Step	<b>Annotated Algorithm:</b> $C := AB + C$ where A is symmetric and stored in the lower triangle
1a	$\left\{C = \widehat{C}\right\}$
4	Partition $A  o \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$ , $B  o \left( \begin{array}{c c} B_T \\ \hline B_B \end{array} \right)$ , $C  o \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right)$
	where $A_{TL}$ is $0 \times 0$ , $B_T$ has 0 rows, $C_T$ has 0 rows
2	$\left\{ \left( \frac{A_{TL}B_T + C_T = \widehat{C}_T}{C_B = \widehat{C}_B} \right) \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \left( \left( \frac{A_{TL}B_T + C_T = \widehat{C}_T}{C_B = \widehat{C}_B} \right) \right) \land (m(A_{TL}) < m(A)) \right\}$
5a	Repartition
	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c}B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c}B_0 \\ \hline B_1 \\ \hline B_2 \end{array}\right), \left(\begin{array}{c}C_T \\ \hline C_B \end{array}\right) \rightarrow \left(\begin{array}{c}C_0 \\ \hline C_1 \\ \hline C_2 \end{array}\right) $ where $A_{11}$ is $b \times b$ , $B_1$ has $b$ rows, $C_1$ has $b$ rows
6	$ \left\{ \left( \frac{A_{00}B_0 + C_0 = \hat{C}_0}{C_1 = \hat{C}_1} \right) \right\} $ $ C_2 = \hat{C}_2 $
8	$C_1 = \hat{C}_1 + A_{10}B_0 + A_{11}B_1$ $C_0 = \hat{C}_0 + A_{10}B_1$
5b	Continue with
	$\left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array}\right)$
7	$ \left\{ \left( \frac{A_{00}B_0 + A_{10}B_1 + C_0 = \widehat{C}_0}{A_{10} + B_0 + A_{11}B_1 + C_1 = \widehat{C}_1} \right) \right\} $ $ C_2 = \widehat{C}_2 $
2	$\left\{ \left( \frac{A_{TL}B_T + C_T = \widehat{C}_T}{C_B = \widehat{C}_B} \right) \right\}$
	endwhile
2,3	$\left\{ \left( \left( \frac{A_{TL}B_T + C_T = \widehat{C}_T}{C_B = \widehat{C}_B} \right) \right) \land \neg \left( m(A_{TL}) < m(A) \right) \right\}$
1b	$\left\{ C = AB + \hat{C} \right\}$

Partition 
$$A o \left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$$
,  $B o \left( \begin{array}{c|c} B_T \\ \hline B_B \end{array} \right)$ ,  $C o \left( \begin{array}{c|c} C_T \\ \hline C_B \end{array} \right)$ 

where  $A_{TL}$  is  $0 \times 0$ ,  $B_T$  has 0 rows,  $C_T$  has 0 rows while  $m(A_{TL}) < m(A)$  do

# Repartition

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \to \left(\begin{array}{c|c}
A_{00} & A_{01} & A_{02} \\
\hline
A_{10} & A_{11} & A_{12} \\
\hline
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c}
B_T \\
\hline
B_B
\end{array}\right) \to \left(\begin{array}{c}
B_0 \\
\hline
B_1 \\
\hline
B_2
\end{array}\right), \left(\begin{array}{c}
C_T \\
\hline
C_B
\end{array}\right) \to \left(\begin{array}{c}
C_0 \\
\hline
C_1 \\
\hline
C_2
\end{array}\right)$$

where  $A_{11}$  is  $b \times b$ ,  $B_1$  has b rows,  $C_1$  has b rows

$$C_1 = \hat{C}_1 + A_{10}B_0 + A_{11}B_1$$
$$C_0 = \hat{C}_0 + A_{10}B_1$$

## Continue with

$$\left(\begin{array}{c|c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|c|c}
A_{00} & A_{01} & A_{02} \\
\hline
A_{10} & A_{11} & A_{12} \\
\hline
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c|c}
B_T \\
\hline
B_B
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
B_0 \\
\hline
B_1 \\
\hline
B_2
\end{array}\right), \left(\begin{array}{c}
C_T \\
\hline
C_B
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
C_0 \\
\hline
C_1 \\
\hline
C_2
\end{array}\right)$$

