

Step	Annotated Algorithm: $C := AB + C$ where A is symmetric and stored in lower triangle
1a	$\{C = \widehat{C}\}$
4	<b>Partition</b> $A \rightarrow \left( \begin{array}{c c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right), B \rightarrow \left( \begin{array}{c} B_T \\ B_B \end{array} \right), C \rightarrow \left( \begin{array}{c} C_T \\ C_B \end{array} \right)$ <b>where</b> $A_{TL}$ is $0 \times 0$ , $B_T$ has 0 rows, $C_T$ has 0 rows
2	$\left\{ \left( \begin{array}{c} A_{TL}B_T + C_T = \widehat{C}_T \\ \hline C_B = \widehat{C}_B \end{array} \right) \right\}$
3	<b>while</b> $m(A_{TL}) < m(A)$ <b>do</b>
2,3	$\left\{ \left( \left( \begin{array}{c} A_{TL}B_T + C_T = \widehat{C}_T \\ \hline C_B = \widehat{C}_B \end{array} \right) \right) \wedge (m(A_{TL}) < m(A)) \right\}$
5a	<b>Repartition</b> $\left( \begin{array}{c c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c c c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ b_1^T \\ B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ C_B \end{array} \right) \rightarrow \left( \begin{array}{c} C_0 \\ c_1^T \\ C_2 \end{array} \right)$ <b>where</b> $\alpha_{11}$ is $1 \times 1$ , $b_1$ has 1 row, $c_1$ has 1 row
6	$\left\{ \left( \begin{array}{c} A_{00}B_0 + C_0 = \widehat{C}_0 \\ \hline c_1^T = \widehat{c}_1^T \\ \hline C_2 = \widehat{C}_2 \end{array} \right) \right\}$
8	$c_1^T = \widehat{c}_1^T + a_{10}^T B_0 + \alpha_{11} b_1^T$ $C_0 = \widehat{C}_0 + a_{10} b_1^T$
5b	<b>Continue with</b> $\left( \begin{array}{c c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c c c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ b_1^T \\ B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ C_B \end{array} \right) \leftarrow \left( \begin{array}{c} C_0 \\ c_1^T \\ C_2 \end{array} \right)$
7	$\left\{ \left( \begin{array}{c} A_{00}B_0 + a_{10}b_1^T + C_0 = \widehat{C}_0 \\ \hline a_{10}^T B_0 + \alpha_{11} b_1^T + c_1^T = \widehat{c}_1^T \\ \hline C_2 = \widehat{C}_2 \end{array} \right) \right\}$
2	$\left\{ \left( \begin{array}{c} A_{TL}B_T + C_T = \widehat{C}_T \\ \hline C_B = \widehat{C}_B \end{array} \right) \right\}$
	<b>endwhile</b>
2,3	$\left\{ \left( \left( \begin{array}{c} A_{TL}B_T + C_T = \widehat{C}_T \\ \hline C_B = \widehat{C}_B \end{array} \right) \right) \wedge \neg (m(A_{TL}) < m(A)) \right\}$
1b	$\{C = AB + \widehat{C}\}$

**Algorithm:**  $C := AB + C$  where  $A$  is symmetric and stored in lower triangle

**Partition**  $A \rightarrow \left( \begin{array}{c|c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right)$ ,  $B \rightarrow \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right)$ ,  $C \rightarrow \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right)$

**where**  $A_{TL}$  is  $0 \times 0$ ,  $B_T$  has 0 rows,  $C_T$  has 0 rows

**while**  $m(A_{TL}) < m(A)$  **do**

**Repartition**

$$\left( \begin{array}{c|c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c|c|c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$$

**where**  $\alpha_{11}$  is  $1 \times 1$ ,  $b_1$  has 1 row,  $c_1$  has 1 row

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$$c_1^T = \hat{c}_1^T + a_{10}^T B_0 + \alpha_{11} b_1^T$$

$$C_0 = \hat{C}_0 + a_{10} b_1^T$$


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**Continue with**

$$\left( \begin{array}{c|c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c|c|c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$$

