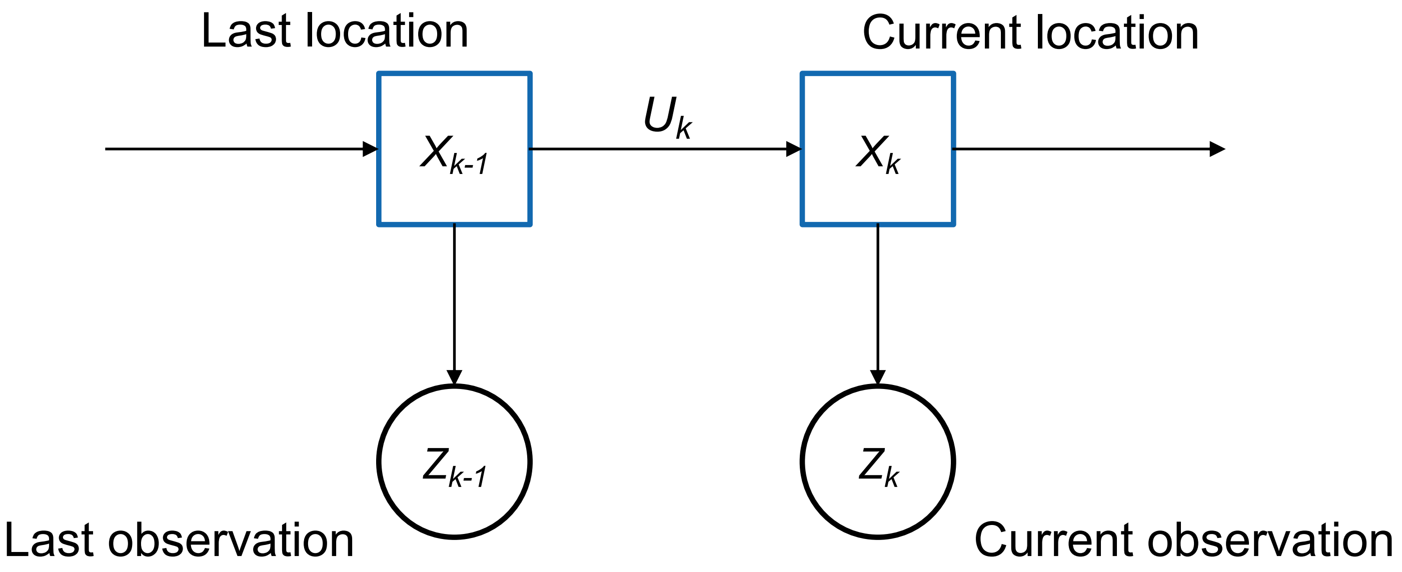
Computer Vision HS 2020
Lab Session 3 - Particle Filter

Zuoyue Li

Fri, 9 Oct 2020

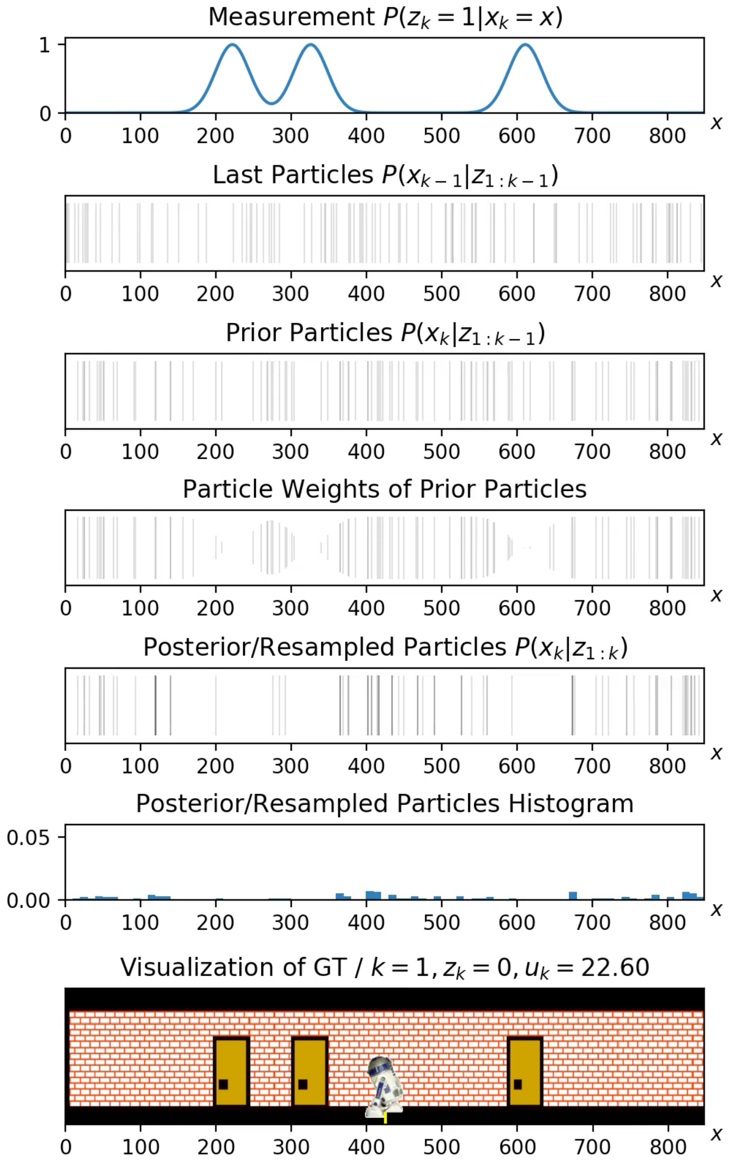
Goal: 1D Localization of a Robot

- Hidden Markov Model (HMM)

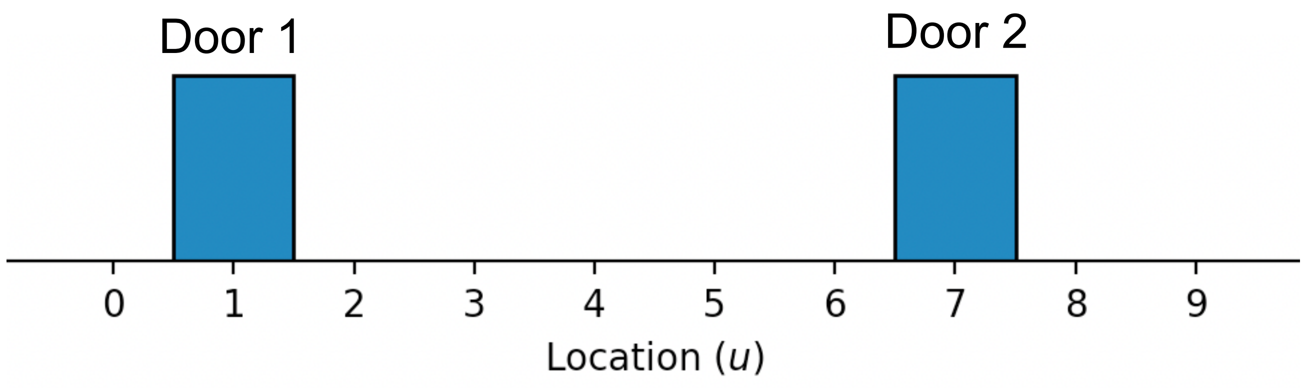


Goal: 1D Localization of a Robot

- What is known
  - Current odometry U_k
    How far travelled since last step
  - Door sensor observation Z_k
    Whether the sensor detects a door
- What we need to compute
  - Posterior: the current location distribution based on the observations so far $P(X_k = x \mid Z_{1:k})$
  - Prior: the current location distribution based on the observations until the last step $P(X_k = x \mid Z_{1:k-1})$
  - Alternating update



Example



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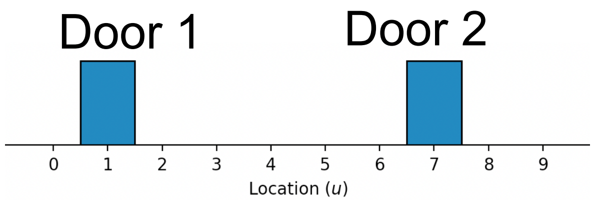
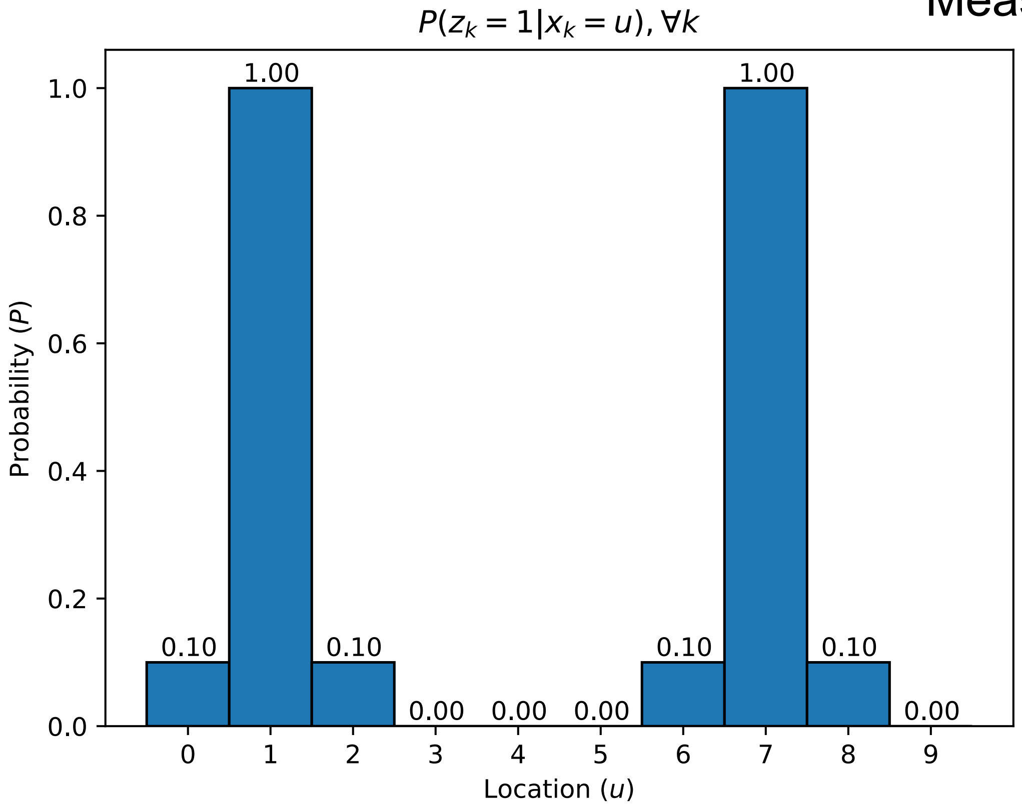
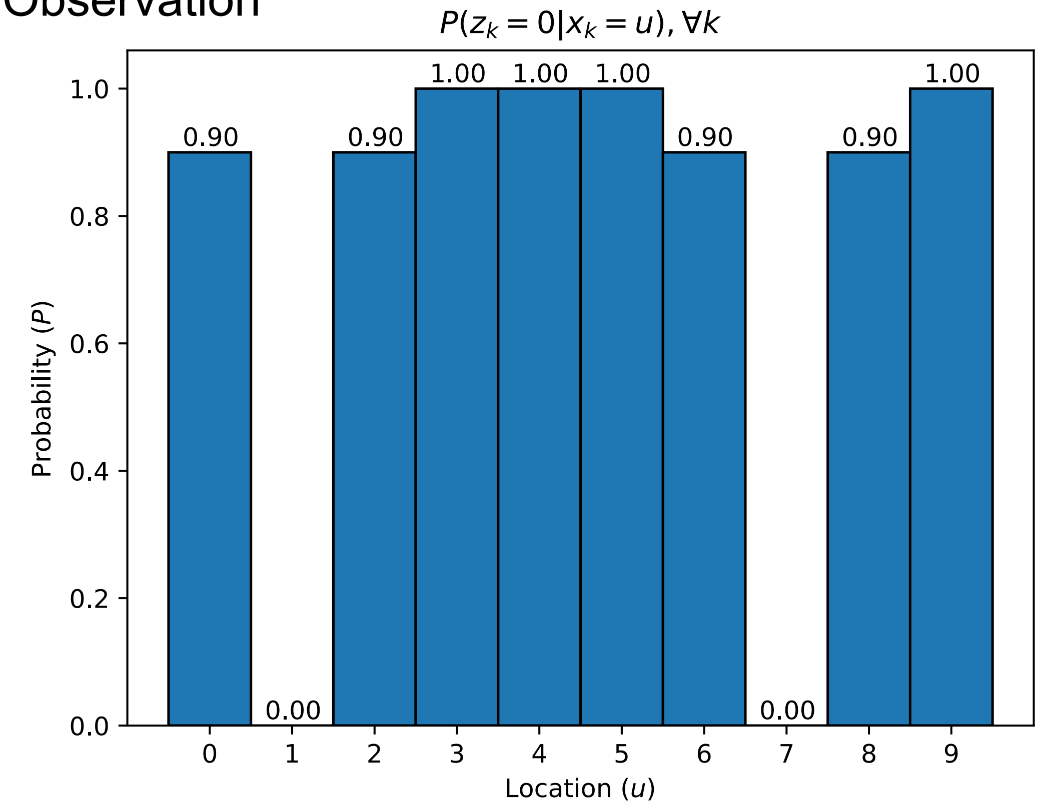


