

| Step | Annotated Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangle |
|------|--|
| 1a | $\{C = \widehat{C}\}$ |
| 4 | Partition $A \rightarrow \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$, $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right)$, $C \rightarrow \left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right)$ where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows |
| 2 | $\left\{ \left(\begin{array}{c} A_{TL}B_T + C_T = \widehat{C}_T \\ \hline C_B = \widehat{C}_B \end{array} \right) \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\left(\begin{array}{c} A_{TL}B_T + C_T = \widehat{C}_T \\ \hline C_B = \widehat{C}_B \end{array} \right) \right) \wedge (m(A_{TL}) < m(A)) \right\}$ |
| 5a | Repartition $\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$ where A_{11} is $b \times b$, B_1 has b rows, C_1 has b rows |
| 6 | $\left\{ \left(\begin{array}{c} A_{00}B_0 + C_0 = \widehat{C}_0 \\ \hline C_1 = \widehat{C}_1 \\ \hline C_2 = \widehat{C}_2 \end{array} \right) \right\}$ |
| 8 | $C_1 = \widehat{C}_1 + A_{10}B_0 + A_{11}B_1$ $C_0 = \widehat{C}_0 + A_{10}B_1$ |
| 5b | Continue with $\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$ |
| 7 | $\left\{ \left(\begin{array}{c} A_{00}B_0 + A_{10}B_1 + C_0 = \widehat{C}_0 \\ \hline A_{10} + B_0 + A_{11}B_1 + C_1 = \widehat{C}_1 \\ \hline C_2 = \widehat{C}_2 \end{array} \right) \right\}$ |
| 2 | $\left\{ \left(\begin{array}{c} A_{TL}B_T + C_T = \widehat{C}_T \\ \hline C_B = \widehat{C}_B \end{array} \right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\left(\begin{array}{c} A_{TL}B_T + C_T = \widehat{C}_T \\ \hline C_B = \widehat{C}_B \end{array} \right) \right) \wedge \neg(m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{C = AB + \widehat{C}\}$ |

Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangle

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$, $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right)$, $C \rightarrow \left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right)$

where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows

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Repartition

$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$

where A_{11} is $b \times b$, B_1 has b rows, C_1 has b rows

$$C_1 = \hat{C}_1 + A_{10}B_0 + A_{11}B_1$$

$$C_0 = \hat{C}_0 + A_{10}B_1$$

Continue with

$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$

endwhile

| Step | Annotated Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangle |
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| 2 | $\left\{ \left(\begin{array}{c} A_{TL}B_T + A_{BL}^T B_B + C_T = \widehat{C}_T \\ \hline C_B = \widehat{C}_B \end{array} \right) \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\left(\begin{array}{c} A_{TL}B_T + A_{BL}^T B_B + C_T = \widehat{C}_T \\ \hline C_B = \widehat{C}_B \end{array} \right) \right) \wedge (m(A_{TL}) < m(A)) \right\}$ |
| 5a | Repartition $\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$ where A_{11} is $b \times b$, B_1 has b rows, C_1 has b rows |
| 6 | $\left\{ \left(\begin{array}{c} A_{00}B_0 + A_{10}B_1 + C_0 = \widehat{C}_0 \\ \hline C_1 = \widehat{C}_1 \\ \hline C_2 = \widehat{C}_2 \end{array} \right) \right\}$ |
| 8 | $C_1 = \widehat{C}_1 + A_{10}B_0 + A_{11}B_1$ $C_0 = \widehat{C}_0 + A_{20}B_2$ |
| 5b | Continue with $\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$ |
| 7 | $\left\{ \left(\begin{array}{c} A_{00}B_0 + A_{10}B_1 + C_0 = \widehat{C}_0 \\ \hline A_{10} + B_0 + A_{11}B_1 + C_1 = \widehat{C}_1 \\ \hline C_2 = \widehat{C}_2 \end{array} \right) \right\}$ |
| 2 | $\left\{ \left(\begin{array}{c} A_{TL}B_T + A_{BL}^T B_B + C_T = \widehat{C}_T \\ \hline C_B = \widehat{C}_B \end{array} \right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\left(\begin{array}{c} A_{TL}B_T + A_{BL}^T B_B + C_T = \widehat{C}_T \\ \hline C_B = \widehat{C}_B \end{array} \right) \right) \wedge \neg (m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{C = AB + \widehat{C}\}$ |

Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangle

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$, $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right)$, $C \rightarrow \left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right)$