**endwhile**

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2	$\left\{ \left( \begin{array}{c} A_{TL}B_T + A_{BL}^T B_B + C_T = \widehat{C}_T \\ \hline C_B = \widehat{C}_B \end{array} \right) \right\}$
3	<b>while</b> $m(A_{TL}) < m(A)$ <b>do</b>
2,3	$\left\{ \left( \left( \begin{array}{c} A_{TL}B_T + A_{BL}^T B_B + C_T = \widehat{C}_T \\ \hline C_B = \widehat{C}_B \end{array} \right) \right) \wedge (m(A_{TL}) < m(A)) \right\}$
5a	<b>Repartition</b> $\left( \begin{array}{c c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c c c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ b_1^T \\ B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ C_B \end{array} \right) \rightarrow \left( \begin{array}{c} C_0 \\ c_1^T \\ C_2 \end{array} \right)$ <b>where</b> $\alpha_{11}$ is $1 \times 1$ , $b_1$ has 1 row, $c_1$ has 1 row
6	$\left\{ \left( \begin{array}{c} A_{00}B_0 + a_{10}b_1^T + C_0 = \widehat{C}_0 \\ \hline c_1^T = \widehat{c}_1^T \\ \hline C_2 = \widehat{C}_2 \end{array} \right) \right\}$
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5b	<b>Continue with</b> $\left( \begin{array}{c c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c c c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ b_1^T \\ B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ C_B \end{array} \right) \leftarrow \left( \begin{array}{c} C_0 \\ c_1^T \\ C_2 \end{array} \right)$
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2	$\left\{ \left( \begin{array}{c} A_{TL}B_T + A_{BL}^T B_B + C_T = \widehat{C}_T \\ \hline C_B = \widehat{C}_B \end{array} \right) \right\}$
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2,3	$\left\{ \left( \left( \begin{array}{c} A_{TL}B_T + A_{BL}^T B_B + C_T = \widehat{C}_T \\ \hline C_B = \widehat{C}_B \end{array} \right) \right) \wedge \neg(m(A_{TL}) < m(A)) \right\}$
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**Repartition**

$$\left( \begin{array}{c|c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c|c|c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$$

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**Continue with**

$$\left( \begin{array}{c|c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c|c|c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$$

**endwhile**

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2	$\left\{ \left( \begin{array}{c} A_{TL}B_T + C_T = \widehat{C}_T \\ \hline A_{BL}B_T + C_B = \widehat{C}_B \end{array} \right) \right\}$
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5a	<b>Repartition</b> $\left( \begin{array}{c c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c c c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$ <b>where</b> $\alpha_{11}$ is $1 \times 1$ , $b_1$ has 1 row, $c_1$ has 1 row
6	$\left\{ \left( \begin{array}{c} A_{00}B_0 + C_0 = \widehat{C}_0 \\ \hline a_{10}^TB_0 + c_1^T = \widehat{c}_1^T \\ \hline A_{20}B_0 + C_2 = \widehat{C}_2 \end{array} \right) \right\}$
8	$C_0 = \widehat{C}_0 + a_{10}b_1^T$ $c_1^T = \widehat{c}_1^T + \alpha_{11}b_1^T$ $C_2 = \widehat{C}_2 + a_{21}b_1^T$
5b	<b>Continue with</b> $\left( \begin{array}{c c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c c c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$
7	$\left\{ \left( \begin{array}{c} A_{00}B_0 + a_{10}b_1^T + C_0 = \widehat{C}_0 \\ \hline a_{10}^TB_0 + \alpha_{11}b_1^T + c_1^T = \widehat{c}_1^T \\ \hline A_{20}B_0 + a_{21}b_1^T + C_2 = \widehat{C}_2 \end{array} \right) \right\}$
2	$\left\{ \left( \begin{array}{c} A_{TL}B_T + C_T = \widehat{C}_T \\ \hline A_{BL}B_T + C_B = \widehat{C}_B \end{array} \right) \right\}$
	<b>endwhile</b>
2,3	$\left\{ \left( \left( \begin{array}{c} A_{TL}B_T + C_T = \widehat{C}_T \\ \hline A_{BL}B_T + C_B = \widehat{C}_B \end{array} \right) \right) \wedge \neg (m(A_{TL}) < m(A)) \right\}$
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$\left( \begin{array}{c|c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c|c|c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right)$ ,  $\left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right)$ ,  $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$

