Temperature Control System (Smart Thermostat) Final Project

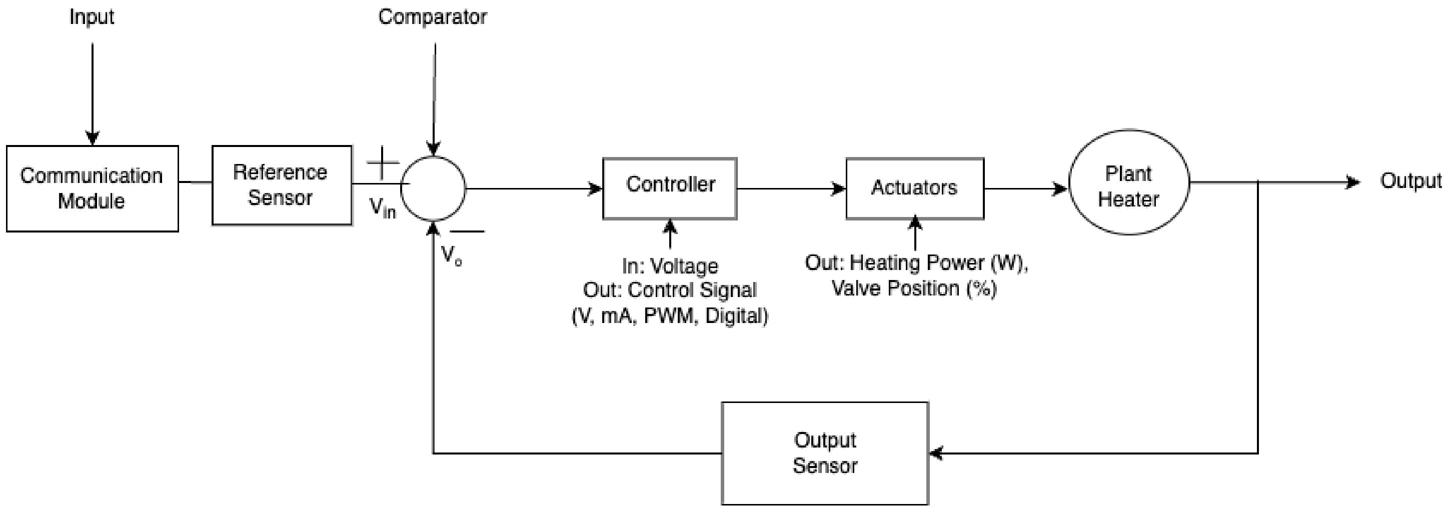
Topic Description - Smart Thermostats

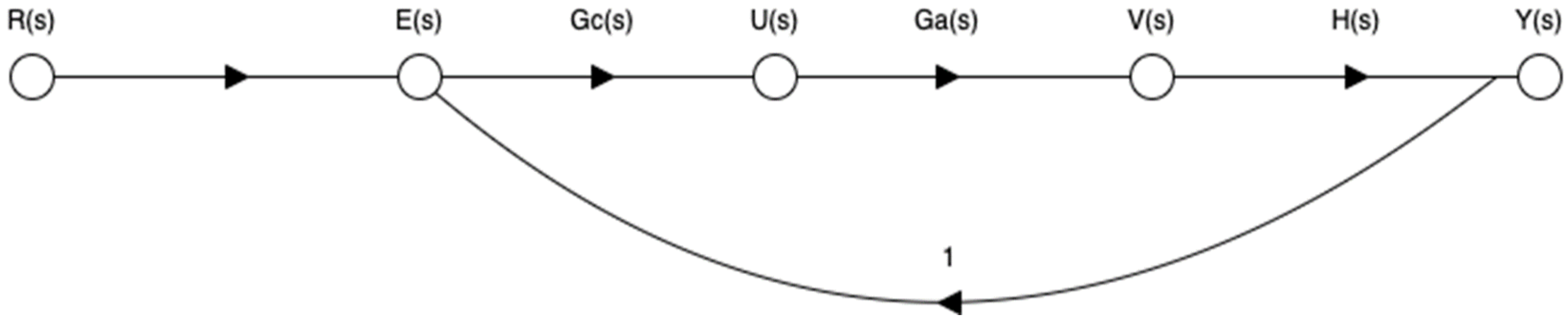
Smart thermostats are Wi-Fi-enabled devices that can automatically adjust your home's heating and cooling temperatures. They can be controlled remotely and provide superior functionality, convenience, and significant energy-saving potential compared to traditional thermostats.

Motivation

The motivation for selecting smart thermostats as the project topic stems from the increasing demand for energy-efficient solutions in both residential and commercial buildings. Advancements in Internet of Things (IoT) technology has enabled smart thermostats to provide a unique combination of convenience, significant energy savings, and enhanced control over indoor climate settings.

