Introduction to wind turbines: physics and technology

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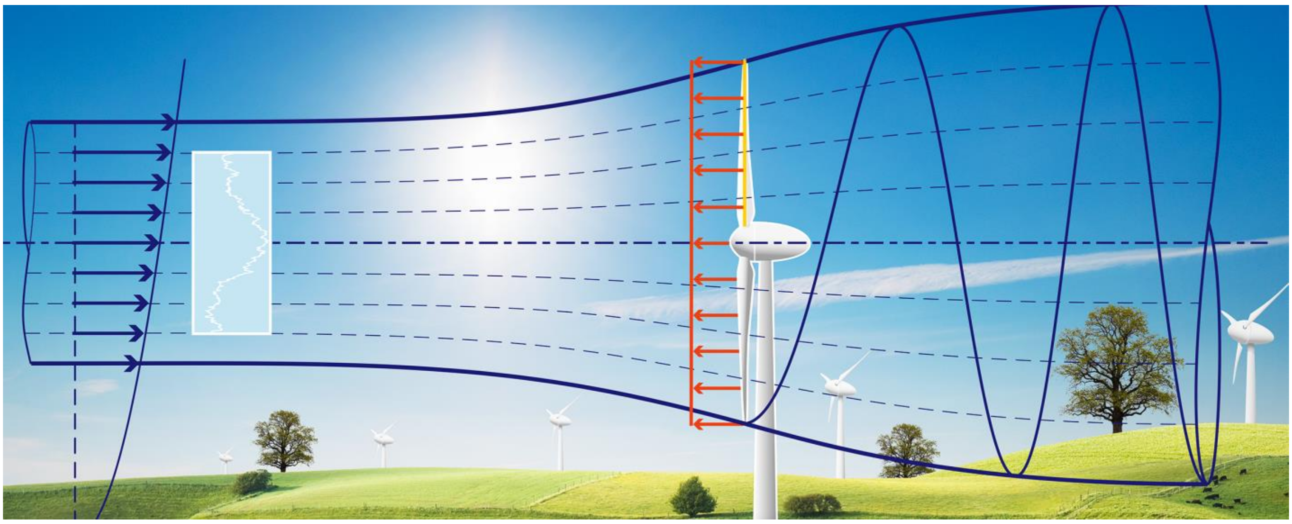

Table of Contents

0 Course information

0.1 Course description

What this course is about

Wind turbines involve many technical aspects: rotor blades, gearboxes, generators, control systems, etc. These are based on many physical disciplines, such as dynamics, statics, aerodynamics and electricity. Anyone who wants to apply their scientific or engineering skills to wind turbines should have an overview of the technology with which they interact. This course provides such an overview and helps you understand how the various components of a wind energy system act together.

This course gives you an overview of the multidisciplinary aspects of wind turbines without too much detailed study of the literature of all these aspects. It will explain the relevant technologies at a similarly accessible level. The following seven subjects are treated over the course of seven weeks:

Week 1: Wind turbine technology
A brief history of wind turbines for electricity production and an overview of the configurations and components of modern wind turbines.

Week 2: Wind climate and energy yield
The mechanisms that create wind, the variation of wind over time and in space, the characterisation of wind speed at a certain site and the estimation of the electricity production of a wind turbine at that site.

Week 3: Rotor aerodynamics
The governing equations of wind energy extraction by an arbitrary device, flow and forces for a rotor, calculation of the performance of a rotor and of a simple rotor design.

Week 4: Drive train and control
The characteristics of the main drive train components, torque and rotational speed in the drive train, control of maximum power in low wind speeds and constant power in high wind speeds.

Week 5: Dynamics of a wind turbine
The principles of dynamics, dynamic properties of wind turbines, and the analysis of variable loading and response of wind turbines.

Week 6: Structural analysis
The principles of failures of wind turbines structures, prediction of failure for extreme loading conditions, estimation of fatigue damage and assessment of blade tip deflections.

Week 7: Wake effects
A description of the wind conditions in the wakes of wind turbines and the calculation of wind speeds and turbulence behind wind turbines in a wind farm.

Expected prior knowledge

Students are expected to have mastered the fundamental concepts of a related engineering or science discipline. This includes satisfactory command of the fundamentals of calculus and physics needed at the BSc or BEng level.

The course is most suitable for students with a background in physics, aerospace engineering, mechanical engineering, electrical engineering or similar disciplines.

What you need to bring is some basic understanding of engineering sciences and we will get you up to speed with the relevance and application to wind turbine physics and technology. Each week a different aspect of wind energy is highlighted with online lectures, readings and slides. Weekly exercises and examples of previous exam questions help you master the subject. You can discuss the topics with fellow learners in the discussion forum where you can help each other. The lecturers will also assist in answering questions. At the end of the course, you can demonstrate what you have learned in a proctored written exam.

0.2 Learning objectives

By the end of this course you will be able to understand what a turbine is and how it interacts with the environment: what it is like, why it is like that and how to calculate values for its characteristic properties. Specifically, you will be able to:

- describe the components and configurations of wind turbines
- describe the characteristics of wind, use models that quantify wind speed variations and estimate the energy yield of a wind turbine
- describe the aerodynamic processes at work in wind energy conversion, calculate the forces and power generated and produce a simple rotor design
- explain the working principles of drive train components and determine the operational conditions that affect power, torque, pitch angle etc.
- identify the drivers of dynamic behaviour and calculate their typical values
- describe the principles of structural analysis of wind turbines and carry out preliminary predictions of structural failure
- describe the effect of wind turbines on downstream conditions and quantify wind speed and turbulence in wakes

A detailed list of knowledge and skills that you will obtain during this course is provided in 'Course information > Assessment'. You can use that as a checklist for the exam. The list refers to the week and to most relevant modules in which these learning objectives are addressed. Per week, the relevant learning objectives are repeated, so you get reminded of the objectives of that particular week.

0.3 Course structure and dates

Overview
Launch of the course: 11 November 2019
Christmas break: 23 December 2019 – 3 January 2020
Exam: Thursday 23 January 2020
Resit of the exam: Thursday 9 April 2020

Release dates
The content of the course will be made available on Brightspace per subject, per week.

Week 1: Wind turbine technology – 11 November 2019
Week 2: Wind climate and energy yield – 15 November 2019
Week 3: Rotor aerodynamics – 22 November 2019
Week 4: Drive train and control – 29 November 2019
Week 5: Dynamics of a wind turbine – 6 December 2019
Week 6: Structural analysis – 13 December 2019
Week 7: Wake effects – 20 December 2019

Contact moments
On campus lecture hours: See mytimetable.tudelft.nl
Live sessions online learners: Information will be announced

Study load
A study load of approximately 7-9 hours/week is expected. In addition, 40-50 hours of preparation for the exam is expected, which can be planned throughout the quarter at your own convenience.

The content of the first week is limited, to allow you to familiarise yourself with the set-up of the course, the teachers, your fellow learners and the learning environment (if this is new to you).

Self-study, lectures and online sessions
The course is organised so you can study at your own pace, at your own place. The theoretical course material and exercises are intended to be self-explanatory.

For on-campus learners (all students registered at TU Delft) there are lecture hours scheduled every week and for online learners (all participants from outside TU Delft) there will be online sessions. These contact moments are interactive and meant to address difficulties in the course material or exercises, to answer questions or to discuss issues more in depth, depending on your wishes.

To make the contact moments most effective, you can influence what will be treated. Questions or requests that are posted in the forum and that get most votes will be addressed during the lecture or online session. Therefore, it is most useful to study the material and do the exercises in the week that the subject is released. It is also expected that the forum for a particular subject is most lively in that week. Therefore, if you keep up with the weekly releases you have the largest chance that you can learn from or help other participants.

0.4 Assessment

Set up of the assessment

The exercises and questions provided throughout the course are only intended to get practical experience with the subjects. They are not mandatory and do not count in any way toward your final grade.

At the end of the course your knowledge and skills will be tested with a proctored exam. The exam will consist of multiple choice questions and open questions. Below, you'll find a checklist of the competences that are expected of you at the exam.

At the exam you get a list of equations as published in week 1 on Brightspace. You need to understand the equations good enough to know what each of the symbols means, since a list of symbols is not provided.

The exam is 'closed book', so you may not bring any course material with you. The use of a pocket calculator is allowed. This may be a graphical pocket calculator, but that is certainly not necessary. A calculator that can connect to internet is not allowed.

Checklist competences for the exam

	Week	Main module(s)	Learning objective
			Know (or understand enough to derive from first principles)
	1	1.1	Relate elements of the big picture of 'modern' wind turbine history (the large steps and the underlying reasons - no exact dates and data)
	1	1.2	Recognise different configurations (particularly with multi-stage gearbox, with single stage gearbox and direct drive) and describe the consequences for rotational speed, torque and generator size
	1	1.3	Identify and name the main components in a wind turbine and explain their function
	1	1.2-1.3	Know of the existence of the uncommon configurations and components of wind turbines (as far as treated in the slides)
	2	2.1	Describe and explain global and local patterns in wind speed and direction
	2	2.2	Know the meaning of 'potential wind speed' and the value of the standard surface roughness length z_0 that corresponds to it
	2	2.2.2, 2.3.3	Name the characteristics and sources of turbulence
	2	2.3	Describe the principles of separation of wind conditions in long-term and short-term conditions
	2	2.3.3	Reproduce the mathematical definition of turbulence intensity
	2	2.4	Identify and name characteristic elements in the power curve and their significance
	2	2.5	Reproduce the mathematical definition of capacity factor
	3	3.1.1, 3.1.2	Describe the direction of the lift and drag force relative to the flow and recognise their characteristic curves
	3	3.1.1, 3.1.2	Recognise the region where the aerofoil is in stall in the lift curve
	3	3.3.2	Reproduce the mathematical definition of induction factor a
	3	3.3.2	Give the equation of c_P as function of the induction factor (= $4a(1-a)^2$)
	3	3.3.2	Give the equation of c_T as function of the induction factor (= $4a(1-a)$)
	3	3.3.3	Give the value of the induction factor and c_P at the maximum aerodynamic performance (Betz optimum)
	3	3.4.2	Reproduce the mathematical definition of tip speed ratio λ

	3	3.4.2, 3.4.3	Draw the diagram of a cross-section of the blade with the vectors of wind and forces
	3	3.4.2	Recognise and describe what is: the inflow angle, the angle of attack, the (structural) twist angle, the (full span) pitch angle
	3	3.4.3, 3.4.4	Reproduce the expressions of the normal force dF_n and tangential force dF_t in blade element theory
	4	4.2.2	Reproduce the mathematical definition of gearbox ratio
	4	4.2.2	Give the gearbox ratio as function of the geometric parameters of a parallel gearbox stage
	4	4.2.2	Reproduce the terminology of gearboxes, the layout of a (typical) gearbox and the rationale for this layout
	4	4.2.3	Describe and recognise the working principle of a 3-phase permanent magnet generator
	4	4.4, 4.5	Describe the principles of power control in partial load and in full load (what are the control objectives, which measured parameters can be used, which control actions are undertaken based on these measurements)
	4	4.4, 4.5	Recognise the shapes of the curves of rotational speed and pitch angle as a function of wind speed and give the reasons why they have these shapes
	5	5.5.1, 5.3	Reproduce the equation of motion of a single degree of freedom system (with translation or rotation)
	5	5.5.1, 5.3	Express the natural frequency of an undamped single degree of freedom system (with translation or rotation) as a function of stiffness and inertia
	5	5.5.1	Reproduce and recognise the shape of the DAF as a function of excitation frequency and describe qualitatively how it is affected by damping
	5	5.3	Identify for a vibration of the rotor whether it is in-plane, flapping, out-of-plane, lead-lag, collective and/or differential
	5	5.5.1	Know the meaning of soft-soft, soft-stiff and stiff-stiff when used to classify a support structure
	6	6.1.2	Express the normal stress due to a longitudinal force as a function of cross-sectional area
	6	6.2.1	List the four failure modes and limit states that are evaluated for wind turbines
	6	6.3.4	Describe the role of the Campbell diagram in the limit state analysis as a pre-assessment for fatigue
	7	7.1.1	Describe the structure and development of the wake of a wind turbine
	7	7.1.1	Describe the effect of meandering of a wind turbine wake and its possible causes
	7	7.1.1	Describe why wakes are important for wind turbines in a wind farm
	7	7.1.1, 7.1.2	List at least three important differences between the near wake and the far wake regions
	7	7.2	Reproduce the geometry and velocity profile description of the Jensen model for wind speed deficit
	7	7.2	List at least 3 assumptions of the Jensen model that are clear simplifications of reality
	7	7.4.2	Give the equation for the equivalent velocity on a rotor with a single partial wake incidence
	7	7.5	Describe deep array effects and that they are treated with adjusted farm ambient turbulence and wind speed.
	7	7.5	Describe the modelling principles of deep array effects with surface roughness and inner boundary layers
			Understand and be able to work with
	1	1.4.1	Interpret wind energy terminology correctly
	1	1.4.2	Interpret symbols used in equations correctly
	2	2.3.2	Understand the principle behind determining a Weibull distribution based on measurements of 10-minute average wind speeds and the interpretation of a wind speed histogram (e.g. to judge from a description whether it is correct or incorrect)
	2	2.3.3	Understand the difference and conversion between standard deviation and turbulence intensity
	2	2.3.4	Understand the principles of recreating short-term time series of wind speed for load case analysis (e.g. to judge from a description whether it is correct or incorrect)
	2	Hands-on experience	Understand that the wind shear profile can also be used to obtain average wind speeds at different heights, even though it represents the profile of wind speeds at a particular moment in time
	3	3.5	Understand the principles of connecting blade element theory and momentum theory in

			'Blade Element Momentum theory' (BEM) and the principles of solving induction factor (a) and thrust (T) in this theory (e.g. to judge the description of how someone implemented it in software)
	3	3.6.1	Recognise a c_p-λ curve and understand how it is created and how it can be used to determine the performance of a rotor at certain operating conditions
	3	3.6.2	Understand how designing for maximum aerofoil performance and for maximum power coefficient leads to twist in the blade
	4	4.6	Understand the implications of the different control regions (e.g. to argue how they relate to the state of the turbine (e.g. partial load, maximum power coefficient or constant power operation))
	4	Hands-on experience	Understand when to use rad/s and when to use Hz for rotational speeds and frequencies
	5	5.1.1	Understand when to use rad/s and when to use Hz for rotational speeds and frequencies
	5	5.1.1	Understand the principles of mode shapes and natural frequencies as solutions for undamped free vibration (e.g. to recognise that the number of mode shapes equals the number of degrees of freedom, that each mode shape is associated with a natural frequency (but several mode shapes may have the same natural frequency) and that the mode shape and natural frequency separate a vibration into function of position and a (sinusoidal) function of time)
	5	5.1.1	Understand the principle of superposition of harmonic loading and response and its use in frequency domain analysis of a linear system (e.g. to sketch the response spectrum if the excitation spectrum and transfer function are given)
	5	5.4	Understand the origin of loading of a wind turbine and which loading causes 1P, 2P, 3P, … excitations on the blades and on the hub and tower (e.g. use the principle of decomposition of a periodic signal in time (sum of sinusoidal signals with base frequency and integer multiples of that) to argue the existence of 4P flap bending moment variations)
	5	5.5.2	Understand the principles of aerodynamic damping (e.g. to argue whether or not tower side-to-side motion is damped by it)
	6	6.2.3	Understand the principles of representing lifelong conditions by load cases (e.g. to judge whether a suggested load case is sensible)
	6	6.3.3	Understand the principles of tip deflection and tower top displacement calculations (e.g. to judge a description of an approach for correctness)
	6	6.3.4	Understand the difference between the use of a dynamic amplification factor (DAF) for single degree of freedom systems and for complex structures
	7	7.2	Understand the use of the conservation of mass over a control volume to derive the Jensen model
	7	7.2	Understand the conversion between induction factor and thrust coefficient in the Jensen model
	7	7.3	Understand the relation between wake decay coefficient and turbulence (more turbulence is higher k)
	7	7.3	Understand the difference between turbulence intensity in the wake and wake added turbulence intensity
	7	7.4.2	Understand the principles of area weighted wind speed averaging for a rotor impacted by multiple wakes
	7	7.4.3	Identify for which turbines in a wind farm the wakes need to be considered in a fatigue analysis
			Apply and analyse
	1-7	[1.4.3 and 1.4.4 for some equations]	Select, combine and manipulate relevant equations to get them in the desired form, using basic mathematics
	1-7	[1.1 for concepts]	Translate what has been treated for the state-of-the-art turbines qualitatively to similar but different concepts to explain their working principles
	2	2.2.1	Determine the wind speed as a function of height for different types of terrain and from different types of information
	2	2.3.2	Determine the probability of occurrence of the wind speed between two values (e.g. between 7.7 and 8.3 m/s)
	2	2.5	Use the definition of capacity factor for various applications, such as the determination of capacity factor for a given site and turbine or to check with typical values of the capacity

			factor that an energy yield calculation gives a reasonable result
	2	2.5	Determine the energy yield as a function of the wind speed distribution and the power curve and (as an intermediate step) the contribution of wind speeds between two values to the annual energy yield (e.g. determine how much energy is produced by wind speeds between 7.7 and 8.3 m/s)
	3	3.1.2	Determine the design angle of attack and design lift coefficient (corresponding to the best 'lift over drag ratio' (c_l/c_d))
	3	3.3	Derive the momentum theory for an actuator disc and the maximum possible wind energy conversion according to Betz
	3	3.4	Derive (parts of) blade element theory
	3	3.5	Apply intermediate steps of Blade Element Momentum theory (BEM) (e.g. calculate the thrust force for a given induction factor from momentum theory and from blade element theory to check whether the induction factor is correct)
	3	3.1.2, 3.3.3, 3.4.4, 3.6, Hands-on experience	Apply knowledge about optimum energy conversion from momentum theory, knowledge about optimal aerofoil performance and knowledge from blade element theory to design a rotor blade
	4	4.3	Combine and rewrite the characteristic equations of component behaviour to determine how the rotor and drive train system behave in certain conditions (e.g. to determine rotational speed, forces and powers at different places in the wind turbine)
	4	4.2-4.6	Use curves for c_p-λ, c_Q-λ, P-V, Q-Ω (for the rotor, as well as for the generator) to determine or predict the behaviour of the system
	4	4.4	Determine the optimal mode gain for partial load control and assess consequences of deviating from the theoretical optimum value
	4	4.4, 4.5	Determine the operational points of a controlled drive train in partial load and in full load (e.g. pitch angle, rotational speed and torque)
	5	5.1.1	Determine the undamped natural frequency of a single degree of freedom system, given the equation of motion (e.g. for the drive train), by recognition of the inertia and the stiffness terms in the equation
	5	5.3	Estimate the natural frequency of the for-aft or side-to-side vibration of the tower+rotor+nacelle, given basic information about the stiffness of the tower and mass of the rotor+nacelle
	5	5.3	Determine the effect of centrifugal stiffening on the natural frequency of the rotating blade as a function of rotational speed
	5	5.5.1	Perform analysis with a Campbell diagram (both by interpretation and analysis of a given diagram, as well as by creating one from basic information)
	6	6.3.1	Estimate the thrust coefficient of a rotor (operating at the Betz optimum), in order to estimate the thrust force on the rotor
	6	6.1.2, 6.3.1	Determine forces and moments in the wind turbine structure with simplified, static load calculations
	6	6.1.2, 6.3.2	Determine stress in a structure due to forces and moments, including the summation of stress from multiple loads and stress concentration (stress concentration factors will be given when needed)
	6	6.3.3	Solve the tip deflection of tower top displacement when the analytical integrations are straightforward (depending on loading conditions and stiffness distribution)
	6	6.3.4	Apply dynamic amplification to static loads as a model for dynamics
	6	6.1-6.6	Assess the limit state for ultimate strength, tip deflection and fatigue (assuming the material is characterised by a Wöhler S-N curve and not by a Goodman diagram) with hand calculations or provided data/graphs from simulations
	7	7.2, 7.4.2	Determine the wind speed at any position in the wake of one or more wind turbines, for given a or c_T
	7	7.2, 7.4.2	Determine the equivalent velocity on a rotor in single or multiple (partial) wakes
	7	7.3	Determine the wake added turbulence intensity and the total turbulence intensity at any position in a wake (when necessary, translating standard deviation to turbulence intensity)
	7	7.4.3	Determine the probability that a turbine experiences wake added turbulence for a wind farm

0.5 Resources and tools

Overview

All study material for this course is made available through Brightspace. The main course material is organised per subject, with modules numbered according to the lecture week and named after the subject. The material of the next week is released on the day after the on-campus lecture (see 'Course information > Course structure and dates' for the release dates). The structure of the weekly content is the same each week and is described in more detail below.

Below the weekly content, you'll find examples of questions from previous exams in the module 'Exam(ple) questions'. These examples are collected in one module, but they will also be released in batches of questions that relate to the subject of each week. In the last week, also examples of questions that cover more than one subject will be made available.

You can download a reader of transcripts of all video lectures and voice-overs, together with the slides and documents. This pdf-file is provided below the modules and also contains this 'Course information' module. The content of the reader is identical to the online material provided in 'Resources' every week. It can be used as alternative for the online course material or for later reference. You therefore don't need to check whether you miss anything when you decide to learn (primarily) from the reader. However, be aware that the exercises and exam questions are not included in the reader.

Several exercises in 'Hands-on experience' use Excel. Several Excel files are provided with data and with preparations for the calculations.

Structure of weekly content

Each subject/week has the following items:

Introduction

Description of the subject and details of what you can do and learn this week.

Hands-on experience

Each week an exercise is given that integrates various aspects of the subject of the week. This gives you hands-on experience with the course material and helps you master the subjects. These exercises can have significant differences with exam questions. By using calculations in Excel, they give you a better overview and understanding than would be possible with examples of exam questions, which allow only limited calculations.

The answers and explanation of the exercise are also given. Try not to peak! The exercise itself contains some hints to help you along.

Resources

The resources provide the theoretical course material. This consists of video lectures, slides with voice-over for explanations and documents.

Practice and discuss

After each topic you can test what you have learned with a selection of previous exam questions. After answering a question, you can click on 'Answer' to go to the correct answer and explanation. To further improve your understanding and application of the theory, you'll find a link in this module to the relevant section in the collected previous exam questions in the module 'Exam(ple) questions'.

Every week a new platform is provided to discuss the current topic with others. Here you can ask questions to your fellow learners, provide insight and help others. The discussion is moderated, so frequently asked questions or misconceptions can be addressed by the teachers, but the primary goal is that learners help each other.

On this forum, you can vote for posts that address a question or subject that also has your interest. That can for instance be about an element from the theoretical material, part of the hands-on experience or an exam question. Posts with the most votes will get attention during the contact hours of the corresponding week (lecture hours for on-campus students or online session for online learners) or may be answered online by one of the teachers (see 'Course information' > Course structure and dates' > 'Self-study, lectures and online sessions').

Note: you can also access the forums to discuss topics and to vote items for the lectures and online sessions via 'Collaboration > Discussions' in the course main menu.

0.6 Staff and support

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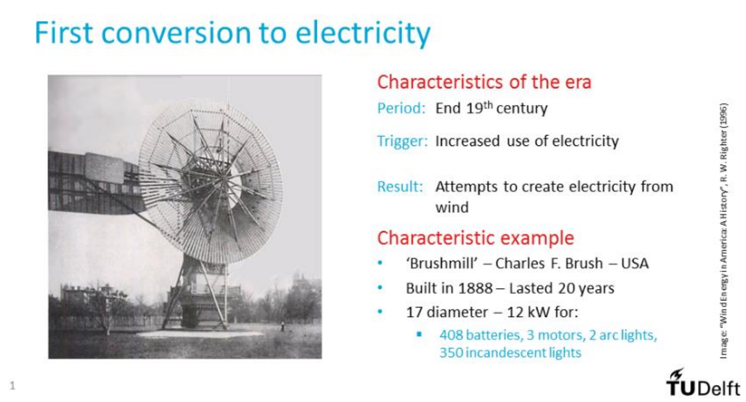
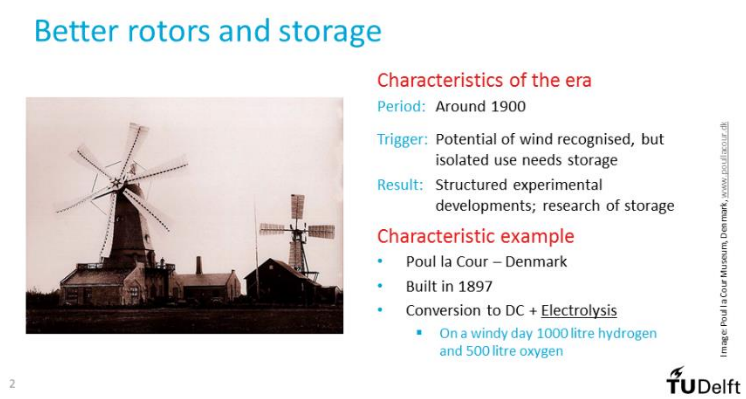
1 Wind turbine technology

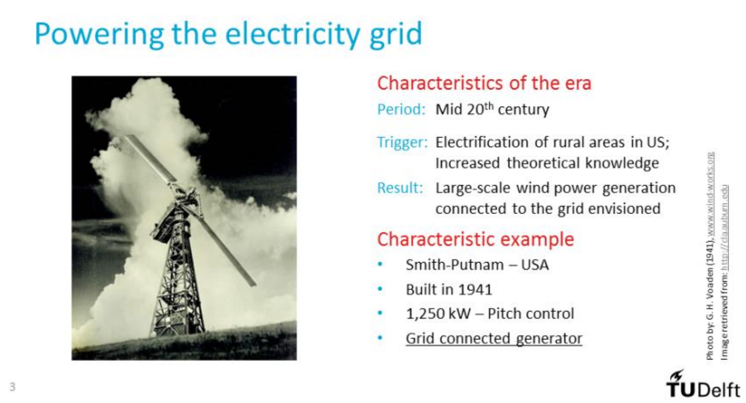
1.1 History of modern wind turbines



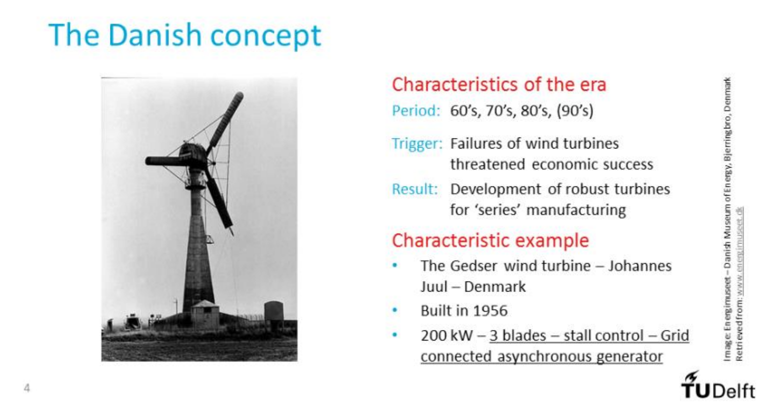
This history starts at the end of the 19th century when the first turbine was made that converted wind energy into electricity. This is the era in which electricity and electrical devices we're upcoming. Remember that this is the time of Thomas Edison. Charles Brush had an estate in which he used such electrical devices and he made what is often called the brush mill to create the electricity for that.



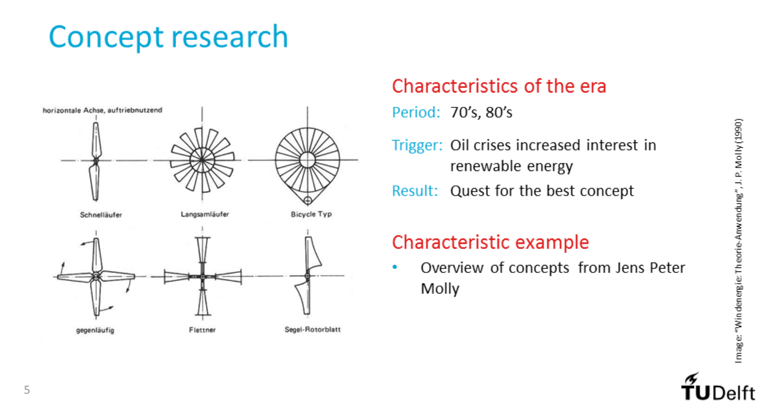
Following the initiative of Charles Brush other people in other countries also tried to create electricity from wind energy. A pioneer in this respect is Poul la Cour from Denmark. He investigated in a mostly experimental way how to improve the aerodynamic design of the windmills and he also investigated forms of storage. In this time there was no electricity grid so all the demand and supply were local and for that storage was needed. Poul la Cour investigated particularly whether electrolysis was a good way of storing electricity.



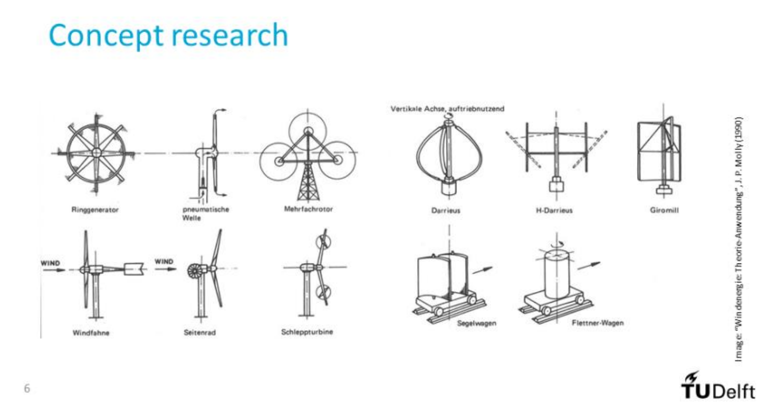
In the middle of the 20th century the electrification in the public grids of the cities expanded to the rural areas in the US. This led to the idea to generate electricity from wind energy in rural areas and transport the electricity to the cities where most of the consumers were. This required also larger turbines to make a significant impact on the electricity supply. An example of a prototype built in this period is the Smith-Putnam turbine. It has a capacity of over one megawatt and most importantly it had a grid-connected generator so the electricity that was generated was immediately fed into the grid and transported to the consumers. It also had other technological features such as pitch control.



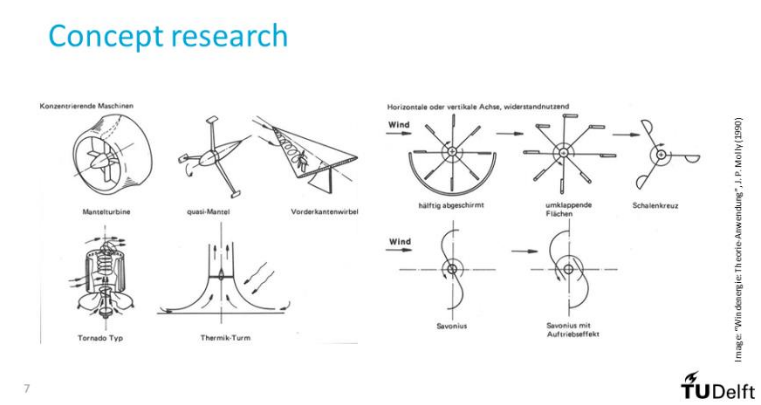
Up to about the '60s of the previous century a lot of turbines were built as single units and often suffered from early failures and breakdown. This made them economically unattractive. In response in Denmark a concept was created that focused on reliability and series production. These turbines were a lot smaller than a lot of the prototypes that were built earlier and used as least advanced technology as possible. They used three blades and stall control meaning that the blades cannot be pitched. They also selected a grid-connected asynchronous generator which is one of the most robust types of generators.



In the '70s and '80s the oil crises triggered the interest in wind energy in many more people, some of these people wondered whether the Danish concept was actually the best concept, they devised various alternative concepts leading to a lot of experiments and a lot of new prototypes being built and tested. This slide and the next two slides show the variation in concepts that were devised from the book of Jens-Peter Moly.



So here you see various alternative concepts with horizontal axis of rotation, vertical axis of rotation and even cars driving over a track.



This shows some more alternatives, the alternatives on the left side aim mostly at concentrating the wind around the rotor and the alternatives on the right side use drag forces of the wind instead of lift forces.

Upscaling failures

Characteristics of the era

Period: 80's

Trigger: Oil crises increased interest in renewable energy

Result: Multi-MW turbines from big companies and institutes

Characteristic example

- WTS-4 – USA
- Built in 1982
- 4 MW
- Pitch-controlled

Photo by: Kurt S. Hansen

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8

The oil crises didn't only trigger the quest for the best concept but also re-established the idea that wind turbines should be much bigger if they were going to make an impact on the electricity supply. This also attracted the interest of larger companies and institutes such as Boeing and NASA. They were mostly focused on how to build very large turbines in the range from 2 to about 4 megawatts.

Upscaling failures

Characteristics of the era

Period: 80's

Trigger: Oil crises increased interest in renewable energy

Result: Multi-MW turbines from big companies and institutes

Characteristic example

- FloWind Darrieus Turbine – USA
- Built in 1980's
- 300-400 kW

Image: www.windsofchange.dk

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9

This and the next slide shows some more characteristic examples from this era that also show that the quest for the best concept was still not finished.

Upscaling failures

Characteristics of the era

Period: 80's

Trigger: Oil crises increased interest in renewable energy

Result: Multi-MW turbines from big companies and institutes

Characteristic example

- NASA/DOE MOD-5B – Hawaii – USA
- Built in 1987
- 3.2 MW
- Partial pitch-controlled

Photo by: Paul Gipe
Image retrieved from: www.nasa.gov

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10

We are still seeing both horizontal axis and vertical axis turbines and as you can see here two bladed rotors are also still in the picture.

Upscaling failures

Characteristics of the era

Period: 80's

Trigger: Oil crises increased interest in renewable energy

Result: Multi-MW turbines from big companies and institutes

Characteristic example

- Megawatt Heidelberg Rotor - Germany
- Built in 1994

Image: www.heiner-doerner-windenergie.de

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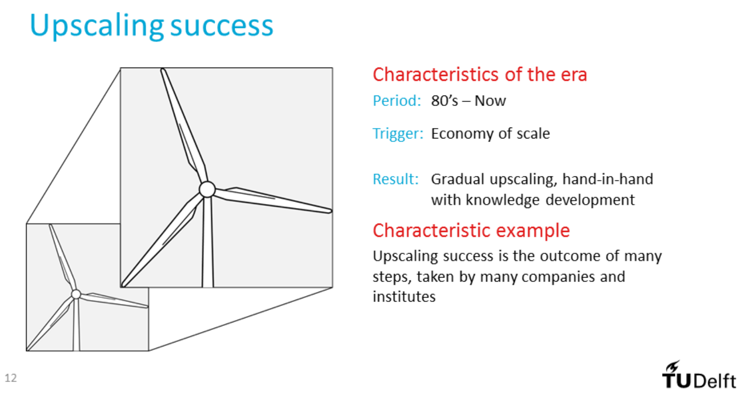
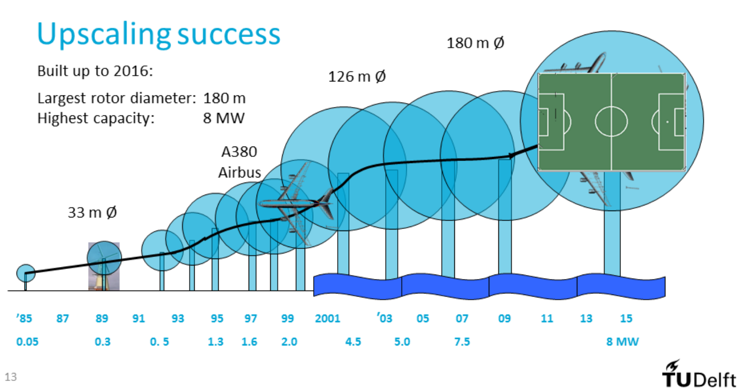
11

This example shows that also for vertical axis turbines various rotor concepts were considered. The outcome of this development of multi megawatt turbines is twofold. On the one hand a lot of knowledge was gained and advanced technologies were developed, on the other hand none of these large turbines became economically successful, they were either too expensive to manufacture or they didn't reach the kind of reliability that would be necessary.



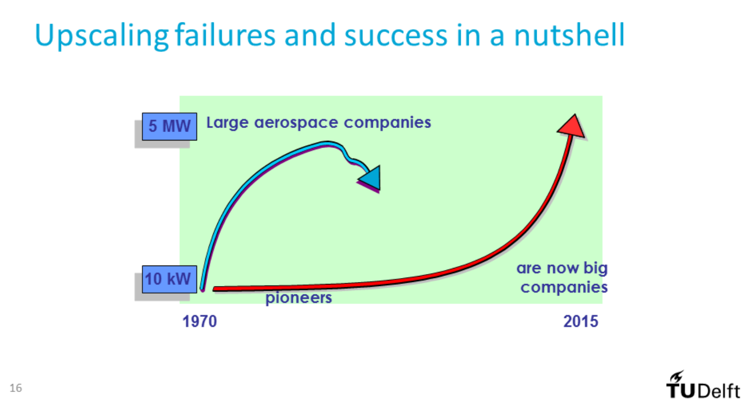
While the big companies were trying to make a big wind turbine in one go, the smaller companies that already had smaller turbines in their portfolio, increased the size of their product step by step. By experimenting on their prototypes and getting feedback from their customers they gradually increase their knowledge about the forces on their turbines and gradually increase the scale of the rotors and the generators.



This diagram illustrates the success of this gradual upscaling process, on the left hand side we start in about 1985 with turbines of 50 kilowatts and their size increases to around one to two megawatt scales at the end of the last century which is a turbine of about the same size as the wingspan of Airbus A380. Nowadays we have prototypes of about eight megawatts and rotor diameters of 120 meters. This can be compared with three Airbuses when they are rotating making wind turbines the largest rotating mechanisms on earth. You can also compare this size with the size of the soccer field. The large turbines say in the range from five to eight megawatt are particularly developed for offshore applications.

This picture illustrates how big an eight megawatt turbine actually is, compare the size of this nacelle with the group of people on the right hand side.

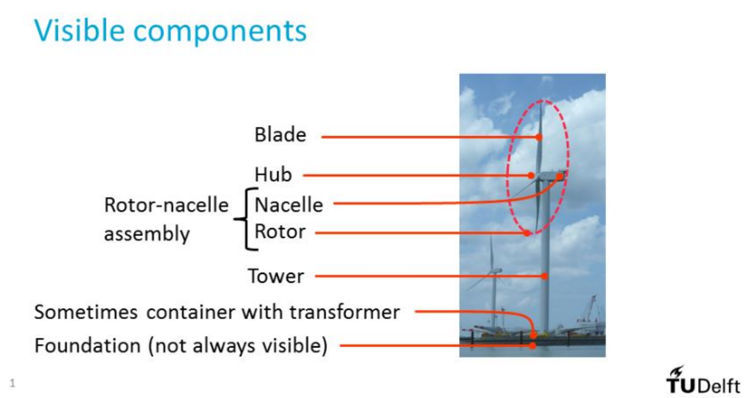
And here you see the size of an 83.5 meter blade. Currently the largest blade is actually already five meters longer. A lot of these blades are developed for the offshore market and therefore most manufacturers move their manufacturing facilities more and more to riversides, in coastal areas so they can do the transport over water.



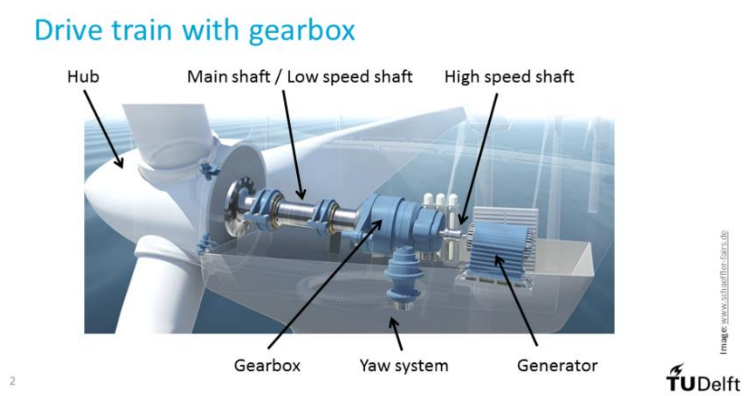
The developments of turbine size can be summarized in this diagram, around the 70’s and 80’s the large companies tried and failed in doing a discontinuous increase in size while the pioneers increased their products gradually and are now the leading companies in turbine manufacturing.

1.2 Atomy of wind turbines

Welcome to this learning unit, in which you’ll get an overview of the outside and inside of the most common types of wind turbines. On the whole, these wind turbines look very much the same. However, you’ll see that there are also some important variations.

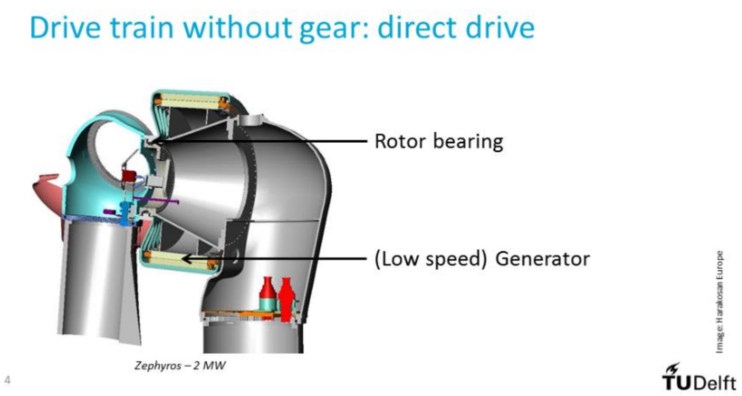


First I’ll give you some terminology of the components that can be seen from the outside. As you all know, these moving parts are the blades. The blades connect to the hub and together they form the rotor. The rotor connects to the nacelle, which houses the machinery. The combination of the rotor and nacelle is aptly called the rotor-nacelle assembly, which is often abbreviated to RNA. The rotor-nacelle assembly is supported by the tower and this rests on the foundation. The tower and foundation together are called the support structure. Sometimes you can recognise the housing of the transformer at the tower base. For an offshore wind turbine this can be on the platform above the boat landing. Some offshore turbines have the transformer at the rear below the nacelle.

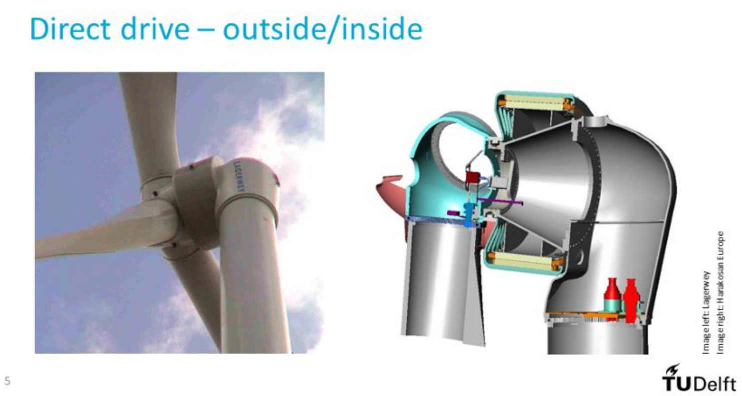


If we open up the nacelle, we can see the drive train. The drive train is the assembly of all rotating components that are involved in the energy conversion. What you see here is traditionally the most common type of drive train: the drive train with a gearbox. On the left-hand see you see the hub that is connected to a low speed shaft. The generator that is used in this drive train is a more-or-less off-the-shelf product and needs to rotate at a much higher speed than the rotor. Therefore, it is connected to the low speed shaft through a gearbox. For multi-megawatt turbines the gearbox increases the rotational speed by a factor of about 100. The nacelle connects to the tower top through the yaw system, which enables the turbine to align itself with the wind.

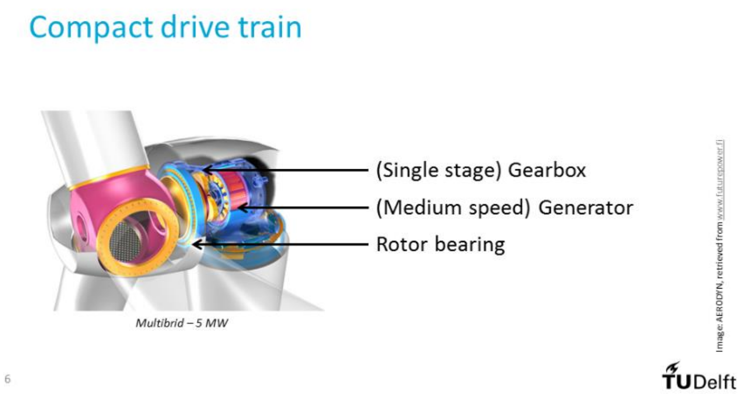
You will recognised the outside of this type of turbine as many turbines of this type have been built on land. It has the shape of a camper van but be aware that it is usually much bigger. The drive train is an elongated assembly of several medium sized components and therefore the nacelle is relatively long, but not so wide and high.



The next drive train has been a minority for a long time, but is growing in popularity. The hub connects directly to the generator and there is no gearbox. This configuration is therefore called a direct drive. The direct drive is particularly gaining popularity for the offshore market. It is expected to have a higher reliability, due to its lack of a gearbox, and that should lead to less downtime and lower maintenance costs. The generator needs to be much bigger because of its low rotational speed and it is therefore much more integrated into the structure. As a consequence, this type of turbine doesn't have an identifiable low speed shaft, but instead a large bearing to carry the rotor.

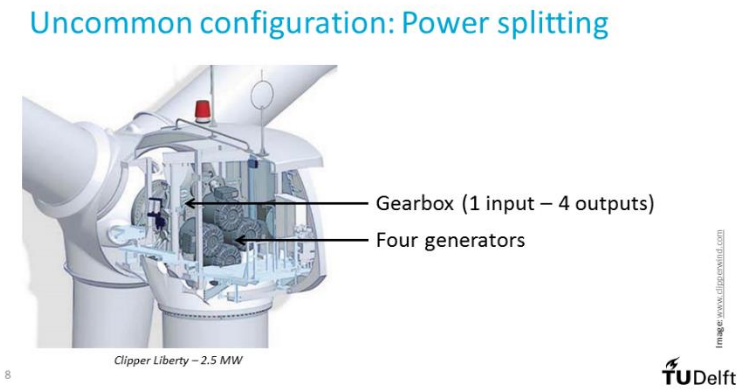


As mentioned on the previous slide, the direct drive generator is very large. This can be seen on the outside, because it invariably leads to nacelles with rounded forms. The large diameter of the generator is clearly visible, while the nacelle is shorter for the lack of a low speed shaft and gearbox.



The third configuration for the drive train is a hybrid of the previous two. It does have a gearbox, but a much smaller one. The rotational speed of the rotor is only increased by a factor of about 10, so 10 times less than in a traditional drive train. Therefore, the generator speed is higher than in a direct drive, but lower than in a fully geared system. This leads to an intermediate size for the generator. At a first glance this system seems to inherit the disadvantages of both previous concepts. It still has a gearbox that can fail and the generator is not an off-the-shelf product. However, this type of drive train can more easily be scaled to larger powers, without excessive increase in drive train mass. For the two previous configurations the mass of either gearbox or generator would increase very much with such scaling.

The hybrid drive train has a more equal distribution of its volume over width, height and length and is therefore very compact. Although it does have potential for large offshore wind turbines, there aren't many around at the moment, so you won't commonly spot them in the field.

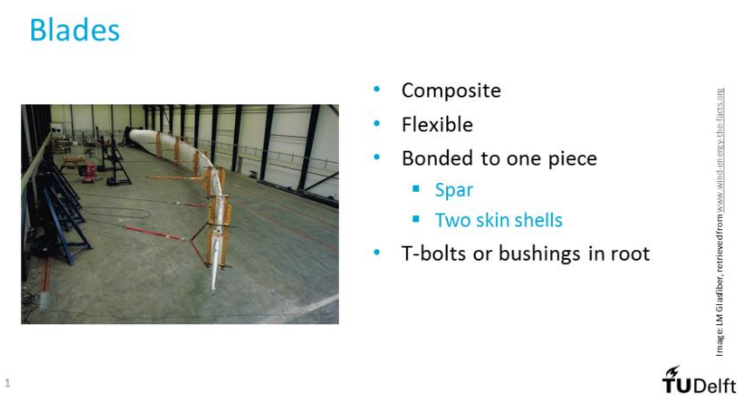


The last drive train configuration that I show here is the exception to the rule that all components are aligned sequentially. Here you see four generators that are connected in parallel to four outgoing shafts of the gearbox. Each generator has a quarter of the power rating of the turbine. In case one of the generators fails, the turbine can continue operation with a slightly reduced performance. I show this drive train to you, to make you aware that still new ideas keep popping up or old ideas are dusted off. These ideas especially deal with particular demands for offshore turbines. Every once in a while such ideas get taken a step further than just the drawing board and we may therefore see some more changes in the future.

In this video you have seen several configurations of drive trains. As you have seen, these drive trains have many components in common, but in different configurations. When we are treating the individual components in the next learning unit, you'll be able to visualise how they fit in these configurations. I hope you enjoyed this topic. Thank you very much for your attention.

1.3 Wind turbine components

1.3.1 Rotor and main shaft



Let's first look at the blade. Blades are usually made of composite material out of glass fibre reinforced plastics. Glass fibres are used instead of for instance more advanced materials such as carbon fibres because it is much cheaper. Everything in the wind turbine has to be as cheap as possible to make them affordable. Blades are very flexible as you can see here in this test rig. In the end they are one piece but they are usually build up of three elements, two shells, an upper shell and a lower shell and a spar. The spar is a beam in the middle which takes much of the loads to transfer them to the blade root. In the blade root T-bolts or bushings are laminated into the material to make the connection to the pitch bearing.

Blade twist

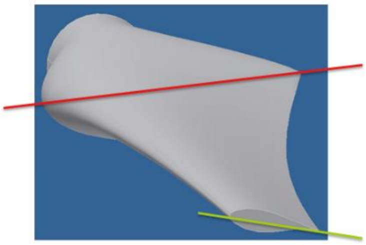


Image: user dsale on http://forums.autodesk.com

2

TUDelft

Wind turbine blades are twisted, this means that if you draw a line through the aerofoils of the blade then the lines are not parallel, as you can see here the green line which is close to the blade tip is not parallel to the red line which is close to the blade root. We'll learn later when we are looking at the aerodynamics why blades are twisted.

Large blades: pre-bending in mould

Image left: www.renvicoenergy.com
Image right: Source unknown

3

TUDelft

Large blades or multi megawatt wind turbines are also curved. On the right hand side you see the mould in which the blade is being manufactured and this mould already has a curvature. On the left hand side you can recognize this curvature in the blade. This blade is being prepared for installation and therefore it curves to the left. However when the turbine is in operation this blade will pitch over about 90 degrees and then the blade tip curves into the wind. By having this pre-bend shape the blade tip is farther away from the tower and that will give it more clearance because the blade will bend towards the tower under the wind loads. For a multi megawatt turbine with blades of 60, 70 or 80 meters we need this blade tip clearance because otherwise the tip will hit the tower.

Uncommon configuration: Split blade

Enercon E-126 – 2011 – 7.5 MW / Magdeburg-Rothensee (DE)

Enercon E-126 – 2008 – 6 MW / Emden (DE)

Image left: www.astroman.com.pl
Image right: www.enercon.de

4

TUDelft

Again I show you an exceptional configuration. This is a blade that is split in two parts. An inner part of about one quarter or a third of the blade length and an outer part. The second exception for this blade is that the inner part is made of sheet metal instead of composite material. The reason to divide the blade into parts is that it makes it more transportable. These turbines that you see here are very large but they are meant for the onshore market and that means that the elements have to be transported over land.

The blades are connected to the hub.

Hub

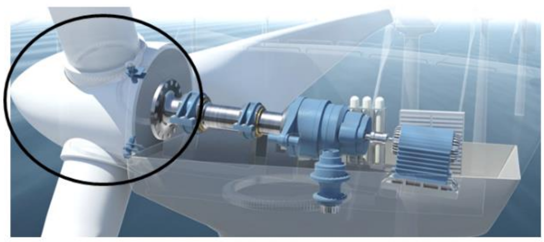
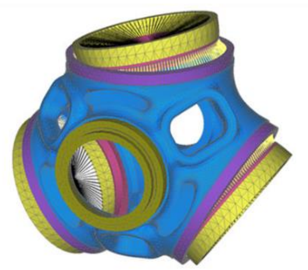


Image: www.schaeffler-fairs.de

5

TUDelft

Hub – Load carrying structure



- Transfer loads from blades to main shaft
- Typically cast iron
- Optimised for low mass

The hub consists of two parts. The first part carries the loads from the blades to the main shaft. This part is often made of cast iron because that can be very precisely optimized. This picture shows a finite element model of a hub and we can use that to determine which part of the hub receives most stresses. In places with high stress we can add material and in places with low stress we can reduce the material. Because this is a cast iron component we can play around with the distribution of the material to minimize its mass.

Hub – Nose cone / Spinner

- Aerodynamic cover
- Typically of composite (or sheet aluminium)

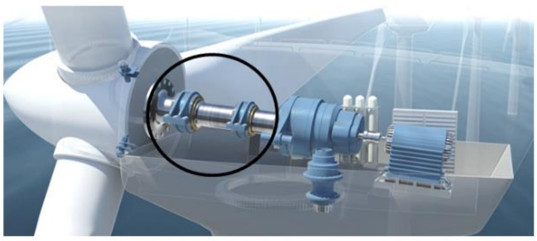
The second part of the hub is the nose cone or spinner. This is an aerodynamic cover that guides the air around the nacelle and that protects the hub. It is typically made of composite but sometimes is also made of sheet metal, such as aluminium.

Hub – Blade pitch assembly

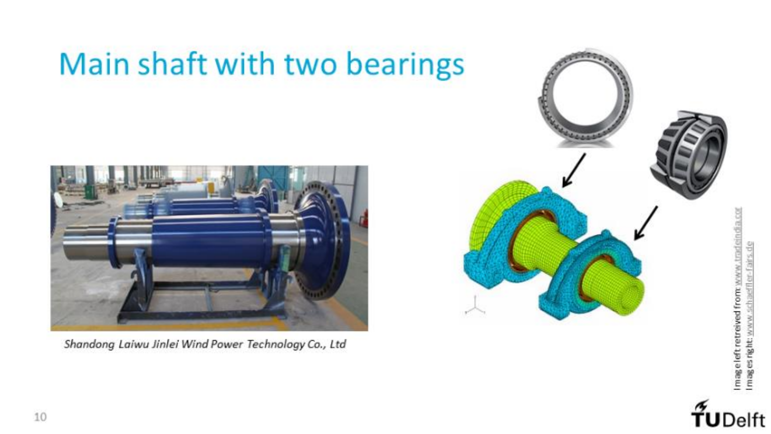
- Bearing
- Drives and gears
- Pitch brakes
- Speed and position sensors

The pitch system provides the interface between the blades and the hub. It consists of bearings that connect the blades to the hub and drives with gears that are used to set the pitch position in the proper angle. Pitch breaks and speed and position sensors are used to control the pitching of the blade.

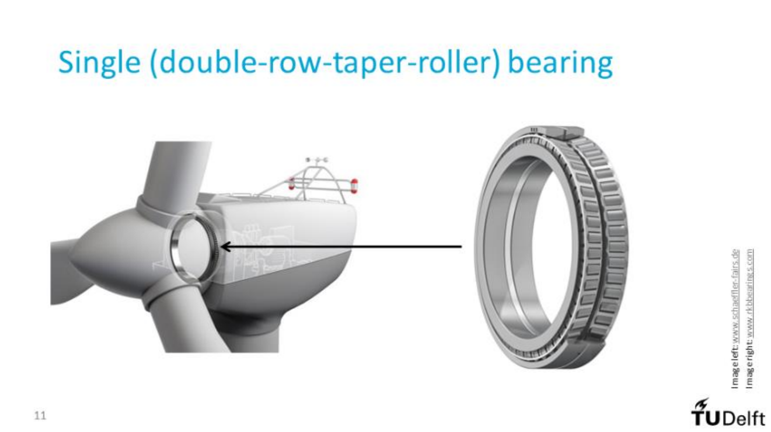
Low speed shaft



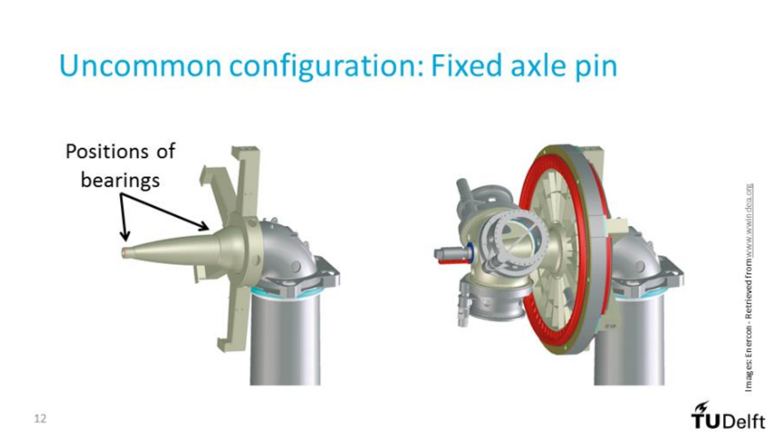
The hub connects to the low-speed shaft which we typically find in the geared system.



Main shafts are milled iron structures that are carried by two bearings. One of the bearings can take axial loads in the wind direction and vertical loads. The other bearing will only take vertical loads. This allows for elongation and shrinking of the main shaft under temperature variation. Together the two bearings carry the weight of the main shaft and the rotor as well as the bending moments.

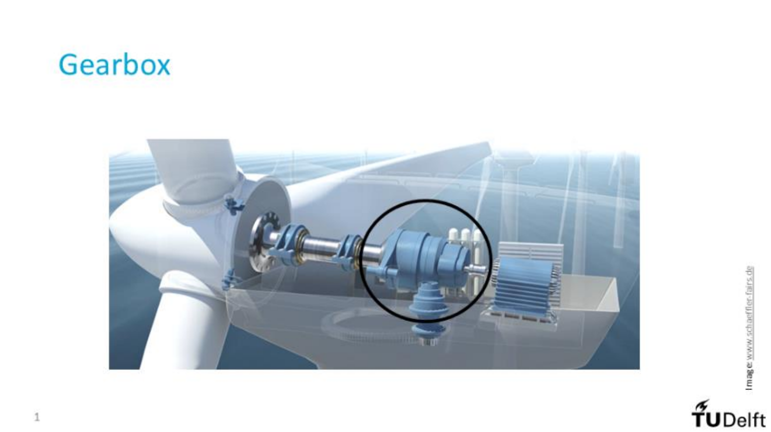


Instead of a main shaft we can also have a single bearing with a double row roller. This type of bearing can take loads in vertical direction, in axial direction and because of its diameter it can also take large bending moments. To take the bending moments in the axial forces we need a double row of rollers.

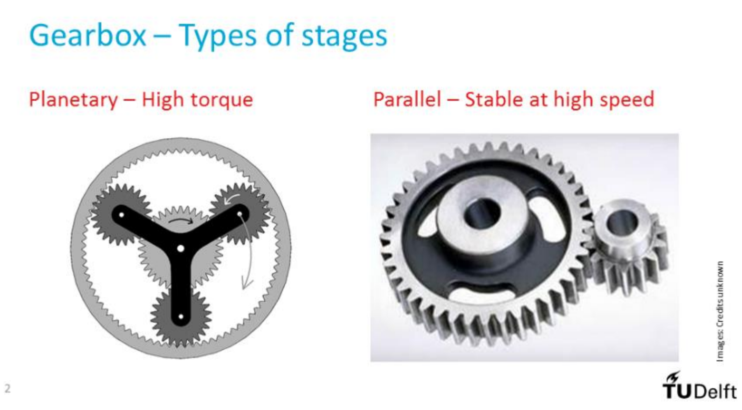


The previous configurations add a rotation inside the bearings. To show you an exception to this rule there is also a configuration with a fixed axle pin with bearings on it, and a rotation outside the bearings. This is sometimes used for direct drive systems because of the large generator. The large generator will rotate around the fixed axle pin, as well as the hub. On the right hand side you see both the hub and the generator being on the outside of the axle pin.

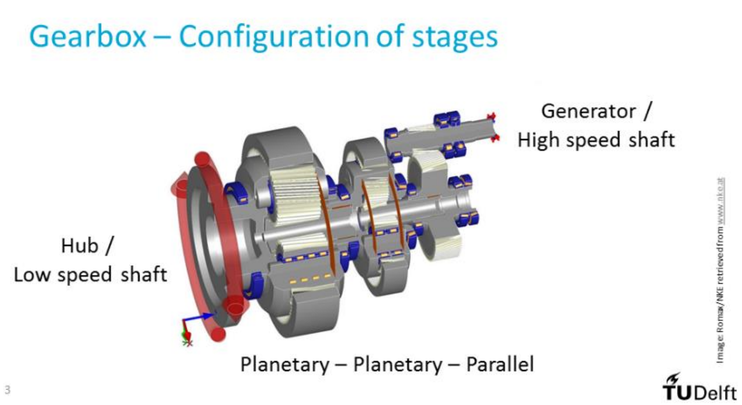
1.3.2 Drive train



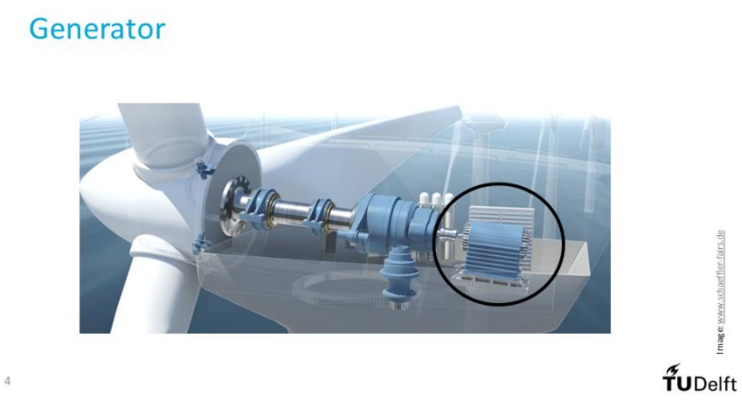
The gearbox if there is any connects the main shaft to the high-speed shaft.



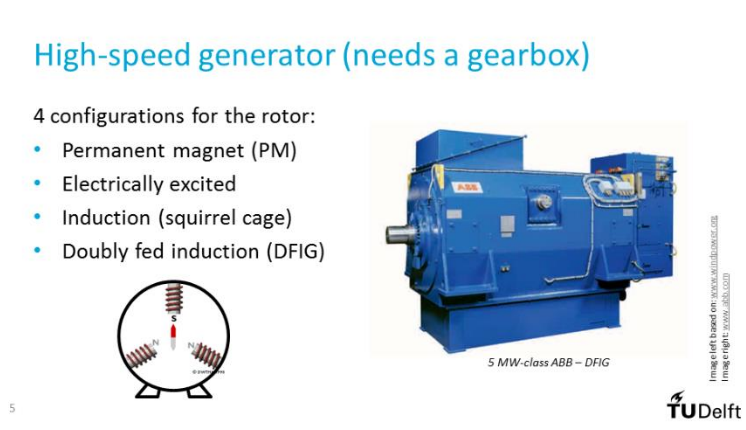
The gearbox consists of multiple stages. There are two different types of configurations that are fundamentally different, one is the planetary stage and the other is the parallel stage. In the planetary stage we have a ring wheel which is the outer wheel that is connected to the frame, so it is fixed in space. On the inside we have the planets which are the three darker wheels and the Sun which is the wheel in the middle. The planets are connected through a Y frame. The Y frame will be connected to the rotor of the turbine because if we turn that once the planets will turn multiple times and therefore also the Sun will turn multiple times, so the Sun rotates at a higher speed than the Y frame. This type of system is suitable for high torque but it is less suitable for high-speed operation. In the parallel stages we have one large wheel and a small wheel, the larger wheel is called the gear and the smaller wheel is called pinion, both have their own shafts that are kept in place by their own bearings. This makes them suitable to high-speed operation. However, there is only one point of contact between the gear and the pinion and this makes them less suitable for high torque operation. In case of the planetary gear we see that there are three points of contact making them more suitable for high torque.



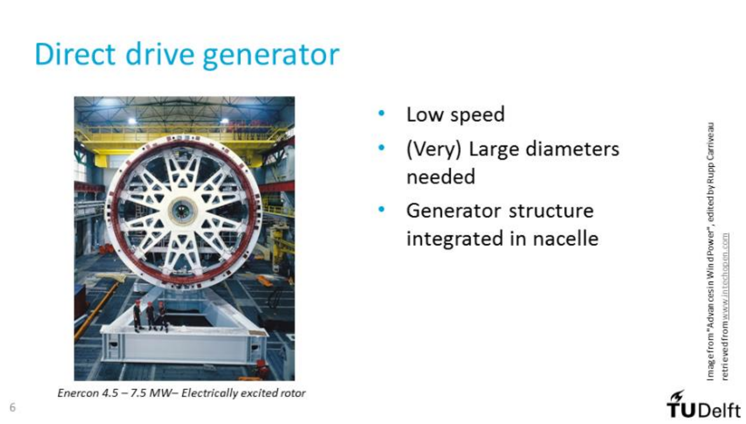
The gearbox of a multi megawatt turbine needs several stages, it needs to step up the speed from about 15 rotations per minute to 1500 rotations per minute, this cannot be achieved by a single stage, each stage can step up the speed by about five to eight times. Typically we will see about three stages in the gearbox. On the rotor side we will see planetary stages because they are suitable for low speed and high torque, on the generator side we will see parallel stages because they are suitable for high speed and low torque. In the middle we can have either a parallel stage or a planetary stage depending on the design and on the power of the turbine.



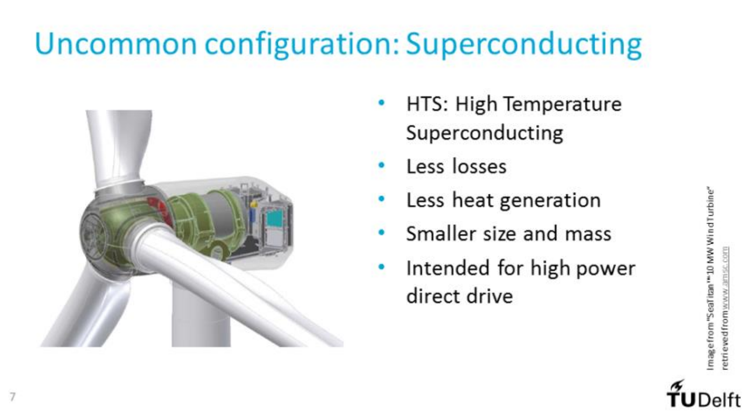
The last component in the drivetrain is the generator.



Large wind turbines use three phase generators, this means that each generator has three coils and each coil is connected to a different phase of the electricity grid. On the inside we have a rotating magnet. The rotating magnet will cause a variation in the North and South Poles at the position of the coils and this variation will create a variation in the magnetic field in the coils. The variation in the magnetic field leads to an induced voltage and current in the coils, this causes the transfer of mechanical energy from the rotating part to electrical energy in the coils. There are four ways to create the magnetism in the rotating part, the most straightforward is to use a permanent magnet in which case we have a fixed North and South Pole created by magnetism of the material. We can also use an electromagnet, and in that case we have what we call an electrically excited generator. The electromagnet is made by using a coil instead of a permanent magnet and feeding this coil with a direct current. If we do not feed this coil with a direct current but we make a short circuit so one side of the coil is directly connected to the other side of the coil, then we get induced voltages and currents in that coil also creating a magnetic field. In this case we speak of an induction generator or as squirrel cage generator. The most used configuration for wind turbines is however a doubly fed induction generator, it uses the induction principle but it has a more complicated configuration allowing it to better control the speed of the turbine.



This is the generator of a direct drive, it is much larger than a generator of a geared system. The generator on the previous slide was about one and a half to two meters, this generator has a diameter of about 10 meters as you can induce from the size of the people in front of it. This size of generator is needed because we operate it at a low speed. If we want to achieve a certain power of a generator at the low speed we need to apply a larger torque. And the larger the torque that is required from the generator the larger the volume that is needed. Because direct drives have such large generators the generator structure is integrated in the nacelle structure so each of these generators has to be designed specifically for the turbine in which they are going to be used.



Again I will show you an uncommon configuration, in this case a superconducting generator. Because direct drive needs such large generators, they are less suitable to upscale to high powers. The superconducting generator combats this effect, if you have a superconducting generator, so with superconducting wiring, then you have less losses in the wiring and that means that we have less heat generation and we need less cooling. As I mentioned before a direct drive generator needs to operate at a high torque and we achieve this high torque by using high currents in the generators, so if you have less losses we can increase the currents without increasing the need for cooling. Of course there's also a catch here which is why we can't yet commercially buy this type of turbine and that is that despite the name high temperature superconducting it is still operating at the temperature far below zero degrees Celsius, so first we need to cool down the temperature of the generator to very low temperatures and only then we operate in the superconducting regime. However, once we have cooled down the generator we don't need to have a lot of additional cooling because we don't have a lot of losses.

Drive train brakes

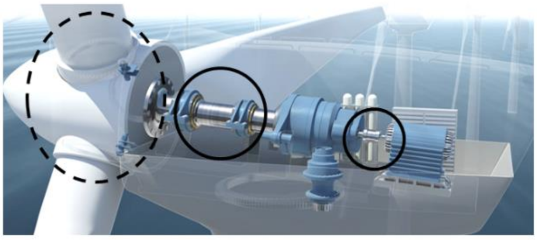


Image www.schaeffler-fairs.de

8

There will be times when we need to slow down the rotor and put it to stand still for instance in case of an emergency but also in the normal conditions when the wind speed is exceeding the speed at which we want to operate. For that we have several options for brakes.

Mechanical brake

Two types of disc brakes

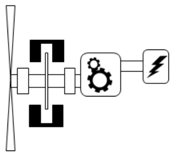
- Failsafe brakes
  - Spring applied
  - Hydraulic relieved
- Active brakes
  - Hydraulic applied
  - Spring relieved

Image www.windpowerengineering.com

9

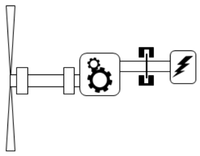
One of the options is to apply a mechanical brake, mechanical brake consists of the brake disc and the brakes themselves, with clamps. There are two types of mechanical brakes, the fail-safe brake or an active brake. In case of a fail-safe brake the braking force is applied by springs and we need hydraulic pressure to relieve the brakes. If the hydraulics fail then the brake automatically kicks in. In an active brake we need to apply hydraulic pressure to brake and the brake shoes are relieved by springs, if we want to make this fail-safe then we will have to have a redundant hydraulic system.