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$$C_0 = \hat{C}_0 + a_{10}b_1^T$$

$$c_1^T = \hat{c}_1^T + \alpha_{11}b_1^T$$

$$C_2 = \hat{C}_2 + a_{21}b_1^T$$


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**Continue with**

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8	$c_1^T = \widehat{c}_1^T + \alpha_{11}b_1^T + a_{21}^T B_2$ $C_2 = \widehat{C}_2 + a_{21}b_1^T$
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2	$\left\{ \left( \begin{array}{c} A_{TL}B_T + A_{BL}^T B_B + C_T = \widehat{C}_T \\ \hline A_{BL}B_T + C_B = \widehat{C}_B \end{array} \right) \right\}$
	<b>endwhile</b>
2,3	$\left\{ \left( \left( \begin{array}{c} A_{TL}B_T + A_{BL}^T B_B + C_T = \widehat{C}_T \\ \hline A_{BL}B_T + C_B = \widehat{C}_B \end{array} \right) \right) \wedge \neg(m(A_{TL}) < m(A)) \right\}$
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**while**  $m(A_{TL}) < m(A)$  **do**

**Repartition**

$\left( \begin{array}{c|c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c|c|c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right)$ ,  $\left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right)$ ,  $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$

**where**  $\alpha_{11}$  is  $1 \times 1$ ,  $b_1$  has 1 row,  $c_1$  has 1 row

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$$c_1^T = \hat{c}_1^T + \alpha_{11} b_1^T + a_{21}^T B_2$$

$$C_2 = \hat{C}_2 + a_{21} b_1^T$$


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**Continue with**

$\left( \begin{array}{c|c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c|c|c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right)$ ,  $\left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right)$ ,  $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$

**endwhile**



Step	Annotated Algorithm: $C := A B + C$ where $A$ is symmetric stored in lower triange
1a	$\{C = \widehat{C}\}$
4	<b>Partition</b> $A \rightarrow \left( \begin{array}{c c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right)$ , $B \rightarrow \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right)$ , $C \rightarrow \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right)$ <b>where</b> $A_{TR}$ is $0 \times 0$ , $B_B$ has 0 rows, $C_B$ has 0 rows
2	$\left\{ \left( \begin{array}{c} C_T \\ \hline A_{BR}B_B + C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right) \right\}$
3	<b>while</b> $m(A_{TR}) < m(A)$ <b>do</b>
2,3	$\left\{ \left( \left( \begin{array}{c} C_T \\ \hline A_{BR}B_B + C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right) \right) \wedge (m(A_{TR}) < m(A)) \right\}$
5a	<b>Repartition</b> $\left( \begin{array}{c c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c c c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right)$ , $\left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right)$ , $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$ <b>where</b> $\alpha_{11}$ is $1 \times 1$ , $b_1$ has 1 row, $c_1$ has 1 row
6	$\left\{ \left( \begin{array}{c} C_0 = \widehat{C}_0 \\ \hline c_1^T = \widehat{c}_1^T \\ \hline A_{22}B_2 + C_2 = \widehat{C}_2 \end{array} \right) \right\}$
8	$C_2 = \widehat{C}_2 + a_{21}b_1^T$ $c_1^T = \widehat{c}_1^T + \alpha_{11}b_1^T + a_{21}^T B_2$
5b	<b>Continue with</b> $\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c c c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right)$ , $\left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right)$ , $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$
7	$\left\{ \left( \begin{array}{c} C_0 = \widehat{C}_0 \\ \hline \alpha_{11}b_1^T + a_{21}^T B_2 + c_1^T = \widehat{c}_1^T \\ \hline a_{21}b_1^T + A_{22}B_2 + C_2 = \widehat{C}_2 \end{array} \right) \right\}$
2	$\left\{ \left( \begin{array}{c} C_T \\ \hline A_{BR}B_B + C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right) \right\}$
	<b>endwhile</b>
2,3	$\left\{ \left( \left( \begin{array}{c} C_T \\ \hline A_{BR}B_B + C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right) \right) \wedge \neg(m(A_{TR}) < m(A)) \right\}$
1b	$\{C := AB + \widehat{C}\}$

