Basic Industrial Math

Study Unit

Fractions, Percents, Proportions, and Angles

Preview

In this study unit, you'll encounter a method of expressing part of a whole number—fractions. Not all things are measured in wholes. Therefore, we have a way of mathematically dealing with partial quantities. That's where fractions come in.

For example, in electronics you'll find resistors with ratings of $\frac{1}{8}$W (watt), and on blueprints you'll often find measurements that include fractions such as $\frac{3}{4}$ inch. In fact, a good working knowledge of fractions is a must for many occupations.

The first part of this study unit covers fractions—beginning with a simple definition and working up to performing mathematical operations involving fractions.

In the second section of this study unit, you'll learn how to solve problems involving percents, proportions, and angles. Sounds challenging, doesn't it? Well, it is, but with a few simple rules, you should be able to deal with all the material in this study unit with little difficulty.

When you complete this study unit, you'll be able to

- Define the following terms: fraction, proper fraction, improper fraction, lowest common denominator, percent, ratio, and proportion
- Add, subtract, multiply, and divide fractions
- Change fractions to decimals and decimals to fractions
- Solve problems involving percent
- Work with ratios and equivalent ratios
- Solve proportion problems
- Use a protractor to measure angles
- Lay out templates for checking angles
- Use a calculator to solve percent problems, to convert fractions to decimals, and to calculate missing terms in proportions

Contents

Fractions, Percents, Proportions, and Angles

FRACTIONS

What Is a Fraction?

The word *fraction* means broken. A fraction represents part of a whole that's been broken into the same size pieces.

Fractions crop up so often in our everyday dealings that sometimes we use them without really thinking about them. Even small children use fractions. When one child has a cookie, for example, and a friend wants part of the cookie, the child must break the cookie in half in order to share it. Each child will get one half ($\frac{1}{2}$) of the cookie, and $\frac{1}{2}$ is a fraction.

Fractions are written in different ways.

$$\frac{1}{4} \qquad {}^{1}\!/_{4} \qquad 1/4$$

All of these ways are correct. It doesn't matter which way you write your fractions. As long as you write the numerator, then a line, then the denominator, you'll be correct. In this study unit, you'll see fractions written these ways.

Industrial workers face problems with fractions daily. Even the simple directions "rotate $\frac{1}{8}$ turn" or "tighten $\frac{1}{2}$ turn" show the need for a basic understanding of fractions. You'll probably meet such directions, plus others with far more difficult fractions, in your daily work.

Let's take a look at the fraction $\frac{1}{5}$ (one fifth). The top number (1) is the *numerator,* and the bottom number (5) is the *denominator.* The line that separates the two numbers is the *fraction bar.* The denominator tells you how many equal parts the whole unit is divided into. In the fraction $\frac{1}{5}$, the denominator of 5 tells you that the whole has been divided into 5 equal parts. The numerator tells you how many of these equal parts are represented by the fraction. The fraction $\frac{1}{5}$ represents one of the five parts. Figure 1 graphically represents fractions with the denominator of 5.

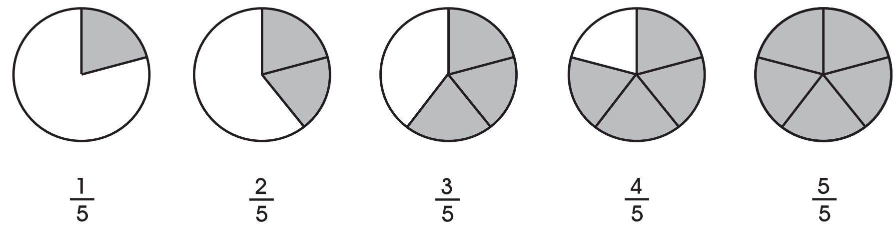


FIGURE 1—The shaded area in each circle graphically represents the fraction shown below it.

Kinds of Fractions

There are two kinds of fractions: proper and improper. If the numerator of a fraction is less than its denominator, then the fraction is less than 1 and is called a *proper fraction.* If the numerator is equal to or greater than its denominator, the fraction is an *improper fraction.* If the numerator of a fraction equals its denominator, the fraction equals 1. If the numerator is greater than the denominator, the fraction represents an amount greater than 1. Proper and improper fractions are illustrated for you in Figure 2.

A third type of fraction that you'll encounter is a *mixed fraction,* which consists of both a whole number and a fraction, such as $1\,\frac{3}{4}$.