Mechanical brake



- Brake at main shaft
  - High torque
  - Large disc / clamping force

and/or



- Brake at high speed shaft
  - Low torque
  - Loads through gearbox

10

In a wind turbine we can apply the mechanical brake in two positions on the low speed shaft or on the high-speed shaft. If we apply it on the low speed shaft we need a high torque and therefore a large disc and clamping force. If we apply it on the high-speed shaft we only need a low torque. However, the thing we want to slow down is the rotor which has the largest inertia and that means that the forces to decelerate the rotor have to go through the gearbox, and the gearbox is a sensitive system for load variation so that's the disadvantage of having the mechanical brake on the high speed shaft.

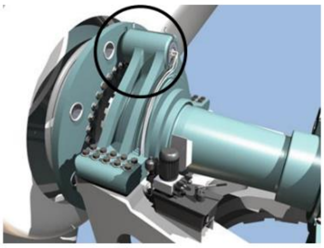
Aerodynamic brake

- Blades pitched to invert aerodynamic torque
- One blade pitch system may fail: redundancy
- Mechanical brake added at low rotational speed

11

Fortunately, nowadays wind turbines have pitchable blades, so we can pitch the blades in such a way that the aerodynamic forces are reversed, so instead of having a driving torque we pitch the blades in such a way that we get a decelerating torque because we have three blades, one of the blade pitch systems can actually fail and we can still lower the speed of the rotor. This is essential because the standards require that we have two independent braking systems. Nevertheless, once the rotor has been slowed down significantly their aerodynamic forces are reduced, so in the end we still need a mechanical brake to fully stop the rotor.

Rotor lock



- Lock bolt
- Lock disc (with holes)
- Used to park drive train (e.g. during maintenance)

Image: www.svendborg-brakes.com

12

But sometimes we will want to totally stop the rotor and be sure that it won't start rotating again, particularly when service personnel is in the nacelle. To achieve this, we have a lock bolt system, that is a disk with holes and the lock bolt will fall into the holes when it is activated.

1.3.3 Supports

Main frame / bedplate

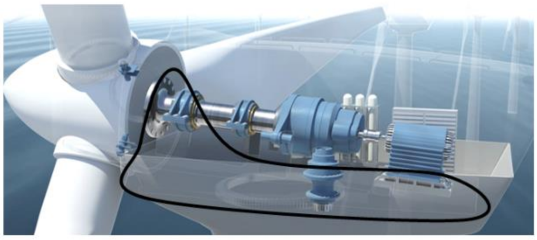


Image: www.schaeffler-fairs.de

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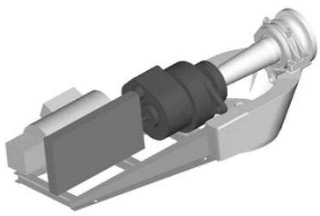
All the components are hold in place by the mainframe or bed plate.

Traditional bedplate



- Cast-iron mainframe for rotor loads (optimised)
- Welded frame to carry other components

Image: Credit unknown

2

The traditional bed plate consists of a cast iron frame and a welded rear frame. The front frame will carry the loads from the hub through the main shaft and bearings to the yaw system into the tower, so the load path from rotor to tower goes through this frame. Therefore, we also want to optimize this frame in the same way as we want to optimize the hub. Using cast iron gives us the flexibility for this optimization. The rear frame only carries the auxiliary components such as the gearbox and the generator and perhaps power converting electronics. This requires far less optimization and therefore it's easier just to use a welded frame.

Compact frames

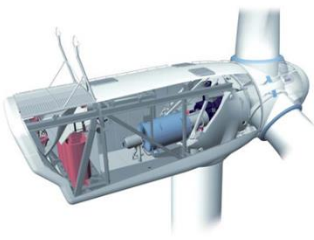
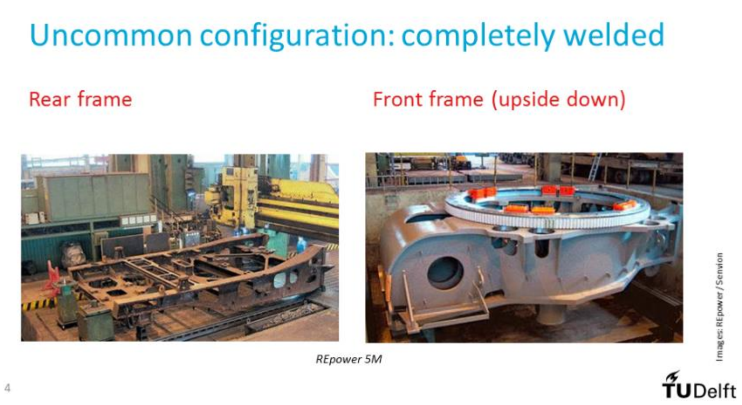


Image left: www.vestas.com
Image right: Zephyros, retrieved from www.peeraer.com/zephyros

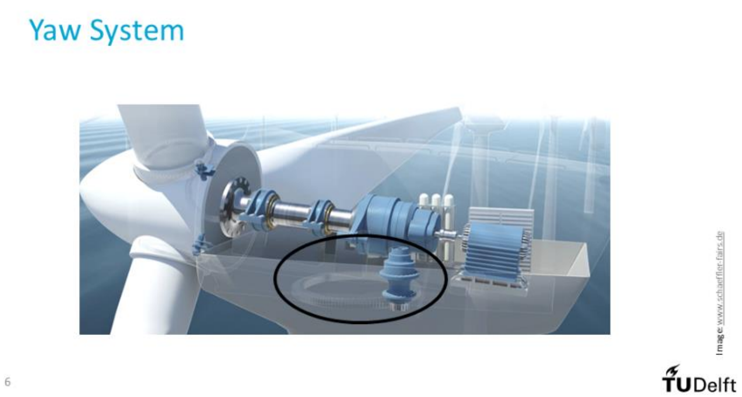
3

To keep the mass of multi megawatt wind turbines in check there is a tendency to make frames more and more compact. One of the ways to achieve this is to use a single bearing instead of using a main shaft. As you can see in these examples for a gear system in a direct drive the frame is kept as compact as possible to reduce the weight and to directly transfer the loads from the rotor to the tower.



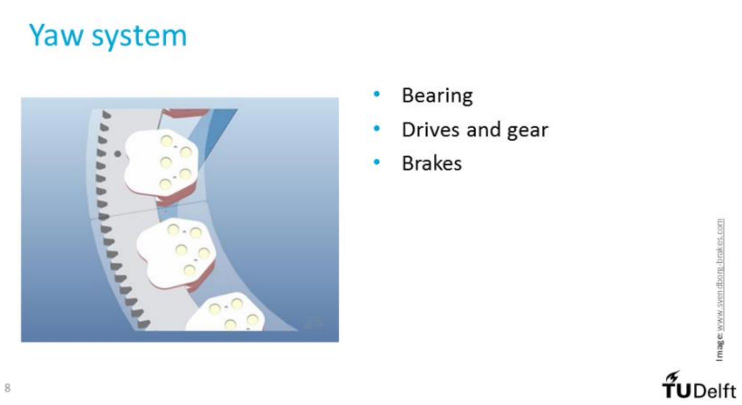
As an exception to the rule this slide shows a completely welded frame. Both the rear and the front are welded, therefore the front is not optimized for mass, however the welding of the frame can be done in many more places than making such large cast iron structures, this will get more competition but also make the manufacturer less reliant on certain suppliers which can be an advantage if you want to step up or slow down the pace at which you produce. The example that you see here of the Repower five megawatts is indeed actually one of the heaviest turbines in its capacity class. Nevertheless they have a large flexibility for finding their suppliers.

And here you see the structure assembled.

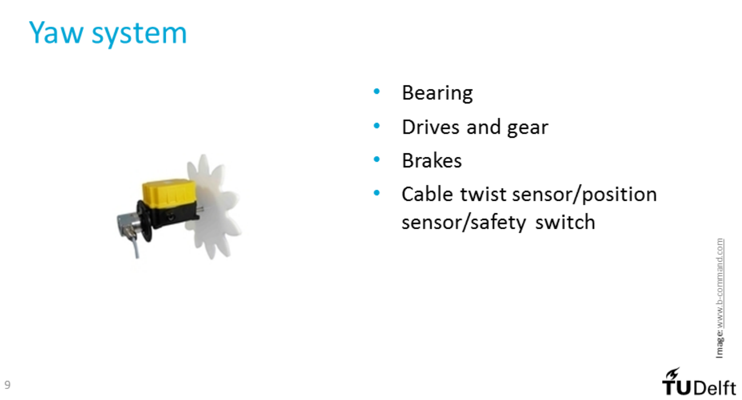


The last component that I'll treat is the yaw system which aligns the rotor with the wind. The yaw system connects the tower to the RNA.

The main component of the yaw system is its bearing, that forms the direct connection between tower and mainframe. There also drives and gears to align the turbine with the wind.



The yaw system also has brakes. When the wind turbine is aligned with the wind the brakes can be applied to keep the turbine in position. When the turbine is yawing the brakes can be lightly applied to give a little friction, if we have a little friction then the yawing goes a bit smoother.



There's also a sensor that measures how many turns the rotor nacelle assembly has made around the tower. If the rotor nacelle assembly turns then it will just the electrical cable that goes down into the tower. This cable can only twist a few times before it will break. The sensor will determine if we have reached that point and if so shut down the turbine and detwist the cable by turning the other way. This is normally done in periods with little wind when possible. If this system fails there is usually a safety switch that will turn off the turbine if the cable twists too much.

1.4 Terminology, symbols and equations

1.4.1 Terminology and abbreviations

This list will NOT be made available during the exam!

This list provides an overview of terms and abbreviations that are used in this course. The explanations are not meant as formal definitions and do not need to be ‘learned’. The list can be used to help understand the meaning of the words or what they identify. When the words are used in an exam question, you are expected to understand what is asked. E.g. in the question “Is the 3P frequency present in the stresses in the blade root?” you have to understand the meaning of ‘3P’ and to which part of the turbine the ‘blade root’ refers to. The list is not comprehensive and many terms for which the meaning follows from the equations in which they are used are not included.

1P, 2P, 3P, …

The terms 1P, 2P, 3P, etc. indicate the frequency or period of repetitive variations, particularly in loads or in the response. The terms can be read as once-Per-revolution, twice-Per-revolution, three-Per-revolution, etc. The frequency related to 1P is thus one divided by the period of one rotation and the frequency related to 3P is three times as high.

Actuator disc

An actuator disc is an imaginary disc that can exert a distributed force (=pressure) on a flow and that is used in flow analysis as a representation of a rotor (or other device for aerodynamic power conversion).

Aerofoil or airfoil

An aerofoil is a geometric shape with the property that an object with a cross-section with this shape can experience lift under certain orientations in a flow.

Angle of attack and design angle of attack

The angle of attack is the angle between the (local) flow direction and the chord line (see 'Chord'). The design angle of attack is the angle of attack at which the ratio of lift coefficient to drag coefficient is largest. The blade is designed to have this angle of attack when it is operated at the design tip speed ratio, because then the losses due to drag are lowest.

Blade element momentum theory

The blade element momentum theory is the combination of the momentum theory and the blade element theory, by equating the forces and the induction factors in the two theories. The momentum theory derives the induction factor and power extraction for a given thrust force from the momentum and energy balance for a flow through an actuator disc. The blade element theory derives the forces and power extraction for a given induction factor from the local flow conditions at a blade element and the theory of lift and drag of an aerofoil.

Blade root

The blade root is the part of the blade that connects to the hub. The other end of the blade is usually called the blade tip.

Capacity factor

The capacity factor is the ratio of the actual energy output over a year, to the potential output if it were possible to operate at rated power throughout the entire year. In other words, the capacity factor is the percentage of time needed for a generator that operates continuously at rated power to achieve the same energy production as the turbine does with its variation in power due to variation in wind speed.

Chord

The trailing edge of an aerofoil is the sharp end of the aerofoil, which is typically on the downwind side. The leading edge (or nose) is the opposite side of the aerofoil, with the more-or-less elliptical shape. The chord line is the line that connects the leading edge with the trailing edge. The chord or chord length is the distance between the leading edge and the trailing edge.

Cut-in, cut-out and rated (wind speed) and rated power

The cut-in wind speed is the lowest wind speed at which the turbine will be taken in operation. At this wind speed the turbine will have an output power somewhat larger than zero, taking into account all the losses. The (average) wind speed will need to be higher than the cut-in wind speed for some time, to avoid starting and stopping due to wind speed variations. The cut-out wind speed is the highest wind speed in which the turbine will be in operation. The turbine will shut down when the (average) wind speed exceeds the cut-out wind speed for some time. Rated wind speed is the lowest wind speed at which the turbine achieves its maximum (average) power, called the rated power. Due to gusts and turbulence the power may exceed the rated power for short periods of time.

Direct drive

A direct drive turbine is a turbine without a gearbox. The rotor is directly coupled to the generator, so rotor and generator rotate at the same speed and form a short drive train.

Drive train

The drive train is the group of components that transfers the mechanical power. The drive train typically exists of a low speed shaft, gearbox, high speed shaft and generator.

Dynamic amplification factor

The dynamic amplification factor is a measure for the difference in the response of a system under dynamic loading and under static loading. It is defined as the ratio of the amplitude of the displacement under harmonic loading to the displacement under static loading, when the amplitude of the harmonic load equals the static load.

Flapping, flapwise, edgewise and lead-lag

The words flapwise and edgewise define forces, movements or displacements relative to the orientation of the aerofoil of the blade. ‘Edgewise’ is in the direction in parallel with the chord line (see ‘Chord’). ‘Flapwise’ is the direction perpendicular to the chord line, so in the direction that is associated with the thickness of the aerofoil. Aerofoils in a wind turbine blade have their chord nearly aligned with the plane of rotation. Due to this orientation, ‘edgewise’ is almost the same as ‘in-plane’, while ‘flapwise’ is almost the same as ‘out-of-plane’. There is a small difference, because of the twist angle, which is non-zero. The word ‘lead-lag’ is sometimes used to indicate motion in edgewise or in-plane direction, without being very strict about which of the two is actually meant (e.g. as in ‘the lead-lag mode of vibration’).

Gust

A gust is a change in wind speed over a short period of time, typically seconds or tens of seconds.

Hub

The hub is the central part of the rotor, to which the blades are connected. The hub is connected to the low speed shaft or to the flange of the single bearing in a compact drive train.

In-plane and out-of-plane

In-plane and out-of-plane are used to indicate the direction of forces, movements or displacements relative to the plane of rotation of the rotor. For instance, the velocity of the blade tip due to rotation is an in-plane velocity and the thrust force is an out-of-plane force.

Inboard and outboard

The half of the blade on the side of the blade root is referred to by ‘inboard’. ‘Outboard’ refers to the outer half of the blade, on the side of the blade tip.

Induction and axial induction factor

The induction is the change in wind speed caused by the presence of the (rotating) rotor (which may be represented by an actuator disc). The axial induction is the change in wind speed in the direction of the axis of rotation of the rotor, for a horizontal axis wind turbine. When the rotor is aligned with the wind, this is the change of the wind speed in the wind direction. For a wind turbine, the change in wind speed is actually a reduction in wind speed, because the thrust force causes the wind to slow down. The axial induction factor is the change in wind speed divided/normalised by the undisturbed wind speed. There is also tangential induction, which is the change in wind speed in the direction of rotation of the blade. This is caused by the in-plane force of the blade on the wind. The tangential induction factor is not treated in this course.

Inflow angle

The inflow angle is the angle between the (local) flow direction and the plane of rotation. The local flow direction at a blade element is determined by the undisturbed wind speed, the apparent wind speed caused by rotation of the blade and the axial induction (and to a lesser extend the tangential induction).

Lift and drag (coefficient)

Lift is the force exerted by a flow on an object in the direction perpendicular to the (local) flow direction. Drag is the force in parallel with, but opposite to, the flow direction. The lift or drag coefficient is obtained by normalising the force per meter of the blade/wing with the chord and the dynamic pressure.

Low speed shaft and high speed shaft

The low speed shaft, or main shaft, is the shaft to which the hub is attached. This shaft rotates at the (low) speed of the rotor. The high speed shaft is the shaft that is connected to the generator, which rotates at a higher speed than the rotor. These two terms are only relevant for drive trains with a gearbox and not for direct drives.

Nacelle

The nacelle is the housing on top of the tower for the components and equipment, such as the gearbox, generator and electrical power converters.

Out-of-plane and in-plane

In-plane and out-of-plane are used to indicate the direction of forces, movements or displacements relative to the plane of rotation of the rotor. For instance, the velocity of the blade tip due to rotation is an in-plane velocity and the thrust force is an out-of-plane force.

Outboard and inboard

The half of the blade on the side of the blade root is referred to by 'inboard'. 'Outboard' refers to the outer half of the blade, on the side of the blade tip.

Rotational sampling

Rotational sampling is the phenomenon of variation of the local wind conditions at the position of a blade element that is caused by its rotational motion. As the blade element describes a circle, it moves through locations with different wind speeds, e.g. due to wind shear. The blade element experiences this as a variation of the wind speed in time.

Spanwise

The spanwise position means the radial position along the blade. Spanwise positions are often given as a fraction of the rotor radius, so as r/R. Using this notation, spanwise positions between 0 and 0.5 are inboard and spanwise positions between 0.5 and 1 are outboard.

Thrust and thrust coefficient

The force exerted by the rotor (or actuator disc) on the flow, in the direction perpendicular to the plane of rotation (or actuator disc), is called the thrust. The thrust coefficient is obtained by normalising the thrust with the dynamic pressure and the rotor swept area.

Tilt (angle), cone (angle) and pre-bending

The tilt angle is the angle between the axis of rotation of the rotor (and drive train) and the horizon. The cone angle is the angle between the axis of the blade bearing with a plane perpendicular to the axis of rotation of the rotor. Pre-bending is the curved shape of the blade when it is not loaded, resulting in larger distance between the blade tip and the tower.

Tip speed ratio, tip speed and design tip speed ratio

The tip speed is the (linear) velocity of the blade tip, which equals the angular velocity times the radius. The tip speed ratio is the ratio between the tip speed and the undisturbed wind speed (so the wind speed far upstream from the rotor, which is not affected by induction). The design tip speed ratio is the tip speed ratio for which the rotor is designed. When the rotor operates at the design tip speed ratio, it will have the highest power (coefficient) that can be achieved with that rotor.

Tower shadow

The reduction of wind speed in front of the tower is called tower shadow. The tower does not only lead to lower wind speeds in the wake behind it, but also just upstream of the tower. The blade that passes in front of the tower experiences this wind speed reduction and the change in angle of attack and lift force that results from it.

Turbulence (intensity)

Turbulence is the continuous variation of wind speed and wind direction over time and position. The changes happen over a wide range of time scales and a wide range of spatial scales (so they include small, quick changes and large, slow changes). The turbulence intensity is a measure of turbulence and is defined as the standard deviation of the wind speed variations divided by the average wind speed. Both standard deviation and average are determined for a 10-minute period.

Twist, section twist, structural pitch and full span pitch (angle)

The terms twist angle and pitch angle both refer to the angle between the chord line and the plane of rotation. Part of this angle is associated with the geometry of the blade, which has larger twist angles near the blade root than near the blade tip. As a consequence, the blade is 'twisted', which can be seen when looking along the length of the blade. Another part of this angle is associated with the rotation of the entire blade with the blade bearing at the blade root. The angle associated with the twist is sometimes also called section twist or structural pitch, while the rotation of the entire blade is called blade pitch or full span pitch. The division of the twist angle over these two contributions depends on the arbitrary offset chosen for the full span pitch, but typically the blade tip is chosen to have zero section twist angle. With this choice, the full span pitch angle during operation at the design tip speed ratio equals the designed twist angle at the tip. Often, 'full span pitch angle' or 'blade pitch angle' is simply called 'pitch angle', which may be confused with the use of pitch angle as given in the first sentence. However, the meaning can usually be derived from the context.

Wake, near wake and far wake

The wake of a wind turbine is the area behind the rotor where the average wind speed and wind speed variations are influenced by the interaction between the rotor and the flow. The average wind speed is smaller than the undisturbed wind speed and the turbulence is increased. The near wake is the region up to about 2 or 4 rotor diameters behind the turbine, which is dominated by the reduction in wind speed due to the induction that is related to the energy conversion process. The far wake is the region behind the near wake, where the wind in the wake mixes with the faster surrounding wind and recovers speed.

Wake meandering

The wake behind a turbine is generally not moving downwind along a straight line. While propagating downstream, it moves laterally and up-and-down as well. This motion of the wake is called wake meandering.

Wind shear

Wind shear refers to the variation in wind speed with height. Particularly, at greater height the wind speed is larger than at lower height. This effect is caused by the friction that the wind experiences at the earth's surface, which creates a boundary layer in the atmosphere.

Wind speed deficit and wake added turbulence

The reduction in the average wind speed in the wake behind a wind turbine is referred to by wind speed deficit. The wind speed deficit increases in the near wake, which means that the average wind speed decreases. In the far wake the wind speed recovers, so the wind speed deficit decreases. The added turbulence intensity is an indicative parameter for the increase in turbulence in the wake. It isn't the turbulence in the wake itself, because that is determined by the combination of the ambient turbulence and the added turbulence.

Yaw and yaw misalignment

Aligning the rotor axis with the wind direction is called yawing. The yaw misalignment is the mismatch between the rotor axis and the wind direction.

1.4.2 List of symbols

This list of symbols will NOT be made available during the exam!

Some symbols have more than one meaning. Meaning should be understood from the context in which the symbol is used. When a self-explanatory (textual) index is used, it is omitted in this list.

A, A_r, A_w, $A_{w,i}$	area, area of rotor, area of wake incidence, area of incidence of wake from turbine *i*
a	induction factor
a	scale factor of Weibull distribution
a'	tangential induction factor
B	magnetic flux density
$\mathbf{C}$	damping matrix
c	chord length
c	damping coefficient
c_D	drag coefficient of (3D) object

c_d	drag coefficient of 2D aerofoil
c_l	lift coefficient
c_P	power coefficient
c_T	thrust coefficient
c_t	damping coefficient for torsion
c_Q	torque coefficient
cf	capacity factor
D	drag force
D	fatigue damage
D	(rotor) diameter
D_w	wake diameter
DAF	dynamic amplification factor
d	diameter (of cylinder)
d_{tip}	tip deflection
E	energy production
E	modulus of elasticity
E_y	yearly energy production
$\mathbf{F}$	force vector
F	force
F_n	normal force
F_t	tangential force
$f(\ )$	probability density
f_{AC}	frequency of alternating current
g	gravity constant
H	transfer function in frequency domain
h, h_{ref}	height above the ground, reference height
I	area moment of inertia
I	current
I	mass moment of inertia
I	turbulence intensity
I_a	turbulence intensity of ambient wind
I_w	added turbulence intensity in the wake
i	interest rate
$\mathbf{K}$	stiffness matrix
K_n	Southwell coefficient
k	shape factor of Weibull distribution
k	stiffness
k	wake decay coefficient
k_{opt}	optimal mode gain
k_t	torsion stiffness
L	lift force

l	length
$\mathbf{M}$	mass matrix
M	moment
m	inverse slope (of S-N curve)
m	mass
m	mass flow
N	number of cycles to failure
N_f	number of failures (per turbine per year)
n	number of poles of generator
P	power
P_{el}	electrical power (output of the turbine)
p	pressure
p_0	environmental pressure (far away from rotor disc)
Q	torque
q	dynamic pressure
R	resultant force (sum of force components)
R	(rotor) radius
r	gearbox ratio
r	radial coordinate / radial distance
r	real interest rate
S	stress amplitude (half the stress range)
s	dimensionless distance (behind or between turbines)
SCF	stress concentration factor
T	thrust force
T_y	duration of a year
t	thickness
t	time
U	(undisturbed) wind speed
U_{10}	10-minute average wind speed
U_{avg}	(annual) average wind speed
U_{ci}	cut in wind speed
U_{co}	cut out wind speed
U_e	wind speed far downstream of the rotor disc
U_{eq}	equivalent wind speed for a rotor with partial wake incidence
U_r	wind speed at rotor disc
U_{res}	resultant wind speed at aerofoil
U_w, $U_{w,i}$, $U_{w,m}$	wind speed in the wake, in wake of turbine *i*, and in mixed wakes
UCS, UTS	ultimate compressive strength, ultimate tensile strength
V	wind speed (used occasionally – preferred symbol in this course is U)
V	velocity
V	voltage level
X	response function in frequency domain
$\mathbf{x}$	displacement vector
x	coordinate of displacement
x	coordinate (along neutral axis of bending)

x	distance (behind or between turbines)
Y	force function in frequency domain
y	coordinate (perpendicular to neutral axis of bending)
z	number of teeth
z_0	surface roughness
α	angle of attack
α	exponent of power law for wind shear
ε	strain
φ	inflow angle
φ	phase angle of harmonic signal (excitation or response)
$\Gamma(\ )$	gamma function
γ	safety factor
λ	tip speed ratio
μ	distributed mass (mass per unit length)
ρ	(air) density
σ	(normal) stress
σ	standard deviation of wind speed (over 10 minutes)
σ_a	standard deviation of ambient wind speed
σ_w	standard deviation characteristic for the added turbulence in the wake
θ	blade twist angle
ψ	angular coordinate
Ω	rotational speed of the rotor
ω	excitation frequency
ω	rotational speed
ω_n	natural frequency

1.4.3 Equations - provided at exam

These equations will be made available during the exam

Logarithmic boundary layer law:

$$U(h) = U(h_{ref}) \cdot \frac{\ln(h/z_0)}{\ln(h_{ref}/z_0)}$$

Power law: (α will be given)

$$U(h) = U(h_{ref}) \cdot \left(\frac{h}{h_{ref}}\right)^{\alpha}$$

Weibull distribution:

$$f(U) = \frac{k}{a}\left(\frac{U}{a}\right)^{k-1} \cdot e^{-\left(\frac{U}{a}\right)^k}$$

Cumulative Weibull distribution:

$$F(U) = 1 - e^{-\left(\frac{U}{a}\right)^k}$$

Weibull scale factor:

$$a = \frac{U_{avg}}{\Gamma(1+1/k)}, \text{ with } \Gamma(1.5) \approx 0.886$$

Lift and drag coefficient definition: $c_l = \frac{L}{½\rho V^2 lc}$, $c_d = \frac{D}{½\rho V^2 lc}$

Thrust coefficient definition: $c_T = \frac{T}{½\rho A U^2}$

Power coefficient definition: $c_P = \frac{P}{½\rho A U^3}$

Torque coefficient definition: $c_Q = \frac{Q}{½\rho U^2 \pi R^3}$

Torque coefficient: $c_Q = \frac{c_P}{\lambda}$

Transmission ratio planetary stage: $r \equiv \frac{\omega_{sun}}{\omega_{planet\ carrier}} = 1 + \frac{z_{ring}}{z_{sun}} = 2\left(1 + \frac{z_{planet}}{z_{sun}}\right)$

Rotational speed generator: $\omega_{generator} = \frac{4\pi f_{AC}}{n}$

Optimal mode gain $k_{opt} = \frac{½\rho c_{P,\max} \eta_{gearbox} \pi R^5}{r_{gearbox}^3 \lambda_{design}^3}$

Centrifugal stiffening: $\omega_{n,rotating}^2 = \omega_{n,non-rotating}^2 + K_n \Omega^2$

Dynamic amplification factor: $DAF = \frac{k}{\sqrt{\left(k - m\omega^2\right)^2 + \left(c\omega\right)^2}}$

Stress due to a moment: $\sigma = \frac{M\,y}{I_x}$

Area moment of inertia of a cylinder: $I_x = \frac{\pi}{64}\left(d_{outer}^4 - d_{inner}^4\right)$

Curvature, for small deflections: $\frac{d^2 y}{dx^2} = \frac{M(x)}{EI(x)}$

Ultimate limit state for stress: $\gamma_f \sigma_{\max} \le \frac{1}{\gamma_n \gamma_m} \min\left(\sigma_{yield}, \sigma_{ultimate}\right)$

Ultimate limit state for fatigue: $D \le 1$

Ultimate limit state for tip deflection: $\gamma d_{tip,max} \le Clearance\ distance\ to\ tower$

S-N curve (Wöhler) for compression: $\log(S) = \log(UCS) - \frac{1}{m}\log(N)$

(m = inverse slope of the curve)

Relative wind speed deficit in wake: $$1-\frac{U_w}{U}=\frac{1-\sqrt{1-c_T}}{(1+2ks)^2}$$

Combination of ambient and added turbulence: $$\sigma_{a+w}=\sqrt{\sigma_a^2+\sigma_w^2}$$

Wake added turbulence intensity: $$I_w=\frac{0.95}{\left(1.5+\frac{0.8\cdot s}{\sqrt{c_T}}\right)}$$

Wind speed deficit in mixed wake: $$\left(1-\frac{U_{w,m}}{U}\right)^2=\sum_{i=1}^{N_{wakes}}\left(1-\frac{U_{w,i}}{U}\right)^2$$

Equivalent wind speed on rotor in mixed wakes: $$\left(1-\frac{U_{eq}}{U}\right)^2=\sum_{i=1}^{N_{wakes}}\left(\frac{A_{w,i}\cdot\left(1-\frac{U_{w,i}}{U}\right)^2}{A_r}\right)$$

1.4.4 Equations - not available at exam

These equations will NOT be made available during the exam!

The purpose of this list of equations is to provide an overview of the most important equations that you need to know by heart or need to be able to derive yourself.

Induction factor definition: $$a=\frac{(U-U_r)}{U}$$

Thrust coefficient: $$c_T=4a(1-a)$$

Power coefficient: $$c_P=4a(1-a)^2$$

Tip speed ratio definition: $$\lambda=\frac{\Omega R}{U}$$

Capacity factor definition: $$cf=\frac{E_y}{P_{rated}\cdot T_y}$$

Annual energy yield: $$E_y=T\int_{U_{ci}}^{U_{co}}P(U)\cdot f(U)\,dU$$

Normal force dF_n: $$dF_n=dL\cdot\cos\varphi+dD\cdot\sin\varphi$$

Tangential force dF_t: $dF_t = dL \cdot \sin\varphi - dD \cdot \cos\varphi$

Gearbox ratio definition: $r \equiv \frac{\omega_{out}}{\omega_{in}}$

Transmission ratio for parallel gear stage: $r \equiv \frac{\omega_{pinion}}{\omega_{gear}} = \frac{z_{gear}}{z_{pinion}} = \frac{d_{gear}}{d_{pinion}}$

Equation of motion of SDOF system:

$k \cdot x + c \cdot \dot{x} + m \cdot \ddot{x} = F$ (translation)

$I\ddot{\psi} + c_t\dot{\psi} + k_t\psi = M$ (rotation)

Natural frequency undamped SDOF system:

$\omega_n = \sqrt{\frac{k}{m}}$ (translation

$\omega_n = \sqrt{\frac{k_t}{I}}$ (rotation)

Normal stress due to longitudinal force: $\sigma = \frac{F}{A}$ (as a function of cross-sectional area)

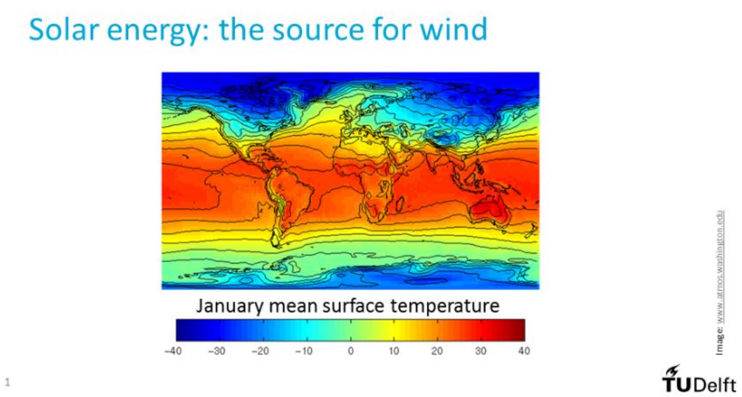
Turbulence intensity definition: $I = \frac{\sigma}{U_{10}}$

Equivalent wind speed partial wake incidence: $U_{eq} = \frac{U_w \cdot A_w + U\left(A_r - A_w\right)}{A_r}$

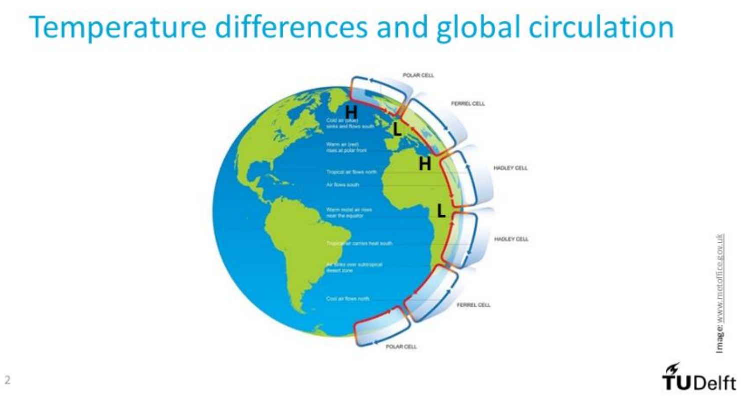
2 Wind climate and energy yield

2.1 The origin of wind

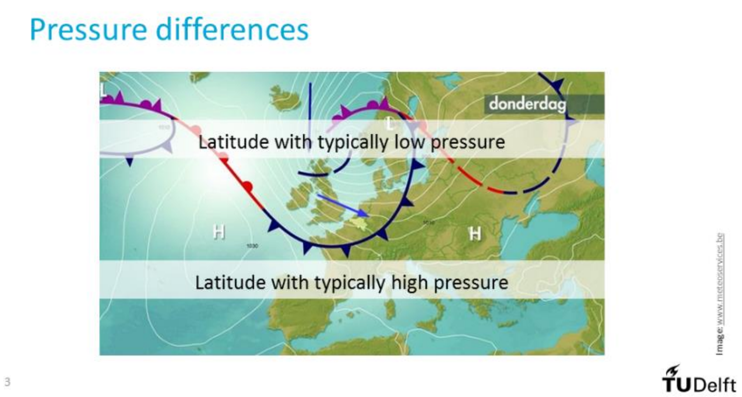
As you all know, the sun is the ultimate origin of wind. In this video, you'll see how heating of the earth surface leads wind and which patterns the wind will consequently follow.



Wind is created through temperature differences on the earth's surface. Here you see a map of the mean surface temperature in January. Obviously, the temperatures are higher around the equator than around the poles and this is driving the global patterns of wind.



If you look at the earth's atmosphere from the side, you can see how the temperature variation over the earth's surface leads to circulation cells. Around the equator warm air rises. Higher up in the atmosphere, this air has to make place for new rising air and therefore it moves either north or south. Eventually, it will hit colder air coming from the direction of the poles. At this point, the two air flows sink back to the earth's surface. A similar, but opposing circulation can be seen at the poles. Cold air at the north pole sinks and has to move to the south. There it encounters air coming from the south and these two flows have to rise. Because of the thickness of the atmosphere, there is room for three of those cells between the equator and each pole. At points where the air rises, it leads to a low pressure region in the lower atmosphere. Where the air sinks, the lower atmosphere becomes a high pressure region. Because of the size of the circulation cells, the north-west of Europe typically experiences low pressure regions.



This typical latitude of high and low pressure regions is visible on this map of a particular day. The high-pressure region is around the Mediterranean and the low-pressure region is around Scandinavia. The white lines represent isobars, or lines with equal pressure. The wind moves along those lines. In first instance, you would actually expect the wind to go directly from high pressure regions to low pressure regions. But we'll see later how the earth's rotation causes the wind to deviate from this direct route and follow the isobars. Because the low-pressure regions are in the north and the high-pressure regions are in the south, we'll see that this causes patterns with mainly westerly wind in north-west Europe.

Why is wind parallel to isobars (1)

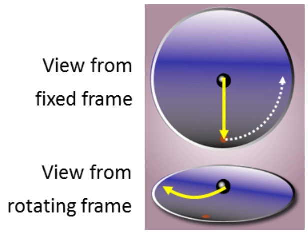


Image: www.de.wikipedia.org under GFDL

4

TUDelft

Let's have a look at this rotating disc and a ball that is moving along a straight line in a fixed frame of reference. If we would look at the motion of this ball from a frame of reference that is fixed to the disc, it would appear to be curved to the right. You can see this in the lower part of the image.

Why is wind parallel to isobars (2)

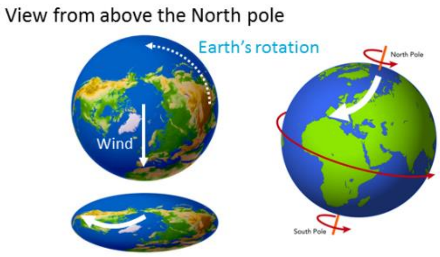


Image: Credits unknown

5

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The same happens if we would have wind moving in a straight line from the north pole to the equator. Looking at the earth from the top, so at the north pole, the earth's rotation is counter clockwise. While the wind is moving from the pole to the equator, the earth underneath it therefore moves to the right. Looking at the wind from a frame of reference fixed to the earth, the wind would then appear to curve to the right. The fictitious force that causes this curvature is called the Coriolis force.

The law of Buys Ballot

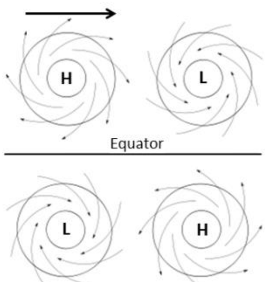


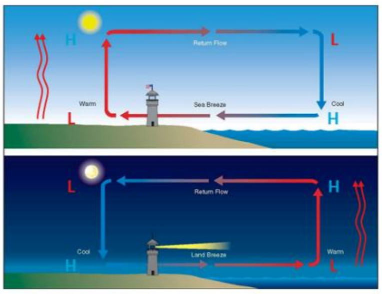
Image right: www.commons.wikimedia.org

6

TUDelft

If we were to look at the Coriolis force in more detail, we would find the law of Buys Ballot. This law states that on the northern hemisphere wind turns clockwise around high pressure regions and counter clockwise around low pressure regions. This explains our pattern of westerly winds in north-west Europe. The upper two circles show the patterns of wind around high and low pressure regions on the northern hemisphere. Imagine the low pressure region on the right to be north of the high pressure region. The wind will then move from left to right between the two pressure regions, so in westerly direction.

Local wind generation (1)



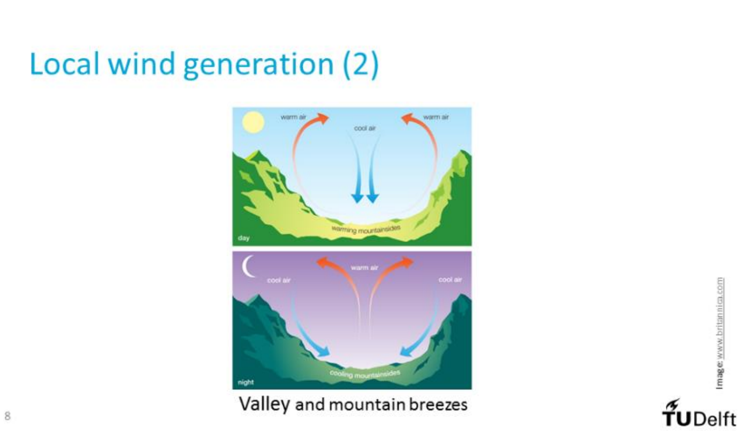
Sea and land breezes

Image obtained from: www.nymetroweather.com

7

TUDelft

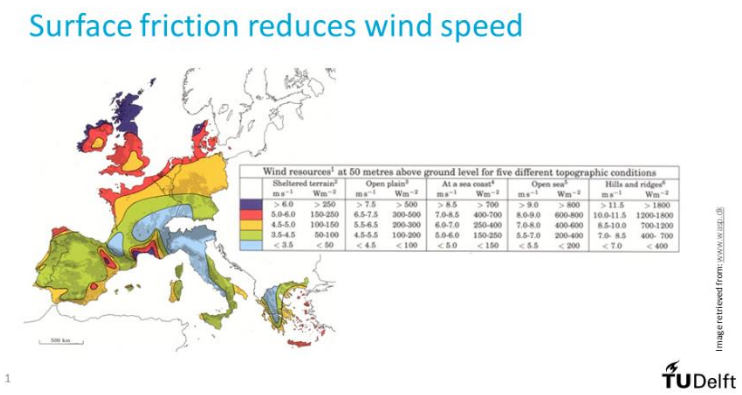
So far, we've looked at global patterns in temperature variation over the Earth's surface leading to large patterns in the wind. Now we're going to look at some local temperature differences leading to local patterns in the wind. If we have a coastal region with land on one side and sea on the other side, we also get temperature differences. During the day, the land heats up faster than the ocean which has quite a constant temperature during the season. This means that the air rises over land, leading to a low-pressure region, and it drops over sea, leading to a high-pressure region. This causes a sea breeze coming from the sea to the land. At night, the earth's surface cools down while the sea keeps its same temperature and we get a reversal in the pattern. You can recognise this pattern in the Netherlands. The sea breeze increases the predominant westerly wind during the day and at night the wind drops due to the reversed flow.



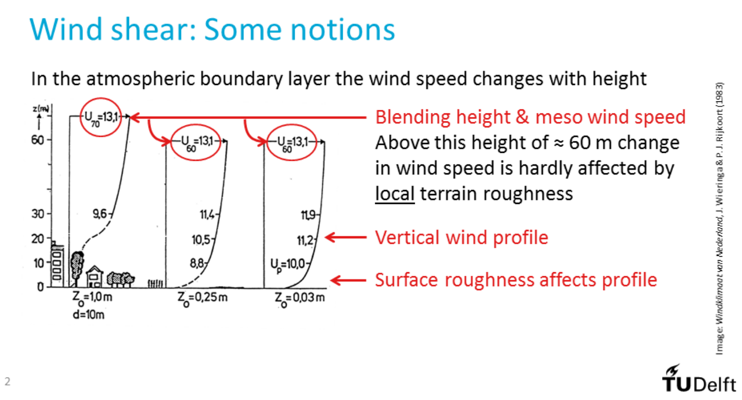
A similar effect can be found in mountainous regions. In principle, the higher you go the colder it becomes. However, during the day the surface of the mountain peaks will heat up faster than the surface of the valleys. This causes air to rise against the mountain slopes and to drop in the valleys. At night, the mountain slopes will cool down quicker than the valleys and also here the pattern reverses. With this last example of local creation of wind from solar energy, we will close off this overview of the origin of wind.

2.2 Wind speed variation in space

2.2.1 Wind shear

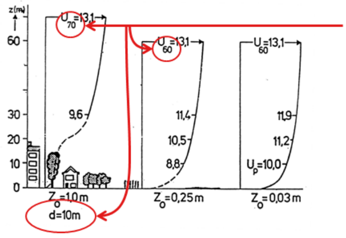


This map shows wind speeds at 50 meters height over a large part of Europe. The more purple-reddish the colour the higher the wind speeds, and the more blueish-green the lower the wind speeds. As you can clearly see, closer to the ocean the wind speeds are higher than further inland. This is caused by the friction of the earth surface as wind is moving over its surface. This takes out some of the energy from the wind and, therefore, the further inland you go the lower the average wind speeds will be.



Locally the friction of the Earth's surface creates what we call the atmospheric boundary layer, which is a vertical wind profile that goes from zero wind speed at the earth surface to the atmospheric wind speed higher up. The shape of this atmospheric boundary layer depends on how much roughness we have on the surface or how much friction. You can imagine if we have houses and trees we have more friction than if we just have meadows. At some height, which we call the blending height, the local effect of earth surface roughness doesn't have any influence anymore on the boundary layer profile. There is still an increase of wind speed with height, but the shape of that increase is no longer dependent on what we have on the Earth's surface exactly.

Wind shear: Model



Offset of blending height by d = effective height of impenetrable roughness (further ignored in this course)

Image: Windklimaat van Nederland, J. Wieringa & P.J. Rijkoort (1983)

3

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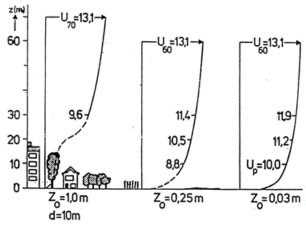
If you have higher obstacles such as trees their effect on the boundary layer profile will reach higher heights or the blending height in those cases is higher than in for instance the case of a meadow. This is expressed by an offset in the Earth's surface by the parameter d the effective height of impenetrable roughness. In other words, we assume that less wind is reaching the Earth's surface. We will not use this expression for the offset in this course and will simply assume a blending height of 60 meters for all conditions.

Wind shear: Logarithmic wind profile



$$U(h) = U(h_{ref}) \frac{\ln\left(\frac{h}{z_o}\right)}{\ln\left(\frac{h_{ref}}{z_o}\right)}$$

h_{ref} is the **reference height** at which the wind speed is known (measured)

z_o is the **surface roughness length**

Image: Windklimaat van Nederland, J. Wieringa & P.J. Rijkoort (1983)

4

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To estimate the wind speed at different heights and the effect of local surface roughness, the atmospheric boundary layer is modeled with the logarithmic wind profile, for which you see the equation here. It expresses the wind speed at an altitude h by the wind speed at a reference height href and it uses a logarithmic description which is a function of this height and a function of Z0. Z0 is the surface roughness length, so that expresses how much friction we have on the Earth's surface.

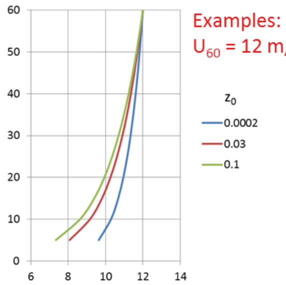
Terrain roughness classification → z_0

very open landscape
z_0 ~ 0.01-0.03 m

open landscape
z_0 ~ 0.05-0.1 m

rough landscape
z_0 ~ 0.2 m

very rough landscape
z_0 ~ 0.4 m



Examples:
U_{60} = 12 m/s

at sea $z_0 \approx$ 0.0002 m, depending on wave conditions

Image left: Windklimaat van Nederland, J. Wieringa & P.J. Rijkoort

5

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The surface roughness length is not a direct measurement of the size of the obstacles. Instead, we use a classification of Z0 for different types of landscape that gives the best fit of measurements of the atmospheric boundary layer to the logarithmic profile. For instance, with a Z0 of about 1 to 3 centimeters, we match the data very well for very open landscape. Here you see four types of landscape and their representative Z0 . Over sea the friction is much lower and, therefore, we have a very very low Z0. This Z0 over sea does depend on wave conditions, but we'll consider it to be constant for the purpose of this course, because there is much less difference between the Z0 over sea than over land. Here you see an example of different surface roughness lengths for a wind speed at 60 meters height of 12 meters per second. So, the lower the surface roughness length the less influence we have from the friction over land or over sea and, therefore, you see that, for in this case the blue line, for conditions over sea the wind speed doesn't drop as much as for the green line, which is the wind speed over a rough landscape.

Roughness map of the Netherlands

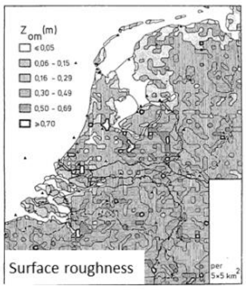


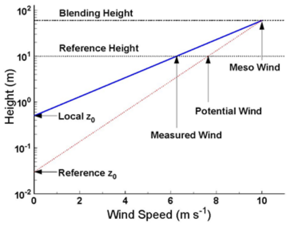
Image: Windklimaat van Nederland, J. Wieringa & P.J. Rijkoort (1983)

6

TUDelft

The best way of guessing the surface roughness somewhere is to simply go there and look at the terrain. However, you could also use a map such as this one. What is informative from this map as well, is that the surface roughness has been determined per area of five by five square kilometers. This is the size of the area that influences the local atmospheric boundary layer shape. These kind of maps are not often used and as you might see this is already a quite old map from 1993. It's much more common to use actual local data.

Potential wind speed



Potential wind speed is:

- hypothetical wind speed
- at 10 m height
- assuming very open landscape

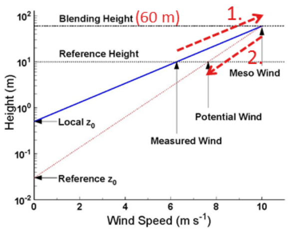
Used to eliminate influence of terrain at place of measurements

Image: http://projects.knmi.nl

7

So, local surface roughness influences local wind speeds and, therefore, also measurements that are always taken at a certain position. To get rid of this influence we use the potential wind speed. The potential wind speed is a hypothetical wind speed that we would measure at 10 meters height, if we would have very open landscape. So, it is effectively the kind of wind you would expect in the most ideal conditions. We use the potential wind speed to make sure that in different positions close together we will use the same wind speed irrespective of the local surface friction.

Determining potential wind speed



1. Translate measured wind speed to 60 m with local surface roughness length
2. Translate down to 10 m with standard surface roughness length

over land: z_0 = 0.03 m
at sea: z_0 = 0.0002 m

Image: http://projects.knmi.nl

8

Now, let's take a look at the graph of how we determine this potential wind speed. The graph is again the logarithmic profile of wind speed variation with height, but now with the height expressed on a logarithmic scale. Therefore, the profile now looks as a straight line. The blue line indicates the actual profile at a certain site, so influenced by the local surface roughness. We measure the wind speed at a reference height, in this case 10 meters, and then we use the logarithmic profile to translate that wind speed up to the blending height of 60 meters. Now, to get the hypothetical wind speed at 10 meters for very open landscape, we translate this wind speed at blending height, so the meso wind speed, down to 10 meters, using the standard surface roughness length or the length of the roughness over very open terrain, which is 0.03 on land. We will use the standard surface roughness length of 0.0002 for situations over sea.

Annual average potential wind speed NL

Potential wind speed averaged over a year

Notice the 'smoothness' in the variation

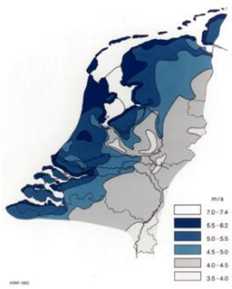
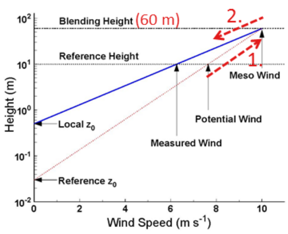


Image: www.knmi.nl

9

When this procedure has been performed for many places we can make a map of the potential wind speed, and here you see the potential wind speed over the Netherlands. What you notice is that the variation is very smooth and this is because we have now taken out the local effects of surface roughness. So, effectively we are only seeing the effect of friction over a larger region because of the decrease of wind energy by the friction of the earth. But we don't see any more local effects because of trees and houses and bushes etc.

Using potential wind speed



1. Translate potential wind speed to 60 m with standard surface roughness length
2. Translate down to height of interest with local surface roughness length

Image: http://projects.knmi.nl

10

With the potential wind speed from such a map or a numerical value of potential wind speed that has been given, we can determine the actual local wind speed by reversing the process that we have used to determine the potential wind speed in the first place. So, first we translate the potential wind speed from 10 meters height, using the standard surface roughness length, up to the meso wind speed at 60 meters height. After that, we use the local surface roughness to translate the wind speed at 60 meters height down to the height in which we are interested.

Wind shear: Power law wind profile

At higher altitudes (above blending height of 60 m) the logarithmic wind profile deviates from the actual wind profile

- Better model for that region: power law
- $U(h) = U(h_{ref})\left(\frac{h}{h_{ref}}\right)^{\alpha}$
  - $\alpha \sim 0.143$ over land
  - $\alpha \sim 0.11$ over open water (sea)

11 TUDelft

So far we've looked at the variation of wind speed with heights or altitudes lower than 60 meters. Here we could use the logarithmic wind profile, because it expresses the effect of local surface roughness by its parameters Z0. As said before, above 60 meters we do not see the influence of the local surface roughness anymore, so there the logarithmic profile is not suitable. In this region, we'll be using the power law. The power law also expresses the wind speed at the height h with the wind speed at the reference height, which we'll multiply with the ratio of the heights to the power alpha. For alpha, we'll use constant values which differ for wind speeds over land and wind speeds over sea. There will be a transition in a coastal region but for the purpose of this course, we'll simply use the constant value.

How to use the different wind profiles

- The logarithmic profile is most suitable between 10-20 m height
- The power law profile is most suitable above 100 m height
- In between they both work fairly well

Recommendation (used in this course):

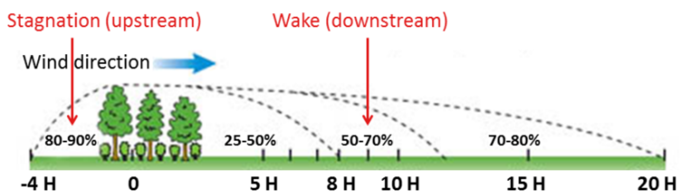
- Use logarithmic law below the blending height (60 m) with the appropriate surface roughness length
- Use power law above the blending height with the appropriate α
- Use both laws sequentially by determining the meso wind speed at 60 m as an intermediate result

12 TUDelft

So, now we have two mathematical models for the wind speed variation with height. The logarithmic profile in the power law. At lower altitudes, say between 10 and 20 meters height, the logarithmic profile is most suitable, because it expresses the influence of local surface roughness by its parameters Z0. At higher altitudes, say 100 meters height, the power law is more suitable, because it is independent of the local surface roughness. In between, around the blending height these two profiles more or less overlap and they work both equally well. Therefore, we recommend to use the logarithmic law below 60 meters height and the power law above that. If we want to translate the wind speed from a height below 60 meters to a height above 60 meters we have to use both laws. First, we translate the wind speeds to the blending height, using the logarithmic law and then we use this wind speed at 60 meters height as the new reference to be substituted in the power law.

2.2.2 *Obstacles and turbulence*

Wind in front and behind obstacles



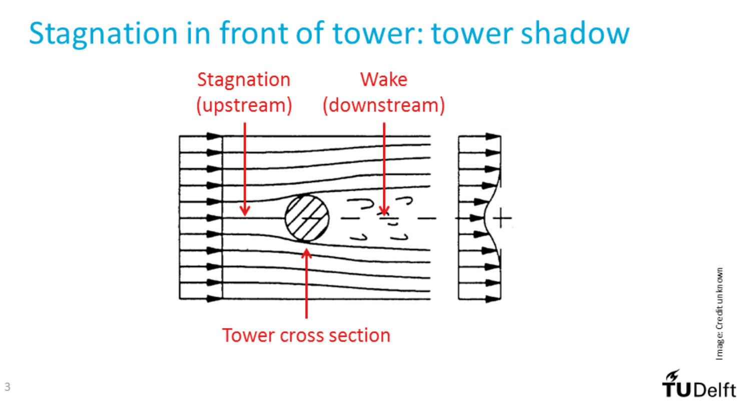
Zones of reduced wind velocity due to obstacle as percentage of open field velocity. Vertical scale exaggerated.

Image: Credit unknown

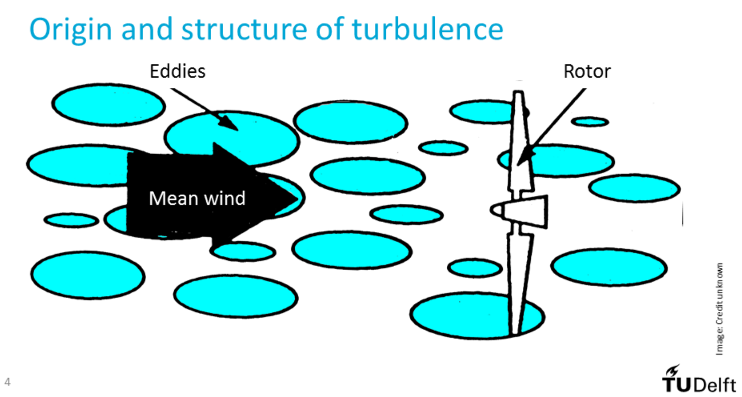
1 TUDelft

This slide zooms in on an obstacle to illustrate how it affects wind speed locally. Obstacles do not only influence the boundary layer profile and therefore the variation of wind speed with height. They also affect the wind speed variation in the horizontal direction. Directly in front of the obstacle there is a stagnation zone, and this zone extends to about 4 times the height of the obstacle. Behind the obstacle, the area of influence is much larger, up to about 20 times the height of the obstacle. This region is called the wake.

A wind turbine itself is also an obstacle, which influences the wind speed at downstream turbines. Here you see a famous aerial photograph of the Horns Rev wind farm. The wakes are visible as vapour trails, clearly showing that they extend up to and beyond other turbines in the wind farm. The photograph also shows that a turbine in a wind farm can be in the wake of multiple turbines, leading to a cumulative reduction in wind speed.



Another particular obstacle that we need to consider for a wind turbine is its own tower. As any obstacle it has stagnation upstream of it and a wake downstream. The wake is usually not an issue because it doesn't affect the possible downstream turbines. By the time it gets to a downstream turbine, the wind speed has already been completely recovered. But the upstream stagnation creates a lower wind speed just in front of the tower which influences the blades when they are passing the tower. When the blade is in front of the tower, it experiences a slightly lower wind speed. This lower wind speed will lead to a reduction in the force on the blade during its passage in front of the tower.

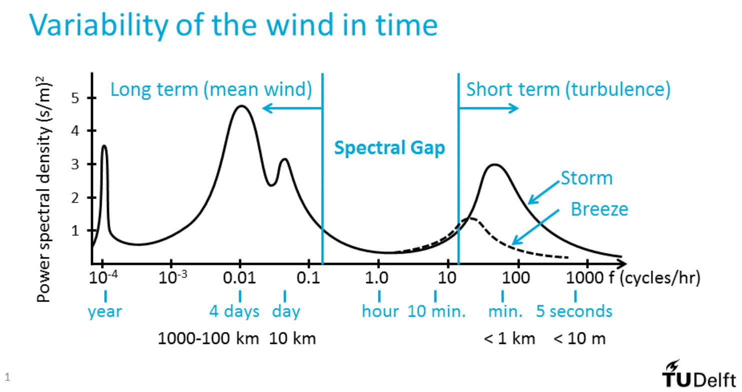


All the phenomena that are described before, such as rising hot air, friction of the earth's surface and individual obstacles, lead to a variation of wind speed in space in certain patterns. However, the patterns that have been shown so far are averages. If we look closely at a snapshot of the wind at a particular moment in time, we will see that there is a large local variation in wind speeds on top of these patterns. This is what we call turbulence. Turbulence itself also appears in patterns or structures, which we call eddies. A visible example of an eddy is the swirl of a river that streams around an obstacle, such as a bridge pillar. These eddies appear in different sizes. The largest size corresponds to the type of patterns that we have seen before, for instance, the size of an obstacle. As these eddies move downstream they break down to smaller scales, and this process continues up to a scale of about one millimetre. Eddies move along with the main flow and will pass through the rotor of a wind turbine. During this passage, they cause local variations in wind speed and direction, and therefore variations in the force on the blades. This ends the overview of phenomena that cause variations of the wind speed in space.

2.3 Wind speed variation in time

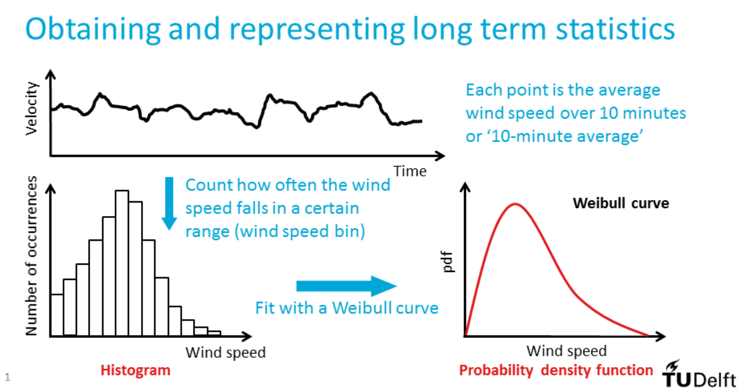
2.3.1 Spectral gap

Having addressed the wind speed variation in space from global scale to millimetre scales, we'll now have a look at the variation of wind speed in time.



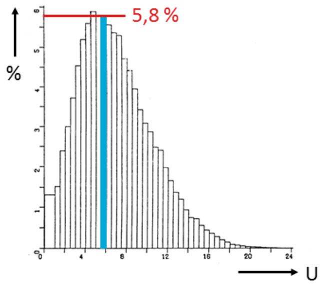
As an introduction to the subject of wind speed variations in time, we will look at the separation into long term variations and short term variations. Here you see the spectrum of wind speeds, which captures the energy content as a function of the frequency, or the period at which the variation occurs. The frequency and period are shown along the horizontal axis, with yearly variations on the left and variations of a few seconds duration on the right. The vertical axis can be interpreted as the contribution to variability of wind related to each time scale. For a time scale of a year, we see the seasonal variations. These are caused mainly by the changes in the temperature distribution over the earth, leading to different distributions of high and low pressure regions for different seasons. These variations have a global scale. The peak in the spectrum for a time scale of about 4 days is caused by the passage of high and low pressure systems. Such pressure systems move slowly and have an effect on the wind speed variations over a region of several hundred kilometres. On a daily basis, we see the regional effects of sea breeze and mountain-slope winds due to the daily pattern in heating and cooling. On a shorter time scale, we see the turbulence created by terrain, obstacles and warm surfaces. Turbulence covers a range of time scales from seconds to minutes, and is associated with spatial variations from several millimetres up to about a kilometre. The energy content of these variations depends on the average wind speed and is evidently higher during storms than during a calm period. The dip in the spectrum between daily variation and turbulence is called the spectral gap. We'll use this dip to distinguish between long term wind climate characterization and short term wind speed variations. The separation is typically put at the 10-minute mark. All variations with a period longer than 10 minutes are part of the long-term climate. The short-term variation is what happens within each 10-minute period. The remainder of this learning unit will show how the long term and short term variations can be characterised.

2.3.2 *Long-term variations*



First, we'll have a look at the long-term statistics of wind speeds. This graph shows the 10-minute average as a function of time. So, every dot on this graph is a measurement of 10 minutes and then the average of that. We can translate this graph to what we call a histogram, which gives the number of occurrences for wind speeds within a certain range. So, each of these bars represents the number of occurrences of what we call a bin of wind speeds (say wind speeds between 4 and 5 meters per second). So, in the time series, we'll look up all the wind speeds that are within 4 and 5 meters per second range, we'll count them and that determines the height of the bar. We will then translate this histogram to a probability density function by fitting it with a Weibull curve, which will be further discussed later on.

Meaning of a histogram

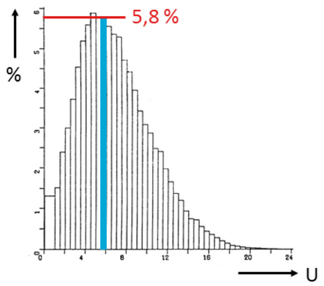


Measurements at Den Helder

- X-Axis
  - Wind speed in *m/s*
  - Divided in bins of *0.5 m/s*
- Y-Axis
  - Percentage of time that wind speeds occurred that fall within that bin

Let's take a closer look at the meaning of the histogram. On the x-axis of the histogram, the wind speed is separated in ranges or in bins, in this case of 0.5 m/s. So, each of the bars covers one of those bins. On the y-axis, we plot the percentage of time that the wind speed occurs in this range. So, if we have a time series and we'll find a wind speed between 5.5 and 6 meters per second, we'll add it to the bin of the blue bar.

Meaning of a histogram



Measurements at Den Helder

In this example there is a *5.8%* probability that *5.5 < U < 6 m/s*

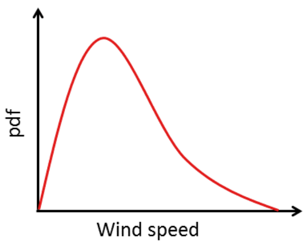
In this example, after processing all the data, we found that 5.8 percent of the samples of 10-minute averages fell into the range of 5.5 to 6 meters per second. So, the probability of occurrence of wind speeds between 5.5 and 6 meters per second is therefore 5.8 percent.

The Weibull distribution *f(U)*

The **Weibull function** *f(U)* is a two-parameter probability density function (pdf)

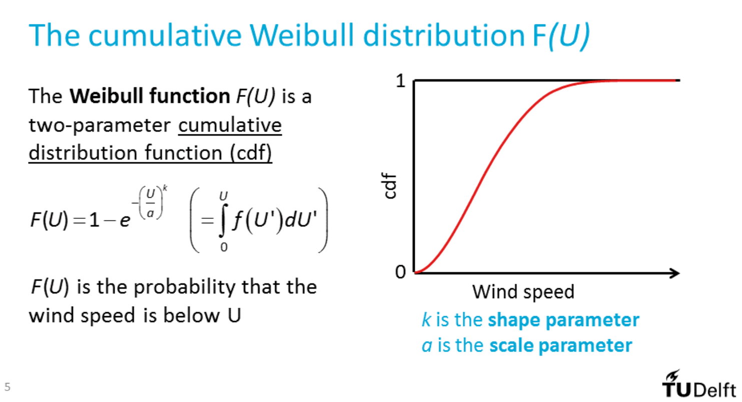
$$f(U)=\frac{k}{a}\left(\frac{U}{a}\right)^{k-1}e^{-\left(\frac{U}{a}\right)^{k}}$$

$f(U)\cdot dU$ is the probability that the wind speed occurs in the range *(bin)* dU

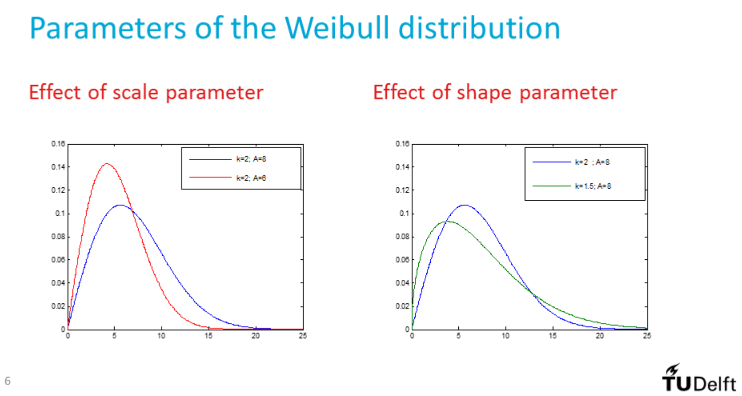


k is the **shape parameter**
a is the **scale parameter**

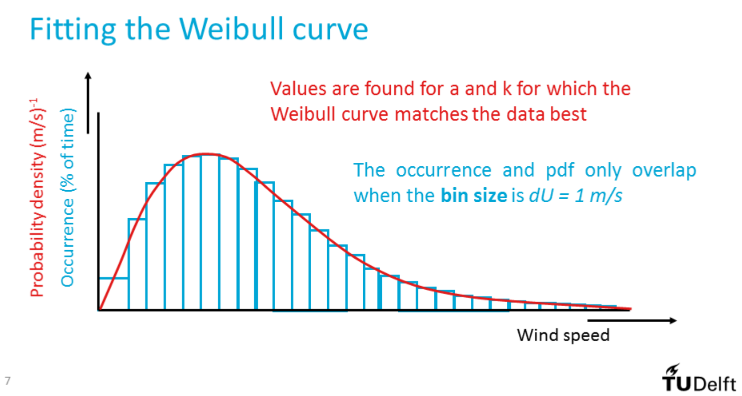
When histograms were made for different places on earth, it was found that they often match very well with a Weibull function. This is a two-parameter probability density function with the parameters k, the shape parameter, and a, the scale parameter. So, by varying these two parameters the histograms could be matched for very different types of wind conditions. The probability density function means that, to get the probability of a wind speed falling in a certain bin or a certain range of wind speeds, we have to multiply the value of the probability density function, f(U), with the width of the bin or the range of wind speeds, dU. So, if we want to compare the probability density function with the histogram, and following our example for Den Helder with wind speed bins of 0.5 m/s, we can estimate the probability of wind speeds falling between 5.5 and 6 meters per second, by taking the value from the probability density function and multiplying it with a dU of 0.5 meters per second.



Sometimes it's more convenient to express the Weibull function in a different form: The cumulative distribution function. This function gives the probability that the actual wind speed at any random moment is smaller than the wind speed on the x-axis of this graph. As you have seen, the probability density function times a small bin width gives the probability that the wind speed falls inside that bin. So, if we integrate this from 0 up to a particular wind speed, we get the probability that the actual wind speed is between 0 and the wind speed up to which we integrated the probability density function. This leads us to the relation that is shown here between the cumulative distribution function indicated with capital F and the probability density function indicated with small f. Because we have this analytical expression for the cumulative Weibull distribution function, we can easily determine the probability that the wind speed is, say, between 5 and 10 meters per second. This is simply the function's value at 10 meters per second minus the function value at 5 meters per second. We can also easily determine the probability that the wind speed exceeds say 25 meters per second that is simply 1 minus the function value at 25 meters per second.

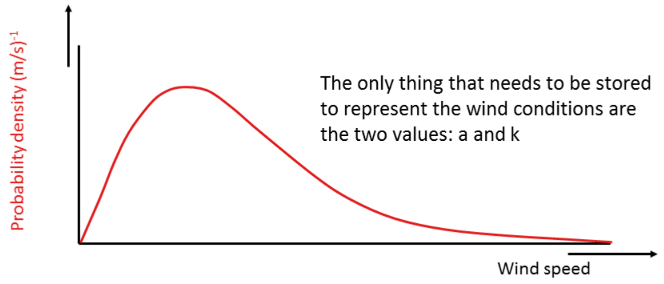


These two graphs show the effect of the scale parameter and the shape parameter on the Weibull curve. The left-hand side shows two graphs with the same value for the shape parameter, but with two different values for the scale parameter: a is equal to 6 and a is equal to 8. In the case of a being equal to 8, we have a higher probability for higher wind speed and, therefore, the blue curve has a higher weight for higher wind speeds. The red curve has a higher peak but this is simply because the integral of the function, which is the area under the curve, has to equal 1. On the right-hand side, you see the effect of the shape parameter. So, here we have two curves with the same value for the scale parameter a, equal to 8, and two values for the shape parameter k, equal to 2 and equal to 1.5. Here, the difference is in the peakedness of the curve. The higher the scale parameter the more peaked the curve will be and this means that there is a higher probability of wind speeds being in a very narrow range.



To get the values for the shape and scale parameters of the Weibull curve we fit it to the data or more precisely we fit it to the histogram. The fit means that we let the curve fall as neatly as possible over the data or, more mathematically, to get the least residuals or the smallest differences between each data point and the value obtained from the Weibull distribution. In this example, the curve falls neatly over the data. However, this can only be expected if the bin size dU is equal to 1 m/s or, in other words, the percentage of occurrence that is given for the histogram corresponds to an occurrence in a range of 1 m/s, say from 4 to 5 meters per second wind speeds. Suppose we would have a wind speed range for the histogram of only 0.5 m/s then the percentage of occurrence for each of the blue bars would drop by about a factor 2, because the bin size has been reduced by a factor 2. So, in that case, the red curve and the blue curve don't overlap anymore. This relates to the fact that the probability density function can be translated to a probability by multiplying it with the step size dU.

