Notes 04: Quadratic Inequalities

Edward Bensley
Email: ebensle1@charlotte.edu

Office Hours: Thursday 1:00 - 3:00
Fretwell 321

1 Quadratic Properties

Remember that a quadratic is of the form ax^2+bx+c. If we set this equal to something, we call it an equation. If we set it not equal - so we might use $\neq$, $\geq$, or $<$ - then we have an inequality. The rules that we used for the intervals in the first week still apply here, and so is the fundamental question: what values of x make the statement true? To answer this question, we need to understand a few properties.

When we graph a quadratic, we say that the shape is a parabola. As x goes from left to right, every parabola will always change directions once. That means it has to stop to turn around; as a consequence, every quadratic will always have either a maximum value or a minimum value, but never both. We can tell based on the sign of the leading coefficient:

1. Positive: decreasing until the vertex, then increasing afterwards; opens up.
2. Negative: increasing until the vertex, then decreasing afterwards; opens down.

The maximum or minimum point is called the **vertex**, and we have a formula to find the x-coordinate:

$$x = -\frac{b}{2a}$$

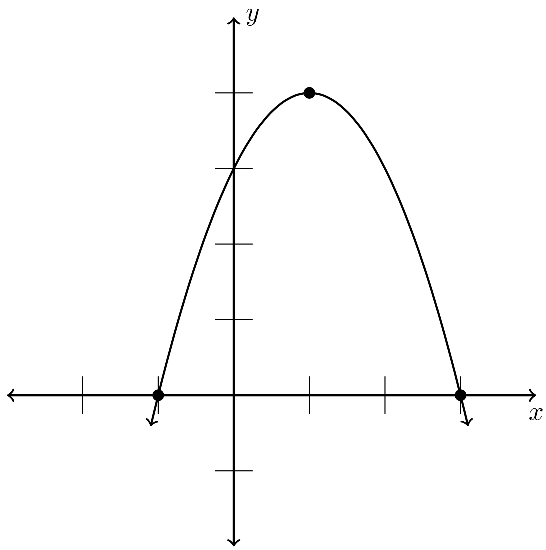
To find the y-coordinate, just plug this into the equation. In fact, that's how you can find any value on any equation - just pick an x, then see what y comes out. For some additional terminology, any x we plug in that results in zero is called *a zero* or a *root*. The vertex will always sit in between the zeroes! This means that the maximum or minimum value will be achieved after the left-most zero and before the right-most; in fact, it will sit directly in the middle.

For example, consider $y = -x^2 + 2x + 3$. We can factor this:

$$\begin{aligned} -x^2 + 2x + 3 &= -1(x^2 - 2x + 3) \\ &= -(x-3)(x+1) \end{aligned}$$

Remember, the factors give us the roots and the y-value corresponding to them is zero.

So we know that $(-1,0)$ and $(3,0)$ lie on the graph and that it opens downward. The formula for the x-coordinate of the vertex gives us $x=1$, so plugging this in gets $y=-(1-3)(1+1)=4$, so the vertex is at $(1,4)$. With these three points, we can graph:

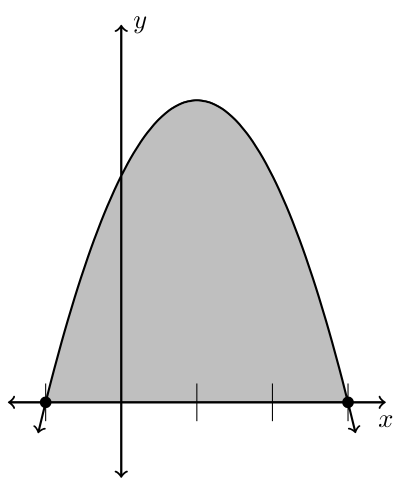


1.1 Solving Inequalities

Now we want to solve a quadratic with inequalities, let's take our example and change it slightly:

$$-x^2+2x+3\geq 0$$

Here's a visual motivation for what we are trying to see: think of the quadratic as "shading in" the appropriate place. Where on the x-axis does the shadow fall on? We know that the zeroes are at $x=-1$ and $x=-3$ and that the quadratic has a maximum value of 4, so we know that everything in between the zeroes is positive. In other words, the inequality is satisfied on the closed interval $[-1,3]$; writing $-1\leq x\leq 3$ is also fine. But graphing describes everything just stated, and is far easier than sitting down and thinking really hard, which means drawing a picture is the best way to arrive at the answer.



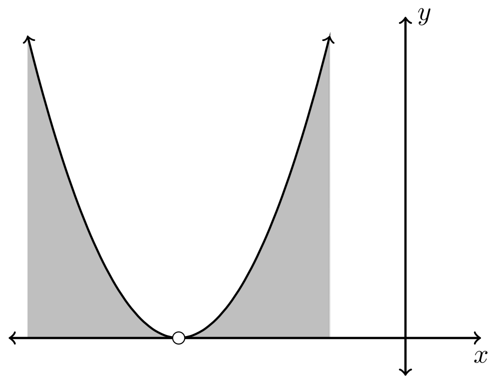
For another example, let's think about

$$x^2 + 6x + 9 > 0$$

This fits the form for a square of sums, so we know this factors to $(x+3)^2 > 0$ and so it has one zero at $x = -3$. We also know that this quadratic has a minimum value because the leading coefficient is positive, and since we want to see where it's greater than zero, we should find what this minimum value is - the y part of the vertex. First,

$$x = -\frac{b}{2a} = -\frac{6}{2} = -3$$

which means our minimum value is zero! So this inequality is satisfied on all real numbers *except* zero! If we graph it, we get



Here's a small list of acceptable answers; the choice isn't important so long as it's clear.

1. $x > -3$ or $x < -3$
2. $x \neq -3$
3. $(-\infty, -3) \cup (-3, \infty)$
4. $x \in \mathbb{R} \setminus \{3\}$