1 Introduction

This is a list of topics that you may choose from to read up on and write about, answer questions, etc. in order to receive full credit for a late assignment. The expectation is that you read more than the first three sentences on Wikipedia and regurgitate them. Mark your selection by writing something like "I.a.ii." to make it clear that you picked Set Theory, topic a, part ii.

2 Topics

I. Set Theory

(a) In your own words, define ordinal and cardinal number and explain some of their similarities and differences.

i. Again in your own words, define limit ordinal and successor ordinal. How are they different?

ii. What is the difference between something being countable and uncountable?

(b) ZFC provides the axioms (the things we assume are true) that are the foundations of math. In plain English, what does the axiom schema of replacement allow for us to do? There is only one axiom that declares a set exists; which is it?

II. Algebra

(a) There are a lot of algebraic structures out there, each having many different features, but they all contain some objects and a way to combine them. For example, rotations of shapes are combined by composition (one after another) while numbers may be added and multiplied. Find one that you find interesting and give an example.

i. Define *magma* (in terms of mathematics, not geology), *associativity*, and *commutativity*. We can think of the game "Rock, Paper, Scissors" as a non-associative but commutative magma where the operation is "Shoot". Below is an example of a few games being played between two people.

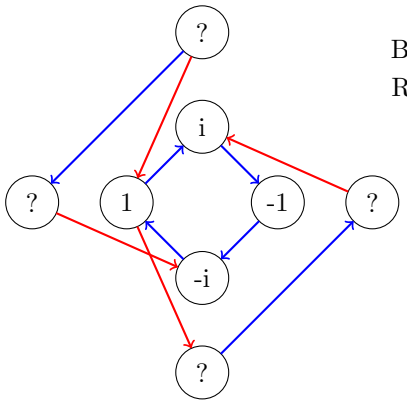
$$r \circ p = p \circ r = p, \quad s \circ r = r, \quad r \circ r = r$$

What do $r \circ (p \circ s)$ and $(r \circ p) \circ s$ result in? Give two more examples with three players.

ii. The quaternions, $\mathbb{H}$, are like four dimensional numbers. A quaternion is of the form $q = a_1 + a_2 i + a_3 j + a_4 k$ where the a are all real numbers. Here's an outline of how the multiplication works between the units i, j, and k:

$$i^2 = j^2 = k^2 = -1, \quad ij = k,\ jk = i,\ ki = j$$

Quaternion multiplication is anti-commutative, so we would have $ji = -k$. Below is an incomplete picture representing the multiplication; you must complete it.



Blue multiplies by i

Red multiplies by ?