Weibull curve captures long term conditions



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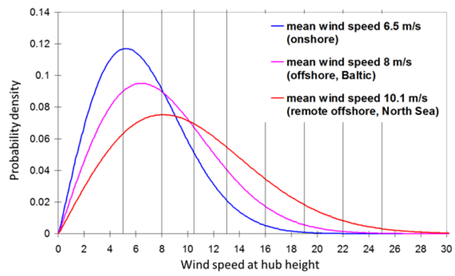
Once we have fitted the viable curves to the data, the only thing we need to store to represent the wind conditions are two values: the scale and the shape factor. So, instead of having a full-time series of data or having a histogram with a lot of bars, we only need to know two values to represent the wind conditions. Since this works on very many places on earth this is a very strong method.

Some typical Weibull distributions



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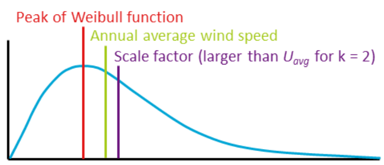
Here you see some examples of Weibull curves obtained for different locations on earth. The blue curve gives the condition for an onshore location with an average wind speed of 6.5 meters per second. The two other curves give conditions at offshore locations in the Baltic, respectively a more remote offshore location in the North Sea. As you can see, the further you go offshore the higher the probability will become for higher wind speeds. So, the curves become lower but more inclined to the right.

Some more properties and values

$$a = U_{avg} / \Gamma(1 + 1/k)$$

- U_{avg} is the **annual average wind speed**
- North-West Europe Coast
  - $k = 2$
  - **Gamma function** $\Gamma(1.5) \sim 0.886$
  - $a \approx U_{avg}/0.886$



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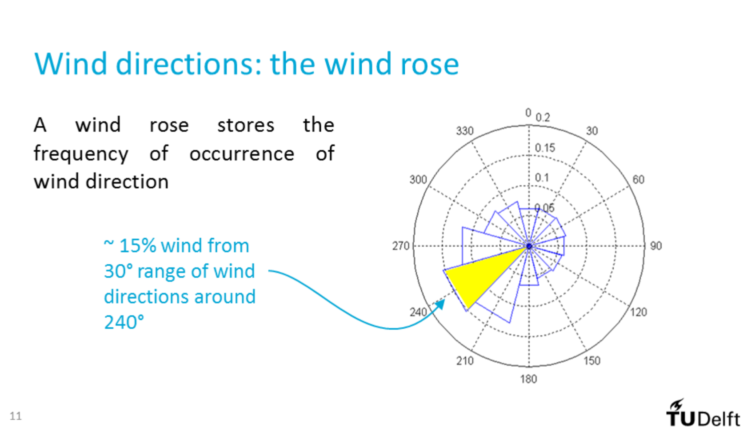
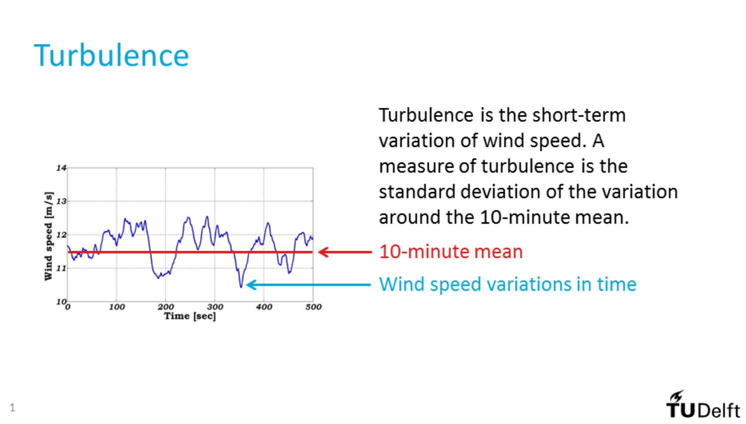
In the previous slides, I mentioned the average wind speed, but this is not directly used by the Weibull curve, which uses the shape and scale parameters. However, there is a relation between the three parameters. The scale parameter is equal to the average wind speed divided by the gamma function of 1 plus 1 over the shape parameter. So, if we do know the average wind speed, we can calculate the scale parameter, if we know the shape parameter as well. Over north-west Europe, the shape parameter doesn't change very much and is approximately equal to 2. So, we can use the gamma function of 1 plus 1 over 2, the gamma function of 1.5, which is approximately equal to 0.886, to find a relation between the scale parameter and the average wind speed. Typically, for cases such as the shape parameter of two, the annual average wind speed is on the right of the peak of the Weibull distribution. So, don't confuse the peak of the Weibull distribution with the annual average. And as the relation between scale factor or scale parameter and annual average wind speed shows, the scale factor is larger than the average wind speed for a shape factor of 2.

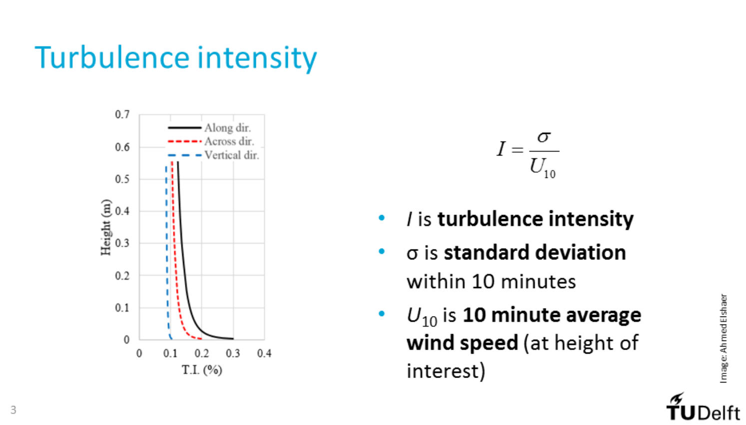


Not only wind speeds change over time, but also wind directions, and this is captured by the wind rose. Much the same as using wind speed bins or ranges of wind speeds to create a histogram for wind speed, we can use sectors or ranges of wind direction to create a histogram of wind direction. The wind rose is actually just a polar version of a histogram. In this example, we have ranges of wind direction of 30 degrees and each time the wind direction falls into this range we will add it to this bin. So, in the example 15 percent of the time the wind was coming from directions in a 30-degree range around 240 degrees.

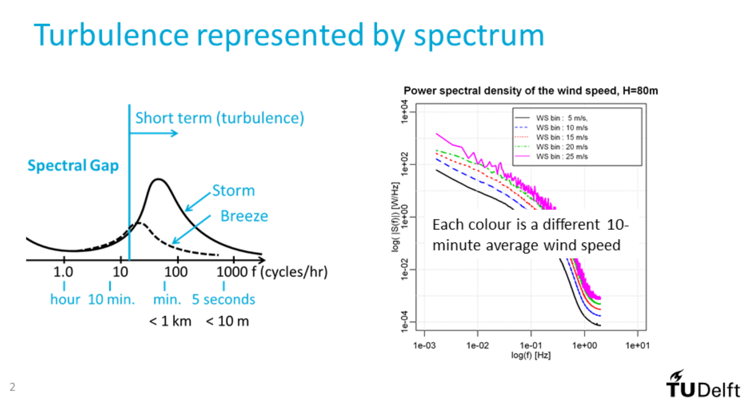
2.3.3 *Short-term variations*



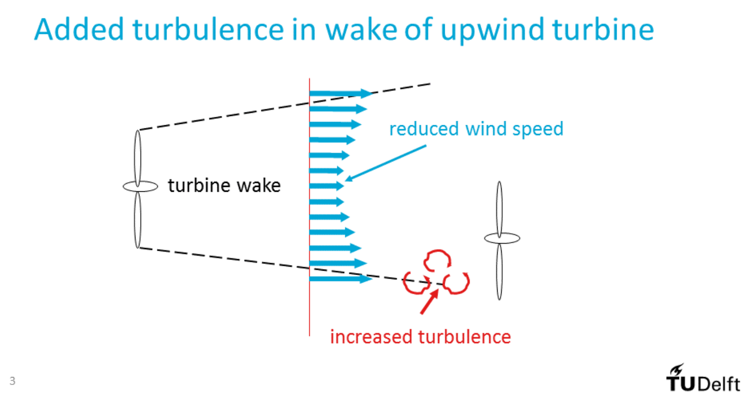
Now that we've captured the long-term character of wind by the Weibull distribution and the wind rose, we'll have a look at the short-term variations. If the 10-minute average wind speed captures the long-term effect, the variation around that 10-minute average is the short-term effect, and this is called the turbulence. The turbulence is often characterised by the standard deviation of this variation around the mean.



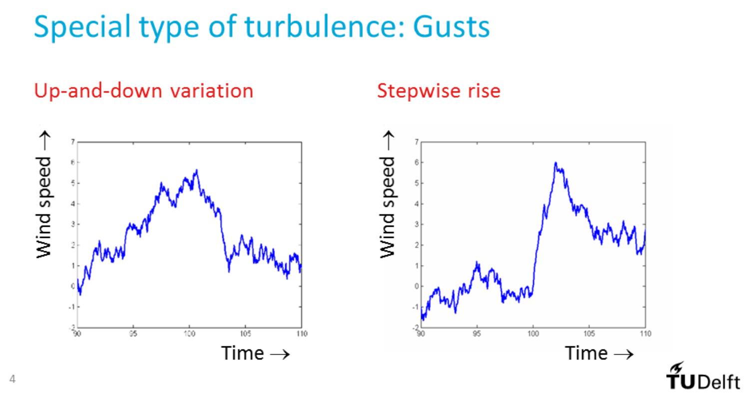
Although the turbulence itself is characterised by the standard deviation of the wind speed variation, the most common parameter to describe turbulence is the turbulence intensity, I. This is the standard deviation of 10 minutes of wind speed variation, σ, divided by the 10-minute mean wind speed, U10. The turbulence intensity is a dimensionless parameter and is either expressed as fraction or as percentage. It is typically between 5% and 30%. Because obstacles are one of the important causes of turbulence, the turbulence intensity is typically larger over land than offshore. Turbulence intensity is also a function of height.



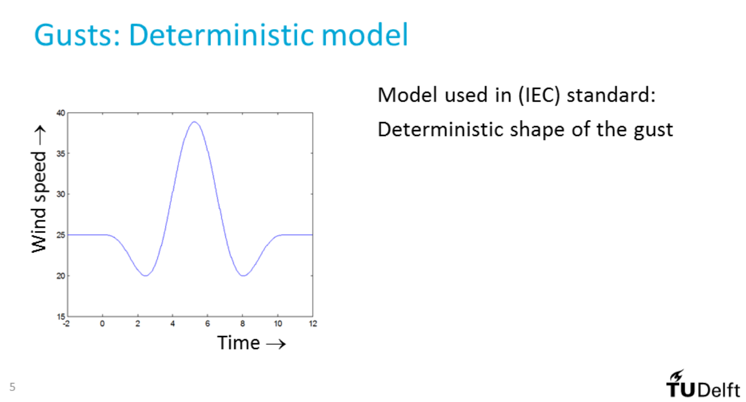
For the long-term variations of wind speeds, you have seen that we do not need to store all measurement data to capture the wind characteristics at a certain location. Instead, we use only the Weibull shape and scale parameters. Similarly, turbulence is usually not represented by extensive data series, but by a few parameters that describe the spectrum of the wind speed variations. You have already seen such a spectrum, when we were looking at the separation of wind speed variation in long-term and short-term variations. The part of the graph on the right-hand side of the spectral graph, is actually the spectrum of turbulence. The standard deviation is an important parameter in the specification of this spectrum. However, we have seen that turbulence also has a spatial component, identified as eddies with different scales. This indicates that turbulence doesn't only have variations in the wind direction, but also in both directions perpendicular to it. It also indicates that there is some level of correlation in turbulence at two nearby locations. Such aspects are important when we want to determine the wind conditions over the rotor plane. However, how all these aspects of turbulence are captured in just a few parameters, is beyond the scope of this course.



We have already seen that different phenomena cause the turbulent structures, and of course this also affects the turbulent velocity changes over time. Let's have a look at the turbine as an obstacle. Not only does it lead to reduced wind speeds in the wake, but also to increased turbulence. Particularly at the edge of the stream tube, which is influenced by the blade tips, we will experience an increase in turbulence. However, the further downstream we go, the more this turbulence is observed everywhere in the wake.



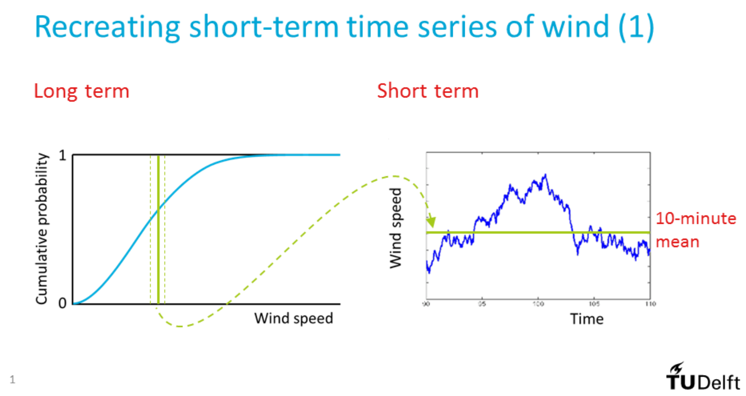
Looking at the structure of turbulence as a variation of wind speed over time, there is a particular type of variation that can be observed, which we call a gust. A gust can be an up-and-down variation in the wind speed, or it can be a stepwise rise in the wind speed. The gust is characterised by the height of the variation and the period over which the variation occurs.



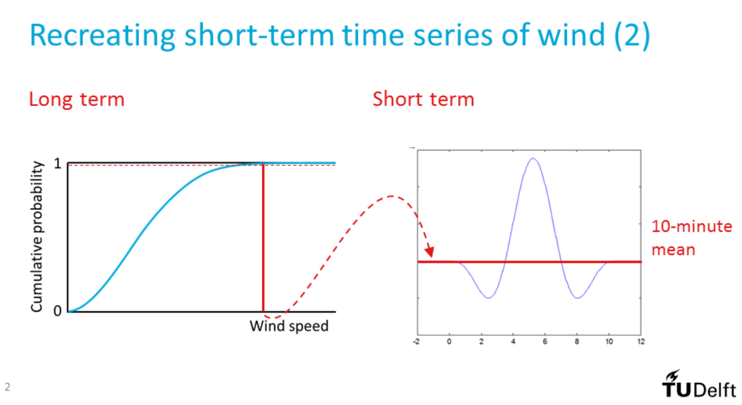
In the standards that prescribe how we have to analyse a wind turbine to get a certificate, the shape of the gust is usually simplified. Here you see how the IEC standard, which is the most commonly used, specifies the shape of the gust as a Mexican-hat shape. So first the wind speed drops a little bit, then it increases, and then at the end of the gust there is again a slight drop in the wind speed before it returns to the average. This is a much simpler and deterministic description of the gust than the random variations that you've seen on the previous slide.

2.3.4 Recreating short-term time series of wind

In the previous explanations, we have separated the wind conditions into the long-term wind climate and the short-term wind speed variations. You have seen how we can characterise them and store information about them. Sometimes, it's enough to only use the long-term description of the Weibull distribution. For instance, when you want to estimate the annual energy yield, you don't need to know the details of the short-term periods. However, for some purposes we need to recombine the long-term and short-term characterisations. The most common case is when you want to predict whether or not a wind turbine will collapse during its lifetime. You normally do this with simulations of the wind turbine motions, for which you need to reproduce reasonable time series of wind speed variations.



Here we'll show two principles that are often applied. The first principle is used when we want to reproduce normal, every-day variations in the wind. On the left-hand side you see the cumulative Weibull distribution. What we want to do is recreate a time series of the wind speed variation for a 10-minute period. Depending on the conditions we want to simulate, we pick a wind speed that occurs regularly. For instance, we pick a 12 meter per second wind speed, during which the turbine is normally in operation and producing electricity. We let this wind speed represent a small range of wind speeds, the wind speed bin. We can determine the probability of occurrences of wind speeds in this bin and use that to estimate how often this situation will occur during the lifetime of the turbine. As you have seen, the wind speed in the Weibull curve represents a 10-minute average. To create the short-term time series of the wind speed, we have to make a realisation of wind speeds that has this average value. For example, we use the definition of the spectrum of turbulence, in which we can substitute the value of 12 meter per second for the average wind speed. A time series created from this spectrum definition will automatically have an average of about 12 meter per second. The time series is created with an inverse Fourier transformation, which is outside of the scope of this course.



The second principle is used when we are interested in extreme wind conditions. From the long-term characterization, we take a wind speed with a low probability, for instance the probability that it is exceeded only once every fifty years. For that 10-minute average, which is a high wind speed, we add the turbulent effect. As in the previous case, this can be done by creating a realisation from the spectrum. In the example shown here, a prescribed deterministic gust is created. This gust is expressed by the period over which the variation takes place and by its height, which is a function of the average wind speed. The x-axis on the right-hand side is not to scale, since the period of such a gust is much smaller than 10 minutes.

Extreme wind conditions

Extremes conditions are a combination of a 10-minute average and a gust amplitude, e.g.

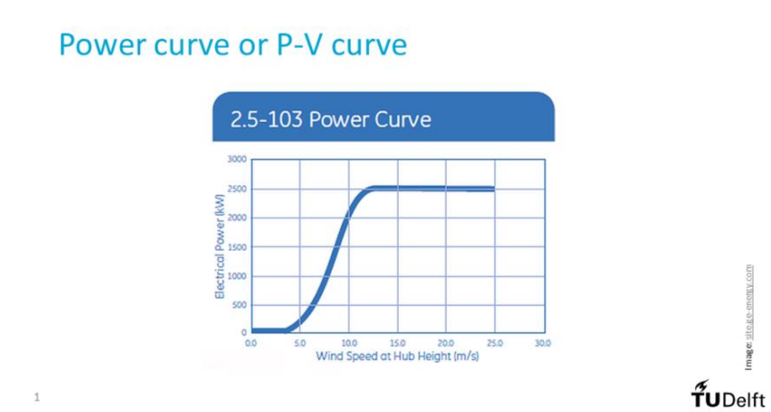
- 10-minute average with 50 year return period, with 'reduced' gust amplitude
- 10-minute average with 1 year return period, with 'extreme' gust amplitude
- Rated wind speed as 10-minute average, with 'extreme' gust amplitude

3 TUDelft

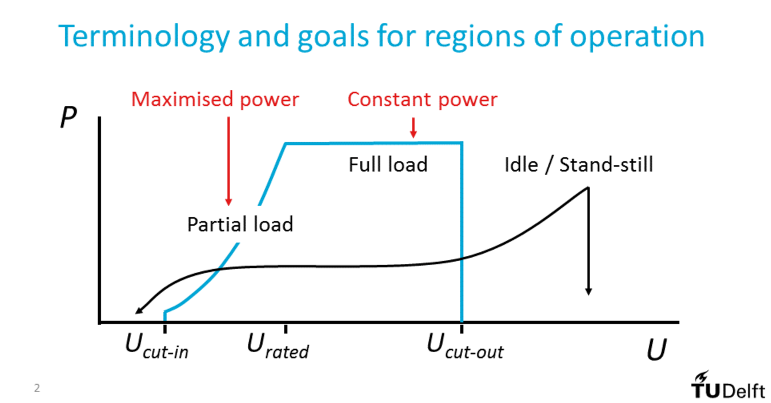
The previous slide gave an example of how the time-series of an extreme gust can be created. It isn't very likely that if we take a 10-minute average wind speed that reoccurs every 50 years that we'll experience a gust with an extreme height inside those ten minutes. A wind turbine typically lasts about twenty years, so it's already fairly unlikely to experience this extreme 10-minute average, and therefore it is also extremely unlikely to have an extreme gust at that one time that we'll experience this 10-minute average. Therefore the standards prescribe that we combine a 10-minute average speed with a 50-year return period with what is called a reduced gust. The height of a reduced gust is slightly lower than that of the extreme gust and therefore it has a higher probability of occurrence. However, if we take the 10-minute average wind speed that is exceeded every year, then this might happen about twenty to thirty times during the lifetime of a wind turbine. Therefore, it's more likely that one of those times we'll experience a gust with extreme height. For this reason the standards prescribe that we combine a 10-minute average extreme wind speed with a one year return period with an extreme gust inside those ten minutes. The common principle for the prescribed combinations of average wind speed and short-term realisation is that the combined probability from the long-term statistics and from the short-term statistics is sufficiently high to be considered realistic. So, after first separating the wind speed fluctuations in long term and short term variations, you now know how these can be recombined to create representative wind conditions.

2.4 The power curve

Before we can estimate the energy yield of a wind turbine at a certain site, there is something we need to know about the performance of that wind turbine. That something is what is called 'the power curve'. This video explains the power curve, showing its typical shape and introducing terminology that is used to characterise it.



Here you see an example of a power curve taken from a brochure for the GE 2.5 megawatt turbine. It is clear from the graph what it means that it is a 2.5-megawatt turbine: At high wind speeds the power doesn't increase beyond its maximum of 2.5-megawatt. At lower wind speeds, we see that it has a lower power output simply because it cannot convert any more energy from the wind. For now, we'll accept that the performance of the turbine can be captured with this type of power curve. However, before the end of the course you will understand why the power curve looks like this and how the behaviour and control of the wind turbine lead to it.



This is a schematic representation of the power curve. We still recognize the region of lower wind speeds, where the power increases with the wind speed, and the region at higher wind speeds, where it remains constant. The difference with real power curves is particularly in the transition between these two regions which is very sharp here. In real power curves, you'll see a more gradual transition. Let's first address some terms for the wind speeds. The lowest wind speed that characterizes the power curve is the cut-in wind speed. This is where the turbine is switched on. At rated wind speed, we achieve our rated power, which is the maximum power. At cut-out wind speed the turbine is shut down. In the region between cut-in wind speed and rated wind speed the power is nearly the maximum power that can be taken from the wind. We call this region the partial load region, because the generator is not operating at its full capacity. In the region between rated wind speed and cut-out wind speed the power is kept constant. This region is called the full load region, because here the generator is operating at full load. Below cut-in and above cut-out, the turbine is not connected to the public grid and the rotor is idling or standing still. Typically, turbines that are not producing power are idling, meaning that they rotate really slowly. They are normally only slowed down to a full stop when personnel needs to access the nacelle.

2.5 Energy yield

Energy yield calculations

Because the wind varies over time, the power also varies over time. To determine the amount of electricity that is produced by the wind turbine over a certain period, we have to integrate this variable power. This would require knowledge of the variation of wind speed over time. It is more convenient to use the statistical properties of this variation, particularly the probability density function of wind speeds, $f(U)$. For the calculation of the long-term energy yield we are not interested in what happens on short time scales, so we can work with the probability density function of the average wind speeds for 10-minute periods.

The probability density function captures how often a 10-minute average wind speed occurs. Of course, if you measure the wind speed over 10 minutes and average it, the probability that it is exactly, for example, 6.000… m/s is zero. The probability density function can be used to determine the probability that the wind speed is in a range dU around this value. This probability is $f(U)dU$. More generally, the probability that the 10-minute average wind speed is between U_1 and U_2, $F(U_1 \le U \le U_2)$, equals:

$$F\left(U_1 \le U \le U_2\right) = \int_{U_1}^{U_2} f\left(U\right) dU \,.$$

The function *f* is called a density, because with have to multiply *f*(*U*) with the range *dU* in the integrand. The unit is therefore $(m/s)^{-1}$, because probability *F* has no unit and *dU* has the unit m/s.

With the power *P*(*U*) from the power curve, the probability of occurrence of wind speeds in a range *dU*, the energy yield *E* in a period of duration *T* can be calculated from:

$$E = T\int_{U_{ci}}^{U_{co}} P(U)\cdot f(U)dU\ .$$

If the period of duration is one year, the energy yield is often called AEP, for annual energy production. If the power is expressed in kW and the duration in the number of hours per year, *T* = 8766 h, then AEP is expressed in kWh.

If you have selected a specific turbine for your project, you can obtain the power curve from the manufacturer. You can use the Weibull distribution as the probability density function of the wind speeds. There are various ways to obtain the shape and scale factor for your site, such as from wind speed measurements, from literature or by estimation from the annual average wind speed.

Capacity factor

The capacity factor, *cf*, expresses the ratio between the actual AEP and the amount of energy that would have been obtained if the turbine had operated the entire year at its rated power. This is the same as the ratio between $T_{equivalent}$ and T_{year}, if $T_{equivalent}$ is the time that the turbine would have to operate at its rated power to obtain the same energy as it actually does in a year. In equations, this reads:

$$cf \equiv \frac{AEP}{P_{rated}T_{year}} = \frac{T_{equivalent}}{T_{year}}\ .$$

Although the capacity factor is often intuitively interpreted as an efficiency, a high capacity factor isn't necessarily good. In Figure 1 you see a turbine with a large rotor and a small generator on the left-hand side. The small generator represents a low rated power. Because the rotor easily drives the generator at its rated power, even for low wind speeds, this configuration has a capacity factor close to one. From an economic perspective, the rotor is too large: we pay too much for the rotor, and do not tap the amount of wind energy it could convert.

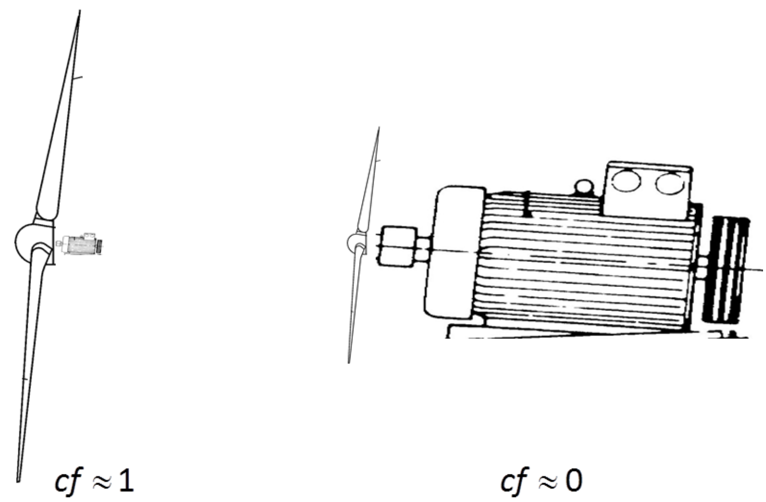


Figure 1 Illustration of a turbine with a high capacity factor and with a low capacity factor

The turbine on the right-hand side has a small rotor and a large generator. This turbine has a low capacity factor, because the generator rarely operates at its rated power. However, almost all the energy that can potentially converted by the rotor is processed by the generator and fed into the grid. The rated power of the generator is rarely a limiting factor. This is also not economically attractive, because we now pay too much for the generator and rarely use it at its full capacity.

The value of the capacity factor depends on size of the rotor, the rated power of the turbine and the wind climate. The best value for the capacity factor depends on what provides the economic optimum. However, high values for the capacity factor do have the advantage that the production of the wind turbine or wind farm is more persistent, because it operates more often close to or on its rated power.

Simple energy yield estimation

If you do not know the wind climate and/or power curve, you can still make an estimate of the annual energy yield, by flipping the definition of the capacity factor:

$$AEP = cf \cdot P_{rated} T_{year} .$$

You would still need to estimate a value for the capacity factor. Figure 2 shows some typical values. However, for each type of site there is a large range, depending on for instance turbine design, hub height and wind conditions.

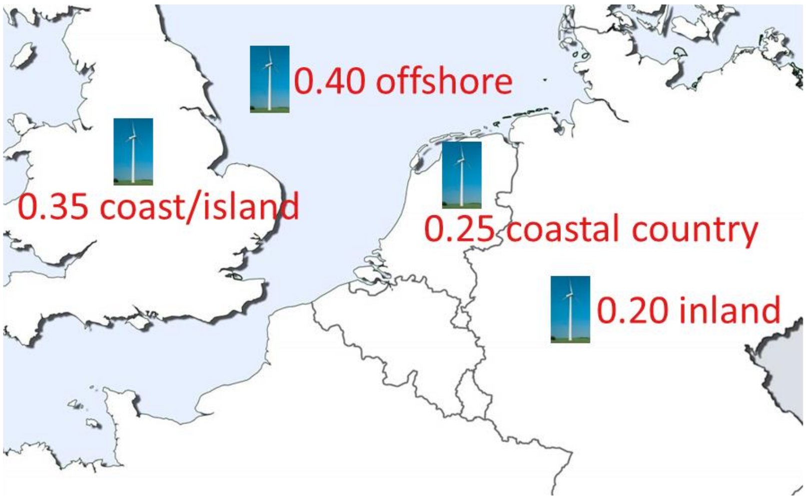


Figure 2 Typical capacity factors for different types of sites

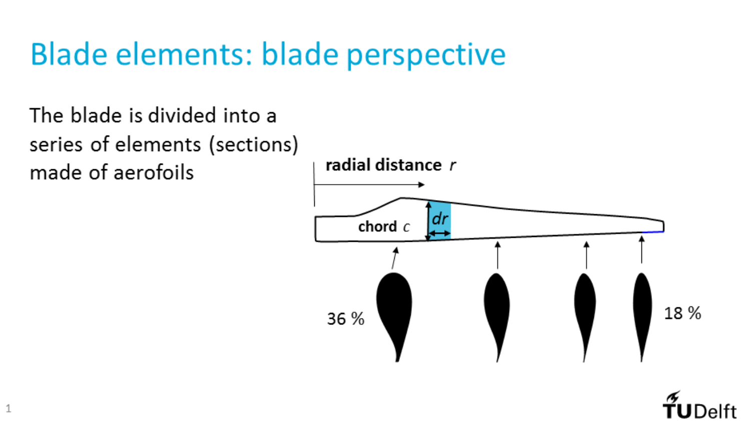
This approach is not accurate, because it doesn't include specific information of the turbine or site. However, it can be used to get ball-park figures (e.g. how many households can approximately be serviced with the wind farm) and to check whether you didn't make a mistake in your detailed energy yield calculations. It is for instance a good estimate to detect a mistake in using kiloWatts instead of MegaWatts.

3 Rotor aerodynamics

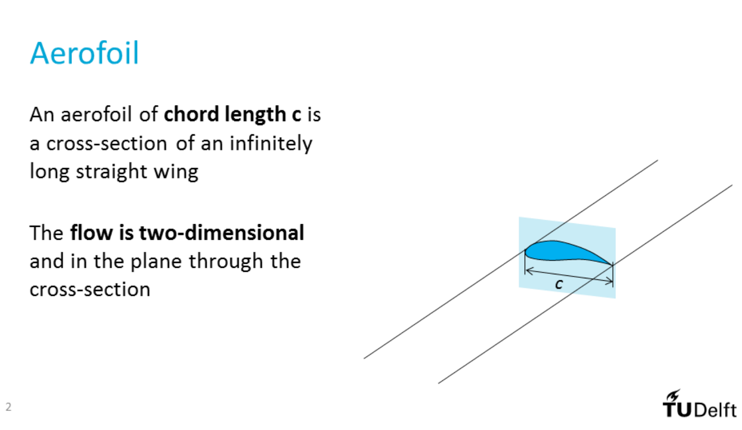
3.1 Background: aerofoil aerodynamics

3.1.1 Lift and drag

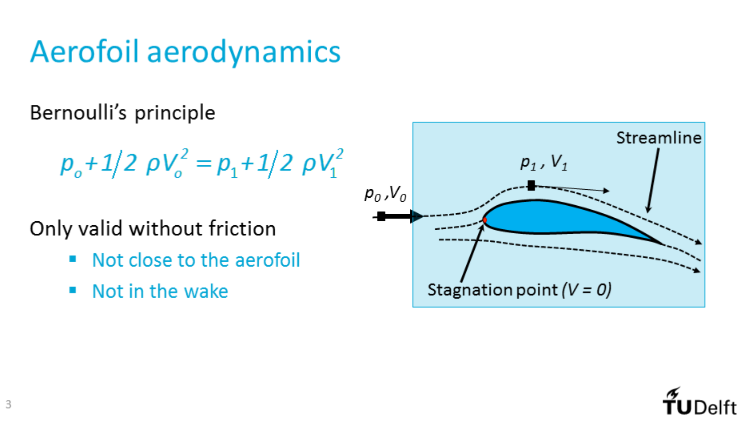
In this video, we will define the concept of blade element and look at some aerodynamics characteristics.



Each blade element is characterised by a certain aerofoil section. These aerofoils are thick close to the hub and become thinner towards the tip.

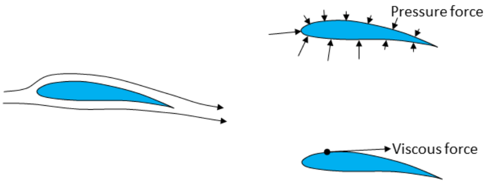


The aerofoils are characterised by a chord "c", which is the distance between the leading edge and the trailing edge of the aerofoil. Around the aerofoils, the flow is assumed to be 2-dimensional and develop in the plane of the aerofoil.



If the fluid is inviscid, Bernoulli principle applies. It states that, along a streamline, the sum of the static pressure and the dynamic pressure remains constant. Note that this is only valid far away from boundaries, since friction cannot be neglected in boundary layers, but too far in the wake, where flow recirculations take place and viscosity also plays an important role, it is not valid neither.

Lift and drag force on an aerofoil

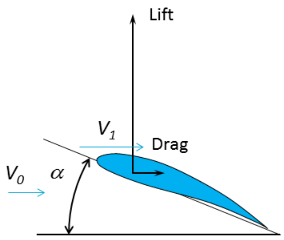


Generally speaking, the force on the aerofoil can be divided into two sources. The pressure force that is the force acting normal to the surface of the aerofoil, and the viscous force that is the force acting tangentially to the surface of the aerofoil.

Lift and drag force on an aerofoil

The **lift** L is perpendicular to the **velocity** V

The **drag** D is parallel to the **velocity** V



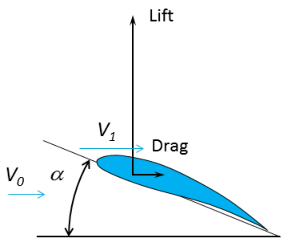
The sum of these two forces can be decomposed into two components: the lift and the drag. By definition, the lift force (L) is the component of the force that is perpendicular to the apparent wind speed seen by the aerofoil, while the drag force (D) is parallel to the apparent wind.

Lift and drag force on an aerofoil

When the **angle of attack** α increases, the **velocity on the upper surface** V_1 increases

The **pressure on the upper surface** p_1 reduces

The **lift** L increases



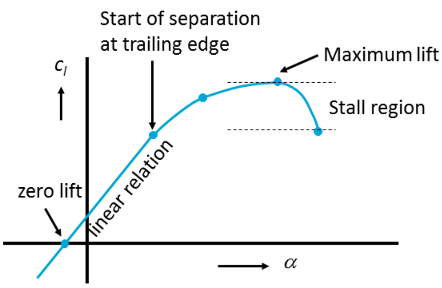
Both lift and drag forces change with the angle of attack alpha, which is the angle between the apparent wind speed seen by the aerofoil and the chord line. As the angle of attack increases, the lift force usually increases because of the over speed happening at the upper surface of the aerofoil. Applying "Bernoulli Principle", if the flow accelerates at the upper surface, then the pressure decreases. Therefore, the pressure at the upper surface becomes smaller than the pressure on the lower surface. This is one reason why lift is generated. As the angle of attack increases, the difference in the pressure also increases meaning that the lift force increases.

Lift curve

The **lift coefficient** c_l

$$c_l = \frac{L}{\frac{1}{2}\rho V^2 c \cdot l}$$

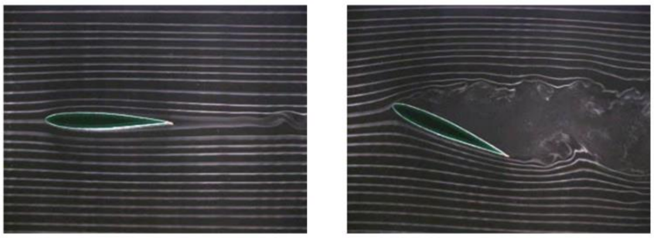
where l is the length of the blade



This is only true for a certain range of angle of attack, as illustrated by this plot of lift coefficient as a function of the angle of attack, alpha. The lift coefficient is defined as the lift force divided by one half rho V-squared times the chord of the aerofoil times the length of the blade. This is a non-dimensionalised way of characterising the lift. The plot shows that, for certain range of alpha, typically up to 10 degrees, the CL varies linearly with the angle of attack. This happens when the flow is fully attached to the aerofoil. At larger values of alpha, the CL still increases but not linearly. This is because flow separation occurs. As the angle of attack increases, the separation region moves from the trailing edge to the leading edge, and the change of lift decreases up to a point where the flow becomes fully separated. This is called stall. In this regime, an increase in the angle of attack leads to a decrease in lift.

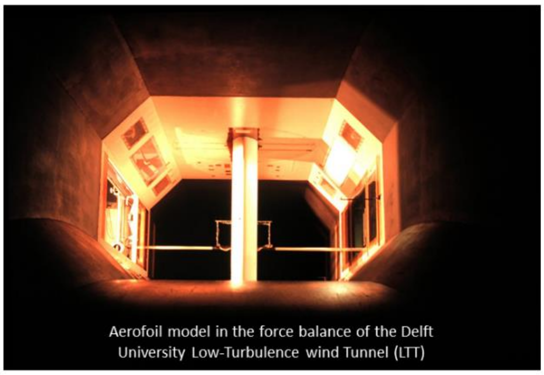
Phenomena that determine lift curve

Attached flow (linear) | Leading edge separation (stall)



8

This is illustrated by these movies. On the left hand side, the flow is fully attached to the aerofoil and the streamlines nicely follow the aerofoil surface. In this case an increase in angle of attack leads to a linear increase in CL. On the right hand side, the flow is fully detached from the leading edge. In that case, an increase in angle of attack leads to a decrease in CL.

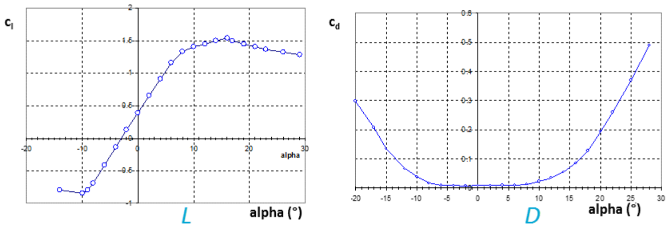


9

The plots of lift and drag forces, as a function of alpha, are typically obtained from experiments. This shows an experiment for the flow field around a blade element. The total force acting on the element is measured. Then, it is projected on the directions parallel and perpendicular to the wind to obtain the lift and drag forces.

3.1.2 *Design angle of attack*

Lift and drag coefficients

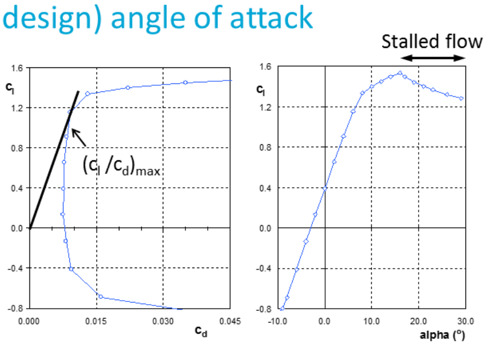


$$c_l = \frac{L}{\frac{1}{2}\rho V^2 c \cdot l} \qquad c_d = \frac{D}{\frac{1}{2}\rho V^2 c \cdot l}$$

1

We saw in the previous video how the curves for lift and drag coefficients can be obtained as a function of the angle of attack. Note that the drag coefficient, shown on the right-hand-side, is close to zero when the flow is fully attached, and largely increases when the flow detaches.

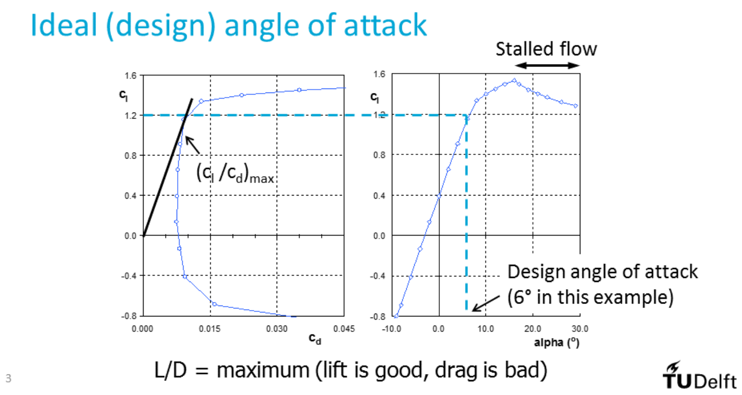
Ideal (design) angle of attack



L/D = maximum (lift is good, drag is bad)

2

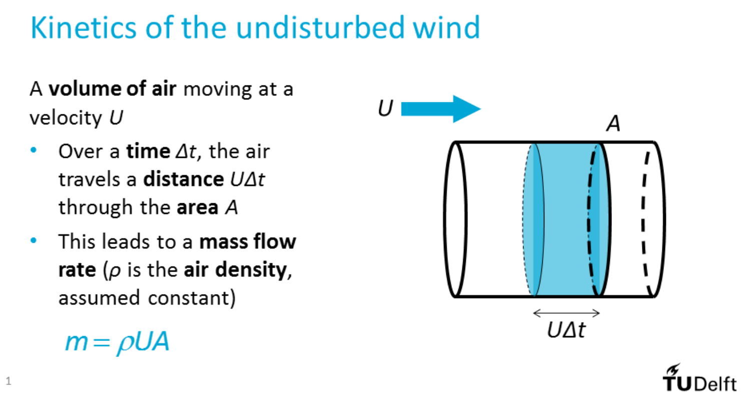
Since Cl and Cd are known as a function of the angle of attack, we can also plot the lift coefficient as a function of the drag coefficient. The design alpha is the angle of attack at which the ratio Cl over Cd is maximum.



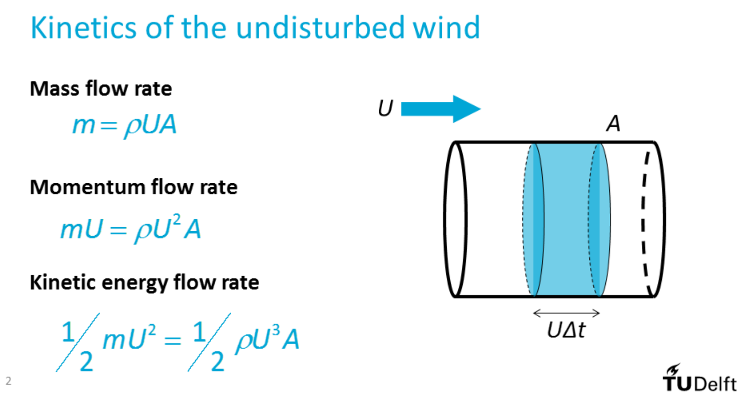
In this example, the optimum alpha is close to 6 degrees. At this angle of attack, the aerodynamic performances of the aerofoil are optimised and so is the power harnessed. In the next unit, we will see that the flow properties vary along the blade and this impacts on the design of the blade.

3.2 Flow through an actuator disc

In this unit, we will look at the momentum theory, which consists in replacing the rotor by a thin actuator disk. Here we will focus on the effect that the disk has on the flow.



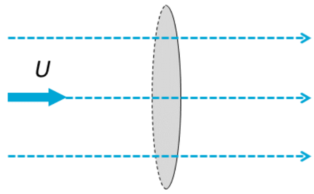
But before we place any wind turbine rotor in the flow, let's first look at what is the power that is available in any given wind. In order to do so, we will consider a volume of air moving at a certain velocity (U). If we assume that the air moves along a pipe of cross-sectional area "A", then over the time interval delta t, the air has travelled a distance which is the velocity of the air (U) times delta t. The associated volume of air, displaced during that time, is equal to the cross-sectional area of the tube times the length U delta t. If we further multiply this volume by the air density rho, we get the mass of air that is displaced. Finally, if we divide this mass by the time delta t, we have a mass flow rate that is the amount of mass per unit of time. Note that here we always consider that the air density is constant.



Thus the mass flow rate (m) is given by rho U A. By definition the momentum flow rate is given by the mass flow rate multiplied by the flow velocity. Therefore, the momentum flow rate is a function of the square of the wind speed. Additionally, the kinetic energy flow rate is given by ½ m U2. If we replace m by its expression, we find that the kinetic energy associated with the displacement of this volume of air is proportional to the cube of the wind speed. This is an important result to keep in mind.

Effect of the rotor on the wind

A rotor represented as an **actuator disk**



3 TUDelft

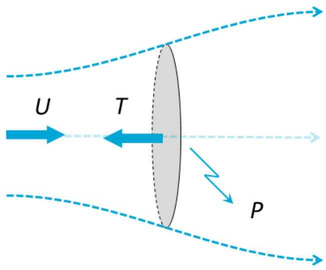
Now let's look at what happens to the flow when we place a disk in it that represents the wind turbine rotor. If the disk does not exert any force on the flow, then the flow field will be completely undisturbed. This means that the streamlines, which are shown by dashed blue lines here, will remain straight and the flow velocity will not change. This is equivalent to the situation we had on the previous slide.

Effect of the rotor on the wind

A rotor represented as an **actuator disk**

It exerts a **thrust force *T*** on the flow

It extracts a **power *P***



4 TUDelft

By contrast, if the disk exerts a certain thrust force T on the flow, then the flow is disturbed. In particular, this force will slow down the flow in the area covered by the disk. Therefore, as we will see on the next slide, the streamtube expands. Furthermore, power can be extracted at the rotor if both the force T and the velocity at the rotor are non-zero.

Wake expansion due to a force on the wind

The **mass flow rate** *m* is conserved,

$$m = \rho U A$$

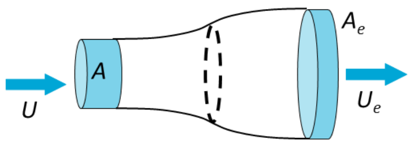
$$m = m_e$$

Because of the **thrust** *T*,

$$U > U_e$$

Therefore,

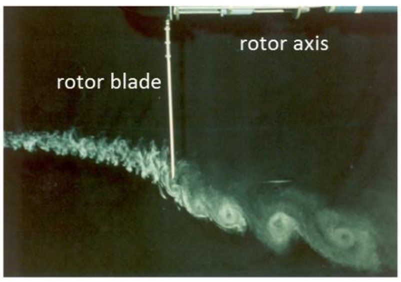
$$A < A_e$$



5 TUDelft

I mentioned that the wake expands due to the presence of the disk and the force that it exerts on the flow. This can be explained by using the principle of mass conservation. Conservation of mass flow rate means that the mass flow should be the same in every cross-section on the stream-tube. In particular, the mass flow rate at the inlet (that is upstream of the disk), equal to rho U A, should be equal to the mass flow rate at the exit of the stream-tube which is rho Ue Ae. Since the flow is slowed down by the disk; the velocity at the outlet of the stream-tube Ue is smaller than the velocity at the inlet of the stream-tube. Considering that the air density is constant, if the velocity decreases then necessarily the cross sectional area of the stream-tube increases by virtue of conservation of mass flow rate. Thus Ae is necessarily larger than A and the stream tube expands.

Visualisation of stream lines through a rotor

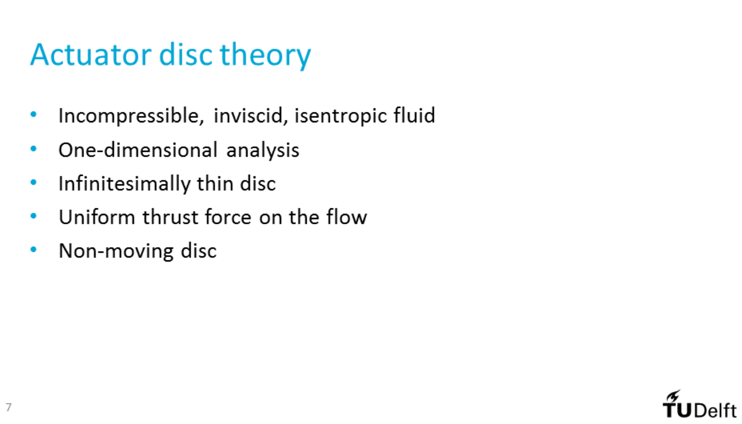


A smoke pattern in a wind tunnel

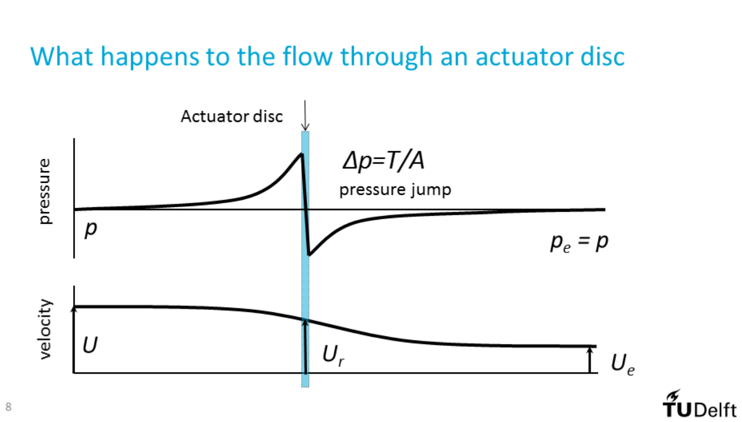
Image: Instituut voor Windenergie, Delft

6 TUDelft

This is nicely visualised in this wind tunnel experiment, in which a wind turbine rotor is placed in an air flow. Note that this is an actual rotor and not a thin disk. The streamline is nicely visualised by ejecting some smoke in the flow. You can clearly see the wake expansion as explained on the previous slide. You can also see large vortices shed from the tip of the blade. These vorticities are not described by the momentum theory.



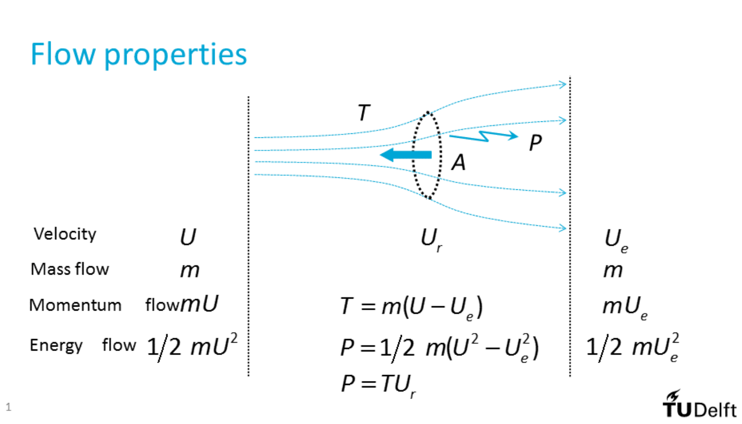
This is because that the momentum theory assumes that the flow is incompressible, inviscid, and isentropic. It further considers the problem to be one-dimensional, meaning that the velocities are identical across the section of the stream tube. It also assumes that the actuator disk is infinitely thin and it generates a uniform thrust force across the disk. Finally, it assumes that the disk is not rotating.



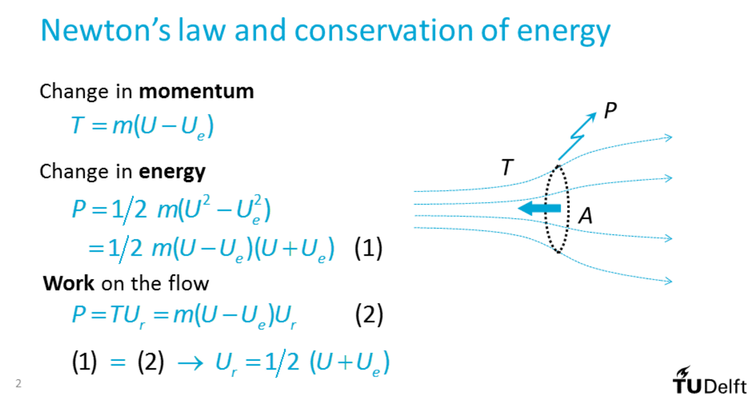
We saw that the actuator disk slows down the flow. But what about the pressure field? Well, when an object is placed in a flow field, a high-pressure region appears at the front of the object and a low-pressure region is formed at the rear of the object. Since the disk is infinitely thin, the regions of high pressure and low pressure are very close to one another Thus the actuator disk creates a pressure jump. The magnitude of that jump actually equals the force T, exerted by the disk, divided by the cross-sectional area of the disk (That is the area over which the thrust force acts). In the next module we will look at the aerodynamic characteristics associated with the flow past this thin actuator disk.

3.3 Momentum theory

3.3.1 Conservation laws



This slide shows a series of streamlines, in blue, that form a streamtube around the actuator disk. Conservation laws dictate that the mass flow rate (m) is identical in every section of the streamtube; which means that the mass flow rate at the inlet, upstream of the disk, is equal to the mass flow rate at the outlet, downstream of the disk. Additionally, the force T slows down the flow, so that the velocity Ue at the outlet of the stream tube is smaller than the velocity U at the inlet of the streamtube. Thus, the momentum flow at the outlet is necessarily smaller than the momentum flow at the inlet. In fact, the momentum loss is caused by the force T. Since momentum should be conserved through the streamtube, the force T necessarily equals the difference between the momentum flow at the inlet and that at the outlet. The same reasoning holds for the energy flow. The energy flow at the inlet is equal to ½ m U squared, and it is larger than at the outlet because the velocity decreases along the streamtube. The loss in energy flow is due to the presence of the disk. By virtue of energy conservation, the power P extracted at the disk equals the difference between the energy in the wind upstream of the disk and that downstream of the disk. The power can also be written as a work performed by the force T; that is the product between T and the velocity of the wind at the rotor, Ur.



Thus, the thrust force is equal to the change in momentum, that is m(U-U¬e). And the power is the change in the energy, thus ½ m (U-squared – Ue-squared). Note that this difference in squared velocities can be re-written as the product between (U – Ue) (U + Ue). Since the power is also the work performed by the force T, it can also be written as P = m (U – Ue) Ur. By equating equation 1 and 2, we find a relationship between the velocity of the wind at the rotor Ur, the incoming velocity U, and the outcoming velocity Ue. In particular, the velocity at the rotor equals one half of the sum of the velocity at the inlet and outlet of the streamtube.

3.3.2 Non-dimensional rotor characteristics

Induction factor

In fluid dynamics, it is convenient to express the quantities of interest in terms of non-dimensionalised numbers. Thus, typically quantities such as the flow characteristics and turbine performance indicators are non-dimensionalised. The induction factor is a non-dimensional parameter that quantifies the loss in velocity induced by the rotor. Defining the loss in velocity ΔU as the difference between the incoming wind speed U and the wind speed at the rotor U_r, the induction factor *a* is defined as

$$a = \frac{U - U_r}{U} = \frac{\Delta U}{\mathrm{U}}.$$

Using this definition, both the velocity at the rotor U_r and the velocity downstream of the rotor U_e, as shown by Figure 1, can be expressed as a function of the incoming wind speed U and the induction factor a, as $U_r = U(1-a)$ and $U_e = U(1-2a)$. These expressions clearly highlight that, if $a \geq 0.5$, then $U_e \leq 0$ and the flow does not exit the streamtube as assumed by the momentum theory. This means that the momentum theory only holds for values of the induction factor smaller than 0.5.

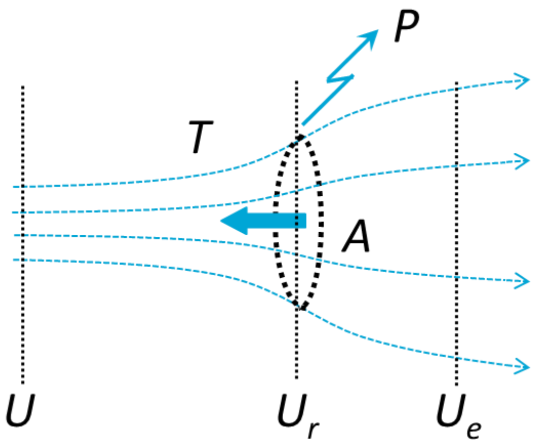


Figure 1 Streamtube around an actuator disk

Parametric representation of the rotor in operation

The expressions of the mass flow rate, the trust force, and the power can also be rewritten as a function of the induction factor. This lead to:

$m = \rho U_r A = \rho U A(1-a),$
$T = m(U - U_e) = ½\rho U^2 A 4a(1-a),$
$P = m(U - U_e)U_r = ½\rho U^3 A 4a(1-a)^2,$

where the thrust force is a function of the wind speed to the power 2 and the power is proportional to the wind speed to the power 3.

Again, it is common practise the express the thrust and power in a non-dimensional way. By definition, the thrust coefficient is given by:

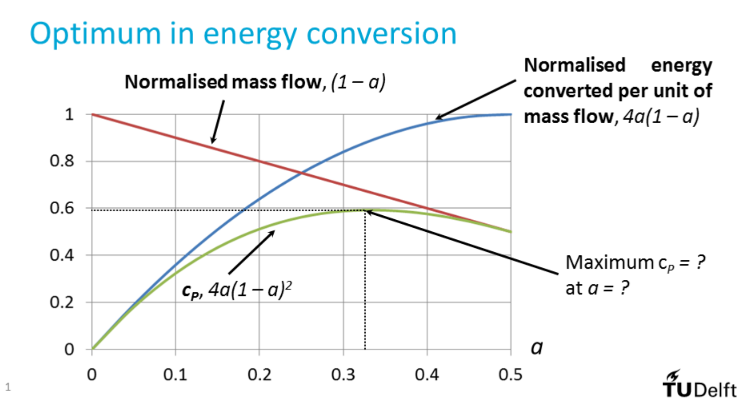
$$c_T = \frac{T}{½\rho U^2 A} = 4a(1-a),$$

and the power coefficient is expressed as:

$$c_P = \frac{P}{½\rho U^3 A} = 4a(1-a)^2,$$

both depending only on the induction factor a.

3.3.3 Optimum energy conversion



This slide shows three different curves as a function of the induction factor a, for values of a smaller than 0.5. The red straight line is the normalised flow rate that is the mass flow rate divided by rho U A, A being the cross-sectional area of the disk. Thus, the normalised flow rate is simply (1-a), and is therefore a straight line. The top blue curve shows the thrust coefficient cT, which is a function of a-squared, and is thus a parabola. The last curve represents the power coefficient, which is a function of a-cubed. From this plot, it is apparent that, when the thrust force increases, the mass flow rate passing through the rotor decreases so that there is an optimal combination of thrust force and mass flow rate that optimises the power. The value of a that maximises cP can be computed by taken the derivative of cP with respect to a and equal it to zero.

Optimum in energy conversion

C_p is maximum when:

$$\frac{dC_p}{da} = 0$$

Or $\frac{dC_p}{da} = 4(1-a)^2 - 8a(1-a) = 4(1-4a+3a^2) = 0$

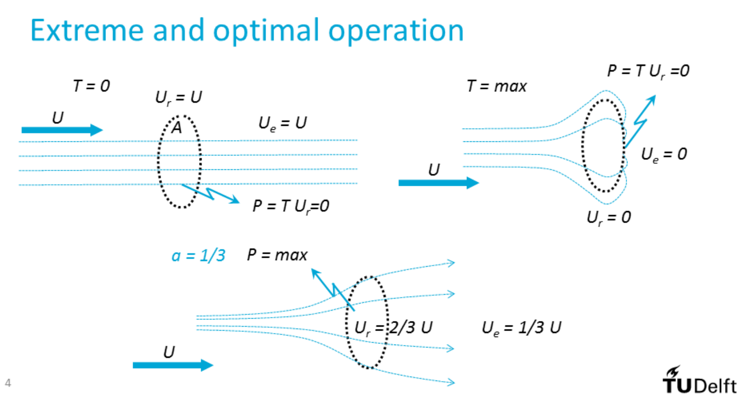
Because $a < 0.5$:

- The only solution is $a = 1/3$, which gives $C_p = 16/27$ or 0.59
- *"This maximum efficiency is known as **the Betz limit**".*

2 TUDelft

By doing this, we obtain a second-order equation for cP as a function of a. This equation is satisfied for two values of a. However, one of them is larger than one half, which does not make sense for the momentum theory. Thus, the only solution of this equation is that a equals one-third. By substituting this value back in the definition of cP, we find that for a = 1/3, cP = 0.59. This is called the Betz limit and it represents the maximum efficiency of a wind turbine rotor.

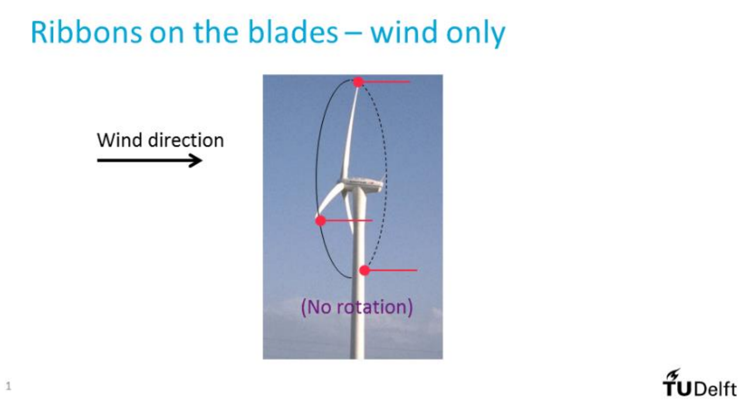
The Betz limit is named after Albert Betz, a German physicist, who derived this limit at the University of Gottingen. He is photographed here next to a wind tunnel.



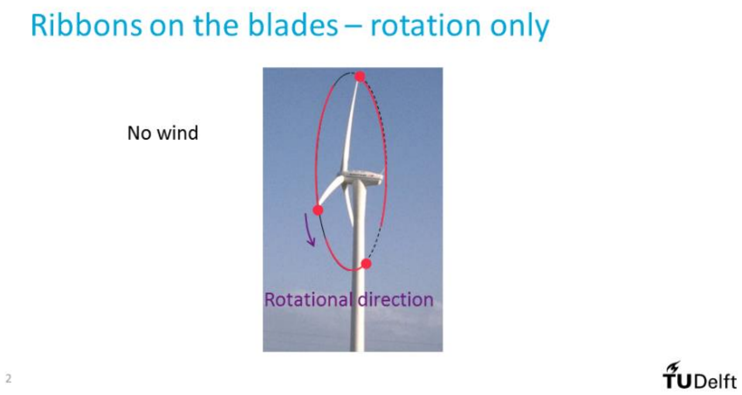
To summarise, we saw in the previous videos that if the disk does not exert any force on the flow, T = 0 and the velocity of the wind is maximum. In that case, the velocity at the rotor is equal to the incoming wind speed and the velocity downstream of the rotor. The power extracted is zero because the force is zero. If the trust force applied on the flow is maximum, then the flow is slowed down to a zero velocity and the power extracted is also zero. Thus, even intuitively, we understand that there is an optimal combination of thrust force that is exerted from the disk on the flow, and flow velocity passing through the disk, for the power to be maximum. In this video, we learned that this optimal combination is obtained for an induction factor of 1/3. This means that the velocity of the wind at the rotor equals 2/3 of the incoming wind speed, and the velocity downstream of the rotor is one third of the incoming wind speed. This is it for the momentum theory. In the next unit, we will represent the rotor slightly more accurately using the blade element theory.

3.4 Blade element theory

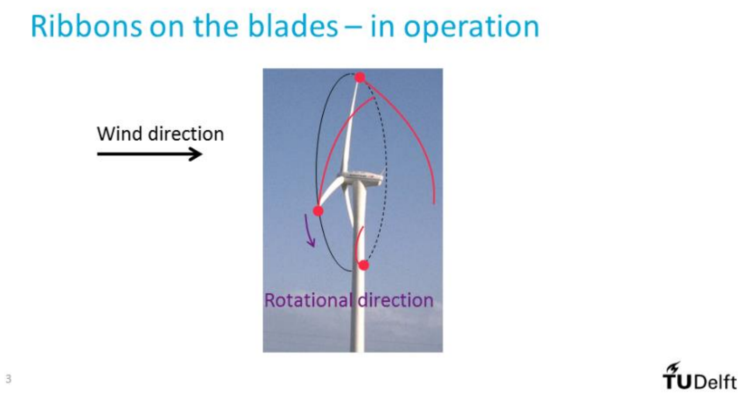
3.4.1 Flow conditions around a blade



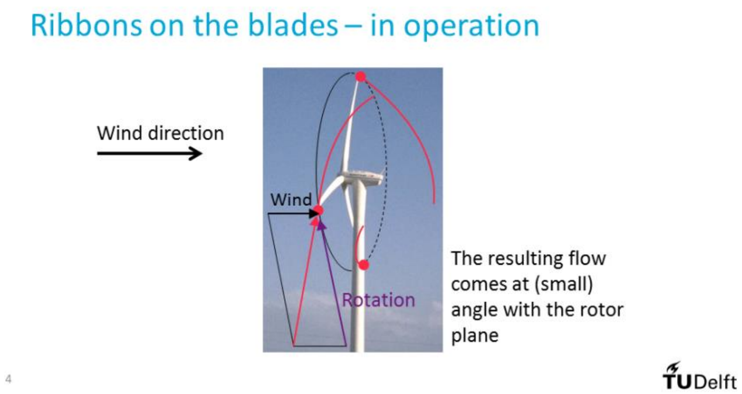
In order to visualise the flow conditions at the rotor, it is convenient to imagine that ribbons are attached at the tip of the blades. This is shown by red straight lines. If the rotor is at standstill and subjected to wind only, the ribbons' motion is according to the local wind velocity. Assuming that the wind flow is normal to the plane of the rotor, the ribbons fly horizontally, in the wind direction, as shown on the slide.



Let’s now consider that there is no wind at all, and that the rotor rotates. Then, the local velocity seen by the ribbons is the rotational velocity of the rotor. Therefore, the ribbons follow the circular path described by the tip of the blade.

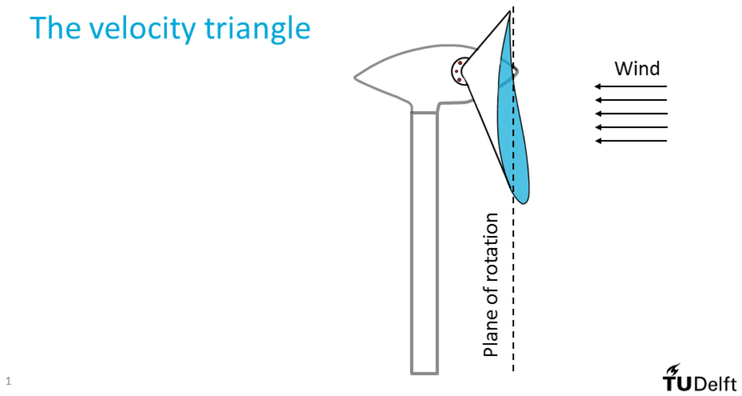


If we combine the two previous situations, that is the rotor rotates and is subjected to a wind flow normal to the rotor plane, then the ribbons experience a velocity coming from both the wind and the rotor’s rotation. Thus, the ribbons describe a helical path.



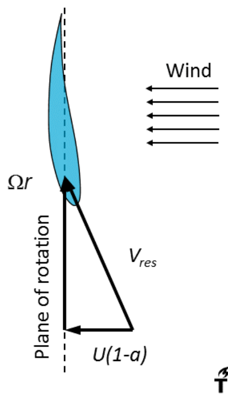
If we look a little bit more closely at what happens at the tip of the blades, we see that the ribbon is subjected to a wind velocity normal to the rotor plane and a rotational speed in the rotor plane (and tangential to the circular orbit described by the blades). The resultant velocity seen by the ribbon is the sum of these two velocities and is tangent to the helical path of the ribbon. Note that because the wind comes perpendicular to the rotor, the resultant wind speed has some angle with the plane of the rotor.

3.4.2 The velocity triangle



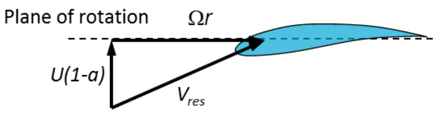
The different velocities experienced by each blade element are represented using a so-called velocity triangle. This slide shows a side view of the wind turbine and the blue shaded area highlights an aerofoil section through the blade. The wind flow is coming from the right. As the rotor rotates, the aerofoil section moves in the plane of rotation, perpendicularly to the wind direction.

The velocity triangle



If we freeze the aerofoil at a certain time, it will see a component of velocity coming from the rotation of the blade. That is Ω r, where Ω is the rotational speed of the shaft and r is the radial location of this aerofoil that is the distance between the aerofoil section and the hub. The aerofoil is of course also subjected to a velocity coming from the incoming wind flow. According to the momentum theory, the velocity of the wind at the rotor equals the far away wind velocity U multiplied by (1-a), where a is the induction factor. Thus the resultant velocity seen by the aerofoil is the sum of the velocity coming from the incoming wind flow and the velocity coming from the rotation of the blade.

The velocity triangle

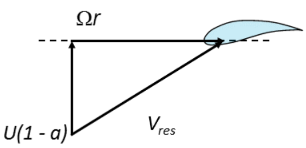


Usually, this velocity triangle is represented with the aerofoil section placed horizontally rather than vertically. Of course, this does not change the velocity triangle.

Tip speed ratio: making rotational speed dimensionless

Tip speed ratio $\lambda = \Omega R / U$,
where R is the rotor radius

Below rated power, control aims at keeping λ constant, that is Ω / U constant.

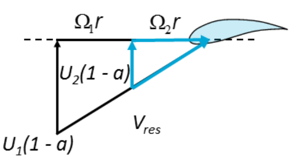


In this field, it is convenient to speak in terms of non-dimensionalised numbers. This also holds for the rotational speed of the rotor. The tip speed ratio is the ratio between the velocity of the tip of the blade Ω R (R being the rotor radius) and the wind speed U. Below rated power, control usually aims at keeping this non-dimensional λ constant by keeping the ratio between the rotational speed Omega and the wind speed U constant.