Measurement/Observation

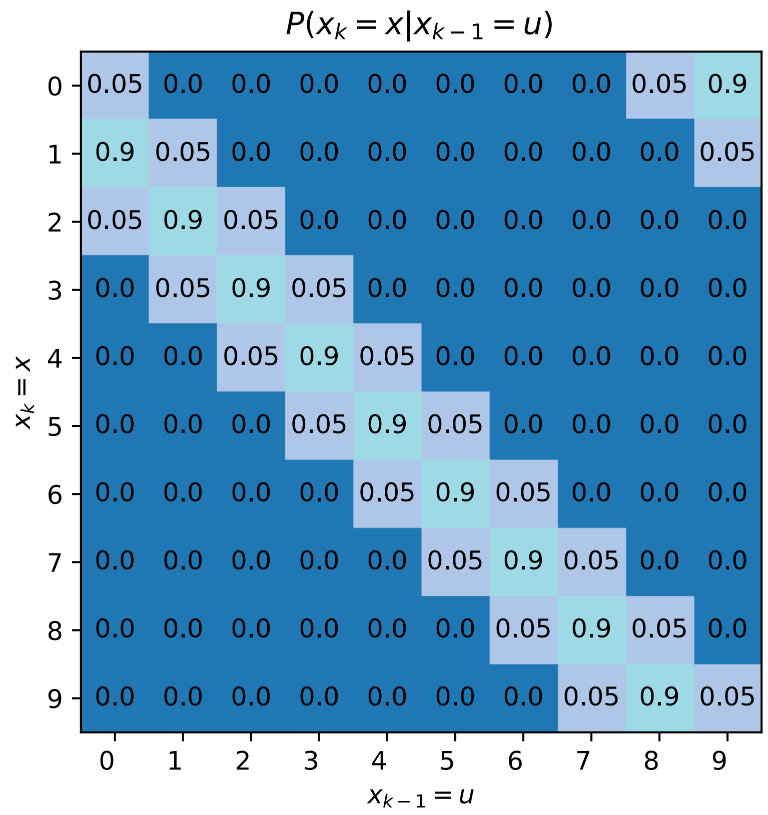




Example

Transition Matrix

P(Remain in the same place) = 0.05
P(Go to the right location) = 0.9
P(Skip the next one) = 0.05



Example

- Compute posterior using prior

Split conditions $P(X_k = x \mid Z_{1:k}) = P(X_k = x \mid Z_k, Z_{1:k-1})$

Bayes' theorem $= \dfrac{P(Z_k \mid X_k = x, Z_{1:k-1}) P(X_k = x \mid Z_{1:k-1})}{P(Z_k \mid Z_{1:k-1})}$

Conditional independence $= \dfrac{P(Z_k \mid X_k = x) P(X_k = x \mid Z_{1:k-1})}{P(Z_k \mid Z_{1:k-1})}$

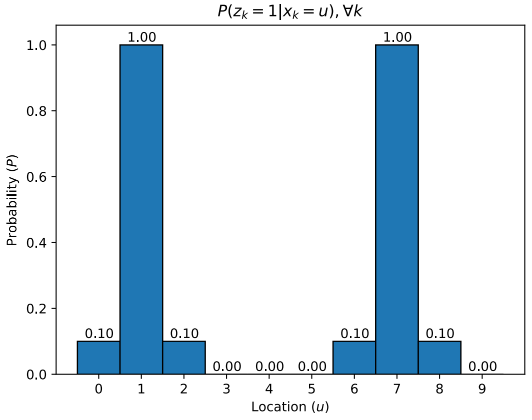
$\propto P(Z_k \mid X_k = x) P(X_k = x \mid Z_{1:k-1})$

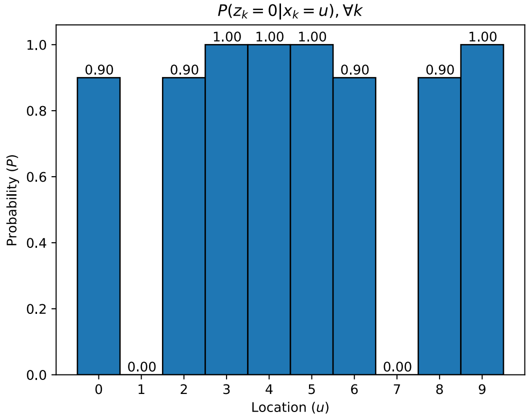
Measurement based on the current observation

Prior

Element-wise product

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$$





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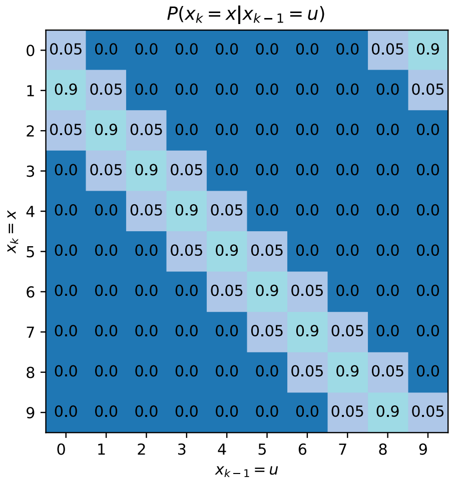


- Compute prior using the last posterior

Marginal distribution $$P(X_k = x \mid Z_{1:k-1}) = \sum_u P(X_k = x, X_{k-1} = u \mid Z_{1:k-1})$$

Conditional distribution $$= \sum_u P(X_k = x \mid X_{k-1} = u, Z_{1:k-1}) P(X_{k-1} = u \mid Z_{1:k-1})$$

Conditional independence $$= \sum_u P(X_k = x \mid X_{k-1} = u) P(X_{k-1} = u \mid Z_{1:k-1})$$

Transition matrix

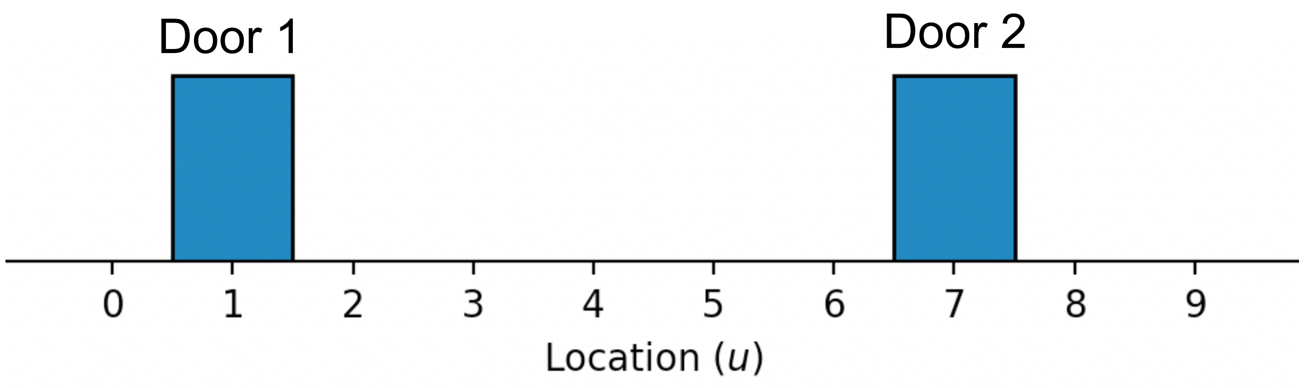
Last Posterior

Matrix product

$$P(AB) = P(A|B)P(B)$$

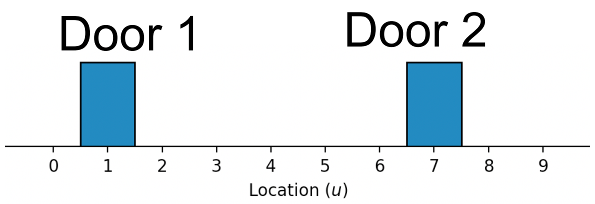
Example

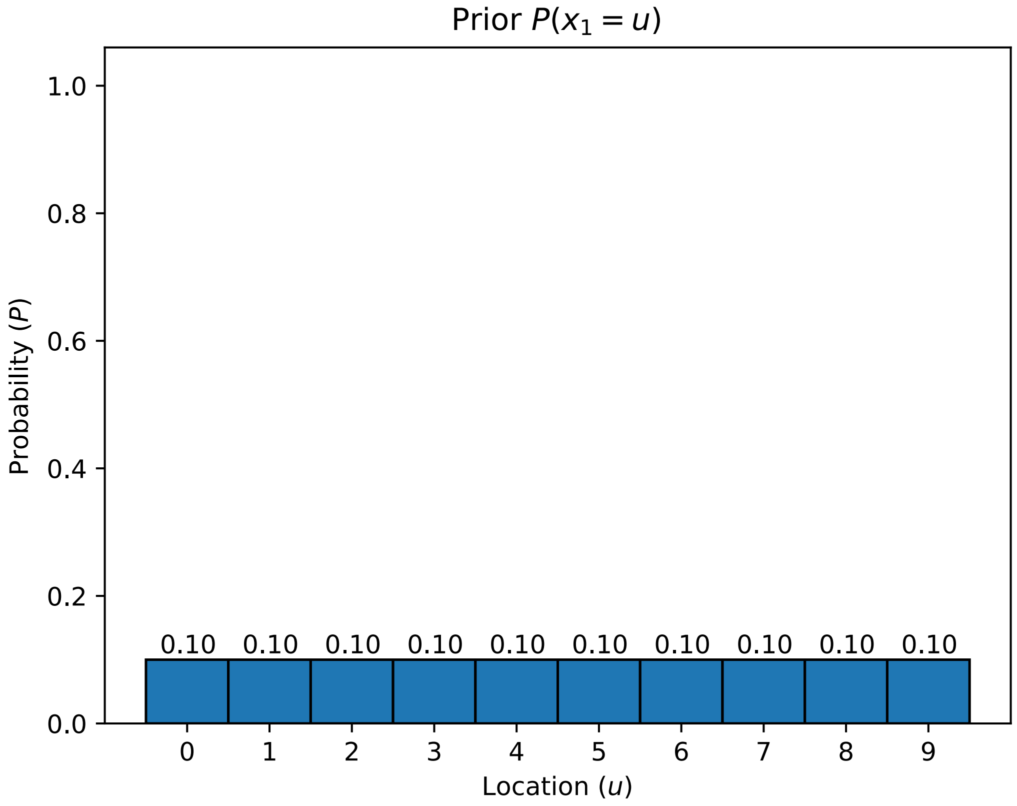
- Door sensor observation: [0, 1, 0, 0, 0, 1]
- Guess what may be the starting location and the current location?