System Block Diagram & Signal Flow Diagram





Heat Transfer/Thermal System Properties

In a thermal system, there are two different ways heat is transferred:

- Thermal Capacitance: $C = \rho c_\rho V$ where C is the thermal capacitance and can be stated as a product of ρ material density, c_ρ specific heat, and Volume V
- Conduction: $q = \frac{kA}{l}\Delta T = D_{1-2}\Delta T$ where q is the rate of heat transfer (flow), k is the thermal conductivity related to material used, A is the area normal to the direction of heat flow, and ΔT is difference between two temperatures. Note that: $D_{1-2} = \frac{1}{\frac{l}{kA}} = \frac{1}{R}$ where R is thermal resistance. This simplifies the heat transfer equation q w/r/t R: $q = \frac{\Delta T}{R}$

TABLE 4-5 Basic Thermal System Properties and Their Units

Parameter	Symbol Used	SI Units	Other Units	Conversion Factors
Temperature	T	°C (Celsius) °K (Kelvin)	°F (Fahrenheit)	°C = (°F − 32) × 5/9 °C = °K + 273
Energy (Heat Stored)	Q	J (joule)	Btu calorie	1 J = 1 N-m 1 Btu = 1055 J 1 cal = 4.184 J
Heat Flow Rate	q	J/sec W	Btu/sec	
Resistance	R	°C/W °K/W	°F/(Btu/hr)	
Capacitance	C	J/(kg °C) J/(kg °K)	Btu/°F Btu/°R	

Table provides the basic units associated with Heat Transfer

Transfer Function Generation

Basic Heat Transfer Equations

Thermal Capacitance $C = \rho c_p V$

Where ρ =density of the material

c_p = specific heat capacity

V = volume

Conduction, $q = \frac{kA\Delta T}{L}$, $R = \frac{L}{kA}$

Where q =rate of heat transfer

k = thermal conductivity

A =cross-sectional area through which heat is transferred

ΔT = temperature difference across the material

L = thickness of the material

R = thermal resistance

Therefore; $q = \frac{\Delta T}{R}$

Transfer Function Generation contd.

Modelling a simple house with one temperature sensor and one heater;

Heat Balance Equation $Q_{in}(t) = Q_{stored}(t) + Q_{loss}(t)$

Store Heat $Q_{stored}(t) = C\frac{dT(t)}{dt}$

Heat Loss $Q_{loss}(t) = -\frac{T_{out}(t) - T(t)}{R}$

Heat Input. $Q_{in}(t) = P_{heater}(t)$

Combine the equations to get

$$C\frac{dT(t)}{dt} = P_{heater}(t) + \frac{T_{out}(t) - T(t)}{R}$$

$$C\frac{dT(t)}{dt} + \frac{T(t)}{R} = \frac{T_{out}(t)}{R} + P_{heater}(t)$$

Taking the Laplace

$$CsT(s) + \frac{T(s)}{R} = +\frac{T_{out}(s)}{R} + P_{heater}(s)$$

$$T(s) = \frac{\frac{T_{out}(s)}{R} + P_{heater}(s)}{Cs + \frac{1}{R}}$$

Assuming $T_{out} = 0$,

Plant Transfer Function

$$\boldsymbol{H(s) = \frac{1}{Cs + \frac{1}{R}}}$$

Transfer Function Generation contd.

From the block diagram we can derive the transfer functions for

- Comparator signal: $E(s) = R(s) - Y(s)$
- Controller, $G_c(s)$: $U(s) = G_c(s) \cdot E(s)$
- Actuator, $G_a(s)$: $V(s) = G_a(s) \cdot U(s)$
- Plant, $H(s)$: $Y(s) = H(s) \cdot V(s)$

To find the overall Transfer Function $\frac{Y(s)}{R(s)}$, substitute each equation into the next

- $E(s) = R(s) - Y(s)$
- $U(s) = G_c(s) \cdot \left(R(s) - Y(s)\right)$
- $V(s) = G_a(s) \cdot \left(G_c(s) \cdot \left(R(s) - Y(s)\right)\right)$
- $Y(s) = H(s) \cdot \left(G_a(s) \cdot \left(G_c(s) \cdot \left(R(s) - Y(s)\right)\right)\right)$

Transfer Function Generation contd.

Solving for $Y(s)$

$$Y(s) = H(s) \cdot G_a(s) \cdot G_c(s) \cdot R(s)$$

Transfer Function $\frac{\mathrm{Y(s)}}{R(s)}$

$$\frac{\mathrm{Y(s)}}{R(s)} = \frac{\mathrm{H}(s) \cdot G_a(s) \cdot G_c(s)}{1+\mathrm{H}(s) \cdot G_a(s) \cdot G_c(s) \cdot \mathrm{R}(s)}$$

