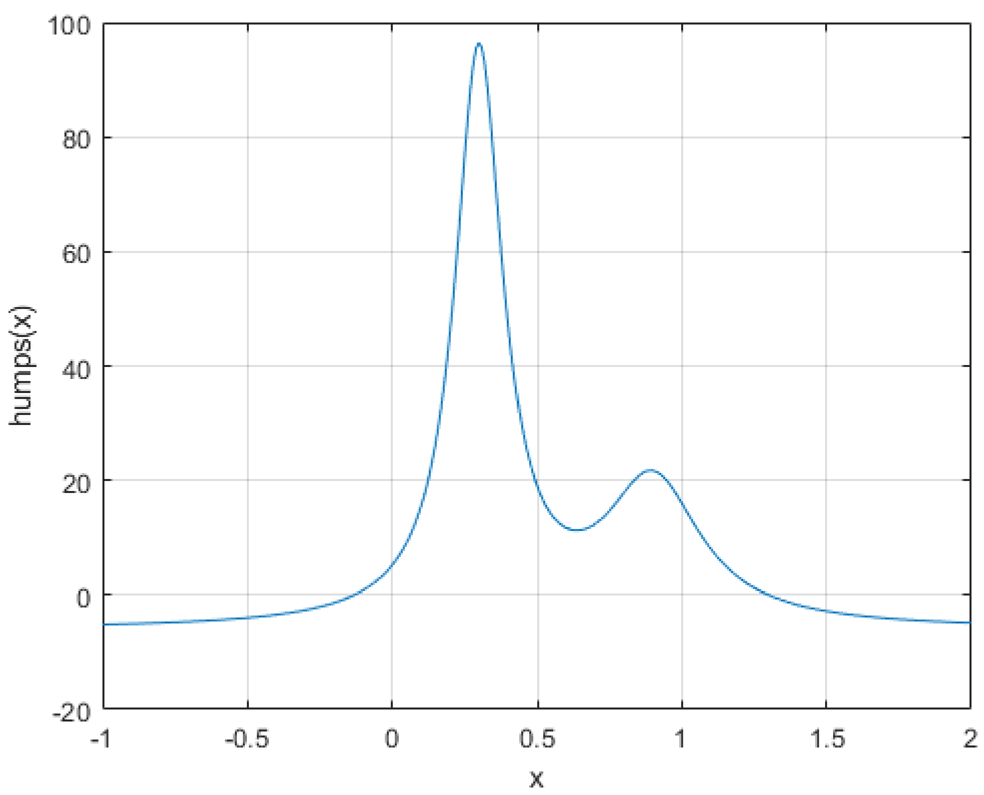
Optimizing Nonlinear Functions

Minimizing Functions of One Variable

Given a mathematical function of a single variable, you can use the `fminbnd` function to find a local minimizer of the function in a given interval. For example, consider the `humps.m` function, which is provided with MATLAB®. The following figure shows the graph of `humps`.

```
x = -1:.01:2;
y = humps(x);
plot(x,y)
xlabel('x')
ylabel('humps(x)')
grid on
```



To find the minimum of the `humps` function in the range `(0.3,1)`, use

```
x = fminbnd(@humps,0.3,1)
```

```
x = 0.6370
```

You can ask for a tabular display of output by passing a fourth argument created by the `optimset` command to `fminbnd`:

```
opts = optimset('Display','iter');
x = fminbnd(@humps,0.3,1,opts)
```

```
 Func-count     x          f(x)         Procedure
    1        0.567376      12.9098        initial
    2        0.732624      13.7746        golden
    3        0.465248      25.1714        golden
    4        0.644416      11.2693        parabolic
    5          0.6413      11.2583        parabolic
    6        0.637618      11.2529        parabolic
    7        0.636985      11.2528        parabolic
    8        0.637019      11.2528        parabolic
    9        0.637052      11.2528        parabolic

Optimization terminated:
 the current x satisfies the termination criteria using OPTIONS.TolX of 1.000000e-04
x = 0.6370
```

The iterative display shows the current value of `x` and the function value at `f(x)` each time a function evaluation occurs. For `fminbnd`, one function evaluation corresponds to one iteration of the algorithm. The last column shows what procedure is being used at each iteration, either a golden section search or a parabolic interpolation. For more information, see Iterative Display.

Minimizing Functions of Several Variables

The `fminsearch` function is similar to `fminbnd` except that it handles functions of many variables. Specify a starting vector x_0 rather than a starting interval. `fminsearch` attempts to return a vector *x* that is a local minimizer of the mathematical function near this starting vector.