Example

- Observation: [0, 1, 0, 0, 0, 1]





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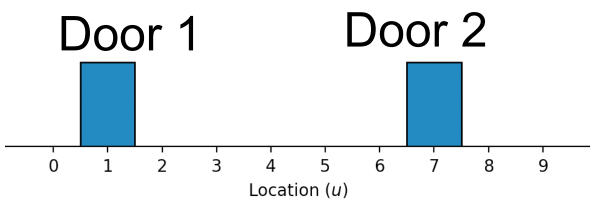

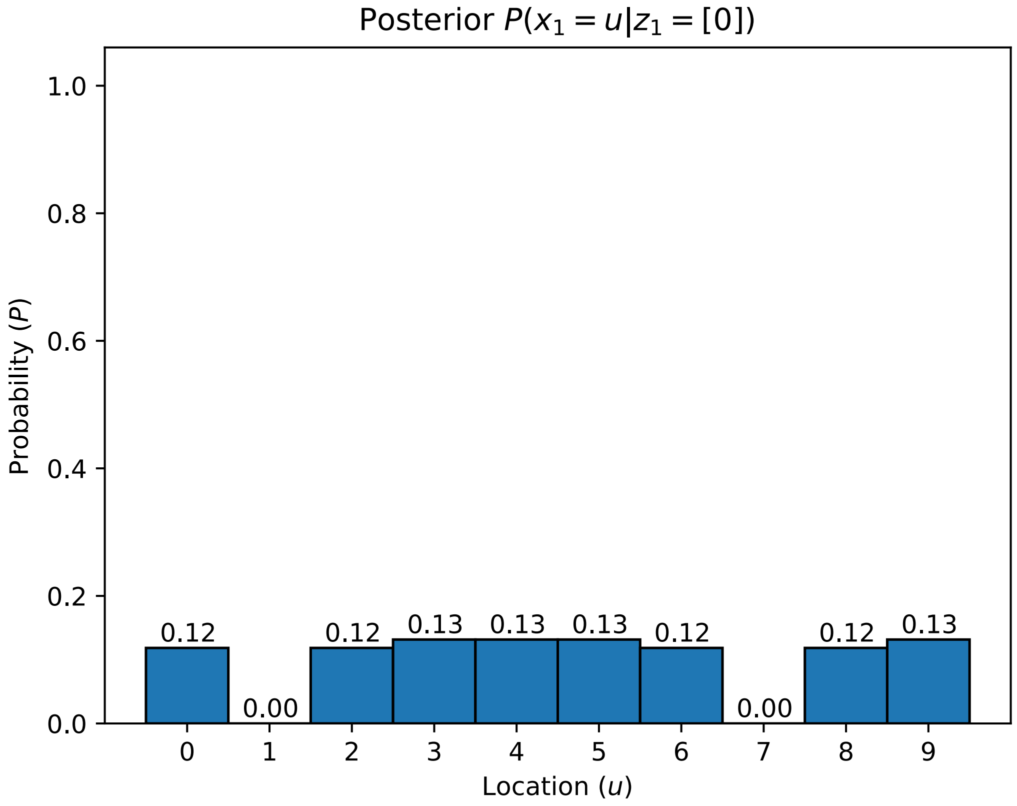

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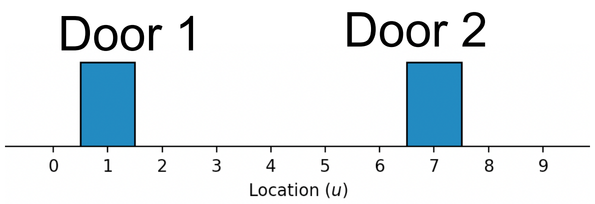

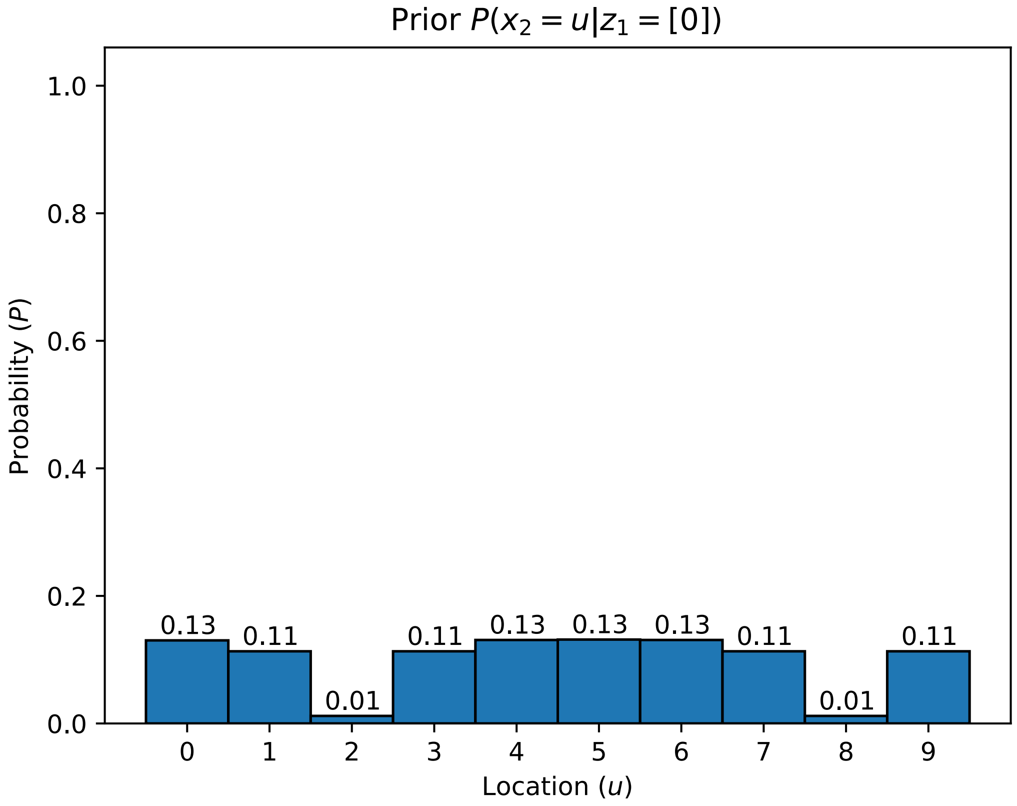

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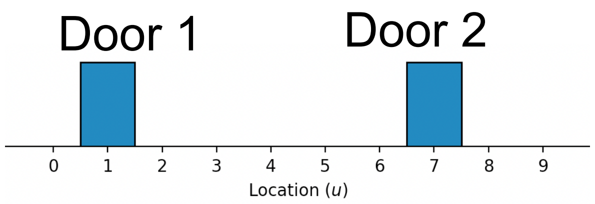

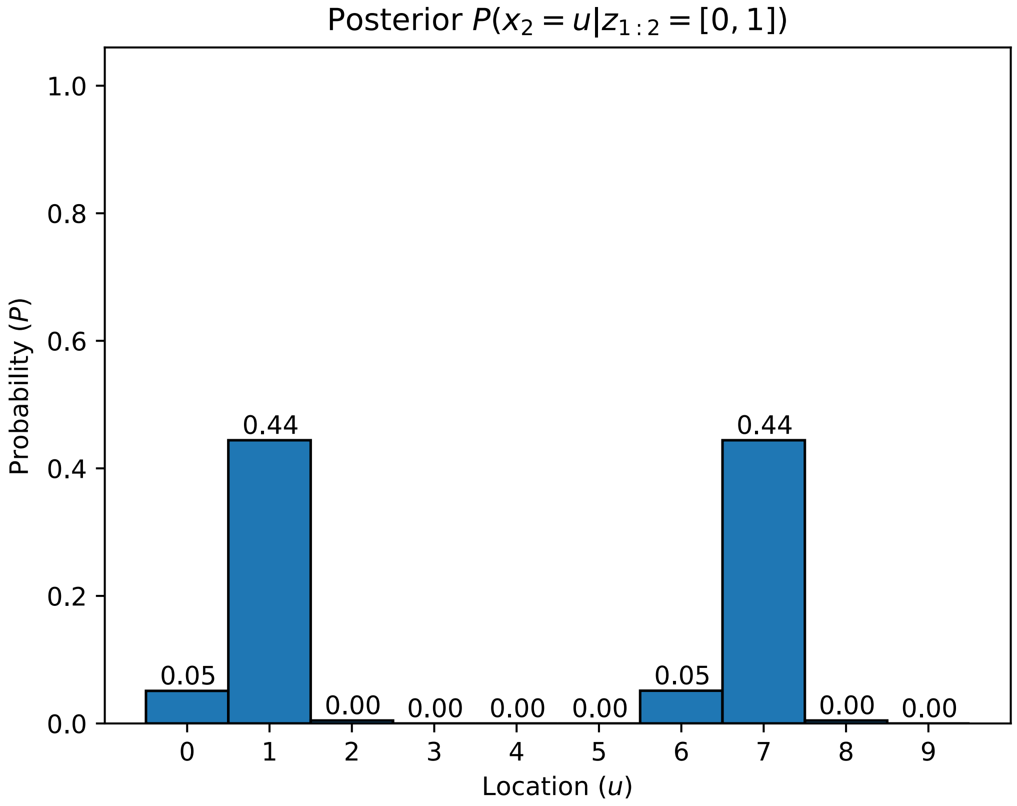

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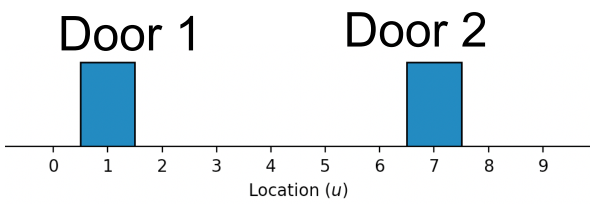