State-Space Equations

State-Variable: Temperature

Equation(s):

$$x(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

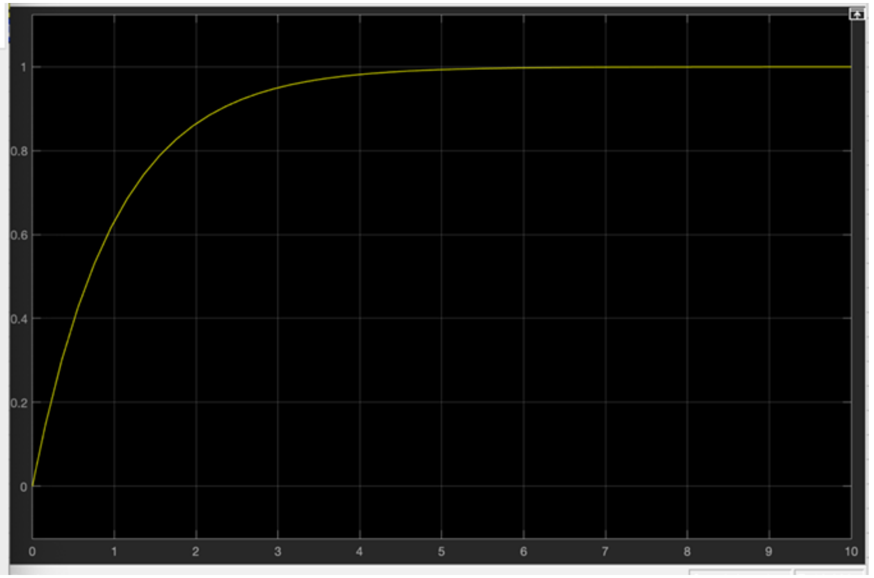
Applied to our Thermal System:

$$A = -\frac{1}{CR}, \qquad \mathrm{B} = \frac{1}{\mathrm{C}}, \qquad \mathrm{C} = 1, \qquad \mathrm{D} = 0$$

Time Domain Analysis in the following slides uses example values of Thermal Capacitance C = 1 and Thermal Resistance = 1

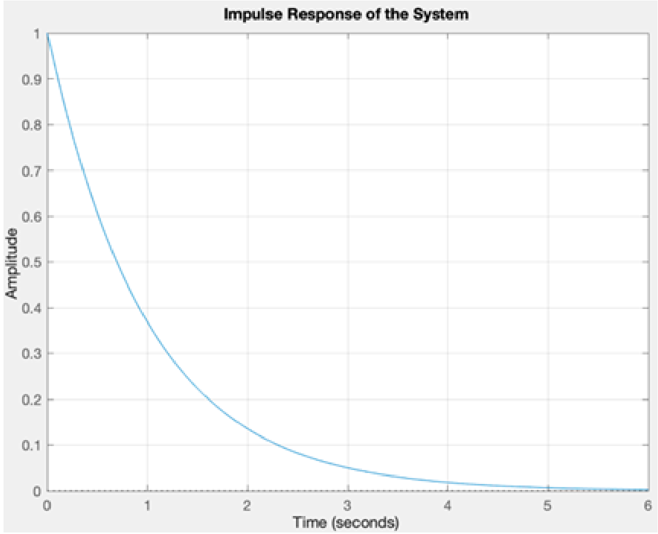
System Block Diagram Design (Simulink)

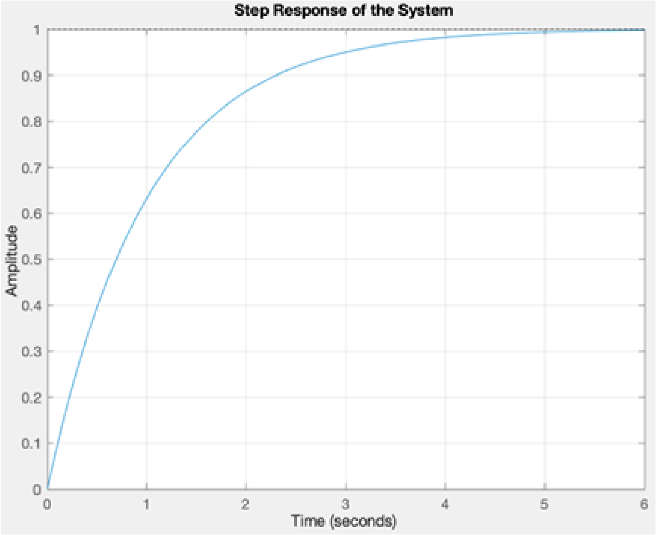
The Thermal System is a simple closed loop feedback system utilizing a PID controller that alters the controller variables to the three example cases: 1. Comfort & Stability, 2. Rapid Response, and 3. Energy Efficiency for different desired thermal outcomes depending on user set function input.



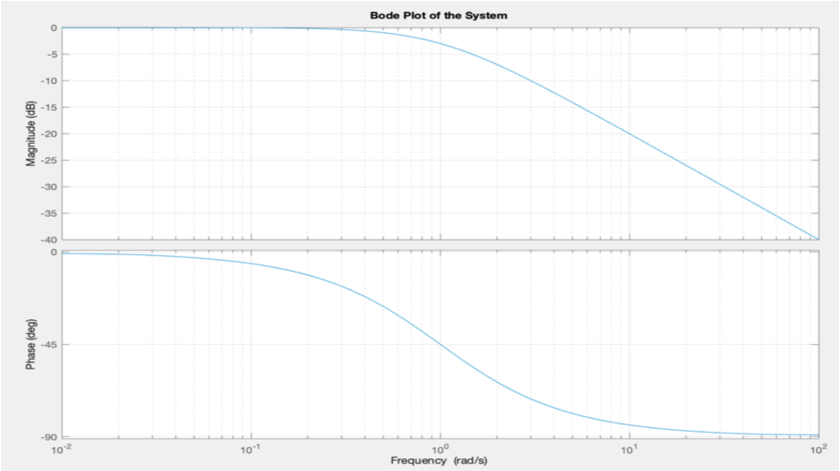
Output graph from the Simulink Scope block seen in the system block diagram