**Algorithm:**  $C := A B + C$  where  $A$  is symmetric stored in lower triange

**Partition**  $A \rightarrow \left( \begin{array}{c|c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right)$ ,  $B \rightarrow \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right)$ ,  $C \rightarrow \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right)$

**where**  $A_{TR}$  is  $0 \times 0$ ,  $B_B$  has 0 rows,  $C_B$  has 0 rows

**while**  $m(A_{TR}) < m(A)$  **do**

**Repartition**

$$\left( \begin{array}{c|c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c|c|c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$$

**where**  $\alpha_{11}$  is  $1 \times 1$ ,  $b_1$  has 1 row,  $c_1$  has 1 row

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$$C_2 = \hat{C}_2 + a_{21} b_1^T$$

$$c_1^T = \hat{c}_1^T + \alpha_{11} b_1^T + a_{21}^T B_2$$


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**Continue with**

$$\left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c|c|c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$$

**endwhile**

Step	Annotated Algorithm: $C := A B + C$ where $A$ is symmetric stored in lower triange
1a	$\{C = \widehat{C}\}$
4	<b>Partition</b> $A \rightarrow \left( \begin{array}{c c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right)$ , $B \rightarrow \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right)$ , $C \rightarrow \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right)$ <b>where</b> $A_{TR}$ is $0 \times 0$ , $B_B$ has 0 rows, $C_B$ has 0 rows
2	$\left\{ \left( \begin{array}{c} C_T \\ \hline A_{BL}B_T + A_{BR}B_B + C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right) \right\}$
3	<b>while</b> $m(A_{TR}) < m(A)$ <b>do</b>
2,3	$\left\{ \left( \left( \begin{array}{c} C_T \\ \hline A_{BL}B_T + A_{BR}B_B + C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right) \right) \wedge (m(A_{TR}) < m(A)) \right\}$
5a	<b>Repartition</b> $\left( \begin{array}{c c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c c c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right)$ , $\left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right)$ , $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$ <b>where</b> $\alpha_{11}$ is $1 \times 1$ , $b_1$ has 1 row, $c_1$ has 1 row
6	$\left\{ \left( \begin{array}{c} C_0 = \widehat{C}_0 \\ \hline c_1^T = \widehat{c}_1^T \\ \hline A_{20}B_0 + a_{21}b_1^T + A_{22}B_2 + C_2 = \widehat{C}_2 \end{array} \right) \right\}$
8	$c_1^T = \widehat{c}_1^T + a_{10}^T B_0 + \alpha_{11} b_1^T + a_{21}^T B_2$
5b	<b>Continue with</b> $\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c c c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right)$ , $\left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right)$ , $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$
7	$\left\{ \left( \begin{array}{c} C_0 = \widehat{C}_0 \\ \hline a_{10}^T B_0 + \alpha_{11} b_1^T + a_{21}^T B_2 + c_1^T = \widehat{c}_1^T \\ \hline A_{20}B_0 + a_{21}b_1^T + A_{22}B_2 + C_2 = \widehat{C}_2 \end{array} \right) \right\}$
2	$\left\{ \left( \begin{array}{c} C_T \\ \hline A_{BL}B_T + A_{BR}B_B + C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right) \right\}$
	<b>endwhile</b>
2,3	$\left\{ \left( \left( \begin{array}{c} C_T \\ \hline A_{BL}B_T + A_{BR}B_B + C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right) \right) \wedge \neg (m(A_{TR}) < m(A)) \right\}$
1b	$\{C := AB + \widehat{C}\}$

**Algorithm:**  $C := A B + C$  where  $A$  is symmetric stored in lower triange

**Partition**  $A \rightarrow \left( \begin{array}{c|c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right), B \rightarrow \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right), C \rightarrow \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right)$

**where**  $A_{TR}$  is  $0 \times 0$ ,  $B_B$  has 0 rows,  $C_B$  has 0 rows

**while**  $m(A_{TR}) < m(A)$  **do**

**Repartition**

$$\left( \begin{array}{c|c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c|c|c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$$