Step	<b>Annotated Algorithm:</b> $C := AB + C$ where A is symmetric and stored in the lower triangle
1a	$\left\{C=\widehat{C}\right\}$
4	Partition $A  o \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$ , $B  o \left(\begin{array}{c c} B_T \\ \hline B_B \end{array}\right)$ , $C  o \left(\begin{array}{c c} C_T \\ \hline C_B \end{array}\right)$
	where $A_{TL}$ is $0 \times 0$ , $B_T$ has 0 rows, $C_T$ has 0 rows
2	$\left\{ \left( \frac{A_{TL}B_T + A_{BL}^T B_B + C_T = \hat{C}_T}{C_B = \hat{C}_B} \right) \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \left( \left( \frac{A_{TL}B_T + A_{BL}^T B_B + C_T = \widehat{C}_T}{C_B = \widehat{C}_B} \right) \right) \wedge \left( m(A_{TL}) < m(A) \right) \right\}$
5a	Repartition
	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ C_2 \end{array}\right) $ where $A_{11}$ is $b \times b$ , $B_1$ has $b$ rows, $C_1$ has $b$ rows
6	$ \left\{ \left( \frac{A_{00}B_0 + A_{10}B_1 + C_0 = \widehat{C}_0}{C_1 = \widehat{C}_1} \atop C_2 = \widehat{C}_2 \right) \right\} $
8	$C_1 = \hat{C}_1 + A_{10}B_0 + A_{11}B_1$ $C_0 = \hat{C}_0 + A_{20}B_2$
5b	Continue with
	$\left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array}\right)$
7	$ \left\{ \left( \frac{A_{00}B_0 + A_{10}B_1 + C_0 = \widehat{C}_0}{A_{10} + B_0 + A_{11}B_1 + C_1 = \widehat{C}_1} \right) \right\} $ $ C_2 = \widehat{C}_2 $
2	$ \left\{ \left( \frac{A_{TL}B_T + A_{BL}^T B_B + C_T = \widehat{C}_T}{C_B = \widehat{C}_B} \right) \right\} $
	endwhile
2,3	$\left\{ \left( \left( \frac{A_{TL}B_T + A_{BL}^T B_B + C_T = \widehat{C}_T}{C_B = \widehat{C}_B} \right) \right) \land \neg \left( m(A_{TL}) < m(A) \right) \right\}$
1b	$\left\{ C = AB + \hat{C} \right\}$

Partition 
$$A o \left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$$
,  $B o \left( \begin{array}{c|c} B_T \\ \hline B_B \end{array} \right)$ ,  $C o \left( \begin{array}{c|c} C_T \\ \hline C_B \end{array} \right)$ 

where  $A_{TL}$  is  $0 \times 0$ ,  $B_T$  has 0 rows,  $C_T$  has 0 rows while  $m(A_{TL}) < m(A)$  do

# Repartition

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \to \left(\begin{array}{c|c}
A_{00} & A_{01} & A_{02} \\
\hline
A_{10} & A_{11} & A_{12} \\
\hline
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c}
B_T \\
\hline
B_B
\end{array}\right) \to \left(\begin{array}{c}
B_0 \\
\hline
B_1 \\
\hline
B_2
\end{array}\right), \left(\begin{array}{c}
C_T \\
\hline
C_B
\end{array}\right) \to \left(\begin{array}{c}
C_0 \\
\hline
C_1 \\
\hline
C_2
\end{array}\right)$$

where  $A_{11}$  is  $b \times b$ ,  $B_1$  has b rows,  $C_1$  has b rows

$$C_1 = \hat{C}_1 + A_{10}B_0 + A_{11}B_1$$
$$C_0 = \hat{C}_0 + A_{20}B_2$$

## Continue with

$$\left(\begin{array}{c|c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|c|c}
A_{00} & A_{01} & A_{02} \\
\hline
A_{10} & A_{11} & A_{12} \\
\hline
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c|c}
B_T \\
\hline
B_B
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
B_0 \\
\hline
B_1 \\
\hline
B_2
\end{array}\right), \left(\begin{array}{c}
C_T \\
\hline
C_B
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
C_0 \\
\hline
C_1 \\
\hline
C_2
\end{array}\right)$$