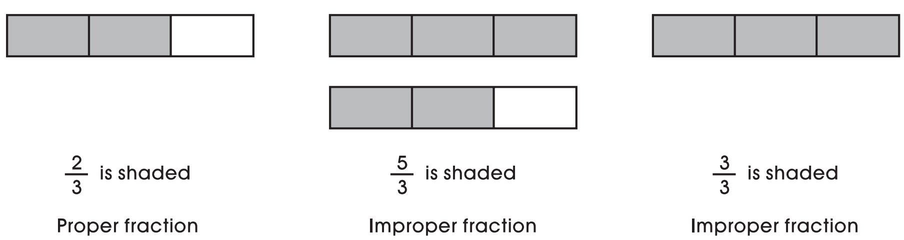


FIGURE 2—Graphic Representation of Proper and Improper Fractions

Reading Fractions

To read a fraction properly, simply say the number in the numerator, and then say the number of parts into which it's been divided (the denominator). To help you understand this, study Figure 3, which shows both commonly used fractions and the words that represent these fractions.

Sometimes, you may be trying to read a fraction with a very large denominator, such as $\frac{95}{167}$. Instead of reading "ninety-five one hundred sixty-sevenths," it's generally easier and clearer to read "ninety-five *over* one hundred sixty-seven."

FIGURE 3—This chart will help you learn how to read fractions properly.

$\frac{1}{2}$	one half	$\frac{1}{8}$	one eighth	$\frac{1}{16}$	one sixteenth
$\frac{1}{3}$	one third	$\frac{1}{9}$	one ninth	$\frac{1}{17}$	one seventeenth
$\frac{2}{3}$	two thirds	$\frac{1}{10}$	one tenth	$\frac{1}{18}$	one eighteenth
$\frac{1}{4}$	one fourth	$\frac{1}{11}$	one eleventh	$\frac{1}{19}$	one nineteenth
$\frac{3}{4}$	three fourths	$\frac{1}{12}$	one twelfth	$\frac{1}{20}$	one twentieth
$\frac{1}{5}$	one fifth	$\frac{1}{13}$	one thirteenth	$\frac{1}{30}$	one thirtieth
$\frac{1}{6}$	one sixth	$\frac{1}{14}$	one fourteenth	$\frac{1}{100}$	one hundredth
$\frac{1}{7}$	one seventh	$\frac{1}{15}$	one fifteenth	$\frac{1}{1,000}$	one thousandth

Uses of Fractions

Fractions have three common uses in everyday mathematics:

1. To stand for part of one whole thing

Example: The top of the ruler in Figure 4 is marked to divide each inch into eight equal parts, or eighths. Suppose you need a screw $\frac{5}{8}$ in. (inch) long to replace a broken one on a machine you're repairing. Notice in Figure 4 that the screw you have reaches the fifth marking on the ruler. Therefore, you know the screw is $\frac{5}{8}$ in. long. The fraction $\frac{5}{8}$ stands for a particular part of one whole inch.

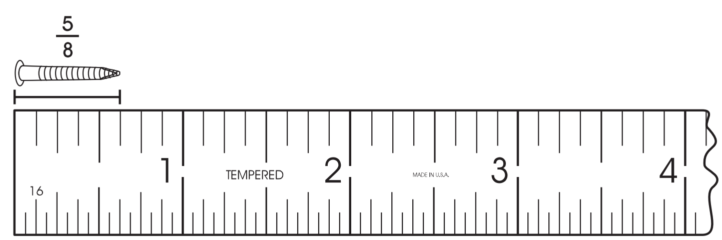
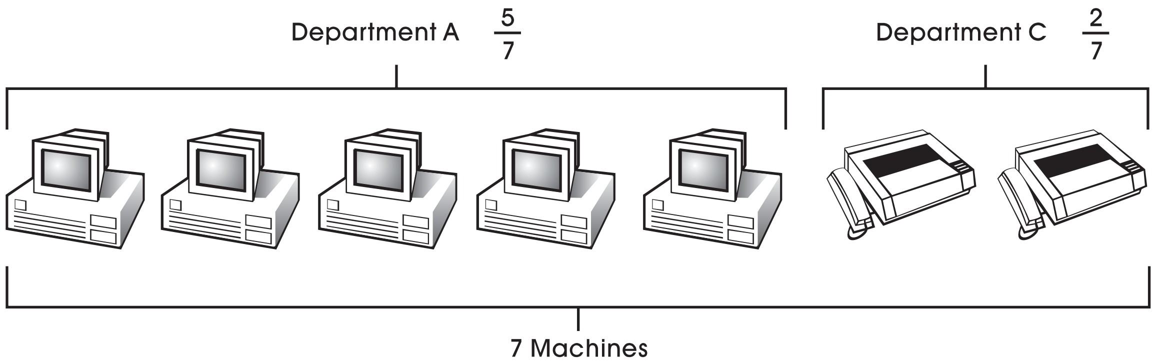


FIGURE 4—Since this ruler is divided into eighths, we can tell that the screw is $\frac{5}{8}$ ***in. long.***

2. To stand for part of a group

Example: You've received requests to service seven machines in your plant. Five machines are in Department A and two are in Department C. What fraction of your work is in Department A?



