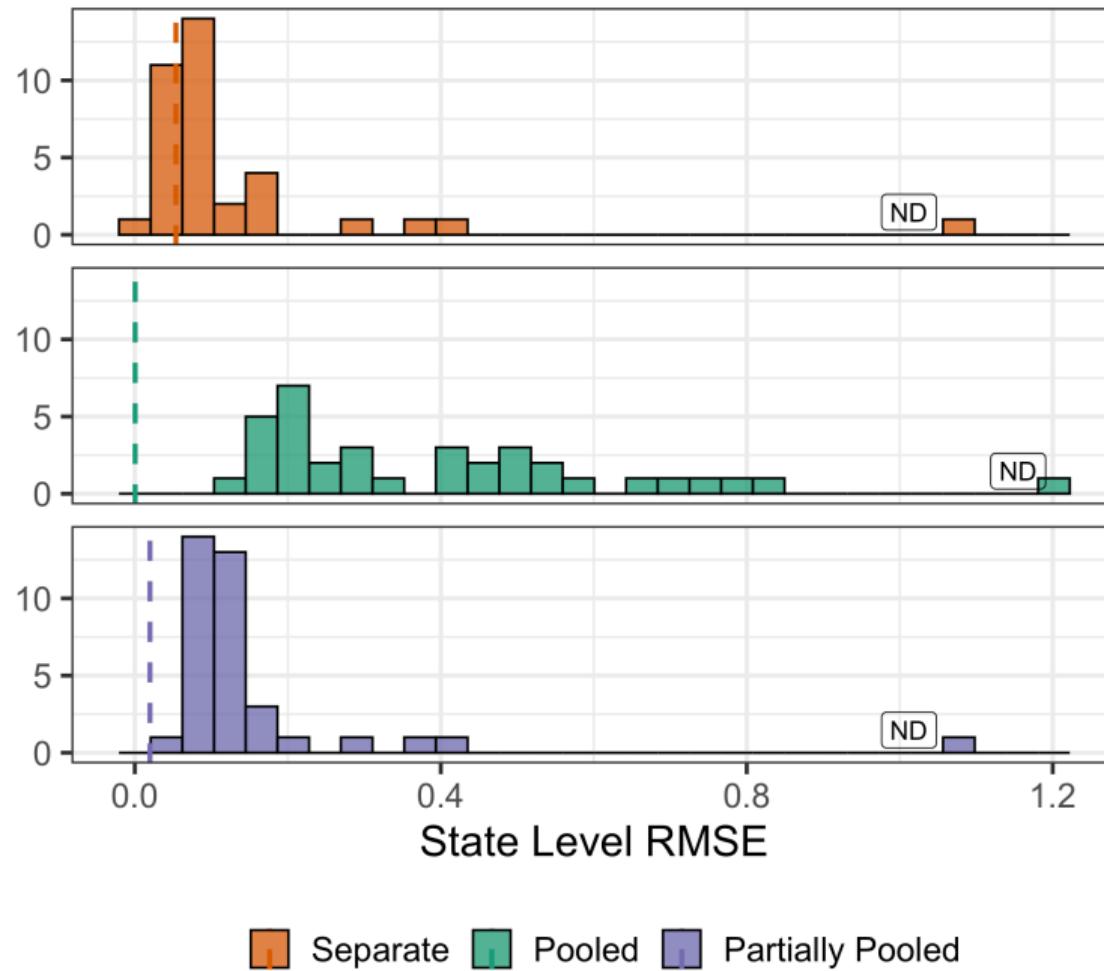
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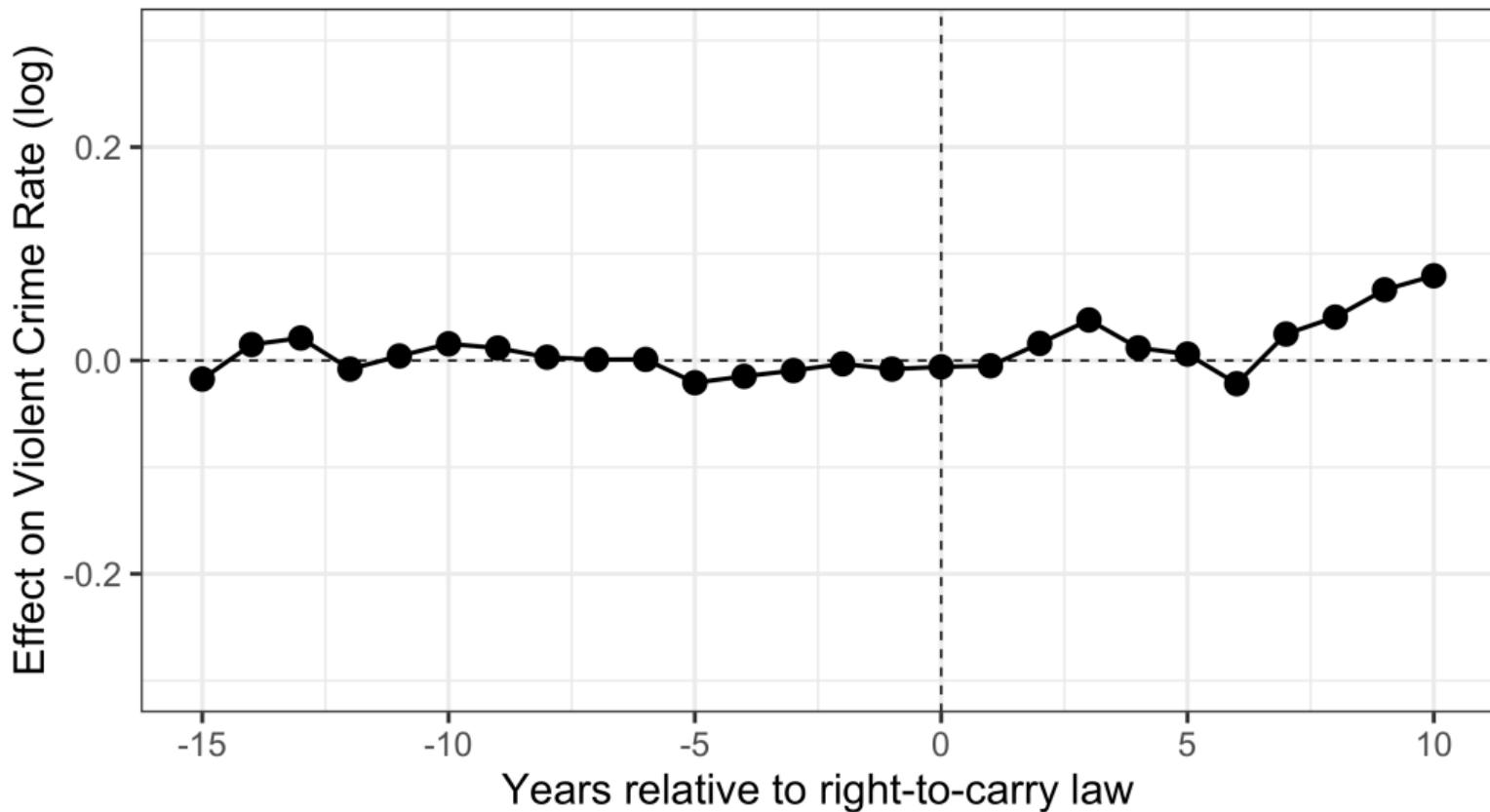
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Heuristic for  $\nu = \frac{\|\text{Pooled Balance}\|_2}{\frac{1}{\sqrt{J}} \sum_{j=1}^J \|\text{State Balance}_j\|_2}$  fit with  $\nu = 0$



## Partially Pooled SCM



# Extensions

## Intercept-Shifted SCM

Adjust for level differences by adding an intercept to the optimization problem

[Doudchenko and Imbens, 2017; Ferman and Pinto, 2018]

$$\hat{Y}_{j,T_j+k}^*(\infty) = \hat{\alpha}_j + \sum_i \hat{\gamma}_{ij}^* Y_{i,T_j+k}$$