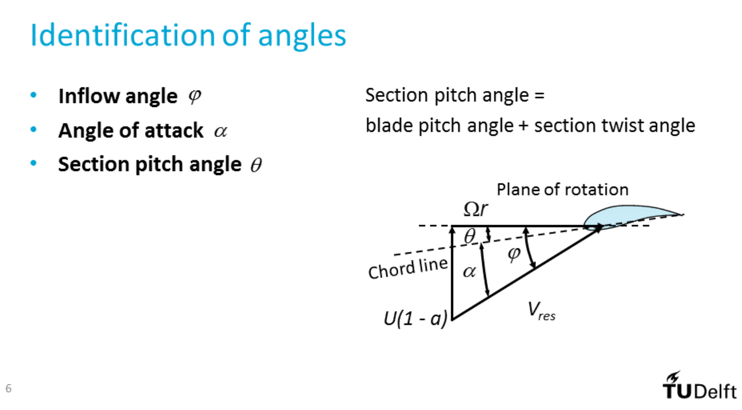
Tip speed ratio: making rotational speed dimensionless

If $\frac{\Omega_2}{\Omega_1} = \frac{U_2}{U_1}$, then $\lambda_1 = \lambda_2$ and

the triangles are similar

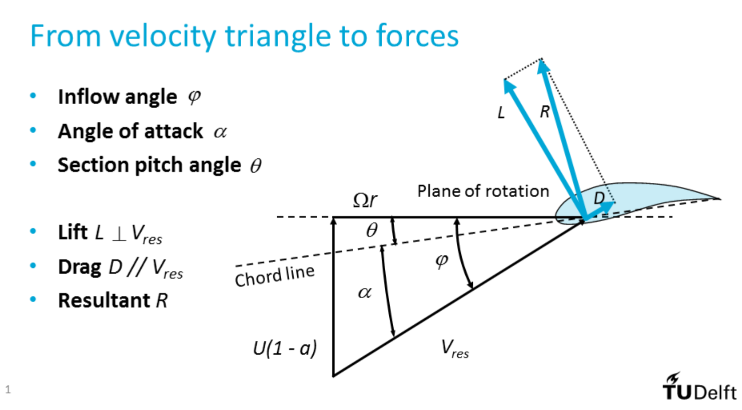


So, if the wind speed decreases by a factor 2, the rotational speed of the rotor is also divided by a factor 2. Then, the resultant velocity is divided by a factor 2 and the velocity triangles at the two wind speeds remain exactly similar.

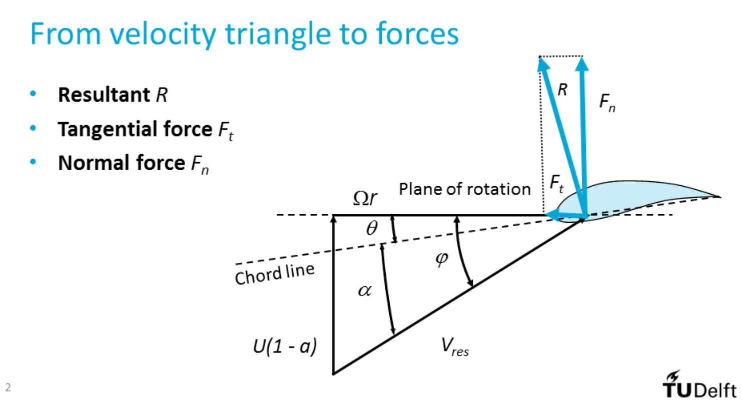


Lastly, let's define a few important angles in the velocity triangle. The inflow angle is defined between the apparent wind speed seen by the aerofoil and the plane of rotation. This angle phi is further decomposed into two separate angles. On the one hand, the angle of attack alpha occurs between the velocity seen by the aerofoil and the chord line. On the other hand, the section pitch angle "theta" is the angle between the chord line and the rotation plane. It is also the sum of the blade pitch angle and the section twist angle. In this unit, we looked at the velocity triangle. In the following unit, we will look at the forces associated with the different velocity components.

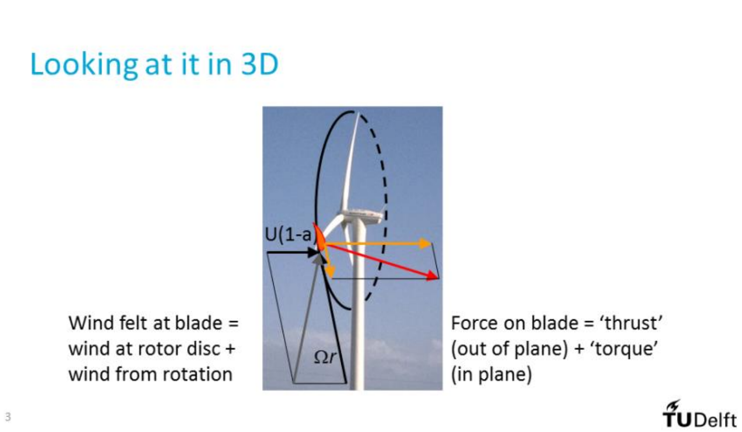
3.4.3 Forces on blade elements (part 1)



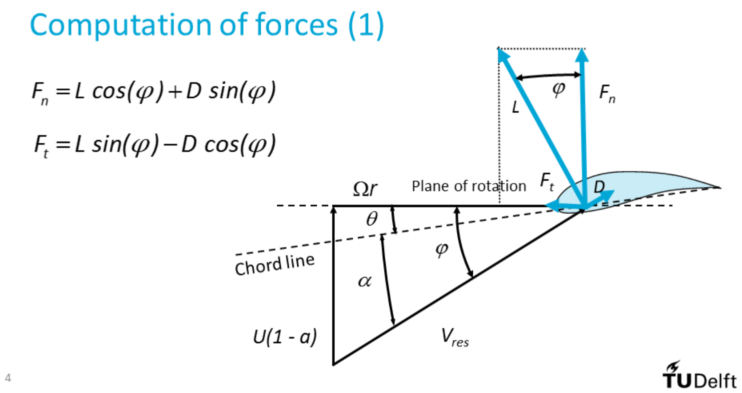
This slide shows the velocity triangle associated with an aerofoil section along the blade. As we saw in the previous unit, the apparent or resultant wind speed seen by the aerofoil, Vres, is the sum of the velocity component coming from the wind U(1-a), and that coming from the rotation of the blade Ωr. The force R denotes the total force acting on the aerofoil. It can be decomposed into a lift and a drag force. The lift is the component of the force perpendicular to the apparent wind speed Vres. The drag force, by contrast, is the component that is aligned with the apparent wind speed.



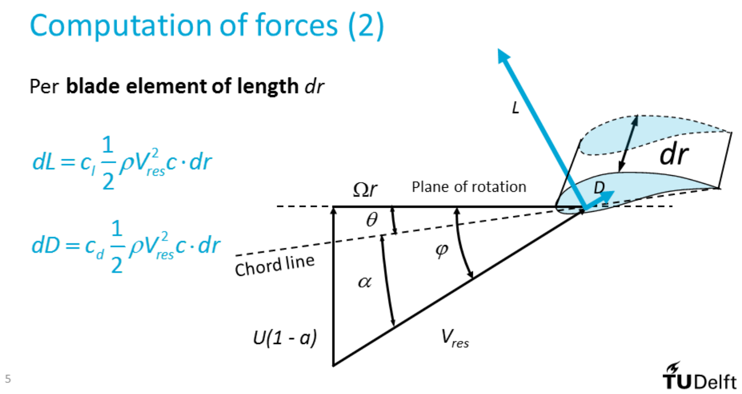
Similarly, we can decompose the total force into components that are normal and tangential to the plane of rotation. Fn is the force that is perpendicular to the plane of rotation, and can therefore be linked to the thrust force that we defined in the actuator disk theory. Ft is the tangential force that lies in the plane of rotation.



This is the same velocity triangle in 3D. The aerofoil section close to the tip of the blade is highlighted in orange. It experiences a velocity U(1-a) coming from the wind and a velocity Ω×r coming from the blade rotation. Forces are associated with these flow velocities. The thrust force is the component normal to the plane of rotation, and the torque (or angular momentum) is the force component in the plane of rotation.



Since the total force can be decomposed as either lift and drag forces or normal and tangential forces, there is a direct relation between these components. In particular, the angle between the lift force L and the normal component Fn is the inflow angle. This is explained by the fact that the inflow angle is also the angle between the apparent wind speed and the plane of rotation, which are perpendicular to L and Fn, respectively. Therefore, by trigonometry, Fn and Ft are directly linked to L and D using the cosine and sine functions of the inflow angle.



Remember that the aerofoil is just one section of a blade element of length dr. Thus, the blade element is subjected to elements of lift and drag forces. dL is the element of lift force and is given by the lift coefficient times ½ rho Vres-squared c dr. This comes from the definition of Cl that we saw in a previous unit, in which the full length of the blade was used instead of dr. Similarly, the element of drag force dD is expressed using the drag coefficient and the length of the blade element dr. Thus, in order to compute the element of lift and drag forces, we need to know the lift and drag coefficients respectively. As mentioned previously, these coefficients are usually tabulated against the angle of attack.

3.4.4 Forces on blade elements (part 2)

The lift and drag forces acting on blade elements are a function of the angle of attack α. From Figure 1, it is clear that α is a function of the inflow angle φ. Additionally, through trigonometry,

$$\tan\varphi = \frac{U(1-a)}{\Omega r}.$$

Knowing the pitch angle θ, the angle of attack is computed as $\alpha = \varphi - \theta$. Since the lift and drag coefficients are typically tabulated against the angle of attack, the knowledge of α leads to the values of lift and drag coefficients on the blade element. The elements of lift and drag forces on a blade element of length dr can then be calculated as:

$$dL = ½ c_L \rho V_{res}^2 c \cdot dr,$$
$$dD = ½ c_D \rho V_{res}^2 c \cdot dr.$$

These expressions can be further used in the element of the force normal to the plane of rotation and tangential to the plane of rotation, as follows:

$$dF_n = dL\cos\varphi + dD\sin\varphi,$$
$$dF_t = dL\sin\varphi - dD\cos\varphi.$$

When substituting the expressions for dL and dD in the relations above, we find that the element of force tangential to the plane of rotation is a function of the lift-to-drag ratio as:

$$dF_t = ½\rho V_{res}^2 c\, c_L \left(\sin\varphi - \frac{\cos\varphi}{c_L/c_D}\right) dr.$$

In particular, as the lift-to-drag ratio c_L/c_D increases, the force also increases. Thus, having a large lift and a small drag is beneficial to the power that a wind turbine harnesses.

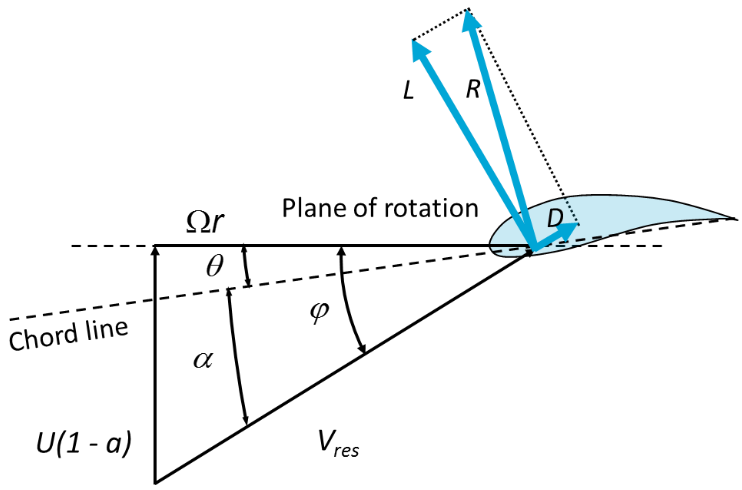
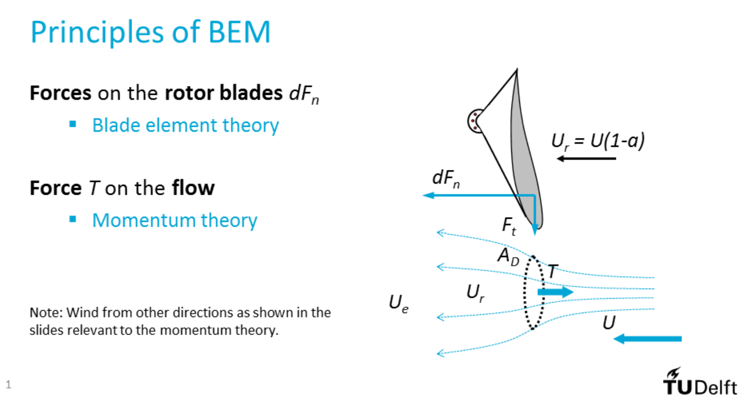


Figure 1 Velocity triangle on a blade element.

3.5 Blade element momentum theory



We saw before that the blade element theory can be used to determine the force on blade elements. In particular, dFn denotes the element of force that is out of the rotor plane. We also saw how the momentum theory can be used to compute the thrust force that an actuator disk would exert on the flow. This thrust force is also perpendicular to the plane of rotation.

Principles of BEM

Forces on the rotor blades dF_n given by the **blade element theory**

$$dF_n = 1/2\ \rho V_{res}^2 c\ (c_l \cos(\varphi) + c_d\ \sin(\varphi))\ dr$$

Force T on the flow given by the **momentum theory**

$$T = 1/2\ \rho U^2 A\ 4a(1-a)$$

Both are a function of the **induction factor a**

2 TUDelft

The blade element theory expresses dFn as a function of the lift and drag coefficients, and the inflow angle phi, and all depend on the angle of attack. Since the inflow angle is a function of the velocity U(1-a), the element of force dFn depends on the knowledge of the induction factor a. In the momentum theory, the thrust force exerted by an actuator disk on the flow was also a function of the induction factor.

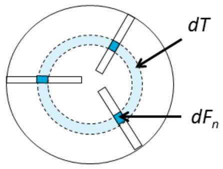
Principles of BEM

Force dF_n on **all the elements** in the annulus

$$dF_n \cdot B$$

Thrust force dT on the flow

$$dT = 1/2\ \rho U^2\ 2\pi r\ dr\ 4a(1-a)$$



B is the **number of blades**

3 TUDelft

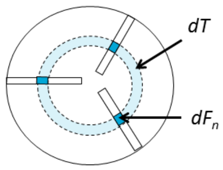
As already mentioned before, the idea behind the blade element momentum theory is to combine blade element theory and momentum theory. The momentum theory assumed that the thrust force T was uniformly spread across a thin actuator disk, whilst the blade element theory divided the blade into a number of elements, each being subjected to a force element dFn. For a given radial location, the force element on one blade is assumed to be the same on all the blades. Therefore, the element of force on one annulus of thickness dr is given by the element of force on one blade element, dFn, times the number of blades. The element of thrust force given by the momentum theory on this annulus is the thrust force written on the previous slide, where the area of the disk is replaced by the area of the annulus, that is 2 π r dr.

Principles of BEM

Equate solutions for forces of the blade element theory and the momentum theory (per annulus)

$$dF_n \cdot B = dT$$

This gives an equation to solve for **the induction factor** *a*.



B is the **number of blades**

4 TUDelft

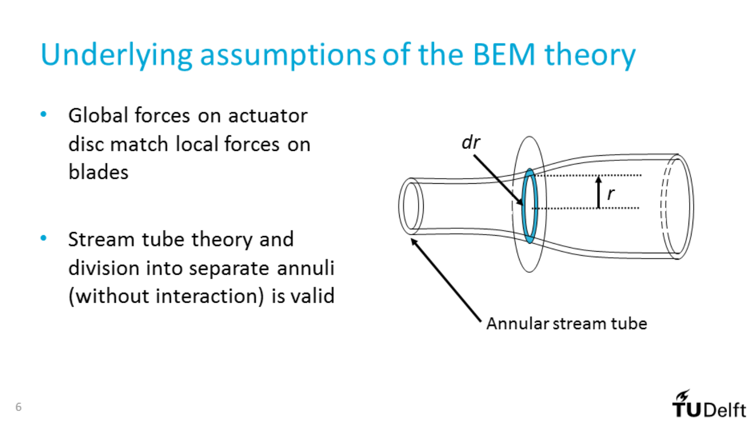
The BEM theory states that these two elements of force should be identical. Thus, by equating their expressions, an equation for the induction factor is obtained.

The iterative nature of solving BEM

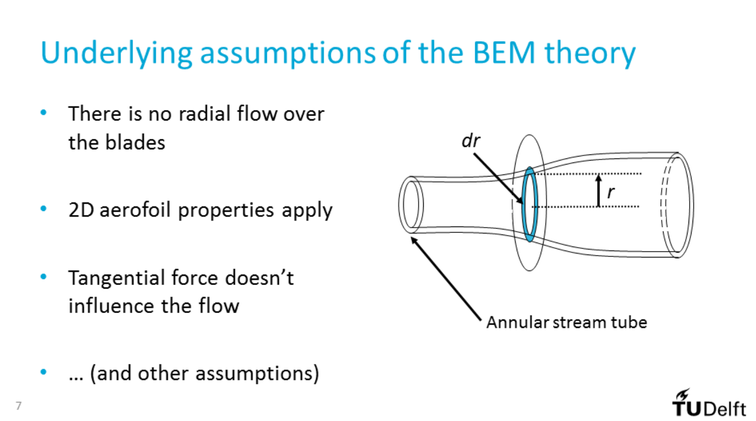
- The blade is divided in several elements, typically 10-20 per element
  - Choose an initial value for the **induction factor** *a*
  - Use this to calculate the **blade element's force** dF_n
  - Update *a* by equating dT to dF_n and solving *a* for this dT
- Continue the above steps until the **induction factor** *a* reaches a constant value
- With the solutions for *a* (and dF_n, dT) for each annulus: integrate the **forces** (and **moments**) on the blade over the **radius** *r*

5 TUDelft

The procedure to solve this equation is iterative. First, an initial value of the induction factor is chosen and the element of force dFn associated to that value is computed on each blade element. The equation for "a" coming from the BEM theory is then solved and the solution is compared with the initial guess. If both values of induction factor differ significantly, then the force is updated and the solving procedure is repeated until the induction factor does not change significantly from one iteration to another. Once convergence is achieved, the element of force on each blade element is computed and their integration gives the total forces and moments on the blade.



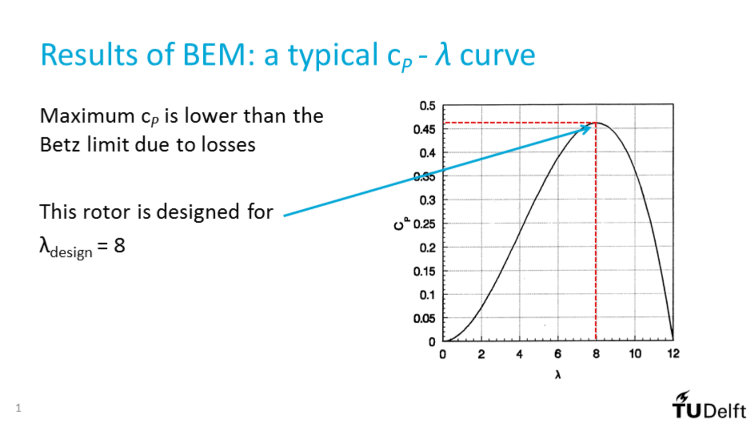
The blade element momentum theory relies on a number of assumptions. It assumes that the global forces on the actuator disk match the local forces on the blades. It also neglects interaction effects between adjacent blade elements.



Similarly, it does not consider any radial flow over the blades and neglects three-dimensional effects, since it only relies on 2D aerofoils. Finally, it does not take into account the tangential component of the force, lying in the plane of rotation. It only equates the normal component to the thrust force. Of course the underlying assumptions of the momentum theory also hold.

3.6 Blade design and performance

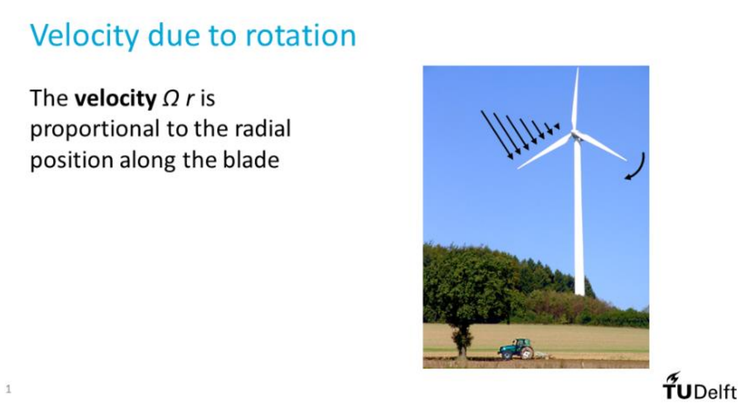
3.6.1 *The c_P-λ curve*



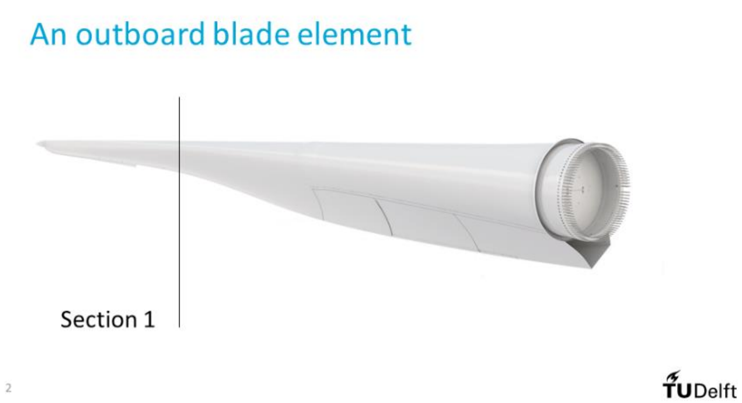
Knowing the forces acting on the rotor, the power that is extracted by the rotor can be computed, and therefore also the power coefficient. The power coefficient is usually represented as a function of the tip speed ratio. For each tip speed ratio, the aerodynamic conditions at each blade section are determined using the blade element momentum theory. From these, the performance of the total rotor can be determined. The CP-λ curve is a non-dimensionalised characteristic of the rotor. It is therefore used in wind turbine design to determine the rotor power for any combination of wind and rotor speed. The shape of the curve can be explained by recalling that at $\lambda = 0$ the rotor does not rotate and hence cannot extract power from the wind. And at very high λ, the rotor runs so fast that it seen by the wind as a completely blocked disc. The wind flows around this "solid" disc without any mass transport through the rotor. Thus, again, the power extracted is zero. Somewhere between these extreme values of λ, there will be an optimum value (here $\lambda=8$) for which the power extracted is maximum. This will be the condition in which the (average) velocity at the rotor disc is 2/3rd of the wind speed according to Betz' law. Note that the maximum CP on this plot is lower than the one given by Betz limit. This is because in practice there are additional losses in the flow than those assumed by the momentum theory. Therefore, the Betz limit is rarely attained for realistic wind turbines. Remember that one mode of control of the wind turbine is to change the rotation speed of the rotor when the wind speed changes, in order to keep the tip speed ratio constant and maximise cp. In practice, the rotation speed of the rotor will therefore be adjusted in order keep lambda as close as possible to the optimal value.

3.6.2 Twist variation along the blade span

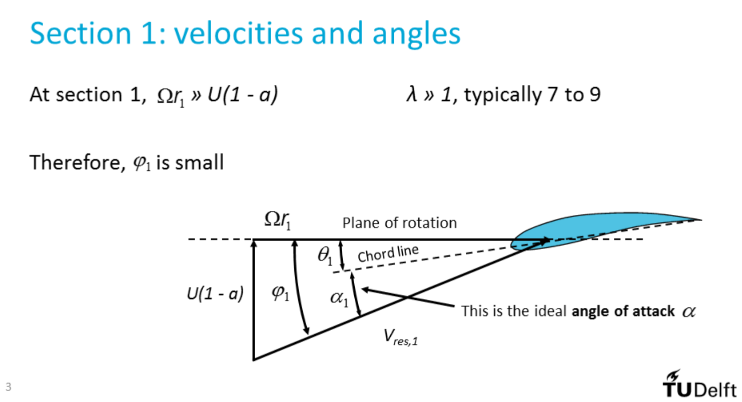
In this unit, we will see how the flow conditions vary along the blade, and what is the impact on the design of wind turbine blades.



Even if the wind turbine is subjected to a uniform wind flow, the velocity seen by the blade varies along its length because the velocity component coming from the rotation of the rotor, that is Omega r, varies linearly with the distance r along the blade. This is why the flow conditions vary along the blade even when the wind flow is uniform.

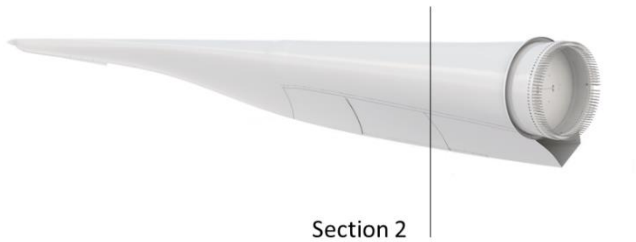


In order to understand the impact of these variations on the blade design, let's first consider a section of the blade located close to the tip. This is section 1.



The velocity triangle at section 1 is shown here. If the distance between the hub and section 1 is called r1, then the velocity induced by the rotation of the rotor equals Omega r1. In normal operation, this velocity is much larger than that component coming from the surrounding wind, that is U(1-a). This is because a wind turbine typically operates at a value of the tip speed ratio between 7 and 9, meaning that the velocity of the tip of the blade is usually seven to nine times larger than the incoming wind speed (U). From the velocity triangle, we see that if Omega r1 is much larger than U(1-a), the inflow angle phi1 is relatively small. Also, in nominal conditions, the control aims at running at the ideal angle of attack that maximises the ratio of lift to drag coefficient. Let's denote that angle alpha_1.

An inboard blade element



Now let's consider a second blade section located close to the hub.

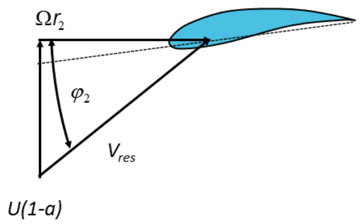
Section 2: Velocities and angles

At section 2, $U(1\text{-}a)$ is identical to the value at section 1

However, r_2 is smaller than r_1

Thus, $\Omega r_2 < \Omega r_1$

And $\varphi_2 > \varphi_1$

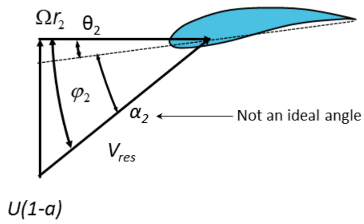


At section 2, we can assume that the wind speed is identical to that at section 1. Thus, the magnitude of the vector U*(1-a) is the same as before. However, the component of the velocity coming from the rotation of the blade is much smaller in section 2 than in section 1, because "r2" is much smaller than r1. Since Omega r2 is much smaller than omega*r1, the inflow angle in section 2 is much larger than that in section 1.

Section 2: Velocities and angles

This means that, if $\theta_2 = \theta_1$, then $\alpha_2 > \alpha_1$

As a result, α_2 is not the ideal angle (assuming the same aerofoil properties)

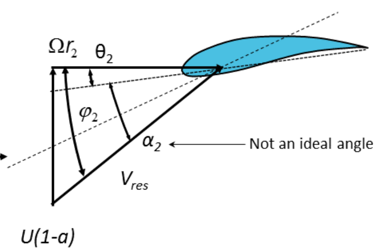


And if the section pitch angles are the same in Section 1 and 2, then the angle of attack at Section 2 is necessarily much larger than the angle of attack at Section 1. Thus, even if the ideal angle of attack is achieved at section 1 close to the tip, the magnitude of the angle of attack at section 2 is far from the optimal value.

Section 2: Velocities and angles

Therefore, θ increases when going towards the blade root in order to have the ideal angle of attack at section 2

Chord line with (ideal) **angle of attack**

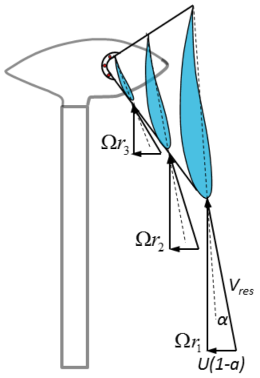


In practise, in order to correct for this, the section pitch angle theta increases from the tip to the hub so that the location of the chord line changes. Then, the ideal angle of attack can also be obtained at section 2.

Velocities and angles

Going from tip (outboard) to root (inboard):

- The radial velocity *Ω r* changes
- The **velocity at the rotor** *U(1 – a)* is constant
- The **angle of attack** *α* is constant



8 TU Delft

Thus, if we consider different blade sections from tip to hub , and assume that U*(1-a) is identical in all the sections, then the pitch angle of the various sections changes in order to keep the angle of attack constant along the blade.

Blade twist

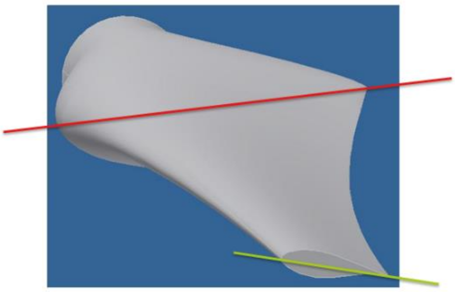


Image: user dsale on http://forums.autodesk.com

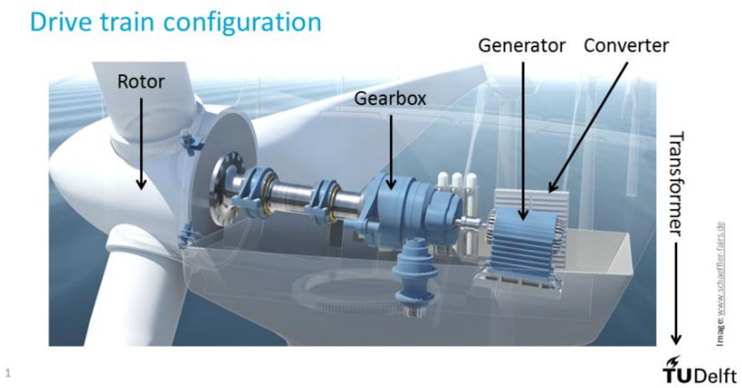
9 TU Delft

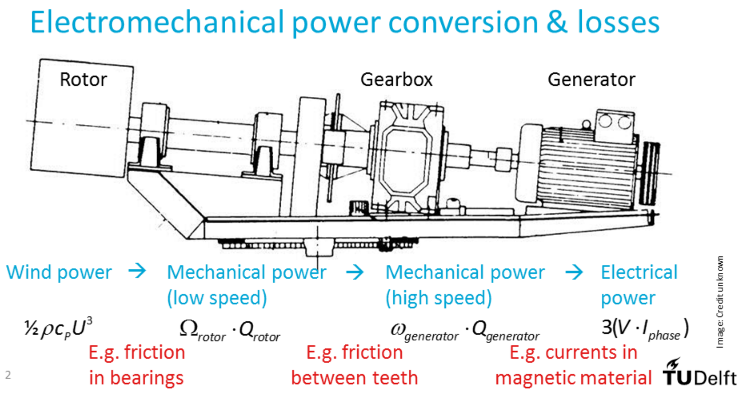
This is why blades are actually twisted.

4 Drive train and control

4.1 Power conversions and losses in the drive train

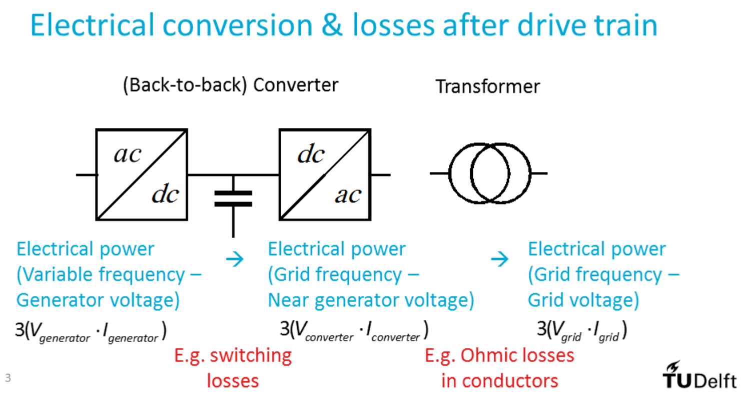
The function of the drive train is to perform a series of power conversions that eventually enables the conversion from wind energy into electricity. Some energy is lost during these conversions. This video describes these power conversions and losses.



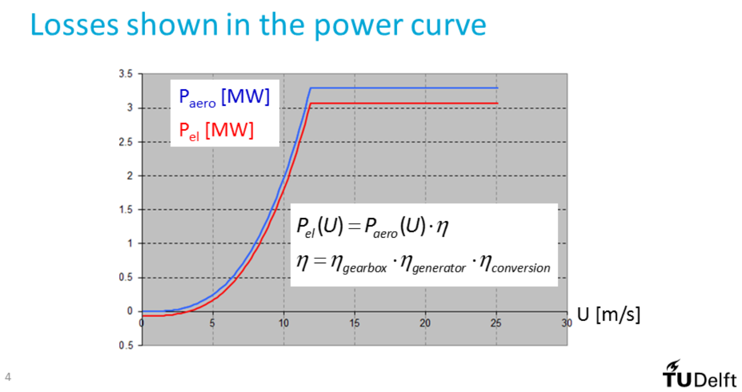


In this video we will focus on the drive train with a gearbox. This means that we have a rotor, a main shaft, a gearbox and a generator. The electricity from the generator goes into a converter, then into a transformer and finally into the public grid. We will look mostly at a permanent magnet generator because the principles of a generator are easiest to explain with this type of generator. This type of generator uses a full electrical power converter which means that all the power that comes out of the generator goes through the converter.

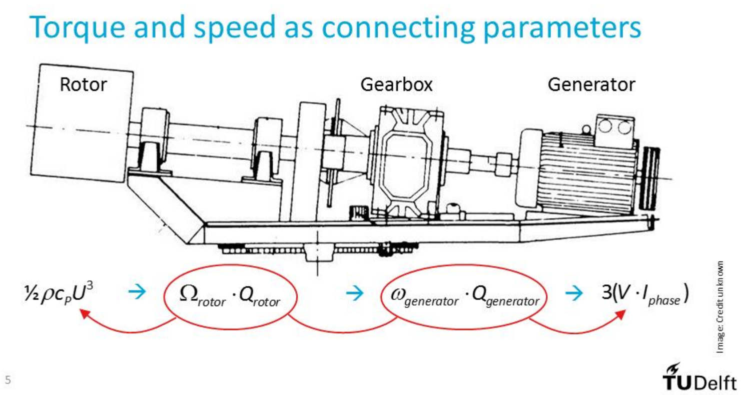
We will first have a look at the conversions and losses in each of the components. The rotor converts wind power to mechanical power at the low speed shaft. The wind power is a ½ rho c_p times the wind speed to the power of three and the mechanical power is the rotational speed of the low speed shaft times to torque in the low speed shaft. The gearbox converts the mechanical power in the low speed shaft to mechanical power in the high speed shaft. Also in the high speed shaft the power can be expressed as the rotational speeds times the torque. The generator converts this into electrical power. Typically a generator in a wind turbine has a 3-phase connection and that means that each of these three phases carries one third of the power. Therefore, the electrical power coming from the generator is three times the voltage times the current in each of the three phases of the output of the generator. For those familiar with three-phase systems, the voltage here is the phase voltage, but that is beyond the scope of this course. In each of these conversion steps some power is lost. The main shaft experiences mechanical losses, such as friction in the bearings. This friction reduces the outgoing torque. There is also friction in the gearbox, for instance between the teeth of the gears and also in the bearings. This reduces the torque in the high speed shaft. The generator has mechanical losses, electrical losses and electro-magnetic losses, such as currents in the magnetic material.



The power that comes out of the generator is carried by an AC or alternating current at a certain frequency. We'll see later that this frequency is determined by the rotational speed of the rotor. The function of the back to back converter is to change this frequency to the frequency of the public grid which is typically about 50 or 60 Hz. During this conversion , there may also be a slight change in the voltage level so the voltage of the converter is a little bit different from the voltage of the generator but the power behind the converter is again three times its voltage times the current. The function of the transformer is finally to step up the voltage level from the converter to the voltage level of the grid. Then we'll end up with a power that is three times the voltage level of the grid times the current in each phase. There are several electrical losses in the converter, such as switching losses. In the transformer we have electrical losses and electro-magnetic losses similar to those in the generator. An example of an electrical loss is the Ohmic loss in the conductor windings.



So far, we have focused on the conversion of wind energy to mechanical energy. So, that gives us the power going into the low speed shaft. In the end, we're interested in the electrical power that is fed into the public grid. The electrical output power is the power that is shown in the power curve of the turbine manufacturer and that is used in annual energy estimations. The losses in the components of the drive train can be translated to efficiencies. Because the power conversions are sequential, we can multiply these efficiencies to get the overall efficiency with which we have to multiply the aerodynamic power to get the electrical power. As shown before, there are different physical processes involved in the losses that lead to these efficiencies. While some losses are independent of operating conditions, other losses can be a function of for instance the rotational speed of the drive train or the currents in the electrical cables. Therefore, the efficiencies are dependent on the operational conditions of the wind turbine. At low wind speeds, the losses may actually be larger than the energy that we can extract from the wind and then we would have to put energy into the system to get the rotor rotating. This is why we have a cut-in wind speed where we are sure that we have a net production of electricity, rather than needing electricity from the grid to get the rotor running.



We have seen that torque and rotational speed in the low speed and high speed shafts play a connecting role in the conversion from aerodynamic power to electrical power. This will reappear in the analysis of drive train behaviour and you will see later that they play an important role in control of the output power of the turbine.

4.2 Component characteristics

4.2.1 Rotor

Aerodynamic power

The behaviour of the rotor can be determined with the blade element momentum theory. One of the most informative ways of capturing the rotor's performance that you have seen so far is the use of the c_P-λ curve. A typical example is repeated in Figure 1.

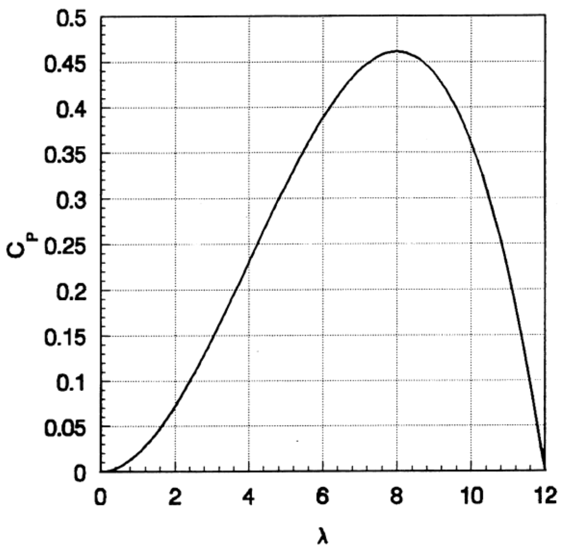


Figure 1Typical c_P-λ curve

The dimensionless coefficients used in this curve are defined by the following equations:

$$\lambda = \frac{\Omega R}{U}\,,\; c_P = \frac{P}{½\rho U^3 \pi R^2}\,.$$

By substitution of the rotor diameter and the actual conditions regarding air density, wind speed and rotational speed in these definitions the actual aerodynamic power can be determined from the curve.

Torque and torque coefficient

The aerodynamic torque of the rotor in the hub can be expressed in the power and rotational speed through the relation

$$Q = \frac{P}{\Omega}\,.$$

As for the thrust and the power, it can be convenient to express the torque in a dimensionless coefficient. For this purpose the torque coefficient is defined as:

$$c_Q \equiv \frac{Q}{½\rho U^2 \pi R^3} = \frac{Q}{½\rho U^2 \pi R^2 \cdot R}\,.$$

You can recognise that the torque is normalised with the rotor radius times the factor that is used to normalise the thrust. This makes sense, since the rotor radius is a representative parameter to use for the lever arm that changes the (representative) force into a moment.

Speed-torque characteristics

The c_P-λ curve relates the dimensionless power to the dimensionless speed, which is convenient to capture the rotor performance for all operational conditions in one curve. A similar characterisation can be determined between the dimensionless speed and the dimensionless torque.

First we'll rearrange the terms in the definition of the torque coefficient and introduce the rotational speed in both numerator and denominator:

$$c_Q \equiv \frac{Q}{½\rho U^2 \pi R^3} = \frac{Q}{½\rho U^3 \pi R^2}\frac{U}{R} = \frac{Q\Omega}{½\rho U^3 \pi R^2}\frac{U}{\Omega R}.$$

In this expression you can recognise the tip speed ratio, $\lambda = \Omega R/U$, and the power, $P = Q\Omega$. Substitution leads to:

$$c_Q = \frac{P}{½\rho U^3 \pi R^2}\frac{1}{\lambda},$$

in which we can recognise the definition of the power coefficient, $c_P = P / (½\rho U^3 \pi R^2)$, which finally gives us a relation between c_Q, c_P and λ:

$$c_Q = \frac{c_P}{\lambda}.$$

If we follow the c_P-λ curve and divide each point on the y-axis by λ, we can make the c_Q-λ curve, shown in Figure 2. Notice that the point where we obtain the highest power coefficient, so at λ = 8, is not at the point where maximum c_Q is obtained.

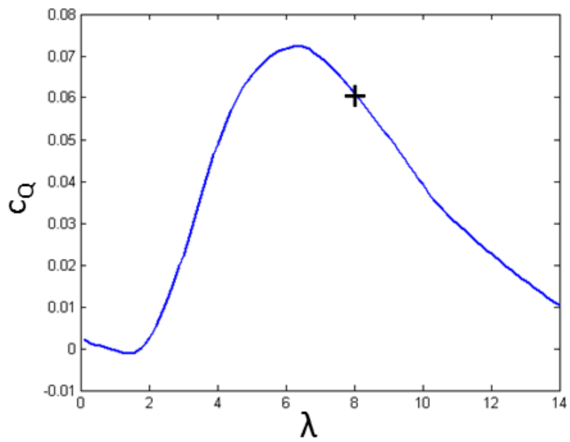


Figure 2 Typical c_Q-λ curve with '+' indication the point of maximum c_P

To make a Q-Ω curve, rewrite the definition of the tip speed ratio and the torque coefficient to:

$$\Omega = \lambda \cdot \frac{1}{R} \cdot U$$
$$Q = c_Q \cdot ½\rho\pi R^3 \cdot U^2$$

These expressions can be used to translate the x, respectively y-axis of the c_P-λ curve. For a specific wind turbine, the middle part of the right-hand side of both expressions is a constant. Therefore, for each wind speed U the Q-Ω curve is different, but they have similar shapes. Examples are given for 10 m/s and 12 m/s wind speeds in Figure 3.

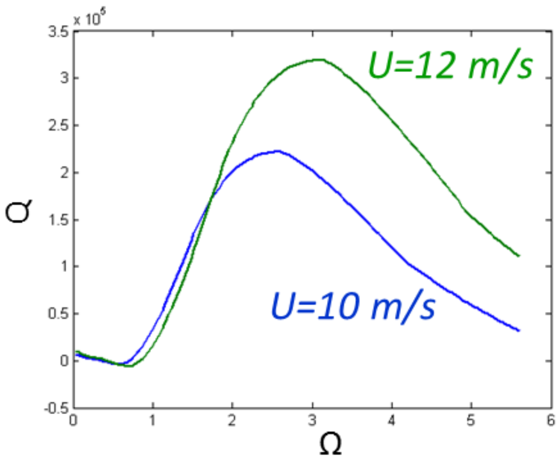
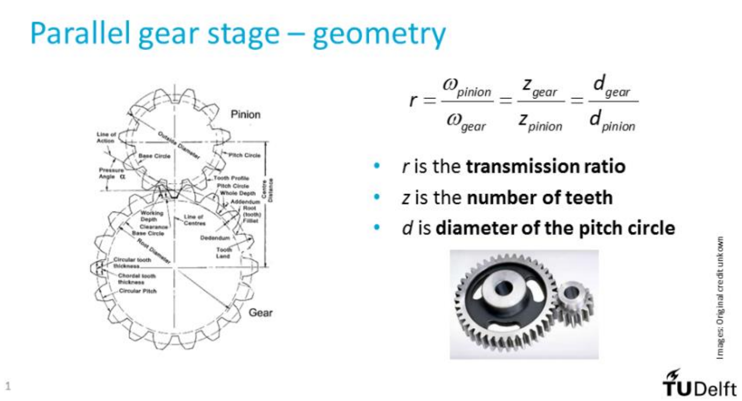
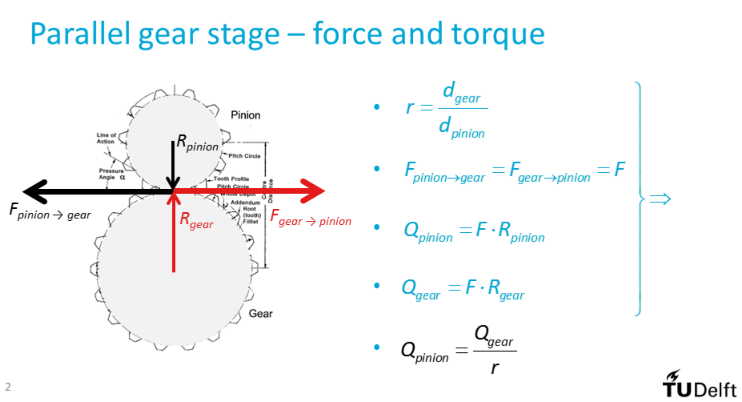


Figure 3 Typical Q-Ω curves for different wind speeds. Values on the x and y-axis depend on turbine size.

4.2.2 Gearbox

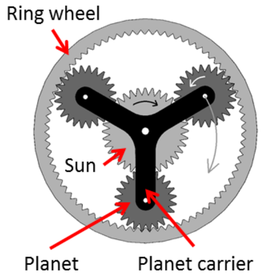


We'll first have a look at the parallel gear stage. The left-hand drawing introduces some terminology. We have two wheels, a smaller one in a bigger one, and the smaller one is called a pinion, while the bigger one is called the gear. The pinion and gear have teeth, and the contact points of these teeth lie on circles for the pinion and the gear, that we call pitch circle. The transmission ratio is, per definition, the rotational speed of the pinion, divided by the rotational speed of the gear. And, through simple geometry, we can see that this equals the number of teeth in the gear, divided by the number of teeth in the pinion. Using the pitch circle diameters, we can also express this as the diameter of the gear pitch circle divided by the diameter of the pinion pitch circle.



The gear stage doesn't only change the rotational speed, but it also changes the torques in the shafts of the pinion and the gear. The force of the pinion on the gear is equal in size but opposite to the force of the gear on the pinion. However, regarding their torques, they have different lever arms: a small lever arm for the pinion, and a larger lever arm for the gear. Therefore, the torque in the gear is higher than the torque in the pinion, by the ratio of the radius of the gear divided by the radius of the pinion. Because of the expression for the transmission ratio in terms of the pitch circles, which is defined by the contact points between the teeth, we can also express this ratio in the transmission ratio, so the torque in the pinion is equal to the torque in the gear, divided by the transmission ratio.

Planetary gear stage – geometry



$$r = \frac{\omega_{sun}}{\omega_{planet\ carrier}}$$

For a fixed ring wheel:

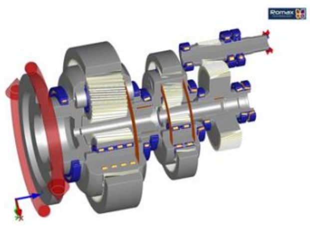
$$r = 1 + \frac{z_{ring}}{z_{sun}} = 2\left(1 + \frac{z_{planet}}{z_{sun}}\right)$$

As for parallel gear, the number of teeth can be replaced by the diameter of the pitch circle

3 TUDelft

Now we'll have a look at the planetary gear stage. Again, the left-hand side drawing gives some terminology. We have a sun, which is the gear in the centre, and we have a ring wheel, which is the outer gear, and in between we have the planets. The planets are carried by the planet carrier, which is the black y frame in the middle. The transmission ratio is again defined as the rotational speed of the sun, in this case, divided by the rotational speed of the planet carrier. For a wind turbine, the ring wheel is usually fixed. For this situation, we have a transmission ratio which is equal to 1, plus the number of teeth in the ring, divided by the number of teeth in the sun. This is slightly more complicated to derive, than for a parallel gear stage, so we'll omit the derivation. We can also express the transmission ratio slightly differently, as 2 times 1 plus the number of teeth in the planet, divided by the number of teeth in the sun. Of course, we can also replace the number of teeth here by the pitch circle diameters, just as we've seen for the parallel gear stage.

Gearbox – planetary & parallel stages



$$r_{gearbox} = r_1 \cdot r_2 \cdot r_3$$

- Planetary stages:
  Typically $r < 7$
- Parallel stages:
  Typically $r < 5$

4 TUDelft

Since the gearbox is simply a sequence of planetary and parallel stages, the transmission ratio of the gearbox is simply a multiplication of the transmission ratios of the individual stages. For planetary stages, we can typically not reach larger transmission ratios than about 7, and for parallel stages no more than about 5. The reason for that, is that the difference in the number of teeth, or the difference in diameter, for the smaller and the larger wheels become too large if we go to larger transmission ratios. We have to take this into consideration, when we determine how many stages we need to get the total transmission ratio.

4.2.3 Generator

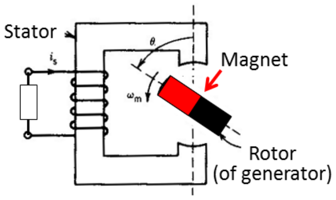
Working principles of a generator

- Two questions
  - How does a generator work?
  - What is the relation between generator speed, torque and the AC frequency to which the generator is connected?
- The three ingredients:
  - Electromagnetism: induced voltage, current and magnetic field
  - Electro-mechanics: forces in (electro-) magnetic field
  - Rotating magnetic field in a three-phase generator

1 TUDelft

To help understand the working principles of a generator, we are going to address two questions. The first one is: how does a generator actually work? The second one is: what is the relation between the speed of the generator, the torque in the high speed shaft, and the AC frequency to which the generator is connected? We'll use three ingredients to explain the working principles, and answer these two questions. The first one is electromagnetism, which relates to the induction of voltages, currents and magnetic fields. The second is electro-mechanics, which relates to the forces on conductors in magnetic fields. The third one is a rotating magnetic field in a three-phase generator.

Ingredient 1: Electromagnetism

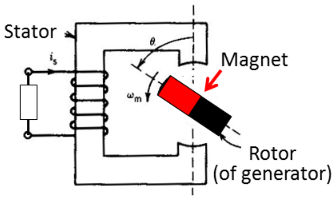


- Changing magnetic field induces a voltage in a coil
- When the circuit is closed, a current is induced
- Current induces a magnetic field that opposes the change

2

So, the first ingredient is electromagnetism. We'll represent the stator of the generator, which is the fixed part of the generator, by this horseshoe shaped iron with a winding around it, and we'll represent the rotor of the generator by a permanent magnet that is able to rotate. If the magnet rotates and the pole moves away or towards the iron, the magnetic field in the iron changes. This change in magnetic field induces a voltage in the winding, and if we attach the winding to a load, in this example to a resistor, we'll induce a current in that winding. The current in the winding will also induce a magnetic field, which opposes the change in the magnetic field caused by the rotation of the magnet.

Ingredient 2: Electro-mechanics



- The induced, opposing, magnetic field creates poles in the stator that exert a torque on the magnet that opposes the rotation
- Power balance without losses:

$$Q_{opposing} \cdot \omega_{magnet} = V_{coil} \cdot I_{coil}$$

3

The second ingredient is electro-mechanics. As we have seen, when the magnet rotates, it causes a change in the magnetic field in the horseshoe, and the winding creates a magnetic field that opposes this change. If the north pole of the magnet is moving away from a cap of the horseshoe, then the opposing magnetic fields will create a south pole on that cap to attract the magnets, and to avoid that rotation. This attraction between the poles is associated with the torque, and the torque of this opposing force times the rotational speed of the magnet, is the work that's done by the horseshoe on the magnets. This work is equal to the power that's taken off by the resistance, which is the voltage of the coil times the current in the coil. So, this is the principle behind the conversion from mechanical energy, the torque times the rotational speed, to electrical energy, the voltage times the currents.

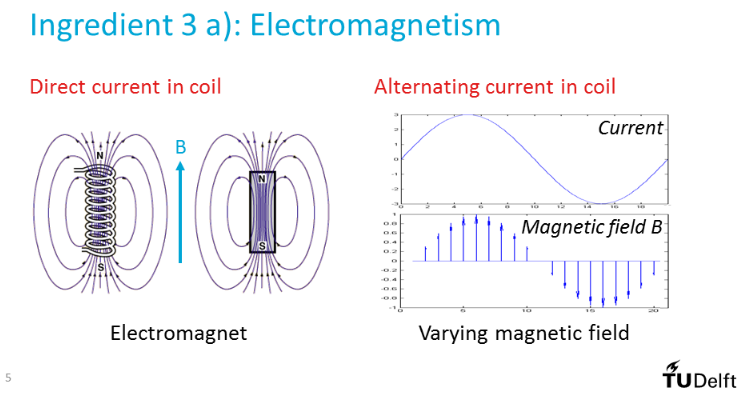
Ingredient 3: Rotating magnetic field

Two parts of the explanation

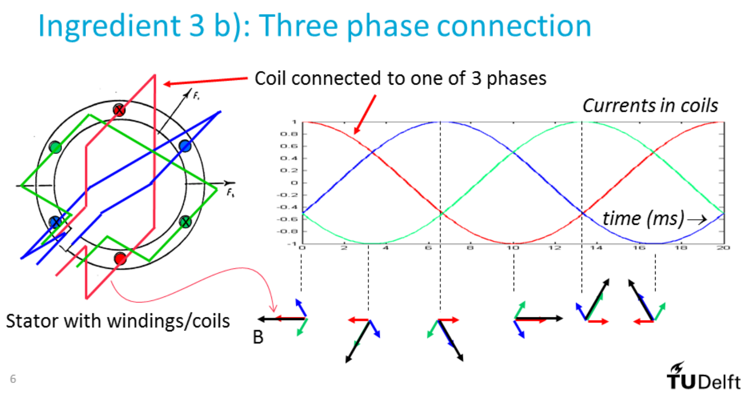
a) Electromagnetism with alternating currents
b) Three phase connection

4

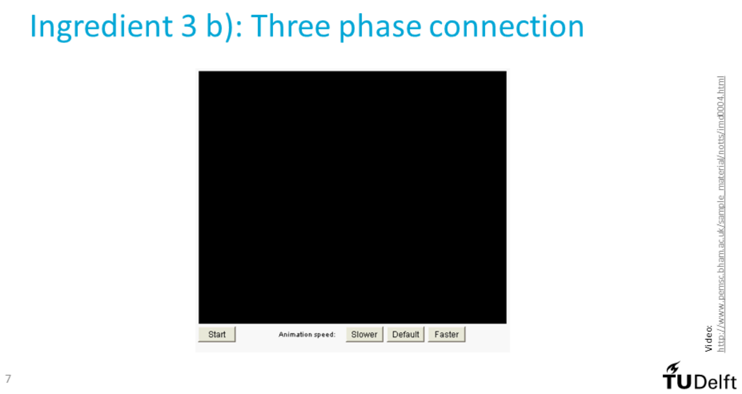
The first two ingredients explain how mechanical energy can be converted into electrical energy, if we have a single winding that is directly connected to a load. However, a generator of a wind turbine is typically connected through multiple windings to the three phase public grid. And this grid is not a simple load, but it is a system operating at alternating currents. Let's have a look what happens then. We will first look into a bit more detail of electromagnetism in case of alternating currents, and then we'll look at the free phase connection of the generator.



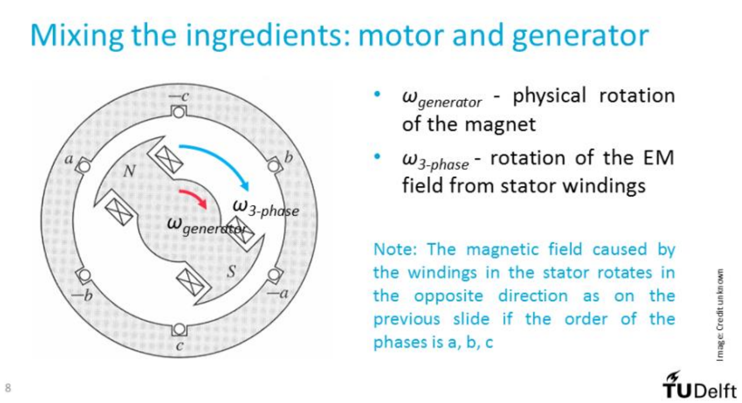
The principle of the electromagnetism in this case is the same as in the previous case: if we apply a current to a coil, then we'll create a magnetic field in that coil. On the left hand side, you see a direct current in a coil and this leads to magnetism inside the coil with a permanent north and south pole. On the right hand side, you see an alternating current which changes sinusoidally over time. If we apply that current to the coil, then also the magnetic fields will change over time, and during half of the period the current will be positive, and the fields will be pointing in one direction, while during the other half of the period the current will be negative, and the magnetic field will be pointing into the opposing direction. Besides that, it will also follow the variation in size.



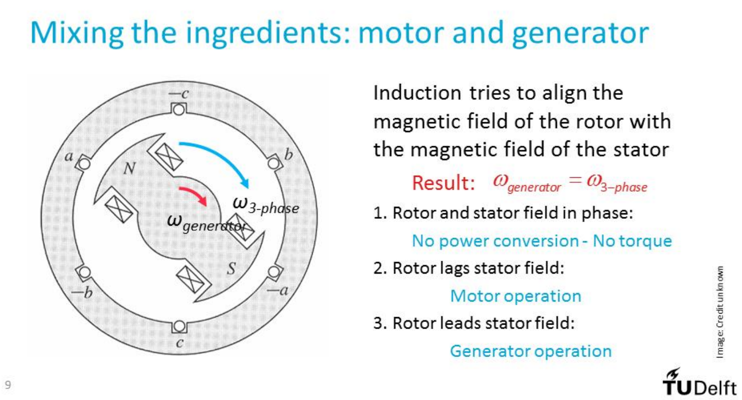
A three phase generator has multiple coils that are connected to different phases of the grid. We'll represent each of these coils with a single winding. Here you see the single winding in red connected to the red current in the right hand graph. Because the winding is oriented vertically, the magnetic field that is induced by the current through this winding is horizontal. At the start of the variation in the current, when it's at its maximum, the magnetic fields points to the left and is at its largest. As the current reduces and becomes negative, the magnetic field also reduces and starts pointing towards the right. The second winding that's shown here is connected to the blue variation of the current. Everything happens here 120 degrees later in the period of the variation of the currents. In addition, the orientation of this winding differs from the orientation of the red winding. So now, the magnetic field is first pointing to the upper-left, and then it's pointing to the lower right, as the current changes. Finally we'll look at the green coil which is connected to again a different phase of the current, and also oriented in a different direction. This magnetic field is first pointing to the lower left, and then pointing to the upper right. The total magnetic field inside the generator is the addition of these magnetic fields through the three different coils. So, by making the vector addition at each point in time, we get the black arrow, and what we see is that the size of the magnetic fields of the total, doesn't change over time, but only its orientation. So in this case the magnetic field is turning counter-clockwise, because of the B phase connection of the three different coils.



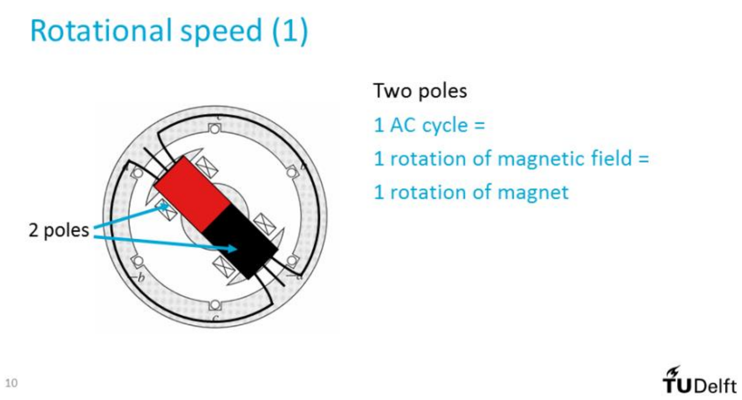
This video shows an animation of the magnetic field inside the generator, that is created by the three phase connection. As explained in the previous slide, the magnetic field rotates without changing in magnitude. This animation also shows that the magnetic field makes one full circle for each period of the variation in the current.



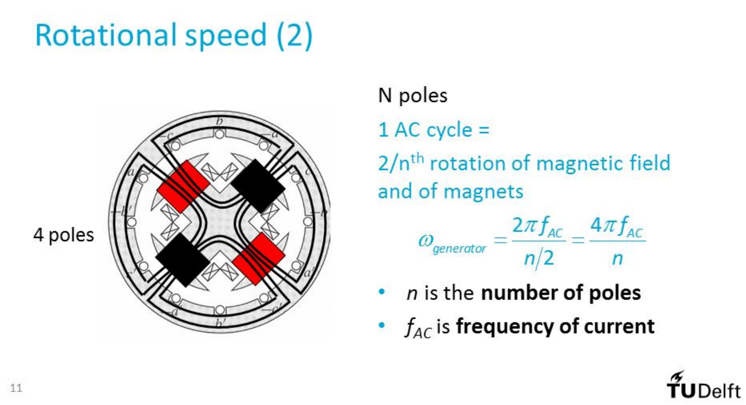
Now we have all the ingredients to understand why a generator generates, and when it operates as a motor. As we have seen, we have a magnet with a fixed north and south pole that's rotating inside the generator. This physical rotation has a rotational speed omega generator. We've also seen that the currents through the windings cause a magnetic field that rotates as well, and this rotates at what we will call omega three-phase, the rotational speed associated with the frequency of variation in the currents.



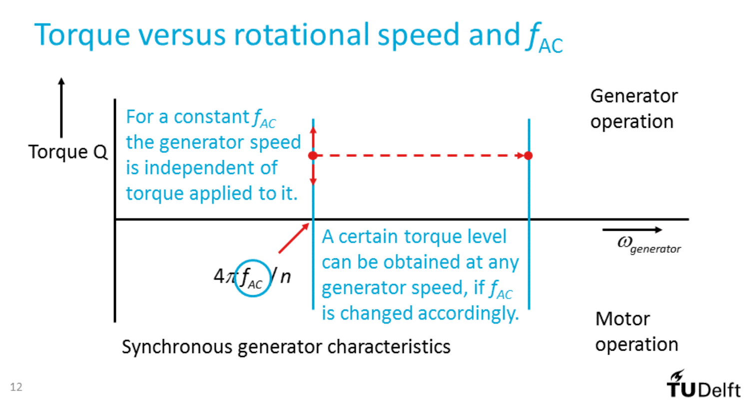
We've seen that in the generator, the stator coils and the permanent magnet of the rotor are two sources of magnetism. Two sources of magnetism will try to align their magnetic fields, which is why the north and south poles attract each other. When the magnetic field of the rotor and the magnetic fields created by the coils in the stator are aligned, the rotational speeds of these two fields are the same. Furthermore, the rotations are in-phase, meaning that they both have their poles aligned vertically at the same time. Due to the equilibrium between the magnetic field of the rotor and the magnetic field created by the stator, there is no electromechanical torque applied to the rotor in this case. In other words, the rotor magnets doesn't induce a current in the stator coils. Both effects imply that there is no energy conversion. No torque means no mechanical energy, and no current means no electrical energy. When the rotor lags the rotating stator field, the stator field will pull on the rotor to align it. This corresponds to a torque in the same direction as the rotational speed, and therefore it performs positive work on the rotor. The rotor induces a current in the stator coils, that is out of phase with its voltage. The pulling torque of the stator field and the out of phase current show that we now have motor operation drawing energy from the grid, and converting it into mechanical energy. When the rotor leads the rotating stator field, the opposite happens. The stator tries to withhold the rotor, causing a torque opposite to the rotational speed. The rotor induces a current in the stator coils, that is in phase with its voltage. This corresponds to generator operation, where mechanical energy is converted into electrical energy.



Now that we've connected the rotational speed of the magnet to the rotational speed of the magnetic field caused by the windings, we can make an assessment of rotational speed of the generator. Let's first have a look at the configuration that we've seen before, where we have a single magnet, with one north and one south pole, and a stator with three windings, each of them connected to one phase of the grid. So, this is also the configuration we've seen before. In this case, as we have seen, during one period of the AC cycle the magnetic field caused by the stator will rotate once, and therefore also the magnet will have to rotate once to keep pace with that. As we will see later, when the magnet is running either in motor operation or in pace with the magnetic field, or in generator operation, it will always rotate at the same speed as the magnetic field. In generator operation it will only be slightly before the magnetic field, and in motor operation it will be slightly behind the magnetic field, but rotating at the same speed. So knowing that one period of the AC cycle corresponds with one rotation of the magnet, we can determine the rotational speed of the magnet.



There are also other configurations of generators with multiple magnets. In this example, you see the configuration with two north poles and two south poles, so this is a four pole generator. In that case we also double the number of windings. So, we have two windings connected to phase A, two windings connected to phase B and two windings connected to phase C. During one period of the AC cycle, the red north pole of the magnet moves half a rotation to the side of the other red north pole. In other words, the magnetic field makes two divided by the number of poles - rotations per AC cycle. Using this, we can calculate the generator speed in the general case, which is 2pi times the frequency of the AC current, divided by half the number of poles, so four pi times the frequency divided by the number of poles.



In this graph, we see the torque as a function of the generator rotational speed. In the previous slides, you have seen that we can calculate the rotational speeds of the stator field, from the number of poles and the frequency of the three-phase connection. You have also seen that the rotor aligns itself with the stator field, giving it the same rotational speed. When we apply a positive torque on the rotor, the rotor will lead a bit on the stator field, and we have generator operation. However, this is only a phase difference between the rotation of the rotor and the rotation of the stator field, and not a speed difference. For a permanent magnet generator, positive or negative torques within the design specifications of the generator, will only lead to a change in the phase and not in the rotational speed. Therefore, the speed-torque characteristic is a straight vertical line. In other words, the electrical frequency determines the speed in the drivetrain, no matter what torque is applied to it. This is why this is called a synchronous generator. However, thanks to the back-to-back converter, the electrical frequency can be changed. Therefore, we can achieve a desired torque level, at any rotational speeds that we want. The ability to manipulate torque and speed of the generator will appear later to be crucial in the control of the turbine.

4.2.4 Electrical components

Back-to-back converter

Figure 1 shows the schematic representation of the back-to-back converter. The function of the back-to-back converter is to match the frequencies of the three-phase connections on either side. It has to match the 50 or 60 Hz of the public grid on one side and the desired frequency on the generator side that is related to the rotational speed of the generator. The frequency on the generator side may both be smaller and larger than the frequency of the grid.

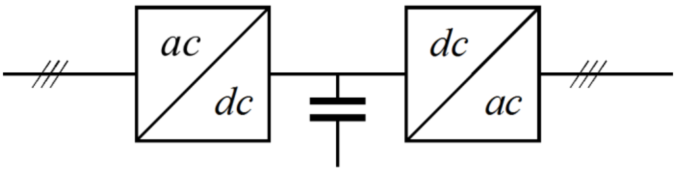


Figure 1 Back-to-back converter

In addition, the DC bus voltage inside the converter and current on the generator side can be controlled. By doing this, it can control the torque of the generator.

Transformer

Figure 2 shows the schematic representation of the transformer. The function of the transform is to step up the voltage from the converter voltage to the grid voltage. Typically, the side of the converter that connects to the transformer has a voltage of several kV, up to around 10 kV. On the grid-side the transformer typically connects to the regional grid, which operates are medium voltage of around 36 kV. Sometimes the turbines need to be connected to a larger grid, or the transmission line of an offshore wind farm, that operates at high voltage. This is usually done by first connecting a group of turbines at medium voltage and then using a larger transformer to go to high voltage.

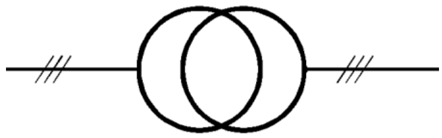
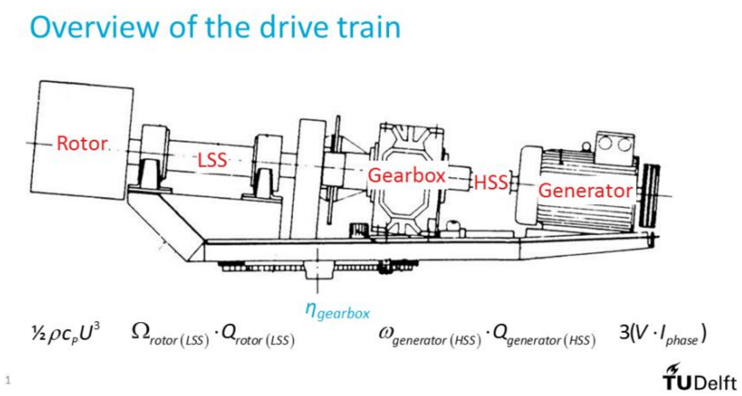


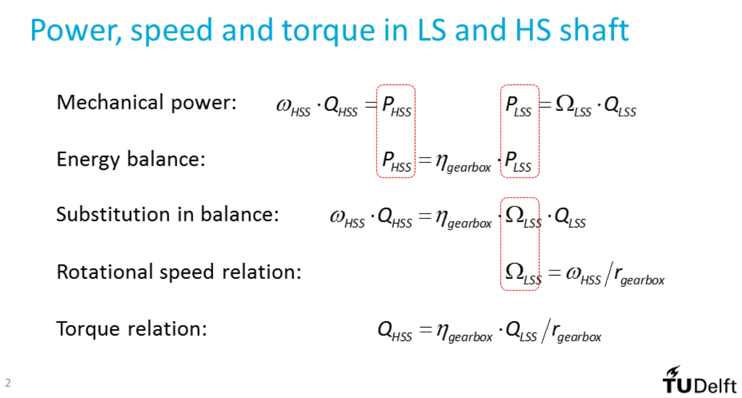
Figure 2 Transformer

For different locations, different transformers can be chosen, depending on the grid voltage to which it connects. Because the back-to-back converter decouples the generator from the transformer as regards frequency, voltage and current, the transformer has no influence on the torque in the drive train.