**where**  $\alpha_{11}$  is  $1 \times 1$ ,  $b_1$  has 1 row,  $c_1$  has 1 row

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$$c_1^T = \tilde{c}_1^T + a_{10}^T B_0 + \alpha_{11} b_1^T + a_{21}^T B_2$$


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**Continue with**

$$\left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c|c|c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$$

**endwhile**

Step	Annotated Algorithm: $C := A B + C$ where $A$ is symmetric stored in lower triange
1a	$\{C = \widehat{C}\}$
4	<b>Partition</b> $A \rightarrow \left( \begin{array}{c c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right)$ , $B \rightarrow \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right)$ , $C \rightarrow \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right)$ <b>where</b> $A_{TR}$ is $0 \times 0$ , $B_B$ has 0 rows, $C_B$ has 0 rows
2	$\left\{ \left( \begin{array}{c} A_{BL}^T B_B + C_T \\ \hline A_{BR} B_B + C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right) \right\}$
3	<b>while</b> $m(A_{TR}) < m(A)$ <b>do</b>
2,3	$\left\{ \left( \left( \begin{array}{c} A_{BL}^T B_B + C_T \\ \hline A_{BR} B_B + C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right) \right) \wedge (m(A_{TR}) < m(A)) \right\}$
5a	<b>Repartition</b> $\left( \begin{array}{c c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c c c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right)$ , $\left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right)$ , $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$ <b>where</b> $\alpha_{11}$ is $1 \times 1$ , $b_1$ has 1 row, $c_1$ has 1 row
6	$\left\{ \left( \begin{array}{c} A_{20}^T B_2 + C_0 = \widehat{C}_0 \\ \hline a_{21}^T B_2 + c_1^T = \widehat{c}_1^T \\ \hline A_{22} B_2 + C_2 = \widehat{C}_2 \end{array} \right) \right\}$
8	$C_0 = \widehat{C}_0 + a_{10}^T B_0$ $c_1^T = \widehat{c}_1^T + \alpha_{11} b_1^T$ $C_2 = \widehat{C}_2 + a_{21}^T B_2$
5b	<b>Continue with</b> $\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c c c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right)$ , $\left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right)$ , $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$
7	$\left\{ \left( \begin{array}{c} a_{10} b_1^T + A_{20}^T B_2 + C_0 = \widehat{C}_0 \\ \hline \alpha_{11} b_1^T + a_{21}^T B_2 + c_1^T = \widehat{c}_1^T \\ \hline a_{21} b_1^T + A_{22} B_2 + C_2 = \widehat{C}_2 \end{array} \right) \right\}$
2	$\left\{ \left( \begin{array}{c} A_{BL}^T B_B + C_T \\ \hline A_{BR} B_B + C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right) \right\}$
	<b>endwhile</b>
2,3	$\left\{ \left( \left( \begin{array}{c} A_{BL}^T B_B + C_T \\ \hline A_{BR} B_B + C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right) \right) \wedge \neg (m(A_{TR}) < m(A)) \right\}$
1b	$\{C := AB + \widehat{C}\}$

**Algorithm:**  $C := A B + C$  where  $A$  is symmetric stored in lower triange

**Partition**  $A \rightarrow \left( \begin{array}{c|c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right)$ ,  $B \rightarrow \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right)$ ,  $C \rightarrow \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right)$

**where**  $A_{TR}$  is  $0 \times 0$ ,  $B_B$  has 0 rows,  $C_B$  has 0 rows

**while**  $m(A_{TR}) < m(A)$  **do**

**Repartition**

$$\left( \begin{array}{c|c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c|c|c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$$

**where**  $\alpha_{11}$  is  $1 \times 1$ ,  $b_1$  has 1 row,  $c_1$  has 1 row

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$$C_0 = \hat{C}_0 + a_{10}^T B_0$$

$$c_1^T = \hat{c}_1^T + \alpha_{11} b_1^T$$

$$C_2 = \hat{C}_2 + a_{21}^T B_2$$


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**Continue with**

$$\left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c|c|c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$$