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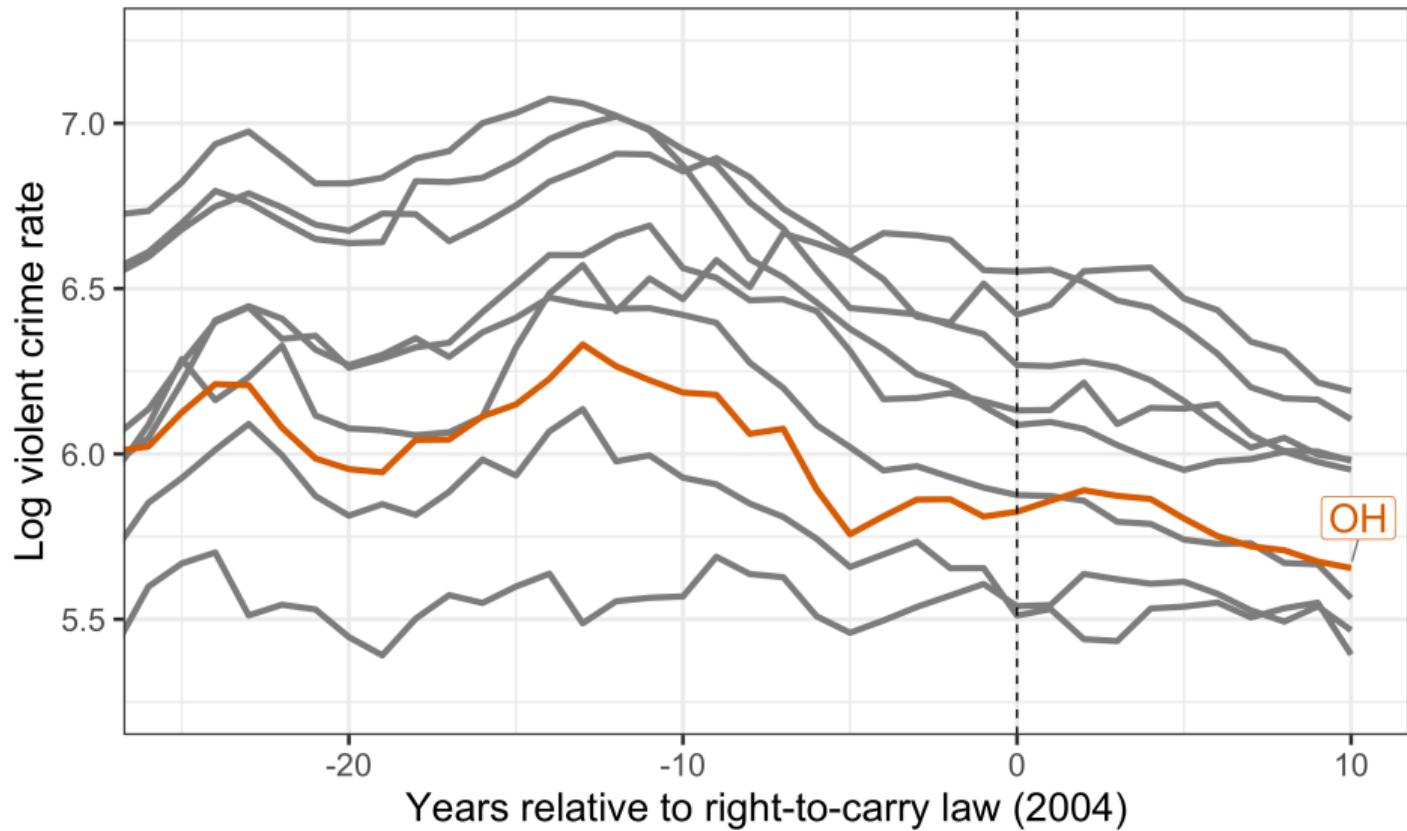
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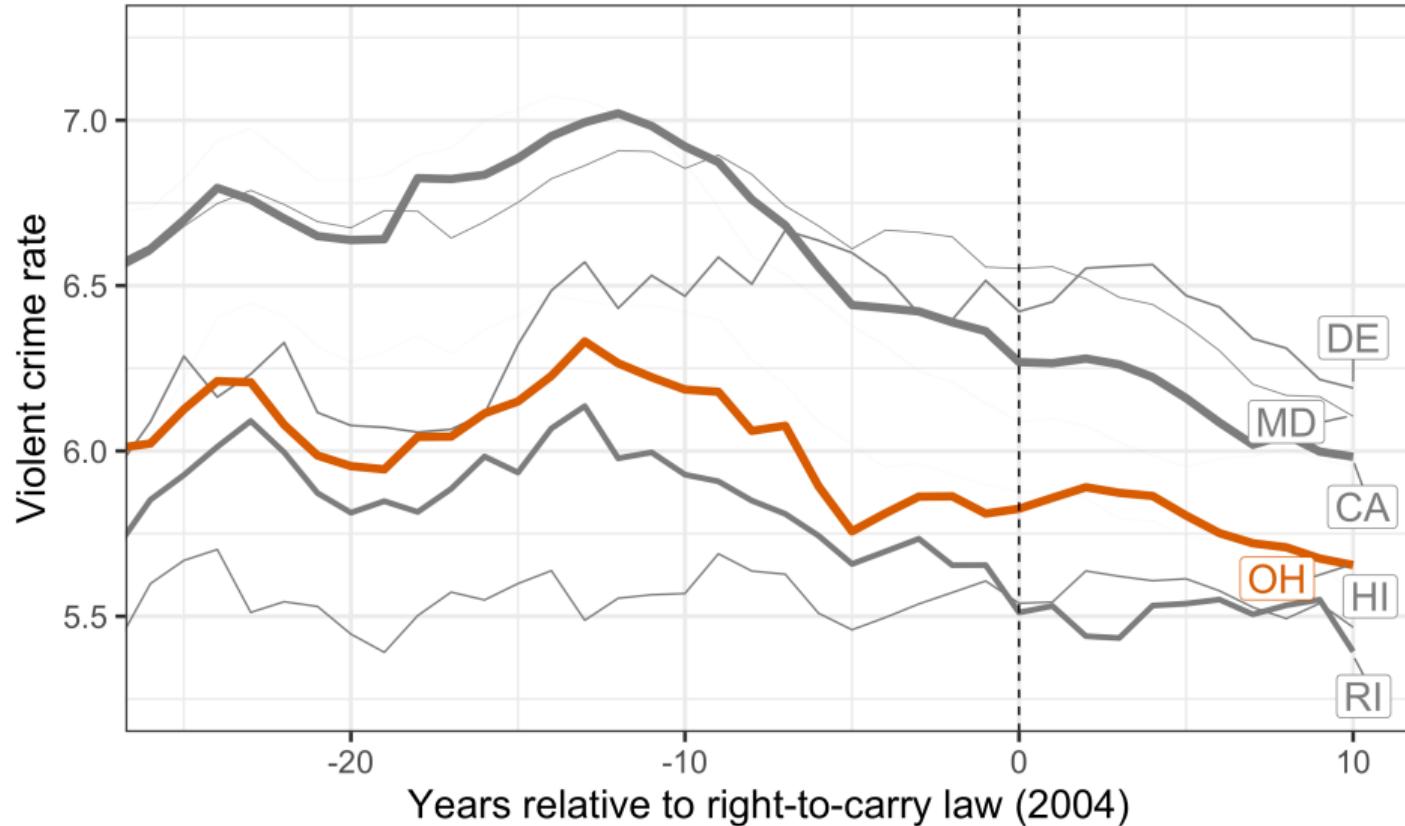
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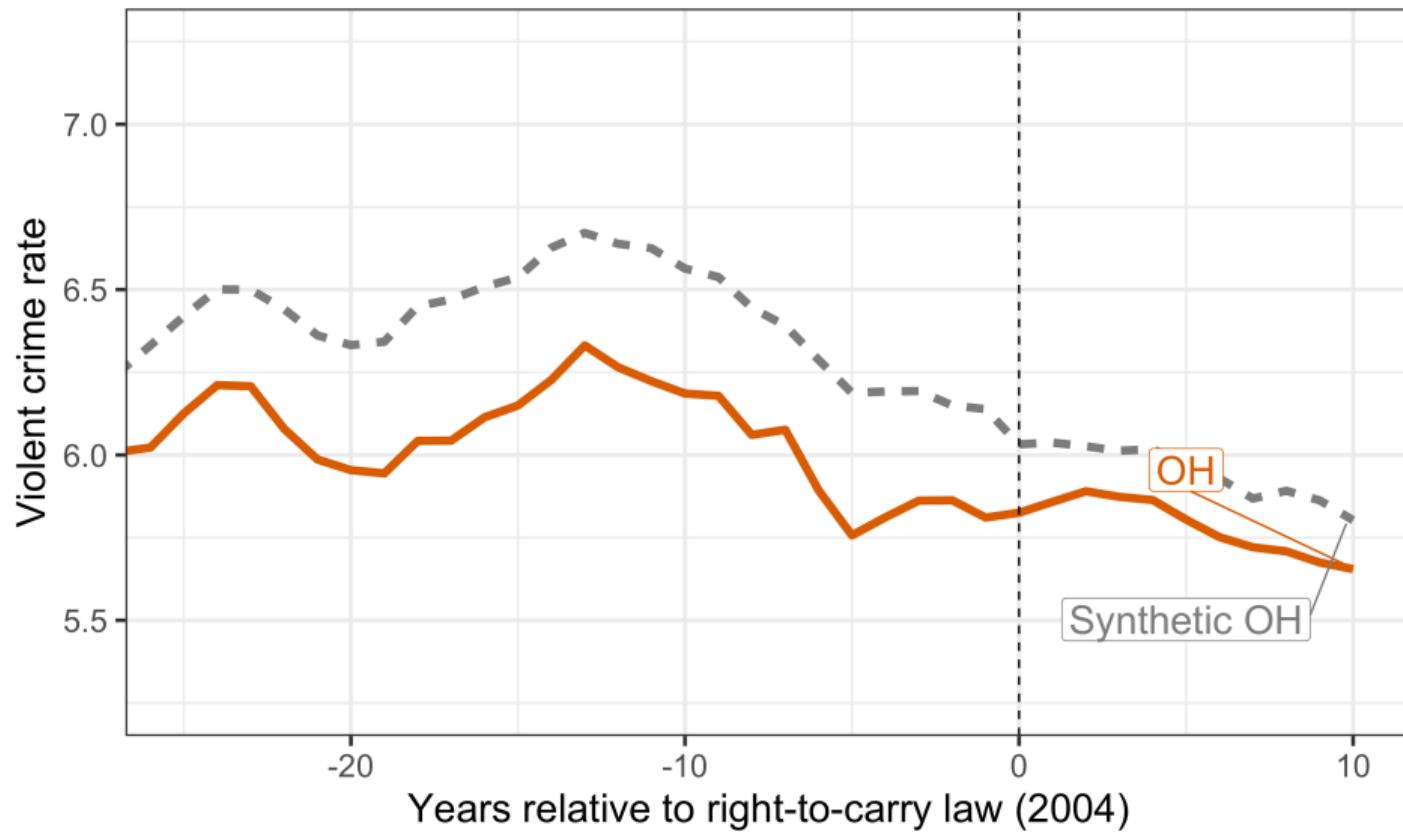
Treatment effect estimate is **weighted difference-in-differences**

$$\hat{\tau}_{jk} = \left( Y_{j,T_j+k} - \bar{Y}_{j,T_j}^{\text{pre}} \right) - \sum_{i=1}^N \hat{\gamma}_{ij}^* \left( Y_{i,T_j+k} - \bar{Y}_{i,T_j}^{\text{pre}} \right)$$

- Uniform weights recover "stacked" DiD [Abraham and Sun, 2018]
- Similar in form to P-score weighted DiD [Abadie, 2005; Callaway and Sant'Anna, 2020]