System Impulse, Step Response, & Bode Plots (Without Controller)

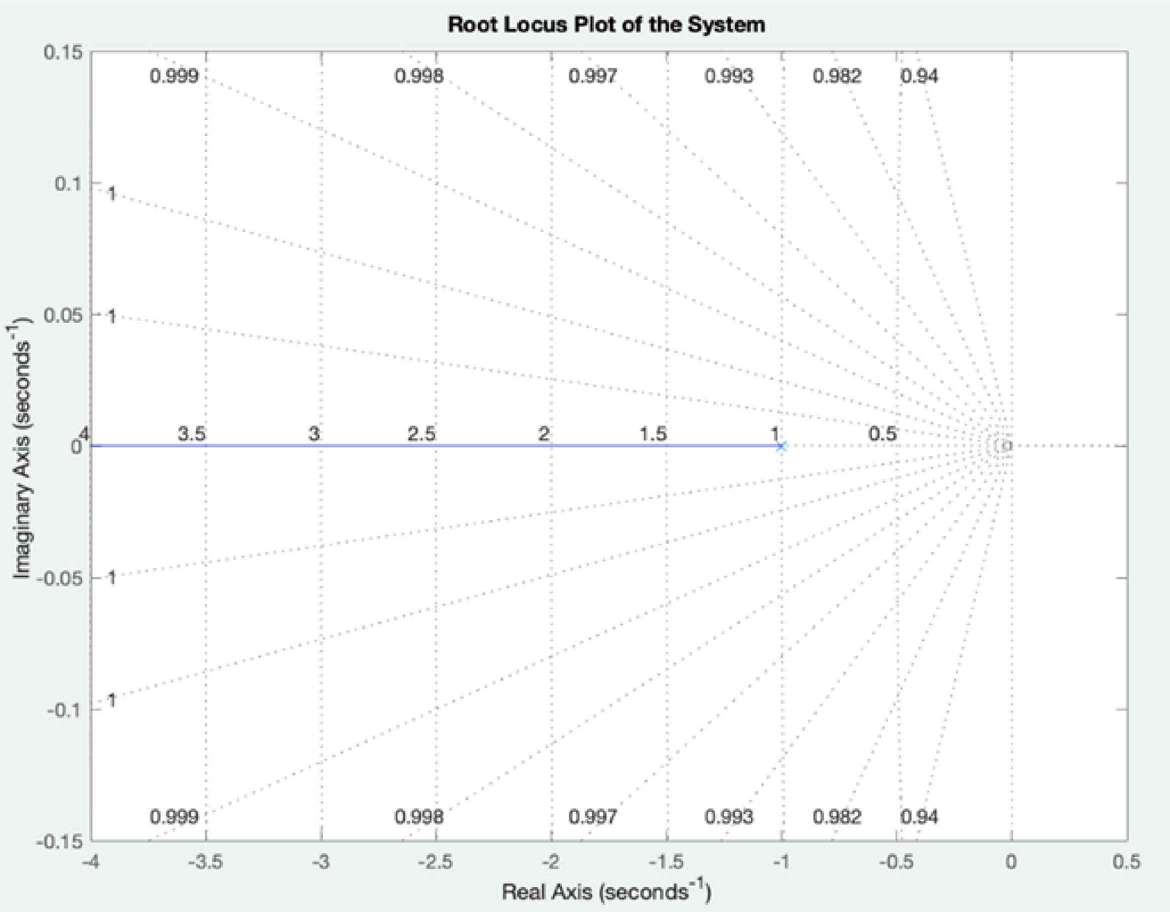




- Overshoot: 0.00%
- Settling Time: 3.91 seconds
- Steady State Value: 1



Root Locus Plot



- Closed-loop zero and pole @ -1
- Shows large region of system stability

Design Criteria, Objectives & Parameters

To evaluate the performance of the smart thermostat system, three distinct design cases are considered:

Case 1: Standard Comfort and Stability

Objective: To achieve a comfortable and stable indoor temperature with minimal overshoot and settling time.

- **Criteria**:
  - **Overshoot**: Less than 10%
  - **Settling Time**: Less than 5 minutes
  - **Steady-State Error**: Close to zero
  - **Phase Margin**: Greater than 45 degrees

Case 2: Rapid Response

Objective: Achieve rapid changes in indoor temperature in response to user input, prioritizing quick adjustment over minimal overshoot.