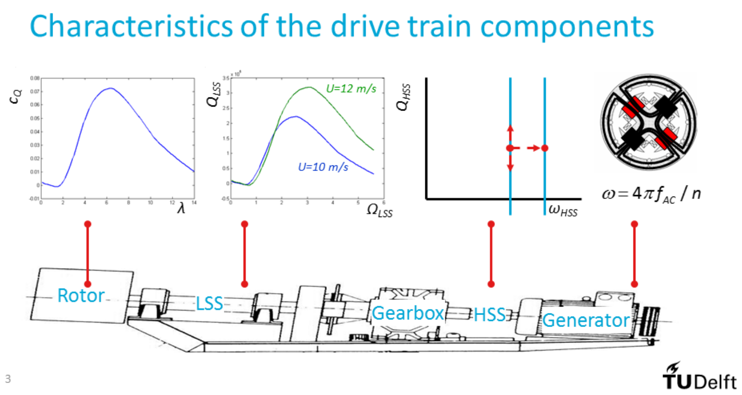
4.3 System behaviour



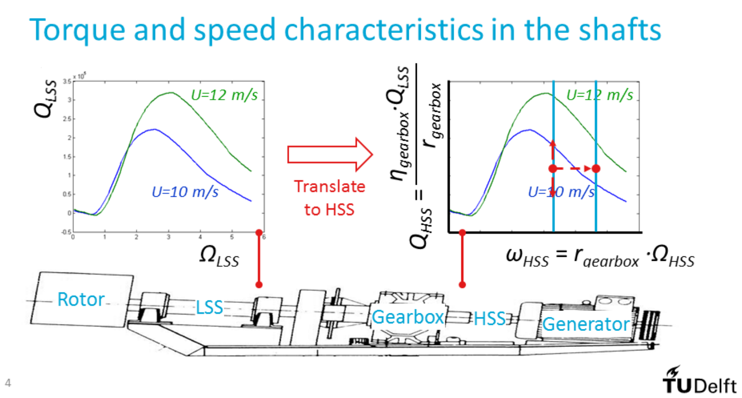
To address the overall behaviour of the drive train, we'll go back to the overview that was provided at the beginning. We have seen how aerodynamic power is converted by the rotor and transmitted as mechanical power through the low speed shaft, the gearbox, and the high speed shaft to the generator, where it is converted into electrical power. For the analysis of the behaviour of the drive train, we will focus on the low speed shaft and on the high speed shaft, for which the power is determined by their rotational speed and torque. Their inputs come from the rotor and generator, respectively, and they connect through the gearbox. Therefore, we'll also look at the efficiency of the gearbox, and how the gearbox changes speed and torque.



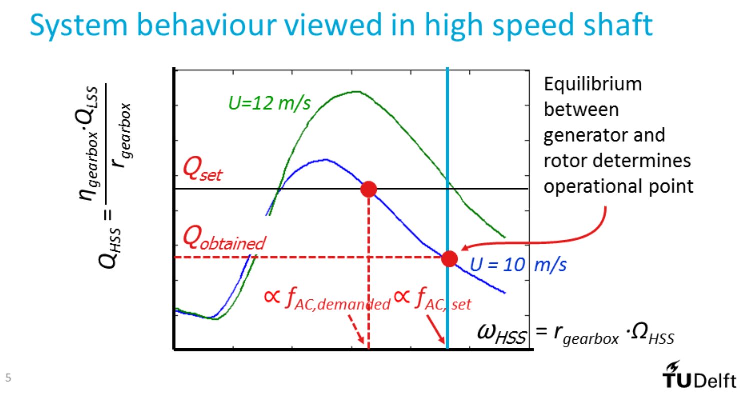
As you have seen before, the mechanical power can be expressed as rotational speed times torque. When we relate the power in the high speed shaft to the power in the low speed shaft, we have to consider the efficiency of the gearbox. In the next step, we substitute the expressions for power, to get a relation between torque and speed in both shafts. Furthermore, the rotational speeds are related through the transmission ratio of the gearbox. The efficiency of the gearbox has no effect on this expression, since it is a purely geometrical relation. Substituting this expression for the rotational speed in the energy balance, leads to a relation between the torque in the high speed shaft and in the low speed shaft. This expression shows that the efficiency directly affects the torque in the high speed shaft. This should not come as a surprise. The losses in the gearbox are caused by friction, which leads to a reduction in torque on the outgoing shaft.



Here you see a recap of the torque speed characteristics of the different components. Neglecting losses in the main bearings, the aerodynamic cQ – λ curve can be directly translated to the speed-torque curves in the low speed shaft. Similarly, the speed characteristics of the generator can be directly translated to the speed-torque characteristic in the high-speed shaft. However, the speed and torque levels in the two shafts differ several orders of magnitude, due to the separation by the gearbox. Therefore, we cannot directly judge from them how the system is going to behave. The next slide will show how the connection of speed and torque through the gearbox properties can help with this.



The torque-speed characteristics in the low speed shaft and high speed shaft are repeated here. For the next step, it is good to realise what these characteristics actually mean. Let's first look at the low speed shaft. These characteristics were based on the aerodynamic properties of the rotor, so it represents the behaviour in the low speed shaft when looking into the direction of the rotor. As a though experiment, disconnect the low speed shaft from the gearbox and instead connect it to a testing machine. This testing machine can be set at any rotational speed. If the wind speed is blowing at 10 m/s, and the testing machine would gradually increase the rotational speed in the low speed shaft, the torque measured by the testing machine would follow the blue curve from left to right. Now we'll look at the high speed shaft. In our thought experiment here, we disconnect the high speed shaft from the gearbox and connect it to the testing machine. Let the testing machine apply a torque on the high speed shaft and measure the rotational speed. It will be clear that the machine will measure the speed as it is set by the electrical frequency. Finally, we get to the crucial step. In the high speed shaft we have looked into the direction of the generator. But what are the torque-speed characteristics if we look from the high speed shaft into the direction of the rotor? In other words, what if we disconnect the high speed shaft from the generator and connect it to the testing machine there? We can achieve these characteristics by using the gearbox properties. These tell us what happens to the torque and speed from the low speed shaft, when they are transferred to the high speed shaft. Using these relations, we can translate the torque-speed curve of the low speed shaft to its equivalent in the high speed shaft, as shown here. Of course, they look similar in shape, but the scales in both x- and y-axis have been changed.

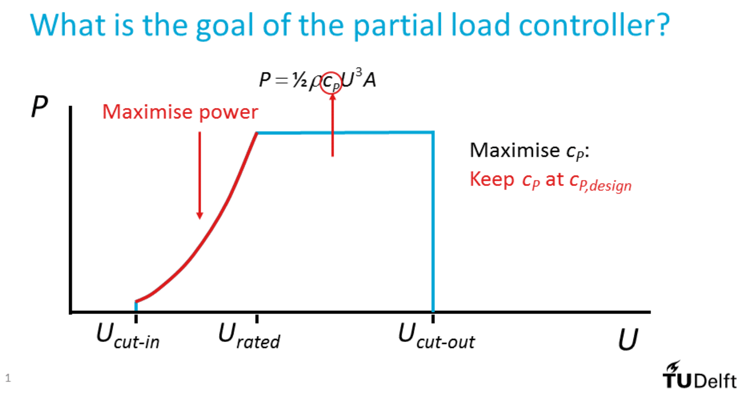


Now that we know the characteristics in the high speed shaft, both looking in the direction of the rotor and in the direction of the generator, we can determine from the combined graphs at which rotational speed and torque the system is going to settle. Remember that the blue curve for 10 m/s wind speed was obtained by replacing the generator by a testing machine. Now that the generator is back, the generator takes the role of the testing machine. It sets the generator curve somewhere, depending on the electrical frequency. The consequential torque in the high speed shaft is where this curve crosses the blue curve. This is the point where the generator torque and rotor torque reach equilibrium in the high speed shaft. If this generator was connected to the 50 Hz of the grid, without back-to-back converter, the operational point would always fall on this vertical line. For different wind speeds it would intersect at different heights with the relevant rotor curve. However, with the back-to-back converter the generator can also be controlled differently. It is also possible to set the torque in the generator and adjust the electrical frequency according to demand. In this case, the operational point is found at the intersection with the horizontal line and the demanded electrical frequency follows from the resulting rotational speed. This analysis shows that it is not the wind or the rotor aerodynamics that determine the speed of the rotor. They do play a crucial role through the cQ – λ curve, but it is the control of the generator that is decisive.

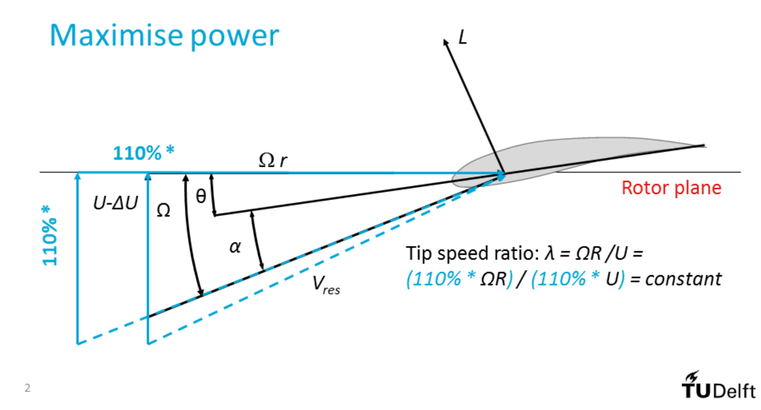
4.4 Partial load control

4.4.1 Goal and working principle

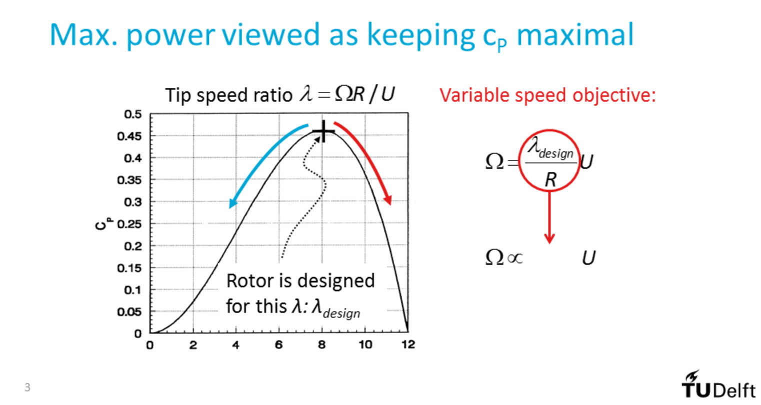
Now that you are familiar with the behaviour of the drive train, we will have a look how the drive train can be controlled to obtain the desired power curve. We'll first look at control in the partial load region.



In the explanation of the power curve it was stated that in the partial load region the output power is maximised. As you know, the power can be expressed as a function of the wind speed and the power coefficient c_P. Actually, the only thing we can influence instantaneously here, is the power coefficient because the wind speed is an external factor and the other parameters are constants. If we want to maximize the power in the partial load region we have to maximize the power coefficient. In other words, we have to keep the power coefficient constant at the maximum achievable value, close to the Betz limit.



What do we need to do to keep the power coefficient at its maximum, if there is a change in wind speed? Suppose that the rotor is operating at maximum performance under operational conditions for which it is designed. At a certain moment the wind speed increases, say by 10 percent. For now, we'll assume that this has the same effect on delta U, so delta U also increases by 10 percent. If we would then increase the rotational speed by 10 percent as well, we would get the same inflow geometry as for the original case. In this case, the angle of attack will remain the same. It can be shown that in this case the thrust coefficient and the induction factor also remain the same. If the induction factor remains the same, our assumption that delta U also increases by 10 percent is valid. Finally, a constant induction factor leads to the conclusion that the power coefficient remains the same. To keep the inflow geometry the same we effectively keep the tip speed ratio the same: for each change in wind speed we apply an equal change in rotational speed, thus keeping the ratio between omega times R and U constant.



We can arrive at the same conclusion from a different perspective. Here you see the c_P – Lamda curve again. The rotor for which this curve is made is designed for a tip speed ratio of eight. Suppose again that the rotor is operating at maximum performance, so at a tip speed ratio of eight. If the wind speed increases and we do not change the rotational speed, the tip speed ratio decreases. This means that the operation of the rotor moves to the left-hand side of the graph, leading to a drop in performance. On the other hand, if the wind speed reduces, then the tip speed ratio increases and operation moves to lower performance on the right-hand side of the graph. To keep the operation of the rotor on the top of the curve, we have to adjust the rotational speed proportional to the wind speed and keep the tip speed ratio constant at its design value of 8. Having established that we need to keep the tip speed ratio constant, the next task we are facing is to determine how that can actually be achieved.

4.4.2 Implementation

Theoretical background

In partial load, the goal of the controller is to keep the power coefficient at its maximum value by keeping the rotational speed proportional to wind speed. A straightforward control strategy therefore seems to measure wind speed and then set the desired rotational speed by controlling the AC frequency of generator connection. However, the wind speed at the rotor cannot be measured accurately. It is influenced by the induction factor and the flow around the nacelle and the measurement may not be representative for the entire rotor, over which the wind speed varies.

To find a solution for a control strategy with observables that can be properly measured, we go back to our original goal, which is to keep c_P and λ constant. Thinking back to the relation $c_Q = c_P / \lambda$, you see that this implies that the control objective can also be interpreted as keeping c_Q constant. How can that be achieved, since none of these dimensionless parameters can be measured directly? Substitute the original definition for C_Q to obtain

$$\frac{Q}{\frac{1}{2}\rho U^2 \pi R^3} = \frac{c_P}{\lambda}.$$

This still contains the wind speed U, but we can substitute this by $U = \Omega R / \lambda$ to get

$$Q = \frac{½\rho(\Omega R)^2 \pi R^3}{\lambda^2}\frac{c_P}{\lambda} = \frac{½\rho c_P \pi R^5}{\lambda^3}\Omega^2 .$$

The coefficient before Ω^2 is a constant, so the controller will have to keep the torque Q proportional to the square of the rotational speed Ω, so $Q \propto \Omega^2$.

Figure 1 demonstrates how this control principle leads to the desired power curve in partial load. On the left-hand side the torque-speed curves of the rotor are shown for various wind speeds. The controller tracks the curve $Q \propto$ to Ω^2 and crosses these curves at the red dots. As an example, consider a wind speed of 10 m/s. The Figure shows that at this wind speed the operational point of the controller will be Ω = 3 rad/s and $Q = 2\cdot10^5$ Nm. Using P = QΩ, this means that the power is $6\cdot10^5$ W, which can be plotted as the power at 10 m/s wind speed, as shown on the right-hand side of the Figure. Doing this for various wind speeds below rated wind speed, we do indeed obtain the expected power curve.

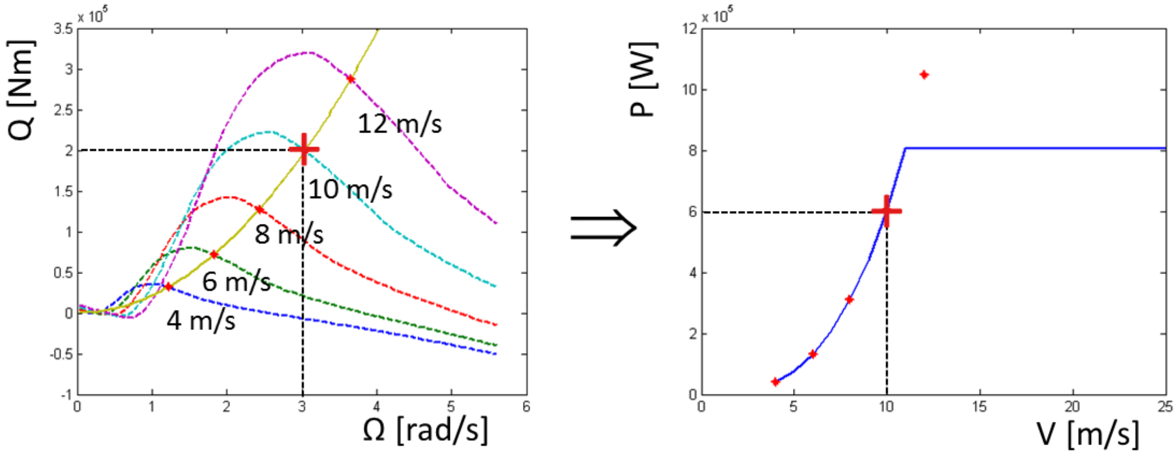


Figure 1 Illustration of how $Q \propto \Omega^2$ – tracking leads to the desired power curve in partial load

Recognise that the control curve crosses the torque-speed curves at the same geometrical position for each wind speed, corresponding to the c_Q at which maximum c_P is achieved.

Control strategy

The relation between torque and rotational speed at the rotor still doesn't provide a convenient means for control. It is possible to measure the rotational speed of the rotor with good accuracy, but the torque in the rotor cannot be set directly. What can be done is setting the torque in the generator. Therefore, it is more convenient to translate the expression to the torque and speed in the high speed shaft. This is done by substitution of

$$\Omega_{LSS} = \omega_{HSS} / r_{gearbox} \text{ and } Q_{LSS} = Q_{HSS} \cdot r_{gearbox} / \eta_{gearbox}$$

in the expression for the curve that needs to be tracked by the controller. This leads to

$$Q_{HSS} \cdot r_{gearbox} / \eta_{gearbox} = \frac{½\rho c_P \pi R^5}{\lambda^3}\left(\omega_{HSS} / r_{gearbox}\right)^2 \Leftrightarrow Q_{HSS} = \frac{½\rho c_P \eta_{gearbox} \pi R^5}{r_{gearbox}^3 \lambda^3}\omega_{HSS}{}^2 .$$

To use this expression in practice, realise that the maximum power is obtained if we substitute the known maximum power coefficient, $c_{P,max}$, and the tip speed ratio at which this is achieve, λ_{design}. The proportionality constant is called the optimal mode gain, k_{opt}, which then becomes:

$$k_{opt} = \frac{½\rho c_{P,\max} \eta_{gearbox} \pi R^5}{r_{gearbox}^3 \lambda_{design}^3} .$$

All parameters in this equation are known, or can be estimated. The maximum power coefficient includes the aerodynamic losses, so it will typically be a bit below the Betz limit. When SI units are used, the unit of the optimal mode gain is $Nm/(rad/s)^2$.

The final controller set-up is shown in Figure 2. The rotational speed is measured and the torque in the generator is set according to the optimal mode gain. This setting is obtained by a (additional) controller for the back-to-back converter.

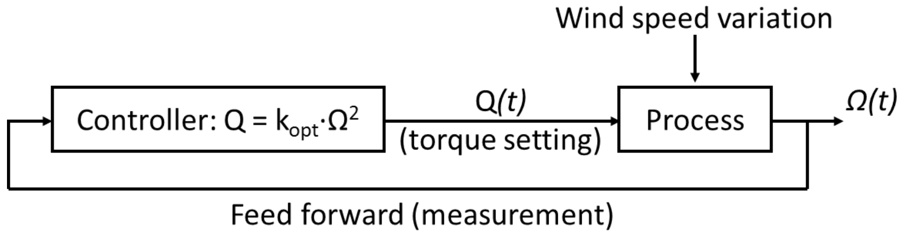


Figure 2 Set-up of partial load controller

Consequential rotor speed curve

Figure 3 shows the rotational speed as a function of wind speed that is obtained for a turbine with this kind of control.

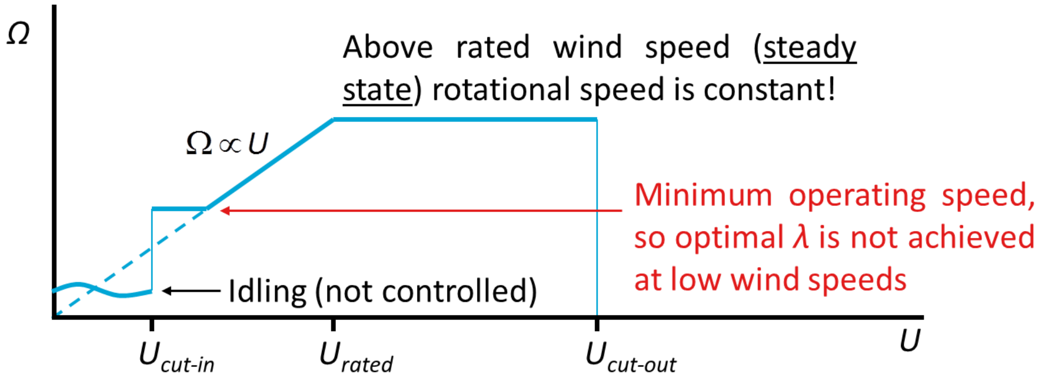
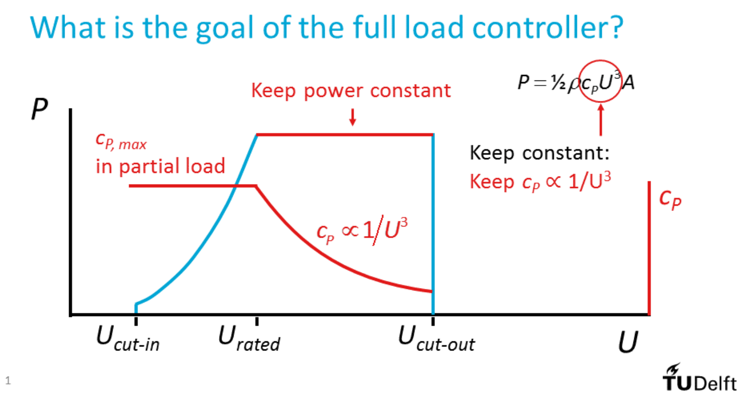


Figure 3 Rotational speed as a function of wind speed

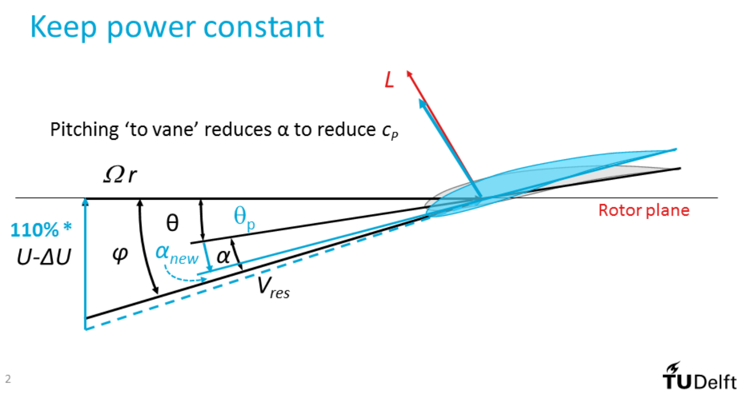
The control that is described is applied in the region where the rotational speed is proportional to the wind speed, which is the goal of this controller. Often, there is a lower limit on the rotational speed, which can have various reasons. The consequence is that the control strategy cannot be applied at low wind speeds and the optimal rotational speed cannot be obtained in the corresponding part of the partial load region. Above rated wind speed the rotational speed is no longer increased with wind speed. In this region the goal is no longer to maximise the power, so it is not necessary to keep operating on the design tip speed ratio. Apart from the lack of reason to keep increasing the rotational speed in this region, the maximum rotational speed is also restricted by noise constraints. Most of the aerodynamic noise is created at the blade tip and this noise increases with the tip speed. Therefore, the tip speed is typically restricted to about 70-80 m/s, or a little higher for offshore wind turbines.

4.5 Full load control

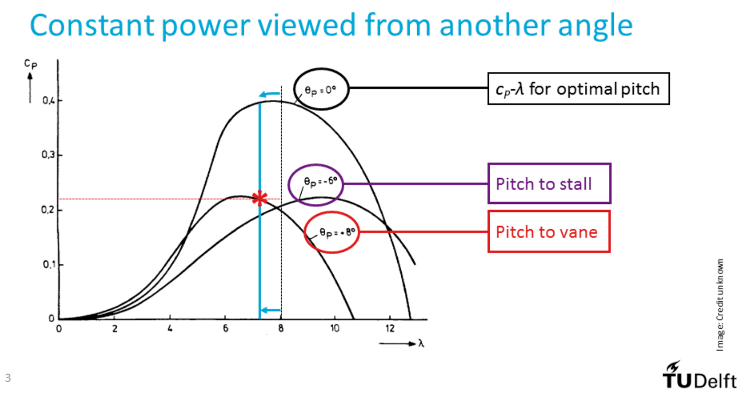
4.5.1 Goal and working principle



The other region of the power curve is the full load region. We already know that the power remains constant at the rated power in this region. In this video you'll see which mechanism is used to achieve this. If we express the power in the well-known equation again, then we see that the power coefficient is still the only parameter we can influence. However, to keep the power constant, the power coefficient times the wind speed cubed has to remain constant. This means that the power coefficient will have to be proportional to one divided by the wind speed cubed. We can also draw this in the graph. We have previously seen that in the partial load region we have the goal to keep the power coefficient at its maximum value. Now we see that for the full load region we want to reduce the power coefficient and to make it the inverse of the wind speed cubed.



To reduce the power coefficient when the wind speed increases what we'll do is pitch the blade. Let's again assume an increase in wind speed of 10 percent and ignore the effect on delta U. When we pitch the blade towards the incoming wind, we reduce the angle of attack. We call this, 'pitch to vane', because the blade is getting more and more aligned with the incoming wind. If we reduce the angle of attack we reduce the lift coefficient. Therefore, we reduce the thrust and because of that the induction factor reduces. Finally, a lower induction factor corresponds to a lower power coefficient. This reduction in power coefficient oppose the increase in the power that we would have because of the increasing wind speed. A lower induction factor also means a reduction in delta U, so the increase in out-of-plane wind speed at the rotor is actually a bit more than 10 percent. This effect needs to be incorporated in the pitching action.



Also in this case, we can look at the c_P – Lamda curve to assess what is happening. The highest curve here gives the c_P – Lamda curve for a pitch angle of zero degrees, for which this blade is designed. Because this is the c_P – Lamda curve for the design conditions, it gives the highest possible c_P value. In this case, the c_P value is maximum at a tip speed ratio of 8. So, this is the design tip speed ratio. If we exceed rated wind speed, the rotational speed is kept constant and, therefore, the tip speed ratio is going down. As a consequence, the power coefficient is also going down because we are now working in off-design conditions for the tip speed ratio. However, this is usually not enough. This graph also shows two examples for other pitch angles: for a pitch angle of minus six degrees and for a pitch angle of plus 8 degrees. A negative pitch angle means that we are increasing the angle of attack and are pitching to stall. This will not be treated further in this course, since this is an uncommon approach. We will be looking at pitching to vane, so at positive pitch angles with which we decrease the angle of attack. If we pitch the blades, we operate the rotor in off-design conditions and, therefore, the c_P – Lamda curve goes down. In this example, pitching the blades by 8 degrees reduces the power coefficient to zero-point-twenty-two at the new tip speed ratio. Does that mean we pitched enough, or did we pitch too much? The challenge for full load control is to determine how much we need to pitch and how this can be controlled dynamically during operation.

4.5.2 Implementation

Theoretical background

In full load, the goal of the controller is to keep the power at the rated power. A straightforward control strategy would therefore be to measure the power and adjust the pitch angle to keep it at the desired value. However, this approach results in control loop with unfavourable properties. Particularly, the stability of the loop cannot be assured, meaning that the power, and other aspects of the turbine, may get large oscillations.

Before addressing a control strategy that avoids this problem, we'll have a look at what the pitch angle settings would need to be theoretically. Figure 1 shows the $c_P - \lambda$ curves for various pitch angles, which can be obtained from a BEM analysis. If we want to know the pitch angle setting for a particular wind speed, we first determine the tip speed ratio for that wind speed from

$$\lambda = \frac{\Omega R}{U},$$

in which the rotational speed is known: above rated wind speed the (steady state) rotational speed remains constant. You can also determine the power coefficient that is needed to obtain the rated power, using

$$c_P = \frac{P}{½\rho U^3 \pi R^2}.$$

This gives us the operational point (λ, c_P) that we want to have. As an example this point is indicated by the red cross in Figure 1. We can see that this point lies on the $c_P - \lambda$ curve for a pitch angle of 10.5. The controller of the back-to-back converter determines the rotational speed and therefore the tip speed ratio. If the controller would set the pitch at the angle we just established, then the equilibrium in the drive train is obtained when the rotor reaches the desired power coefficient. As discussed for partial load, wind speed cannot be measured accurately, so this method doesn't provide a means to control the power in full load operation.

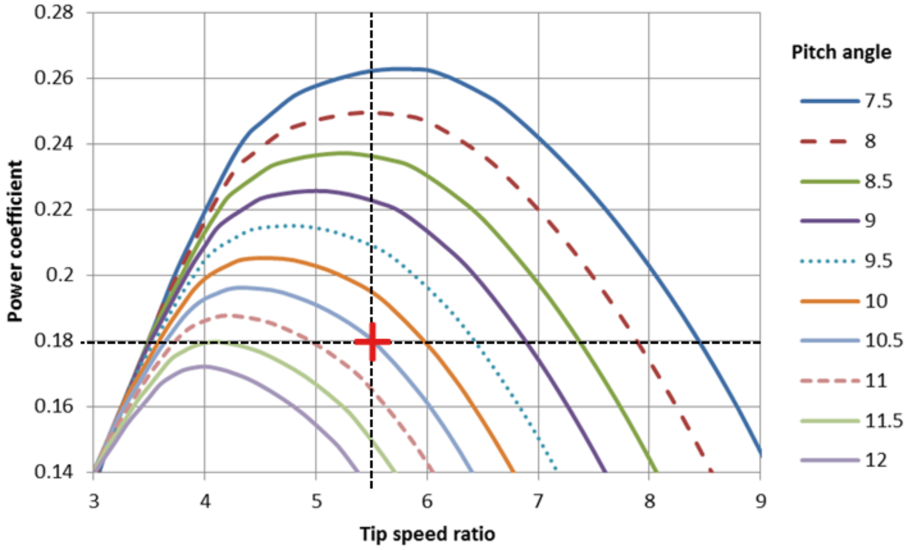


Figure 1 $c_P - \lambda$ curves for various pitch angles

Control strategy

As for the partial load controller, we're going to look at torque and speed as parameters to measure and control. The type of control that is used on full load is a feedback loop: a parameter of the wind turbine is measured and compared with a reference value. The difference between the reference and the measured value will be used by the controller to change the pitch angle settings. As indicated above, P_{rated} cannot be used directly as a reference to track by the controller.

Instead, consider that $P = Q \cdot \Omega$. Either torque or rotational speed can be set independently by controlling the back-to-back converter. The other parameter, and consequently the power, are determined by the performance characteristics of the rotor, which determine at which point equilibrium is obtained in the drive train.

The most common full load control strategy is that the torque is fixed by the control of the back to back converter. We can then determine the rotational speed reference that needs to be tracked by controlling the pitch angle:

$$\Omega_{ref} = \frac{P_{rated}}{Q_{rated}}.$$

The set-up for this control strategy is shown in Figure 2. The rotational speed is measured and compared with the desired reference speed. If the speed is too high, the controller will increase the pitch angle and if it is too low, the controller will decrease the pitch angle.

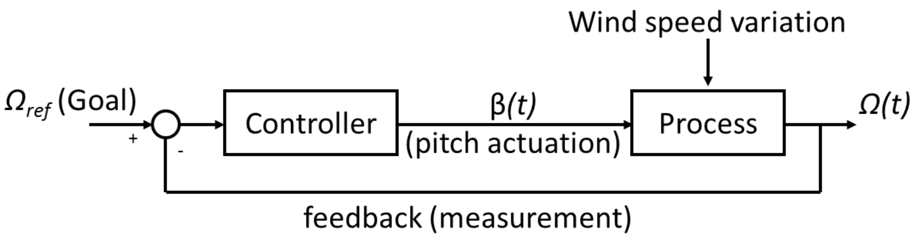


Figure 2 Set-up of full load controller

Consequential pitch angle curve

Figure 3 shows the pitch angle as a function of wind speed that is obtained for a turbine with this kind of control.

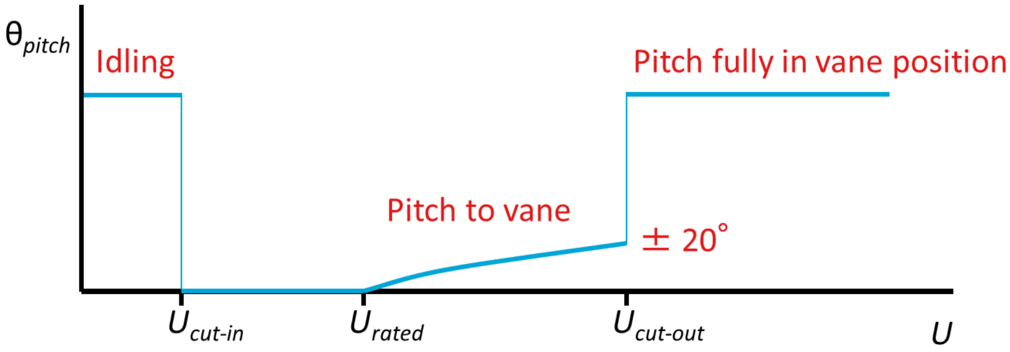
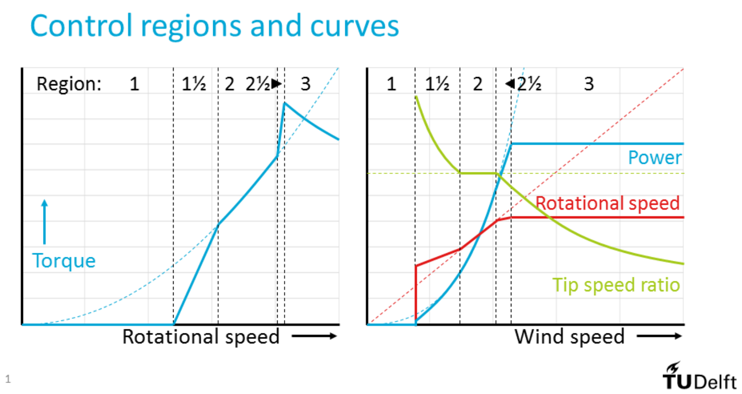


Figure 3 Pitch angle as a function of wind speed

When the turbine is idling, the blades are fully pitched to vane, which is at about 90°. In partial load the pitch angle is fixed at the angle that corresponds with the rotor design for optimal performance. Above rated wind speed the pitch angle increases. The gradient of increase is a bit larger at the beginning of the full load region than at the end. Typically, the pitch angle at the end of the full load region is about 20°.

4.6 Control regions

The power curve is divided into two regions and you now know how the rotor can be controlled to obtain this power curve. However, in reality the controller also has to facilitate the transitions in the power curve: from no power to maximum power coefficient and from maximum power coefficient to constant power. Therefore, for power control purposes also these transition regions are identified. We will have a closer look at that here.



You have seen that torque and rotational speed are essential parameters in power control. Therefore, we address the control regions in a graph of torque versus rotational speed. The dashed line in the graph is the curve for maximum power conversion, which serves as a reference. On the right-hand side of the slide we will follow what happens to several parameters as a function of wind speed. The dashed lines here also serve as references and correspond to what happens if the controller follows the dashed line in the torque-speed diagram. The rotational speed, in red, would increase linearly with wind speed. The tip speed ratio, in green, would remain constant. The power, in blue, would keep increasing, proportional to the wind speed cubed. Below cut-in wind speed, the generator is not connected to the grid, meaning that no torque is applied. The rotor is idling at a very low rotational speed and there is no electricity production. This operation without actual power control is identified here as region 1. In region 2 the controller follows the reference curve according to the principles of partial load control that you have seen before. The rotational speed, tip speed ratio and power therefore also follow their respective reference curves. It is shown here that region 1 and region 2 are not connected, leading to a transition region in the wind speed diagram. This control region is identified as one-and-a-half. The typical shape of the control curve in this region is a straight line, connecting region 1 and 2. In this example, this line has a finite gradient, but it can also be a vertical line. It is clear that the power increases when this curve is followed from low torque to high torque. At cut-in wind speed, the generator is connected to the grid and controlled to follow this curve. The rotor will automatically find the point of torque and speed corresponding to the power that is converted from the wind. The control curve in region one-and-a-half is to the right of the reference curve. This means that the rotational speed is higher than the ideal speed. Consequently, the tip speed ratio is too high and the power is slightly below its reference curve in the right-hand diagram. Full load control is identified as region 3. As you have seen, the rotational speed is kept constant in full load, so region 3 would appear to be only a single point on the control graph. However, you have also seen that full load control is actually performed by measuring small deviations from the target rotational speed. During an increase in wind speed, the rotor is actually allowed to speed up a little bit, which is needed to allow for the time that is required to pitch the blades. During the increase in rotational speed in a gust, the controller follows the curve in region 3. On this curve the torque is inversely proportional to rotational speed. You can easily establish that this corresponds to constant power, even during a gust. While this curve is followed, the controller will pitch the blades to force operation back to the target rotational speed. The rotational speed in region 3 is usually a bit to the left of the reference control curve and thus lower than ideal. Keeping the rotational speed a bit lower reduces the aerodynamic noise that is produced by the rotor. The consequence of this choice is that the tip speed ratio is not ideal anymore when rated wind speed is reached. However, this is not a substantial issue, since for all wind speeds higher than rated wind speed the power coefficient needs to be reduced anyway. The remaining control region is region two-and-a-half, which is the transition to full load control. This region is also typically a straight connecting line, either vertical or with a finite gradient. Because the control curve in region two-and-a-half connects to the rotational speed of region 3, it is also gradually deviating from the ideal tip speed ratio. The consequence of this is a little loss in power in the transition region. It is clear from the curves in this slide that the transition regions add some complexity to the control of wind turbines and can extend over a fair range of wind speeds. However, the effects on the power curve are limited. Therefore, in calculations in this course we will ignore these transition regions, unless specified differently. This means that we will assume constant tip speed ratio operation all the way from cut-in wind speed till rated wind speed. There are various other parameters that are interesting to show as a function of wind speed. You have already seen the pitch angle in full load control. Curves for parameters such as torque, thrust and thrust coefficient can be derived along similar lines as shown here for rotational speed, tip speed ratio and power. This could be a nice exercise for you to test your understanding.

5 Dynamics of a wind turbine

5.1 Background: Introduction to dynamics

5.1.1 Introduction to dynamics

Single degree of freedom systems

In Figure 1 you see a schematic presentation of a single degree of freedom system. The mass, *m*, can only move in one direction, indicated by the x-axis. The mass is connected to a fixed point through a spring with stiffness *k* and a viscous damper with damping coefficient *c*.

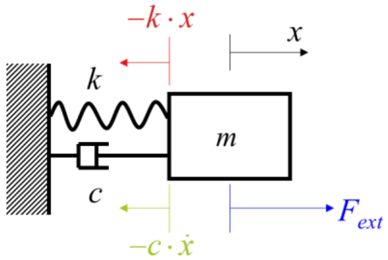


Figure 1 single degree of freedom system with internal and external forces

In this document we'll look at the dynamic behaviour of this simple system, as it helps us to understand the dynamics of the more complex wind turbine system. In the last section we'll get a preview of the similarities between a single degree of freedom system and a flexible structure, introducing the concept 'mode shape' that is important for understanding structural dynamics.

Equation of motion

The spring and the damper exert forces on the mass that oppose displacement and motion, respectively. These forces are internal forces and are comparable to the resisting forces in the material of a tower or a monopile. In addition, there can be an external force, F_{ext}. According to Newton's law, the acceleration of the mass equals the sum of the forces that act on it. We call this the equation of motion:

$$F_{ext} - k\cdot x - c\cdot\dot{x} = m\cdot a = m\cdot\ddot{x}\,.$$

Types of motion

Because the spring and damper forces are internal, it is more convenient to write these on the other side of the equation, to separate the external force from the aspects that characterise the system. This leads to:

$$k\cdot x + c\cdot\dot{x} + m\cdot\ddot{x} = F_{ext}\,.$$

For a flexible system the stiffness is always important, but the damping coefficient can have more or less significance. The structure can also be subject to different kind of external loading. Therefore, we can distinguish several types of motion:

- Static displacement – The external force is constant
- Free vibration – There is no external force
- Force vibration with harmonic excitation – The external force varies sinusoidal in time
- Force vibration with random excitation – The external force varies randomly

The next four sections give some more information about these four types of motion.

Static displacement

A static force will move the mass to a static displacement, where $\dot{x}$ and $\ddot{x}$ are zero and the spring force is in equilibrium with the external force: F_{ext}. = $k \cdot x$. The static displacement is thus:

$$x_{static} = \frac{F_{ext,static}}{k}.$$

Free vibration

When there is no external force, the system can vibrate due to an excitation in the past. In damped free vibration, the motion will gradually decrease while it will continue indefinitely for undamped free vibration. We'll take a closer look at undamped free vibration, because it is easier to solve and reveals an important property of the system: its natural frequency.

The equation of motion for the undamped free vibration is obtained by setting c and F_{ext} equal to zero:

$$k \cdot x + m \cdot \ddot{x} = 0.$$

This is a differential equation for which its known that the solution takes the form

$$x = \hat{x} \cdot \cos(\omega_n t - \varphi),$$

in which $\hat{x}$ is the amplitude of the vibration, ω_n is a frequency that is called the natural frequency (for reasons that will become clear soon) and φ is a phase shift that is used to describe any time delay of the motion with respect to a chosen time frame.

Substituting this solution and its second derivative in the equation of motion yields:

$$k \cdot \hat{x} \cdot \cos(\omega_n t - \varphi) - m \cdot \omega_n^2 \hat{x} \cdot \cos(\omega_n t - \varphi) = 0.$$

Because the amplitude and time dependent term can be scratched, this leads to the analytical equation

$$k - m\omega_n^2 = 0,$$

with the solution for the natural frequency

$$\omega_n = \sqrt{\frac{k}{m}}.$$

This frequency is called the natural frequency, because it is a 'natural' property of the system. In the absence of any external force, the system will vibrate in this particular frequency and in no other frequency, if it has been excited in the past.

We can determine that the unit for ω_n is rad/s and not Hz by looking closely at the derivation. The solution for x given above is periodic for period T = $2\pi/\omega_n$, so with frequency $f = \omega_n/2\pi$.

Forced vibration with harmonic excitation

Forced vibration with harmonic excitation means that the external force varies in time according to

$$F_{ext} = \hat{F} \cdot \cos(\omega t - \varphi_1),$$

in which $\hat{F}$ is the amplitude of the force variation and ω is the excitation frequency. Also for this case it is known that the solution takes the form

$$x = \hat{x} \cdot \cos(\omega t - \varphi_2),$$

with the same frequency ω as the excitation. The equation of motion then becomes:

$$(k - m\omega^2)\hat{x}\cos(\omega t - \varphi_2) - c\omega\hat{x}\sin(\omega t - \varphi_2) = \hat{F}\cos(\omega t - \varphi_1).$$

the solution of this equation is:

$$\hat{x} = \frac{\hat{F}}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}; \quad \tan(\varphi_2 - \varphi_1) = \frac{c\omega}{k - m\omega^2}.$$

The equation on the right-hand side represents a delay in the response. The equation on the left-hand side gives a relation between the amplitude of the response and the amplitude of the force. Remember this relation for the static displacement: $x_{static} = F_{ext,static} / k$.

The Dynamic Amplification Factor (DAF) is defined as the ratio between the dynamic response and the static response, for the same external force (amplitude). So, for $\hat{F}_{ext} = F_{ext,static}$:

$$DAF = \frac{\hat{x}}{x_{static}} = \frac{k}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}.$$

Figure 2 shows the shape of this function, where the excitation frequency is normalised with the natural frequency of the system. At an excitation frequency of 0, which corresponds to static loading, the DAF is 1, as expected. You can clearly see a significant rise in response when the excitation frequency gets close to the natural frequency, which is called resonance. The magnitude of the resonance peak depends much on the magnitude of the damping (here captured by a parameter called 'damping ratio'). For wind turbines the damping ratio of the support structure vibration is even less than the 0.1 in Figure 2, so you see that resonance will lead to a large response.

If damping can be neglected, the DAF can be simplified by dividing both numerator and denominator by k and substituting the earlier found solution for the natural frequency. This clearly helps to see that the response goes to infinity when the denominator goes to zero:

$$DAF = \frac{1}{\left|1 - \frac{\omega^2}{\omega_n{}^2}\right|}.$$

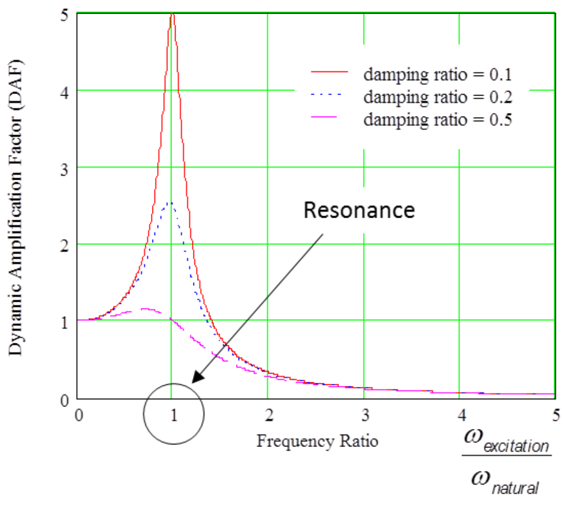


Figure 2 Dynamic Amplification Factor as function of excitation frequency

Forced vibration with random excitation

There are several reasons to pay attention to forced vibration with harmonic excitation, even when the forces in reality are not that regular or when the system has more than one degree of freedom. The forced vibration of a single degree of freedom system with harmonic excitation was shown above to lead to a harmonic response at the same frequency. This behaviour can be easily translated to the behaviour of a more complex system under more complex loading.

A linear system has the following properties:

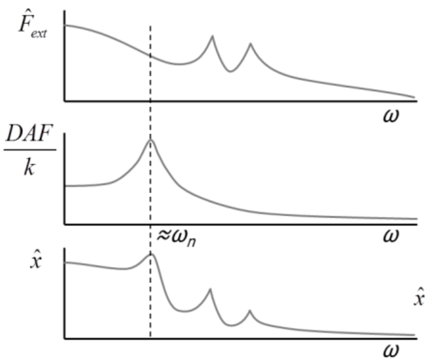
If $F = \hat{F}_1 \cos\left(\omega_1 t + \varphi_1\right)$ leads to $x = \hat{x}_1 \cos\left(\omega_1 t - \varphi_1 '\right)$

and $F = \hat{F}_2 \cos\left(\omega_2 t + \varphi_2\right)$ leads to $x = \hat{x}_2 \cos\left(\omega_2 t - \varphi_2 '\right)$

then $F = \hat{F}_1 \cos\left(\omega_1 t + \varphi_1\right) + \hat{F}_2 \cos\left(\omega_2 t + \varphi_2\right)$ leads to $x = \hat{x}_1 \cos\left(\omega_1 t - \varphi_1 '\right) + \hat{x}_2 \cos\left(\omega_2 t - \varphi_2 '\right)$.

This implies that if we assess the loading and response per frequency, we can simply add the responses per frequency. This can conveniently be done in the frequency domain, which works with the spectra of loads and responses, since the spectra contain the information of excitation or response per frequency.

According to Fourier theory, a random external force can be represented by a spectrum, which is a distribution of harmonic force variations at different frequencies. For each of these frequencies we get a response of *x* at that frequency. For the equation of motion of a single degree of freedom system it is known that the responses of *x* at different frequencies can be determined with the dynamic amplification factor, the DAF. This is illustrated graphically in Figure 3. The uppermost graph represents the spectrum of the external force, so the amplitude of harmonic force variations at different frequencies. At the vertical dashed line we determine the response for an external excitation for a particular frequency, using the DAF. If you do this for all frequencies you get the spectrum of the response amplitude. The middle graph is the transfer function of the system. We get the response spectrum through multiplication of the excitation spectrum with the transfer function, per frequency. This principles also applies if we have more complex transfer functions, e.g. with multiple resonance peaks.



$$\hat{x}(\omega) = \frac{\hat{F}_{ext}(\omega)}{k} \cdot DAF(\omega) \text{ for all } \omega$$

Figure 3 Frequency domain analysis of response to random external force variation (The phase ϕ is not shown, but should also be considered)

A note of attention is that a wind turbine is not a linear system and therefore the transfer-function approach is not generally valid. However, the wind turbine can be approximated by a linear system, provided that we do the linearisation for a wind speed close to the wind speed for which we do the analysis. For different (average) wind speeds a different linear model needs to be made.

Structural dynamics

The single degree of freedom system has only one, discrete flexibility: displacement along the x-axis. In a structure, such as a support structure for a wind turbine, the flexibility is continuous. This means that there is an infinite number of degrees of freedom. Nevertheless, in practice such continuous systems are often modelled as a limited number of connected discrete elements, having a limited number of degrees of freedom. This is illustrated in Figure 4. The degrees of freedom of the elements of a beam are normally captured by three directions of displacement plus rotations about three axes for each end of the element.

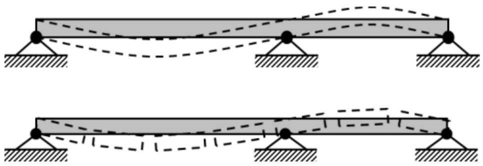


Figure 4 Discretisation of a continuous beam (top) in a limited number of elements (bottom)

The extensions of the information regarding the single degree of freedom system presented above can be summarised by:

- The equation of motion becomes $\mathbf{kx}+\mathbf{c\dot{x}}+\mathbf{m\ddot{x}}=\mathbf{F}_{\mathbf{ext}}$, in which **x** and $\mathbf{F}_{\mathbf{ext}}$ are vectors with one element per degree of freedom and **k**, **c** and **m** are matrices
- The number of natural frequencies becomes equal to the number of degrees of freedom, although some natural frequencies may be duplications of the same value
- Each natural frequency corresponds to a shape of vibration, called a mode shape (see Figure 5 for examples). Duplicated frequencies have different mode shapes
- Which natural frequency and which mode shape appear in free vibration depends on the history of the excitation
- The graph of the DAF has multiple peaks, occurring at the natural frequencies

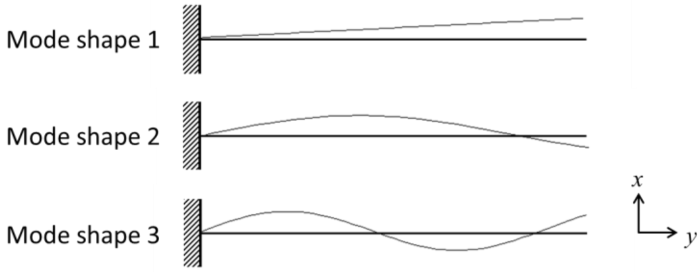
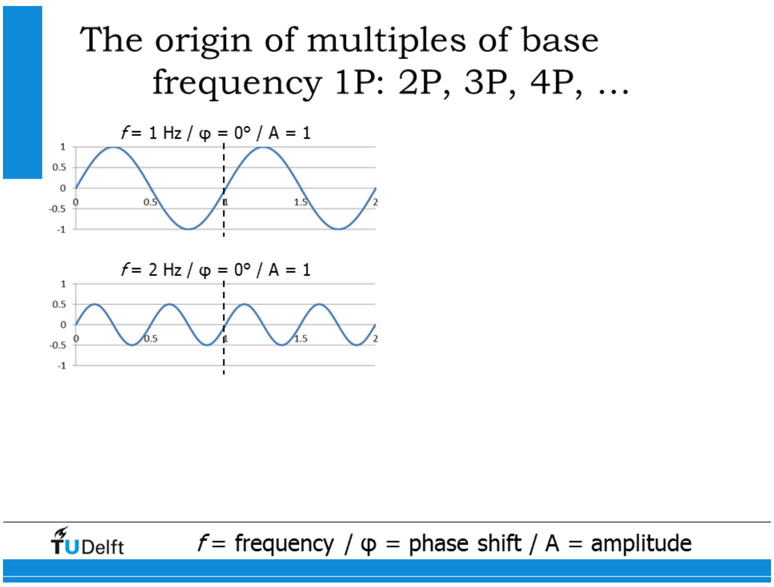
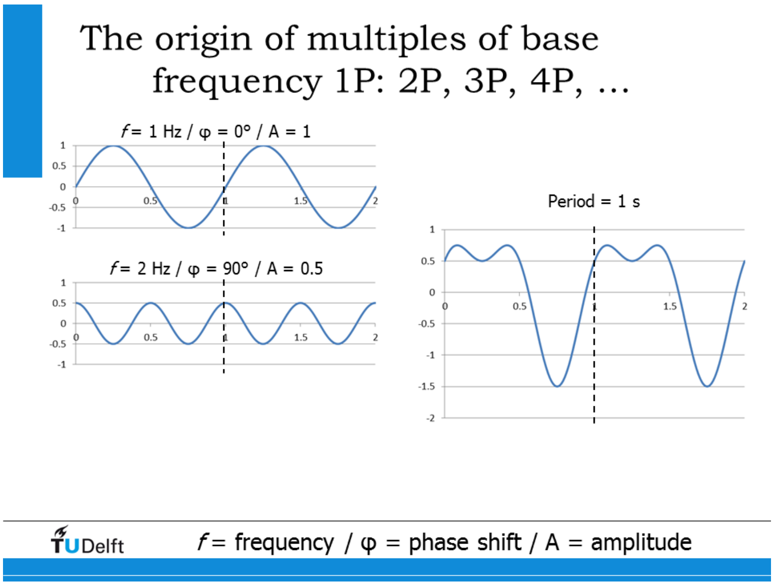


Figure 5 First three mode shapes of a cantilever beam

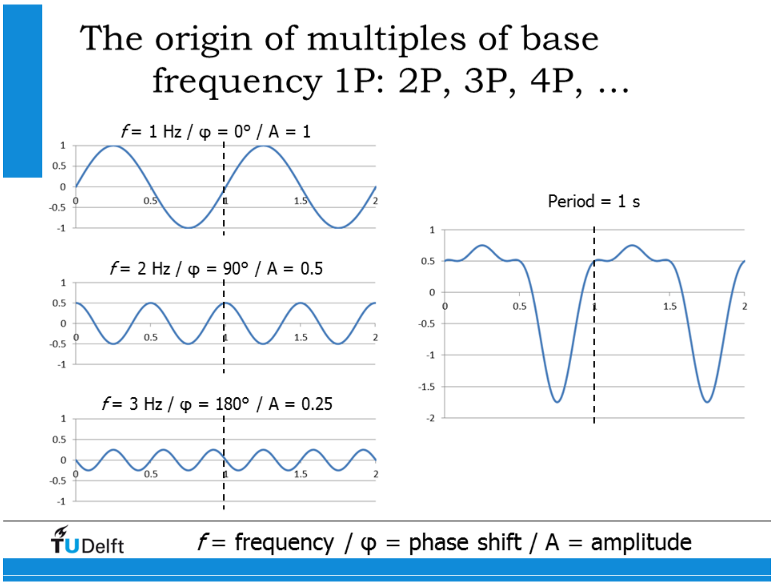
5.1.2 Periodic non-harmonic signals



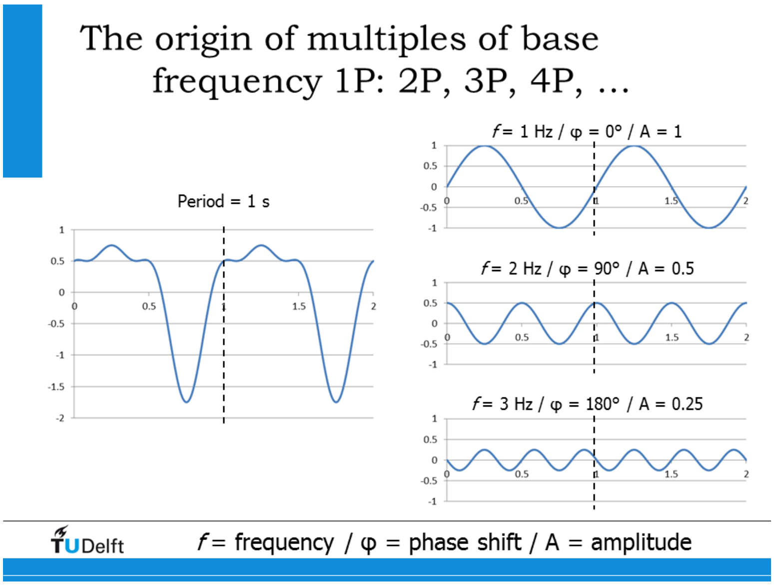
A lot of variations in a wind turbine happen once per rotation or, in brief, at 1P, so once per rotation, and this slide explains how something that changes once per rotation actually also includes frequency components at twice per rotation or 2P, 3P, 4P, etc. And we call these integer multiples of the 1P frequency the harmonics. But let's start with the 1P variation and let's take a nice sinusoidal variation and so this is the period and I'll be calling this the base period. We have a 1 Hertz signal in this case, so this is one second, we have zero phase shift, meaning that we start here at 0, and we have an amplitude of 1. Now if we have a second signal at 2 Hertz, so twice the base frequency, and in this case also zero phase shift, then you see that, if the signal follows 2 cycles, then at this point, where we have also finished one cycle in the base frequency, then we're actually at the same state that we were here. So this part of the 2 Hertz signal is exactly the same as this part of the 2 Hertz signal.



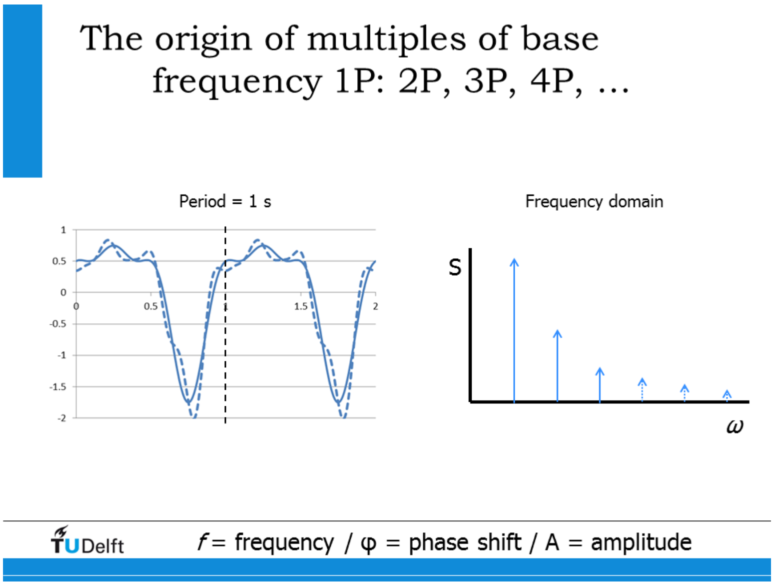
And that's also the case if we apply a phase shift, so it doesn't matter which phase shift we have. Always after one period in the base cycle we return to the same state when we have twice the frequency as that we had here at the beginning of the first period. So if we add these two signals, that automatically means that we get a periodic signal, because the addition of these two signals here will lead exactly to the same result as the addition of the two signals here. And that's indeed what you see here on the right hand side.



Now if we add a third signal at three times the base frequency, so 3 Hertz in this case, I applied a random phase shift of 180 degrees this time, still you see that if we start here at zero and going down, we make three full circles and then we end up here at the same state. So also the addition of these three signals, for the first base period, will lead to exactly the same result as the addition of these three signals, for the second period, as you can see here.



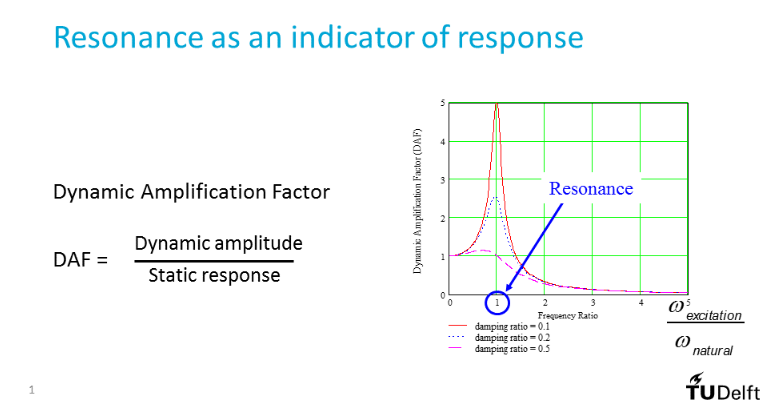
And now, without really proving it, I'm stating that the opposite is actually also true. So if we have a periodic signal, with a certain period that you see here, then this signal can always be decomposed in a series of frequencies, with a base frequency that is the same as the frequency of this periodic signal, and the harmonics. So, like I said, although it's not proven by this, this does clarify the origin of it.



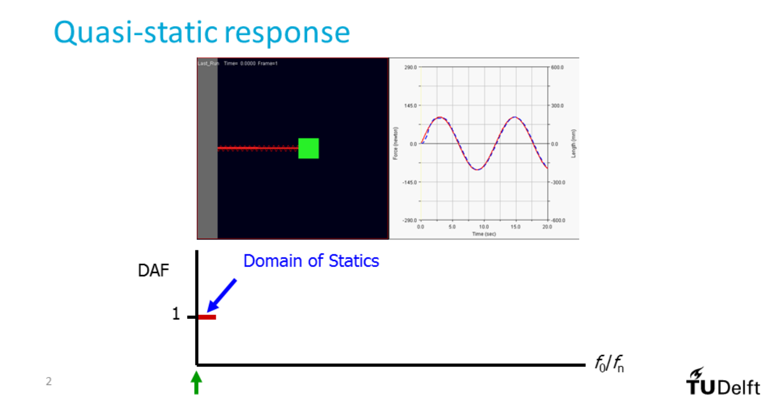
And this can also be shown in the frequency domain, instead of in the time domain. If we have a periodic signal, then in the frequency domain we will see a peak at the base frequency, and we will also see peaks at the harmonics, so at twice the base frequency and, in this example, at three times the base frequency. And for certain periodic signals, there will also be energy in four times the base frequency, five times the base frequency, etc. So that clarifies, that, even though in the time domain you see just a periodic signal with a very long period, so relatively low frequency, in the frequency domain you also see multiples of that, and very high frequencies are actually also present in that signal.

5.1.3 *Dynamic amplification of single-degree-of-freedom system*

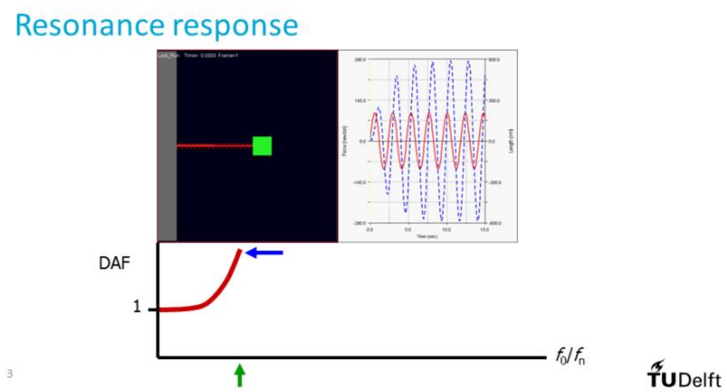
In this video we use an animation of a single degree-of-freedom system to give you a better idea of what is actually happening at different conditions of dynamic amplification.



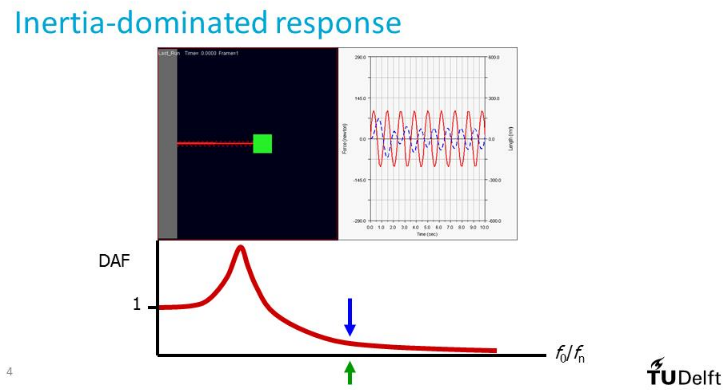
Here you see the graph of the dynamic amplification factor as a function of the ratio between excitation frequency and natural frequency. At very low frequencies the factor is close to one, meaning that the dynamic response is similar to a static response for the same input force. When the ratio between excitation frequency and natural frequency equals one, the system is in resonance and has a high amplification factor. How much higher the dynamic response is depends significantly on the amount of damping. At very high excitation frequencies the dynamics amplification factor drops below one. This means that the dynamic response is less than the response to a static input force with the same magnitude.



This animation shows what happens to the single degree-of-freedom system during low frequency excitation. As the force is growing towards the right, the mass is moving along in that direction. At all instances there is almost equilibrium between the external force and the force in the spring. The inertia of the mass is insignificant to the response. You could say that the mass can easily follow the slow variation in the loading. This can also be observed in the graph, where the movement of the mass in blue nearly overlaps with the force variation in red. Because the inertia is insignificant and there is almost no dynamic effect, we call this quasi-static behaviour.



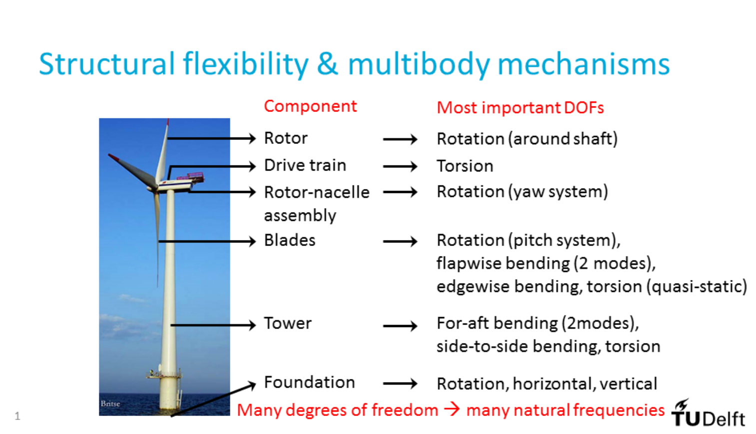
The next animation shows the system under resonance conditions. Compared to the previous animation, the first difference to note is that the motion of the mass is not in phase with the variation of the force. When the mass is at the right-most extreme position, the force is no longer large and pointing to the right. It has already changed direction. The movement is thus lagging on the force variation. This can also been seen in the graph. The consequence is that the direction of the force is now always the same as the direction of the velocity of the mass and not of its position. This means that the force is always performing work on the mass and thus increasing its kinetic energy content. The only reason that the response of the mass doesn't keep increasing is that at some point the average work done by the force comes in equilibrium with the average dissipation of energy in the damper.



The last animation shows very high frequency excitation. Again, something has happened to the phase difference between force and response. The movement is even lagging more than in the case of resonance and the position of the mass is almost a half cycle out of phase with the force. However, that isn't the most important aspect of what is happening here. To get a displacement, the mass first needs to get speed and that requires acceleration. As you know from Newton's law, this acceleration is determined by the net force on the mass. Initially, only the external force can start this acceleration. However, the variation in the external force is so fast that there is no time for the acceleration of the mass to lead to any significant speed, let alone displacement. Before the mass is picking up pace, the force has changed direction and is decelerating the mass again. You could say that in this case the mass cannot follow the fast variations anymore. The velocity and displacement remain small, so the forces in the spring and damper remain insignificant. The external force is continuously in equilibrium with the acceleration of the mass, which is why we call this inertia-dominated response. I hope you enjoyed this video and increased your understanding of dynamic amplification. Thank you very much for your attention.

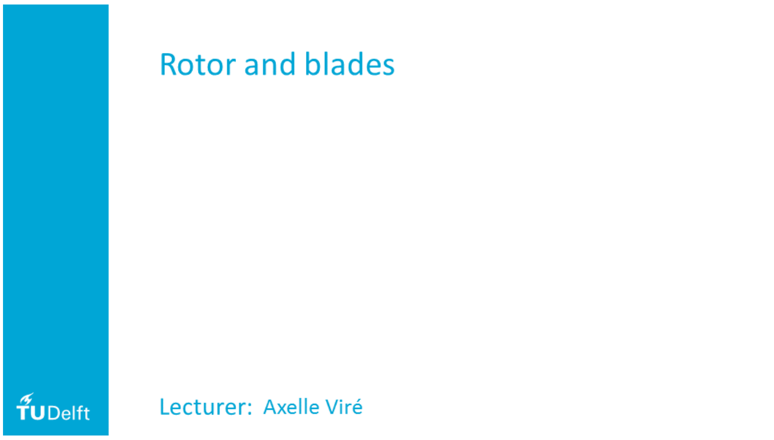
5.2 Flexibility and mechanisms of the system

This short video gives an overview of the modes of deformation of a wind turbine.

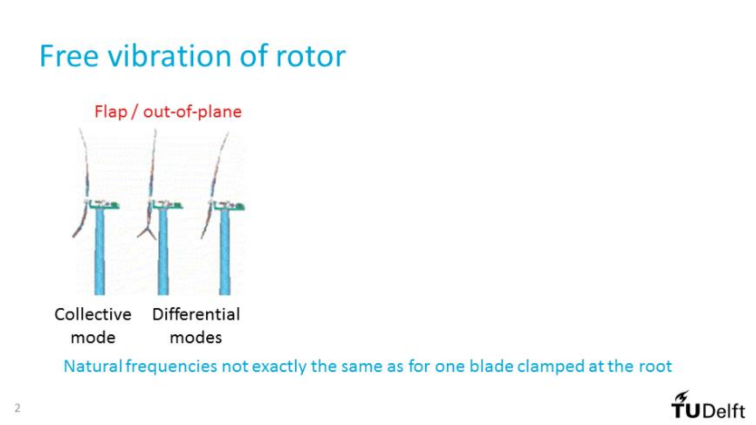


A wind turbine has a lot of different degrees of freedom. This slide shows the most important ones for the main components of the wind turbine. Let's start at the top of the turbine. The rotor of course rotates around the shaft. Thus, its degree of freedom is a rotation. The drive train inside the nacelle also rotates along its main axis, and this creates a torsion. Additionally, the entire rotor nacelle assembly is able to rotate in yaw in order to align the rotor with the main wind direction. The degrees of freedom of the blades are a bit more complex. First, they can rotate around their own axis through the pitch control system. That is a mechanical rotation. There is also a deformation associated with this motion and this is a torsion, usually quasi static. The blades have also two bending motions: the flap-wise bending which occurs out of the plane of rotation, and the edgewise bending which takes place in the plane of rotation. The tower is also able to bend in two different ways: fore-aft, that is against or with the wind direction, and side-to-side thus perpendicularly to the wind direction. For jackets or gravity-based foundations, torsion can also be important. Finally, the foundation that is below the sea level and mounted on the seabed, has degrees of freedom in rotation and translation. The vertical degree of freedom is more relevant for jackets, because the different piles might not be mounted exactly at the same level. Thus a wind turbine has many different degrees of freedom, and therefore, its dynamics is characterised by a lot of different natural frequencies.