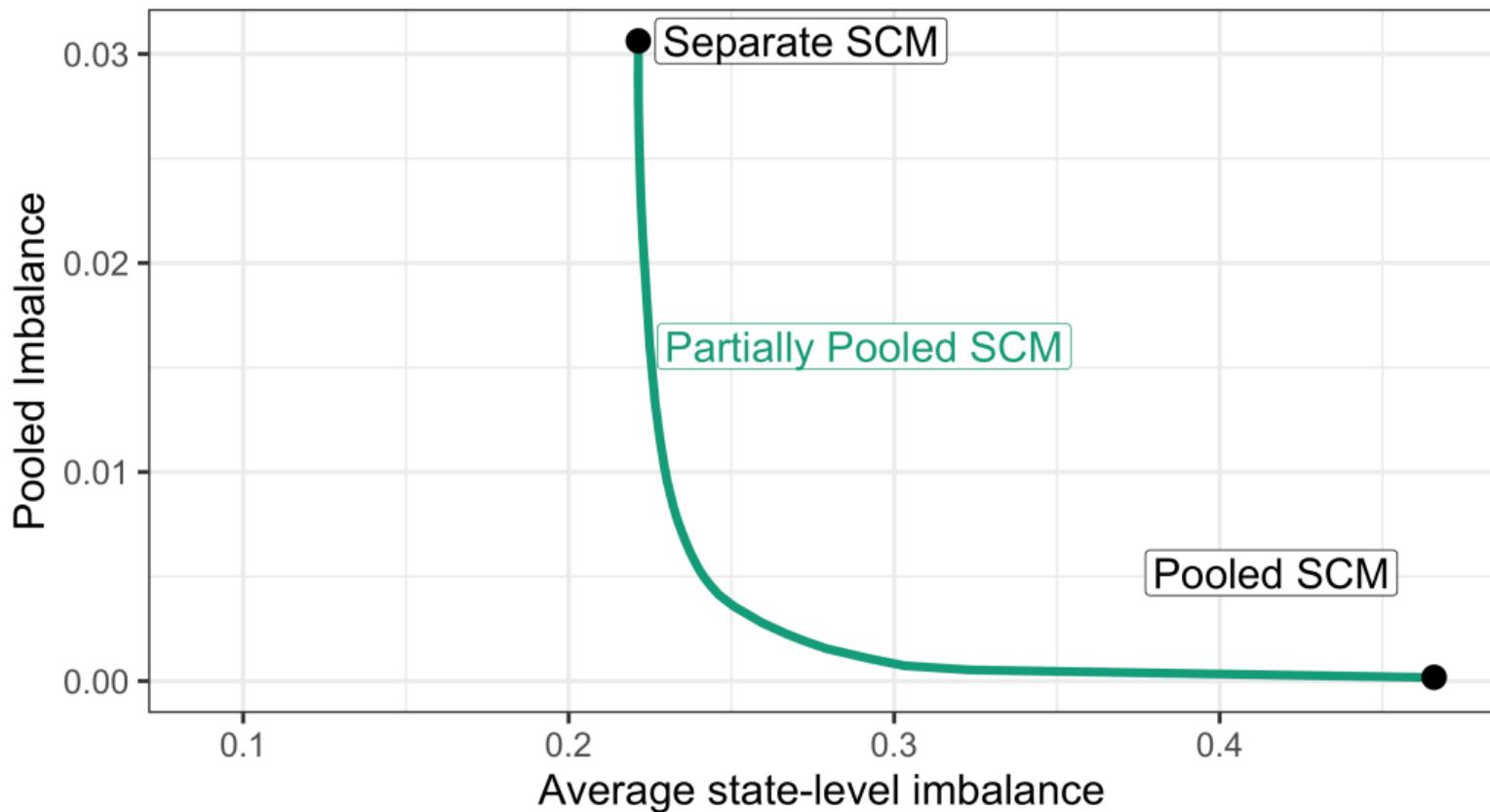




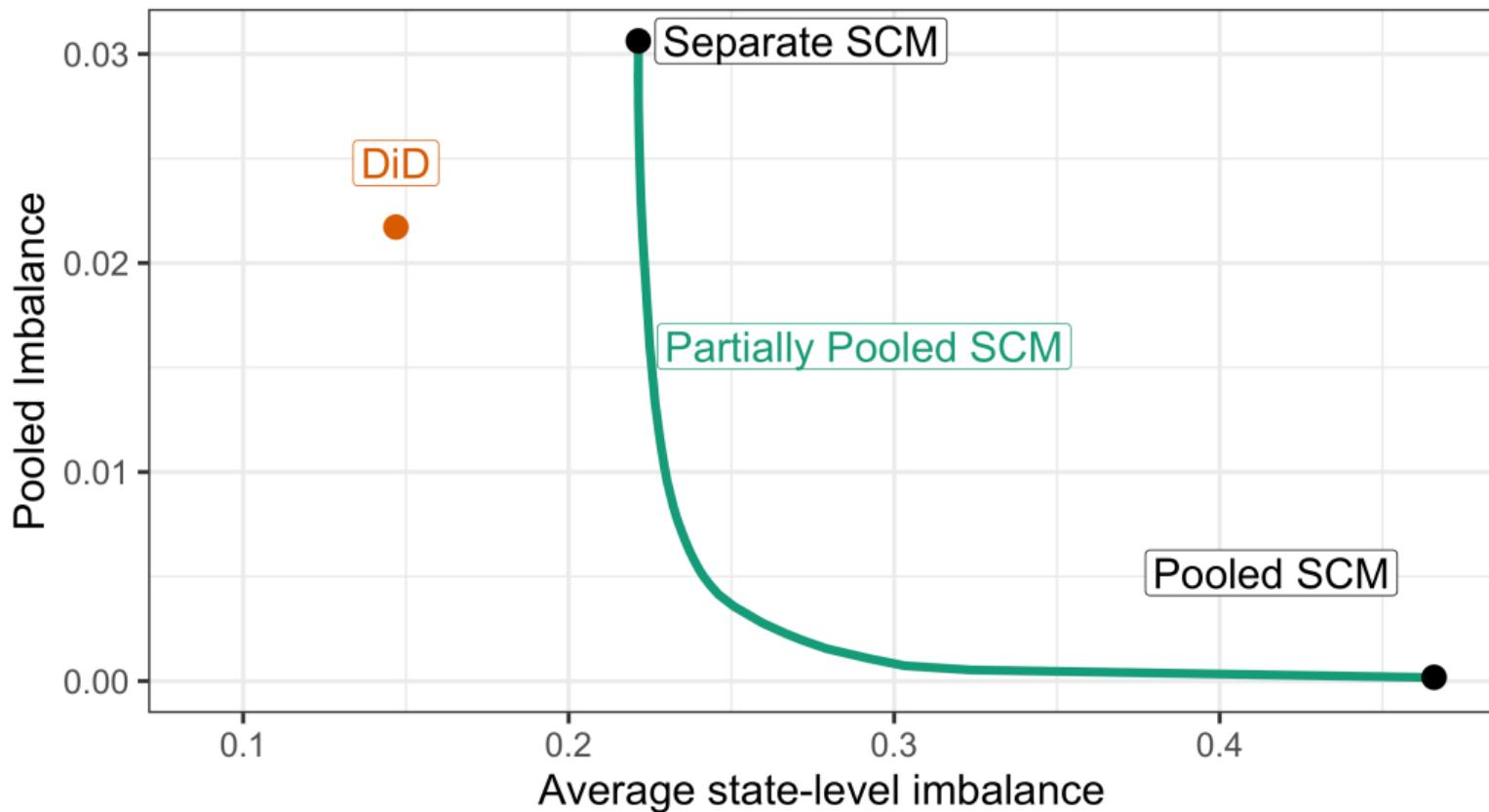




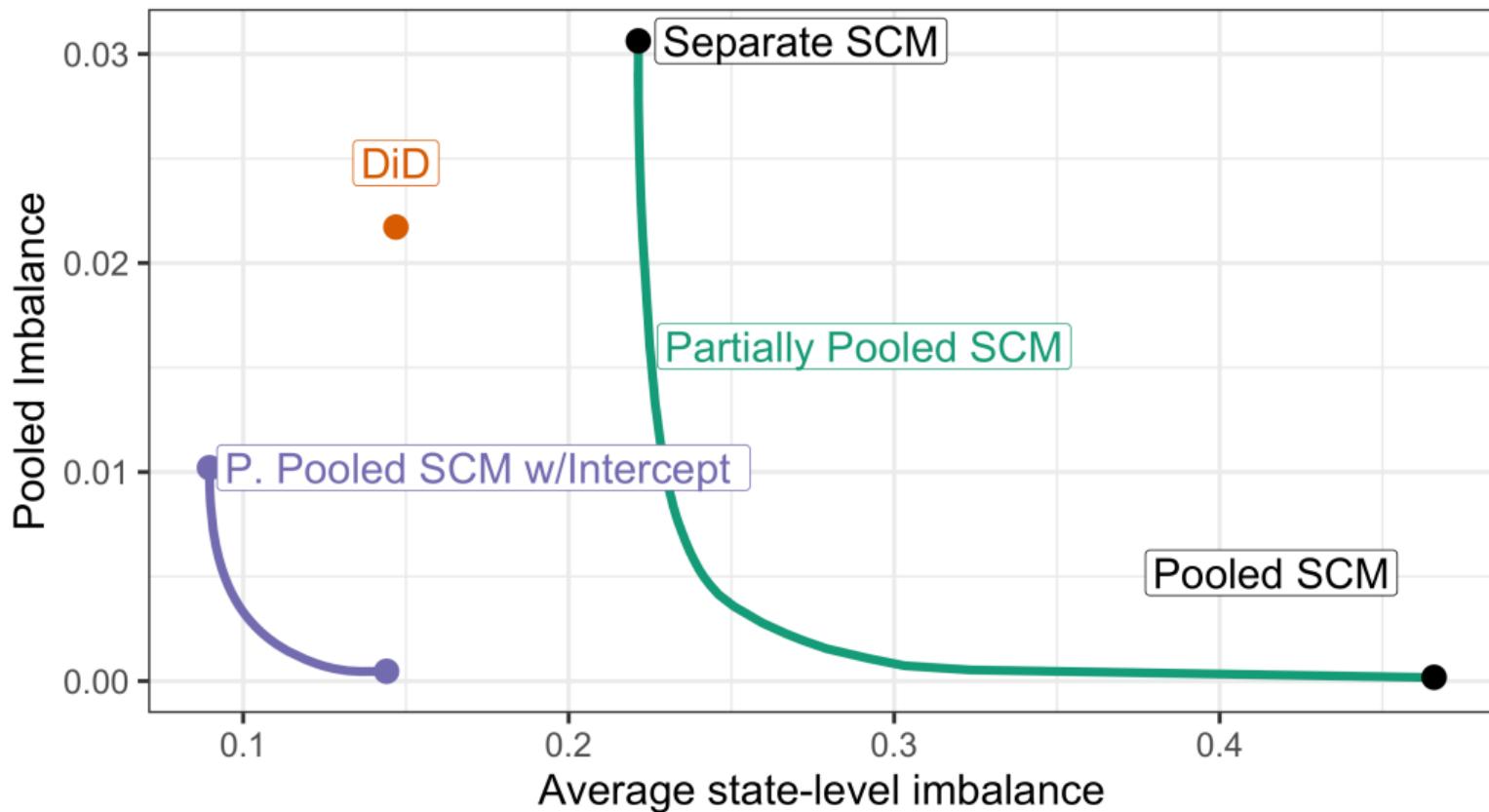
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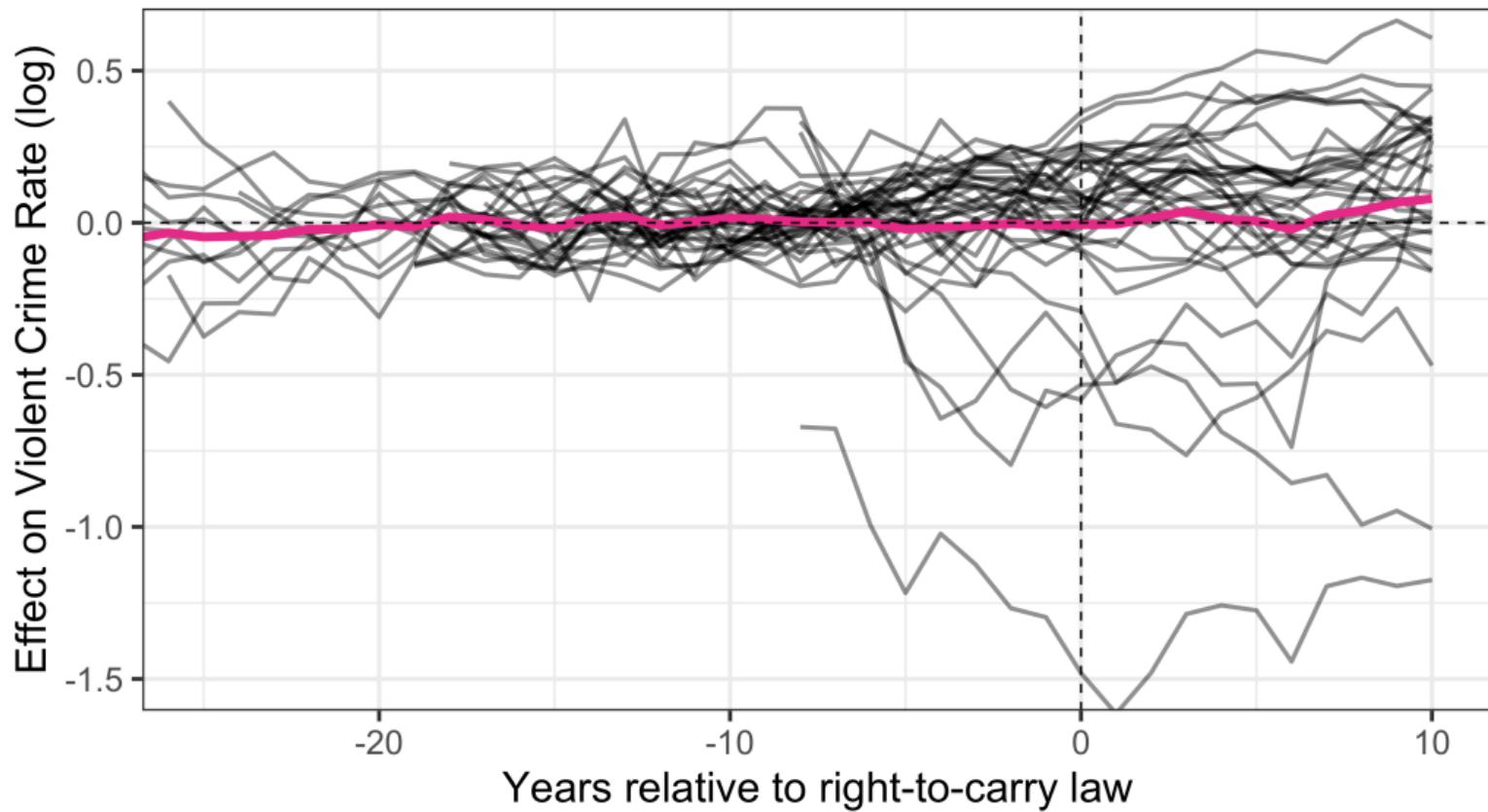