Step	Annotated Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangle
1a	$\left\{C=\widehat{C}\right\}$
4	Partition $A  o \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$ , $B  o \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right)$ , $C  o \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right)$
	where $A_{TL}$ is $0 \times 0$ , $B_T$ has 0 rows, $C_T$ has 0 rows
2	$\left\{ \left( \frac{A_{TL}B_T + C_T = \widehat{C}_T}{A_{BL}B_T + C_B = \widehat{C}_B} \right) \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \left( \left( \frac{A_{TL}B_T + C_T = \widehat{C}_T}{A_{BL}B_T + C_B = \widehat{C}_B} \right) \right) \land (m(A_{TL}) < m(A)) \right\}$
5a	Repartition
	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ C_2 \end{array}\right) $ where $A_{11}$ is $b \times b$ , $B_1$ has $b$ rows, $C_1$ has $b$ rows
6	$ \left\{ \left( \frac{A_{00}B_0 + A_{10}B_1 + C_0 = \hat{C}_0}{C_1 = \hat{C}_1} \right) \right\} $ $ C_2 = \hat{C}_2 $
	$C_0 = \widehat{C}_0 + A10^T B_1$
8	$C_1 = \widehat{C}_1 + A_{11}B_1$
	$C_2 = \hat{C}_2 + A_{21}B_1$
5b	Continue with
	$\left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array}\right)$
7	$ \left\{ \left( \frac{A_{00}B_0 + A_{10}B_1 + C_0 = \widehat{C}_0}{A_{10} + B_0 + A_{11}B_1 + C_1 = \widehat{C}_1} \right) \right\} $ $ C_2 = \widehat{C}_2 $
2	$\left\{ \left( \frac{A_{TL}B_T + C_T = \widehat{C}_T}{A_{BL}B_T + C_B = \widehat{C}_B} \right) \right\}$
	endwhile
2,3	$\left\{ \left( \left( \frac{A_{TL}B_T + C_T = \widehat{C}_T}{A_{BL}B_T + C_B = \widehat{C}_B} \right) \right) \land \neg (m(A_{TL}) < m(A)) \right\}$
1b	$\left\{ C = AB + \widehat{C} \right\}$

Partition 
$$A o \left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$$
,  $B o \left( \begin{array}{c|c} B_T \\ \hline B_B \end{array} \right)$ ,  $C o \left( \begin{array}{c|c} C_T \\ \hline C_B \end{array} \right)$ 

where  $A_{TL}$  is  $0 \times 0$ ,  $B_T$  has 0 rows,  $C_T$  has 0 rows while  $m(A_{TL}) < m(A)$  do

# Repartition

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \to \left(\begin{array}{c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \\ \left(\begin{array}{c|c} B_T \\ \hline B_B \end{array}\right) \to \left(\begin{array}{c|c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array}\right), \\ \left(\begin{array}{c|c} C_T \\ \hline C_B \end{array}\right) \to \left(\begin{array}{c|c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array}\right)$$

where  $A_{11}$  is  $b \times b$ ,  $B_1$  has b rows,  $C_1$  has b rows

$$C_0 = \hat{C}_0 + A 10^T B_1$$

$$C_1 = \hat{C}_1 + A_{11} B_1$$

$$C_2 = \hat{C}_2 + A_{21} B_1$$

## Continue with

$$\left(\begin{array}{c|c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c|c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array}\right)$$