- **Criteria**:
  - **Overshoot**: Acceptable up to 20%
  - **Settling Time**: Less than 2 minutes
  - **Steady-State Error**: Close to zero
  - **Phase Margin**: Greater than 30 degrees

Case 3: Energy Efficiency

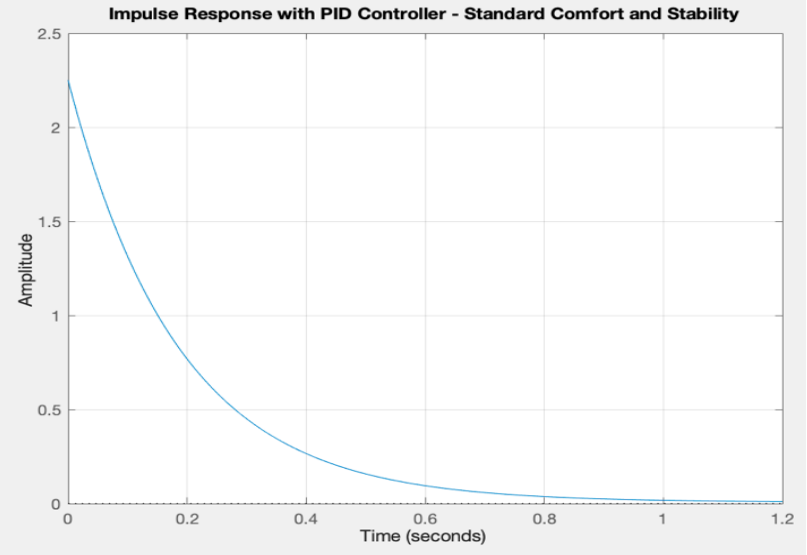
Objective: Optimize energy efficiency by reducing power consumption while maintaining a stable indoor temperature.

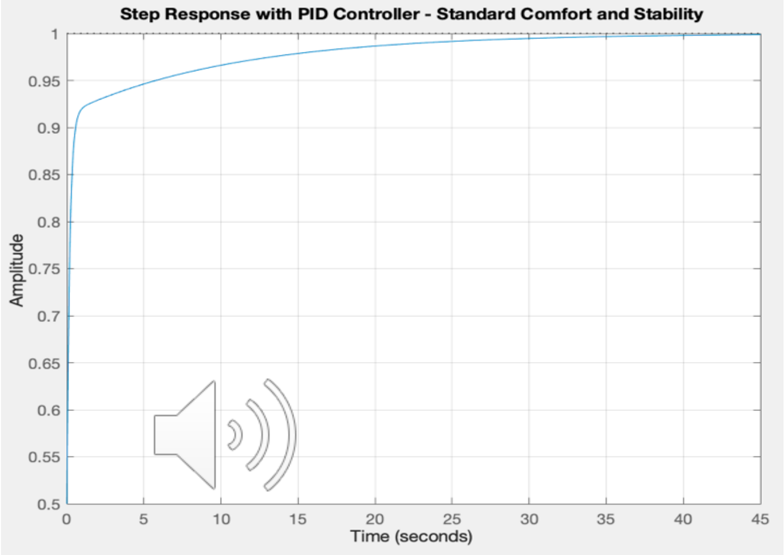
- **Criteria**:
  - **Overshoot**: Less than 5%
  - **Settling Time**: Less than 10 minutes
  - **Steady-State Error**: Minimal
  - **Power Consumption**: Minimized
  - **Phase Margin**: Greater than 50 degrees

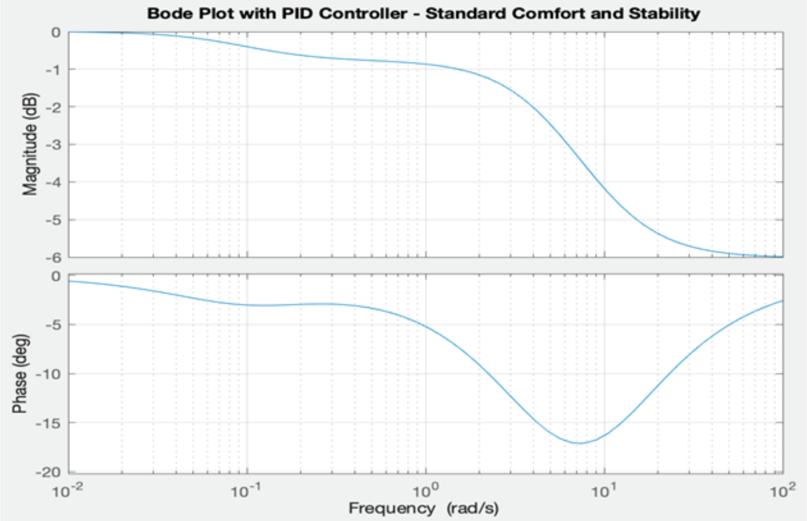
Design Criteria Controller Values:

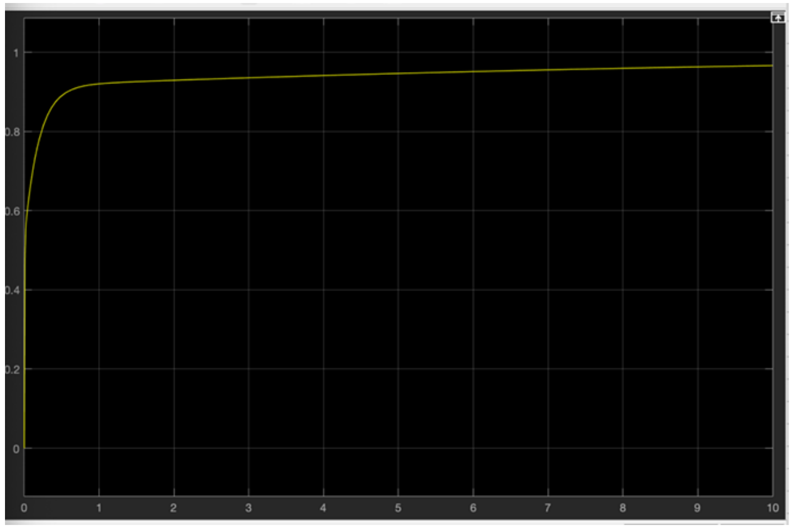
- Case 1: Kp = 10, Ki = 1, Kd = 2
- Case 2: Kp = 15, Ki = 2, Kd = 2
- Case 3: Kp = 8, Ki = 0.5, Kd = 0.5

System Impulse, Step Response, & Bode Plots (With Comfort & Stability PID Controller)



