ST 705 Linear models and variance components
Lab practice problem set 1

December 31, 2021

1. Prove or find a counter example to the following inequality.

$$1 \leq \sum_{k=1}^{\infty} \frac{1}{k^2} \leq 2.$$

2. Let $x_i, y_i \in \mathbb{R}$ for $i \in \{1, \ldots, n\}$, and show that

$$\frac{1}{n} \sum_{i=1}^{n} \sum_{j<i} (x_i - x_j)(y_i - y_j) = \sum_{i=1}^{n} (x_i - \bar{x}_n) y_i = \sum_{i=1}^{n} (x_i - \bar{x}_n)(y_i - \bar{y}_n).$$

   Note the particular case when $x_i = y_i$ for every i.