Step	<b>Annotated Algorithm:</b> $C := AB + C$ where A is symmetric and stored in the lower triangle
1a	$\left\{ C = \widehat{C} \right\}$
4	$\textbf{Partition} \ \ A \rightarrow \left( \frac{A_{TL}}{A_{BL}} \left  \frac{A_{TR}}{A_{BR}} \right  \right), \ B \rightarrow \left( \frac{B_T}{B_B} \right), \ C \rightarrow \left( \frac{C_T}{C_B} \right)$
	where $A_{TL}$ is $0 \times 0$ , $B_T$ has 0 rows, $C_T$ has 0 rows
2	$\left\{ \left( \frac{A_{TL}B_T + A_{BL}^T B_B + C_T = \hat{C}_T}{A_{BL}B_T + C_B = \hat{C}_B} \right) \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \left( \left( \frac{A_{TL}B_T + A_{BL}^T B_B + C_T = \widehat{C}_T}{A_{BL}B_T + C_B = \widehat{C}_B} \right) \right) \wedge (m(A_{TL}) < m(A)) \right\}$
5a	Repartition
	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array}\right) $ where $A_{11}$ is $b \times b$ , $B_1$ has $b$ rows, $C_1$ has $b$ rows
6	$ \left\{ \left( \frac{A_{00}B_0 + A_{10}B_1 + C_0 = \widehat{C}_0}{C_1 = \widehat{C}_1} \atop C_2 = \widehat{C}_2 \right) \right\} $
8	$C_1 = \hat{C}_1 + A11B_1 + A_{21}B_2$ $C_2 = \hat{C}_2 + A_{21}B_1$
5b	Continue with
	$\left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array}\right)$
7	$ \left\{ \left( \frac{A_{00}B_0 + A_{10}B_1 + C_0 = \widehat{C}_0}{A_{10} + B_0 + A_{11}B_1 + C_1 = \widehat{C}_1} \right) \right\} $ $ C_2 = \widehat{C}_2 $
2	$ \left\{ \left( \frac{A_{TL}B_T + A_{BL}^T B_B + C_T = \widehat{C}_T}{A_{BL}B_T + C_B = \widehat{C}_B} \right) \right\} $
	endwhile
2,3	$\left\{ \left( \left( \frac{A_{TL}B_T + A_{BL}^T B_B + C_T = \widehat{C}_T}{A_{BL}B_T + C_B = \widehat{C}_B} \right) \right) \land \neg (m(A_{TL}) < m(A)) \right\}$
1b	$\left\{ C = AB + \widehat{C} \right\}$

Partition 
$$A o \left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$$
,  $B o \left( \begin{array}{c|c} B_T \\ \hline B_B \end{array} \right)$ ,  $C o \left( \begin{array}{c|c} C_T \\ \hline C_B \end{array} \right)$ 

where  $A_{TL}$  is  $0 \times 0$ ,  $B_T$  has 0 rows,  $C_T$  has 0 rows while  $m(A_{TL}) < m(A)$  do

Repartition

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \to \left(\begin{array}{c|c}
A_{00} & A_{01} & A_{02} \\
\hline
A_{10} & A_{11} & A_{12} \\
\hline
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c}
B_T \\
\hline
B_B
\end{array}\right) \to \left(\begin{array}{c}
B_0 \\
\hline
B_1 \\
\hline
B_2
\end{array}\right), \left(\begin{array}{c}
C_T \\
\hline
C_B
\end{array}\right) \to \left(\begin{array}{c}
C_0 \\
\hline
C_1 \\
\hline
C_2
\end{array}\right)$$

where  $A_{11}$  is  $b \times b$ ,  $B_1$  has b rows,  $C_1$  has b rows

$$C_1 = \hat{C}_1 + A11B_1 + A_{21}B_2$$
$$C_2 = \hat{C}_2 + A_{21}B_1$$

Continue with

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
A_{00} & A_{01} & A_{02} \\
\hline
A_{10} & A_{11} & A_{12} \\
\hline
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c}
B_T \\
\hline
B_B
\end{array}\right) \leftarrow \left(\begin{array}{c}
B_0 \\
\hline
B_1 \\
\hline
B_2
\end{array}\right), \left(\begin{array}{c}
C_T \\
C_B
\end{array}\right) \leftarrow \left(\begin{array}{c}
C_0 \\
\hline
C_1 \\
\hline
C_2
\end{array}\right)$$