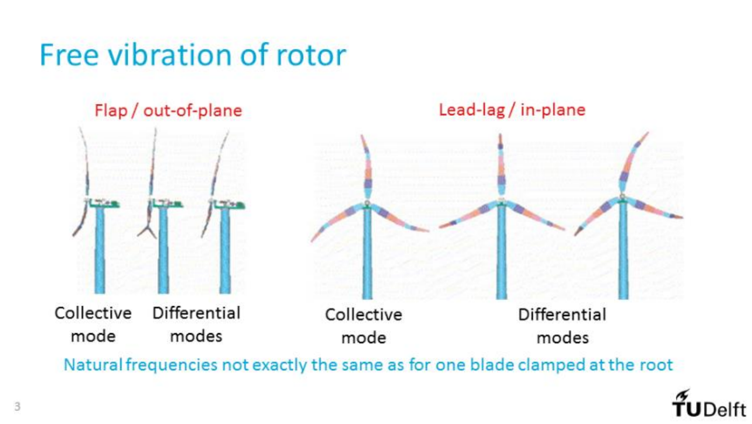
5.3 Dynamic properties of components



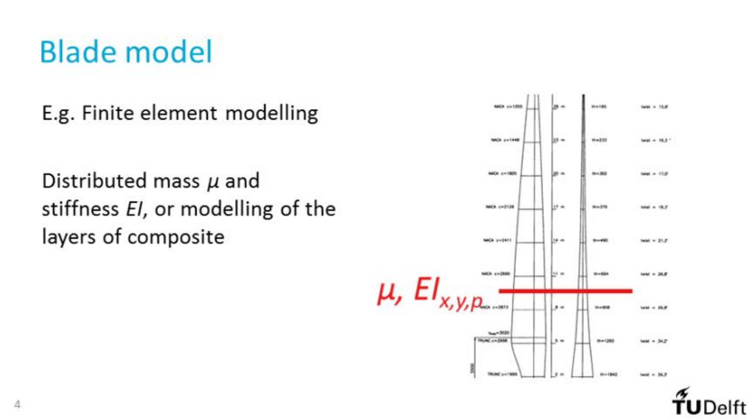
Let's start with the rotor blade assembly.



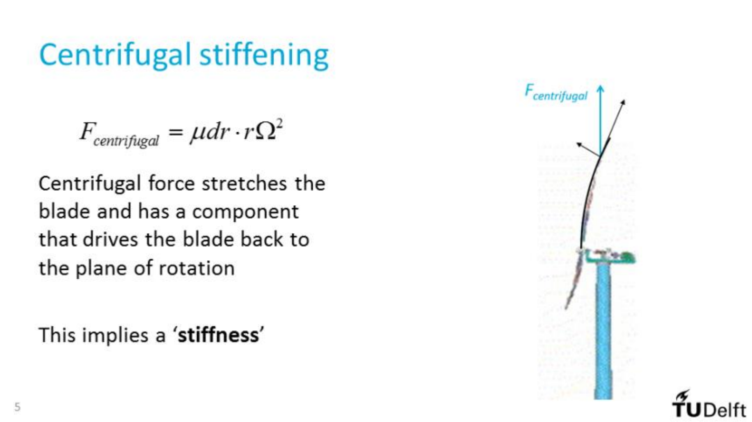
As we already saw before, the first bending mode of the rotor is a flap-wise mode. This corresponds to a bending out of the plane of rotation, as illustrated on the left-hand side of this slide. If all the blades deform in the same way, for example all bending against the wind direction, then it is referred to as a collective mode. Otherwise, it is called a differential mode.



The second bending mode is the edge-wise deformation. It is also called lead-lag motion because the rotor deforms in the plane of rotation, and therefore, may increase the rotation speed, that is the lead case, or decrease it, in the lag case. Again, both collective and differential modes exist depending on whether the blades behave in the same way or not.



In order to model these deformations, the blade can be decomposed into a certain number of elements, for example using a finite element model, each element having a mass property and a stiffness property.



An interesting effect that happens on the rotor is the centrifugal stiffening, which is a stiffening of the blade due to the centrifugal force. When the rotor rotates, rotates it is subjected to a certain centrifugal force, If the rotor is completely un-deformed, that is the blades are completely straight in the plane of rotation, then the centrifugal force tends to stretch the blades because it is in line with the blade and perpendicular to the rotation speed. When the blade deforms, the direction of the centrifugal force remains identical. However, because of the blade deformation, the centrifugal force can be decomposed into two different components: a component that is aligned with the blade and that stretches it, and a component that is perpendicular to the blade, as illustrated on the right-hand side of this slide. The component perpendicular to the blade counteracts the blade deformation, and tends to bring the blade back in its original position. A force that does this acts like a spring element, and is therefore a stiffness. Thus, when the blade deforms, in particular in the flap-wise direction, the centrifugal force tends to bring the blade back in its original position by stiffening the blade. This phenomenon increases the natural frequency of the blade. Since the centrifugal force increases with the rotation speed, the stiffening also increases as the rotor speed increases.

Centrifugal stiffening

Approximate effect on the natural frequency

$$\omega^2_{n,rotating} = \omega^2_{n,non-rotating} + K_n\Omega^2$$

where K_n is the Southwell coefficient

Depending on the blade structure, K_n is about 1.8

TUDelft

As a result, the natural frequency of the rotating rotor is the sum of the squared natural frequency of the non-rotating rotor, and the contribution of the centrifugal stiffening that is proportional to the square of the rotation speed. The coefficient of proportionality depends on the structure of the blade, and its value is usually close to 1.8.

Drive train

TUDelft

Lecturer: Axelle Viré

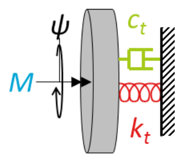
Now let's consider the drive train and its main degree of freedom: the torsion.

SDOF equation of motion (rotation)

$$I\ddot{\psi} + c_t\dot{\psi} + k_t\psi = M$$

Undamped free vibration

$$\omega_n = \sqrt{\frac{k_t}{I}}$$



I: (mass) momentum of inertia
Index t stands for torsional

TUDelft

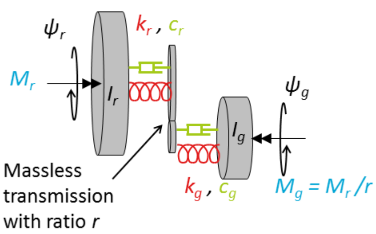
Earlier this week, we expressed the equation of motion of a single degree of freedom system in translation. We can generalise this expression to rotation by using Newton's second law for the angular momentum psi. Denoting I the momentum of inertia of the system, c_t the damping element in torsion, and k_t the stiffness in torsion, the equation of motion of the system is written here, where the right-hand-side M is the sum of all external moments. In the case of undamped free vibrations, the natural frequency of the system is the square root of the stiffness in torsion k_t, divided by the mass momentum of inertia. This is the direct analogy of the natural frequency associated with a single degree-of-freedom system in translation.

Ensemble rotor/gearbox/generator

Undamped free vibration

$$\frac{1}{\frac{1}{I_r}+\frac{1}{I_g r^2}}\ddot{\psi}_t + \frac{1}{\frac{1}{k_r}+\frac{1}{k_g r^2}}\psi_t = 0$$

with $\psi_t = \psi_r - \frac{\psi_g}{r}$



Analogy with mass-spring system to determine ω_n

r: rotor shaft
g: generator shaft

TUDelft

The equation of motion of the ensemble rotor, gearbox and generator, in undamped free vibrations, is written on the left-hand side of the slide. Although you don't need to know the derivation of this equation by heart, it is important to identify the analogy with the equation for a mass-spring system. It follows that the coefficient of psi double-dot is the mass contribution, while the coefficient of psi is the stiffness contribution. Since the natural frequency of the system is expressed as the square root of the stiffness over the inertia of the system or the mass of the system, it is expressed as the square root of the ratio between the coefficient of the second term of this equation, divided by the coefficient of the first term of this equation.

Support structure

Lecturer: Axelle Viré

To conclude this unit, let's look at how the dynamics of the support structure can be modelled.

Tower vibrations (video)

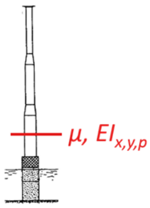
11

This video illustrates that the tower vibration is definitely non negligible. This was filmed for a two-blade rotor, but the same conclusion holds for a three-blade rotor.

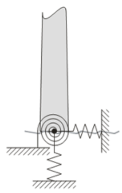
Support structure model

Tower: distributed mass μ and stiffness EI

e.g. finite element model



Foundation: spring or spring-damper model



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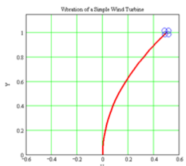
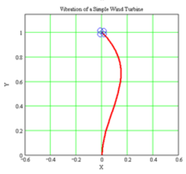
So how can we model its dynamics? The support structure encompasses two parts: the tower on the one hand, and the foundation on the other hand. The tower is a structure with continuous properties for mass and stiffness. Thus, it can be discretised in a series of elements, for example, using a finite element model with discrete values of mass and stiffness on each element. The effect of the foundation, by contrast, is usually approximated using a spring element or a spring-damper model. In this example on the right-hand-side, a spring is used in the horizontal and vertical directions, and a spring or damper element is used for the degree of freedom in rotation.

Mode shapes (Ψ) and superposition

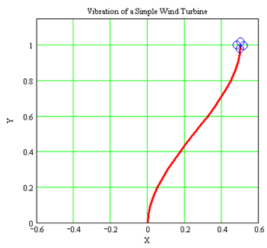
$$x_1(y,t) = \psi_1(y) Z_1(t)$$



$$x_2(y,t) = \psi_2(y) Z_2(t)$$



Superposition



$$u(y,t) = \psi_1(y) Z_1(t) + \psi_2(y) Z_2(t)$$

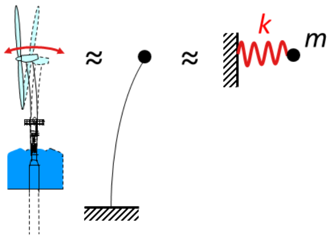
13

Once the model is established, we can solve the equations of motion of that system to obtain its mode shapes. Assuming that the support structure is characterised by linear structural properties, the different mode shapes can be superimposed using the linear superposition principle, in order to describe the response of the structure to certain excitations.

Approximation of the first natural frequency

Estimation of k and m:

- k: Apply force on tower top and get displacement (model/measurement)
- m: Rotor-nacelle mass (+ ~20% of tower mass)



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A very first approximation of the first natural frequency of the system can also be obtained by simplifying the representation of the tower using a cantilever beam. The beam has a certain mass, representing the distributed mass of the tower, and supports a mass point at the top, representing the rotor. The fore-aft motion of the tower can also be mimicked using a spring element. In that case, the first natural frequency of the system is the square-root of k over m, where the m is the mass of the rotor-nacelle assembly plus some contribution of the tower, and the stiffness k of the system is estimated by looking at how the mass behaves when it is subjected to a certain force. This approach, although simplistic, is very useful to get an idea of the first natural frequency of the system.

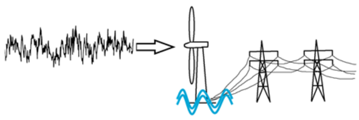
5.4 Characteristics of loading

5.4.1 Sources of wind turbine loading

Origins of loading

Forces of the rotor

- Wind/turbulence/gusts
- Tower shadow (also upstream of tower)
- Wind shear



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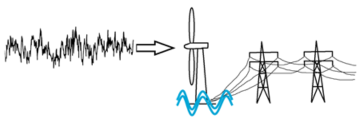
We will enumerate the forces acting on three different parts of the wind turbine: the rotor, the support structure, and the generator. The rotor is of course subjected to wind force, including turbulences and wind gusts. The presence of the tower modifies the flow field around the wind turbine, and thus, generates force variations on the rotor. This effect is called tower shadow, and exists for both downwind and upwind turbines. Finally, the wind velocity increases with the altitude. This wind shear also means that the various parts of the rotor will experience different velocities, and therefore, different forces.

Origins of loading

Force on the support structure

- Wind/turbulence/gusts
- Wind shear
- Waves and current



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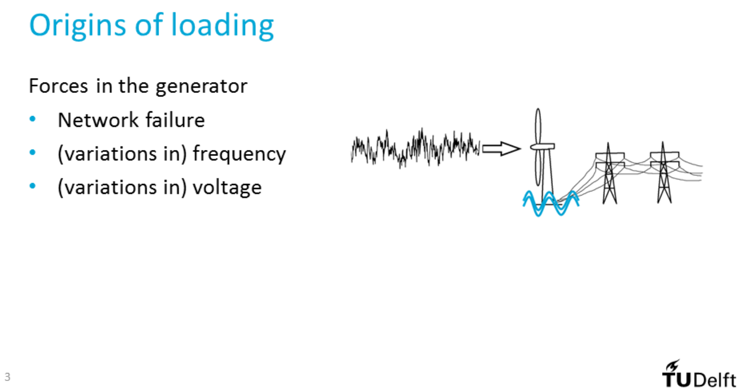
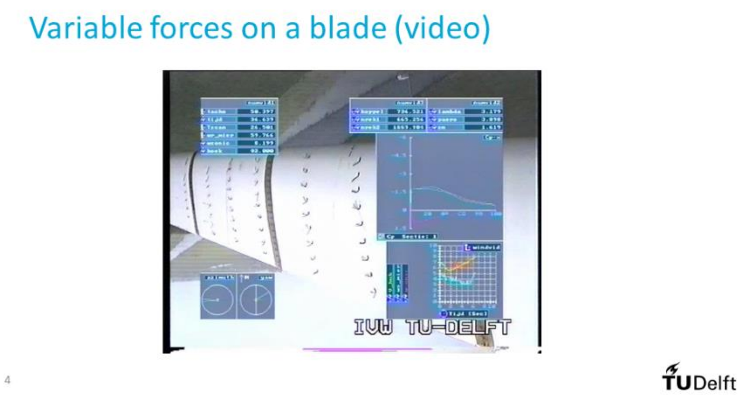
The wind flow, its variations, and shear also affect the loading on the support structure. If the turbine is placed offshore, additional forces come from the presence of waves and currents.



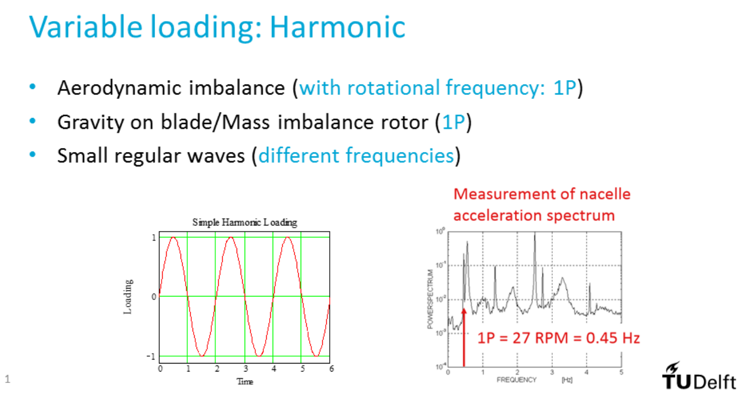
Finally, the generator is subjected to forces coming from failures in the network, and also variations in frequencies or voltage.



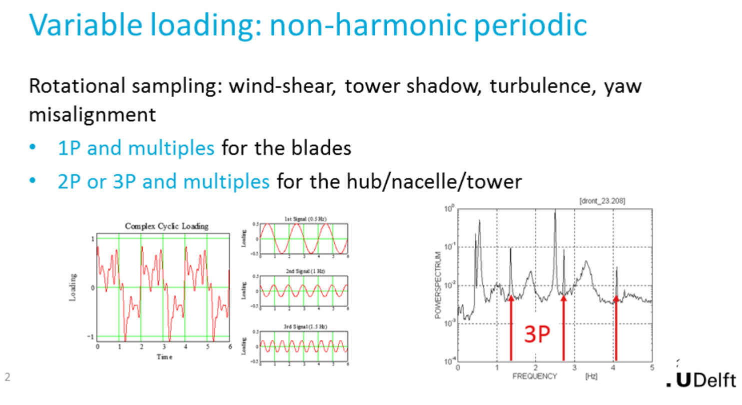
This video clip illustrates the blade vibrations as it rotates. It clearly indicates that the blade deformations and dynamics are significant during operation. The flow is further visualised using ribbons, called tufts, placed on the blade. Tufts are commonly used for flow visualisation because they align with the flow streamlines, and can therefore indicate when the flow is attached or detached at the blade surface.

5.4.2 *Characterisation of loading*

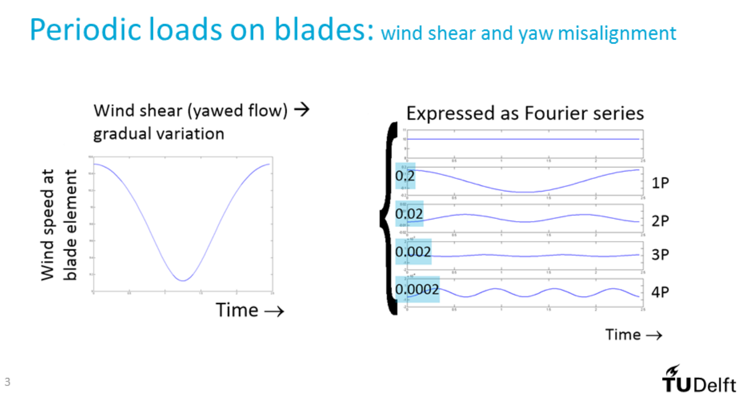
This video aims at characterising the different sources of loading in terms of their periodicity. In particular, we will distinguish between harmonic loading, non-harmonic but periodic loading, and completely random loading.



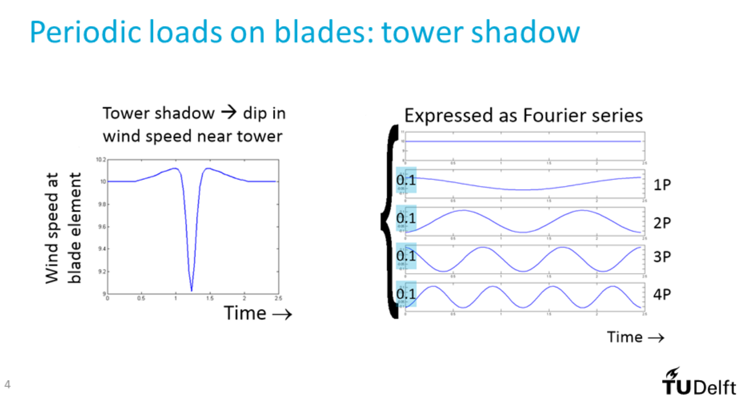
A loading is harmonic when it varies according to a cosine or a sine function. Three sources of wind turbine loading are harmonic. The first one is the so-called aerodynamic imbalance, which occurs for example when the pitch angle of one of the blades is different than the pitch on the other blades. Such a situation shifts the centre of thrust force from the hub (where the centre of rotation is) to one blade. Thus, as the rotor rotates, it experiences a load variation and a moment associated to it. The period of this variation coincides with the rotation period of the rotor. In other words, the period of this loading is 1P, meaning that it equals one times the period of rotation. Another source of harmonic loading is the mass imbalance, which originates from the fact that the mass of each blade might not exactly be distributed in the same way on all the blades. This imbalance shifts the centre of gravity of the rotor from the hub to one blade, which again creates a load variation at the frequency of 1P, once per rotation of the turbine. Finally, for offshore turbines, small regular waves can also be approximated as a sinusoidal function, to which is associated a sinusoidal load. The figure on the right hand side shows the frequency variations of the acceleration of the nacelle measured on a turbine placed in IJsselmeer, which is an inland sea in the Netherlands. We know what the rotation speed of the rotor is, so we know what the 1P frequency is. The plot shows a significant peak at the 1P frequency, which confirms the response of the structure to 1P loadings.



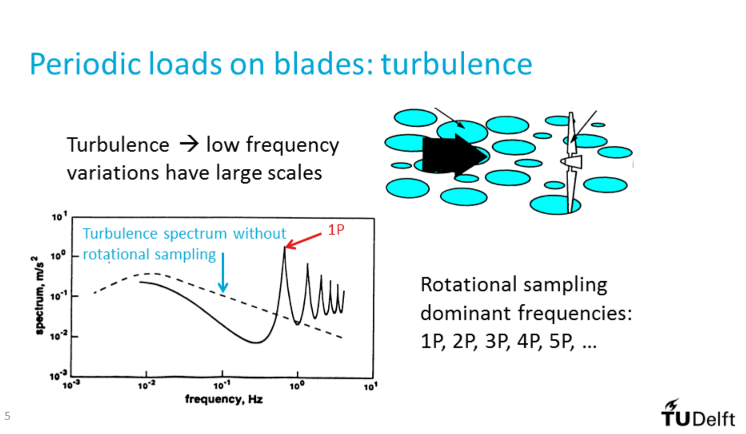
A loading can also be periodic without being harmonic. This is illustrated on the left-hand side figure. The signal representing the time variation of the loading is periodic but it does not coincide with a sine or cosine function. In fact, periodic yet non-harmonic signals can be decomposed into a series of harmonic signals, each one having a different frequency. In this case, the first harmonic is a sine function at a certain frequency and amplitude. The second harmonic is a sine function at a frequency that is twice the frequency of the first harmonic. The third harmonic has a frequency 3 times larger than that of the first harmonic, etcetera. The sum of these harmonics gives a signal that is periodic but not harmonic. Rotational sampling is a good example of phenomenon that produces periodic loadings. It is due to the fact that a blade, when rotating, will sample different characteristics of the wind field. For example, due to wind shear, the wind velocity varies with the altitude. The blade therefore samples, over one revolution, different wind velocities. The frequency of these variations is directly linked to the rotation speed, and is therefore again 1P and all its multiples. These frequencies are apparent when the loading is measured on the blades. However, they are not apparent on the hub or the nacelle, as we will explain in a minute. For the three-blade turbine in IJsselmeer, rotational sampling produces load variations on the nacelle at a frequency of 3P and its multiples, as shown on the right-hand-side plot.



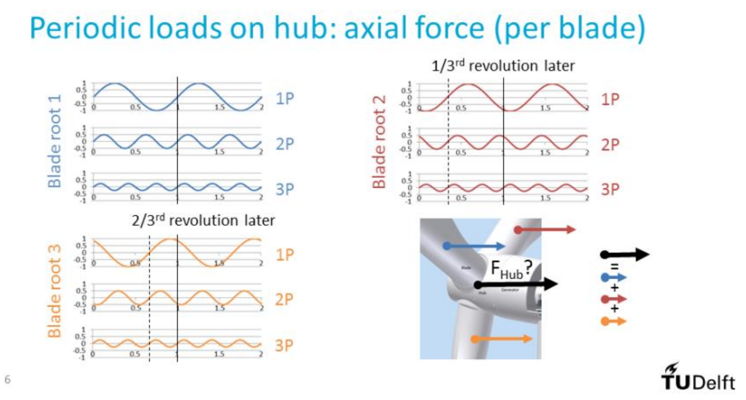
This slide shows, on the left-hand-side, the typical wind speed seen by a blade element as it rotates. This is a periodic signal for example due to the presence of wind shear. This signal can be decomposed into harmonic signals by taking its Fourier transform. As explained before, the frequencies of these harmonics are 1P, 2P, 3P, etc., and their amplitudes decrease as the frequency increases. This is because the original signal for the wind speed is still close to a sine function.



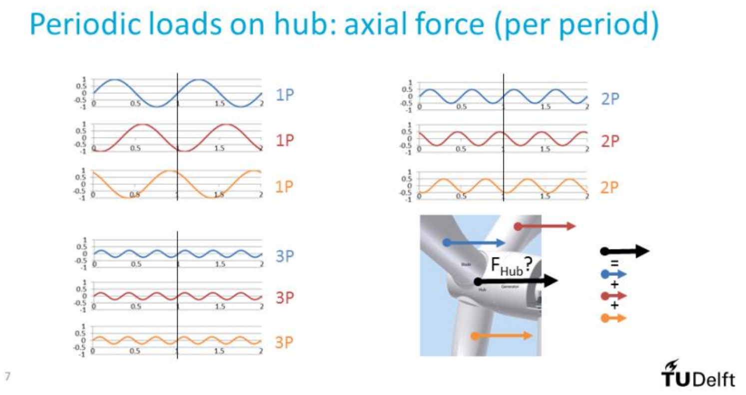
The effect of the tower on the load variations is different because this is a phenomenon that is much more limited in time. This is because the time that the blade spent in front of the tower over one revolution is very small. This is illustrated by the plot of the wind speed on the left-hand-side. The passage of the blade in front of the tower produces a large decrease in wind speed but this occurs over a very short duration. Thus, the overall signal is very far from a sine function. This means that, as opposed to the previous slide, after decomposition in Fourier series, all the harmonics of the signal have similar amplitudes.



Another source of periodic loads is turbulence in the wind. Wind turbulence can be decomposed into different length scales, large ones associated with large time scales, and small ones associated with small time scales. Wind turbulence generates periodic loads on the rotor because the blades will sample these wind perturbations in the wind during a revolution. The continuous line on the plot shows the load variation coming from the flow turbulence on the blade. It has the characteristic frequencies 1P, 2P etc. The dashed line shows the turbulence energy spectrum, in which large scales (which have large time scales, and therefore low frequencies) are more energetic than small scales characterised by large frequencies. Thus, the frequencies dominant in the rotational sampling actually coincide with the smaller scales of the turbulent spectrum.

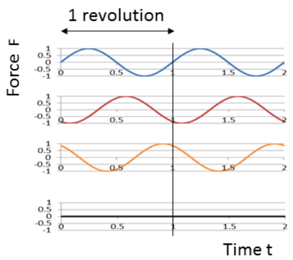


I mentioned earlier that the multiples of the 1P frequencies due to the rotational sampling are not all felt by the nacelle or the hub. In order to explain this, the forces on each blade are plotted on this slide. As mentioned before, periodic loads can be decomposed into harmonics at frequencies 1P, 2P, 3P, etc. This decomposition of loads on the first blade is shown on the top left plot. For a three-blade wind turbine, the second blade will sample the same phenomena as the first blade but a third of a revolution later. Thus, the harmonics associated with the second blade are all shifted by a third of a revolution compared to the signals on blade 1. This is shown on the top right figure. Similarly, the harmonics on the third blade will be shifted by 2/3 of a revolution compared to those on blade 1. The total force on the hub is the sum of all the harmonics on the three blades. So let's group all the harmonics of the same frequencies on one plot.



If we focus on the 1P signals first, on the top left plot, we see that when the 1P signal on blade 1 is zero, it is largely negative on blade 2 and largely positive on blade 3 (with the same magnitude as on blade 2). Thus the sum of the 1P signals is actually zero. The same holds for the 2P signals. By contrast, the 3P signals are all in phase. This is because the 3P signals have a period that is 3 times smaller than the 1P signals, and they are shifted by a third of a revolution. Thus, when summed up, the total 3P harmonic is non-zero on the hub.

Axial force with 1P variation



E.g. due to tower shadow

1st blade

2nd blade (1/3rd revolution later)

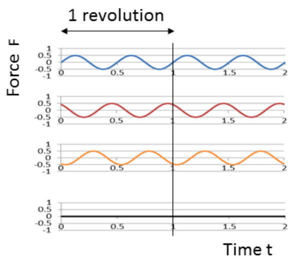
3rd blade (2/3rd revolution later)

Summation: axial force variation on hub at 1P is zero

8

This is shown here again. For periodic loadings, for example due to tower shadow, the loads on each blade have the same amplitudes but are shifted by a third or two thirds of a revolution. Therefore, when summed up, the 1P harmonics on the hub cancels out.

Axial force with 2P variation



E.g. due to tower shadow

1st blade

2nd blade (1/3rd revolution later)

3rd blade (2/3rd revolution later)

Summation: axial force variation on hub at 2P is zero for 3-bladed rotor (for 2-bladed rotor: delay is 1/2nd revolution and not 1/3rd)

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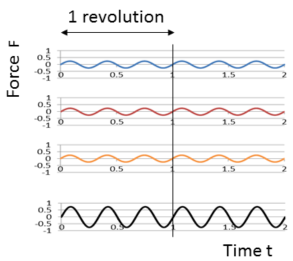
The same holds for the 2P harmonics, if the rotor has three blades. Of course, if the rotor has only two blades, the 2P harmonics sum up to a non-zero value.

Axial force with 3P variation



E.g. due to tower shadow

1st blade

2nd blade (1/3rd revolution later)

3rd blade (2/3rd revolution later)

Summation: axial force variation on hub at 3P

10

For a three-blade rotor, it is the 3P variations of the load that sum up to a non-zero value.

Generalisation of periodic loads (hub, drive train, tower)

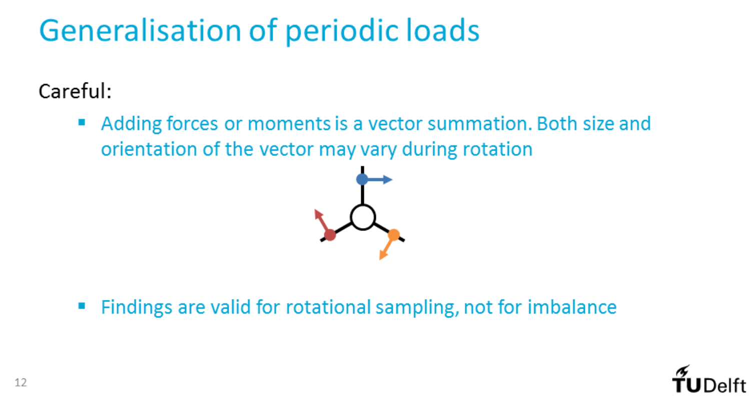
Higher harmonics (3 bladed rotor)
- 4P, 5P, 7P, 8P, 10P etc. cancel in the same way
- 6P, 9P, 12P etc. add up in the same way

Number of blades (B)
- ≠ n·BP cancel, so for 2 bladed: 1P, 3P, 5P, etc.
- n·BP add up, so for 2 bladed: 2P, 4P, 6P, etc.

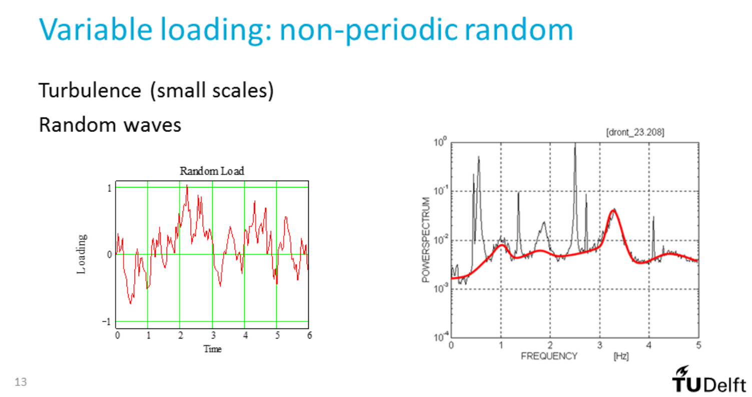
Forces and moments elsewhere (e.g. in drive train or on tower top)
- Same principle as on hub

11

To summarise, the periodic loads on the hub of a three-blade wind turbine exhibit harmonics at the 3P frequency and its multiple. The components at 1P and 2P, and their multiples, cancel out. For a two-blade wind turbine, only the multiples of 2P remain. Thus, if we generalise to a rotor with N blades, all the multiples of the number of blades N times the rotational frequency P add up, while all the non-multiple of this number cancel. This is true for the load felt by the hub and other components such as the drive train, tower, etc. Only individual blades experience all the harmonics of periodic loads.



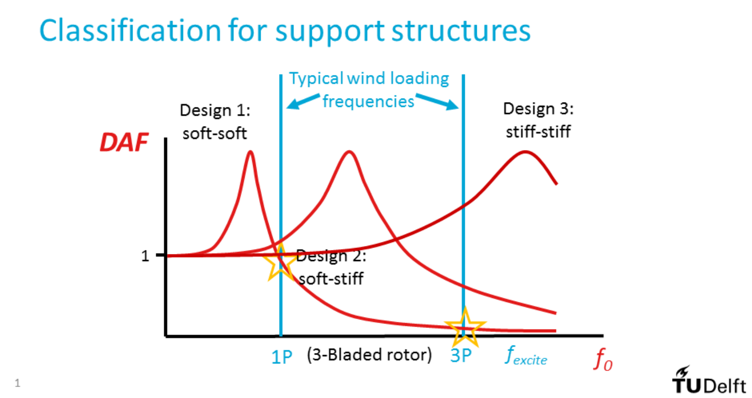
There are two final remarks to make about periodic loads. First, when we talked about adding up the forces, or moments, a vector summation should be considered. This means that both the amplitude and the orientation of the vector, which may vary during the rotation, should be added. Also, it is important to keep in mind that the cancellation of certain harmonics on the hub holds for the rotational sampling but not for imbalances. This is because the loads due to rotational sampling affect all the blades in the same way. Indeed, the effect of wind shear, or tower shadow, will be identical on all the blades, the only difference being the time in a revolution at which this effect occurs. By contrast, imbalances characterise the whole rotor. The imbalance occurs on one blade only and affects the position of the centre of gravity, or thrust, towards one blade. Therefore, imbalances affect the loading on the whole rotor, rather than on individual blades. The consequence is that the hub is subjected to 1P load variations coming from imbalances, but not coming from rotational sampling.



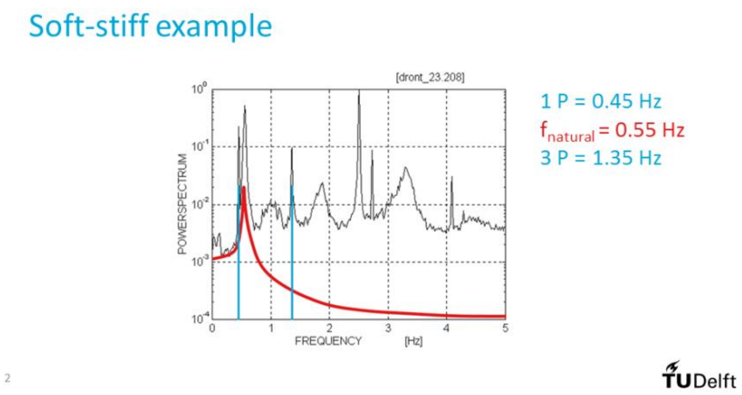
Some loads are neither harmonic, nor periodic: they vary randomly. Random loads typically originate from the smallest scales of turbulence and random waves. These loads produce small random variations in the acceleration spectrum of the nacelle, as shown on the right hand side. This unit focused on the characteristics of the loads experienced by a wind turbine. In the next unit, we will characterise the response.

5.5 Characteristics of response

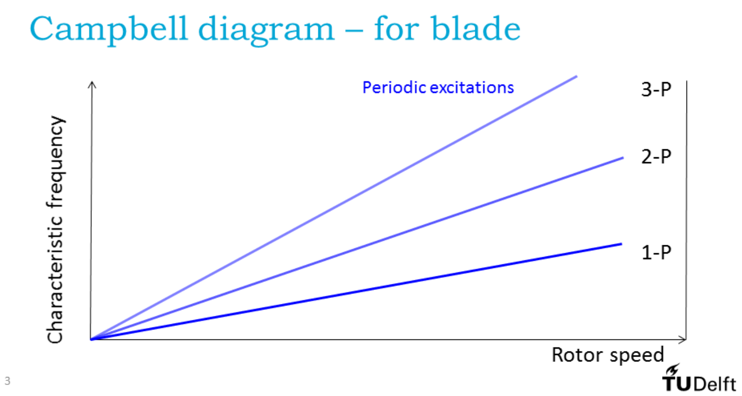
5.5.1 Comparing natural and excitation frequencies



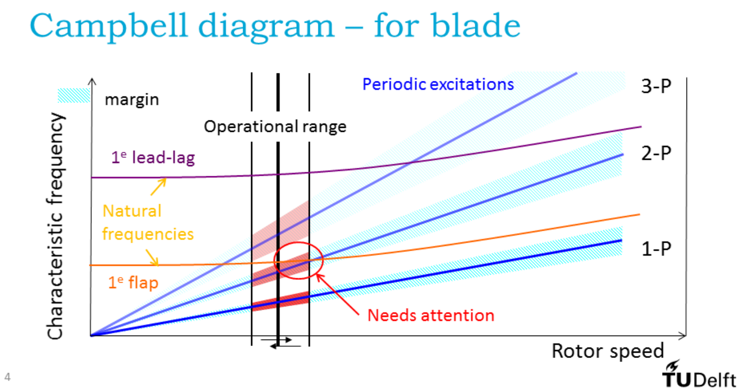
So, let's consider a three-blade rotor. The typical wind loading frequencies for this type of rotor are 1P and 3P; the 1P frequency being due for example to imbalances, and the 3P frequency being caused by rotational sampling. There are three types of dynamic response of the support structure, depending on its natural frequency. If the dynamic amplification factor of the structure peaks at a frequency smaller than 1P and 3P, then the natural frequency of the tower is also smaller than these values. The structure is thus qualified as soft-soft, because it is soft with respect to both the 1P and 3P frequencies. By contrast, if the natural frequency of the support structure is larger than 1P but smaller than 3P, then the structure is soft-stiff because it is soft with respect to the 3P frequency, and stiff with respect to the 1P frequency. Finally, the support structure is stiff-stiff when its natural frequency is larger than both the 1P and 3P frequencies.



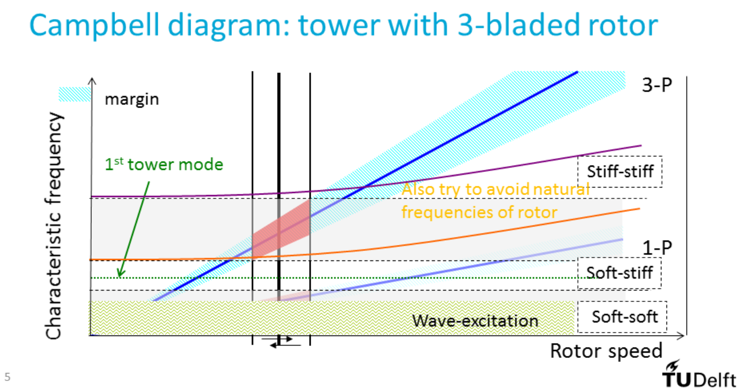
In the case of the wind turbine located in Ijsselmeer and discussed in the previous unit, the first large peaks in the acceleration spectrum indicate the 1P and 3P frequencies. Since the dynamic amplification factor peaks between these two frequencies, the structure is soft-stiff. Note that in this example, the natural frequency is actually dangerously close to the 1P frequency, and therefore, the response of the system to certain excitations could be very large.



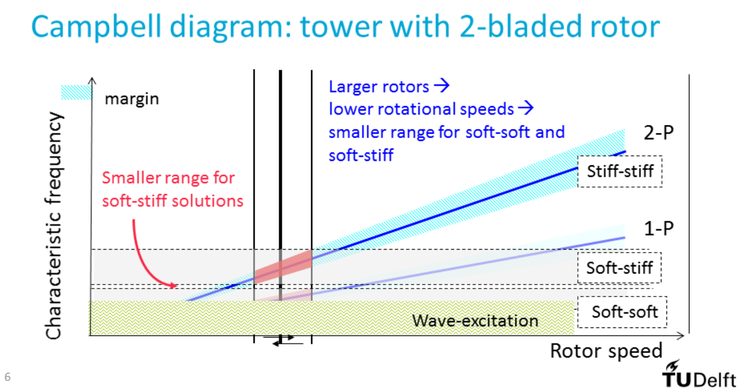
The so-called Campbell diagram is typically used to assess whether a structure is subjected to resonance or not. It shows both the excitation frequencies and the natural frequencies of the system as a function of the rotation speed of the rotor. Let's consider the blade of a wind turbine. We know that it is subjected to loads at the frequencies of 1P, 2P, 3P etc. These frequencies of course depend on the rotor speed, because the P precisely denotes the frequency of rotation. Thus, these periods of excitation are straight lines in the diagram.



Instead of being sharp lines, we usually consider a certain margin around these frequencies. These are not uncertainty margins, because the rotation speed of the rotor is actually known exactly. Rather, they are used because the peak corresponding to the natural frequency in the dynamic amplification factor is not sharp but has a certain width. Thus, a margin around the 1P, 2P and 3P frequencies should be considered for safety. Indeed, even if the structure does not experience resonance, it could still exhibit very large motions when operating in the peak of the amplification factor. The other region of interest in the diagram is the range of speeds at which the rotor operates. Since the rotor speed never exceeds the cut-out value, speeds larger than that value, on the right-hand-side of the right vertical line, can be disregarded. Speeds smaller than the left vertical line can be encountered because the rotor will speed up to the operational regime. However, the time spent at these low speeds is insufficient to create resonance of the rotor. Thus, rotor speeds smaller than the minimal operational value can also be disregarded. The region between the two vertical lines shown on the slide is thus of interest to assess whether the system resonates. In particular, in that region, the natural frequencies of the rotor should not overlap with the margins around the 1P, 2P and 3P frequencies. The natural frequencies can then be added to the diagram. For the blades, the main modes of deformation are the first flap-wise motion and the first edge-wise or lead-lag motion. As shown on the diagram, these natural frequencies increase slightly with the rotor speed, because of centrifugal stiffening. The region that needs attention with respect to resonance is highlighted on the slide. It is the intersection between the first flap-wise natural frequency and the 2P region, which might lead to resonance in the operational regime. It would therefore be safer to stiffen the rotor in the flap-wise mode, that is increasing the flap-wise natural frequency. The mode would then intersect with the 3P region. However, that intersection would occur at a smaller rotational speed, hence leading to smaller loads.

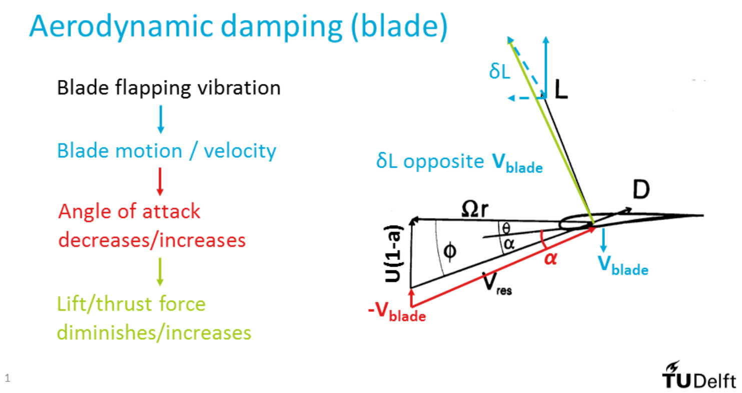


This slide summarises the three types of tower designs for a three-blade rotor. We learned before that the excitation frequencies of interest for the tower of such a rotor are the 1P and the 3P frequencies, since the 2P frequency cancels out. The 1P harmonics only come from imbalances, thus the 3P harmonics dominate. In order to design a tower that is safe for resonance, the regions shown by the shaded red area should be excluded. As explained before, there are three types of design for wind turbine towers: soft-soft if the natural frequency is smaller than 1P and 3P, soft-stiff if it lies between 1P and 3P, and stiff-stiff if it's larger than the 3P frequency. Most wind turbines are soft-stiff, meaning that the first tower mode lies in the soft-stiff region. This is particularly true for offshore wind turbines, for which a small natural frequency would intersect with the wave excitation frequencies. Finally, the first natural frequencies of the tower should also be different from the natural frequencies of the rotor. Thus, it should be well separated from the orange and purple lines on this slide.

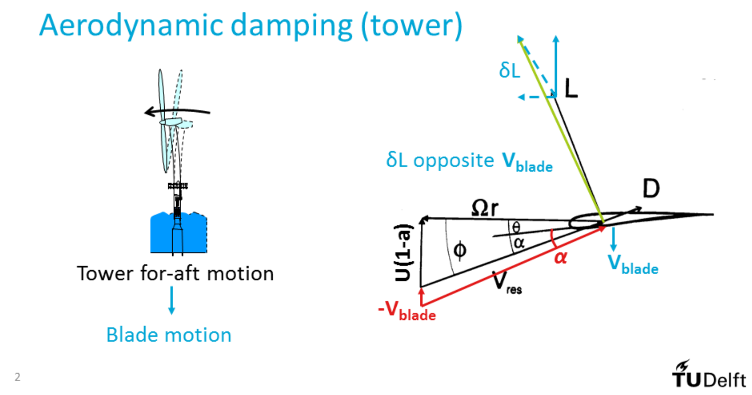


For a two-blade rotor, the excitation frequencies of interest are 1P and 2P and the soft-stiff region is narrower than on the previous slide. This means that it is difficult to design rotors on this soft-stiff region. This is increasingly challenging as the size of the rotors increase. The larger the rotor, the smaller its rotation speed in order to keep a constant tip speed ratio. Thus, the operational range of large rotors is shifted towards smaller rotation speeds. This means that the soft-stiff region narrows even further and moves towards the wave excitation frequencies. This explains why the tower design for an offshore wind turbine with a large rotor is very challenging.

5.5.2 Aerodynamic damping



In this unit, we will explain the concept of aerodynamic damping associated with the blade motion. When a blade vibrates, one of the bending modes is a flapping motion that occurs in the plane perpendicular to the plane of rotation. Thus, during flapping, the blade experiences a velocity V_blade perpendicular to the plane of rotation. This increases the apparent wind speed, and also the angle of attack seen by the blade. If the blade is not in stall, then the lift force experienced by the blade element also increases. This increase in lift delta_L can be decomposed into a component perpendicular to the plane of rotation and another parallel to the plane of rotation. The figure on the right-hand-side illustrates that the component perpendicular to the plane of rotation actually opposes the initial flapping motion of the blade. Thus, the variations of aerodynamic lift force balance the flapping bending of the blade. A force that opposes a velocity, or is proportional to it, is typically a damping force. Therefore, this phenomenon is called aerodynamic damping.



The same damping mechanism may occur with the tower fore-aft motion. In that case, the tower bending with or against the wind also creates a relative velocity at the blade variations of the lift force. Thus the dynamic motions of the system can be partially damped aerodynamically.

5.5.3 Full dynamic wind turbine simulation

When we consider the dynamics of a wind turbine, we have several sources of loading coming from the wind, coming from the waves and also coming from the grid. The wind loading will affect the rotor directly and that will be transmitted to the drive train and to the generator. We also have loading from the waves that will affect mostly the support structure, which is again coupled to the rotor. For example, support structure motion will cause aerodynamic damping. The generator connects to the grid, but when a back-to-back converter is used most of the dynamics of the grid are more-or-less filtered out by the control of the converter. However, for instance a short-circuit in the grid will impact the generator and therefore the drive train and the rotor.

The controllers that determine the operation of the wind turbine look at the state of the wind turbine and then feed that back to pitch actuators in the rotor and generator or converter settings. They may also look at the wind and the grid state and use such information for feed-forward control to obtain better response.

Because of these interactions in the wind turbine, the loading and motion of the various subsystems are strongly coupled. Therefore, full dynamic analysis of the response of a wind turbine is normally done with a simulation tool in which all these subsystems are integrated. Figure 1 shows a diagram of such an integrated wind turbine simulation tool, illustrating the connections between the subsystems. Examples of such tools are Bladed (from DNV-GL), FAST (from NREL), FLEX (from DTU) and Focus (from WMC).

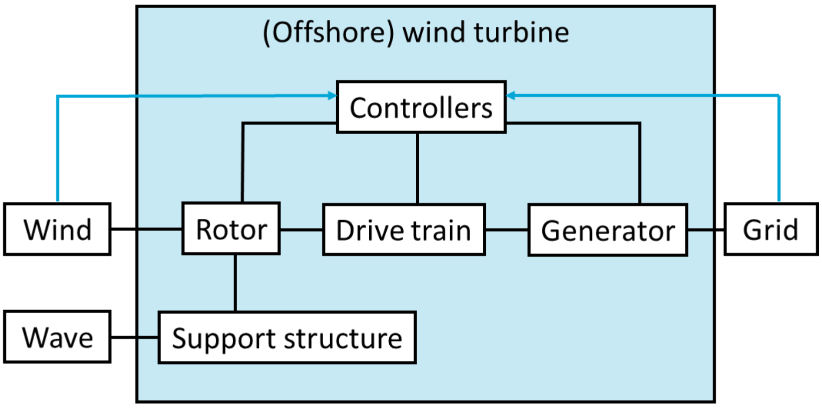


Figure 1 Diagram of wind turbine simulator structure

The outputs of wind turbine simulation tools can be time series of response, but also natural frequencies, mode shapes and spectra of response. Figure 2 shows an example of a time series of response generated with a wind turbine simulation tool. We will not use wind turbine simulation software in this course. However, the preliminary assessments of dynamics form a good basis in understanding the loads and response and to interpret the results of a simulation.

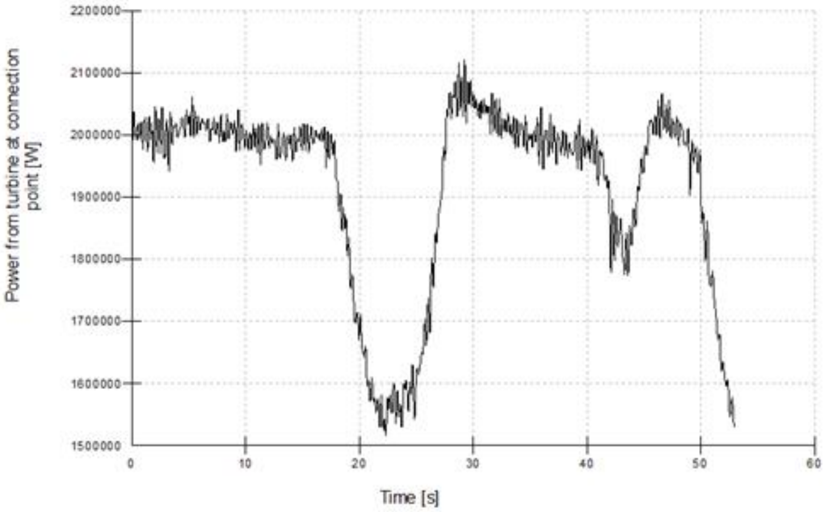
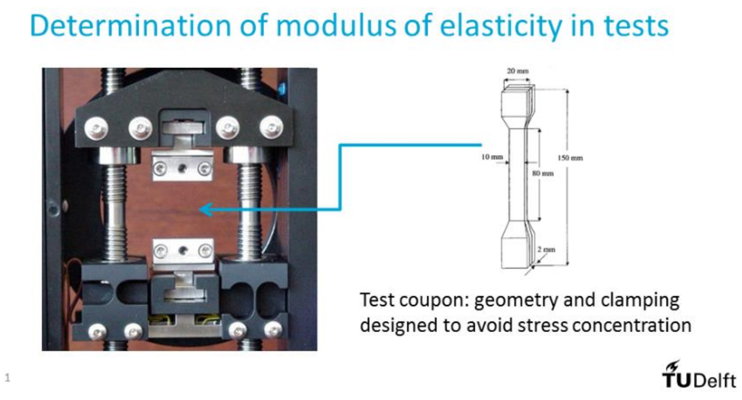


Figure 2 Example of the power obtained through simulation of a wind turbine in operation

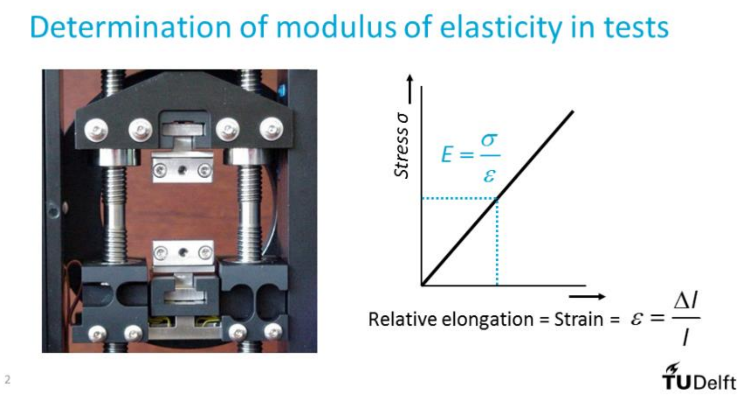
6 Structural analysis

6.1 Background: Introduction to mechanics

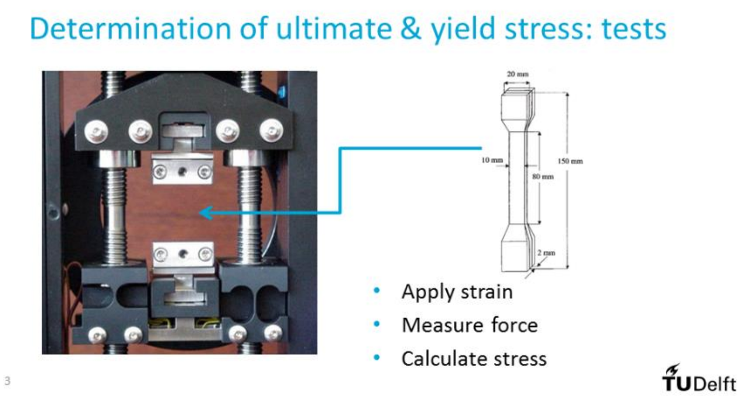
6.1.1 Material properties



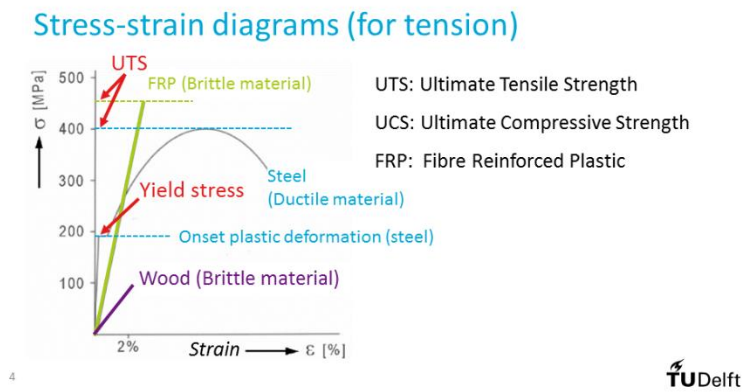
The modulus of elasticity of a material is determined with a testing machine. A coupon is made for which the geometry and the clamping are designed such that we don't have stress concentrations in the centre of the coupon. The testing machine will stretch the coupon and it will measure the force that is needed for this stretching. Because we know the cross-sectional area of the coupon, we can determine the stress from the measured force.



The stretching of the testing machine is expressed as the strain, which is the length of this stretching, delta L, divided by the length of the coupon L. If we plot the stress as a function of the strain, for most materials we'll find a region in which the relation is linear. The gradient of this linear relation is the modulus of elasticity. This linear relationship is called Hooke’s law.

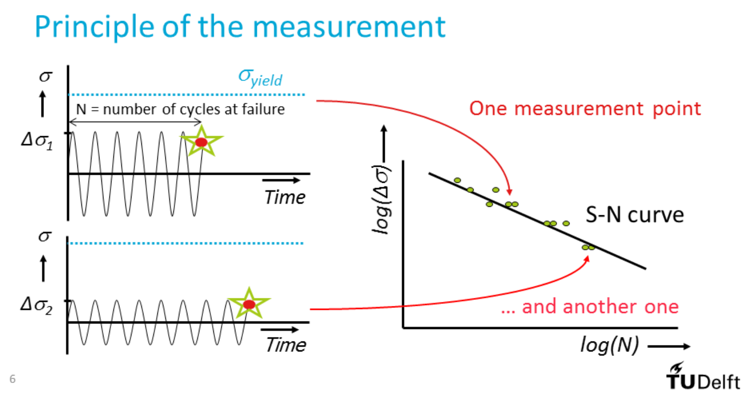


The yield stress and ultimate stress are also material properties, which are determined with the testing machine. Again, we'll use a coupon with no stress concentration, we'll apply a strain and we'll measure the force from which you can calculate the stress.

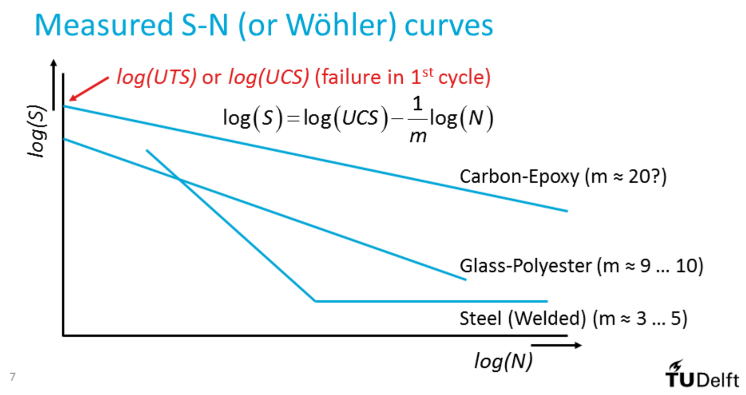


If you plot the stress against a strain we can recognise, for low strain levels, the linear region for Hooke's law. If we keep increasing the strain for steel, which is a ductile material, we'll see that after a certain strain there is no longer a linear increase of the stress, and this is because we have reached the onset of plastic deformation at the yield stress. A further increase in the strain will lead us to a point where the stress doesn't increase anymore, and this is where we find our ultimate tensile strength. For brittle materials such as glass fibre reinforced plastics, used in the blades, we only have a linear region and once we reach the ultimate tensile strength we have failure. We can make similar diagrams by compressing the coupon instead of stretching it, and then we'll find the ultimate compressive strength.

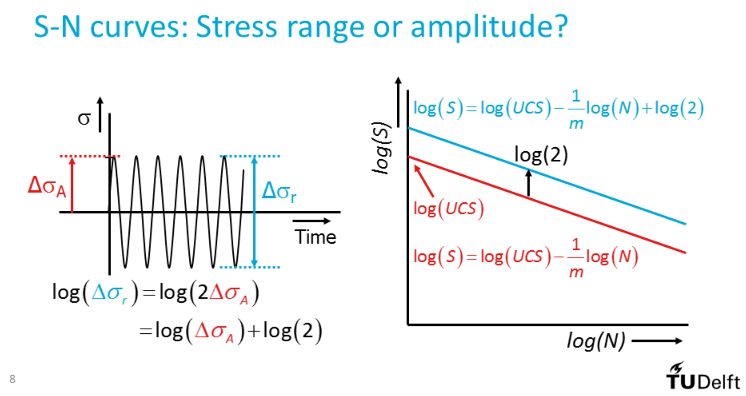
A material can fail under variable loading, even when the loading never exceeds the ultimate tensile or compressive strength. This phenomenon is called fatigue. To determine when the material fails, we use a slightly more complicated testing machine, which can apply a variable loading to the coupon. The coupons can be similar to what we have seen for the static tests of the modulus of elasticity and of the ultimate strength, but sometimes test coupons are used with a specific geometry. The fatigue behaviour can be sensitive to the geometry of the structure, since that can lead to concentrations of stress in certain parts of material.



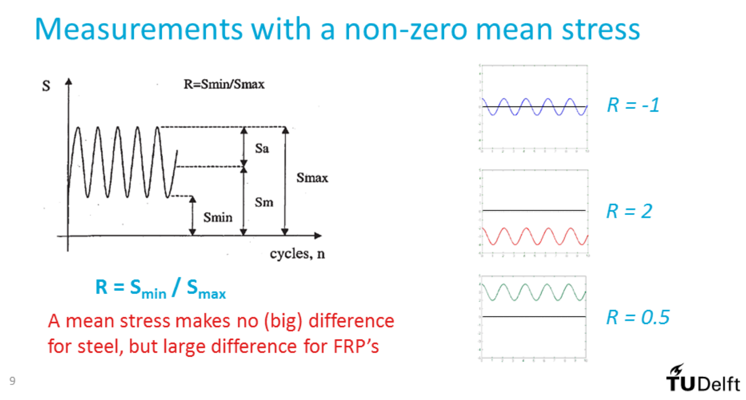
During the test, a variable loading will be applied to the coupon with a constant amplitude. After a number of cycles that is counted, the coupon will fail. This will give us one point in a diagram of the stress amplitude versus the number of cycles until failure. We can repeat this test at a different stress amplitude, and that will give us a different number of cycles until failure. In this case, we have a lower stress amplitude, and therefore a larger number of cycles before failure. This will give us another point in the diagram. If we do this multiple times, and we plot it on a double logarithmic scale, then we'll generally find that the points end up on a straight line. The line through these points is called the S-N curve. In practice, the line is not drawn exactly in the middle of the points, but a little bit to the left of it, such that 95% of the points fall to the right of the line. That means that if we would reach the line for the number of cycles at a certain stress level, we can be 95% sure that we don't have failure yet.



The S-N curve is also often called a Wöhler curve, and for different materials we'll find different Wöhler curves. The difference is found in the slope of the curve and in where it crosses the y-axis. The y-axis corresponds to the logarithm of n equal to zero, so n equal to 1, meaning that we have failure in the first cycle. This corresponds to having an amplitude of the cycle of the ultimate tensile stress, or of the ultimate compressive stress. Therefore, the line crosses the y-axis at the logarithm of the ultimate tensile strength or the logarithm of the ultimate compressive stress. For steel, we see two regions with a different slope. In this course, we will only consider materials that have a single slope.

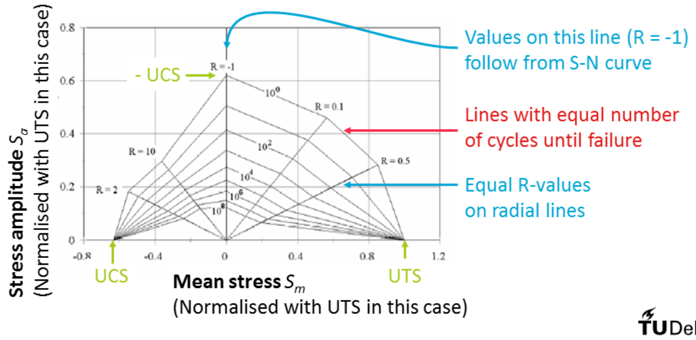


On the y-axis of the S-N curve we can plot either the amplitude of the stress or the stress range. The stress range is the difference between the highest and lowest value of the stress variation, so it's two times the amplitude. Since the stress range is twice as large as the stress amplitude, the logarithm of the stress range is equal to the logarithm of the stress amplitude plus the logarithm of 2. Therefore, the S-N curve for the stress range runs parallel to the S-N curve for the stress amplitude, but is shifted by the logarithm of 2. It therefore crosses the y-axis at the logarithm of twice the ultimate stress. Be aware that there is no unique preference for using stress range or amplitude in literature on fatigue. Check whether it is stated explicitly which is used or see whether it can be concluded from the crossing of the S-N curve with the vertical axis.



In the previous slides, we only considered the amplitude of the load variation, but we might need to consider the mean stress as well. For a material such as steel, the mean stress has almost no effect on the number of cycles until failure for a certain stress amplitude. However, for a material such as glass-fibre reinforced plastic, the mean can have a large effect on the number of cycles until failure. If we are dealing with a variable loading with an amplitude and a mean stress, we often use the parameter R which is the minimum stress divided by the maximum stress. If the minimum stress is negative but of equal size as the maximum stress, we have a zero mean and that corresponds with R equal to minus 1. We can identify different sorts of loading, such as having a negative mean stress or a positive mean stress, and such as having a larger amplitude than the mean or a smaller amplitude than the mean, and these correspond to certain values or ranges of values for R.

Results are given in a Goodman diagram



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You can imagine that if you have a material for which the number of cycles until failure is dependent on the mean stress and on the stress amplitude, we have to do many more tests than in the case it only depends on stress amplitude. And furthermore, we need a different way of representing the results than in a Wöhler curve. In this case, we usually use a Goodman diagram. On the x-axis in the Goodman diagram we plot the mean stress and on the y-axis we plot the stress amplitude. The lines in the Goodman diagram are lines of equal life time, or to be more precise, of an equal number of cycles until failure. The line that is pointed at here has failure at 10 to the power 0, so at exactly one cycle. The radial lines correspond to an equal R value. If we increase the amplitude at the same pace as we increase the mean stress, then the R value remains the same. In this diagram, we can also recognise the points for the ultimate tensile stress, the ultimate compressive stress, and minus the ultimate compressive stress at the y-axis of the diagram. As I mentioned before, the outer line in this diagram is failure at exactly one cycle. So, along the x-axis for the mean stress, we have failure on the right-hand side for the ultimate tensile strength and on the left-hand side for the ultimate compressive strength. Along the y-axis, we also have a failure in the first cycle if we reach the ultimate compressive strength. The ultimate compressive stress is typically smaller than the ultimate tensile strength, and therefore during the cycle the ultimate compressive strength is dominant in when material will fail. Finally, along the y-axis we'll find the values for R is equal to minus 1. As mentioned before, this corresponds to a mean stress of zero, so the values of the number of cycles until failure that are given here correspond to what we have found for the Wöhler curve.

Note about Goodman diagrams

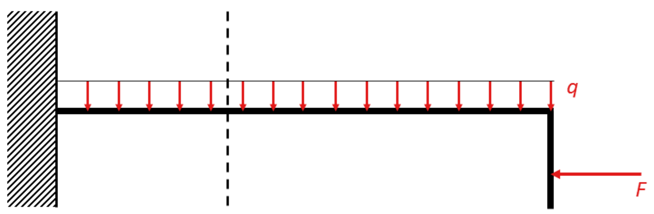
- You don't need to fully understand these for the exam
- You do need to realise:
  - Mean stress has an effect on fatigue of FRP (= material used in blades)
  - The higher the mean stress, the sooner fatigue failure occurs
  - Effect of compressive mean stress is larger than of tensile mean stress
- In this course we will only work with the zero-mean stress S-N curves (the Wöhler curve)

11

As mentioned before, you don't need to understand and remember all the details of the Goodman diagram. What you do need to realise is that for the material used in the blades, the mean stress has an effect on the number of cycles until failure. Also, the Goodman diagram shows us that the higher this mean stress, the lower the number of cycles until failure for a certain stress amplitude. And finally, what we have seen is that the compressive mean stress has a larger effect than tensile mean stress, and this is because glass-fibre reinforced plastics often fail much sooner under compression than under tension. When we are doing calculations in this course, we will only be using the zero-mean stress S-N curve, or the Wöhler curve.

6.1.2 Forces, stresses and deflections

Static forces and moments in cantilever beams



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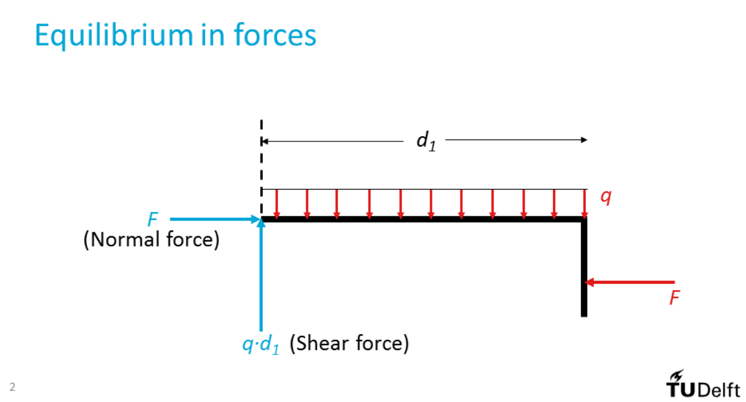
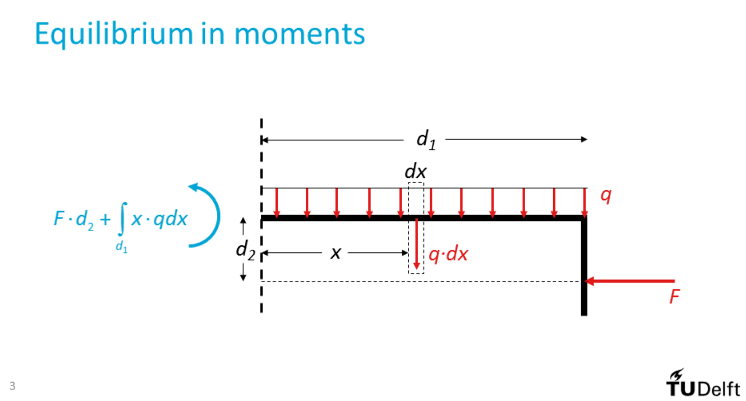
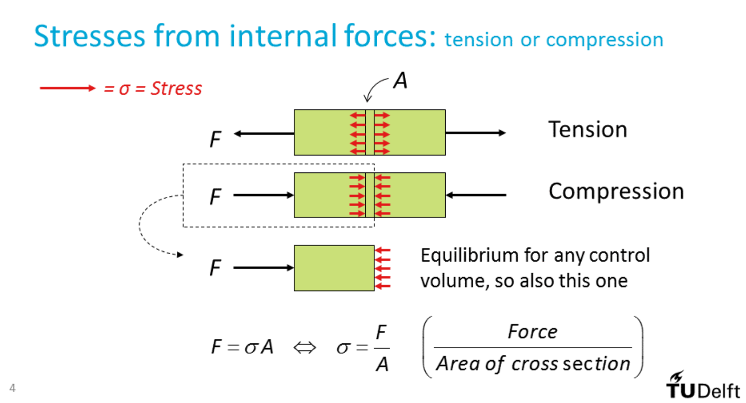
The tower and the blades of wind turbines can be considered to be cantilever beams. This is a beam that is only clamped on one side. The next slides will show how we can determine the forces and moments inside a cantilever beam, when it is subject to external loading. Consider this L-shaped beam, that is clamped on the left and that has two external loads on it. One load is a continuous load distribution q, which is a force per meter. The other is a point force F. Suppose you want to know the forces and moments inside the beam at the cross section that is indicated by the dashed line.



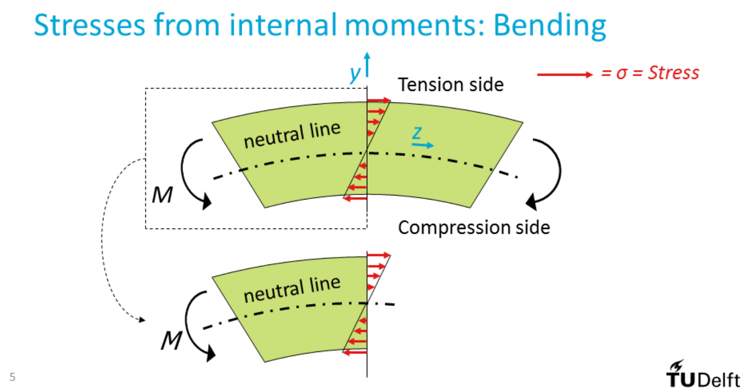
Let’s first look at the forces in the cross section. In our minds, we’ll cut the beam at the cross section and remove the structure on the left hand side. This doesn’t make a difference to the structure on the right-hand side, as long as we apply the same forces and moments in the cross-section that were previously applied by the left-hand structure. We know that the remaining structure will only stay at its current position, when there is an equilibrium in forces. There is an external force F on the structure, so we’ll have to apply an equal, opposite force on the cross section. This is a normal force, since it acts perpendicular to the cross section. The total vertical force of the load distribution q is the integral of q over the entire beam. This equals q times d1. To obtain equilibrium, we also need to apply an equal, opposite vertical force q times d1 on the cross section. This is a shear force, because it acts parallel to the cross section.