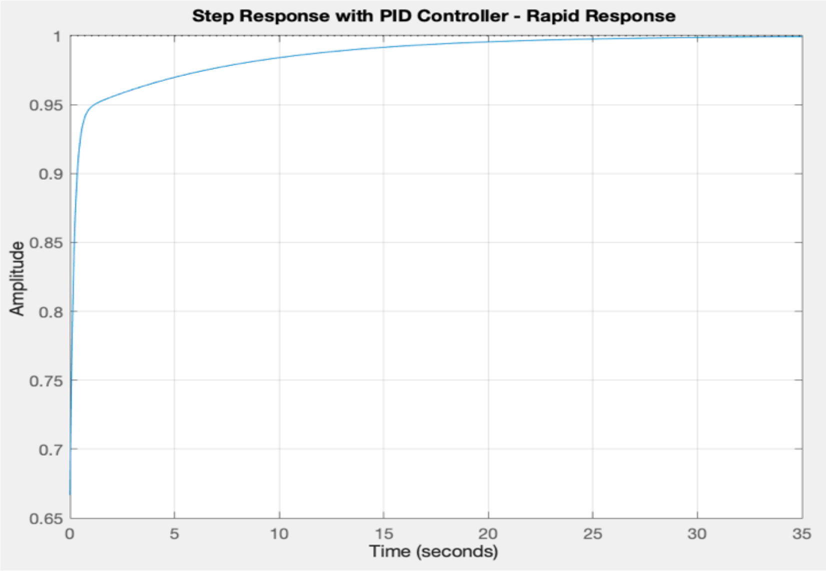


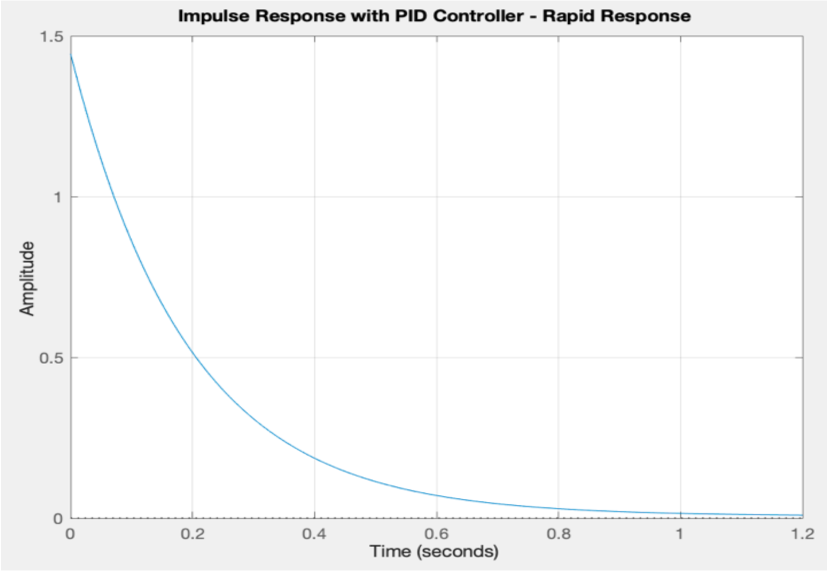


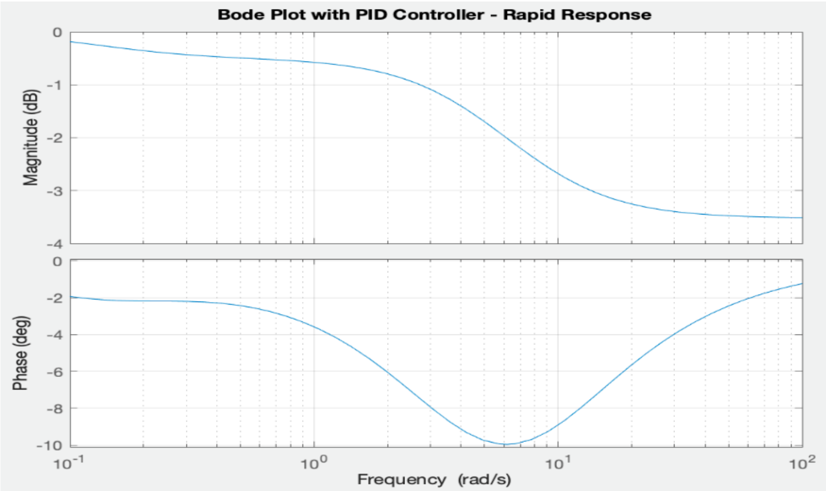


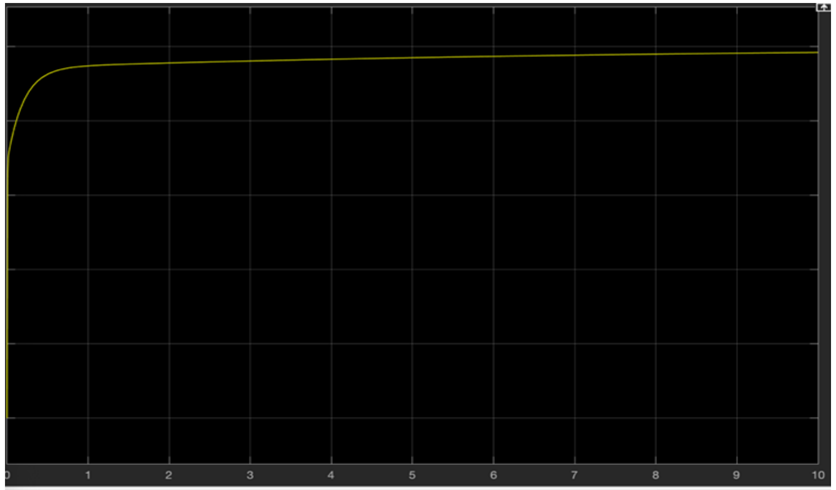
- Overshoot: 0.00% (Criteria of 10.00%)
- Settling Time: 15.70 seconds (Criteria of 300.00 seconds)
- Steady State Value: 1.00

System Impulse, Step Response, & Bode Plots (With Rapid Response PID Controller)