Step	<b>Annotated Algorithm:</b> $C := AB + C$ where A is symmetric and stored in the lower triangle
1a	$\left\{C = \widehat{C}\right\}$
4	Partition $A  o \left( \frac{A_{TL}}{A_{BL}} \begin{vmatrix} A_{TR} \\ A_{BR} \end{vmatrix} \right), B  o \left( \frac{B_T}{B_B} \right), C  o \left( \frac{C_T}{C_B} \right)$
	where $A_{TL}$ is $0 \times 0$ , $B_T$ has 0 rows, $C_T$ has 0 rows
2	$\left\{ \left( \frac{C_T = \widehat{C}_T}{A_{BR}B_B + C_B = \widehat{C}_B} \right) \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \left( \left( \frac{C_T = \widehat{C}_T}{A_{BR}B_B + C_B = \widehat{C}_B} \right) \right) \land (m(A_{TL}) < m(A)) \right\}$
5a	Repartition
	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array}\right) $ where $A_{11}$ is $b \times b$ , $B_1$ has $b$ rows, $C_1$ has $b$ rows
6	$ \left\{ \left( \frac{C_0 = \hat{C}_0}{C_1 = \hat{C}_1} \right) \right\} $ $ \left\{ \left( \frac{A_{22}B_2 + C_2 = \hat{C}_2}{A_{22}B_2 + C_2 = \hat{C}_2} \right) \right\} $
8	$C_2 = \hat{C}_2 + A21B_1$ $C_1 = \hat{C}_1 + A_{11}B_1 + A_{21}B_2$
5b	Continue with
	$ \left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c}B_T \\ B_B \end{array}\right) \leftarrow \left(\begin{array}{c c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array}\right), \left(\begin{array}{c}C_T \\ C_B \end{array}\right) \leftarrow \left(\begin{array}{c}C_0 \\ \hline C_1 \\ \hline C_2 \end{array}\right) $
7	$ \left\{ \left( \frac{C_0 = \widehat{C}_0}{\frac{A_{11} + B_1 + A_{21}B_2 + C_1 = \widehat{C}_1}{A_{21}B_1 + A_{22}B_2 + C_2 = \widehat{C}_2}} \right) \right\} $
2	$\left\{ \left( \frac{C_T = \widehat{C}_T}{A_{BR}B_B + C_B = \widehat{C}_B} \right) \right\}$
	endwhile
2,3	$\left\{ \left( \left( \frac{C_T = \widehat{C}_T}{A_{BR}B_B + C_B = \widehat{C}_B} \right) \right) \land \neg \left( m(A_{TL}) < m(A) \right) \right\}$
1b	$\left\{ C = AB + \hat{C} \right\}$

Partition 
$$A o \left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$$
,  $B o \left( \begin{array}{c|c} B_T \\ \hline B_B \end{array} \right)$ ,  $C o \left( \begin{array}{c|c} C_T \\ \hline C_B \end{array} \right)$ 

where  $A_{TL}$  is  $0 \times 0$ ,  $B_T$  has 0 rows,  $C_T$  has 0 rows while  $m(A_{TL}) < m(A)$  do

Repartition

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \to \left(\begin{array}{c|c}
A_{00} & A_{01} & A_{02} \\
\hline
A_{10} & A_{11} & A_{12} \\
\hline
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c}
B_T \\
\hline
B_B
\end{array}\right) \to \left(\begin{array}{c}
B_0 \\
\hline
B_1 \\
\hline
B_2
\end{array}\right), \left(\begin{array}{c}
C_T \\
\hline
C_B
\end{array}\right) \to \left(\begin{array}{c}
C_0 \\
\hline
C_1 \\
\hline
C_2
\end{array}\right)$$

where  $A_{11}$  is  $b \times b$ ,  $B_1$  has b rows,  $C_1$  has b rows

$$C_2 = \hat{C}_2 + A21B_1$$

$$C_1 = \hat{C}_1 + A_{11}B_1 + A_{21}B_2$$

## Continue with

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
A_{00} & A_{01} & A_{02} \\
\hline
A_{10} & A_{11} & A_{12} \\
\hline
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c}
B_T \\
\hline
B_B
\end{array}\right) \leftarrow \left(\begin{array}{c}
B_0 \\
\hline
B_1 \\
\hline
B_2
\end{array}\right), \left(\begin{array}{c}
C_T \\
C_B
\end{array}\right) \leftarrow \left(\begin{array}{c}
C_0 \\
\hline
C_1 \\
\hline
C_2
\end{array}\right)$$