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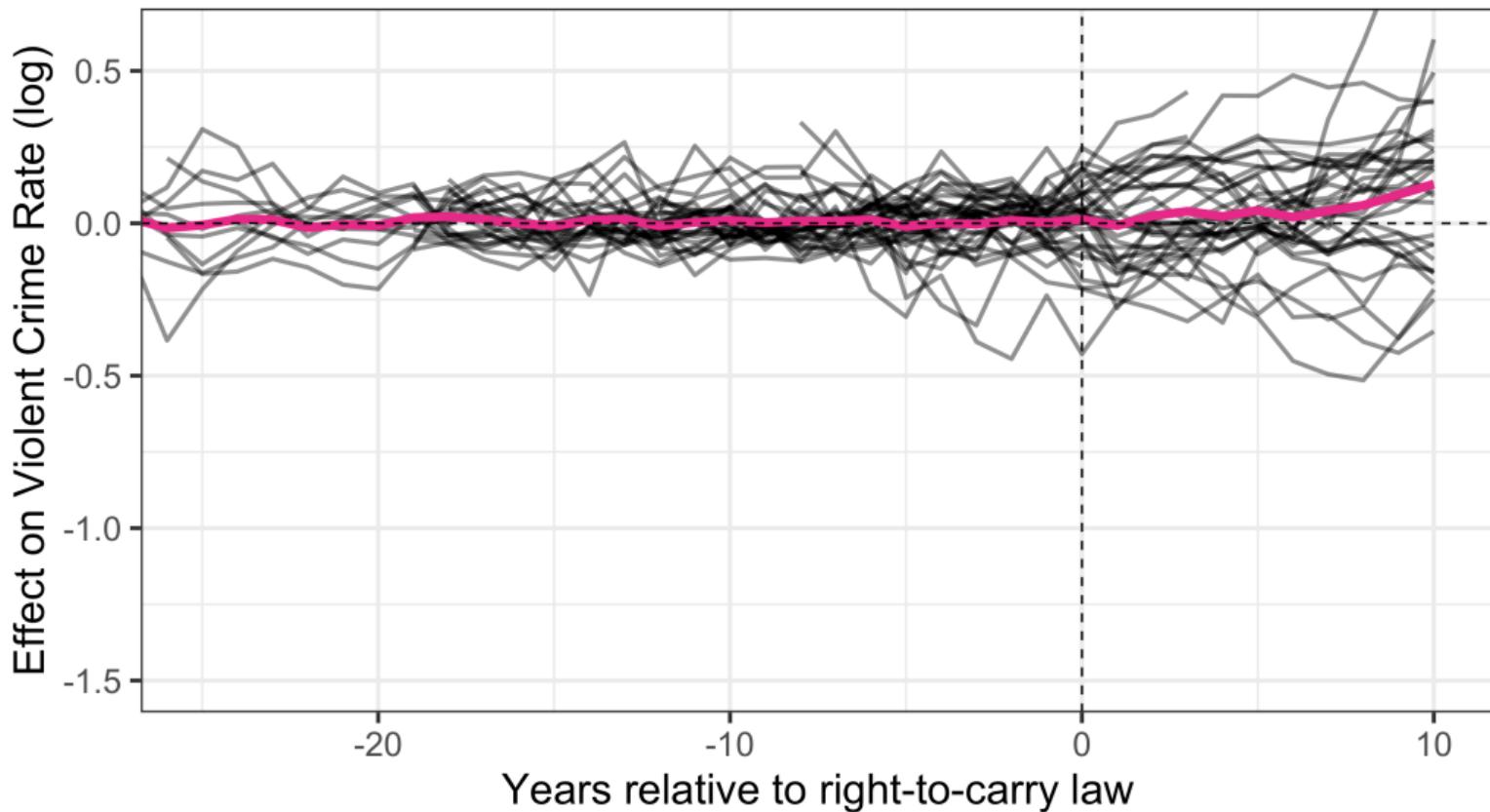
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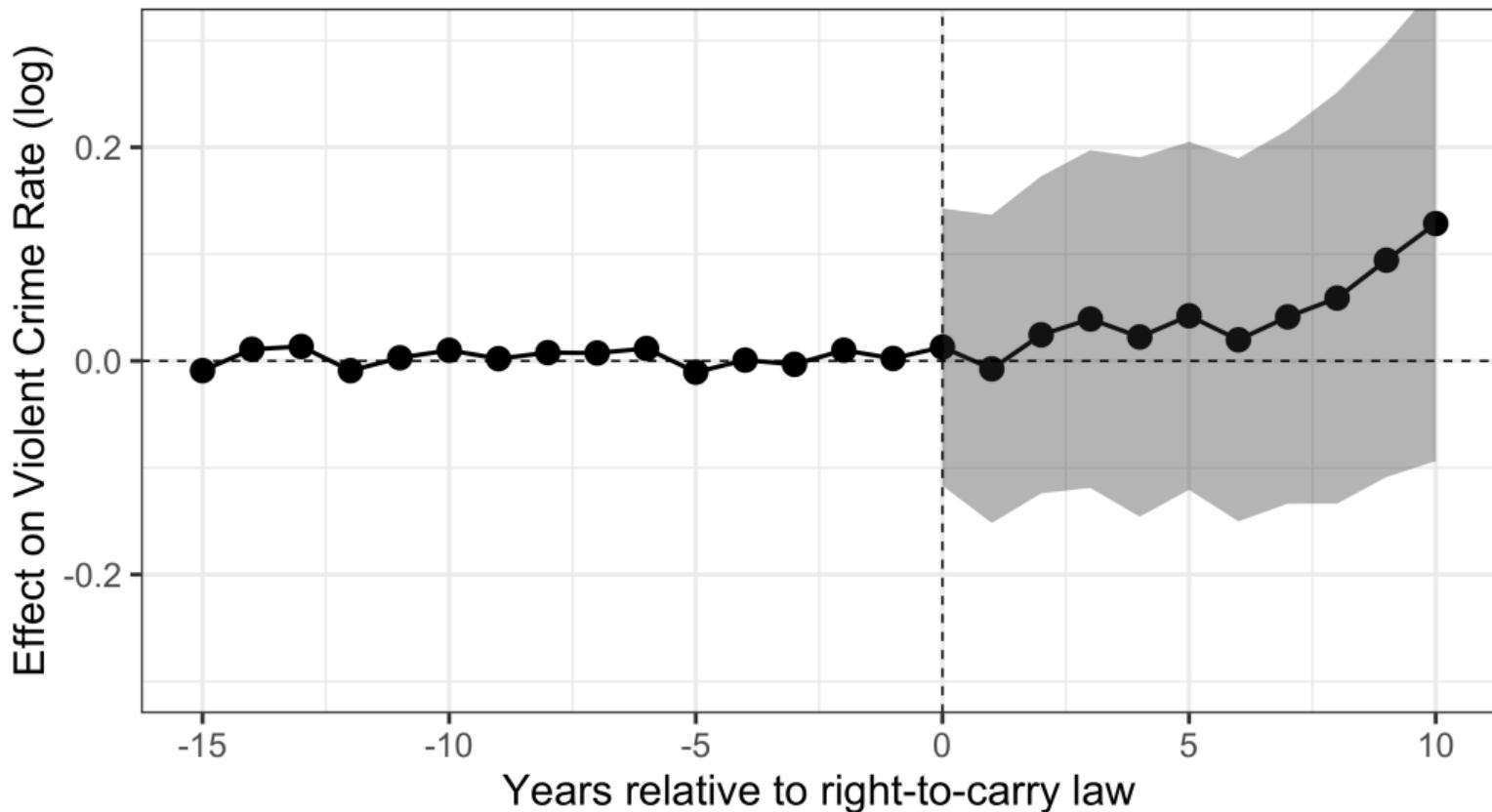
## Partially Pooled SCM



