Notes 07: Function Transformations

Edward Bensley
Email: ebensle1@charlotte.edu

Office Hours: Friday 1:30-3:30
Fretwell 321

1 Functions

Think about these two statements:

1. I had to prove that $\sqrt{2}$ is irrational, and my brother helped me.
2. I had to prove that $\sqrt{2}$ is irrational, but my brother helped me.

The difference between these two is in the implication: from the second one, you might determine that I was struggling and wouldn't have completed it without some assistance. The point here is that, depending on our goal, the same information can be given a different presentation depending on what we want to emphasize.

How does this relate to math? Well, try to find the difference between these two:

1. $y = x^2 - 4$
2. $f(x) = x^2 - 4$

If your guess was "there isn't", then you're spot on. Why bother deciding between the two? It might be confusing to write $y = x^2 - 4$, $y = \sqrt{5x-4}$, and $y = x^x - x^6$ every single time they came up, so it would be best to give them names—$f(x)$, $g(x)$, and $h(x)$ for example—in order to distinguish between them. An aircraft engineer would have an equation for fuel usage in terms of time and also have an equation for aerodynamic stress based on the force, altitude, and shape of the aircraft; these might be written as $f(t) = 2x^2 + x + 0.5$ and $\vec{F}(\rho) = \oint_A \rho \, dA$ because these equations require different inputs. There might even be *multiple* inputs, such as a curve in three dimensions given by $f(x, y) = x^3 + xy$, where new letters can be confused for variables.

For us the most important thing to remember is that $f(x) = y$, meaning that they are **interchangeable**. A basic function—also called a parent function—is what function transformations are built off of; we take a parent function and shift it around to get what we want. All of these must pass through the origin, which is the point $(0, 0)$. This is the most important thing to remember: For any parent function, its "center" or start point is at the origin and $f(0) = 0$. Here's a short, non-exhaustive list of parent functions

1. Linear: $f(x) = x$
2. Quadratic: $f(x) = x^2$
3. Square root: $f(x) = \sqrt{x}$
4. Absolute value: $f(x) = |x|$

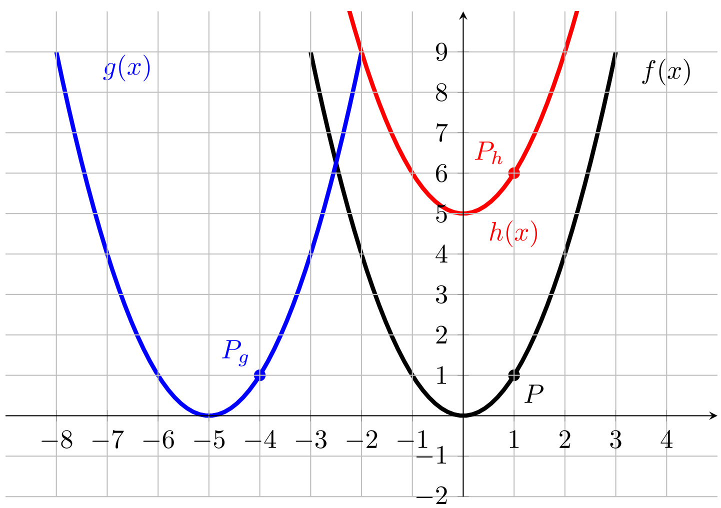
2 Transformations

We can think of functions as taking a number and transforming it to another number; similarly, we can transform a function into another. There are three types of transformations:

- Translation (shifting horizontally or vertically); addition
- Scaling (stretching or compressing); multiply
- Reflection; multiply by a negative

Let's actually make a graph of a basic function $f(x) = x^2$ and transform it. We will do two transformations: one by adding 5 *to* the x, and the other by adding 5 to f as a whole:

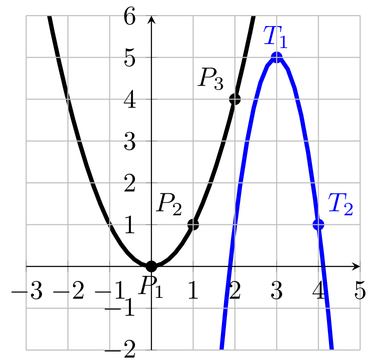
$$f(x) = x^2 \xrightarrow{x+5} g(x) = (x+5)^2$$
$$f(x) = x^2 \xrightarrow{f(x)+5} h(x) = x^2 + 5$$



Notice that the graph shifted *left* when we added to x while shifting *up* when we added to $f(x)$ Here are the rules as to what happens:

1. To x: horizontal and opposite
2. To $f(x)$: vertical and expected

What about finding a transformation from a graph? This is where the idea of "known points" comes in. First, identify your parent function and think about the graph. It's pretty clear that we have a reflection over the y-axis and that we shift up 5, as well as shifting right 3.



Vertically, P_1 and P_2 were 1 away from each other, and the same can be said horizontally. But T_1 and T_2, when 1 away horizontally, are now 4 away vertically! This means that we have a vertical stretch by a factor of 4. But we also could compare P_3 and T_2, since they are both 4 away from P_1 vertically—in this case, T_2 is half the distance from T_1 as P_3 is from P_1. The idea here is that there *is* a relationship between vertical and horizontal stretching. Putting this all together, the equation is

$$g(x) = -4(x-3)^2 + 5 = -(2(x-3))^2 + 5$$

The cautionary tale is that you need to shift *before* you scale.