Step	<b>Annotated Algorithm:</b> $C := AB + C$ where A is symmetric and stored in the lower triangle
1a	$\left\{ C=\widehat{C} ight\}$
4	Partition $A  o \left( \frac{A_{TL}}{A_{BL}} \frac{A_{TR}}{A_{BR}} \right)$ , $B  o \left( \frac{B_T}{B_B} \right)$ , $C  o \left( \frac{C_T}{C_B} \right)$
	where $A_{TL}$ is $0 \times 0$ , $B_T$ has 0 rows, $C_T$ has 0 rows
2	$\left\{ \left( \frac{C_T = \widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + C_B = \widehat{C}_B} \right) \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \left( \left( \frac{C_T = \widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + C_B = \widehat{C}_B} \right) \right) \land (m(A_{TL}) < m(A)) \right\}$
5a	Repartition
	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array}\right) $ where $A_{11}$ is $b \times b$ , $B_1$ has $b$ rows, $C_1$ has $b$ rows
6	$ \left\{ \left( \frac{C_0 = \widehat{C}_0}{C_1 = \widehat{C}_1} \right) \right\} $ $ \left\{ A_{20}B_0 + A_{21}B_1 + A_{22}B_2 + C_2 = \widehat{C}_2 \right\} $
8	$C_1 = \widehat{C}_1 + A_{10}B_0 + A_{11}B_1 + A_{21}^T B_2$
5b	Continue with
	$\left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array}\right)$
7	$ \left\{ \left( \frac{C_0 = \hat{C}_0}{A_{10}B_0 + A_{11} + B_1 + A_{21}^T B_2 + C_1 = \hat{C}_1}{A_{20}B_0 + A_{21}B_1 + A_{22}B_2 + C_2 = \hat{C}_2} \right) \right\} $
2	$\left\{ \left( \frac{C_T = \widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + C_B = \widehat{C}_B} \right) \right\}$
	endwhile
2,3	$\left\{ \left( \left( \frac{C_T = \widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + C_B = \widehat{C}_B} \right) \right) \land \neg \left( m(A_{TL}) < m(A) \right) \right\}$
1b	$\left\{ C = AB + \widehat{C} \right\}$

Partition 
$$A o \left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$$
,  $B o \left( \begin{array}{c|c} B_T \\ \hline B_B \end{array} \right)$ ,  $C o \left( \begin{array}{c|c} C_T \\ \hline C_B \end{array} \right)$ 

where  $A_{TL}$  is  $0 \times 0$ ,  $B_T$  has 0 rows,  $C_T$  has 0 rows while  $m(A_{TL}) < m(A)$  do

# Repartition

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \to \left(\begin{array}{c|c}
A_{00} & A_{01} & A_{02} \\
\hline
A_{10} & A_{11} & A_{12} \\
\hline
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c}
B_T \\
\hline
B_B
\end{array}\right) \to \left(\begin{array}{c}
B_0 \\
\hline
B_1 \\
\hline
B_2
\end{array}\right), \left(\begin{array}{c}
C_T \\
\hline
C_B
\end{array}\right) \to \left(\begin{array}{c}
C_0 \\
\hline
C_1 \\
\hline
C_2
\end{array}\right)$$

where  $A_{11}$  is  $b \times b$ ,  $B_1$  has b rows,  $C_1$  has b rows

$$C_1 = \hat{C}_1 + A_{10}B_0 + A_{11}B_1 + A_{21}^T B_2$$

## Continue with

$$\left( \begin{array}{c|c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right), \\ \left( \begin{array}{c|c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c|c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right), \\ \left( \begin{array}{c|c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left( \begin{array}{c|c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$$