## P. Pooled SCM w/Intercept



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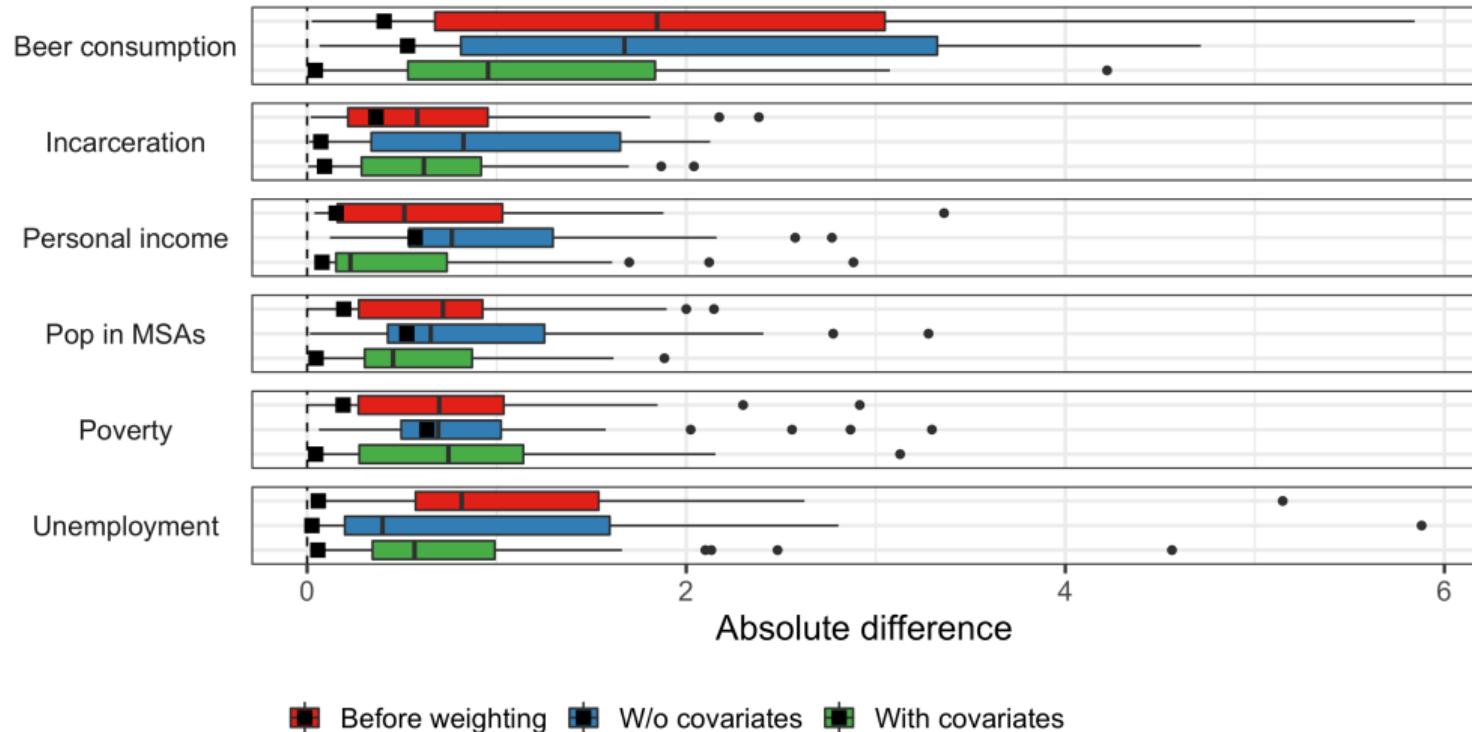
## Incorporating auxiliary covariates

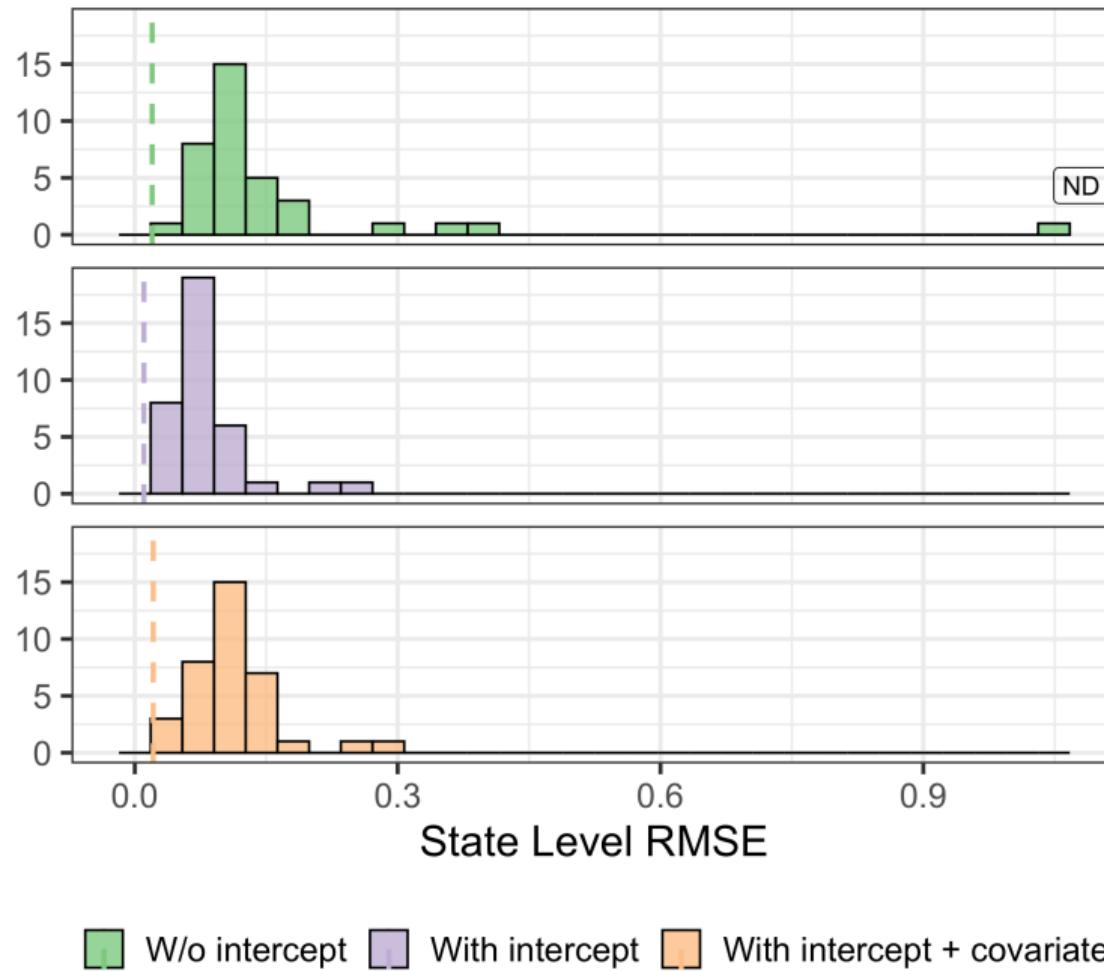
Often have additional covariates other than the main outcome

- E.g. poverty, unemployment, incarceration, and police staffing rates
- Demographics

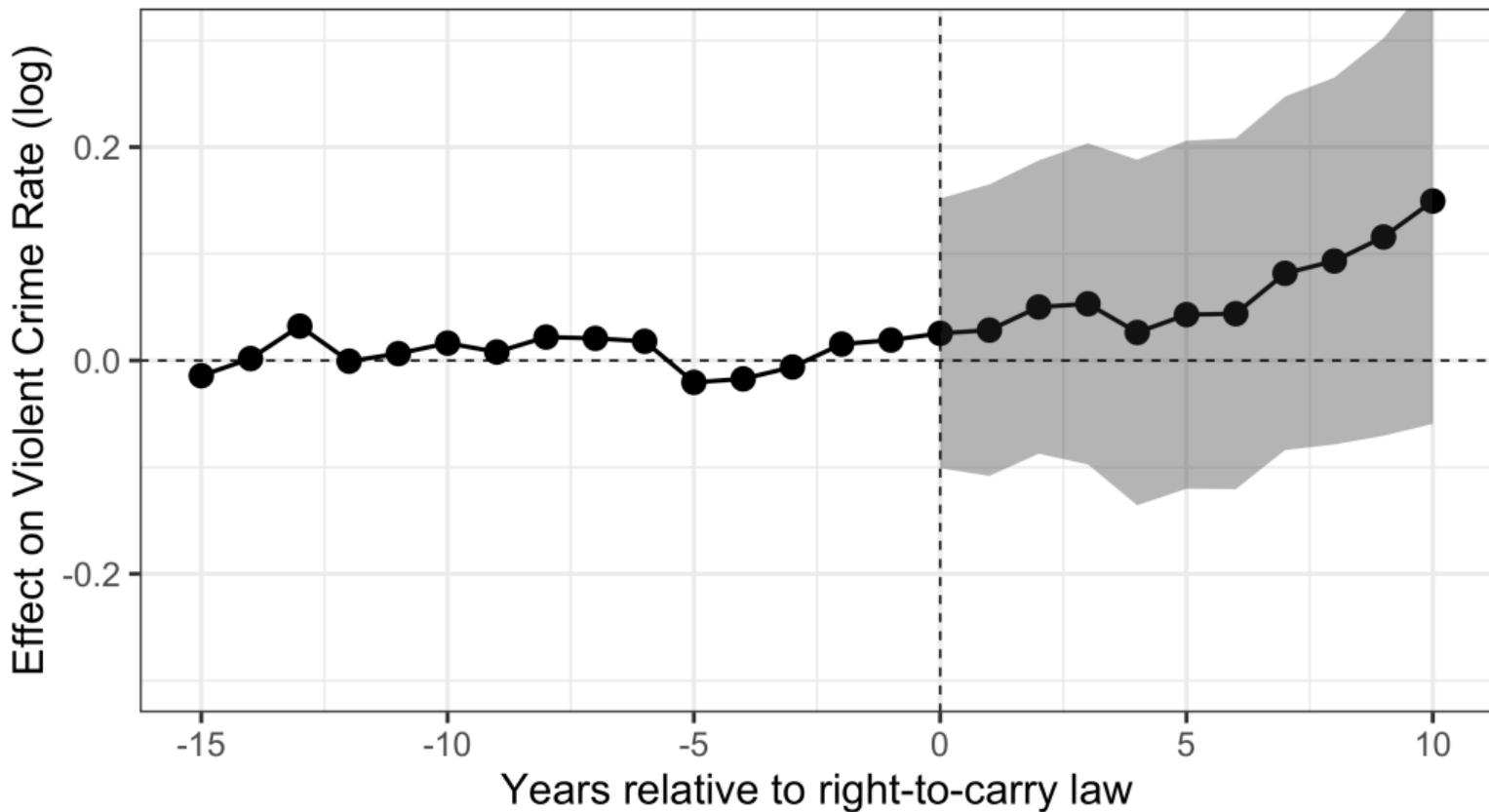
Same trade-off between **State Balance** and **Pooled Balance**

We focus on fixed covariates, but time-varying covariates are similar





## Intercept shift + covariates



## Recap

Many policies we care about have **staggered adoption**

- Need to be careful when estimating effects!

A design-based approach helps clarify the issues

Applying these notions to SCM with staggered adoption

- Find weights that control **State Balance** and **Pooled Balance**
- Include an **intercept** to adjust for level differences
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## Recap

Many policies we care about have **staggered adoption**

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Applying these notions to SCM with staggered adoption

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Thank you!