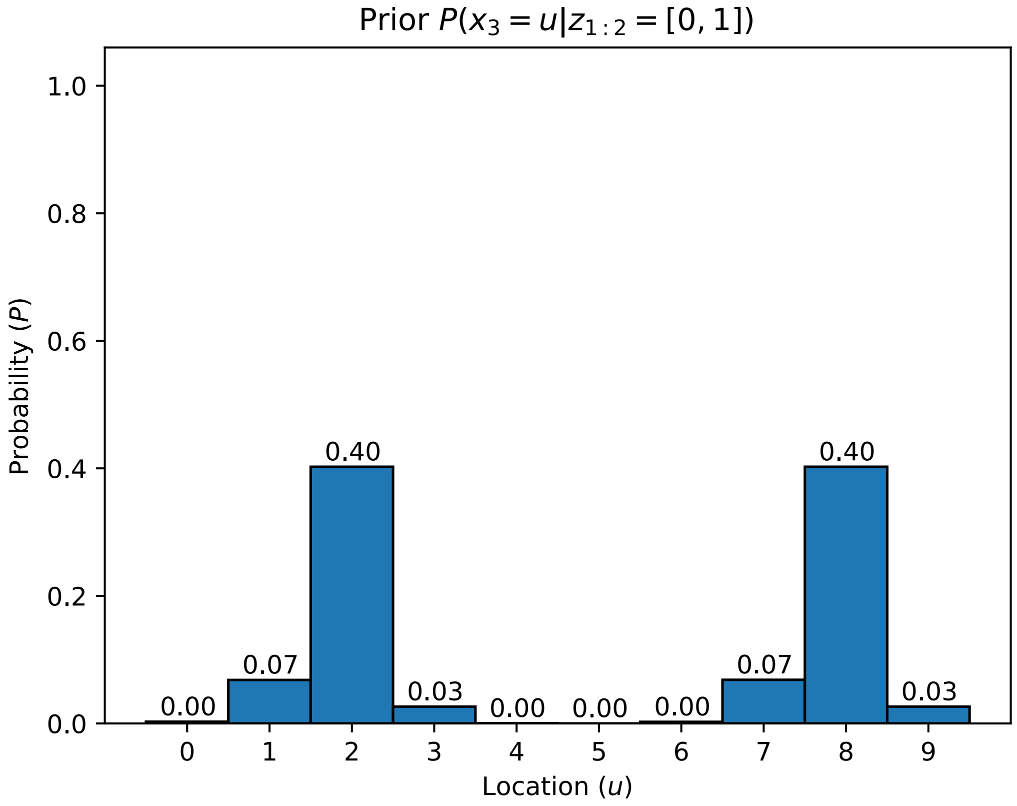

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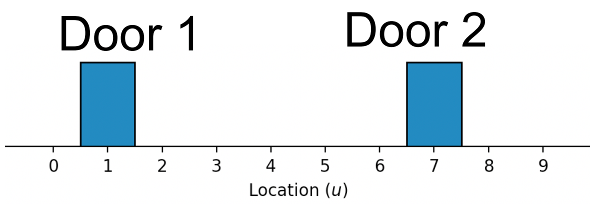

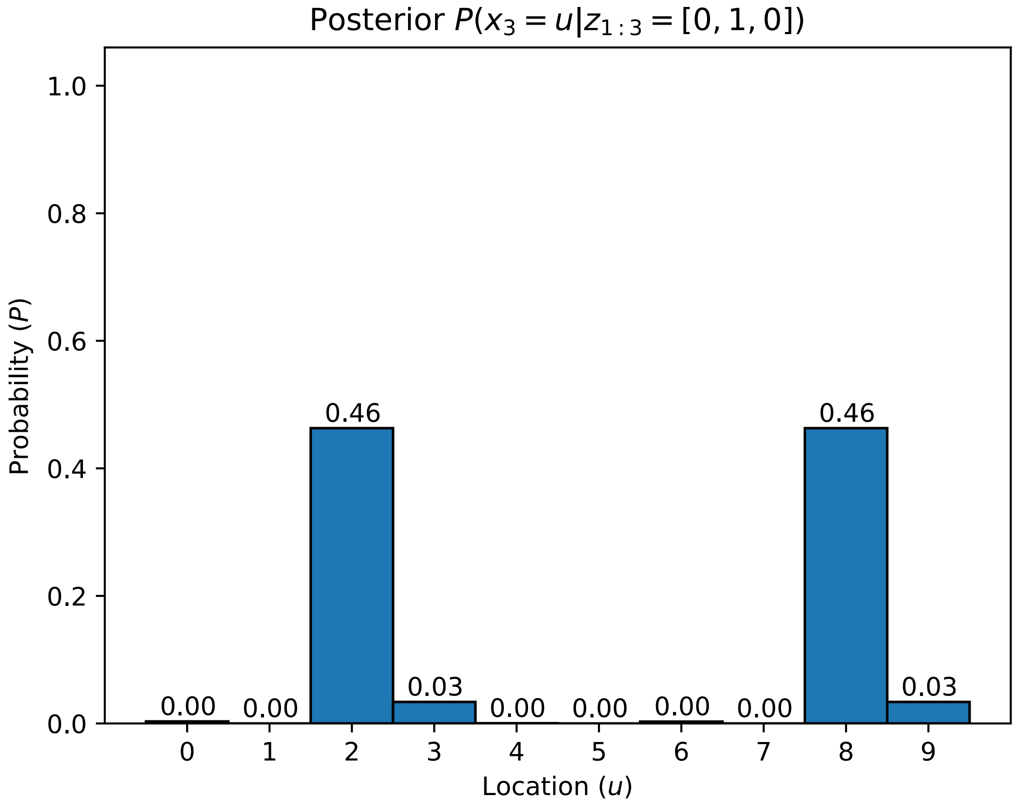

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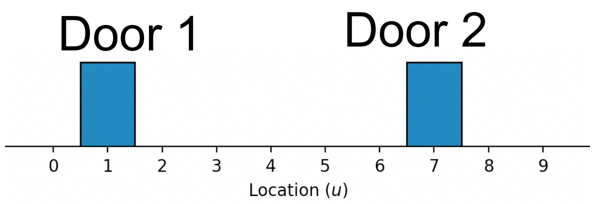

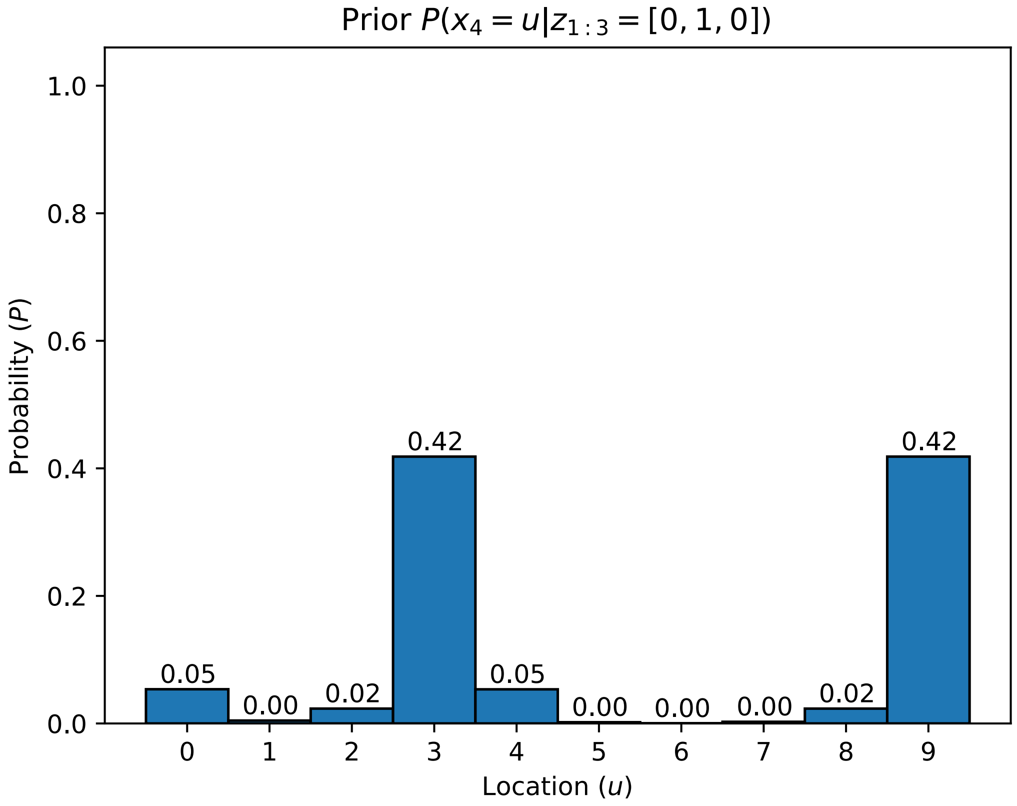

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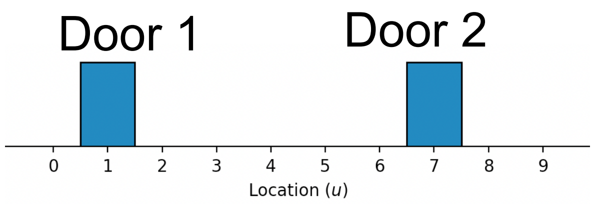

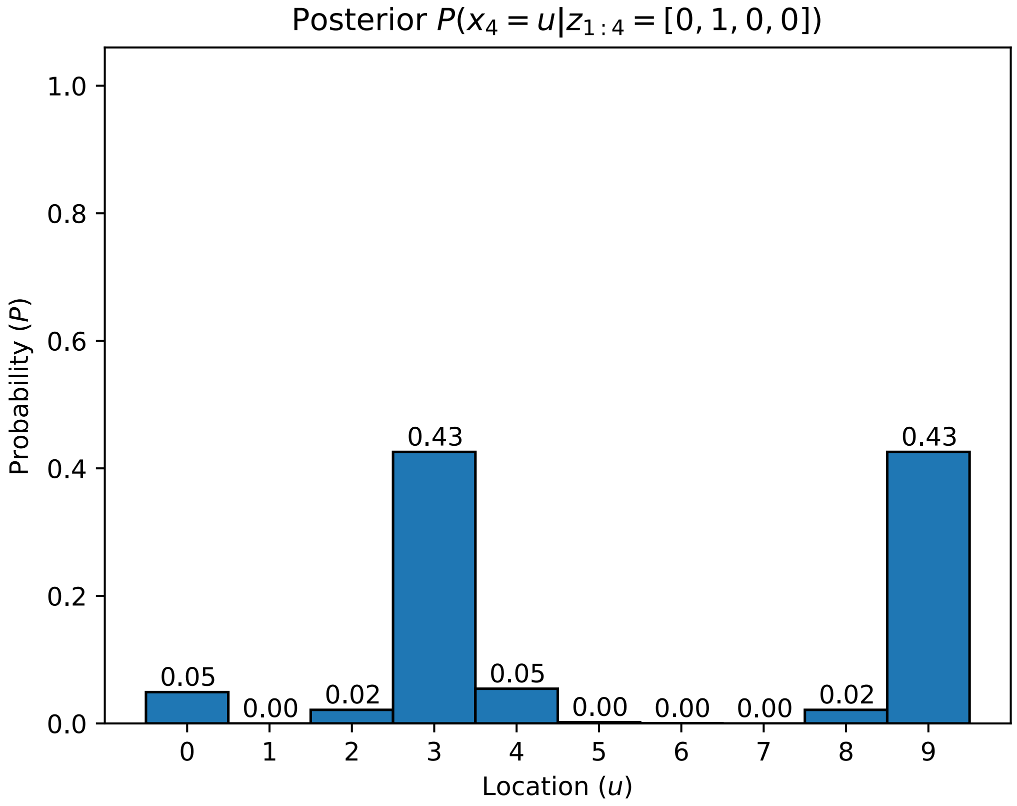

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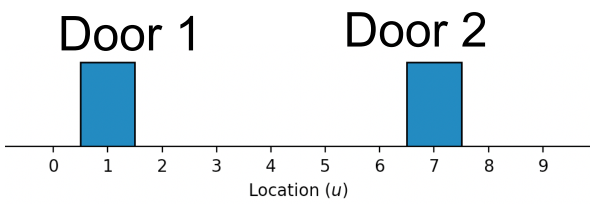