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Repartition

$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$

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$$C_1 = \hat{C}_1 + A_{10}B_0 + A_{11}B_1$$

$$C_0 = \hat{C}_0 + A_{20}B_2$$

Continue with

$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$

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| 2 | $\left\{ \begin{array}{l} A_{TL}B_T + C_T = \widehat{C}_T \\ \hline A_{BL}B_T + C_B = \widehat{C}_B \end{array} \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\begin{array}{c} A_{TL}B_T + C_T = \widehat{C}_T \\ \hline A_{BL}B_T + C_B = \widehat{C}_B \end{array} \right) \wedge (m(A_{TL}) < m(A)) \right\}$ |
| 5a | Repartition $\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$ where A_{11} is $b \times b$, B_1 has b rows, C_1 has b rows |
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| 8 | $C_0 = \widehat{C}_0 + A_{10}^T B_1$ $C_1 = \widehat{C}_1 + A_{11} B_1$ $C_2 = \widehat{C}_2 + A_{21} B_1$ |
| 5b | Continue with $\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$ |
| 7 | $\left\{ \begin{array}{l} A_{00}B_0 + A_{10}B_1 + C_0 = \widehat{C}_0 \\ \hline A_{10} + B_0 + A_{11}B_1 + C_1 = \widehat{C}_1 \\ \hline C_2 = \widehat{C}_2 \end{array} \right\}$ |
| 2 | $\left\{ \begin{array}{c} A_{TL}B_T + C_T = \widehat{C}_T \\ \hline A_{BL}B_T + C_B = \widehat{C}_B \end{array} \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c} A_{TL}B_T + C_T = \widehat{C}_T \\ \hline A_{BL}B_T + C_B = \widehat{C}_B \end{array} \right) \wedge \neg (m(A_{TL}) < m(A)) \right\}$ |
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$$C_0 = \widehat{C}_0 + A_{10}^T B_1$$

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Continue with

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| 3 | while $m(A_{TL}) < m(A)$ do |
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| 8 | $C_1 = \widehat{C}_1 + A_{11}B_1 + A_{21}B_2$ $C_2 = \widehat{C}_2 + A_{21}B_1$ |
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| 2 | $\left\{ \left(\begin{array}{c} A_{TL}B_T + A_{BL}^T B_B + C_T = \widehat{C}_T \\ \hline A_{BL}B_T + C_B = \widehat{C}_B \end{array} \right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\left(\begin{array}{c} A_{TL}B_T + A_{BL}^T B_B + C_T = \widehat{C}_T \\ \hline A_{BL}B_T + C_B = \widehat{C}_B \end{array} \right) \right) \wedge \neg (m(A_{TL}) < m(A)) \right\}$ |
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| 1a | $\{C = \widehat{C}\}$ |
| 4 | Partition $A \rightarrow \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$, $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right)$, $C \rightarrow \left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right)$ where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows |
| 2 | $\left\{ \left(\begin{array}{c} C_T = \widehat{C}_T \\ \hline A_{BR}B_B + C_B = \widehat{C}_B \end{array} \right) \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\left(\begin{array}{c} C_T = \widehat{C}_T \\ \hline A_{BR}B_B + C_B = \widehat{C}_B \end{array} \right) \right) \wedge (m(A_{TL}) < m(A)) \right\}$ |
| 5a | Repartition $\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$ where A_{11} is $b \times b$, B_1 has b rows, C_1 has b rows |
| 6 | $\left\{ \left(\begin{array}{c} C_0 = \widehat{C}_0 \\ \hline C_1 = \widehat{C}_1 \\ \hline A_{22}B_2 + C_2 = \widehat{C}_2 \end{array} \right) \right\}$ |
| 8 | $C_2 = \widehat{C}_2 + A_{21}B_1$ $C_1 = \widehat{C}_1 + A_{11}B_1 + A_{21}^T B_2$ |
| 5b | Continue with $\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$ |
| 7 | $\left\{ \left(\begin{array}{c} C_0 = \widehat{C}_0 \\ \hline A_{11} + B_1 + A_{21}^T B_2 + C_1 = \widehat{C}_1 \\ \hline A_{21}B_1 + A_{22}B_2 + C_2 = \widehat{C}_2 \end{array} \right) \right\}$ |
| 2 | $\left\{ \left(\begin{array}{c} C_T = \widehat{C}_T \\ \hline A_{BR}B_B + C_B = \widehat{C}_B \end{array} \right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\left(\begin{array}{c} C_T = \widehat{C}_T \\ \hline A_{BR}B_B + C_B = \widehat{C}_B \end{array} \right) \right) \wedge \neg (m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{C = AB + \widehat{C}\}$ |

Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangle

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$, $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right)$, $C \rightarrow \left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right)$

where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows

while $m(A_{TL}) < m(A)$ **do**

Repartition

$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$

where A_{11} is $b \times b$, B_1 has b rows, C_1 has b rows

$$C_2 = \hat{C}_2 + A_{21}B_1$$

$$C_1 = \hat{C}_1 + A_{11}B_1 + A_{21}^T B_2$$

Continue with

$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$

endwhile

| Step | Annotated Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangle |
|------|--|
| 1a | $\{C = \widehat{C}\}$ |
| 4 | Partition $A \rightarrow \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$, $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right)$, $C \rightarrow \left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right)$ where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows |
| 2 | $\left\{ \left(\begin{array}{c} C_T = \widehat{C}_T \\ \hline A_{BL}B_T + A_{BR}B_B + C_B = \widehat{C}_B \end{array} \right) \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\left(\begin{array}{c} C_T = \widehat{C}_T \\ \hline A_{BL}B_T + A_{BR}B_B + C_B = \widehat{C}_B \end{array} \right) \right) \wedge (m(A_{TL}) < m(A)) \right\}$ |
| 5a | Repartition $\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$ where A_{11} is $b \times b$, B_1 has b rows, C_1 has b rows |
| 6 | $\left\{ \left(\begin{array}{c} C_0 = \widehat{C}_0 \\ \hline C_1 = \widehat{C}_1 \\ \hline A_{20}B_0 + A_{21}B_1 + A_{22}B_2 + C_2 = \widehat{C}_2 \end{array} \right) \right\}$ |
| 8 | $C_1 = \widehat{C}_1 + A_{10}B_0 + A_{11}B_1 + A_{21}^T B_2$ |
| 5b | Continue with $\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$ |
| 7 | $\left\{ \left(\begin{array}{c} C_0 = \widehat{C}_0 \\ \hline A_{10}B_0 + A_{11} + B_1 + A_{21}^T B_2 + C_1 = \widehat{C}_1 \\ \hline A_{20}B_0 + A_{21}B_1 + A_{22}B_2 + C_2 = \widehat{C}_2 \end{array} \right) \right\}$ |
| 2 | $\left\{ \left(\begin{array}{c} C_T = \widehat{C}_T \\ \hline A_{BL}B_T + A_{BR}B_B + C_B = \widehat{C}_B \end{array} \right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\left(\begin{array}{c} C_T = \widehat{C}_T \\ \hline A_{BL}B_T + A_{BR}B_B + C_B = \widehat{C}_B \end{array} \right) \right) \wedge \neg (m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{C = AB + \widehat{C}\}$ |

Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangle

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$, $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right)$, $C \rightarrow \left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right)$

where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows

while $m(A_{TL}) < m(A)$ **do**

Repartition

$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$

where A_{11} is $b \times b$, B_1 has b rows, C_1 has b rows

$$C_1 = \hat{C}_1 + A_{10}B_0 + A_{11}B_1 + A_{21}^T B_2$$

Continue with

$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$

endwhile

| Step | Annotated Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangle |
|------|--|
| 1a | $\{C = \widehat{C}\}$ |
| 4 | Partition $A \rightarrow \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$, $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right)$, $C \rightarrow \left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right)$ where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows |
| 2 | $\left\{ \left(\begin{array}{c} A_{BL}^T B_B + C_T = \widehat{C}_T \\ \hline A_{BR} B_B + C_B = \widehat{C}_B \end{array} \right) \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\left(\begin{array}{c} A_{BL}^T B_B + C_T = \widehat{C}_T \\ \hline A_{BR} B_B + C_B = \widehat{C}_B \end{array} \right) \right) \wedge (m(A_{TL}) < m(A)) \right\}$ |
| 5a | Repartition $\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$ where A_{11} is $b \times b$, B_1 has b rows, C_1 has b rows |
| 6 | $\left\{ \left(\begin{array}{c} C_0 = \widehat{C}_0 \\ \hline C_1 = \widehat{C}_1 \\ \hline A_{20} B_0 + A_{21} B_1 + A_{22} B_2 + C_2 = \widehat{C}_2 \end{array} \right) \right\}$ |
| 8 | $C_0 = \widehat{C}_0 + A_{10} B_0$ $C_1 = \widehat{C}_1 + A_{11} B_1$ $C_2 = \widehat{C}_2 + A_{21}^T B_2$ |
| 5b | Continue with $\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$ |
| 7 | $\left\{ \left(\begin{array}{c} C_0 = \widehat{C}_0 \\ \hline A_{10} B_0 + A_{11} B_1 + A_{21}^T B_2 + C_1 = \widehat{C}_1 \\ \hline A_{20} B_0 + A_{21} B_1 + A_{22} B_2 + C_2 = \widehat{C}_2 \end{array} \right) \right\}$ |
| 2 | $\left\{ \left(\begin{array}{c} A_{BL}^T B_B + C_T = \widehat{C}_T \\ \hline A_{BR} B_B + C_B = \widehat{C}_B \end{array} \right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\left(\begin{array}{c} A_{BL}^T B_B + C_T = \widehat{C}_T \\ \hline A_{BR} B_B + C_B = \widehat{C}_B \end{array} \right) \right) \wedge \neg (m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{C = AB + \widehat{C}\}$ |

Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangle

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$, $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right)$, $C \rightarrow \left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right)$

where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows

while $m(A_{TL}) < m(A)$ **do**

Repartition

$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$

where A_{11} is $b \times b$, B_1 has b rows, C_1 has b rows

$$C_0 = \hat{C}_0 + A_{10}B_0$$

$$C_1 = \hat{C}_1 + A_{11}B_1$$

$$C_2 = \hat{C}_2 + A_{21}^T B_2$$

Continue with

$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$

endwhile

| Step | Annotated Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangle |
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| 1a | $\{C = \widehat{C}\}$ |
| 4 | Partition $A \rightarrow \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$, $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right)$, $C \rightarrow \left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right)$ where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows |
| 2 | $\left\{ \left(\begin{array}{c} A_{BL}^T B_B + C_T = \widehat{C}_T \\ \hline A_{BL} B_T + A_{BR} B_B + C_B = \widehat{C}_B \end{array} \right) \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\left(\begin{array}{c} A_{BL}^T B_B + C_T = \widehat{C}_T \\ \hline A_{BL} B_T + A_{BR} B_B + C_B = \widehat{C}_B \end{array} \right) \right) \wedge (m(A_{TL}) < m(A)) \right\}$ |
| 5a | Repartition $\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$ where A_{11} is $b \times b$, B_1 has b rows, C_1 has b rows |
| 6 | $\left\{ \left(\begin{array}{c} C_0 = \widehat{C}_0 \\ \hline C_1 = \widehat{C}_1 \\ \hline A_{20} B_0 + A_{21} B_1 + A_{22} B_2 + C_2 = \widehat{C}_2 \end{array} \right) \right\}$ |
| 8 | $C_0 = \widehat{C}_0 + A_{10}^T B_1$ $C_1 = \widehat{C}_1 + A_{10} B_0 + A_{11} B_1$ |
| 5b | Continue with $\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$ |
| 7 | $\left\{ \left(\begin{array}{c} C_0 = \widehat{C}_0 \\ \hline A_{10} B_0 + A_{11} B_1 + A_{21}^T B_2 + C_1 = \widehat{C}_1 \\ \hline A_{20} B_0 + A_{21} B_1 + A_{22} B_2 + C_2 = \widehat{C}_2 \end{array} \right) \right\}$ |
| 2 | $\left\{ \left(\begin{array}{c} A_{BL}^T B_B + C_T = \widehat{C}_T \\ \hline A_{BL} B_T + A_{BR} B_B + C_B = \widehat{C}_B \end{array} \right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\left(\begin{array}{c} A_{BL}^T B_B + C_T = \widehat{C}_T \\ \hline A_{BL} B_T + A_{BR} B_B + C_B = \widehat{C}_B \end{array} \right) \right) \wedge \neg (m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{C = AB + \widehat{C}\}$ |

Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangle

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$, $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right)$, $C \rightarrow \left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right)$

where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows

while $m(A_{TL}) < m(A)$ **do**

Repartition

$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$

where A_{11} is $b \times b$, B_1 has b rows, C_1 has b rows

$$C_0 = \hat{C}_0 + A_{10}^T B_1$$

$$C_1 = \hat{C}_1 + A_{10} B_0 + A_{11} B_1$$

Continue with

$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ \hline C_2 \end{array} \right)$

endwhile