Synthetic Controls with Staggered Adoption

A trial emulation approach for policy evaluations with group-level longitudinal data

<https://github.com/ebenmichael/augsynth>



# Appendix

## The role of State Balance and Pooled Balance

Generalization of parallel trends: Linear Factor Model

$$Y_{it}(\infty) = \phi'_i \mu_t + \varepsilon_{it}$$

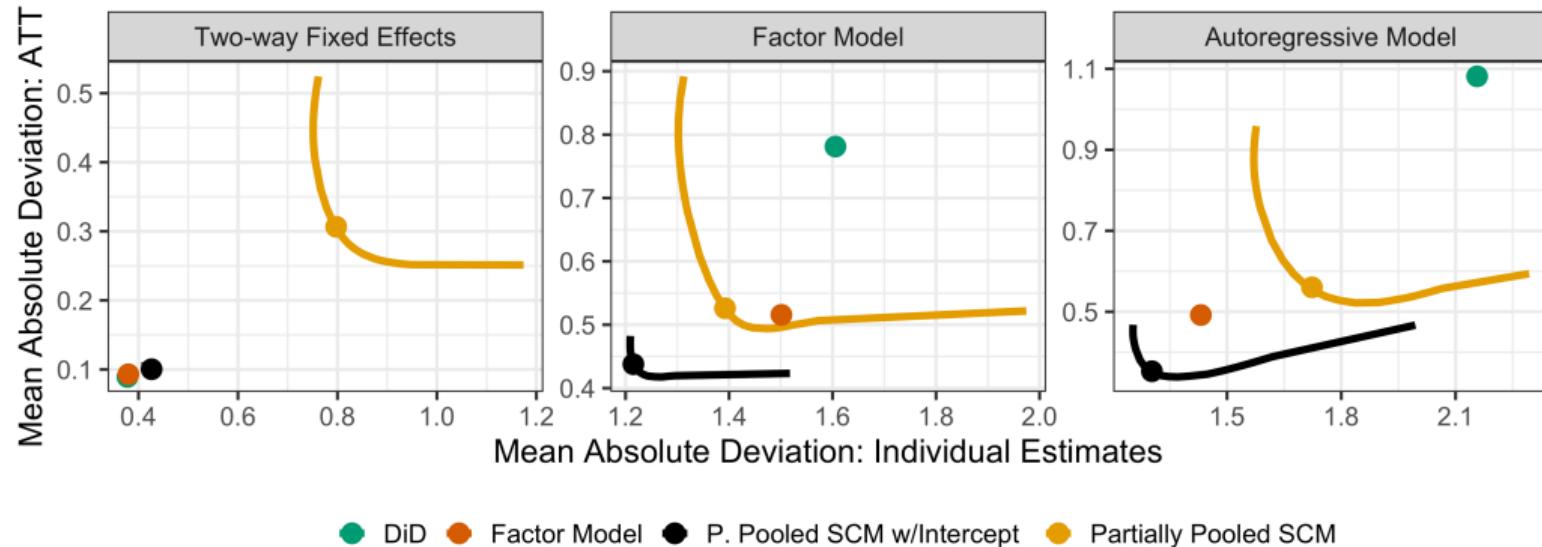
### Error for ATT

$$\left| \widehat{\text{ATT}}_0 - \text{ATT}_0 \right| \lesssim \|\bar{\mu}\|_2 \|\text{Pooled Balance}\|_2 + S \sqrt{\sum_{j=1}^J \|\text{State Balance}_j\|_2^2} + \sqrt{\frac{\log NJ}{T}}$$

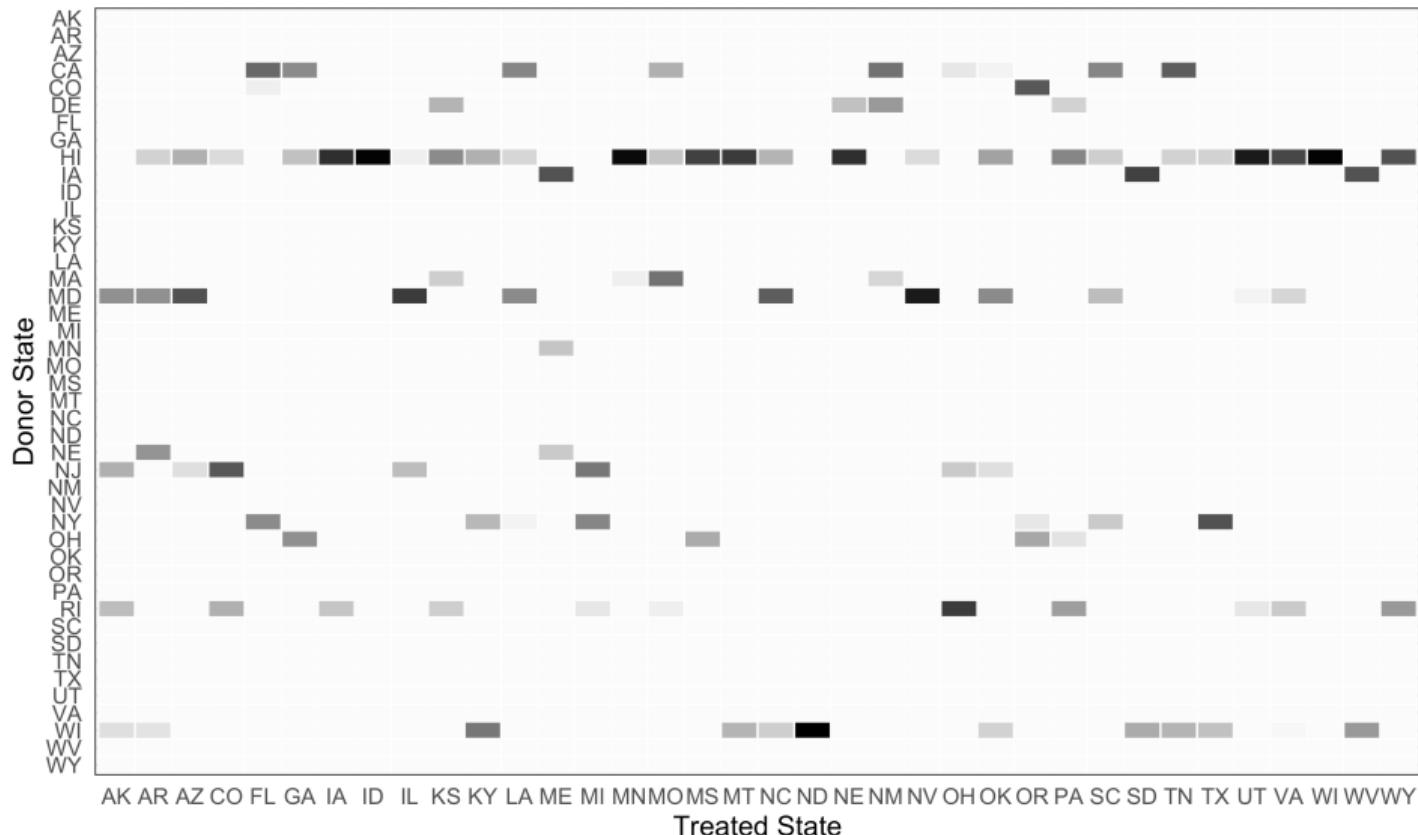
Level of **heterogeneity over time** is important

- $\bar{\mu}$  is the **average factor value** → importance of **Pooled Balance**
- $S$  is the **factor standard deviation** → importance of **State Balance**
- Special case: unit fixed effects, only **Pooled Balance** matters