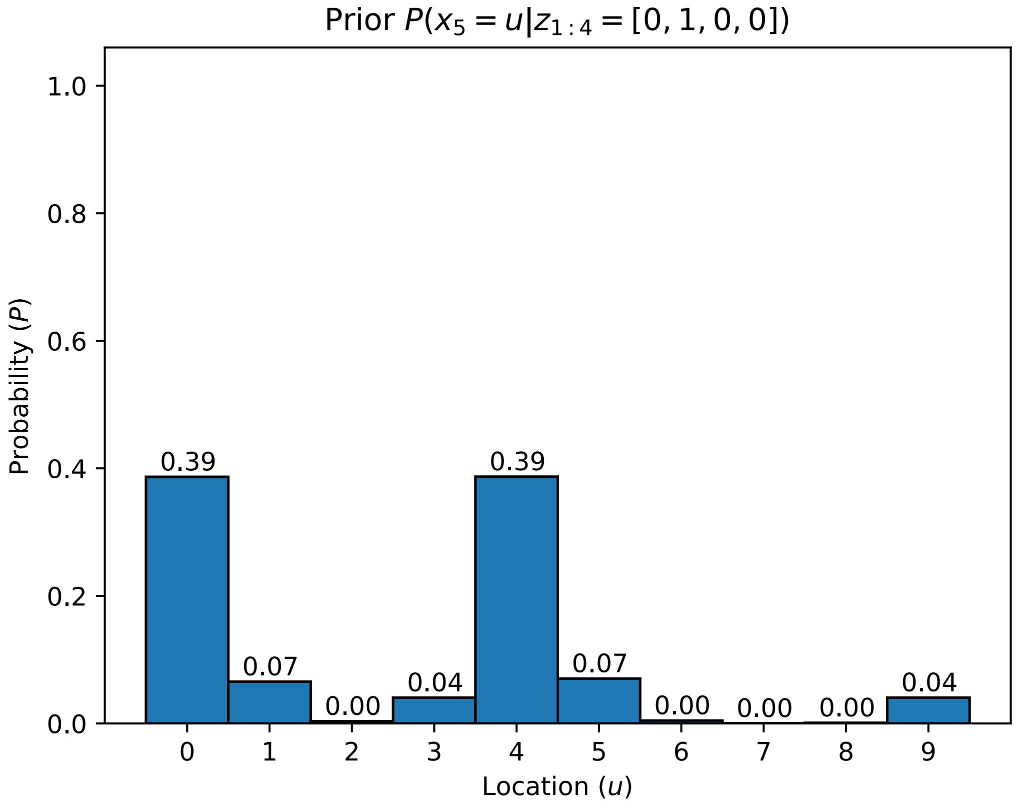

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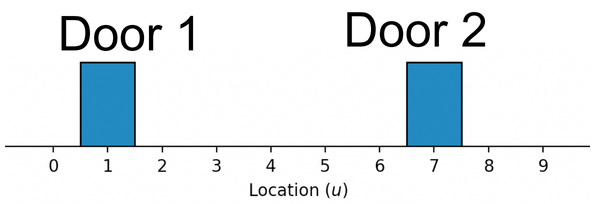

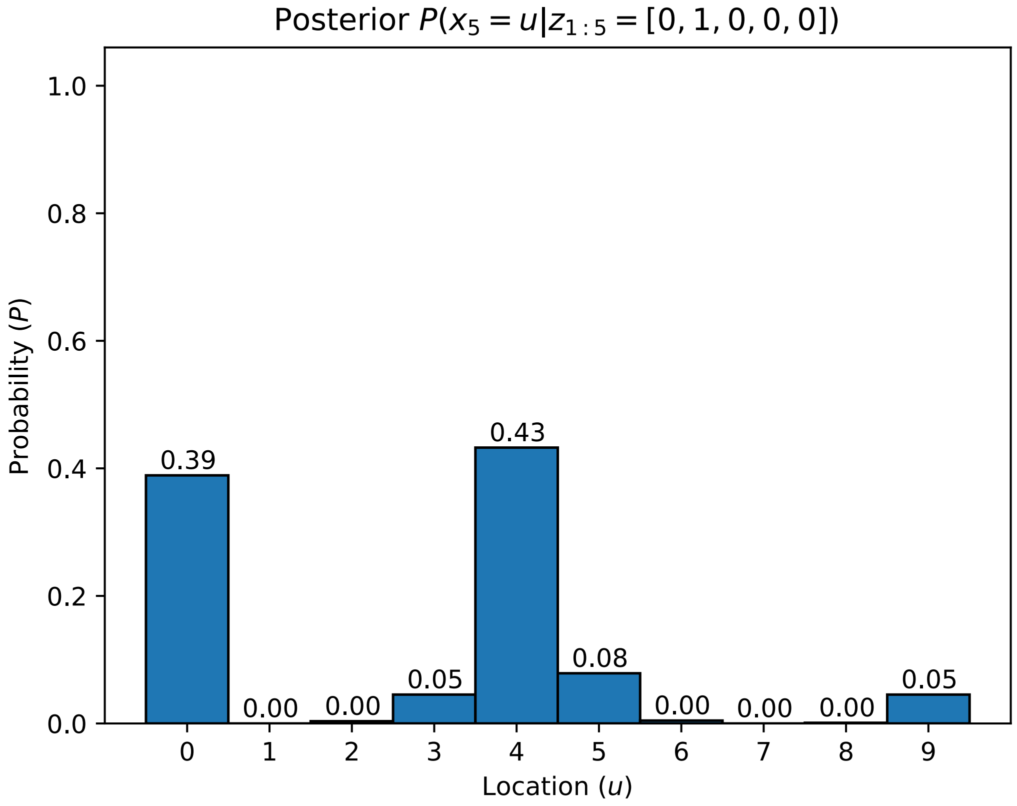

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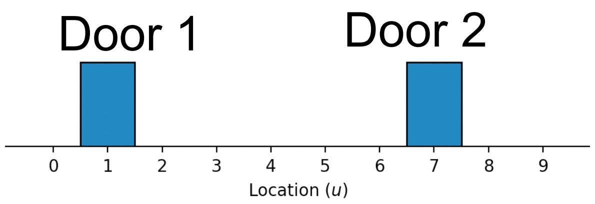

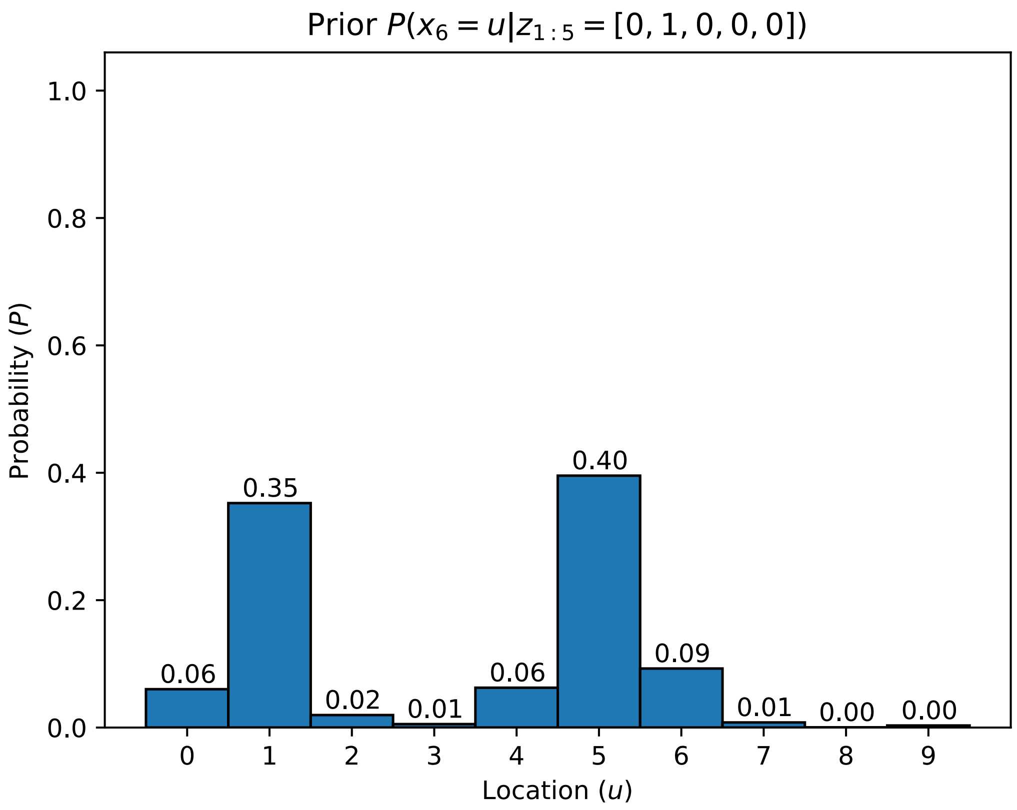

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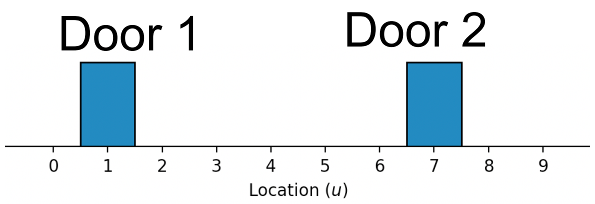
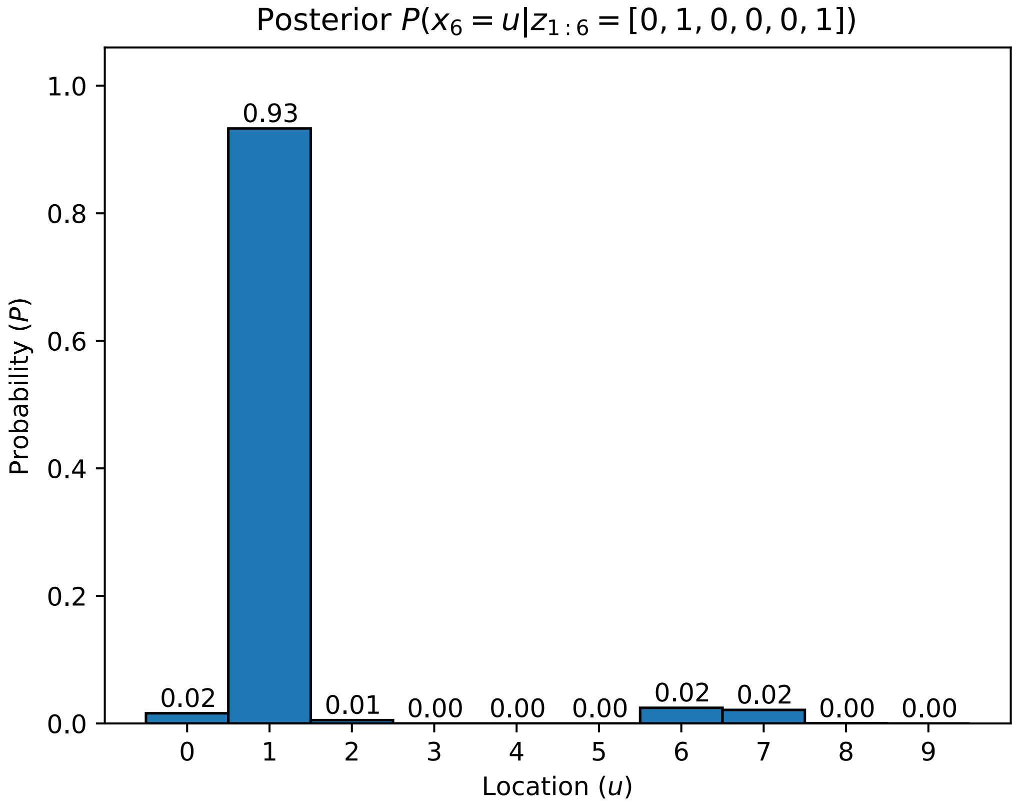
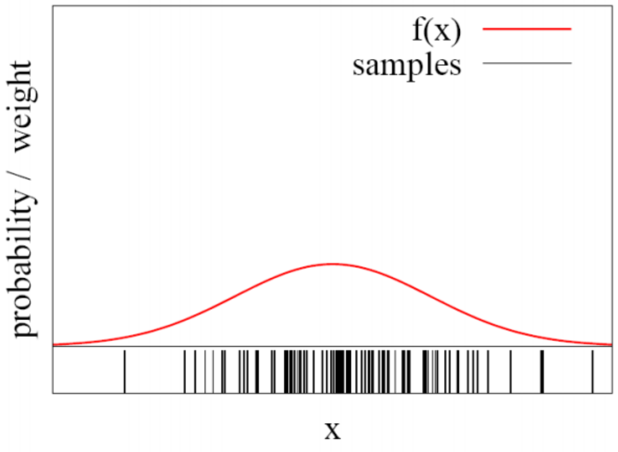
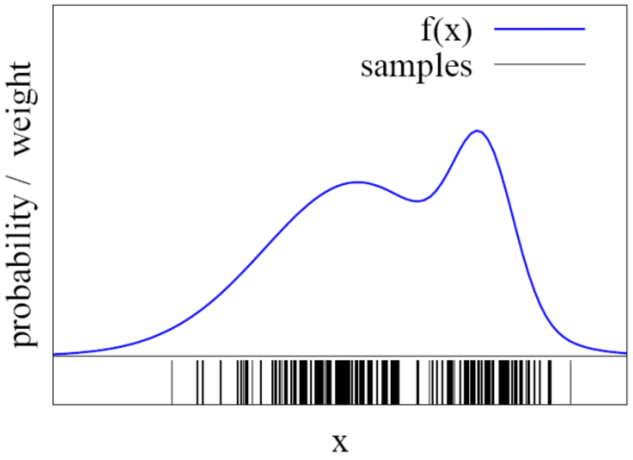




