Notes 01: Domains, Intervals, Radicals

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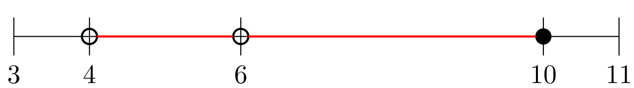
1 Sets and Notation

The English definition of the word "domain" refers to where something lives, and the mathematical definition is no different: the domain of a function refers to the set of all numbers that we may actually plug into it. But this may introduce another question: what is a set? As it is in English, a set in mathematics is a collection of objects - in this class, our focus is entirely on the set of real numbers. We write the set of real numbers as $\mathbb{R}$, which includes the numbers you're familiar with (like 0, 4, and $-4/17$) as well as more exotic numbers like π^2 or e. There are numbers outside of the reals and there are even structures that exist independently of numbers, which have one unifying concept: we use sets to describe them. Using sets allows us to describe all numbers that follow a particular rule. Here's an example.

$$\{x \in \mathbb{R} : 4 < x \leq 10,\ x \neq 6\}$$

From left to right, this is how you read it: all real numbers x such that 4 is less than x but x is less than or equal to ten, and where x does not equal 6. The left side tells you what members of the set look like and the right side tells you the rule that they follow, which is the important part. The rule here is an **inequality**. This is pretty easy to visualize:



Notice that the $4 < x$ means *strictly* greater, not equal, so we exclude $x = 4$ from this set and we represent this as an open circle on a number line. The same applies at $x = 6$ since we've excluded it. But since we have $x \leq 10$, we include $x = 10$ and represent this with a closed circle.

Interval notation is another convenient way of describing sets. We write the lower bound on the left and the upper bound on the right, separated by a comma, with a bracket if the boundary is included or a parenthesis if it is not. Intervals may be open (which don't include the boundary), closed (which do), or half-open. We can also stitch intervals (and sets) together by using the symbol $\cup$, called **cup** or **union** (so please don't write the letter u); $A \cup B$ is the set of stuff in A and the stuff in B, and $(0, 1) \cup [7, 12]$ refers to the numbers in between 0 and 1 as well as everything from 7 to 12. With intervals, the idea is that $\cup$ allows you to jump over a gap between them. You can jump as many times as you like! For our example, we need to include numbers after 4, up to and including 10, and exclude 6. Clearly $(4, 10]$ would be incorrect because 6 lives there, so we need to stitch two intervals together. Writing $(0, 5] \cup [7, 10]$ would be wrong too - what about 5.5 and 6.001? 6 acts as another boundary that we can get very close to but not actually touch in the same way that we can with 4, so we need to use parenthesis (I call them open brackets) for 6. The correct way to write this is $(4, 6) \cup (6, 10]$.

1.1 Domain

It's worth mentioning that the words graph, equation, and function are interchangeable. We typically pick between them when we want to focus on a particular aspect - graph for the way it looks, equation for the numbers we care about, and function when we care about the relationships and properties. On a graph, the **domain** is simply all of the x values that correspond to a point on the graph. The best way to approach this is to take your pen, place it on the x-axis (the horizontal line), and slide it along where the graph starts and ends. This is a convenient number line. If a particular value for x does *not* have a y value above it (i.e. the graph isn't there), then draw a circle to denote a hole in the domain. In general, the best way to think of the domain of a function is all of the values for x that we are *allowed* to use. On the other hand, for the values that we could *actually* get out of the function (the y for a graph), we call that the **range**.

2 Radicals

When we multiply a number by itself, we say that the number is squared - for example, $5 \cdot 5 = 5^2 = 25$. The square root, denoted by the radical symbol, will undo this: $\sqrt{25} = 5$. The best way to approach these problems is to break the numbers down into their factors: what can you multiply to make the number? Whenever you have a number represented twice, you can factor that out and take its square root. For example,

$$\begin{aligned}
\sqrt{48} &= \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3} \\
&= \sqrt{2^2 \cdot 2^2 \cdot 3} \\
&= \sqrt{4 \cdot 4 \cdot 3} \\
&= \sqrt{4^2 \cdot 3} \\
&= \sqrt{4} \cdot \sqrt{3} \\
&= 2\sqrt{3}
\end{aligned}$$

Take care to note that the **only way** to distribute a radical or an exponent is over multiplication. It **does not** distribute over addition. If you try, you get nonsense like this:

$$\begin{aligned}
\sqrt{4} &= \sqrt{1+1+1+1} \\
&= \sqrt{1} + \sqrt{1} + \sqrt{1} + \sqrt{1} \\
&= 1+1+1+1 \\
&= 4
\end{aligned}$$

If multiplication is repeated addition, then exponentiation is repeated multiplication - and if subtraction is the opposite of addition and division is the opposite of multiplication, then "rooting" is the opposite of exponentiation. You wouldn't want to skip a step and fall down the stairs, would you? Here's a small diagram.