Step	Annotated Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangle
1a	$\left\{ C=\widehat{C} ight\}$
4	Partition $A  o \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$ , $B  o \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right)$ , $C  o \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right)$
	where $A_{TL}$ is $0 \times 0$ , $B_T$ has 0 rows, $C_T$ has 0 rows
2	$\left\{ \left( \frac{A_{BL}^T B_B + C_T = \hat{C}_T}{A_{BR} B_B + C_B = \hat{C}_B} \right) \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \left( \left( \frac{A_{BL}^T B_B + C_T = \widehat{C}_T}{A_{BR} B_B + C_B = \widehat{C}_B} \right) \right) \land (m(A_{TL}) < m(A)) \right\}$
5a	Repartition
	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array}\right) $ where $A_{11}$ is $b \times b$ , $B_1$ has $b$ rows, $C_1$ has $b$ rows
6	$ \left\{ \left( \frac{C_0 = \widehat{C}_0}{C_1 = \widehat{C}_1} \atop A_{20}B_0 + A_{21}B_1 + A_{22}B_2 + C_2 = \widehat{C}_2 \right) \right\} $
	$C_0 = \hat{C}_0 + A_{10}B_0$
8	$C_1 = \hat{C}_1 + A_{11}B_1$
	$C_2 = \widehat{C}_2 + A_{21}^T B_2$
5b	Continue with
	$\left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array}\right)$
7	$ \left\{ \left( \frac{C_0 = \widehat{C}_0}{\underbrace{A_{10}B_0 + A_{11} + B_1 + A_{21}^T B_2 + C_1 = \widehat{C}_1}}{A_{20}B_0 + A_{21}B_1 + A_{22}B_2 + C_2 = \widehat{C}_2} \right) \right\} $
2	$\left\{ \left( \frac{A_{BL}^T B_B + C_T = \widehat{C}_T}{A_{BR} B_B + C_B = \widehat{C}_B} \right) \right\}$
	endwhile
2,3	$\left\{ \left( \left( \frac{A_{BL}^T B_B + C_T = \widehat{C}_T}{A_{BR} B_B + C_B = \widehat{C}_B} \right) \right) \land \neg \left( m(A_{TL}) < m(A) \right) \right\}$
1b	$\left\{ C = AB + \widehat{C} \right\}$

Partition 
$$A o \left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$$
,  $B o \left( \begin{array}{c|c} B_T \\ \hline B_B \end{array} \right)$ ,  $C o \left( \begin{array}{c|c} C_T \\ \hline C_B \end{array} \right)$ 

where  $A_{TL}$  is  $0 \times 0$ ,  $B_T$  has 0 rows,  $C_T$  has 0 rows while  $m(A_{TL}) < m(A)$  do

Repartition

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \to \left(\begin{array}{c|c}
A_{00} & A_{01} & A_{02} \\
\hline
A_{10} & A_{11} & A_{12} \\
\hline
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c}
B_T \\
\hline
B_B
\end{array}\right) \to \left(\begin{array}{c}
B_0 \\
\hline
B_1 \\
\hline
B_2
\end{array}\right), \left(\begin{array}{c}
C_T \\
\hline
C_B
\end{array}\right) \to \left(\begin{array}{c}
C_0 \\
\hline
C_1 \\
\hline
C_2
\end{array}\right)$$

where  $A_{11}$  is  $b \times b$ ,  $B_1$  has b rows,  $C_1$  has b rows

$$C_0 = \hat{C}_0 + A_{10}B_0$$

$$C_1 = \hat{C}_1 + A_{11}B_1$$

$$C_2 = \hat{C}_2 + A_{21}^TB_2$$

Continue with

$$\left(\begin{array}{c|c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c|c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array}\right)$$