Why Particles?

- Unable to traverse the full domain
  - Continuous distribution
  - High-dimensional discrete space
- Use particles for function approximation





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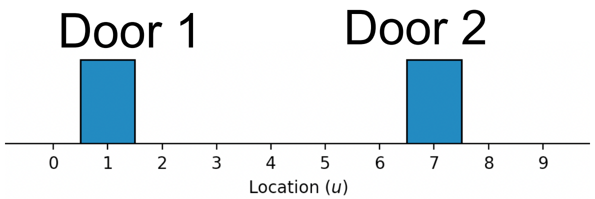

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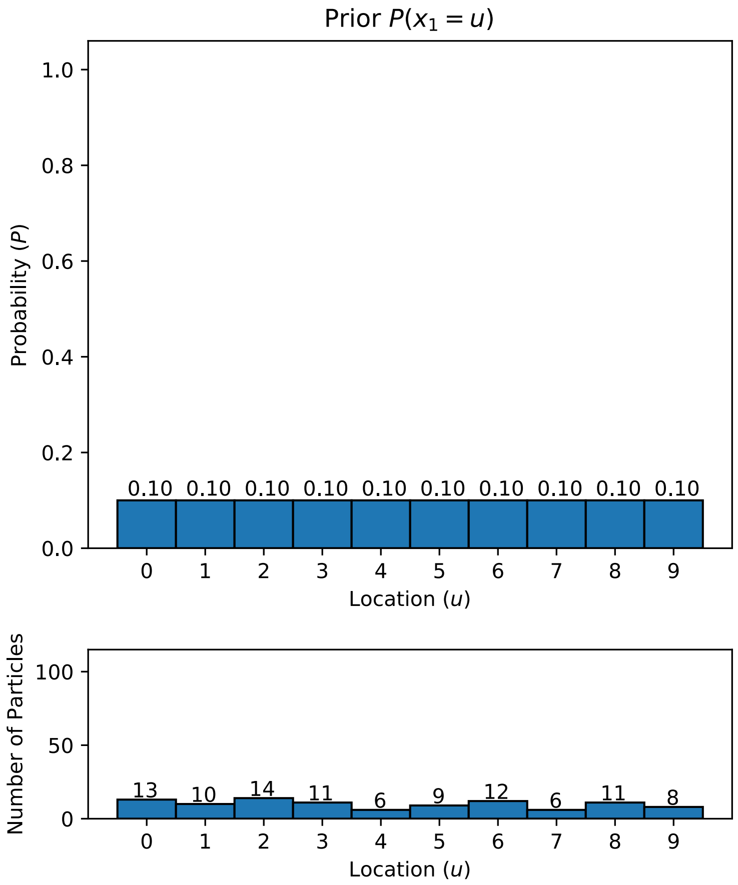

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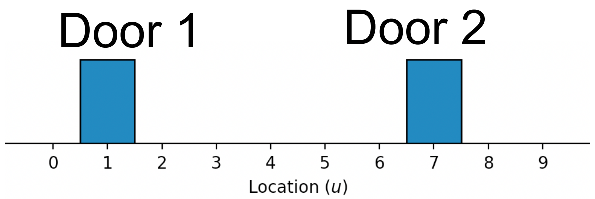

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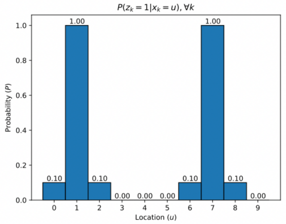
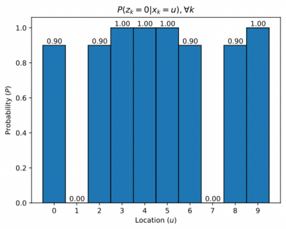
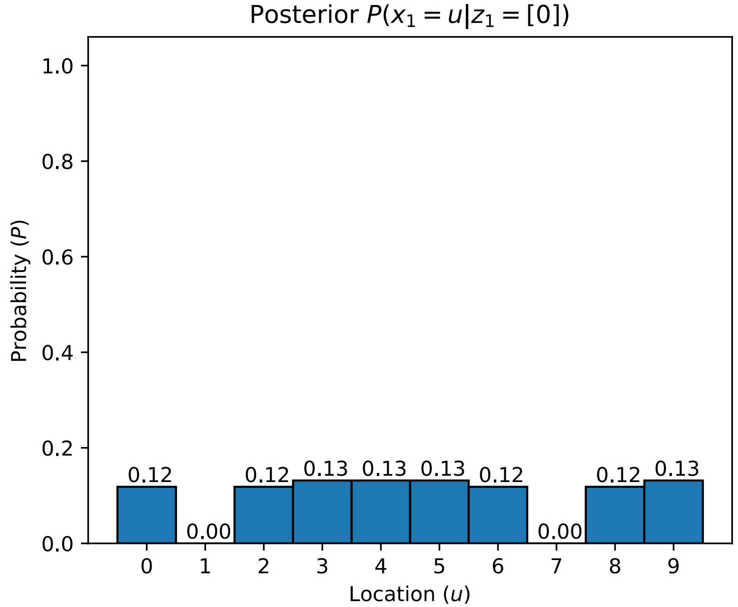
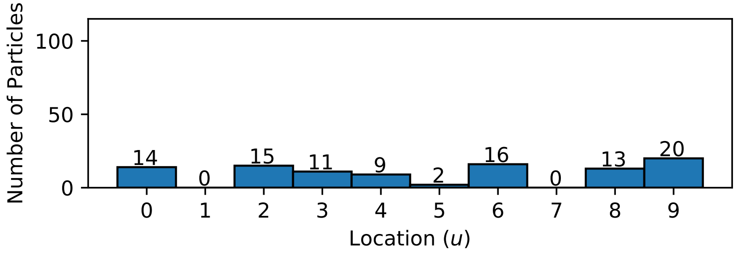








- Compute posterior using prior

Split conditions $P(X_k = x \mid Z_{1:k}) = P(X_k = x \mid Z_k, Z_{1:k-1})$

Bayes' theorem $= \dfrac{P(Z_k \mid X_k = x, Z_{1:k-1})P(X_k = x \mid Z_{1:k-1})}{P(Z_k \mid Z_{1:k-1})}$

Conditional independence $= \dfrac{P(Z_k \mid X_k = x)P(X_k = x \mid Z_{1:k-1})}{P(Z_k \mid Z_{1:k-1})}$

$\propto P(Z_k \mid X_k = x)P(X_k = x \mid Z_{1:k-1})$

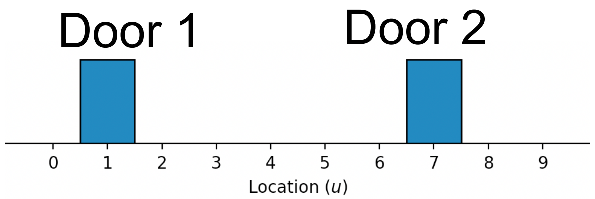
Measurement based on the current observation

Prior

Element-wise production

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$$

Example



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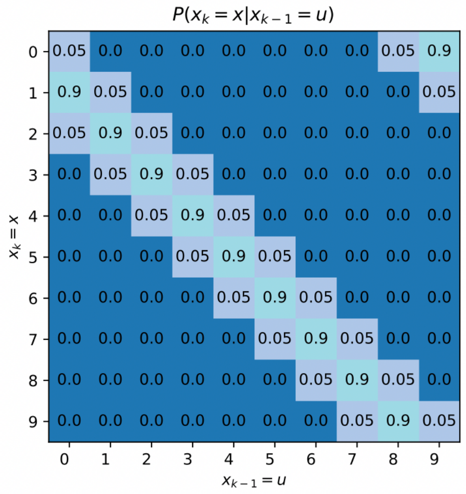
Transition Matrix



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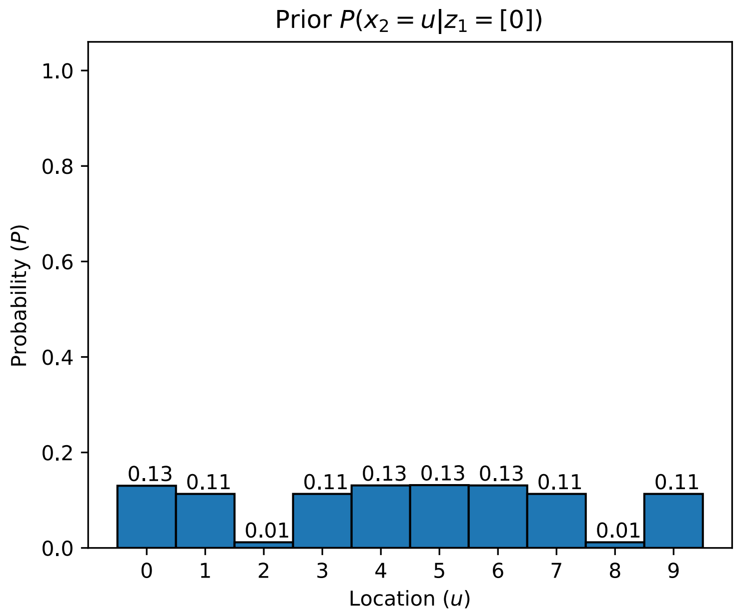

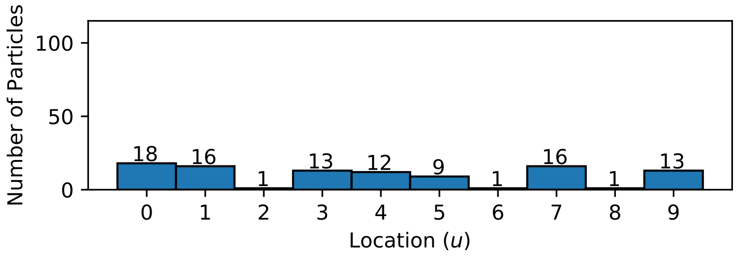

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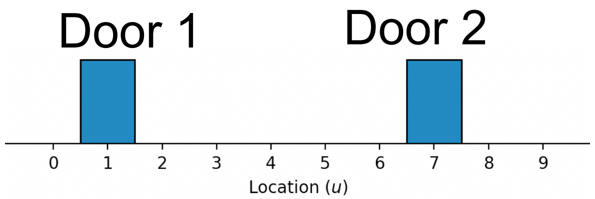