To try `fminsearch`, create a function `three_var` of three variables, `x`, `y`, and `z`.

```
function b = three_var(v)
x = v(1);
y = v(2);
z = v(3);
b = x.^2 + 2.5*sin(y) - z^2*x^2*y^2;
```

Now find a minimum for this function using `x = -0.6`, `y = -1.2`, and `z = 0.135` as the starting values.

```
v = [-0.6,-1.2,0.135];
a = fminsearch(@three_var,v)

a =
    0.0000   -1.5708    0.1803
```

Maximizing Functions

The `fminbnd` and `fminsearch` solvers attempt to minimize an objective function. If you have a maximization problem, that is, a problem of the form

$$\max_x f(x),$$

then define $g(x) = -f(x)$, and minimize g.

For example, to find the maximum of $\tan(\cos(x))$ near $x = 5$, evaluate:

```
[x fval] = fminbnd(@(x)-tan(cos(x)),3,8)

x =
    6.2832

fval =
   -1.5574
```

The maximum is 1.5574 (the negative of the reported `fval`), and occurs at $x = 6.2832$. This answer is correct since, to five digits, the maximum is $\tan(1) = 1.5574$, which occurs at $x = 2\pi = 6.2832$.

`fminsearch` Algorithm

`fminsearch` uses the Nelder-Mead simplex algorithm as described in Lagarias et al. [1]. This algorithm uses a simplex of $n + 1$ points for n-dimensional vectors x. The algorithm first makes a simplex around the initial guess x_0 by adding 5% of each component $x_0(i)$ to x_0. The algorithm uses these n vectors as elements of the simplex in addition to x_0. (The algorithm uses 0.00025 as component i if $x_0(i) = 0$.) Then, the algorithm modifies the simplex repeatedly according to the following procedure.

> **Note**
>
> The keywords for the `fminsearch` iterative display appear in **bold** after the description of the step.

1. Let $x(i)$ denote the list of points in the current simplex, $i = 1,\ldots,n+1$.
2. Order the points in the simplex from lowest function value $f(x(1))$ to highest $f(x(n+1))$. At each step in the iteration, the algorithm discards the current worst point $x(n+1)$, and accepts another point into the simplex. [Or, in the case of step 7 below, it changes all n points with values larger than $f(x(1))$].
3. Generate the *reflected* point

   $$r = 2m - x(n+1),$$

   where

   $$m = \Sigma x(i)/n, \ i = 1\ldots n,$$

   and calculate $f(r)$.
4. If $f(x(1)) \le f(r) < f(x(n))$, accept r and terminate this iteration. **Reflect**
5. If $f(r) < f(x(1))$, calculate the expansion point s

   $$s = m + 2(m - x(n+1)),$$

   and calculate $f(s)$.

   a. If $f(s) < f(r)$, accept s and terminate the iteration. **Expand**

   b. Otherwise, accept r and terminate the iteration. **Reflect**
6. If $f(r) \ge f(x(n))$, perform a *contraction* between m and the better of $x(n+1)$ and r:

   a. If $f(r) < f(x(n+1))$ (that is, r is better than $x(n+1)$), calculate

   $$c = m + (r - m)/2$$

   and calculate $f(c)$. If $f(c) < f(r)$, accept c and terminate the iteration. **Contract outside** Otherwise, continue with Step 7 (Shrink).

   b. If $f(r) \ge f(x(n+1))$, calculate

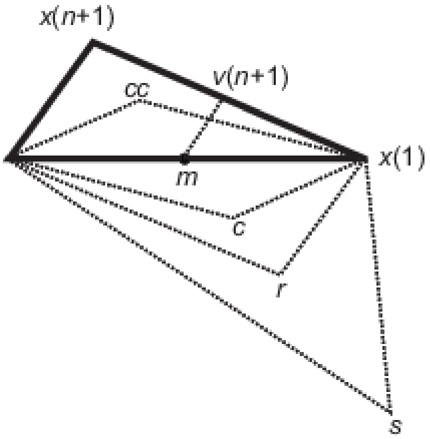
   $$cc = m + (x(n+1) - m)/2$$

   and calculate $f(cc)$. If $f(cc) < f(x(n+1))$, accept cc and terminate the iteration. **Contract inside** Otherwise, continue with Step 7 (Shrink).
7. Calculate the n points

   $$v(i) = x(1) + (x(i) - x(1))/2$$

and calculate $f(v(i))$, i = 2,...,n+1. The simplex at the next iteration is $x(1)$, $v(2)$,...,$v(n+1)$. **Shrink**

The following figure shows the points that `fminsearch` can calculate in the procedure, along with each possible new simplex. The original simplex has a bold outline. The iterations proceed until they meet a stopping criterion.



Reference

[1] Lagarias, J. C., J. A. Reeds, M. H. Wright, and P. E. Wright. “Convergence Properties of the Nelder-Mead Simplex Method in Low Dimensions.” *SIAM Journal of Optimization*, Vol. 9, Number 1, 1998, pp. 112–147.

Related Topics

- Nonlinear Optimization (Optimization Toolbox)
- Curve Fitting via Optimization