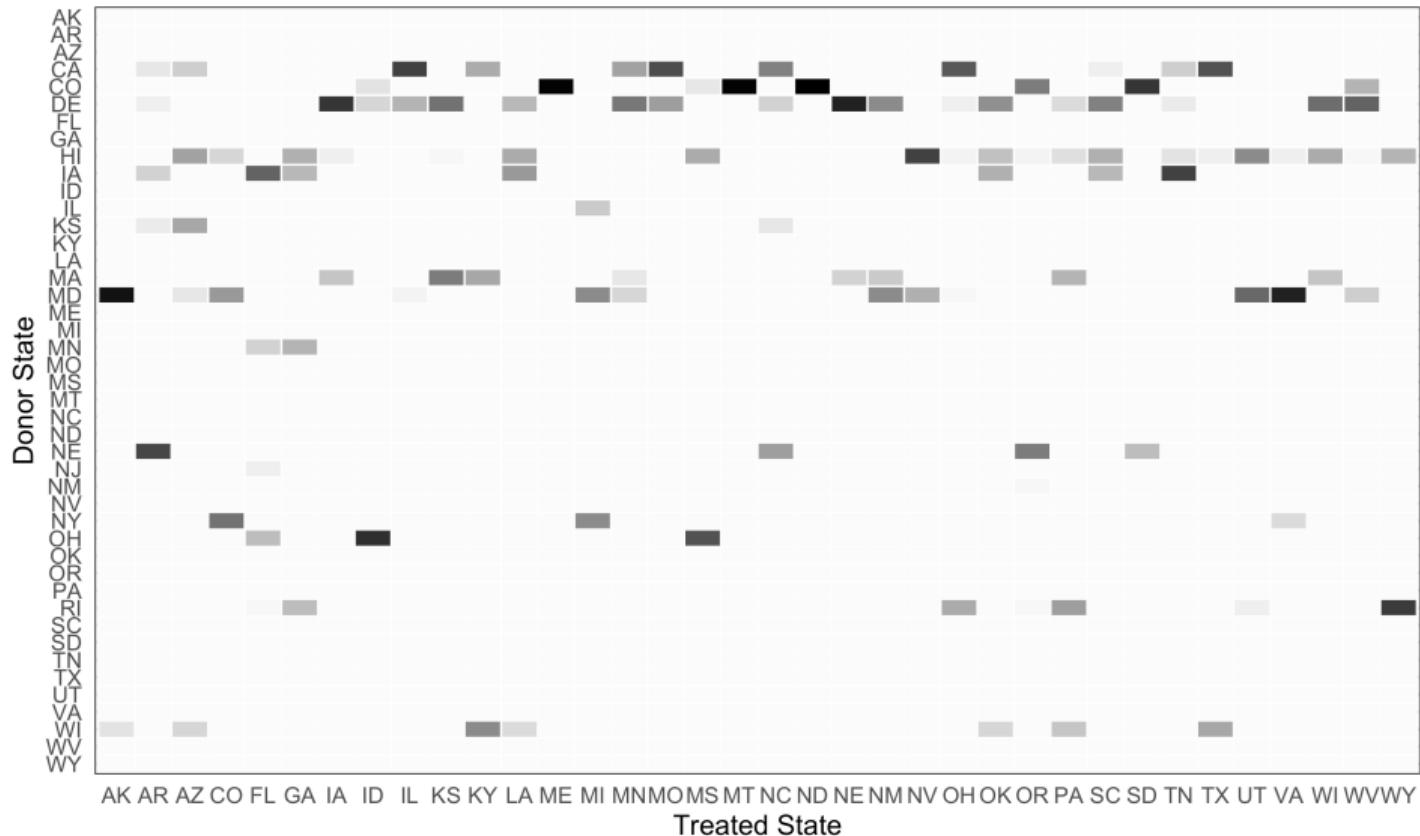
# Simulation study

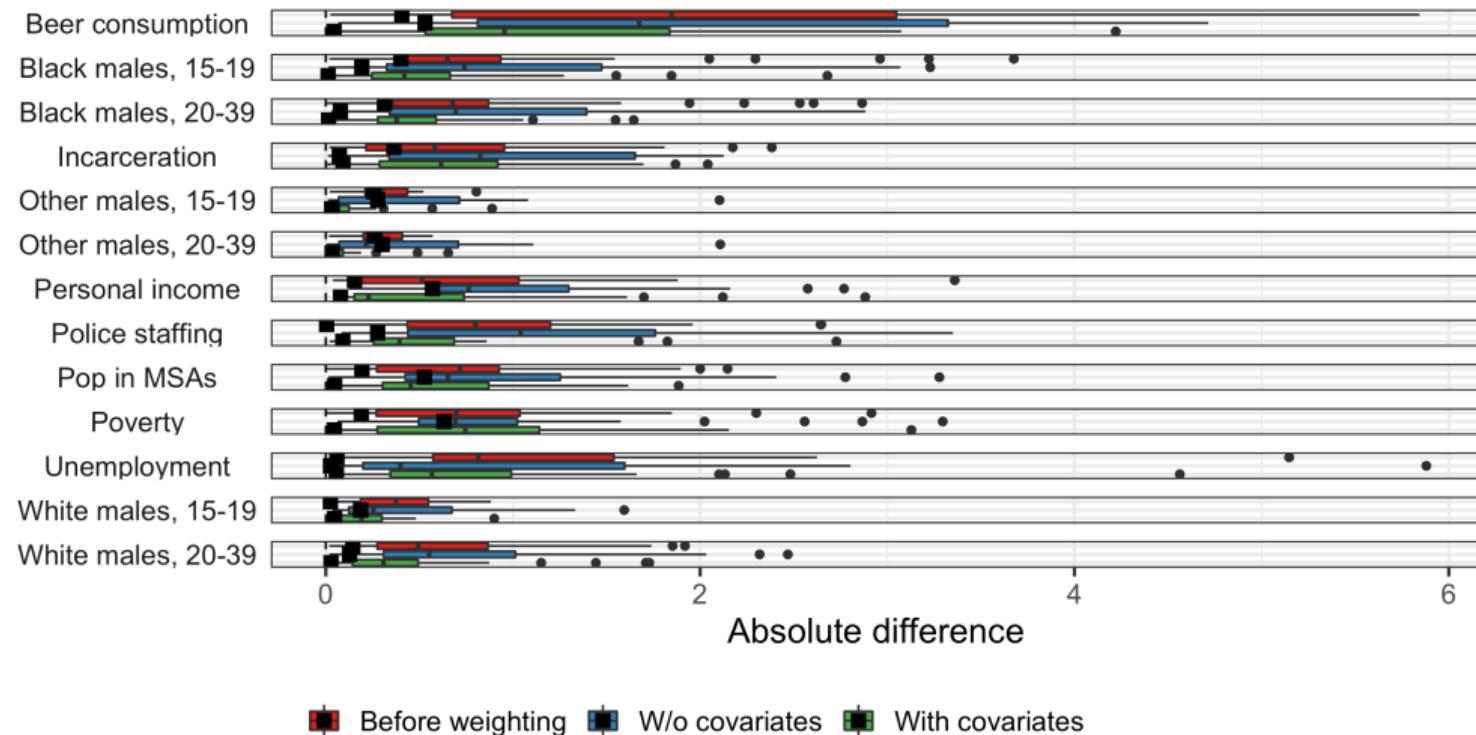


# Partially pooled SCM weights

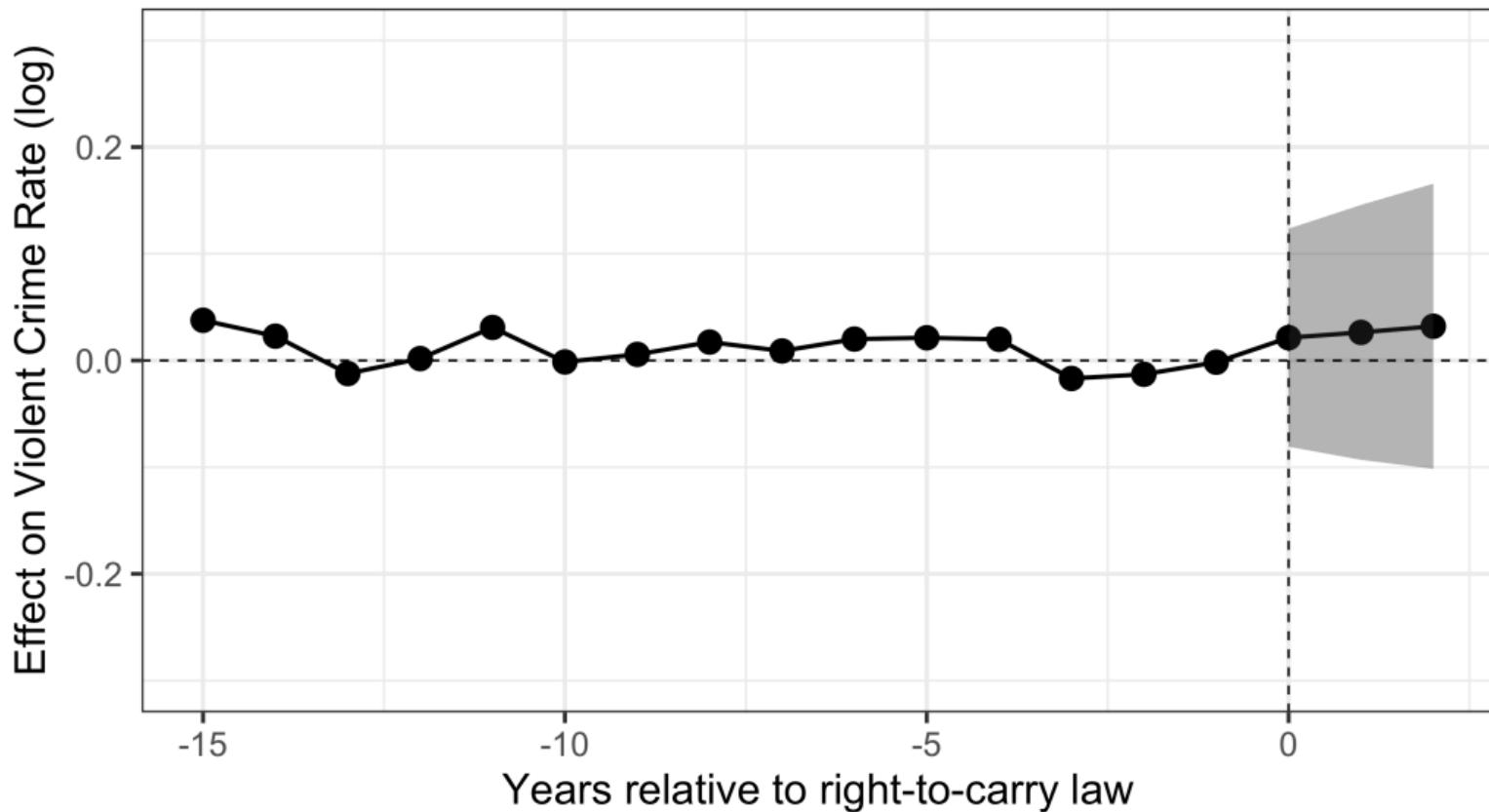


## Weights with intercept

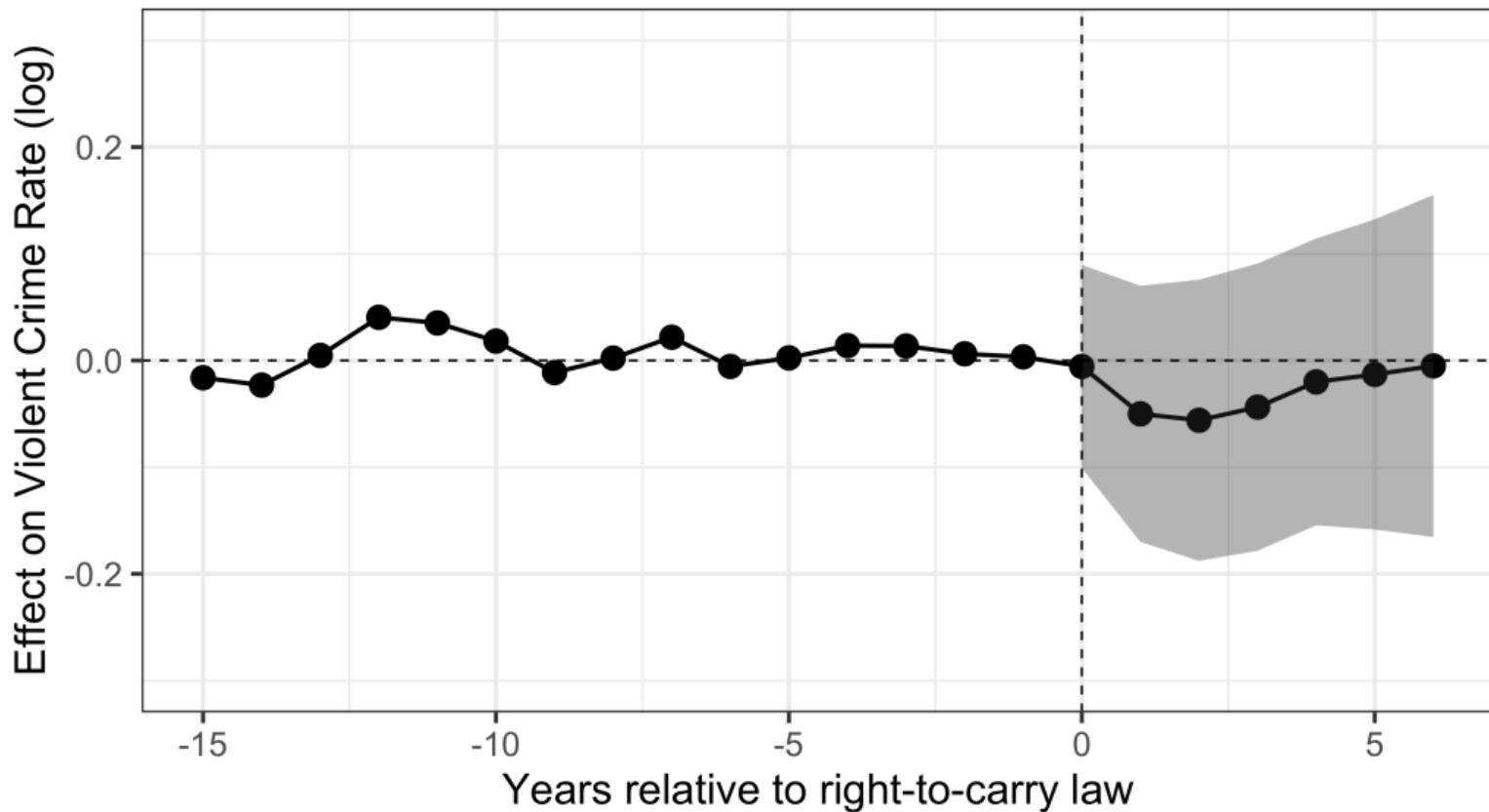




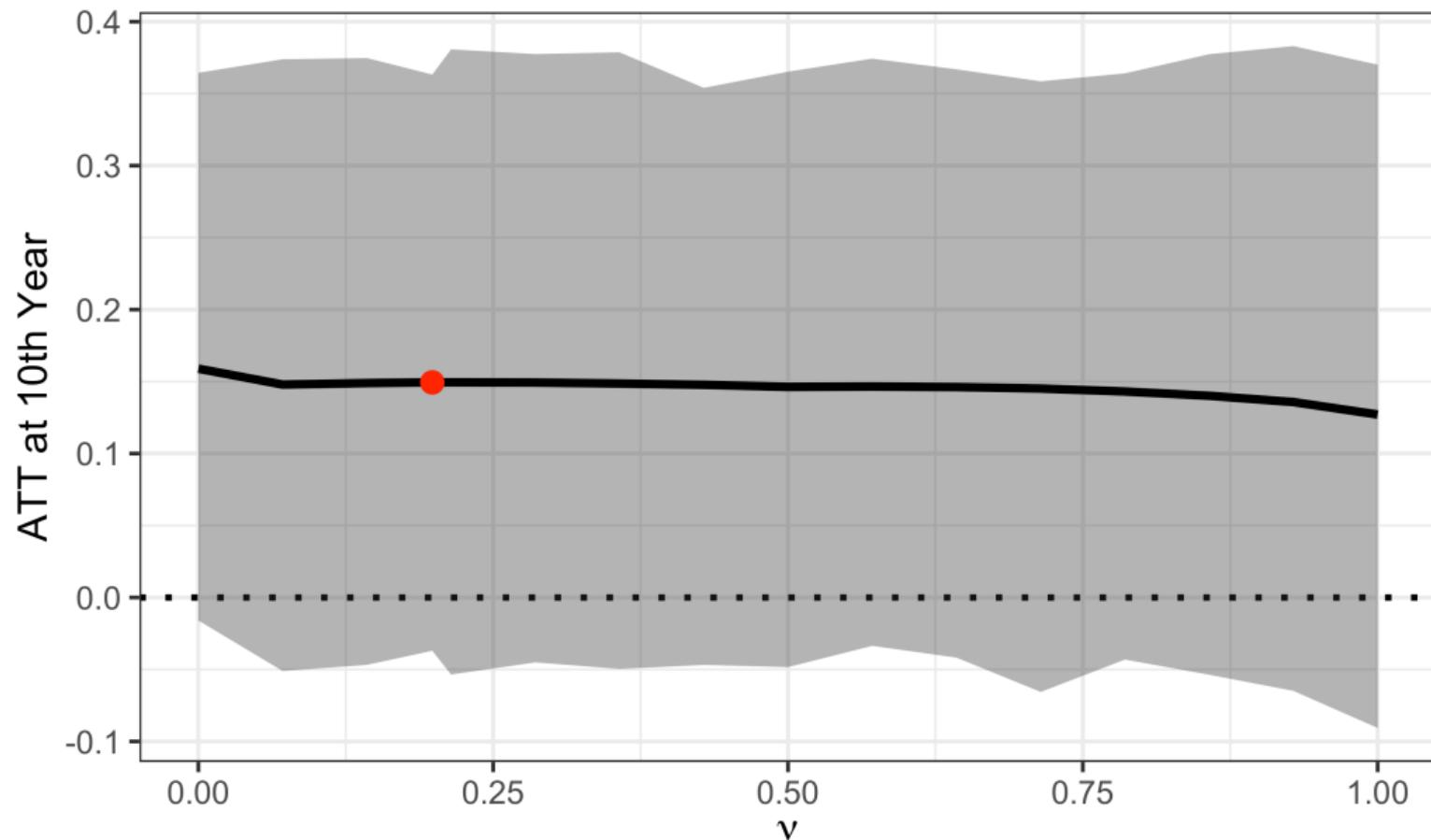