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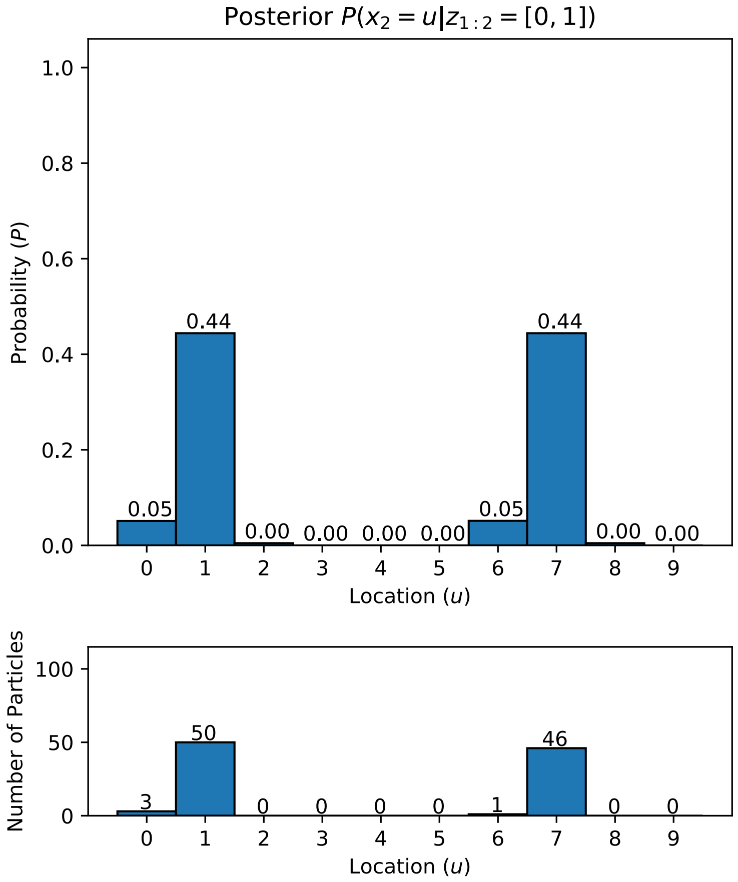

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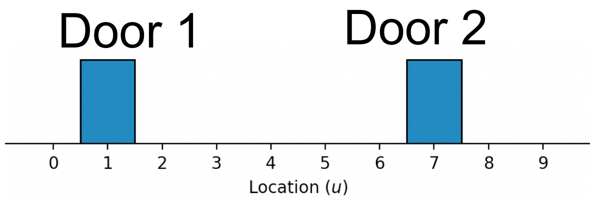

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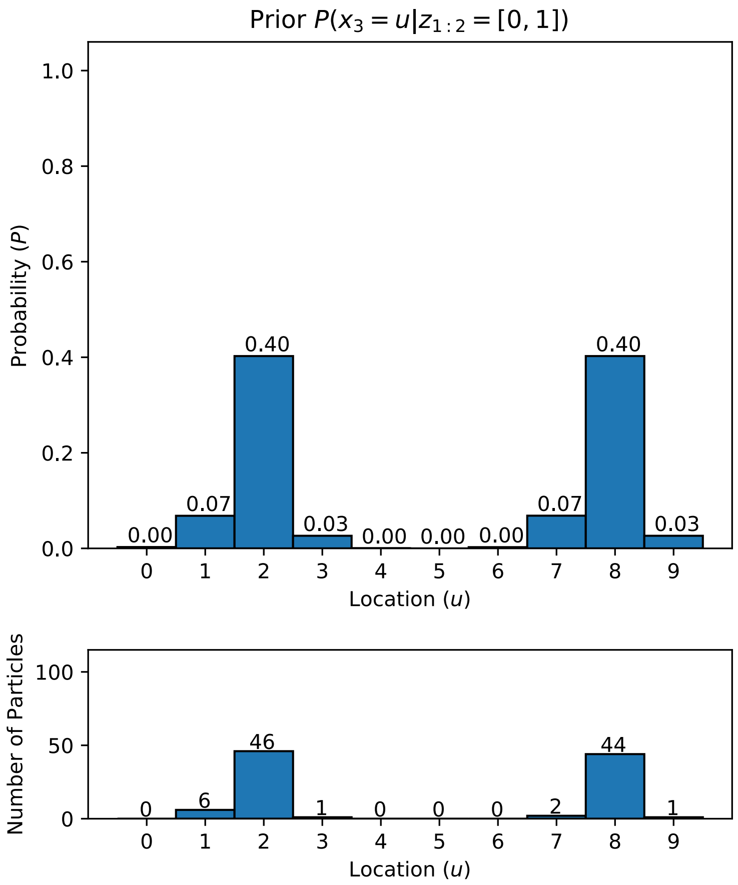

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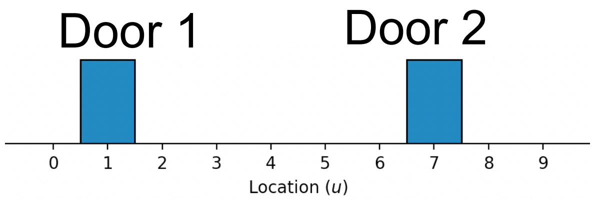

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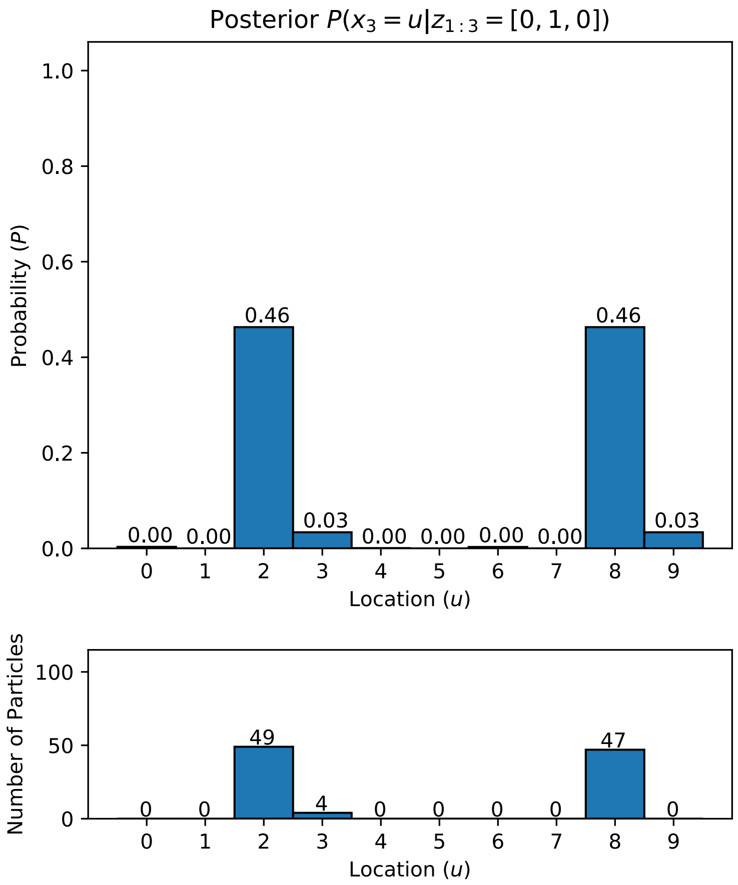

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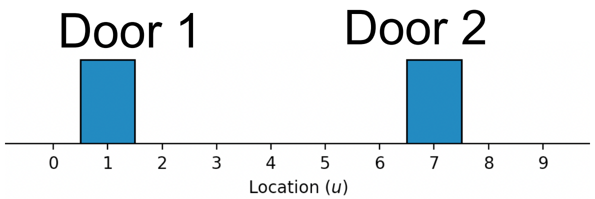

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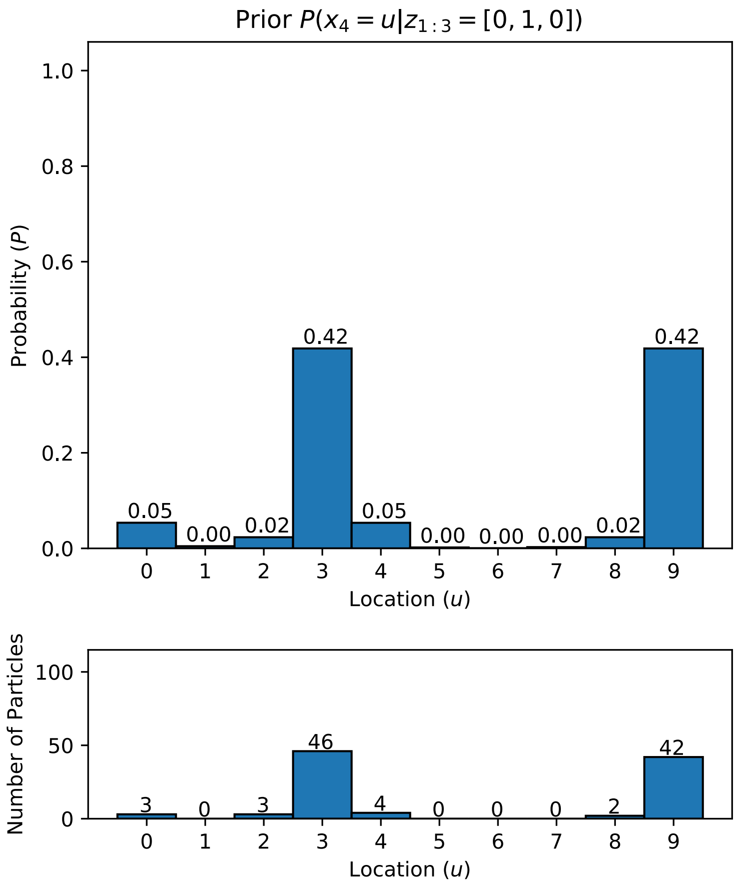

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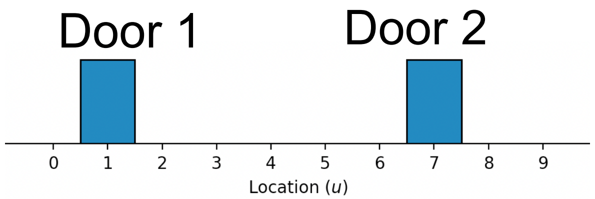