**endwhile**

Step	Annotated Algorithm: $C := A B + C$ where $A$ is symmetric stored in lower triangle
1a	$\{C = \widehat{C}\}$
4	<b>Partition</b> $A \rightarrow \left( \begin{array}{c c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right), B \rightarrow \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right), C \rightarrow \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right)$ <b>where</b> $A_{TR}$ is $0 \times 0$ , $B_B$ has 0 rows, $C_B$ has 0 rows
2	$\left\{ \left( \frac{A_{BL}^T B_B + C_T}{A_{BL} B_T + A_{BR} B_B + C_B} \right) = \left( \frac{\widehat{C}_T}{\widehat{C}_B} \right) \right\}$
3	<b>while</b> $m(A_{TR}) < m(A)$ <b>do</b>
2,3	$\left\{ \left( \left( \frac{A_{BL}^T B_B + C_T}{A_{BL} B_T + A_{BR} B_B + C_B} \right) = \left( \frac{\widehat{C}_T}{\widehat{C}_B} \right) \right) \wedge (m(A_{TR}) < m(A)) \right\}$
5a	<b>Repartition</b> $\left( \begin{array}{c c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c c c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$ <b>where</b> $\alpha_{11}$ is $1 \times 1$ , $b_1$ has 1 row, $c_1$ has 1 row
6	$\left\{ \left( \begin{array}{c} A_{20}^T B_2 + C_0 = \widehat{C}_0 \\ \hline a_{21}^T B_2 + c_1^T = \widehat{c}_1^T \\ \hline A_{20} B_0 + a_{21} b_1^T + A_{22} B_2 + C_2 = \widehat{C}_2 \end{array} \right) \right\}$
8	$C_0 = \widehat{C}_0 + a_{10} b_1^T$ $c_1^T = \widehat{c}_1^T + a_{10}^T B_0 + \alpha_{11} b_1^T$
5b	<b>Continue with</b> $\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c c c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$
7	$\left\{ \left( \begin{array}{c} a_{10} b_1^T + A_{20}^T B_2 + C_0 = \widehat{C}_0 \\ \hline a_{10} B_0 + \alpha_{11} b_1^T + a_{21}^T B_2 + c_1^T = \widehat{c}_1^T \\ \hline A_{20} B_0 + a_{21} b_1^T + A_{22} B_2 + C_2 = \widehat{C}_2 \end{array} \right) \right\}$
2	$\left\{ \left( \frac{A_{BL}^T B_B + C_T}{A_{BL} B_T + A_{BR} B_B + C_B} \right) = \left( \frac{\widehat{C}_T}{\widehat{C}_B} \right) \right\}$
	<b>endwhile</b>
2,3	$\left\{ \left( \left( \frac{A_{BL}^T B_B + C_T}{A_{BL} B_T + A_{BR} B_B + C_B} \right) = \left( \frac{\widehat{C}_T}{\widehat{C}_B} \right) \right) \wedge \neg (m(A_{TR}) < m(A)) \right\}$
1b	$\{C := AB + \widehat{C}\}$

**Algorithm:**  $C := A B + C$  where  $A$  is symmetric stored in lower triange

**Partition**  $A \rightarrow \left( \begin{array}{c|c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right)$ ,  $B \rightarrow \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right)$ ,  $C \rightarrow \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right)$

**where**  $A_{TR}$  is  $0 \times 0$ ,  $B_B$  has 0 rows,  $C_B$  has 0 rows

**while**  $m(A_{TR}) < m(A)$  **do**

**Repartition**

$$\left( \begin{array}{c|c} A_{TL} & * \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c|c|c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$$

**where**  $\alpha_{11}$  is  $1 \times 1$ ,  $b_1$  has 1 row,  $c_1$  has 1 row

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$$C_0 = \hat{C}_0 + a_{10} b_1^T$$

$$c_1^T = \hat{c}_1^T + a_{10}^T B_0 + \alpha_{11} b_1^T$$


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**Continue with**

$$\left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c|c|c} A_{00} & * & * \\ \hline a_{10}^T & \alpha_{11} & * \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$$

**endwhile**