Logical Relations in Coq

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Introduction to Logical Relations

Theorem

Normalization of STLC: For all terms e, if $\vdash e : \tau$*, then there exists a value v s.t.* $e \to^* v$*.*

Proof.

By induction on the typing derivation?

Case T-App

$$\frac{\vdash e1 : \tau_2 \to \tau \quad \vdash e2 : \tau_2}{\vdash e1\, e2 : \tau}$$

By IH,

$$e1\, e2 \to^* (\lambda x : \tau_2.e')\, e2 \to^* e'[v2/x]$$

IH is too weak!

Introduction to Logical Relations

Define a relation N_τ:

$$
\begin{aligned}
N_{\mathsf{bool}}(e) &\equiv \quad \vdash e : \mathsf{bool} \wedge \exists v.e \to^* v \\
N_{\tau_1 \to \tau_2}(e) &\equiv \quad \vdash e : \tau_1 \to \tau_2 \wedge \exists v.e \to^* v \wedge \forall e'.N_{\tau_1}(e') \Rightarrow N_{\tau_2}(e\,e')
\end{aligned}
$$