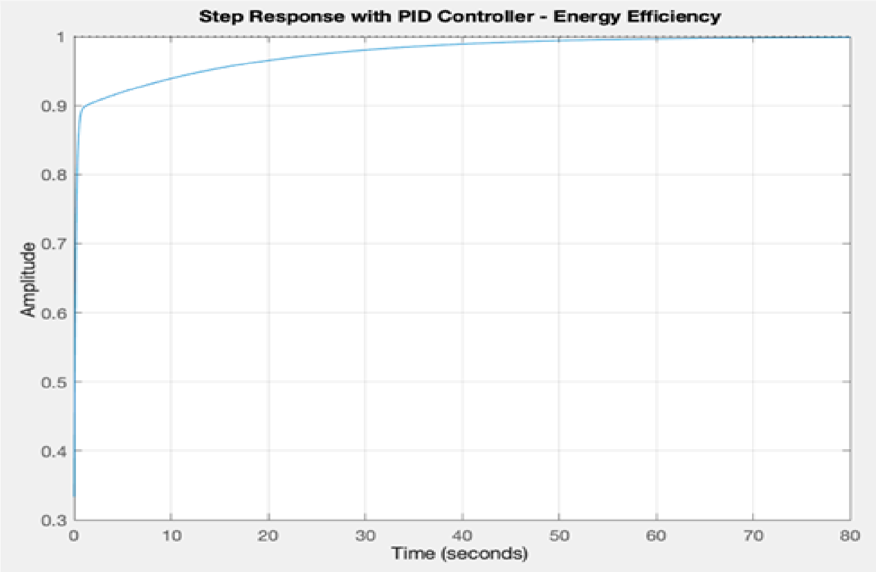


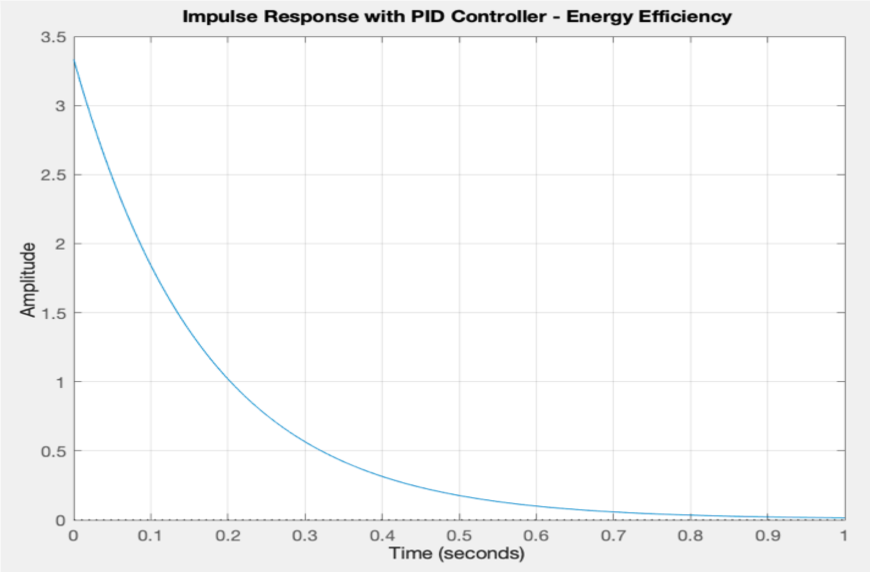


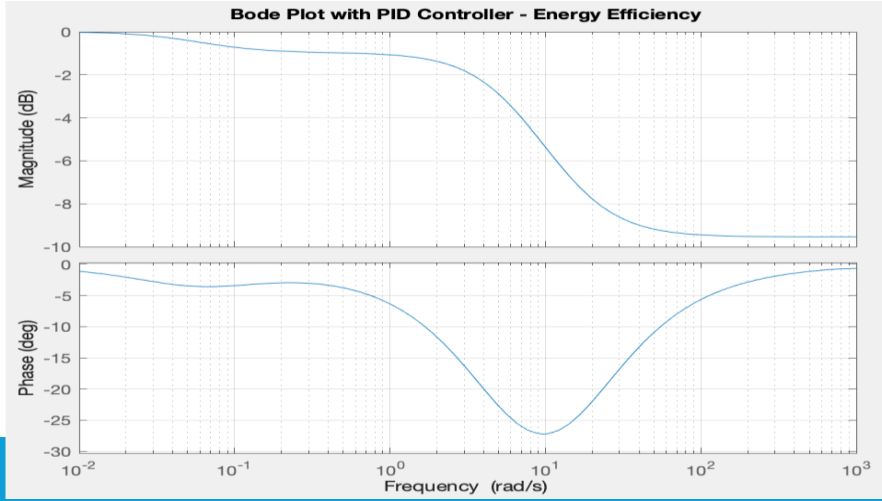


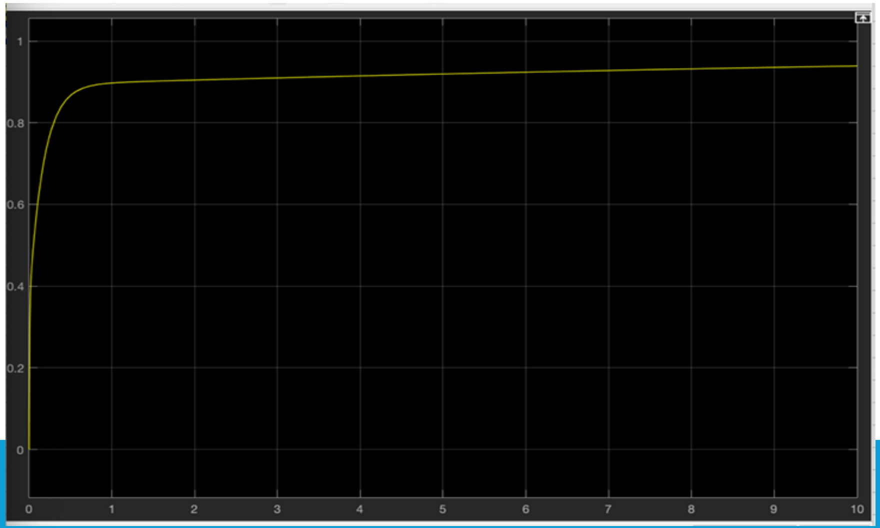
- Overshoot: 0.00% (Criteria of 20.00%)
- Settling Time: 8.21 seconds (Criteria of 120.00 seconds)
- Steady State Value: 1.00

System Impulse, Step Response, & Bode Plots (With Energy Efficiency PID Controller)









- Overshoot: 0.00% (Criteria of 5.00%)
- Settling Time: 29.89 seconds (Criteria of 600.00 seconds)
- Steady State Value: 1.00

Conclusion

This project has successfully demonstrated the use of mathematical modeling and control theory in designing and testing PID controllers for a smart thermostat system. By applying the Root Locus method, the system was able to achieve specific performance criteria across various scenarios, including standard comfort and stability, rapid response, and energy efficiency.

MATLAB and Simulink were used in modeling, simulating, and analyzing the system's behavior, allowing for effective adjustments to the controllers. This project has provided a deeper understanding of the practical applications of control theory and the challenges associated with optimizing system performance.

References

- *Thermal Model of a House*. (2024). MathWorks. photograph. Retrieved 2024, from https://www.mathworks.com/help/simulink/slref/thermal-model-of-a-house.html
- Golnaraghi, F., & Kuo, B. C. (2017). Chapter 4 Theoretical Foundation and Background Material: Modeling of Dynamic Systems. In *Automatic Control Systems* (9th ed., pp. 147–223). essay, John Wiley & Sons Inc.