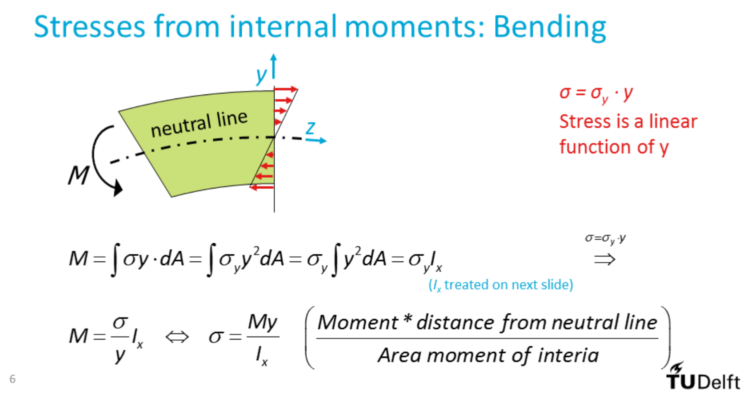
Application of the forces that obtain equilibrium is not yet sufficient to keep the remaining beam in position. It would start to rotate under the moment of the external forces. Therefore, we also need to apply a counteracting moment to the cross section. We can use any reference point to determine that we have an equilibrium in all the moments. However, it is convenient to pick the centre of the cross section, because the cross sectional forces we have just determined have no moment with respect to that point. What remains are the moments due to the force F, due to the distributed load q and the moment we apply to the cross section. The moment of the force F is simply F times distance d2. For the distributed load we have to perform an integration again. For each contribution of the load, q times dx, we have to take its distance to the point of reference. So we have to integrate x times q-dx over the beam length d1. The two contributions of the external forces to the moment around the reference point have the same direction, so they add up. To obtain equilibrium, we have to apply an equal counteracting moment. So now we know which forces and moments the left-hand side of the structure applies to the right-hand side of the structure at the position of the cross section. Since this method assumes equilibrium, it is actually only valid for static loading. If the loading changes in time, the dynamic response will have an effect on what the internal forces and moments will be. This could be determined through simulation, but that is outside the scope of this course.



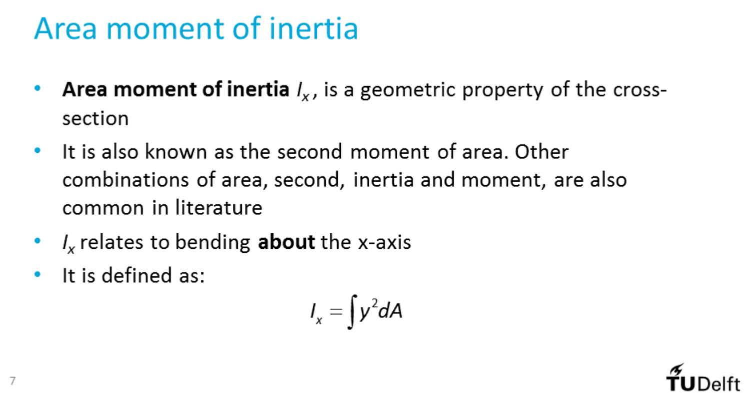
Now that we've determined the forces, the next step is to determine the stresses that are created by these forces. The forces can create either tension or compression in the material, and to determine how much tension or compression we have, we use a control volume. For any control volume, we assume that we have static equilibrium between all the forces that go into that control volume. So, if we pick a volume like this, with the external force on one side and the stress on the other side, we have to have equilibrium between this stress and the force. Therefore, the stress times the area to which it is applied has to be equal to the force, and we can use that to determine the stress as being equal to the force divided by the area of the cross section.



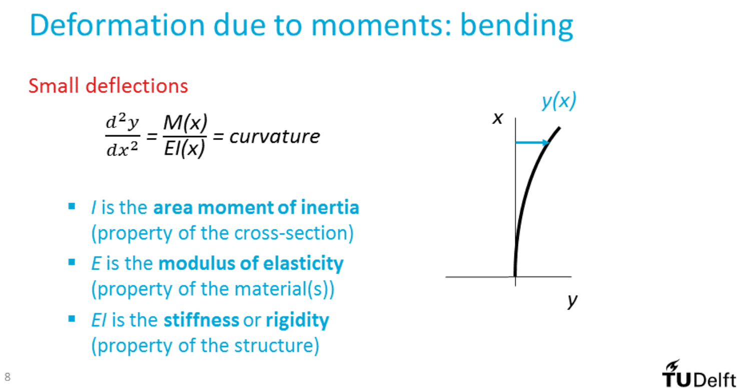
In many parts of the turbine we do not only have pure tension or compression forces, but we also have moments, and these create bending. Here we see an element with a moment that creates pure bending, because the moment on the left hand side and the right hand side are equal but opposite. In this example, you see that on the lower side of the beam there is a shortening of the beam, and therefore a compression region, and on the other side of the beam there is an elongation of the beam, and therefore we have a tension side there. In the middle you see the dash-dotted line, which is neither stretched nor compressed, and therefore this is called the neutral line. Also in the case of bending, we can define a control volume with on one side the external moment, and on the other side the cross section with the stresses. Also in this case there has to be an equilibrium between this external moment and the moment created in the cross section by the stresses.



The drawing already implies that the stress is not uniform over the cross-section, and it can't be, because we have a tension side and a compression side. So, somewhere the sign of the stress has to change. If we look into this in more detail, we can find that the stress is actually linearly proportional to the distance to the neutral line. This is caused by the assumption that the cross section doesn't deform in pure bending. For now we'll just take this linear variation for granted. And we'll use it to express the moment M in terms of the stress on the cross section. Each part of the stress distribution on the cross section contributes to a moment, by taking the local stress times the lever arm y, times the local area dA, and then integrate it over the entire cross section. If we substitute the linear function of the stress as a function of y into this, then we can separate it into the constant sigma y and an integration of y squared dA, which is only a function of the geometry of the cross section. This will be treated on the next slide. If we then substitute the linear equation back into this, we can express the moment M in terms of the local stress divided by the distance y, times this property of the cross section which we call the area moment of inertia. From this we can also get an expression for the local stress as a function of y, and as a function of the moment and the area moment of inertia.

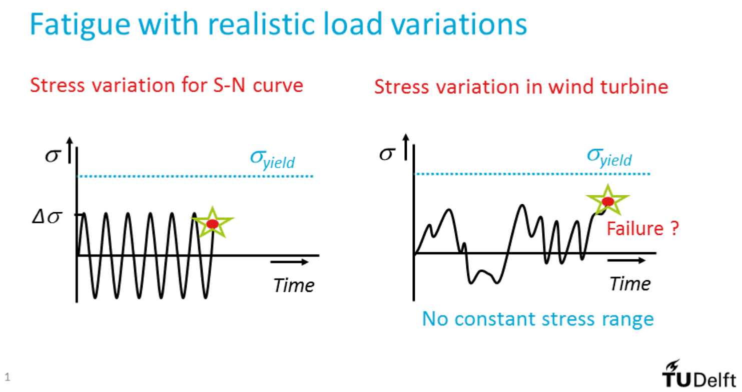


As mentioned on the previous slide, the area moment of inertia Ix is a pure geometric property of the cross-section. As it is defined as the integral of the y-coordinates squared times dA, you can consider the y-coordinate squared to be a weighing factor for the different parts of the area in the cross section. The more areas dA the cross section has far away from the neutral line, the larger the area moment of inertia will be. The area moment of inertia is also called differently in literature, with different combinations of the words area, second, inertia and moment. So, beware of that those difference words can refer to the same thing. However, also be aware that the moment of inertia also refers to the mass moment of inertia relating to the resistance of an object against rotation. The x index in Ix relates to bending about the x-axis. A cross section will have a different area of moment of inertia for different directions of bending.

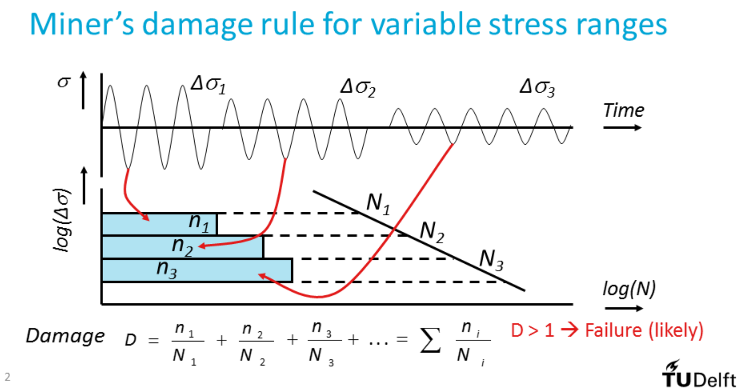


For small deflections, the second derivative of the deflection as a function of the distance x is equal to the curvature, and it can be derived that it is proportional to the moment, and inversely proportional to the modulus of elasticity and the area moment of inertia. So, the multiplication of the area moment of inertia and the modulus of elasticity is a measure of the resistance against bending, and therefore this is called the stiffness or the rigidity. In the expression that we have here, we see all aspects that are relevant for the state of the turbine. We have the moment M, which is a function of the external conditions, we have the area moment of inertia I, which is a function of the geometry of the blade, and we have the modulus of elasticity, which is a property of the material.

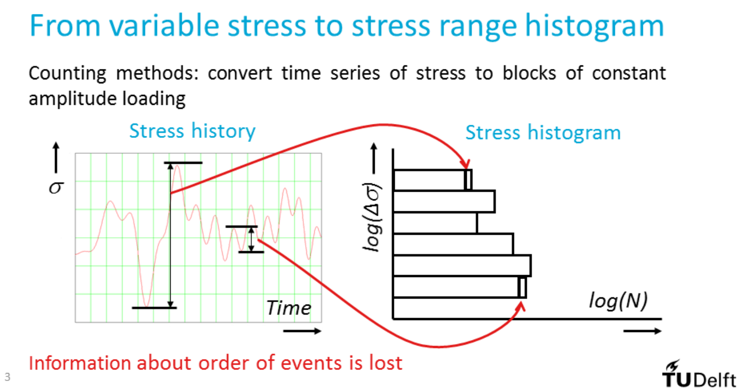
6.1.3 Fatigue



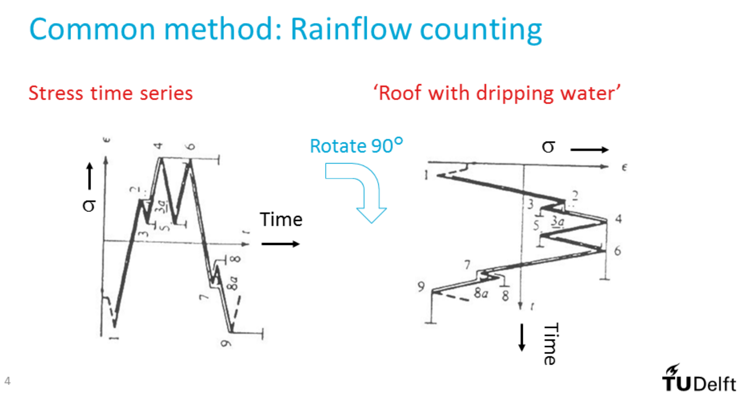
The fatigue properties of a material are determined with a variable loading with constant amplitude. However, the variable loading on many structures, including wind turbines, doesn't have a constant amplitude. Instead, the load variations are stochastic, such as in the graph on the right-hand side.



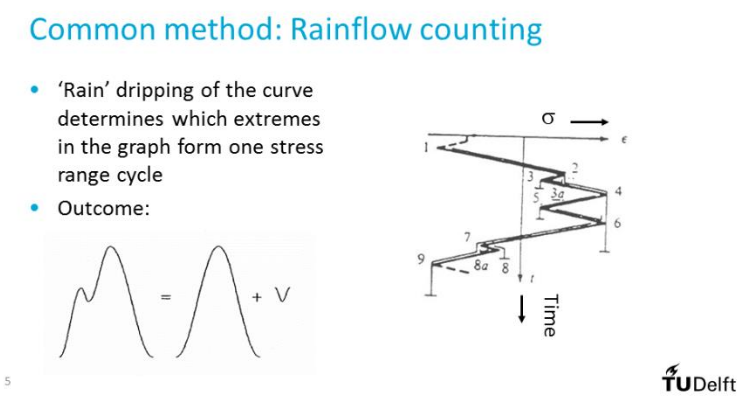
To bridge the gap between the measurements of the material properties for fatigue, and the reality of the variable loading for a wind turbine, we're first going to have a look at dealing with blocks of variable loading at an equal amplitude, with different of such blocks over time. Suppose we first have a variable loading at an amplitude delta sigma 1. We count the number of cycles at this variable loading, and that's equal to small letter n1. We compare that small letter n with capital N1, which is the number of cycles at failure for this stress amplitude. The ratio between the two is what we call the damage, and you can consider this also as being the lifetime that has been consumed for the fatigue. If the damage is 1, or if we have consumed the full lifetime of damage, then we have failure. After this, we could have a variable stress at a different stress amplitude delta sigma 2. Again we can count the number of cycles that we have, divide it by the number of cycles of failure at this stress amplitude, and add that to the damage or add it to the consumption of lifetime. And we could do this again and again for different stress amplitudes: count the number of cycles and add the ratio to the damage. As long as the damage remains smaller than 1, then it's unlikely that we have a failure.



But in reality, we also don't have these blocks of equal stress amplitude, but we have a continuously variable stress. So, the last step we have to take is to determine for each individual stress range which stress level it belongs to, and add it to the count for that stress level. So here you see an example of a large stress range or a large amplitude, and we add 1 to that stress level in the histogram. Another example given here is a stress variation with a low amplitude or a low range, and we add 1 to the bin that corresponds to that low stress range. And we have to go through the entire signal and add each of the stress variations to one of these bars in the histogram.



To determine which part of the load variation in the time series corresponds to which bin in the stress histogram, a commonly used method is the rainflow counting method. Here, and on the next slide, we'll show the principles of how it works just so you get an idea of how it's done. On the left-hand side, you see a schematic version of the stress time series, where we have only kept the extremes, and we've drawn straight lines between these extremes. In the rainflow counting method, we rotate this diagram over 90 degrees, and we interpret the result as if it is a drawing of a roof. In each of the extremes we release a little bit of water and we let the water drip down the roof.



Let's follow the flow of water that has been released in point number 1. It flows down the roof to point number 2, where it falls and it hits the next level of the roof, where it continues flowing in the direction of point number 4. Here it falls again. If we follow the curve further down it moves to the left, and at some point, it moves farther to the left than point number 1, where the water has been released. This is the point where we stop the flow of the water that has been released in point number 1, and this completes the cycle of the stress variation that started in point number 1. So, this would correspond to a stress variation between the stress level at point number 1 and the stress level at point number 4 or number 6. We can do this for the entire diagram, and effectively we then decompose the continuous variation into individual variations at different stress levels, and these individual variations can then be added to the histogram.

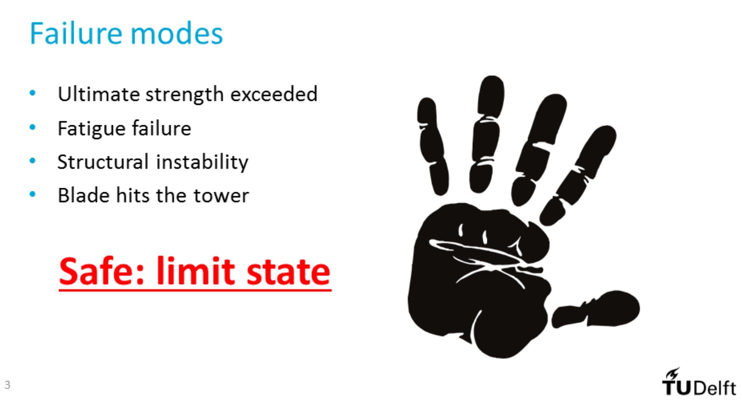
6.2 Principles of structural analysis

6.2.1 Why perform structural analysis

Before diving into the details of structural analysis, we ask ourselves in this video why we would need to perform such analysis at all.

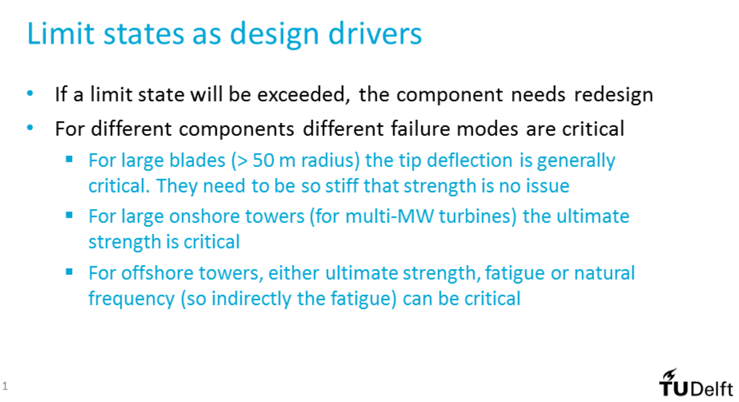
Why we want to perform a structural analysis is most clearly illustrated by pictures of turbines that have failed. On the left-hand side, you see a wind turbine that failed due to structural instability, in this case, buckling of the tower. On the right-hand side is another example of structural instability, but in this case, overturning of the foundation. It is clear that we don’t want this to happen. We therefore want to assess in advance under which conditions these kinds of failures can be expected, so that we can avoid them.

The rotor in this video is rotating at a much too high speed. This is only possible if the generator has been disconnected from the public grid and not applying a torque on the drive train. This could have been caused by a fault in the turbine or by an inadequate response to a fault in the grid. At this speed the brakes no longer have sufficient capacity to stop the rotor. As you can see, the blades can sustain these conditions despite the high speed. However, at some point the aerodynamic forces let the blades bend so much that one of them hits the tower. The tower is no able to endure this impact and the entire structure fails.

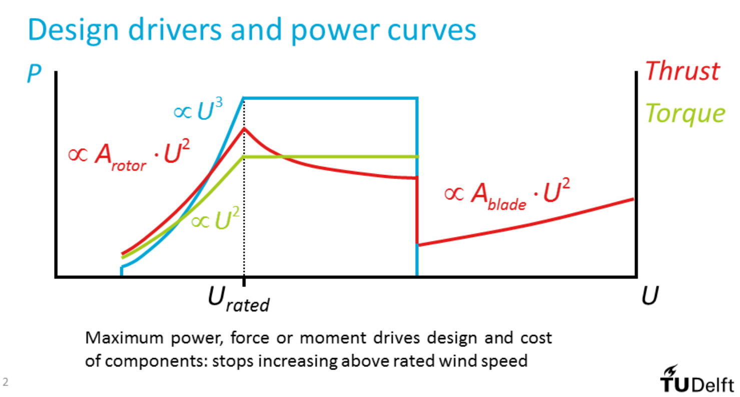


Looking at failures of wind turbines and other structures in the past, it has been observed that there are four failure modes that are relevant for wind turbines. The four conditions that can lead to failure are: the ultimate strength is exceeded, fatigue leads to failure, the structure becomes unstable, or as we have seen in the previous video, the blade hits the tower. The meaning of these four failure modes will be explained later, when we address how each of them is analysed. Because we want to avoid such failures, designers and standards have introduced what is called the limit state. The limit state is a condition of the turbine in which it is still safely away from these failures. In other words, it’s a state of the structure beyond which it no longer has a significant margin until the failure mode is reached. Before addressing the failure modes and limit states, we will therefore first address how the state of a wind turbine can be assessed.

6.2.2 Structural analysis and design



Now you know that the wind turbine will need to be assessed on various failure modes, to establish whether or not the limit states will be exceeded during the lifetime of the turbine. However, it is clear that we will not be done when we find out that the limit state will indeed be exceeded. In that case, we'll have to redesign the wind turbine. The limit states that dominate the design of a component are called the design drivers. Not all components have the same design drivers. Obviously, the tip deflection can only be a design driver for the blades. Indeed, for large blades the tip deflection is most critical. When the limit state for tip deflection is not exceeded, most parts of the inboard section of the blade will be strong enough to withstand ultimate loading and fatigue. A large part of the tower of an onshore wind turbine is designed to make sure that the ultimate strength is not exceeded during its lifetime, but at the same time the local buckling needs to be taken into account. However, for an offshore wind turbine the fatigue limit state may also be a design driver for the tower. This is a consequence of the hydrodynamic loading, which leads to extra vibrations. In the end, we want to be sure that all components stay below all limit states. However, during the design process we can focus per component on the design drivers, which become apparent after the initial structural assessments.



Without knowing yet how structural analysis is performed, the role that the design drivers play in the design of components answers a question that we promised to address a long time ago. Here you see the schematic power curve with which you are by now familiar. In the partial load region we maximise the power conversion, but in the full load region we keep the power constant. As a consequence, we are losing energy that could have been converted if we kept on operating near the Betz limit. So why do power curves have this shape? From the aerodynamics lecture, we know that in the region below rated wind speed, the power that is available and therefore the power that we convert is proportional to the wind speed to the power of three. Why do we not continue this increase with wind speed to the power of three beyond rated wind speed? We also know from the aerodynamics lecture, that the thrust force is proportional to the wind speed squared and to the area of the rotor. So, what happens beyond rated wind speed if we keep the power constant? In that case, the thrust will decrease with increasing wind speed. As you can also see from the aerodynamics lecture, the power that is converted through the rotor is the wind at the rotor, multiplied with the thrust force. So, if the wind at the rotor increases, we have to decrease the thrust force to keep the power constant. Beyond the cut-out wind speed the thrust force, or in this case it's a drag force, is still proportional to the wind speed to the power of two, but now it's proportional to the area of the blade, instead of the area of the rotor, because the rotor is not turning anymore. Effectively, what we're missing here is that the lift force created by a rotating rotor by its own motion, by the omega r, is dropped out. So, what happens to the torque in the drive train? Below rated wind speed, the torque in the drive train is proportional to the wind speed squared, as we'll see later. Above rated wind speed the torque is constant. Because the power is constant and also the rotational speed is constant, the relation between power and rotational speed and torque explains that the torque is constant, because power is equal to rotational speed times torque. So, what we see in the region beyond rated wind speed, is that the power and the torque remain constant and that the thrust force decreases. This means that the design of components based on the power, for instance the power electronic converter, is dimensioned for the conditions at rated wind speed. The drivetrain, for instance the generator and the gearbox, are dimensioned for the torque. And the structural components, such as the tower and the bed plate, are designed for the loads, so for the thrust. And as we can see, these no longer increase beyond rated wind speed, and therefore they are designed for the conditions at rated wind speed rather than at conditions at higher wind speeds. And this is beneficial, because otherwise we have to pay more for the generators and the structures and the drivetrain, just to get a little bit extra energy. What we lose in terms of energy in this region, is actually fairly limited.

6.2.3 Approaches for lifetime assessment

Approaches for limit state analysis

There are several design standards that describe the limit state analysis of wind turbines. An example is the IEC 61400-1 Ed. 3 (2005) *Wind turbines – Part 1: Design requirements*. The IEC standard defines classes, for which the wind conditions are specified and for which the limit states of the turbine can be checked. There are three categories for annual average wind speed (I, II and III) and three categories for turbulence intensity (A, B and C). These categories are independent, so a turbine can, for instance, be certified for class IIA.

This certification approach is based on the expectation that the rotor-nacelle assembly will not exceed the limit states at locations with similar or milder wind conditions than those specified by the class. When a turbine is selected for a project, the developer must demonstrate that the site-specific conditions are indeed mild enough.

There are various approaches to perform the limit state analysis. DNV-GL standards distinguish:

- design by partial safety factor method
- design assisted by testing
- probability-based design

Partial safety factor method

The partial safety factor method applies a safety factor to the loads (external forces and dynamic response), a safety factor to the resistance (material properties and geometry) and a safety factor to account for the severity of failure. The analysis of the loads and resistance is performed for specified conditions and the consequential results are considered to be representative. This is therefore called a deterministic approach.

The magnitude of the safety factor on loading depends on the expected uncertainty and whether the loading is favourable or unfavourable. E.g. the safety on gravity loading is lower than on aerodynamic loading, because it can be determined with high precision.

Failure of the support structure has large consequences, as other parts of the wind turbine may be damaged or there may even be total loss. Therefore, the safety factor for severity is high for support structure limit state analysis. If a failure only leads to limited component damage and downtime, a lower safety factor is used.

Design assisted by testing

For some phenomena the accuracy of model predictions is low, requiring much conservatism in the design (high safety factors). In such cases tests on a scale model or on a full scale prototype may be used to calibrate models or to demonstrate that design requirements are satisfied.

Probabilistic analysis

The starting point for a probabilistic approach is the notion that there is a probability distribution for the loading and for the resistance. The aim of the approach is to estimate these distributions and to obtain an acceptable probability for the realisations in which the loads exceed the resistance. This is visualised in Figure 1. The full analysis of the overlap of the two probability distributions may be cumbersome and imprecise. A practical implementation of the approach is to determine the loading for which the change that this load will be exceeded during the lifetime of the turbine is below a predefined (low) value.

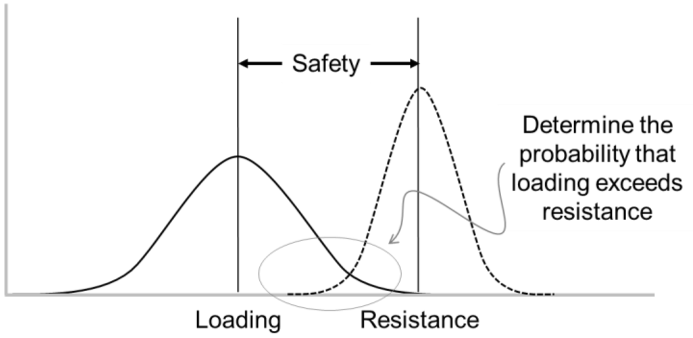


Figure 1 Illustration of the principle of probabilistic analysis

Representation of lifetime loading by load cases

Wind turbines are designed for a lifetime of 20 years or more. The analyses of the limit states has to be representative for the conditions that can occur during this entire lifetime. Figure 2 illustrates the response of the wind turbine to these external conditions to identify several issues for this analysis.

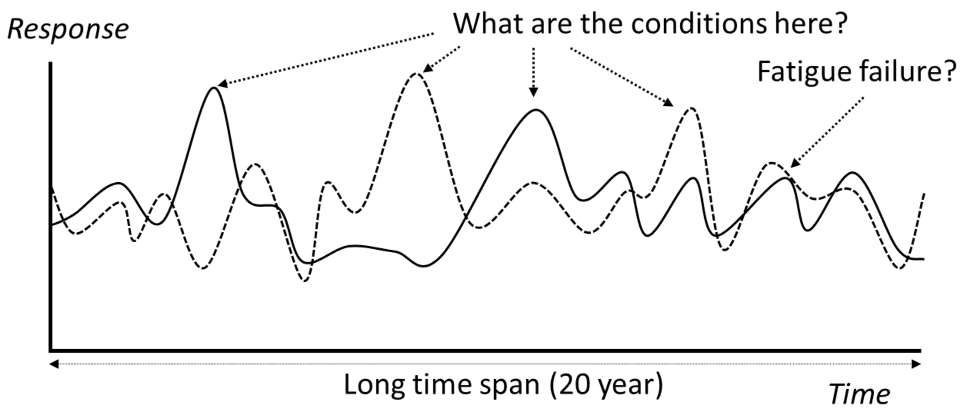


Figure 2 Possible responses of a wind turbine during a lifetime of variable external conditions

The issues that are illustrated by Figure 2 are:

- Simulation of the entire long lifespan would require much computational resources
- External conditions are stochastic and many realisations are possible
- The response is a function of external conditions, operation and dynamics. Therefore, it is unknown what the conditions are that lead to the largest response
- For fatigue assessment the load variation over the entire lifespan needs to be represented, including conditions with lower response

To deal with these issues, load analysis is performed with load cases. A load case is a combination of external conditions and operational conditions of the turbine that specify a situation that can occur during the lifetime of the turbine. Instead of trying to simulate the lifetime of the turbine sequentially, we analyse a set of load cases. The types of conditions that can be specified by a load case are shown in Figure 3.

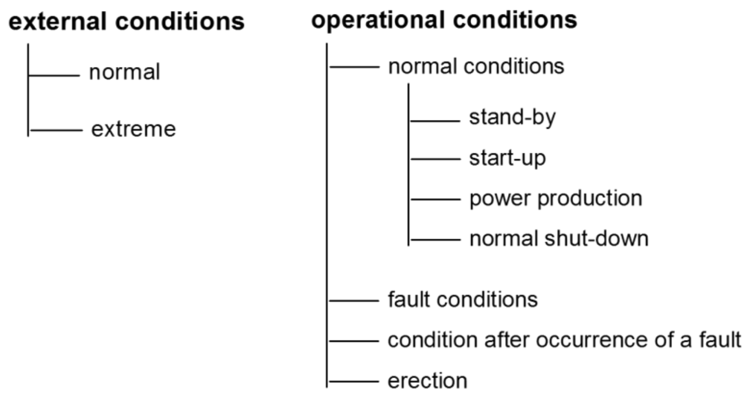


Figure 3 Types of external and operational conditions that can be specified by load cases

If we would analyse all possible load cases, the design would become unnecessarily conservative. The analysis can be restricted to load cases with a reasonable likelihood of occurrence. For instance, it is not likely that a 50-year extreme wave height and a 50-year extreme gust coincide, let alone that this would happen at the exact moment of failure of the pitch system.

The standards prescribe which load cases should be considered. As an example, Figure 4 shows the table with load cases defined by in IEC 61400-1. The rationale used by the developers of the standards is that correlated events are likely to occur simultaneously, while uncorrelated events are unlikely to occur simultaneously. As an example for an offshore wind turbine, the significant wave height in a sea state and the average wind speed are correlated phenomena, as waves are (partly) generated by the wind. Therefore, some load cases consider the simultaneous occurrence of a high significant wave height and a high average wind speed. However, the height of individual waves and individual gusts in that sea state is not correlated. Therefore, these two extreme external conditions are not combined in one load case. This example cannot be seen in Figure 4, since that table is for onshore turbines.

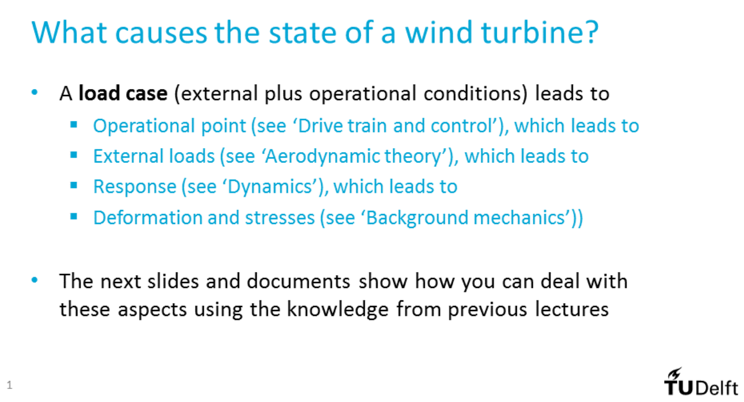
Design situation	DLC	Wind condition	Other conditions	Type of analysis	Partial safety factors
1) Power production	1.1	NTM $V_{in} < V_{hub} < V_{out}$	For extrapolation of extreme events	U	N
	1.2	NTM $V_{in} < V_{hub} < V_{out}$		F	*
	1.3	ETM $V_{in} < V_{hub} < V_{out}$		U	N
	1.4	ECD $V_{hub} = V_r - 2$ m/s, V_r, V_r +2 m/s		U	N
	1.5	EWS $V_{in} < V_{hub} < V_{out}$		U	N
2) Power production plus occurrence of fault	2.1	NTM $V_{in} < V_{hub} < V_{out}$	Control system fault or loss of electrical network	U	N
	2.2	NTM $V_{in} < V_{hub} < V_{out}$	Protection system or preceding internal electrical fault	U	A
	2.3	EOG $V_{hub} = V_r \pm 2$ m/s and V_{out}	External or internal electrical fault including loss of electrical network	U	A
	2.4	NTM $V_{in} < V_{hub} < V_{out}$	Control, protection, or electrical system faults including loss of electrical network	F	*
3) Start up	3.1	NWP $V_{in} < V_{hub} < V_{out}$		F	*
	3.2	EOG $V_{hub} = V_{in}$, $V_r \pm 2$ m/s and V_{out}		U	N
	3.3	EDC $V_{hub} = V_{in}$, $V_r \pm 2$ m/s and V_{out}		U	N
4) Normal shut down	4.1	NWP $V_{in} < V_{hub} < V_{out}$		F	*
	4.2	EOG $V_{hub} = V_r \pm 2$ m/s and V_{out}		U	N
5) Emergency shut down	5.1	NTM $V_{hub} = V_r \pm 2$ m/s and V_{out}		U	N
6) Parked (standing still or idling)	6.1	EWM 50-year recurrence period		U	N
	6.2	EWM 50-year recurrence period	Loss of electrical network connection	U	A
	6.3	EWM 1-year recurrence period	Extreme yaw misalignment	U	N
	6.4	NTM $V_{hub} < 0,7\ V_{ref}$		F	*
7) Parked and fault conditions	7.1	EWM 1-year recurrence period		U	A
8) Transport, assembly, maintenance and repair	8.1	NTM V_{maint} to be stated by the manufacturer		U	T
	8.2	EWM 1-year recurrence period		U	A

Figure 4 design load cases specified in IEC 61400-1 ed. 3

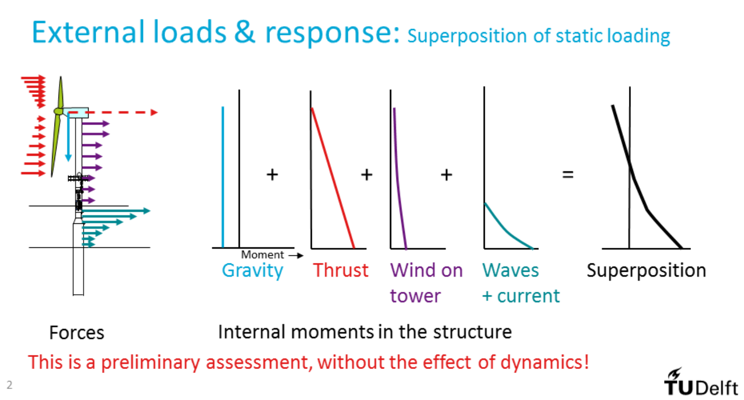
For the analysis of ultimate strength all load cases are considered either separately (in the partial safety factor method) or in a statistical analysis (in the probabilistic approach). For the analysis of fatigue, you need to determine a probability of occurrence for each load case. With this probability you can determine how often the load case re-appears over the lifetime of the turbine. This is used to determine the cumulative damage of variable loading.

6.3 The state of the wind turbine

6.3.1 The loading situation



For the specified external and operational conditions for a particular load case, we can determine what the operational point is of the wind turbine. For this, we can use our knowledge from drivetrain and control. If we know the operational point, then we can determine the external loads. Particularly, the loads on the rotor which we have seen in aerodynamic theory. Once you have the external loads, we can determine the response, for which we also need to include the dynamic effect that we have seen in dynamics. And finally, if we have the response, we can determine the deformation, so the tip deflection, and the stresses inside the structure. How we can do this is dealt with in the next slides.



Let's first have a look at the loads. On the left hand side, you see a support structure and there we will look at the different forces that are applied to it. On the right hand side, we are going to look at the effect of these forces on the internal moments in the tower. So, let's first take the gravity loading on the rotor nacelle assembly. The gravity loading is applied at the centre of gravity, which is usually a bit offset from the axis of the tower, and it's typically also a bit upfront of it. If we define counter clockwise moments to be negative, we can plot the gravity moment as a function of the height along the tower, as a constant negative value. The effect of the thrust force or the aerodynamic loading on the blades is a constantly increasing moment on the tower. At the tower top, where the thrust force is applied, it doesn't create a moment in the tower, but the further down we go, the larger the lever arm of this force, so at the tower base we have a very large moment caused by the thrust. Similarly, for the aerodynamic drag on the tower. The aerodynamic drag on the tower is usually smaller than the thrust force, and therefore its moment is also smaller, but it increases more than linearly, because it is a distributed force. The further down we go, the bigger the lever arm for the higher drag loads, but there's also additional drag loads on the lower sections of the tower. The same applies for hydrodynamic loading in case we have a support structure in the sea, so, in case we have an offshore wind turbine. In a static analysis, we use the principle of superposition and simply add the moments created by these different forces, and that will lead us to the total moment distribution over the height of the tower. Be aware that in this case, we do not include the dynamic effects, so this is a purely static analysis.

Preliminary static load analysis

- Gravity
  - $F = m \cdot g$
- Rotor aerodynamics
  - Find operational point and use BEM (complex due to iterations)
  - Simplified (operational): use $c_T \approx 8/9$ (Betz) at U_{rated}
- Wind on nacelle and tower
  - Simplified: use c_D of nacelle and cylinder
- Submerged tower hydrodynamics
  - Not treated here

3 TUDelft

The determination of the gravity force and consequentially the moment created by it is very straightforward, as the gravity force is equal to the mass times the gravity acceleration. To determine the rotor aerodynamics or the thrust force, we could use blade element momentum theory but this is rather complex for manual calculations, because the induction factor depends on the thrust force, and the thrust force depends on the induction factor. Therefore, we would need to solve this iteratively. We can also simplify the assessment by assuming that we are operating at the Betz limit, up to the rated wind speed. We've seen before that the thrust force, if we analyse it statically, is largest at the rated wind speed. So, we could use a thrust coefficient of 8 over 9 which corresponds with the Betz limit, and apply it at rated wind speed to get an indication of the maximum possible thrust. For the assessment of the wind loading on the nacelle and the tower, we can use the drag coefficient of the nacelle and of the cylindrical tower. This will be further shown on the next slide. The forces due to hydrodynamics on the structure are not treated here.

Using c_D

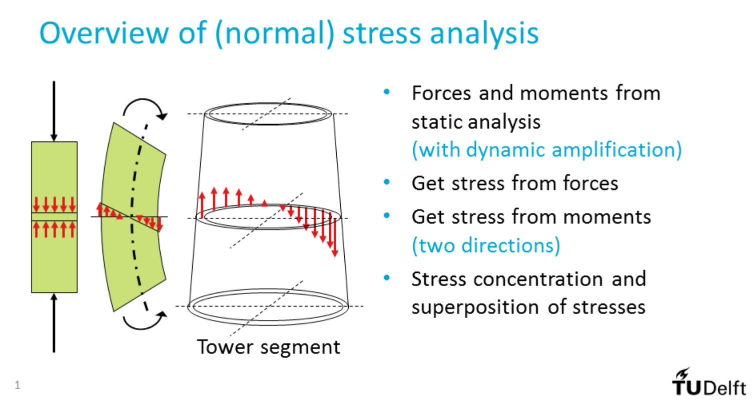
$$dF = \tfrac{1}{2}\rho c_D U_{local}^2 dA$$

	c_D	U_{local}	dA
Nacelle	~ 1.2	From wind shear model	Frontal area
Tower	Drag coefficient of a cylinder ~ 1.2	From wind shear model	Diameter times the segment length

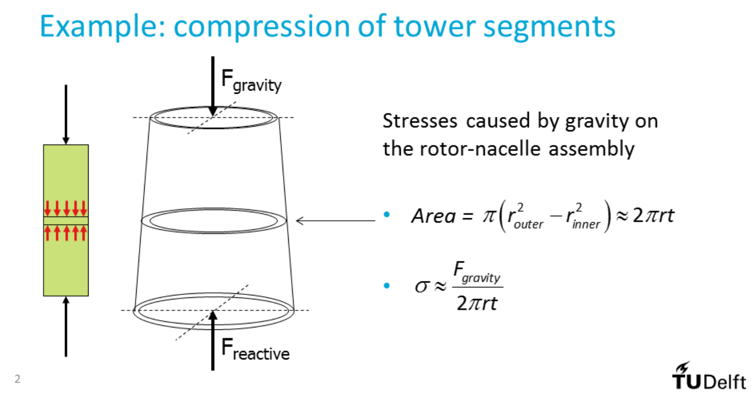
4 TUDelft

The equation that we use to determine the drag force on the nacelle and the tower, is very similar to the equation that we use to determine the thrust force. It is 1/2 times the density of air, times the drag coefficient, times the local wind speed squared, times the area. So, the difference with the determination of the thrust force, is that we use a drag coefficient here, instead of a thrust coefficient. The drag coefficient for both the nacelle and the tower can be approximated by 1.2. The exact value of the drag coefficient depends for instance on the shape of the nacelle, or on the Reynolds number, but for a simple calculation we can assume it to be 1.2. The local wind speed can be determined from a wind shear model, because the wind speed depends on the height. The frontal area is the relevant area for the nacelle, and for the tower the relevant area is the diameter times the segment length. We need to divide the tower in separate segments, because the wind speed is a function of height.

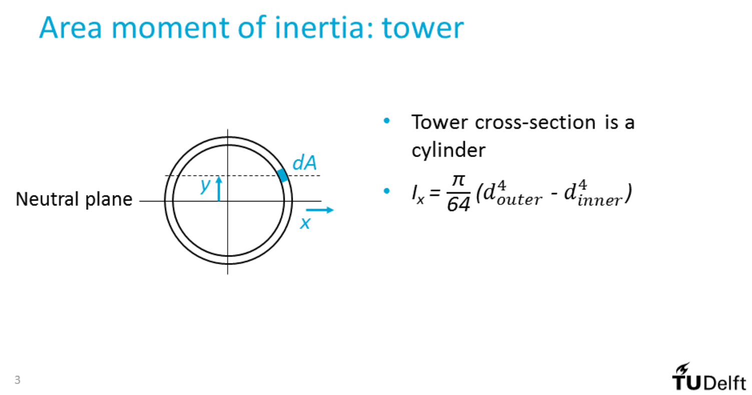
6.3.2 Determination of stresses



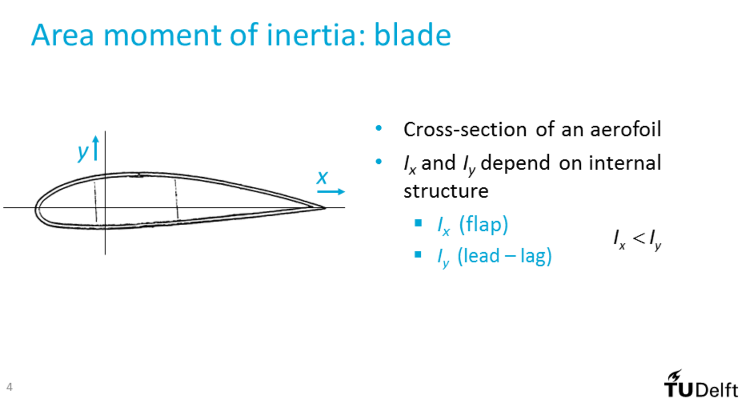
Having obtained the internal forces and moments from the preliminary load analysis, the next step is to determine the stresses. We will focus on the normal stress, which is perpendicular to the cross-section and can be visualised by tearing the two sides of the cross-section apart. In our preliminary analysis, we will neglect the shear stress. This stress is parallel to the cross-section and can be visualised by sliding between the two sides of the cross-section. In blades and towers of wind turbines the stresses are dominated in most places by the bending moments and therefore the normal stress is much larger than the shear stress. We will take three steps in the determination of the stresses. First, we'll determine the stress from the tensile or compressive forces, which are aligned with the tower or blade longitudinal axis. Then we'll determine the stresses from bending. Because the bending moment can have two components, in two perpendicular directions, we may have to do this twice. Finally, we'll add the stresses from the different load origins. In this step we will also consider whether there is stress concentration. What this means will be explained when we get there.



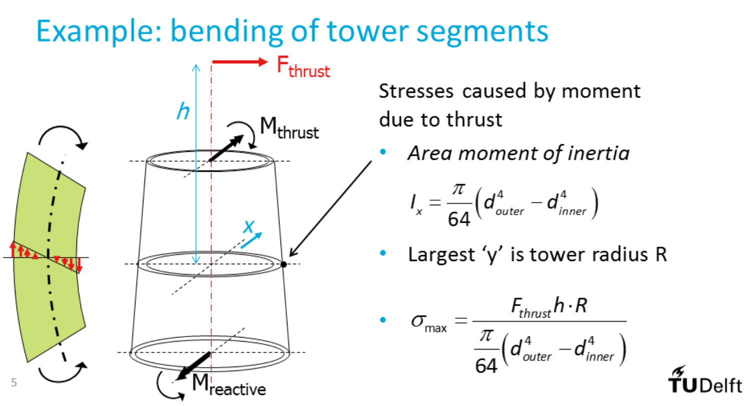
As an example, we're going to consider the compression of a tower segment. On the far left, you see the element that we used to derive the relation between stress and force, and compare that with the tower segment. At the top of the tower segment, you have a gravity force, and if we ignore the weight of the segment itself, then we have an equal reactive force at the base of this segment. The area of the cross section is approximately two times pi times the radius of the segment times the thickness of the segment. So, we can determine the stress in this case, as being approximately equal to the gravity force divided by this area.



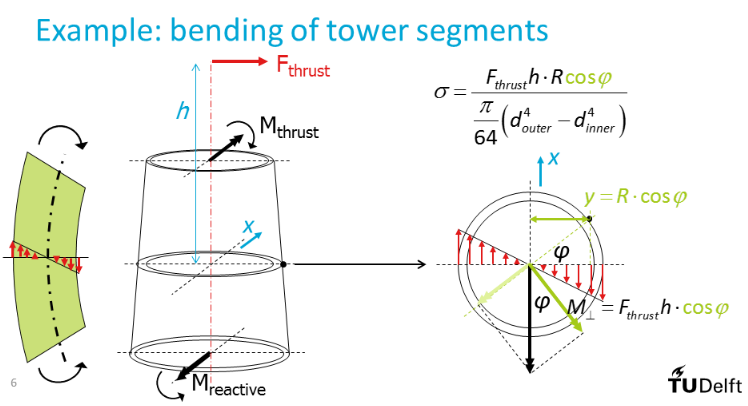
For the analysis of stresses caused by moments, we will need the area moment of inertia. For many geometries of cross-sectional areas, the area-moment of inertia can be found in literature, for instance in handbooks. For the tower, we have a circular cross-section and for that the area-moment of inertia is as given in the expression shown here.



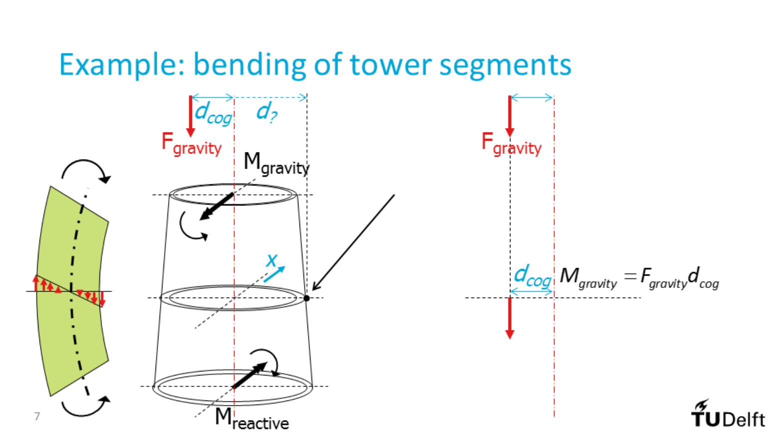
For the area moment of inertia of the blade, we cannot give a straightforward analytical expression, because it depends on the aerofoil shape and also on the geometry of the spar caps and the shear webs inside the structure. However, we can state that the area moment of inertia around the x-axis, so for flapping, is typically smaller than the area moment of inertia around the y-axis for lead-lag. You can see this from the drawing, because if you look at the areas dA that contribute to the area moment of inertia, you see that these areas are typically much closer to the x-axis, than they are to the y-axis.



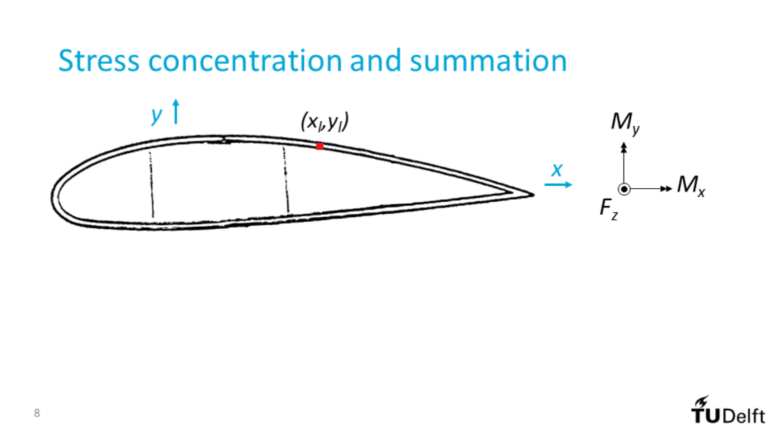
As an example of the stress caused by bending, we'll have a look at the tower segment again and we'll have a look at the moment created by the thrust force. On the far left, we have our beam element again that we used to derive this stress as a function of the external moment. The external moment in this case, is the thrust force times the lever arm h. We are interested in the cross-section of the tower, which is circular and has an area moment of inertia as given here. The neutral plane for bending is spanned up by the tower centreline and the dashed line that you see in the middle of the tower segment. The largest distance to this neutral line or to this neutral plane is equal to the tower radius R. So on the right-hand side, where you see the arrow pointing from the bullet 'Area moment of inertia to the cross-section', that is our point with the largest y, which is equal to the tower radius. Therefore, this is the point where we have the largest stress and this stress can be derived by substituting the moment, substituting the tower radius and substituting the area moment of inertia.



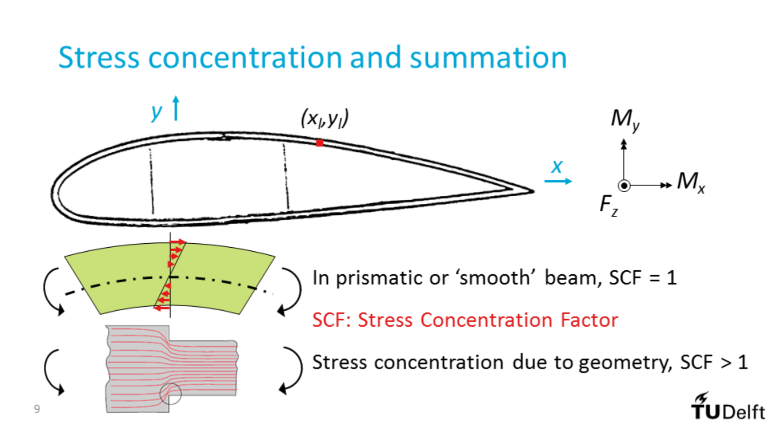
We might not only be interested in the maximum stress, but also in stresses at other points in the cross-section. The vertical plane through the x-axis of this figure is the neutral plane, so there the normal stress due to bending is zero. The stress increases linearly with the distance to the neutral plane. This distance is equal to the tower radius time the cosine of phi. We can also look at this differently, by decomposing the bending moment in two directions: one parallel to the line through the point that we're interested in and one perpendicular to that. For the component parallel to the connecting line, the point of interest is on the neutral plane and it therefore experiences no stress from this component. For the component perpendicular to the connecting line it experiences the maximum stress. This component is the moment times cosine phi, so with either way of looking at it the cosine of phi appears in the expression for the stress.



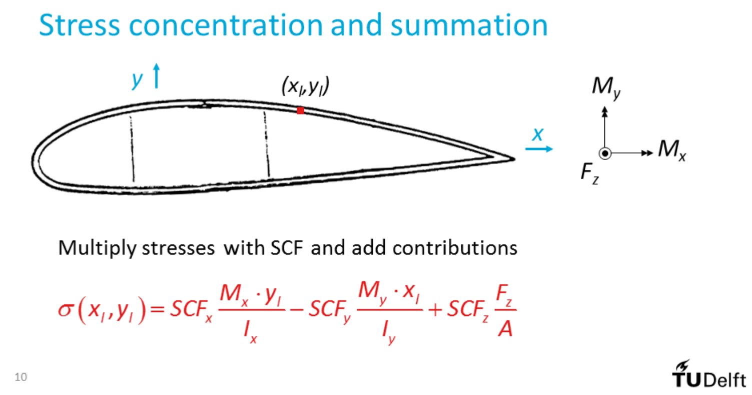
Sometimes there appears to be confusion about the determination of stresses due to the gravity force. The confusion is caused by the dimensions of the tower. To determine the moment caused by the gravity force, you should not take the lever arm to the point in the cross-section. The determination of the moments in cross-sections is based on the assumption that the beam is infinitesimally small. Therefore, think of the tower as only a centreline in this part of the process. This is shown on the right-hand side. Then it is clear that the moment caused by gravity is the gravity force times the distance of the centre of gravity to the tower centreline.



In the previous examples we've only looked at a single force or a single moment, but a cross-section can experience moments in two directions, in addition to the force, and they can appear simultaneously. So, here we are going to address how to sum the stresses caused by the different moments and the force, and also we're going to address the effect of stress concentration.



Let's first have a look at stress concentration. A prismatic beam is a beam in which the cross-section is the same everywhere along the beam. In this case, we have a smooth load path and there is no stress concentration, and the only variation of the stress through the cross section of the beam, is because of the distance to the neutral axis. In this case, the stress concentration factor is equal to 1, meaning that there is no concentration. However, if we have a beam with, for instance, a certain drop in the cross-sectional area, then you can see that the load paths have to go from the outside of the beam, in the case of a wide cross sectional area, to the inside of the beam in the case of a smaller cross sectional area. This leads to stress concentration in the corners, and that means that we have a stress concentration factor larger than 1.



Now we're going to combine the effect of stress concentration and the summation of stresses due to different moments and forces. First the effect of stress concentration. We simply multiply the stress that we calculate, with the stress concentration factor. The stress that we calculate is M times y divided by the area moment of inertia, and we use simple superposition to add the stresses due to different moments and forces. However, we have to be careful with the signs during this addition. All tensile forces add, but a tensile and a compressive force have to be subtracted. In this example, you see a point of interest (xl, yl) and it's at a certain distance from the y-axis, the distance yl, and it's at a certain distance from the x-axis, with a distance xl. If we calculate the stress due to the moment Mx, it will be Mx times the distance yl, divided by the area moment of inertia about the x-axis. In this case, if the moment Mx is positive, then we have a tensile force in the point of interest and we'll use a positive sign for that as well. So, the first term is a positive term. Now, we are going to add the stress created by the moment My. We have to multiply the moment My with the distance xl, and divide it by the area moment of inertia about the y-axis, but in this case if the moment My is positive, then we have a compressive stress in the point of interest, and we'll use a negative sign for that. So, we have a tensile stress due to the moment Mx, and we have a compressive stress due to the moment My, which has to be subtracted from that. Finally, we add the effect of the force FZ, and in this case if FZ is positive when it is pointing towards you, outside out of the screen, then we have a tensile force in the point of interest, so again we use a positive sign for FZ divided by A.

6.3.3 Determination of tip deflections

The deflection *y* as a function of radial position *r* is obtained by integration of the gradient of the deflection:

$$y(r) = \int_0^r \frac{dy}{dx} dx + c_1 .$$

The boundary condition *y* = 0 at *r* = 0 leads to c_1 = 0. The gradient of the deflection is not directly known, but it is known that it depends on the curvature along the blade, which in turn depends on the moment, which in turn depends on the force distribution over the blade. Therefore, before looking for a solution to find the tip deflection, Figure 1 shows that we will use different symbols for the locations of these three ingredients along the blade.

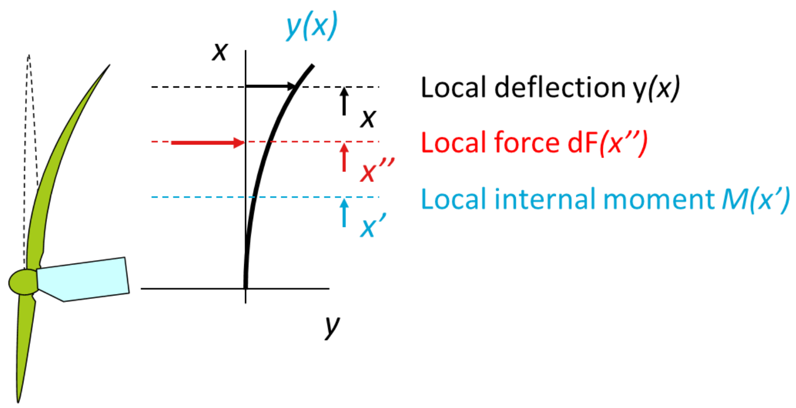


Figure 1 Schematic representation of the deflected blade, defining the coordinates x, x' and x''.

The gradient of y as a function of x is obtained by integration of the curvature:

$$\frac{dy}{dx} = \int_0^x \frac{d^2 y}{dx'^2} dx' + c_2 .$$

The boundary condition $dy/dx = 0$ at $x = 0$ leads to $c_2 = 0$. The coordinate x' is used, because we can express the curvature in the internal moment $M(x')$ through

$$\frac{d^2y}{dx'^2} = \frac{M(x')}{EI(x')}.$$

The modulus of elasticity, E, and the area moment of inertia, I, at position x' are properties of the blade that are known. The internal moment can be determined from a static analysis, by integration of the contributions of the distributed force. Figure 2 illustrates the lever arm of this contribution Fdx'' at point x'' to the point x' in the blade, where the internal moment has to balance the moment of the external force.

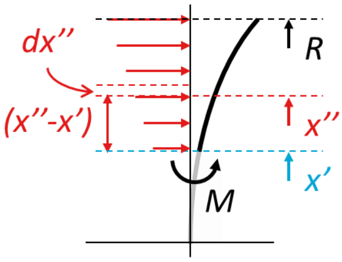


Figure 2 Schematic representation of the blade to determine the internal moment

Using the scheme of Figure 2, it can be seen that the internal moment can be determined from:

$$M(x') = \int_{x'}^{R} F(x'')(x''-x')\,dx''.$$

The last step is to substitute the expression for the moment in the expression for the curvature, substitute this result in the expression for the gradient of the deflection and finally substitute this result in the expression for the deflection. This gives the triple integral:

$$y(r) = \int_0^r \int_0^x \int_{x'}^R \frac{F(x'')(x''-x')}{EI(x')}\,dx''dx'dx.$$

The tip deflection is the evaluation of this equation for $r = R$.

6.3.4 Dynamics in preliminary structural analysis

Dynamic response

The effect of dynamics on the tip displacement, stresses and fatigue is usually determined through simulations. In a simulation the equations of motion are integrated, using the varying external conditions as inputs. In a preliminary analysis, the effect of dynamics can be incorporated through the notion of dynamic amplification.

Figure 1 repeats the single degree of freedom system that was used to explain the principles of dynamics. Under variation in load F_{ext} the response in position x may experience amplification or reduction compared to the response to a static external force. The internal force in the spring, F_{int}, is linearly proportional to the displacement, as expressed by the equation $F_{int} = k \cdot x$. This implies, that the internal forces and stresses experience an amplification or reduction equal to that of the displacements.

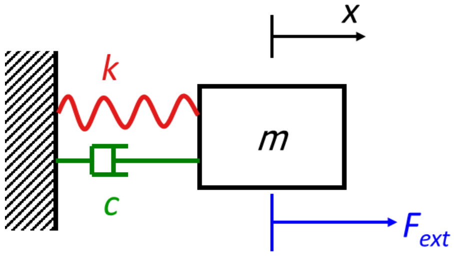


Figure 1 Single degree of freedom system

For a single degree of freedom system and a harmonic variable loading at a single frequency the effect of dynamics can be easily determined. In this case we can multiply the displacement or stress from the static analysis with the DAF at that frequency to get the dynamic response. However, for a wind turbine there are several aspects that complicate the analysis of the effect of dynamics.

First, the loading is not purely harmonic and contains a broad band of excitation frequencies. The amplifications at different frequencies differ and it is not possible to determine an effective amplification by taking e.g. the mean or the maximum amplification. Second, the dynamic amplification around resonance is concentrated in the mode shape that is associated with the corresponding natural frequency. The mode shape determines which parts of the structure get the largest deflection or bending (and thus stress). Since the mode shapes differ at different resonance frequencies, different parts of the structure experience the corresponding dynamic amplification. Furthermore, these mode shapes differ from the deflection shape of the static analysis, making the dynamic amplification factors of the mode shapes not entirely representative for the amplification of the static results.

The practical approach to dynamic amplification is to compare simulation results to results under static loading. Typically, the extreme dynamic response is found to be between around 1 and 2 times as large as the static response. Therefore, a reasonable dynamic amplification factor for random loading is between 1 and 2.

Role of Campbell diagram in limit state analysis

As you have seen, the limit states that need to be assessed for certification of a wind turbine are related to four failure types: the ultimate strength is exceeded, failure due to fatigue, structural instability or the blade hits the tower. None of these assessments directly refers to conditions regarding the excitation or natural frequencies. So what role does the analysis of the Campbell diagram have in the structural analysis?

Resonance, which occurs when the excitation frequency is close to the natural frequency, is not a failure mode. It doesn't immediately compromise the safety or stability of the structure. However, it leads to dynamic amplification and to many, large variations in the stress.

The dynamic amplification may lead to extreme stresses or to extreme tip deflections. However, such extremes typically occur during transient conditions. Examples of transient conditions are a gust, a sudden change in wind direction or a short circuit in the grid. Resonance and large dynamic amplification relate to harmonic excitations that persist for some time. It takes some time to build up the large response. This persistence of harmonic loading is less present in transient conditions and therefore resonance is usually not a dominant consideration in the analysis of ultimate strength or tip deflection.

The many and large variations in the stress during resonance contribute significantly to the fatigue damage. If resonance occurs regularly in the structure, this would become apparent in the analysis of the fatigue limit state. However, the fatigue analysis is time consuming and we know in advance that it would be better to avoid resonance. The Campbell diagram can be used in a preliminary analysis as an indicator of potentially large fatigue damage. Based on the outcome of the analysis, a partial redesign of the structure or operation of the turbine can be considered to avoid large fatigue damage, depending on the expected severity of the dynamic response.

Figure 2 shows an example of a Campbell diagram. It identifies possible resonance of the first flap mode. Several aspects can be used to judge the severity of the response, by answering several questions. E.g. does resonance occur at low wind speeds or high wind speeds (which can be judged from the position in the operational range)? How important is the excitation frequency, considering cancellation and energy content of higher harmonics? How often are the conditions expected to occur during the lifetime of the turbine? In this example, resonance occurs at high rotational speeds, so at high loading conditions near and above rated wind speed. 2P excitations are relevant for blade resonance and leads to fatigue damage in the blade structure. Because resonance would occur for all operational wind speeds above rated conditions, this would occur very often during the lifetime. In this case a redesign would be preferable, to avoid this coincidence of the natural frequency and 2P excitation frequency.

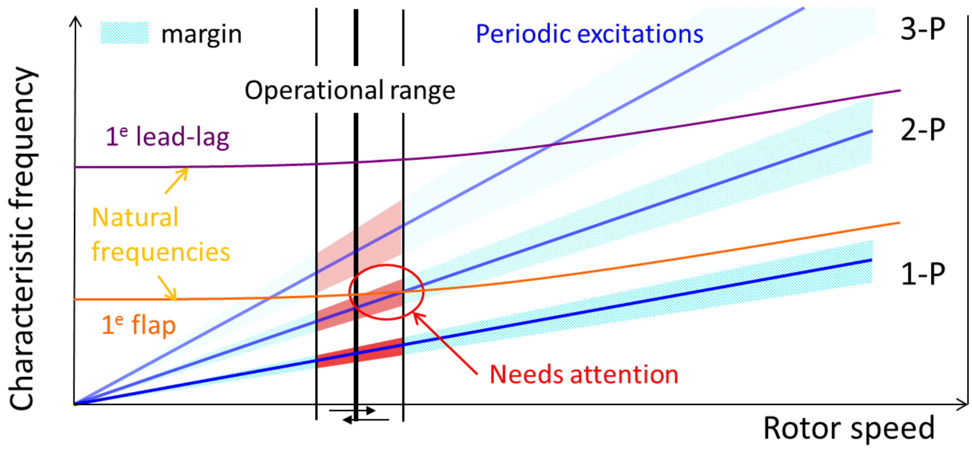


Figure 2 Example of Campbell diagram, with identification of point of attention

There are several changes in the design or operation that can be considered to avoid resonance. The basis is to either change the natural frequency, to change the occurrence of the excitation frequency or to change at which wind conditions resonance occurs. Here are several options:

- Change the mass that is in motion in the vibration
- Change the stiffness of the structure
- Reduce the operational speed range
- Change the design tip speed ratio, to shift the operational speed range
- Avoid the rotor speed that causes resonance by 'skipping' it, using the partial load controller

Different options have different consequences on the cost, energy yield and/or impact on the design process. E.g. changing the design tip speed ratio will affect many parts of the design and will therefore be unfavourable at a late stage in the development.

When the severity of the resonance is expected to be low or when a redesign is very unfavourable, it can be considered to allow resonance to happen. In this case the Campbell diagram indicates where in the fatigue assessment extra attention is due: in which load cases, at which wind speeds, in which components, etc. The extra attention entails for instance:

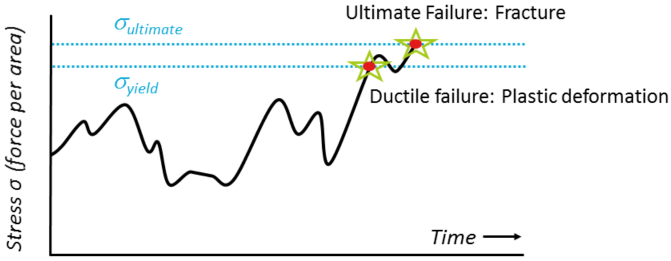
- more precise determination of the damping coefficient, for instance with experiments
- sensitivity analysis for several uncertain factors, such as damping and stiffness
- more realisations for simulations of load cases with resonance
- assessment in more points in the structure

6.4 Ultimate strength

6.4.1 Principles of ultimate strength assessment

In this video we will address the failure mode in which the ultimate strength is exceeded.

What is the ultimate strength failure mode?



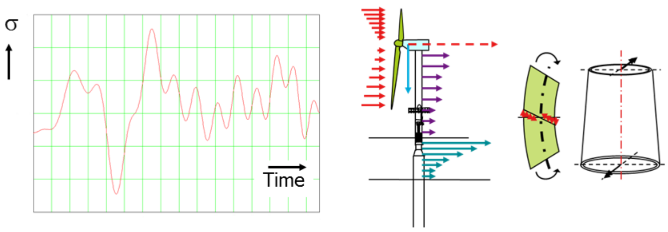
The first question is, what is this failure mode? Here you see a graph of the variation of the stress over time. For a wind turbine this variation is stochastic. If the stress exceeds the tensile stress, we have ductile failure, which means that we have plastic deformation. Unlike elastic deformation that we have at lower stresses, plastic deformation is irrecoverable. Therefore, the structure will be changed, which is not acceptable. If the stress further increases, we may reach the ultimate stress, and we will have ultimate failure or fracture. Some materials, such as glass fibre reinforced plastic, don't have a ductile region, but immediately reach the ultimate stress. These materials are called brittle materials. The two ingredients that are needed for the analysis of the limit state are the stress at failure and the obtained stress in the structure. The stress at failure is a material property. Depending on the type of material, it is the yield stress or, for brittle materials, the ultimate stress. The obtained stress differs for each point in the wind turbine structure, because each point has a different response and also different cross-sectional properties. The stress therefore needs to be determined for multiple points where high stresses can be expected. The rest of this video will focus on the determination of the obtained stress for one point.

Define external and operational conditions

Design situation	DLC	Wind condition	Other conditions	Type of analysis	Partial safety factors
1) Power production	1.1	NTM $V_{in} < V_{hub} < V_{out}$	For extrapolation of extreme events	U	N
	1.2	NTM $V_{in} < V_{hub} < V_{out}$		F	*
	1.3	ETM $V_{in} < V_{hub} < V_{out}$		U	N
	1.4	ECD $V_{hub} = V_r - 2$ m/s, V_r, V_r +2 m/s		U	N
	1.5	EWS $V_{in} < V_{hub} < V_{out}$		U	N
2) Power production plus occurrence of fault	2.1	NTM $V_{in} < V_{hub} < V_{out}$	Control system fault or loss of electrical network	U	N
	2.2	NTM $V_{in} < V_{hub} < V_{out}$	Protection system or preceding internal electrical fault	U	A
	2.3	EOG $V_{hub} = V_r \pm 2$ m/s and	External or internal	U	A

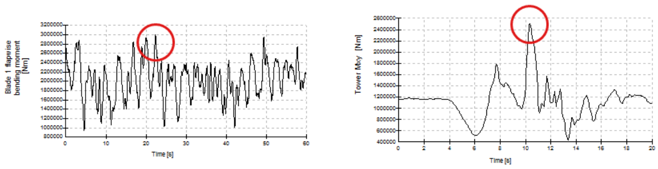
For the analysis of the extreme stress you have to go back to the principles of structural analysis of wind turbines. To deal with the long lifespan of the turbines the loading conditions are represented by load cases. The standards for wind turbine certification provide a list of load cases and specify which ones need to be assessed to estimate the largest stress during the lifetime. Here you see part of this list from the IEC standard, which is commonly used. The IEC identifies this type of analysis with a 'U' for 'Ultimate'. For example, the first row of the load cases in the IEC standard shows design load case, or DLC, 1.1. This specifies that power production during normal turbulent wind conditions needs to be assessed for several average wind speeds between cut-in and cut-out.

Determine displacements & stresses



For each load case we have to do a simulation of the loading and response of the turbine. Each simulation gives you a variation of the stress over time as shown in the graph on the left-hand side. The typical simulation period is 10 minutes in accordance with the separation of long term and short term wind speed variations around the spectral gap. In a preliminary analysis, we can determine the stress from a hand calculation of the quasi-static state of the turbine.

Deterministic analysis



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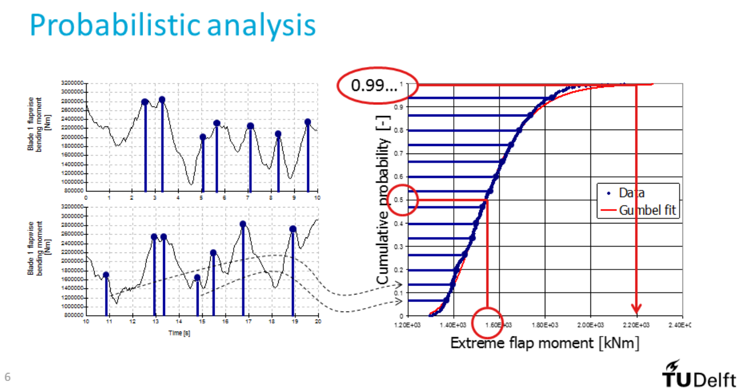
From the simulation results of all load cases we determine the maximum stress that has occurred. This can be done by deterministic analysis or probabilistic extreme stress extrapolation. Here you see the principle of the deterministic approach. These two graphs are examples of simulation results. The graph on the left shows the blade flap bending moments for a normal power production load case and the graph on the right shows the bending moment in the tower during a gust. In both time series we can identify the maximum bending moment from which the maximum stress can be determined. If you do this for all load cases, you find the maximum stress for the entire lifetime of the turbine. This analysis method is called deterministic because the environmental conditions are prescribed and we do not look at the statistics of the resulting stress.