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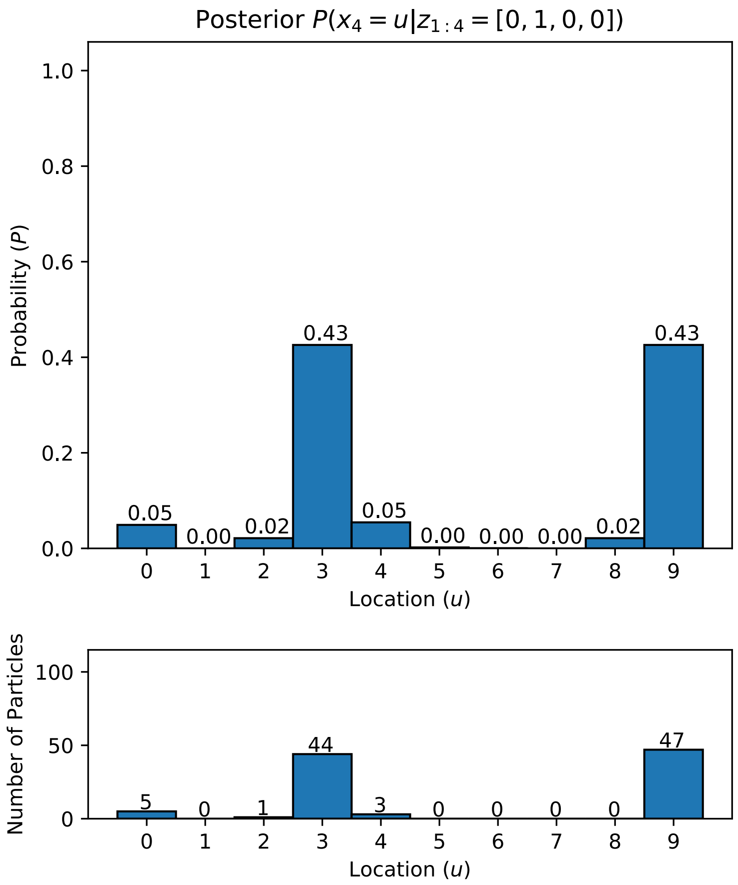

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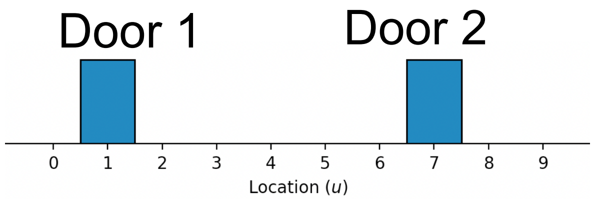

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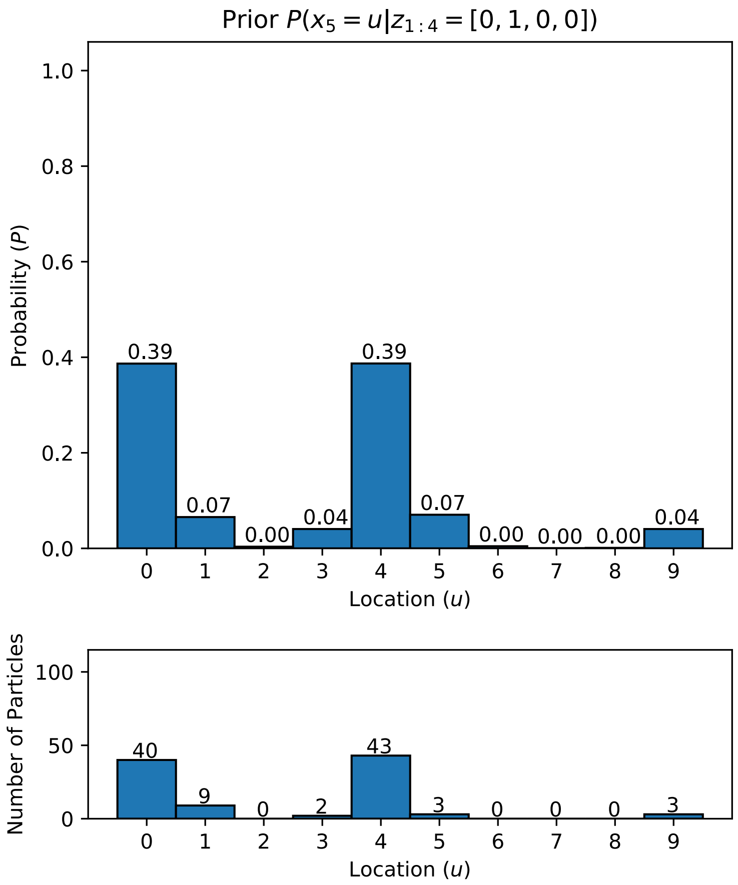

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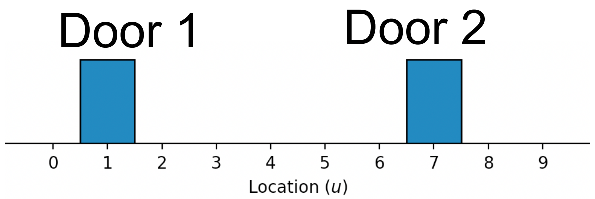

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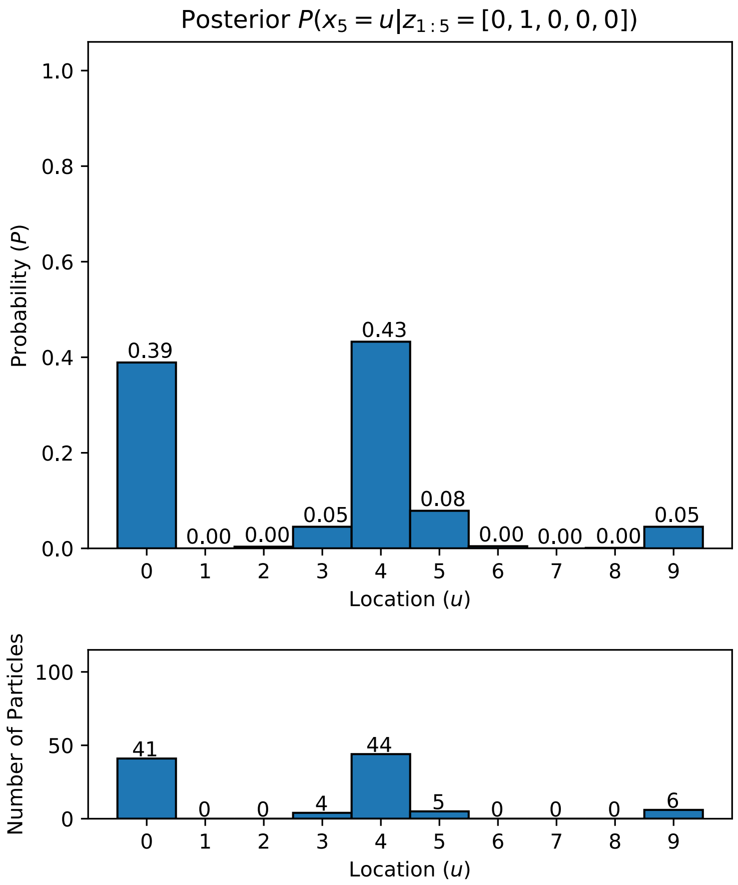

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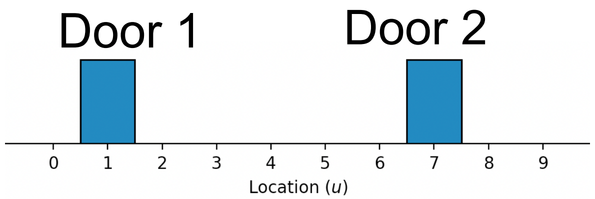
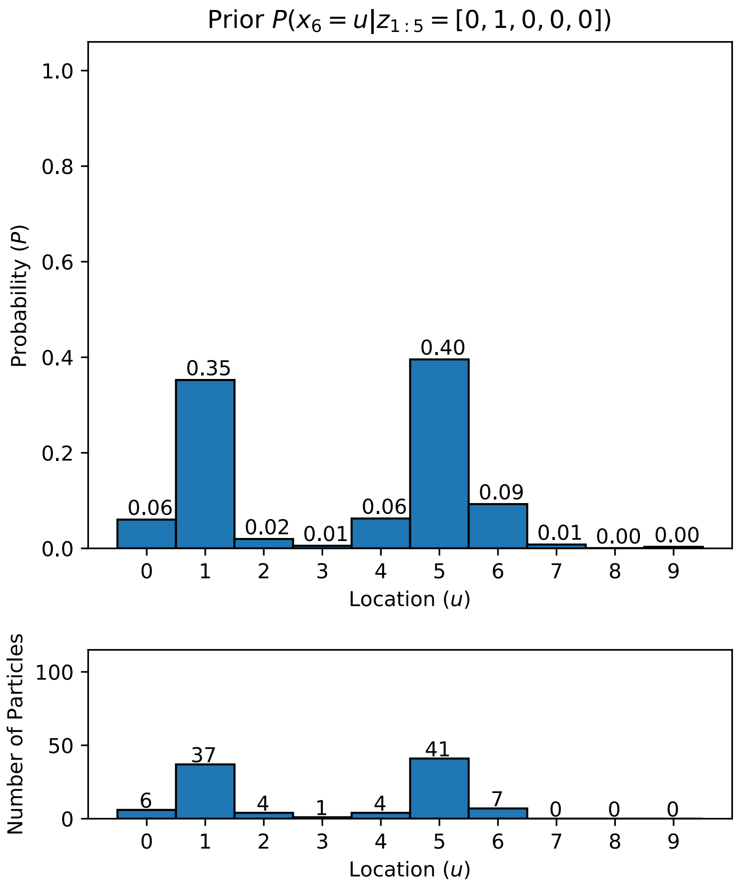
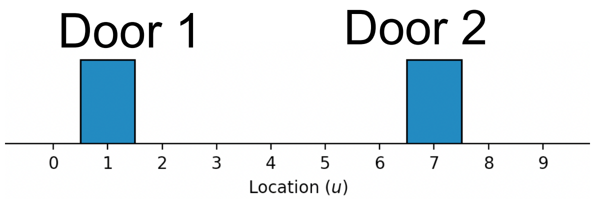


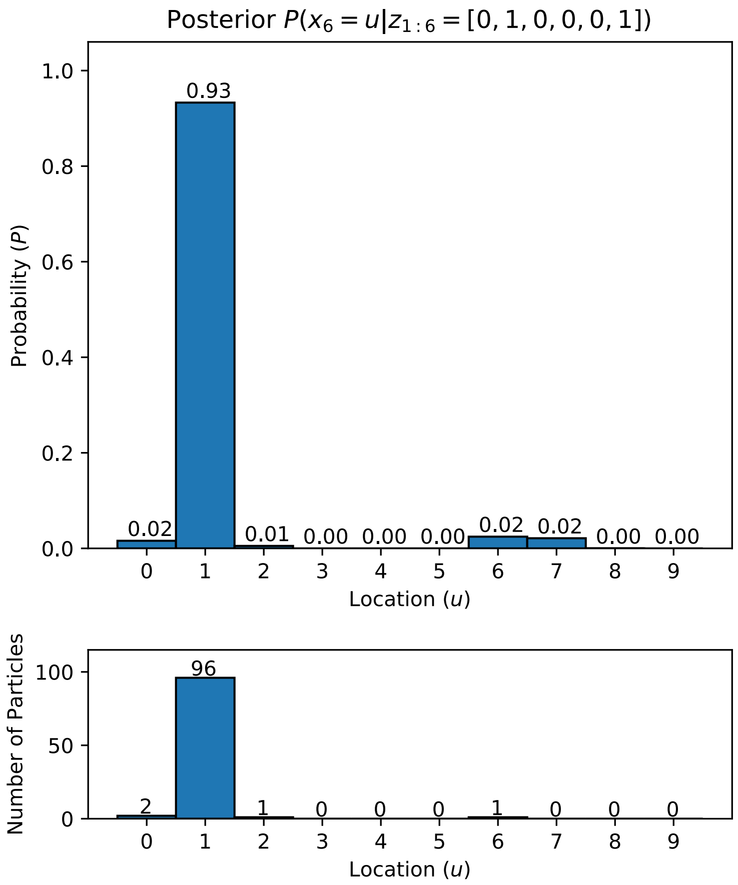
- Observation: [0, 1, 0, 0, 0, 1]



Example



- Observation: [0, 1, 0, 0, 0, 1]



Thank you!