Laborprotocol BMo2-1

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Abstract: This protocol for L1 and H1 is concerned with the experimental setup. A qualified model of the dynamics were derived and missing parameters of the model were identified.

1. WORK TASKS

First, to be familiar with the experimental setup. A strategy should be develpoed to solve the main task of the lab course. In addition, a system need to be modeled with an adequate precision and linearized.

2. SIMULATION MODEL OF THE HELICOPTER

In order to be able to design controllers during L2, the helicopter system has to be modeled. Because the complete model of the helicopter is very complicated, simplification of the model while keeping the model complexity within an acceptable range is necessary.

Figure 1 shows the simplified model with the corresponding parameters.

Applying the law of angular momentum to the axes a,b and c, the following equations are derived.

$$\Theta_a \ddot{\alpha} = -\cos(\beta) L_2 \sin(\gamma)(F_f + F_b) \quad (1)$$
$$\Theta_b \ddot{\beta} = \cos(\gamma) L_2 (F_f + F_b) - \cos(\beta)(mL_1 - ML_2)g \quad (2)$$
$$\Theta_c \ddot{\gamma} = \frac{L_3}{2}(F_f - F_b) \quad (3)$$

Θ_a, Θ_b and Θ_c are the moment of inertia with respect to the axes a,b and c. Their values can be calculated by the following equations.

$$\Theta_a = mL_1^2 + ML_2^2 \quad (4)$$
$$\Theta_b = mL_1^2 + ML_2^2 \quad (5)$$
$$\Theta_c = \frac{1}{6} m (\frac{L3}{2})^2 \quad (6)$$

The state vector $x = \begin{bmatrix} \alpha & \beta & \gamma & \dot{\alpha} & \dot{\beta} & \dot{\gamma} \end{bmatrix}^T$ and the input vector $u = [F_f \; F_b]^T$. Therefore, $\dot{x} = \begin{bmatrix} \dot{\alpha} & \dot{\beta} & \dot{\gamma} & \ddot{\alpha} & \ddot{\beta} & \ddot{\gamma} \end{bmatrix}^T$ can be displayed as follows.

$$\dot{x} = \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \\ \frac{-\cos(\beta) L_2 \sin(\gamma)(F_f + F_b)}{mL_1^2 + ML_2^2} \\ \frac{\cos(\gamma) L_2 (F_f + F_b) - \cos(\beta)(mL_1 - ML_2)g}{mL_1^2 + ML_2^2} \\ \frac{\frac{L_3}{2}(F_f - F_b)}{\frac{1}{6} m (\frac{L3}{2})^2} \end{bmatrix} \quad (7)$$

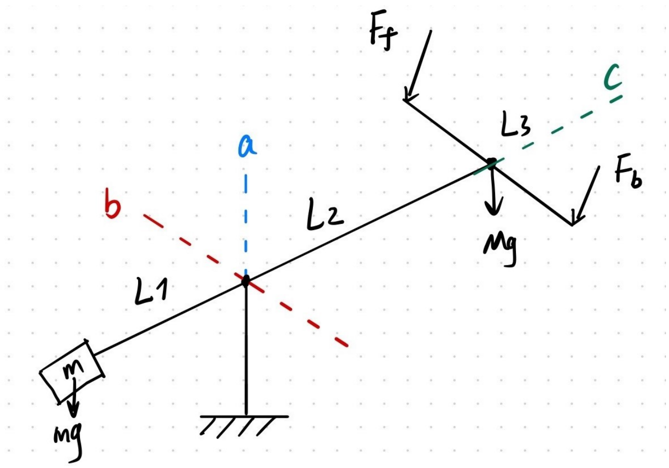

Fig. 1. simplified model of the helicopter

3. VOLTAGE MODEL

Data of the dependency between the voltage and the corresponding forces of the propeller were provided. Analysis of the data shows a quadratic curve relationship between the Forces and the applied voltage. Using curve fitting methods (*polyfit* Matlab command), the exact parameters of the curve are obtained to be able to model the force dependency for any given decimal voltage. The derived coefficients and the corresponding equation the force as a function of the voltage is:

$$F_f(V) = 6.156 * V^2 + -0.1342 * V - 0.13917 \quad (8)$$
$$F_b(V) = 4.704 * V^2 + 0.00510 * V - 0.278 \quad (9)$$

A plot of the forces over a voltage spectrum is illustrates in Figure 2.

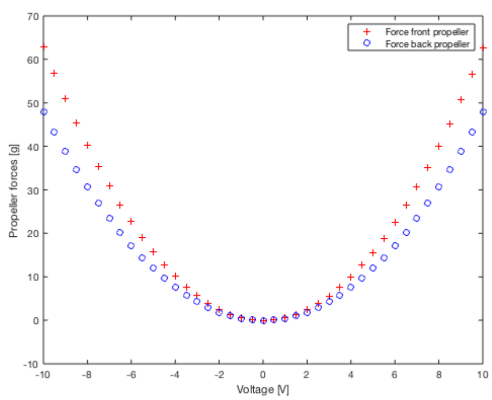

Fig. 2. Plot of propeller forces over voltage

4. PLANT MODEL

Using the equation of the simplified physical model of the helicopter discussed in *Simulation model of the helicopter*{section 2}, the mathematical state space model of the plant is derived by linearization of $\dot{x}$ w.r.t. x to derive the A matrix and $\dot{x}$ w.r.t. u to derive the B matrix.

The resulting computation in Matlab yields:

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -\frac{(\sin(\beta)\sin(\gamma)L_2(F_b+F_f)}{mL_1^2+ML_2^2} & -\frac{\cos(\beta)\cos(\gamma)L2(F_b+(F_f)}{mL_1^2+ML_2^2)} & 0 & 0 & 0 \\ 0 & -\sin(\beta)gL_2M - L_1m & -\frac{\sin(\gamma)L_2(F_b+F_f)}{mL_1^2+ML_2^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\frac{\cos(\beta)\sin(\gamma)L_2}{mL_1^2+ML_2^2} & \frac{\cos(\beta)\sin(\gamma)L2}{mL_1^2+ML_2^2} \\ \frac{\cos(\gamma)L_2}{mL_1^2+ML_2^2} & \frac{\cos(\gamma)L_2}{mL_1^2+ML_2^2} \\ \frac{12}{mL_3} & -\frac{12}{mL_3} \end{bmatrix}$$

For the next Lab session, the following draft working plan detailed in Table 1 will be used Table 1.

5. COMMENTS

Since the physical model of the helicopter has been simplified, our validation of the model demonstrated deviations from the provided blackbox model. These simplifications will be considered in the design of the controller.

Table 1. Working Plan for L2.

Time	**Duration**	**Goal**	**Task**	**Preparation**
14:00	30 min.	Test File to actuate motor	Create Quarc Simulink File	Read Quarc documentation
14:30	1,5 hour	Design a controller	Choose a suitable controller	Review KRT lecture notes
15:30	1,5 hour	Trajectory planning	Develop optimal trajectory for helicopet tasl	Review KRT lecture videos and script

REFERENCES

Quanser Inc. (2011). 3-DOF Helicopter: User Manual'.

Control of a 3-DOF Helicopter(2020). Handbook for Laboratory Course "Concepts of Automatic Control". Corona Edition.