1 (continued)

0 $\nabla \cdot \phi_1 + \Sigma_a \phi_0 = S_0$

1 $\left(\frac{2}{3}\right) \nabla \phi_2 + \left(\frac{1}{3}\right) \nabla \phi_0 + \Sigma_t \vec{\phi}_1 = \Sigma_{s1} \vec{\phi}_1$

2 $\left(\frac{3}{5}\right) \nabla \cdot \vec{\phi}_3 + \left(\frac{2}{5}\right) \nabla \cdot \vec{\phi}_1 + \Sigma_t \phi_2 = \Sigma_{s2} \phi_2$

3 $\left(\frac{4}{7}\right) \nabla \phi_4 + \left(\frac{3}{7}\right) \nabla \phi_2 + \Sigma_t \vec{\phi}_3 = \Sigma_{s3} \vec{\phi}_3$

4 $\left(\frac{5}{9}\right) \nabla \cdot \vec{\phi}_5 + \left(\frac{4}{9}\right) \nabla \cdot \vec{\phi}_3 + \Sigma_t \phi_4 = \Sigma_{s4} \phi_4$

5 $\left(\frac{6}{11}\right) \nabla \phi_6 + \left(\frac{5}{11}\right) \nabla \phi_4 + \Sigma_t \vec{\phi}_5 = \Sigma_{s5} \vec{\phi}_5$

isotropic source $S_{\ell'} = 0$ for $\ell > 0$

gaussian quadrature

2) a/b/c

$$\int_{4\pi} d\hat{\Omega} |\Omega| = 4\pi = \sum_{i=1}^{n} w_i |\Omega| \leftarrow 1$$

sum across

over full surface normalize by $\frac{4\pi}{8}$ for $\frac{4\pi}{8} \sum_{b=1}^{N(n)} w_b = 4\pi$

		n	μ_n	w_n^b
for	S_4	1	0.3500212	.333
		2	0.8688903	
	S_6	1	0.2666355	0.1761263
		2	0.6815076	0.1570711
		3	0.9261808	

for one octant

a) for S_4 $\int_{4\pi} d\Omega |\Omega| = 4\pi = \frac{4\pi}{8}\left(8 \sum w_n^b\right)$

By figure 4.3 in L&M

$= \frac{4\pi}{8}\left(8(.333 \times 3)\right)$

$= 4\pi$

b) for S_6 $\int_{4\pi} d\Omega |\Omega| = 4\pi = \frac{4\pi}{8}\left(8 \sum w_n^b\right)$

$= \frac{4\pi}{8}\left(8(0.1761263 \times 3 + 0.1570771 \times 3)\right)$

$= 4\pi$

1) $\left(\frac{l'+1}{2l'+1}\right)\frac{d}{dx}\phi_{l'+1}(x)+\left(\frac{l'}{2l'+1}\right)\frac{d}{dx}\phi_{l'-1}(x)+\Sigma_t(x)\phi_{l'}=\Sigma_{sl'}(x)\phi_{l'}(x)+S_{l'}(x)$

✓ $\phi_{-1}=0$? $\phi_{N+1}=0$ or $\frac{d}{dx}\phi_{N+1}=0$

use second order form w/ Marshack bounds

$$-\frac{d}{dx}D\frac{d\phi_0}{dx}+\Sigma_a(x)\phi_0(x)=S_0(x),\quad 0<x<X$$

$$\frac{1}{2}\phi_0(0)-D\frac{d\phi_0}{dx}(0)=2J^+(0)$$

$$\frac{1}{2}\phi_0(X)+D\frac{d\phi_0}{dx}(X)=2J^-(X)$$

where $D=\frac{1}{3[\Sigma_t(x)-\Sigma_{s1}(x)]}$

generalize to 3-D by following

Replace 1-D diffusion Operator $\frac{d}{dx}D\frac{d}{dx}$

with $\nabla\cdot D\nabla\equiv\frac{\partial}{\partial x}D\frac{\partial}{\partial x}+\frac{\partial}{\partial y}D\frac{\partial}{\partial y}+\frac{\partial}{\partial z}D\frac{\partial}{\partial z}$

In the boundary conditions replace the derivative terms

$\pm\frac{d}{dx}$ with $\vec{n}\cdot\nabla$

From these we obtain standard 3-D diffusion (P_1) equations

$$-\nabla\cdot D\nabla\phi_0(\vec{r})+\Sigma_a(\vec{r})\phi_0(\vec{r})=S(\vec{r}),\quad \vec{r}\in V$$

$$\frac{1}{2}\phi_0(\vec{r})+D\vec{n}\cdot\nabla\phi_0=2J^-(\vec{r}),\quad \vec{r}\in\partial V$$

For odd values of l', $\phi_{l'}$ is replaced by a vector by several SP_N equations

$$\phi_{l'}\rightarrow\vec{\phi}_{l'}=(\phi_{l'}^x,\phi_{l'}^y,\phi_{l'}^z)$$

in even l' equations derivative in x replaced by divergence

$$\frac{d}{dx}\rightarrow\nabla\cdot$$

In odd l' equations the x derivatives is changed to a gradient

$$\frac{d}{dx}\rightarrow\nabla$$

this allows rewrite first order form of SP_N eqn

$$\nabla\cdot\vec{\phi}_1+\Sigma_a\phi_0=S_0$$

$$\left(\frac{l'+1}{2l'+1}\right)\nabla\phi_{l'+1}+\left(\frac{l'}{2l'+1}\right)\nabla\phi_{l'-1}+\Sigma_t\vec{\phi}_{l'}=\Sigma_{sl'}\phi_{l'}+S_{l'}\quad\text{for odd } l'$$

$$\left(\frac{l'+1}{2l'+1}\right)\nabla\cdot\vec{\phi}_{l'+1}+\left(\frac{l'}{2l'+1}\right)\nabla\cdot\vec{\phi}_{l'-1}+\Sigma_t\phi_{l'}=\Sigma_{sl'}\phi_{l'}+S_{l'}\quad\text{for even } l'>0$$

Continued for SP_3 on next page