# Notes 06: Rational Expressions and Equations

Edward Bensley
Email: ebensle1@charlotte.edu

Office Hours: Friday 1:30-3:30
Fretwell 321

## 1 Fractions, but with Polynomials

In the beginning of the semester, we started by working with fractions - how to add, how to multiply, and so on. Since then, we've been building familiarity on working with polynomials. For the most part, these polynomials have just been quadratics, where the greatest exponent is 2, or else they can be fairly easily turned into one; for example, $x^4 - 3x^3 - 18x^2$ factors to $x^2(x^2 - 3x - 18)$, which is now easier to factor. Last week we learned about fractions made out of polynomials, also known as rational functions; this week, we will apply our usual arithmetic. But you already know how to do this because *nothing* changes between rational functions and fractions! To make life easier, it's worth the effort to provide a step-by-step reminder on how to do arithmetic with fractions:

- Addition and subtraction

  **Step 1:** Multiply to achieve a common denominator

  **Step 2:** Combine the numerators: add or subtract

- Multiplication and division

  **Step 1:** If dividing, do keep, change, flip: ignore the left side, change $\div$ to $\cdot$, then flip the right side

  **Step 2:** Combine the numerators: multiply across

  **Step 3:** Cancel the terms that are shared between the numerator and denominator

The only difference with rational functions is that at some point we will need to factor. However, this is really "baked in" to the last steps for simple fractions, but doing it sooner rather than later can prevent you from having to do a lot of work.

**Example. (Simplifying Expressions)**

$$\begin{aligned}\frac{x^2+4x-5}{x^2+1} \div \frac{x-1}{4x^2+4} &= \frac{(x+5)(x-1)}{x^2+1} \cdot \frac{4(x^2+1)}{x-1} \\ &= \frac{4(x+5)(x-1)(x^2+1)}{(x^2+1)(x-1)} \\ &= 4(x+5) \\ &= 4x+20\end{aligned}$$

☕

**Example. (Solving Equations)**

$$\frac{2x+2}{x^4-x^2} = \frac{6}{x^2} - \frac{5}{x^2-1}$$

Factor the left hand side:

$$\frac{2(x+1)}{x^2(x^2-1)} = \frac{6}{x^2} - \frac{5}{x^2-1}$$

If we multiply everything by each denominator, then denominators will disappear. But we can see that the left hand side's denominator shares the right hand side's denominators, so we only need to multiply everything by $x^2(x^2-1)$.

$$\begin{aligned}
\frac{2(x+1)\cdot(x^2(x^2-1))}{x^2(x^2-1)} &= \frac{6\cdot(x^2(x^2-1))}{x^2} - \frac{5\cdot(x^2(x^2-1))}{x^2-1} \\
2x+2 &= 6(x^2-1) - 5(x^2) \\
2x+2 &= x^2-6 \\
0 &= x^2-2x-8
\end{aligned}$$

Since this problem has been reduced to factoring, we'll call it done. ☕

It's also worth mentioning that you can find the factors by using the quadratic formula. For example, using it on $21x^2-22x-8$ gives us $r_1 = 4/3$ and $r_2 = -2/7$ for the roots. Now just flip the signs and put an $x$: $(x-4/3)(x+2/7)$. Since nobody writes roots like that, take the denominator and stick them in front of the $x$ and we have it factored into $(4x-3)(7x+2)$.