Step	<b>Annotated Algorithm:</b> $C := AB + C$ where A is symmetric and stored in the lower triangle
1a	$\left\{ C=\widehat{C} ight\}$
4	Partition $A  o \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$ , $B  o \left( \begin{array}{c c} B_T \\ \hline B_B \end{array} \right)$ , $C  o \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right)$
	where $A_{TL}$ is $0 \times 0$ , $B_T$ has 0 rows, $C_T$ has 0 rows
2	$\left\{ \left( \frac{A_{BL}^T B_B + C_T = \widehat{C}_T}{A_{BL} B_T + A_{BR} B_B + C_B = \widehat{C}_B} \right) \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \left( \left( \frac{A_{BL}^T B_B + C_T = \hat{C}_T}{A_{BL} B_T + A_{BR} B_B + C_B = \hat{C}_B} \right) \right) \wedge (m(A_{TL}) < m(A)) \right\}$
5a	Repartition
	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c}B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c}B_0 \\ \hline B_1 \\ \hline B_2 \end{array}\right), \left(\begin{array}{c}C_T \\ \hline C_B \end{array}\right) \rightarrow \left(\begin{array}{c}C_0 \\ \hline C_1 \\ \hline C_2 \end{array}\right) $ where $A_{11}$ is $b \times b$ , $B_1$ has $b$ rows, $C_1$ has $b$ rows
6	$ \left\{ \left( \frac{C_0 = \hat{C}_0}{C_1 = \hat{C}_1} \right) \right\} $ $ \left\{ A_{20}B_0 + A_{21}B_1 + A_{22}B_2 + C_2 = \hat{C}_2 \right) $
8	$C_0 = \hat{C}_0 + A_{10}^T B_1$ $C_1 = \hat{C}_1 + A_{10} B_0 + A_{11} B_1$
5b	Continue with
	$\left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array}\right)$
7	$ \left\{ \left( \frac{C_0 = \widehat{C}_0}{\frac{A_{10}B_0 + A_{11} + B_1 + A_{21}^T B_2 + C_1 = \widehat{C}_1}{A_{20}B_0 + A_{21}B_1 + A_{22}B_2 + C_2 = \widehat{C}_2} \right) \right\} $
2	$\left\{ \left( \frac{A_{BL}^T B_B + C_T = \widehat{C}_T}{A_{BL} B_T + A_{BR} B_B + C_B = \widehat{C}_B} \right) \right\}$
	endwhile
2,3	$\left\{ \left( \left( \frac{A_{BL}^T B_B + C_T = \widehat{C}_T}{A_{BL} B_T + A_{BR} B_B + C_B = \widehat{C}_B} \right) \right) \land \neg (m(A_{TL}) < m(A)) \right\}$
1b	$\left\{ C = AB + \hat{C} \right\}$

Partition 
$$A o \left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$$
,  $B o \left( \begin{array}{c|c} B_T \\ \hline B_B \end{array} \right)$ ,  $C o \left( \begin{array}{c|c} C_T \\ \hline C_B \end{array} \right)$ 

where  $A_{TL}$  is  $0 \times 0$ ,  $B_T$  has 0 rows,  $C_T$  has 0 rows while  $m(A_{TL}) < m(A)$  do

Repartition

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \to \left(\begin{array}{c|c}
A_{00} & A_{01} & A_{02} \\
\hline
A_{10} & A_{11} & A_{12} \\
\hline
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c}
B_T \\
\hline
B_B
\end{array}\right) \to \left(\begin{array}{c}
B_0 \\
\hline
B_1 \\
\hline
B_2
\end{array}\right), \left(\begin{array}{c}
C_T \\
\hline
C_B
\end{array}\right) \to \left(\begin{array}{c}
C_0 \\
\hline
C_1 \\
\hline
C_2
\end{array}\right)$$

where  $A_{11}$  is  $b \times b$ ,  $B_1$  has b rows,  $C_1$  has b rows

$$C_0 = \hat{C}_0 + A_{10}^T B_1$$
  

$$C_1 = \hat{C}_1 + A_{10} B_0 + A_{11} B_1$$

## Continue with

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
A_{00} & A_{01} & A_{02} \\
\hline
A_{10} & A_{11} & A_{12} \\
\hline
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c}
B_T \\
\hline
B_B
\end{array}\right) \leftarrow \left(\begin{array}{c}
B_0 \\
\hline
B_1 \\
\hline
B_2
\end{array}\right), \left(\begin{array}{c}
C_T \\
C_B
\end{array}\right) \leftarrow \left(\begin{array}{c}
C_0 \\
\hline
C_1 \\
\hline
C_2
\end{array}\right)$$