## In-time placebo (2 years)



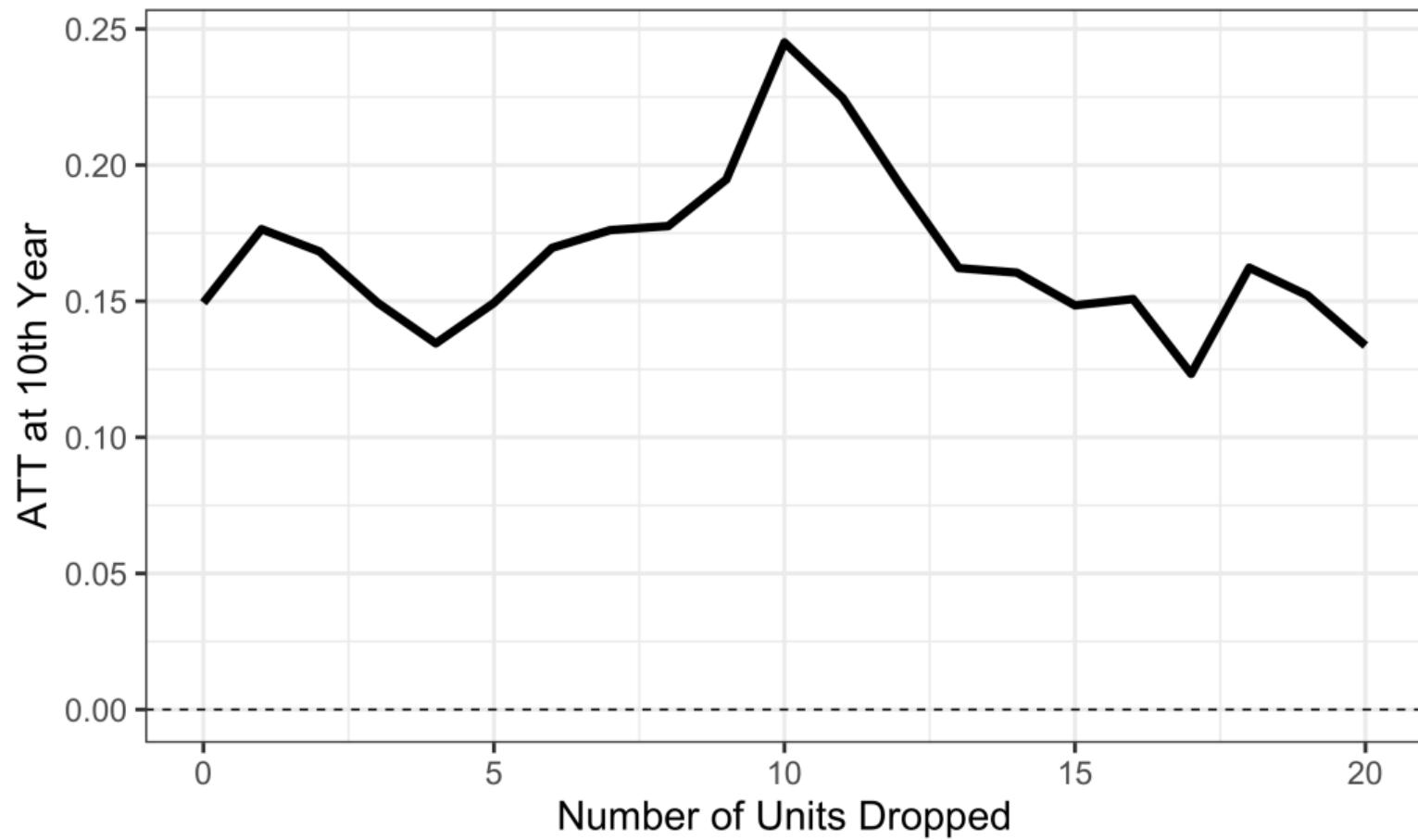
## In-time placebo (6 years)



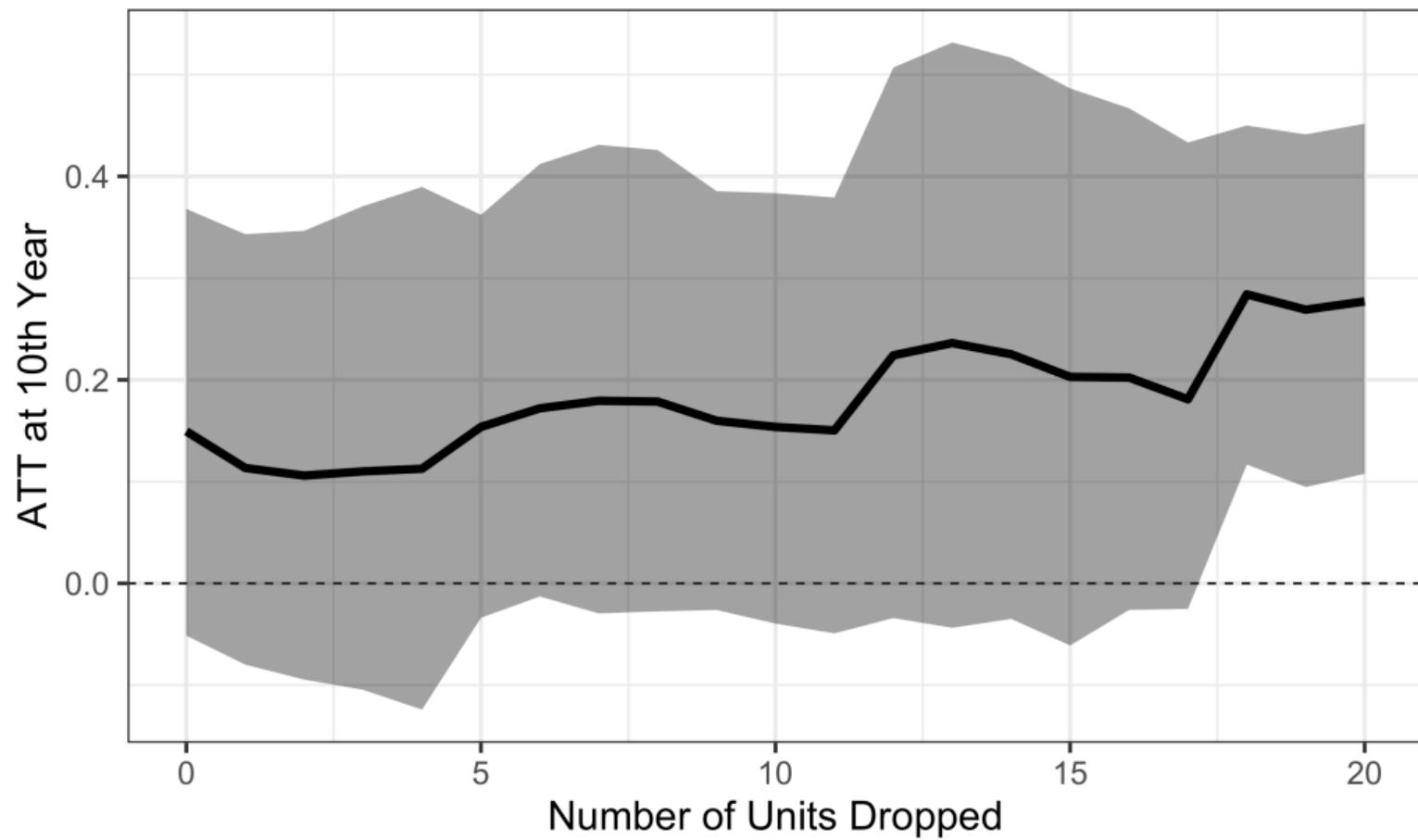
## Sensitivity to choice of $\nu$



## Dropping worst-fit units: P. Pooled SCM



## Dropping worst-fit units: P. Pooled SCM + Intercept + Covariates



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