Probabilistic analysis

Design situation	DLC	Wind condition	Other conditions	Type of analysis	Partial safety factors
1) Power production	1.1	NTM $V_{in} < V_{hub} < V_{out}$	For extrapolation of extreme events	U	N
	1.2	NTM $V_{in} < V_{hub} < V_{out}$		F	*
	1.3	ETM $V_{in} < V_{hub} < V_{out}$		U	N
	1.4	ECD $V_{hub} = V_r - 2$ m/s, V_r, V_r +2 m/s		U	N
	1.5	EWS $V_{in} < V_{hub} < V_{out}$		U	N
2) Power production plus occurrence of fault	2.1	NTM $V_{in} < V_{hub} < V_{out}$	Control system fault or loss of electrical network	U	N
	2.2	NTM $V_{in} < V_{hub} < V_{out}$	Protection system or preceding internal electrical fault	U	A
	2.3	EOG $V_{hub} = V_r \pm 2$ m/s and	External or internal	U	A

5

Because the environmental conditions and loading are highly stochastic, there is a need for a probabilistic analysis of the maximum stress. This probabilistic approach is called extreme value extrapolation. The fundamental idea of the probabilistic approach is that we cannot perform a sufficient amount of simulations for conditions that happen very often. Therefore, the extreme stress that could occur during such conditions may be missed by the simulations. The most common condition for a wind turbine is power production during normal turbulent wind conditions. The IEC therefore only requires that simulations of DLC 1.1 are analysed with extreme value extrapolation.



The probabilistic approach starts with the execution of many simulations. Two examples of the resulting blade root bending moments are shown here. In the results we identify the extreme values according to certain criteria. As you see here, we can identify more than one maximum per simulation. The next step is that we list these extremes in order. So, first we identify the smallest of the extremes. Then we identify the second smallest and ordering the list is shown here as stacking the values for the extremes. If we continue this process, we get a pile of values from smallest to largest. Next, we assume that each of these samples of the extremes has an equal probability of occurrence. Therefore, the values are stacked here at equal vertical distances. This enables us to draw a cumulative distribution function through these samples. The values of the extremes are plotted on the x-axis and the probability that other samples of the extremes will be below this value is plotted on the y-axis. To understand the meaning of this graph it helps to realize that half of our samples are below a cumulative probability of 0.5, halfway the y-axis. This means that there is a 50% probability that a sample of the extreme values is below the value at this point. As you can see, the highest extreme value does not have a cumulative probability of 1. This is the consequence of assuming a probability of occurrence of one divided by the number of extremes plus one, instead of one divided by the number of extremes. The basis for this assumption is that our simulations do not yet contain the highest extreme value that can be expected. In other words, there is a probability that the largest extreme that we found in the simulations can be exceeded in reality. That brings us to the region of interest in this graph. We want to know the extreme value that is not likely to be exceeded during the lifetime of the wind turbine. This implies that we are looking for an extreme value with a very high probability that other samples of extreme values are lower. The standards prescribe which probability should be used, but it will be very close to 1. Since we have no data for such high probabilities, we fit an analytical distribution function to the data and extrapolate this to the probability that is specified. Finally, we find the extrapolated extreme value on the x axis. The outcome of this approach is that there is a very high probability that the actual extreme experienced during the lifetime of the turbine will remain below the extrapolated extreme value. With the ingredients provided in this video you are now able to apply the check on the limit state for ultimate strength.

6.4.2 Ultimate strength check

The criterion for the ultimate limit state check for the stresses in the structure reads:

$$\gamma_f \sigma_{\max} \le \frac{1}{\gamma_n \gamma_m} \min\left(\sigma_{yield}, \sigma_{ultimate}\right).$$

The maximum stress σ_{max} is the outcome of the lifetime ultimate loading analysis. For ductile materials the yield stress σ_{yield} is relevant, but for brittle materials the ultimate stress $\sigma_{ultimate}$ is used, since they don't have a yield stress.

The safety factors provide the margin to failure and are respectively:

γ_f safety factor for uncertainty in loads/stresses

γ_m safety factor for uncertainty in material properties

γ_n safety factor for severity of effect of failure

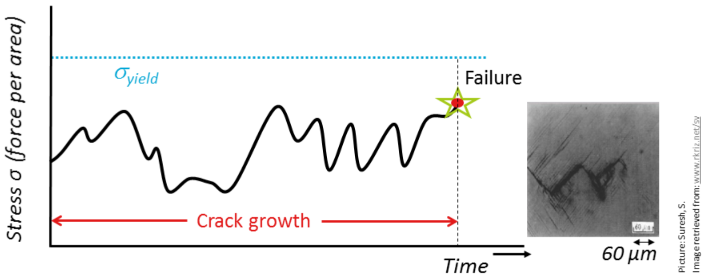
This design check has to be performed for all points in the structure.

6.5 Fatigue analysis

6.5.1 Principles of fatigue assessment

In this video we will address fatigue.

What is fatigue failure?



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Also here we first ask ourselves what failure due to fatigue is. To answer that question we go back to the variation of stress over time. We can have a variation of stress and never exceed the yield stress, and still at some point be surprised by failure. This phenomenon is called fatigue failure. Every material has some imperfections such as very small cracks. The stress will concentrate at the tip of such a crack, and if we have a variable loading, then the crack will grow over time. At some point in time, the cracks will be so large that the residual strength of the structure is no longer enough to resist even a small load. Our traditional analysis of the stress in the structure fails to predict this, because it doesn't consider how the cracks increase the stress. The first ingredient that is needed for the analysis of the fatigue limit state is a measure for the amount of crack growth. A common measure for wind turbines is the fatigue damage according to Miner's rule. The damage according to Miner's rule depends on the variation of stress over time and therefore on the loading and response of the turbine. However, it also depends on the material properties, as captured by for instance a Wöhler curve or a Goodman diagram. When no safety margin is considered, failure is expected when the damage reaches the value one. The damage differs for each point in the wind turbine structure and therefore needs to be determined for multiple points. The rest of this video will focus on the determination of the damage for one point.

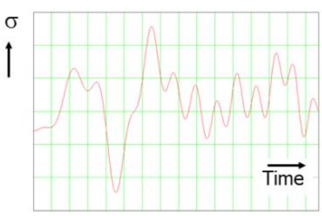
Define external and operational conditions

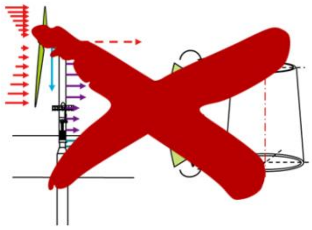
Design situation	DLC	Wind condition	Other conditions	Type of analysis	Partial safety factors
1) Power production	1.1	NTM $V_{in} < V_{hub} < V_{out}$	For extrapolation of extreme events	U	N
	1.2	NTM $V_{in} < V_{hub} < V_{out}$		F	*
	1.3	ETM $V_{in} < V_{hub} < V_{out}$		U	N
	1.4	ECD $V_{hub} = V_r - 2$ m/s, V_r, V_r +2 m/s		U	N
	1.5	EWS $V_{in} < V_{hub} < V_{out}$		U	N
2) Power production plus occurrence of fault	2.1	NTM $V_{in} < V_{hub} < V_{out}$	Control system fault or loss of electrical network	U	N
	2.2	NTM $V_{in} < V_{hub} < V_{out}$	Protection system or preceding internal electrical fault	U	A
	2.3	EOG $V_{hub} = V_r \pm 2$ m/s and	External or internal	U	A

2

The procedure to determine the fatigue damage has much in common with the procedure for the analysis of ultimate stress. It starts with the definition of the external conditions and operational conditions for the load cases, which can again be found in the standards. The IEC identifies the appropriate load cases with 'F' for 'Fatigue'. Only load cases that occur frequently are selected for reasons that will become clear later. As a consequence, none of the load cases with fault conditions is used.

Determine stress time series





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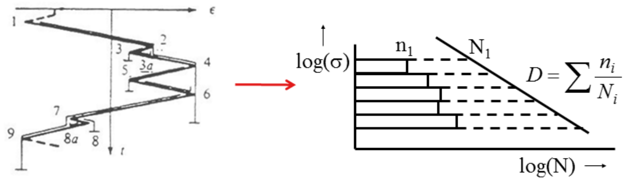
Next we perform the simulations for all load cases to get the stress variation over time. For the analysis of the ultimate limit state we could start with hand calculations in the preliminary stage of design. However, it has been seen in the past that preliminary calculations of stress variations are difficult to make and they are too inaccurate for fatigue assessment. We therefore cannot avoid doing simulations for fatigue analysis.

Stress histogram and damage per load case



After performing the simulations, we have to use a method such as rain flow counting to determine the stress histogram. The stress histogram then gives us the number of cycles at different stress amplitudes and we can compare these with the number of cycles until failure. Having determined the number of cycles at a certain stress level we can apply Miner's rule to get the total damage for this particular load case. As you have seen before, in Miner's rule we add the damage created at each load level, where this damage is obtained by taking the ratio of the actual number of cycles to the number of cycles until failure.

Accumulate lifetime fatigue damage

Situation	Wind speed	Probability (Weibull)	Nr. 10-min. periods	Damage per 10 min.	Damage of situation
Production	5-7 m/s	0.194	204,073	$1{\cdot}10^{-7}$	$2.04{\cdot}10^{-2}$
	…				…
	23-25 m/s	0.002	2,104	$2{\cdot}10^{-6}$	$2.10{\cdot}10^{-3}$
Start ups					…
Shut downs					…
Idling					…
				Total	…

Having done the previous steps for all load cases, we know the damage for each 10-minute simulation period. In this table, this damage per simulation is written down in the second column from the right. Each simulated situation represents many 10-minute periods with similar conditions. We can calculate the total number of 10-minute periods occurring during the lifetime of the wind turbine. If we multiply that value with the probability of having similar conditions, we find the number of periods represented by the simulation. In this table the probability that the wind speed is between 5 and 7 meters per second has been determined from a Weibull distribution to be a bit less than one-fifth. For a lifetime of 20 years, that corresponds to 204 thousand and 73 10-minute periods. If we now multiply this number of 10-minute periods with the damage per simulation, we obtain the lifetime damage due to this type of situation. This is effectively the application of Miner's rule, which simply states that damage can be summed linearly. The total lifetime damage is obtained by adding the damages for all situations. Load cases for exceptional situations, such as fault conditions, represent very few 10-minute periods. Therefore, they would not contribute significantly to the total damage and are not selected for the analysis. With the ingredients provided in this video you are now able to apply the check on the limit state for fatigue.

6.5.2 Accumulated lifetime fatigue damage

The criterion for the fatigue limit state check reads:

$$D \leq \frac{1}{\gamma_n \gamma_m}.$$

The damage *D* is the outcome of the lifetime fatigue analysis. There is no safety factor on the loads, since the fatigue is caused by the variations in the stress. There is no logical consideration to assess what would entail under-prediction of this variability.

The safety factors provide the margin to failure and are respectively:

γ_m safety factor for uncertainty in material properties

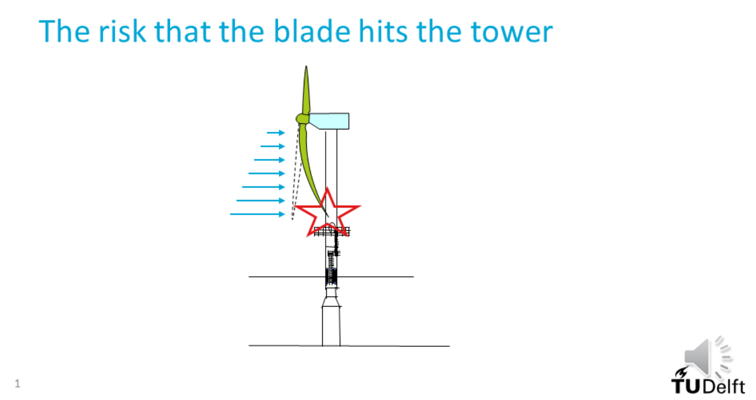
γ_n safety factor for severity of effect of failure

This design check has to be performed for all points in the structure.

6.6 Blade tip deflection

6.6.1 *Principles of tip deflection limit state*

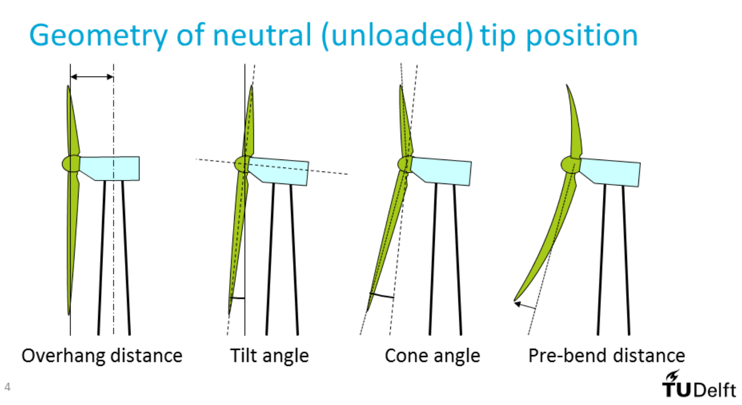
In this video we will address the failure mode in which the blade hits the tower.



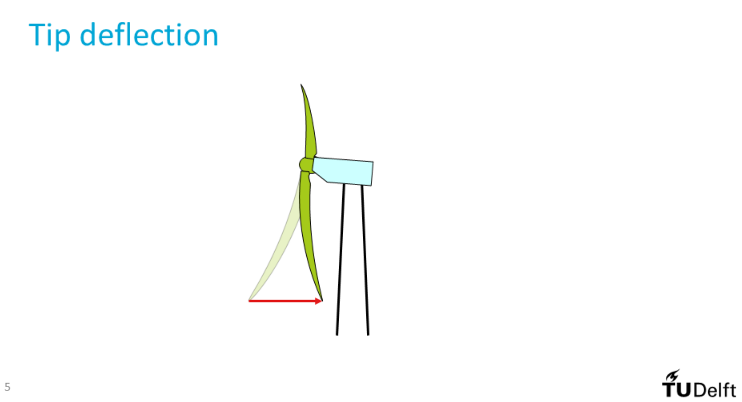
The failure mode of the blade hitting the tower is quite elementary. We have aerodynamic forces on the blade and these cause the blade to bend. At some point the tip deflections may become so large that the tip hits the tower. The two ingredients that are needed for the analysis of the limit state are the tip deflection and the clearance distance between the blade tip and the tower when there are no forces on the blade.

Although it may be evident that we need to perform a check that the tip is not hitting the tower, we may question whether it's important for the design of the blade. The answer to that is a qualified yes, because for multi-megawatt turbines the tip deflection is the main design driver. This means that we have to design the structure and add material to make the blade stiff enough so that it doesn't deflect too much. When the blade is designed for stiffness, it will often automatically be strong enough to avoid fatigue or ultimate failure. Only the outer part of the blade, that doesn't contribute much to the tip deflection, may need to be designed for strength.

First we need to determine what the clearance distance is between the blade tip in the unloaded position and the tower. This clearance distance provides the space for the blade in which it can bend.



The first contribution to the clearance is the overhang distance between the hub and the tower centreline. The space that is occupied by the tower itself needs to be considered, so in case of a cylindrical tower its radius needs to be subtracted from the overhang distance. The next contribution is due to tilting of the nacelle and the rotor axis. This tilting rotates the blades away from the tower. The third contribution is a cone angle. There is a cone angle when the blades are not connected perpendicularly to the axis of rotation. When the blades are connected at a slightly forward angle, the rotating blades describe a cone rather than a flat plane. The final contribution to the blade tip clearance is pre bending of the blades. This is achieved by manufacturing the blades in a curved mould.



The only thing that remains, to check that the limit state is not exceeded, is the tip deflection. The same approach is applied here as for the limit state for ultimate strength: many simulations are run for various load cases to determine the largest expected tip deflection. Actually, the simulations that were performed for the ultimate strength analysis are directly used for this. Either a deterministic or a probabilistic analysis can be performed on the simulated extreme tip deflections. As a preliminary analysis, the tip deflection can be determined from a quasi-static analysis of the state of the turbine. Be aware that the blade will only hit the tower when it is in a downward position. When the blade is at its highest point its deflection is not relevant. With the ingredients provided in this video you are now able to apply the check on the limit state for tip deflection.

6.6.2 Calculation of tip deflection limit state

The criterion for the ultimate limit state check for the tip deflection reads:

$$\gamma_f y_{tip} \leq \frac{1}{\gamma_n \gamma_m} d_{\max} .$$

The maximum tip deflection y_{tip} is the outcome of the lifetime ultimate loading analysis and only needs to be taken for periods that the blade passes the tower. The clearance distance d_{max} is the distance from the tip of an unloaded blade to the tower, when the tip is in front of the tower. In a more detailed assessment the 3D geometry of the tower structure and blade tip motion can be considered.

The safety factors provide the margin to failure and are respectively:

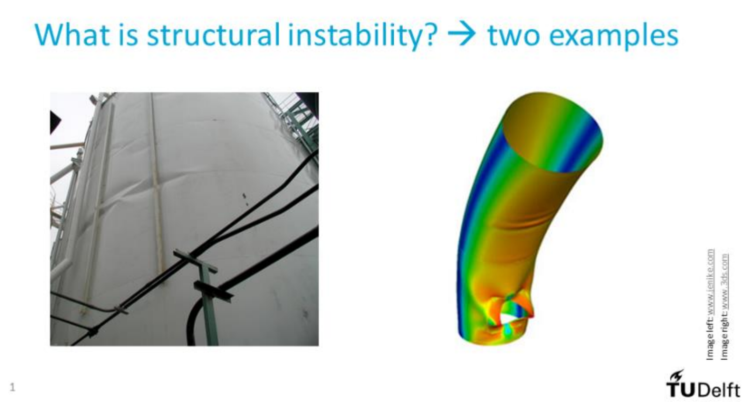
γ_f safety factor for uncertainty in loads/deflection

γ_m safety factor for uncertainty in elastic properties, including geometric and material uncertainties

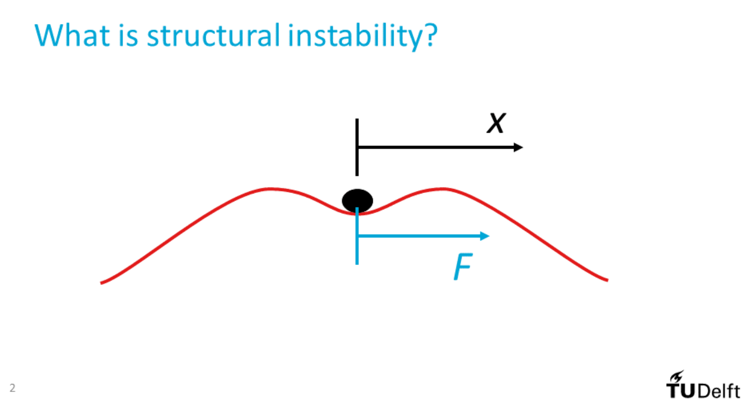
γ_n safety factor for severity of effect of failure

6.7 Structural stability

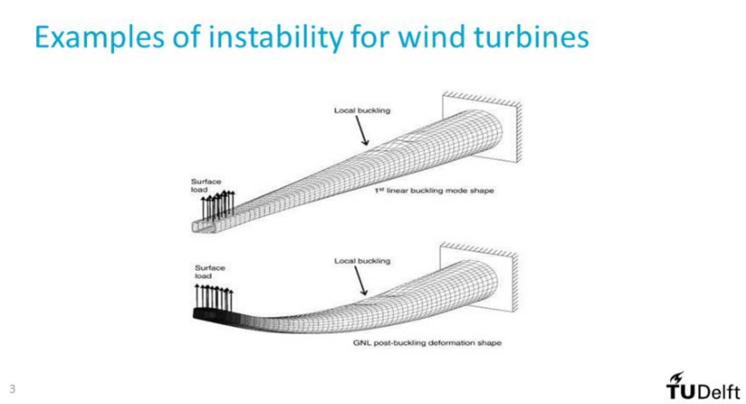
In this video we will address the failure mode in which the structure becomes instable.



The most typical form of structural instability for a wind turbine is buckling and here you see two examples of this. On the left-hand side you see the buckling of a silo under gravity loading, and on the right-hand side buckling of a tower under bending. What is happening can be best understood if we look at the silo. The gravity loading is purely vertical. However, we see a deformation out of the plane of the shell material, even though there is no horizontal loading on it. This effect is caused by imperfections in the geometry or a small horizontal load that triggers it. Imagine that the shell is not entirely flat, but there may be some wiggle in it. If we apply a vertical force on the shell, then this wiggle may grow, so it starts to bend out of plane. At some point the stiffness of the material may be insufficient to counteract this bending. This example shows that buckling is actually not a strength problem. The yield stress is not exceeded at the moment that the structure starts to fail. Buckling is initially a stiffness problem, because it is characterised by divergence in a deformation.



The fundamental characteristic of structural instability can be illustrated by a marble in the hole at the top of a hill. If we apply a force F on the marble, it will pull it out of the centre of the hole, but gravity will pull it back. However, if the force F becomes too large, then the marble will exceed the crest of the hill and it will roll down. In other words, x will increase unlimitedly or it will diverge. Something similar applies to buckling. The stiffness of the plates creates a restoring force, but at some point this restoring force reduces and fails to be enough. Typically for buckling, as we have seen, the deformation is out of the plane or line of the loading. Since buckling is associated with imperfections, the structure may appear stable until for instance a very small out-of-plane loading creates bending in the blade material that cannot be restored and the structure collapses.



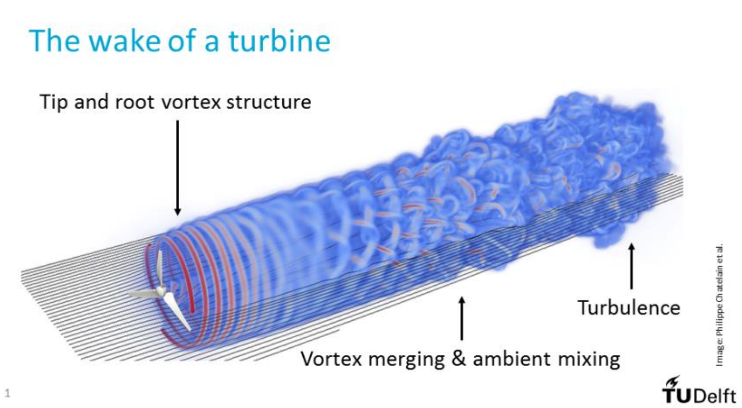
As mentioned before, buckling is an important type of structural instability for wind turbines. For certain types of buckling and for certain simple geometries, there are some approximation methods to make an assessment of buckling. However, for more complicated structures, such as the blades, we need to use more complicated approaches. What you see here is a final element model to assess whether or not buckling occurs. Particularly the shells of towers and blades are susceptible to this failure mode and need to be designed accordingly. For this course, we will not go into details of buckling or methods to check the limit state for it. This brief introduction is only meant to complete the overview of relevant limit states and to make you aware of the importance of buckling.

7 Wake effects

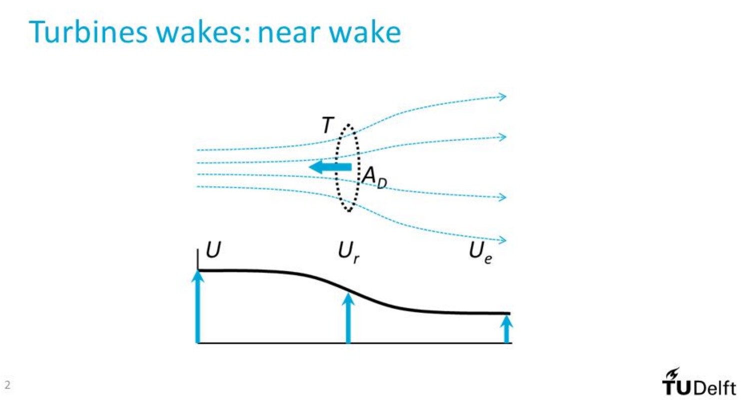
7.1 Wind turbine wakes

7.1.1 Description of the wake phenomenon

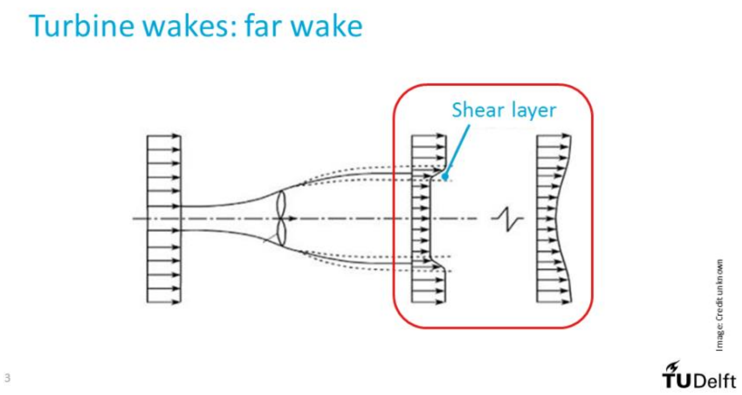
As you have learned before, a wind turbine has a wake, just as any other obstacle. In the wake, the wind is influenced over large distances. In this video we will address the nature of wakes behind wind turbines and the importance of these wakes.



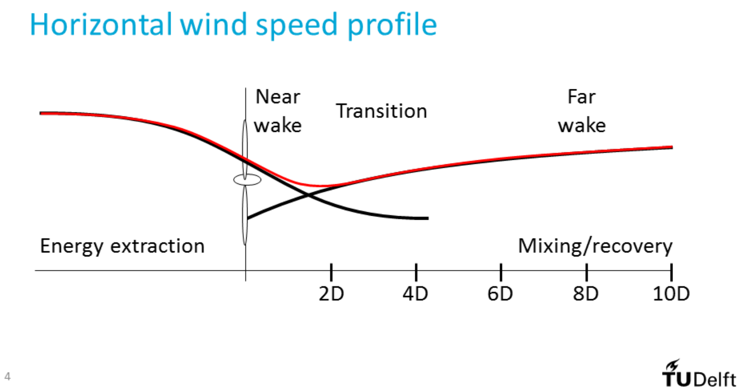
'Blade element momentum' theory tells us something about the wind speed at the rotor and somewhere downstream of the rotor, where the stream tube stops expanding. However, BEM assumes that the wind speed is uniform inside each cross section of the stream tube. In reality, the effect of each individual blade on the flow can be recognised as what are called vortices. At each blade tip and blade root a strong vortex is shed, which moves downstream with the wind. Because the blades rotate, the root and tip vortices describe helixes. The helix structure remains visible until several rotor diameters behind the turbine. Then, the structure starts to collapse. The vortices affect each other's propagation and they start to roll over one another. Also, the vortex structure starts to mix with wind surrounding the wake, due to turbulence in the ambient wind. Eventually, the structure is completely gone and a highly turbulent flow remains, due to the rumpled vortex lines.



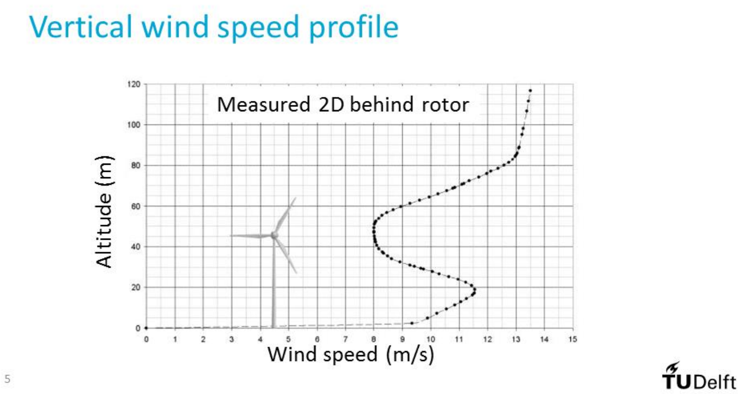
Let's go back to momentum theory to see what happens around the rotor. The thrust force reduces the wind speed, which is parameterised with the induction factor. At the rotor the dimensionless wind speed has dropped by the induction factor and it drops by an equal amount downstream of the rotor. The length over which the wind speed drops behind the rotor is typically 2 to 4 times the rotor diameter. This region is called the near wake. It is characterised by the wind speed drop and by the clear structure of the tip and root vortices. The drop in wind speed in the wake is referred to by wind speed deficit.



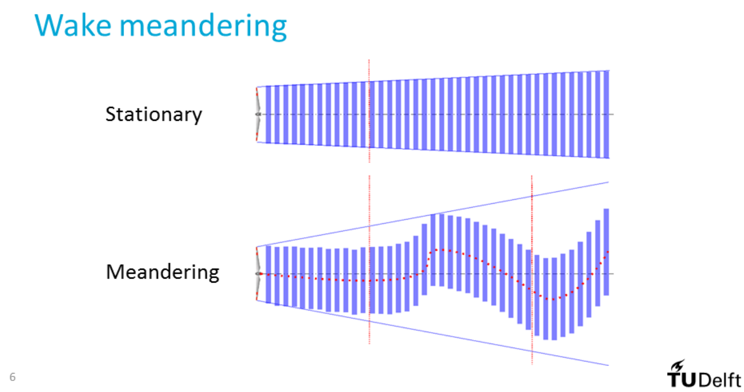
If the induction factor is nearly equal for all annuli, then the wind speed is nearly uniform over the cross-section of the stream tube downstream of the rotor. This means that we have the undisturbed wind speed outside the stream tube and a sharp transition to the lower wind speed inside the stream tube. The region with the wind speed transition is called shear layer. The stronger the wind speed gradient in the shear layer, the larger the transport of momentum from the high speed region to the low speed region. Much of this transport is caused by turbulence. In other words, due to turbulent mixing, the wind speed in the stream tube is regenerated by the undisturbed wind. Due to the momentum transport in the shear layer, the wind speed gradient reduces and the shear layer stretches over an increasing region. Eventually, the shear layer covers the entire original stream tube and even the wind speed at the centre recovers. This region in which the wind speed is regenerated is called the far wake. It is characterised by the decrease in wind speed deficit and by the increased turbulence from the broken down vortex structure.



Here you see the wind speed as it passes through the rotor and in the wake. You can recognise the wind speed reduction in the near wake and the wind speed recovery in the far wake. The wind speed increases asymptotically to the undisturbed wind speed, with most of the recovery within 10 diameters behind the rotor. There is a transition between the near wake and the far wake, in which there is a bit of all the phenomena mentioned before.



Here you see a measurement of the vertical wind speed profile at twice the diameter behind a rotor. The wind speeds are averaged over a period of time, to remove irregularities caused by turbulence. You see the large wind speed deficit and the distinctive shear layer. You can also observe that the reduced wind speed is superimposed on the boundary layer profile.



Many simple wake models assume that the wake propagates along a straight line in the wind direction. However, for many conditions the wake also has lateral motion, both horizontally and vertically. This motion is called wake meandering. There are different ideas about the origin of this meandering effect. One idea is that the large turbulent structures of the wind surrounding the wake contain cross winds that push the wake around. Another idea is that the flow inside the stream tube itself creates disturbances in the lateral direction. Either way, the phenomenon can be observed in reality and might need to be considered.

Wind turbines are often installed in pairs, clusters or wind farms. This means that for certain wind directions turbines might be in the wake of their companions. There are two reasons why the wake effects require careful consideration. First, the lower wind speed in the wake reduces the power in the downstream turbines. In the short term, this causes variation in the power when the wind is changing direction, moving the wake along. This is a disadvantage for the balance between electricity production and consumption in the public grid. Over the long term, this means that the energy yield will be lower than the yield of an equal amount of solitary turbines. The second reason to consider wakes is that they increase the load variation in downstream turbines. The increased turbulence in the wake is by itself a source of additional load variation. Besides, when the wind direction is changing the wake may move over the rotor area of a downstream turbine. While this happens, the wind speed can change between ambient conditions and full wind speed deficit or the blade may pass through regions with different wind speeds over each revolution. The increased load variation causes additional fatigue damage. Because of the importance of wakes, there are many models that try to represent them. A few simple models will be presented in this course, to perform initial assessments of the wake effects.

7.1.2 Summary near and far wake characteristics

The following table summarises the characteristics of the near wake and the far wake.

Table 1 Characteristics of near wake and far wake

	Near wake	Far wake
Physical phenomenon	Energy extraction	Radial momentum transfer
Effect on wind	Wind speed reduction	Wind speed recovery in wake
Radial velocity profile	Creation of shear layer	Reduction of shear
Structure	Clear (vortex) structure	Turbulent wind field
Location	up to 1D … 3D	3D up to 10D … 20D

7.2 Wind speed deficit

Wind speed deficit models

The wind in the wake of a turbine differs from the undisturbed wind speed in terms of its average and in terms of the turbulence intensity. The reduction in average wind speed is called the wind speed deficit. Since wind turbines are usually separated over distances of at least several rotor diameters, the wind speed deficit in the far wake is most important for the determination of the wind conditions at downstream turbines. Many models have been developed to assess the average wind speed in the far wake. The complexity of these models ranges from simple explicit analytical expressions of wind speeds to computational fluid dynamics (CFD) approaches that solve (variants of) the full flow equations: the Navier-Stokes equations.

In this course, we'll introduce and use one of the earliest and simplest models: the Jensen model. Although it is simple, and as you'll see it deviates in many aspects from the actual physics in the wake, it gives sufficiently good solutions for many applications.

Model description

In the Jensen model, the geometry of the wake is prescribed. Furthermore, the profile of the average wind speed in the wake is prescribed. This predefined description of the wake structure is shown in Figure 1 and will be explained below. The control volume is not part of the wake description, but it will be used later in the derivation of the model equations.

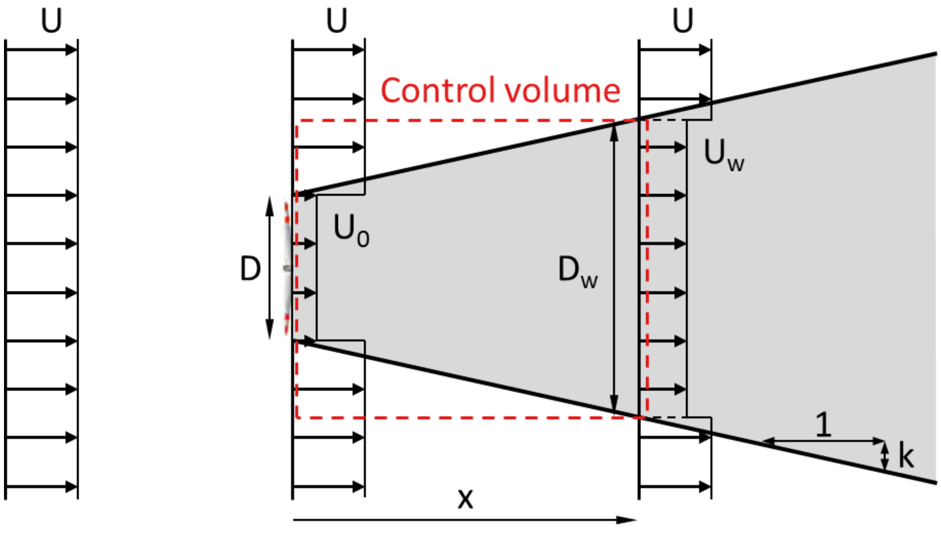


Figure 1 Geometry and wake description of the Jensen model

On the left-hand side of the figure, you see the undisturbed wind speed, far upstream of the rotor. The wake starts at the rotor, with a diameter equal to the rotor diameter *D*. It expands linearly, meaning that the radius of the wake increases with *k* for every unit of distance in downwind direction *x*. Inside the wake, the wind speed deficit is not a function of the radial distance from the centre of the wake. This corresponds with the 'top-hat' wind speed distribution that is shown at the rotor and at distance *x*. The wind speed inside the wake at the position of the rotor is U_0. At other places in the wake it is U_w, which is a function of the distance x, but which is constant within the wake diameter D_w. Outside the wake, the wind speed remains the undisturbed wind speed *U*.

Derivation of the model equations

By prescribing the wake geometry and the wind speed profile in the wake, we don't have to solve these from the equations that describe the flow. This gives us the advantage that it becomes easier to solve the unknown parameters in the model: *k*, D_w, U_0 and U_w. The disadvantage is that this implies that this solution will not be consistent with all physics processes, unless the prescribed conditions are actually the true solution to the flow problem. It is clear that this is not the case, since it is known that downstream the shear layer becomes wider and the wind profile loses its top-hat shape.

The fundamental equation that was used by Jensen to derive his model was the conservation of mass. Let us apply this to a cylindrical control volume as shown in Figure 1. At the outer side of the control volume, so the top and bottom in Figure 1, the wind flow is parallel to the boundary of the volume. Therefore, there is no mass flow in or out through this boundary. What remains are the mass flow into the volume on the left-hand side and out of the volume on the right-hand side. These have to be equal, as given by the following expression:

$$\rho U_0 \frac{1}{4}\pi D^2 + \rho U \frac{1}{4}\pi\left(D_w^2 - D^2\right) = \rho U_w \frac{1}{4}\pi D_w^2.$$

The first term on the left is the mass flow through the rotor area. The second term on the left is the mass of the flow through the ring surrounding the rotor, up to the wake diameter D_w. The right-hand side is the mass flow through the wakes circumference at distance *x*.

It's clear that we can scratch many factors in this equation and end up with a relation between diameter and velocities only:

$$U_0 D^2 + U\left(D_w^2 - D^2\right) = U_w D_w^2 .$$

Jensen substituted $U_0 = U\left(1-2a\right)$, or in words: the initial wind speed in the wake is equal to the end velocity given by momentum theory. After rearranging the terms, this leads to:

$$\left(1-2a\right)D^2 + \left(D_w^2 - D^2\right) = \frac{U_w}{U} D_w^2 \Leftrightarrow \left(1-\frac{U_w}{U}\right) = \frac{D^2}{D_w^2} 2a .$$

The wake diameter can be determined from the prescribed geometry, resulting in:

$$1-\frac{U_w}{U} = \left(\frac{D}{D+2kx}\right)^2 2a = \frac{2a}{\left(1+2ks\right)^2},$$

with $s = \frac{x}{D}$, the dimensionless distance behind the rotor.

Since it is more convenient to use the thrust coefficient than the induction factor, the last step is the substitution of $a = ½ - ½\sqrt{1-c_T}$, giving the final expression:

$$1-\frac{U_w}{U} = \frac{1-\sqrt{1-c_T}}{\left(1+2ks\right)^2}.$$

If the undisturbed wind speed is known, you can estimate the thrust coefficient by calculation. For real turbines the thrust coefficient is often tabulated as a function of wind speed and you can look it up.

The only remaining unknown parameter is the wake decay coefficient *k*. This parameter is used to calibrate the outcome of the model against measurements. The following guidelines have been found to work well:

Onshore $k \approx 0.075$ [-]
Offshore $k \approx 0.05$ [-]

In more general terms, the wake decay coefficient is a function of the turbulence intensity. The more turbulence, the higher the mixing with the undisturbed wind outside the wake, the higher the wake expansion and thus the higher *k*. Higher mixing also means that recovery of the wind speed in the wake goes faster and you can confirm that U_w increases quicker when *k* is larger. Turbulence is larger onshore than offshore, hence the difference between the two recommended wake decay coefficients.

Evaluation of the model

In the previous two sections, the description of the wake and the steps in the derivation of the model are given without reflection. The list below summarises the implicit and explicit assumptions made in the model, after which some implications of these choices are discussed:

- Linear wake expansion
- Top-hat shape wind speed profile
- Mass flow conservation
- Boundary condition 1: diameter at $x = 0$ is equal to the rotor diameter

- Boundary condition 2: wind speed in the wake at $x = 0$ equals the end speed according to momentum theory
- Wake decay coefficient is determined empirically

The conservation of momentum and energy is not enforced in the model. Since the prescribed wake expansion and velocity profile in the wake do not correspond to reality, it isn't likely that the model conserves these quantities in the wake. Certainly, these quantities are not conserved if we look at the flow before and after the rotor. The boundary conditions for the wake diameter and wake velocity are applied at $x = 0$: $D_w = D$, and $U_0 = U_e$. The end velocity according to momentum theory actually applies to a point further downstream of the rotor and for a larger streamtube diameter. Therefore, not even the mass flow is consistent with the upstream inflow conditions, which would give $U_0 = U\,(1 - a)$ at the rotor. Later variants of the model compensate for this inconsistency caused by $D_w = D$. They use the diameter of the end of the streamtube from momentum theory as the initial diameter of the wake.

Because of the inconsistencies in the mass flow, momentum flow and energy flow at the start of the wake, the model is not valid for small *s*. It is recommended no to use it for $s < 3$, so within three rotor diameters. This is no problem in practice, since turbines are usually not placed that close together. This would give too much turbulence on the downstream rotor, and consequently too high fatigue loading. Furthermore, the model is not expected to work well for small *s*, since it is a far wake model. The model description only starts at $x = 0$ to be able to solve the equations.

The model results have been compared by Jensen with measurements from Vermeulen et al. This is shown in Figure 2. As can be seen, there are regions where Jensen's model over-predicts the wind speed and regions where it under-predicts, but on average it works fairly well.

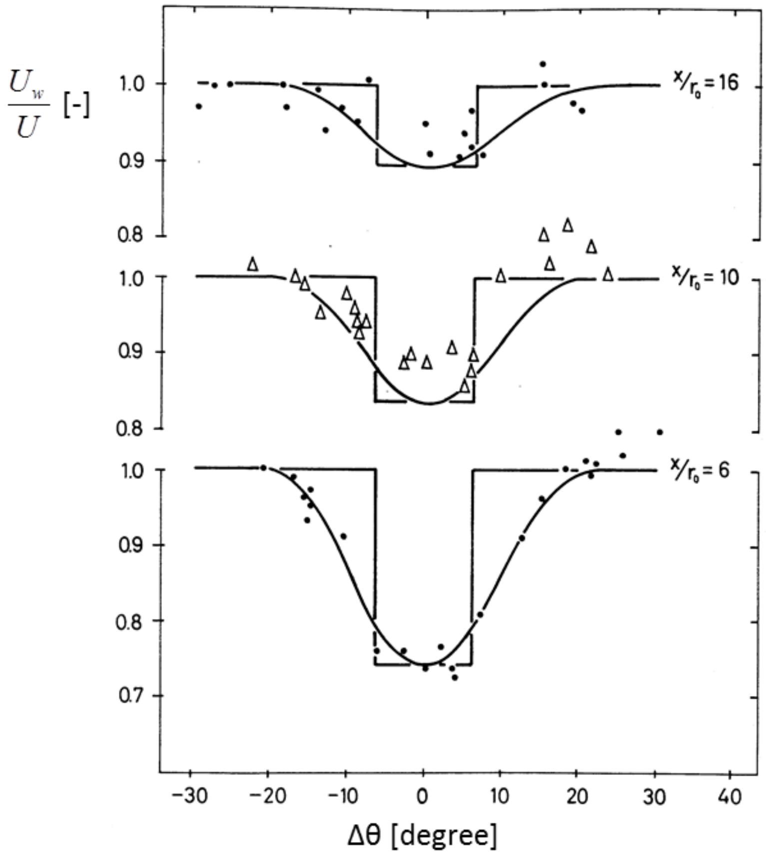


Figure 2 Measured wake profile at 3 different distances behind the rotor, with the Jensen model (rectangular profile) and a smoother model (bell-shaped profile). Radial distance from centreline given as angle w.r.t. rotor centre.

7.3 Turbulence in wakes

Ambient turbulence and wake added turbulence

The second effect of the rotor on the wind conditions in the wake is the increase in turbulence. The turbulence intensity inside the wake of a wind turbine depends on the turbulence in the undisturbed wind and on the turbulence that is added by the rotor. The turbulence in the undisturbed wind is called the ambient turbulence. Although the turbulence added by the rotor depends on the interaction of the rotor with the ambient wind, the added turbulence in the wake is generally considered separately from the ambient turbulence and then combined. Because both the ambient turbulence and the wake added turbulence are stochastic variations in the wind speed, the standard deviation of the turbulence in the wake is not the linear superposition of the standard deviation of the ambient turbulence and that of the wake added turbulence. A reasonable estimate of the combined turbulence is obtained from the root-sum-square:

$$\sigma_{a+w} = \sqrt{\sigma_a^2 + \sigma_w^2}\,.$$

where the indices *a* and *w* indicate *ambient* and *wake* respectively. The turbulence intensity in the wake and the added turbulence intensity are defined by their respective standard deviations, normalised with the ambient 10-minute average wind speed. They are not normalised with the average velocity in the wake, which is affected by the wind speed deficit. Following this definition, the turbulence intensity in the wake becomes:

$$I_{a+w} = \frac{\sigma_{a+w}}{U_{a,10}} = \sqrt{\frac{\sigma_a^2}{U_{a,10}^2} + \frac{\sigma_w^2}{U_{a,10}^2}} = \sqrt{I_a^2 + I_w^2}\,.$$

Added turbulence intensity in the wake

As for the wind speed deficit in the wake, there are various models to determine the added turbulence intensity in the wake. Most of these models are simple and are explicit analytical equations. Evidently, a full solution of the Navier-Stokes equation from a CFD model would not only give the wind speed deficit, but also the total turbulence. In CFD models the turbulence in the wake is not separated into ambient turbulence and wake added turbulence.

The IEC standard for wind turbines, IEC 61400-1, provides the following model for the added turbulence intensity, as function of the dimensionless distance *s*, downstream of the rotor:

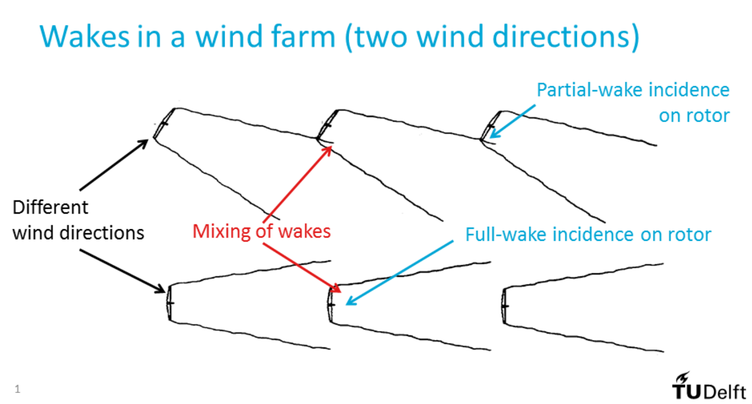
$$I_w = \frac{0.95}{\left(1.5 + \frac{0.8 \cdot s}{\sqrt{c_T}}\right)}\,.$$

As for the wind speed deficit, the thrust coefficient is an important parameter for this wake effect. Again, you can estimate its value, or look it up from tabulated data.

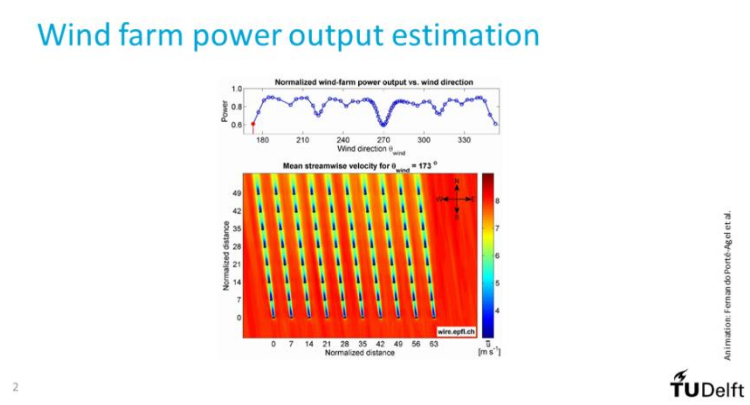
7.4 Partial and mixed wake incidence

7.4.1 What it is and why it is important

The wake models that have been presented describe the wind speed or turbulence in the wake of a turbine. This is typical of many wake models. However, in the end we are interested in the effect of the wakes on the downstream turbines in clusters and farms.

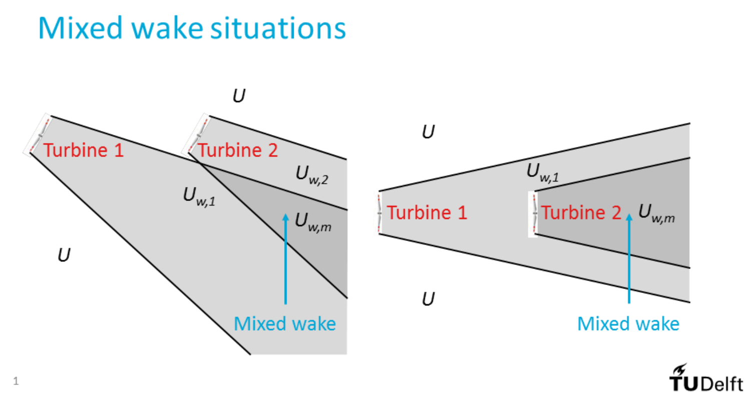


The first thing to consider is the incidence of the wake on the downstream rotors. Evidently, this depends on the wind direction. A particular rotor can be outside the wake, it can be fully covered by the wake, or there can be partial wake incidence, in which only part of the rotor area is inside the wake. The second thing to consider is that a rotor may be in multiple wakes from different turbines. This is typically the case when the wind direction is more or less aligned with a series of turbines. In this case, turbines may create wakes while themselves being in the wake of upstream turbines. It is also possible that wakes from different turbines overlap due to the expansion of the wakes. Even though the wakes are initially separated, they get closer and closer together by their expansion.

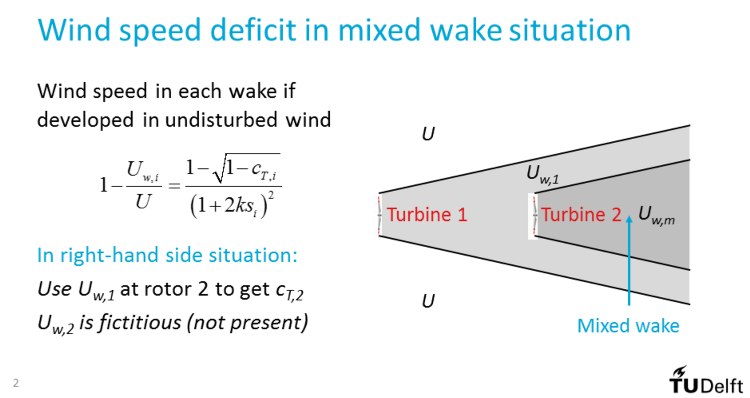


The most common application where you need to consider the effect of partial wake incidence and mixed wakes is the assessment of the wind farm power output as a function of wind direction. Travelling through wind directions from 0 to 360 degrees there will be more, or less effect of the wakes on the total power. Evidently, when the wind is aligned with the rows in a wind farm, there will be full wake incidence of mixed wakes on many turbines. For other wind directions there may be less wake incidences, coming from turbines at much further upstream distances. Similar assessments are made for the turbulence intensity, to estimate whether turbines in the farm will not succumb to fatigue during their lifetime. The following course material provides methods to deal with full and partial wake incidence as well as with wake mixing for such assessments.

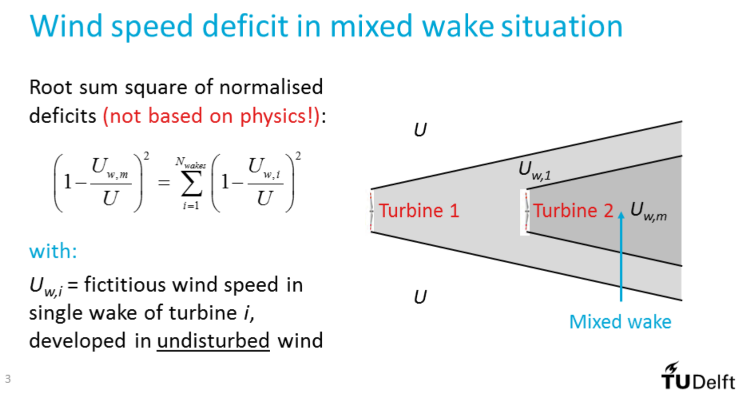
7.4.2 Model for wind speed deficit



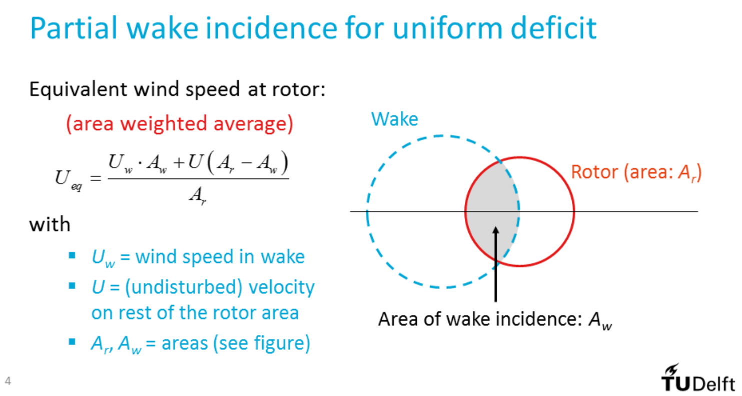
It has become clear that in a wind farm we can have partial wake incidence and mixing of wakes before they reach the next turbine. In these slides we will first address wake mixing, then partial wake incidence and finally the combination of both phenomena. Here you see two possible situations for mixing of the wakes of two turbines. On the left-hand side, the two wakes of the two turbines are initiated independently, but they merge somewhere downstream. On the right-hand side, the second turbine and its wake are entirely submerged into the wake of the first turbine. In both situations the wakes are not surrounded by undisturbed wind everywhere. In the situation on the right the second rotor is also not surrounded by undisturbed wind. Therefore, its operational conditions, in terms of for instance its power and its thrust coefficient, are determined by the wind speed in the wake of turbine 1.



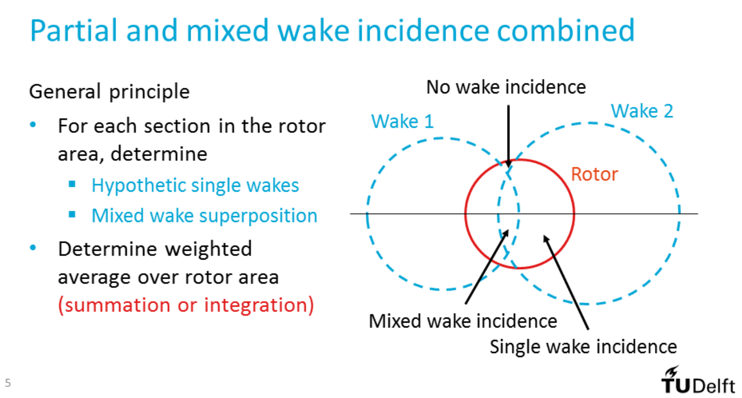
The first approximation that is made in the assessment of the mixed wake situations is to model the wind speed deficit in each wake as if it is individually recovering. Furthermore, it is assumed that this recovery is equal to the recovery in a wake that is fully surrounded by the undisturbed wind speed. This is obviously not entirely correct, since at least a part of the wake connects to another wake, in which the wind speed is lower. However, that wake has usually recovered sufficiently to allow for this simplification. However, in case the rotor itself is inside a wake, the conditions inside the wake have to be considered for the operation of that turbine. Operation of turbine 2 is influenced by the wind from a few rotor diameters upstream to a few rotor diameters downstream of its position, because that is the extent of the streamtube over which the energy conversion takes place. The wind speed in the wake varies over this range. Nevertheless, it is assumed that the wind speed at the position of the rotor itself is representative, to determine the power and thrust coefficient of this turbine. With these assumptions, we can apply for instance Jensen's model to wake 1 and wake 2, although it is clear that the single wake solution for wake 2 is actually nowhere present. Inside wake 2 we have mixed conditions everywhere.



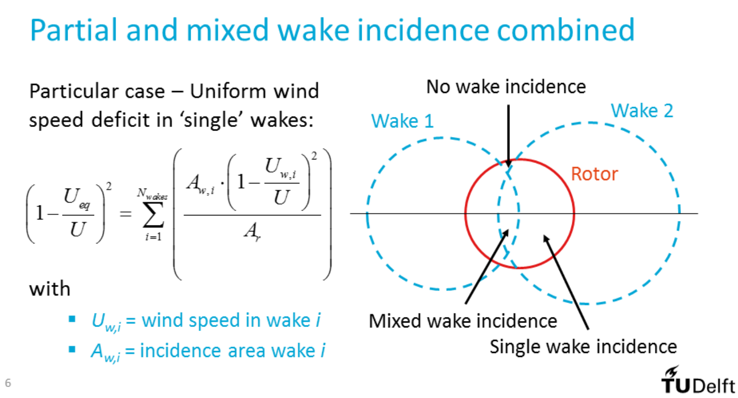
The final step in the determination of the wind speed in a mixed wake is to take the root-sum-square of the relative wind speed deficits of the individual wakes. Instead of taking the root of the right-hand side of this equation, often the left-hand side is squared to maintain a clearer symmetrical equation. As the equation shows, it is applicable to more than two wakes as well. Each of these wakes is assumed to develop in the same way as when it would have been surrounded by undisturbed wind, but the relevant thrust coefficient has to be determined from the local wind speed at each of the turbines.



To address partial wake incidence, we first consider a single wake again. In this figure you see a rotor that is partly inside a wake and partly outside of it. The most common model to estimate an equivalent wind speed for the operation of this turbine is take the linear average of wind speeds over the rotor area. Also here, we consider the wind speed at the position of the rotor, even though the wind speed in the wake and the size of the wake vary over the upstream and downstream region that influences the energy conversion process. When we use the Jensen model for the wind speed deficit, averaging the wind speed over the rotor area is simple. In that case the wind speed inside the wake is constant and of course the undisturbed wind speed is also constant over the rotor plane. Therefore, the equivalent wind speed is then the average of these constant speeds, weighed by the area over which they cover the rotor.

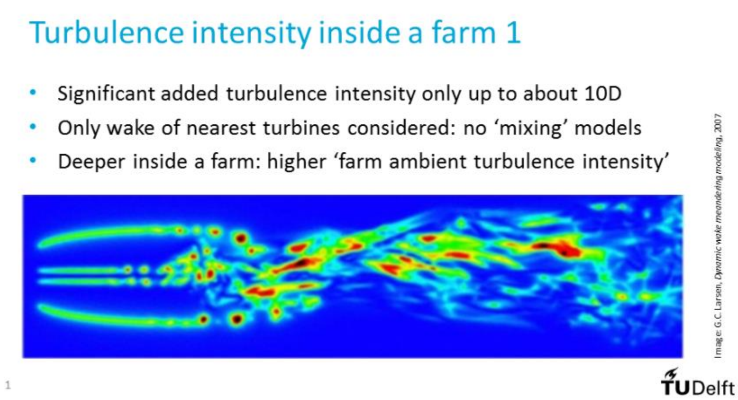


The combination of mixed wakes and partial wake incidence is treated as a straightforward extension of the principles we've just addressed for these phenomena separately. In the figure you can see that we can divide the rotor in areas with undisturbed wind speed, with single or with multiple wake incidence. The first step in the determination of the equivalent wind speed for this situation is to determine the velocities in each of the wakes separately. For the areas of mixed wake incidence, we then determine the mixed wake velocity, according to the previous method. Finally, we determine the average velocity over the rotor area. If we use Jensen's model for the wakes, also here each identified area in the figure has a constant velocity and we can take the weighted average. If we use a wake model in which the velocity is not constant over its cross-section, we would have to do a numerical integration and determine the velocity at each point on the rotor plane. However, the models and procedures are already so far simplified and crude, that this integration can also be fairly coarse.



In case of the Jensen model, so with uniform wind speed deficit in the individual wakes, the equivalent velocity over the rotor can be determined with this equation. You can see that the summation is done over all wakes that impact the rotor. It is not a weighted summation over areas with no, single or multiple wake incidence, as described on the previous slide. In case an area of the rotor is in the wake of multiple turbines, you see that this area appears multiple times in the summation. However, the difference in the final result is very small and both approaches can be used. With this expression for the equivalent velocity we have all ingredients to determine the wind speed at any point in a wind farm, and therefore to estimate the power output of a wind farm with wake effects. For all turbines in the farm, determine which wakes of upstream turbines impact their rotor area. To determine the equivalent velocity, and thus the power of this turbine, you have to know the thrust coefficients of the upstream turbines. Therefore, you first have to determine the equivalent velocity for them. This shows that it is convenient to start this assessment at the most upwind turbine and then determine the conditions at the other turbines while moving gradually downwind. Since you usually want to do this for multiple wind directions and wind speeds, this is normally not something to do manually for a big farm. Once the farm power output is known for all wind directions and wind speeds, we can combine it with the probabilities from the wind rose and the Weibull distribution to determine the annual energy yield.

7.4.3 Model for turbulence intensity

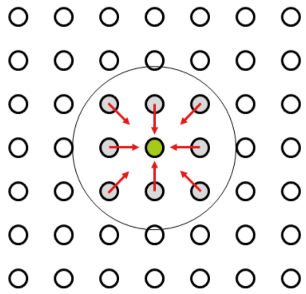


When analysing the turbulence intensity inside a wind farm, we also have to consider the influence of multiple turbines and the effect of partial wake incidence, same as for the wind speed deficit. However, the treatment of these conditions differs. It can be observed from measurements, as well as from the models, that the change in turbulence intensity in the wake is only significant up to about 10 times the rotor diameter downstream of the turbine. With typical separations between turbines of 6 or 7 times the rotor diameter, this implies that the added wake turbulence is only relevant at the first downstream turbine. The second downstream turbine, where mixing of two wakes has occurred, is then roughly 12 to 14 rotor diameters away from the most upstream turbine. Therefore, only the effect of the nearest neighbours is considered and normally no mixing models are applied. In principle, something similar applies to the wind speed deficit. It is indeed true that in mixed wakes the effect of the nearest upstream turbine dominates the effect of the farther upstream turbine. However, for wind speed deficit we aim for a higher precision in our assessment, since the farm power output is an important parameter for the economic viability of the farm. The turbulence intensity is used to determine the loading conditions inside the farm, and it is thus relevant for the limit state analysis. The use of mixing models for turbulence intensity would complicate this analysis, while not adding accuracy. There are many other uncertainties and errors in the models used for limit state analysis that are of more significance. Nevertheless, the large number of wakes inside a farm does leave a trail of increased turbulence further downstream. Therefore, deeper inside a farm this is often represented by a higher farm ambient turbulence intensity.

Turbulence intensity inside a farm 2

Wind farm configuration	N
2 wind turbines	1
1 row	2
2 rows	5
Inside a wind farm with more than 2 rows	8

Example more than 2 rows: Neighbouring wind turbines taken into account for fatigue analysis



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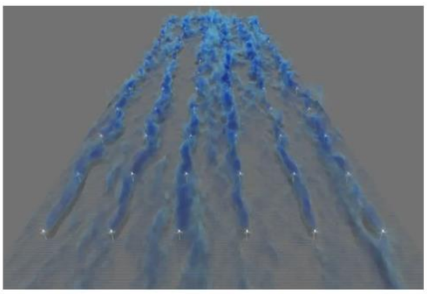
As mentioned on the previous slide, only the wake added turbulence of the nearest turbines needs to be considered. The IEC standard for wind turbines, IEC 61400-1, provides a guideline how to interpret this. This table shows for several configurations of wind farms with aligned turbines, how many turbines need to be considered. The figure on the right gives an example for a larger farm, for which 8 neighbours need to be included in the limit state analysis. The red arrows indicate for which wind direction each neighbour is relevant. It is clear that there are no situations in which the wakes of multiple turbines impact the green turbine at the same time. The importance of considering 8 neighbours is primarily for which percentage of time they increase the turbulence at the green turbine and by which magnitude. The first aspect follows from the geometry of the farm and the wind direction probabilities from the wind rose. The second aspect follows from the distance between the turbines in the rows, columns and diagonals of the configuration. For similar reasons as for the omission of wake mixing models, partial wake incidence is also not modelled for turbulence. Instead, any partial wake incidence is considered to be equal to full wake incidence. If the rotor area has regions inside a wake and regions inside undisturbed wind, the variation of average wind speed over the rotor is considered at least equally bad as added wake turbulence. Furthermore, due to wake meandering the wake will sweep over the rotor area, also with negative effects on loading. Keep this in mind when determining how often a turbine in a farm is influenced by the wakes of neighbouring turbines.

7.5 Deep array wake effect

So far, we have looked at single wakes and at mixing of multiple wakes that overlap on the rotor area of a downstream turbine. This works well when we have only a few turbines. However, far downstream in a large wind farm this modelling doesn't suffice anymore. For these regions deep inside a wind farm so called deep array models have been developed.

Wake effects deep inside the farm



Animation: David Bock and Richard Stevens

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As the wakes propagate, they meander, mix and fall apart. Wakes of far upstream turbines can no longer be clearly identified individually, such as is assumed when using single wake models. However, some of the turbulence remains and deep inside the wind farm the wind surrounding the wakes can no longer be considered to be undisturbed. Deep array models try to capture these conditions in adjusted wind farm ambient turbulence and adjusted wind farm ambient wind speed.

The wind farm as surface roughness

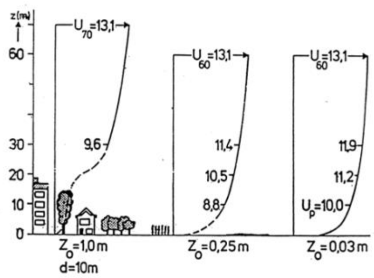
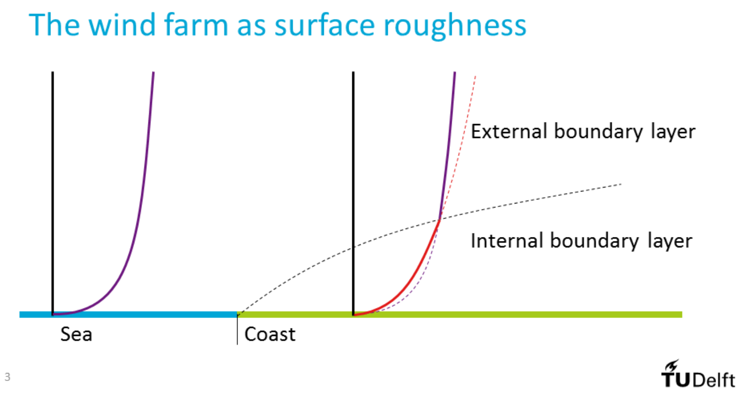


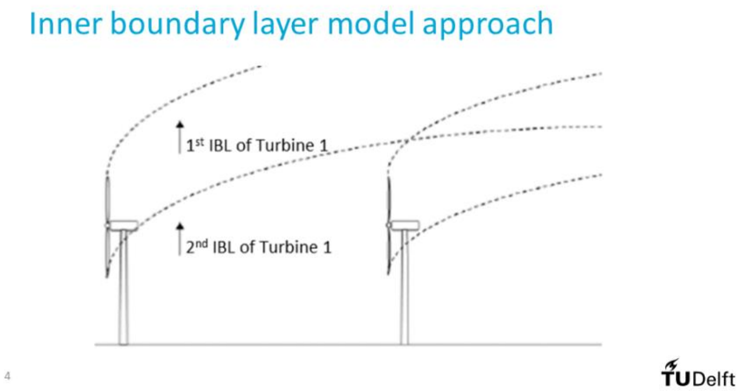
Image: Windklimaat van Nederland, J. Wieringa & P.J. Rijkoort (1983)

2

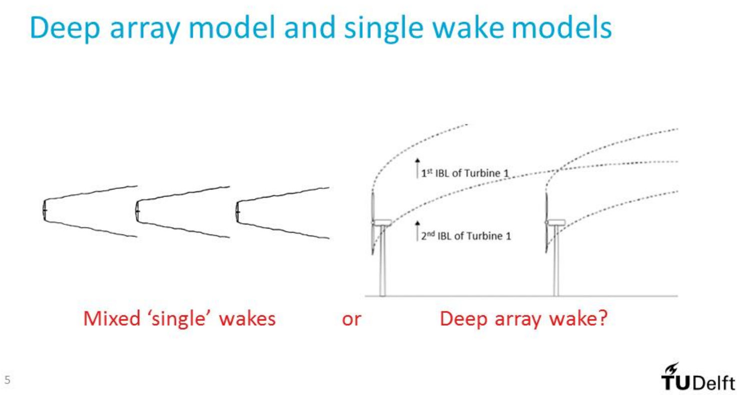
The common principle in deep array models is to consider the wind turbines as an added surface roughness. As you have seen, the larger the surface roughness, the larger the wind shear in the atmospheric boundary layer and the lower the near surface wind speed. You may also recall that surface roughness is a source of turbulence. Treating turbines as surface roughness therefore enables to capture their large scale effect on the wind, without going into the details of individual wakes.



The effect of different surface roughness categories on the wind shear profile, such as captured in the logarithmic law, only applies for fully developed boundary layers. This development requires a larger distance than the depth into the wind farm that we are interested in. Therefore, several deep array models use the concept of an inner boundary layer. In this concept a new boundary layer develops inside an existing boundary layer at a point where the surface roughness changes. This is for instance the case at the transition from sea to land. The inner boundary layer grows in height and the new surface roughness determines the wind shear inside this new boundary layer. The original boundary layer keeps its original wind shear. At the top of the inner boundary layer the wind speed must be continuous, which is used to connect the wind speed between the inner and outer boundary layer.



As an example, here you see the development of an internal boundary layer based on the deep array model proposed by Frandsen. This model is used to determine the adjusted wind farm ambient wind speed. Frandsen proposed a model along similar lines for the adjusted wind farm ambient turbulence. Since only the rotor affects the wind speed, this model introduces two inner boundary layers. One starts at the top of the rotor and for this boundary layer the rotor is modelled as surface roughness. Below the rotor, an inner boundary layer is started that develops according to the original surface roughness of the earth surface. Also here, at each transition between boundary layers continuity of wind speed is assumed.



The deep array models by themselves are not sufficient to determine the conditions at turbines deep inside the wind farm. These turbines can still be affected more by the wakes of nearby turbines than by the changed ambient conditions. Therefore, the deep array models are combined with the individual wake models that were presented earlier. There are suggestions to accumulate the effects that are determined with each type of model. However, there are also suggestions to test the deep array model and the individual wake models and then pick the most conservative result. The deep array models are too complex for hand calculations. Therefore, this video only provides descriptions of some modelling principles. The mathematical expressions and algorithms are beyond the scope of this course.