The answer is $\frac{5}{7}$. The fraction $\frac{5}{7}$ stands for the part of the group of machines you'll service that are in Department A.

3. To show division

Example: One way to represent division is to place one number over the other with a line between them. And that's just what a fraction is! All fractions actually represent division problems.

$$\frac{12}{3} = 12 \div 3 = 4 \qquad \frac{1}{4} = 1 \div 4 = .25 \qquad \frac{5}{5} = 5 \div 5 = 1$$

Equivalent Fractions

Sometimes two fractions have different numerators and denominators but still have the same value. Look at the rulers in Figure 5. The top part of the ruler in A is divided into sixty-fourths. That is, each mark between inches divides the inch into 64 equal parts. The bottom of the ruler in A is divided into thirty-seconds. The top of the ruler in B is divided into eighths, and the bottom is divided into sixteenths.

Notice the vertical line drawn between these rulers. If you read the measurement on the top and bottom of each ruler, you would come up with the following fractions: $\frac{32}{64}$, $\frac{16}{32}$, $\frac{4}{8}$, and $\frac{8}{16}$.

As you can see in Figure 5, these fractions represent the same part of one inch. In other words, these fractions are equivalent, or equal. *Equivalent fractions* may have different numerators and denominators, but represent the same value.

To show that these fractions are in fact equal, you must learn the following rule: *If you multiply both the numerator and denominator of a fraction by the same number, the value of the fraction doesn't change.*

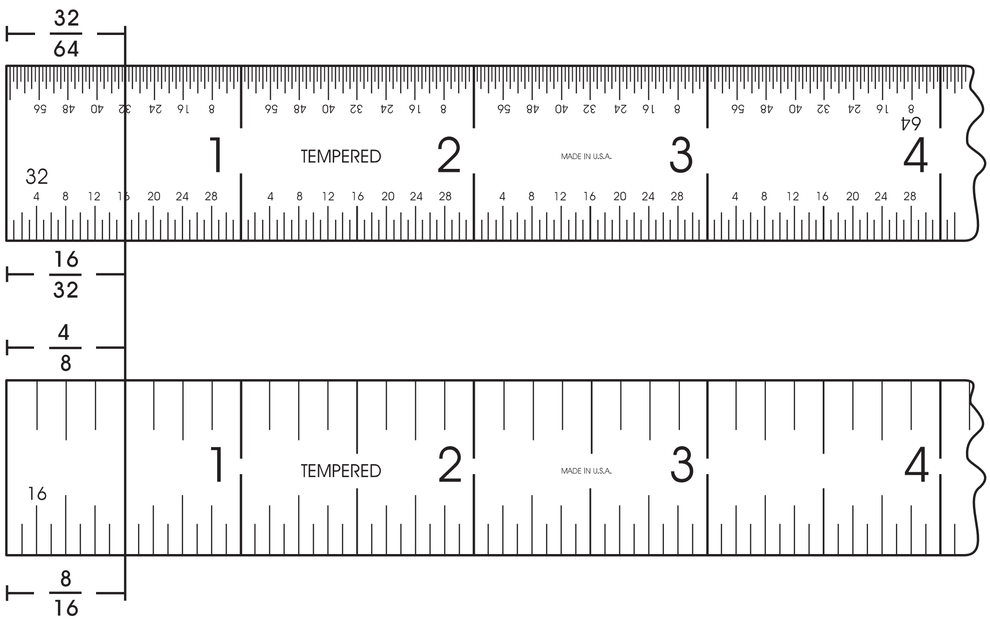


FIGURE 5—The two rulers have markings for eighths, sixteenths, thirty-seconds, and sixty-fourths. The vertical line shows that the fractions $\frac{4}{8}$, $\frac{8}{16}$, $\frac{16}{32}$, and $\frac{32}{64}$ are equivalent.

Let's see how this works with the fractions $\frac{32}{64}$, $\frac{16}{32}$, $\frac{4}{8}$, and $\frac{8}{16}$.

$\frac{4}{8}$ — First, take the fraction $\frac{4}{8}$.

$\frac{4 \times 2}{8 \times 2}$ — Multiply both the numerator and the denominator by the same number (2). According to the rule, this won't change the value of the fraction.

$\frac{4 \times 2}{8 \times 2} = \frac{8}{16}$ — The new fraction is $\frac{8}{16}$, which is equivalent to $\frac{4}{8}$.

From this you can see that the fractions $\frac{4}{8}$ and $\frac{8}{16}$ are equivalent.

$\frac{4}{8}$ — Now, take the fraction $\frac{4}{8}$ again.

$\frac{4 \times 4}{8 \times 4}$ — This time multiply the numerator and denominator by 4.

$\frac{4 \times 4}{8 \times 4} = \frac{16}{32}$ — The new fraction is $\frac{16}{32}$, which is equivalent to both $\frac{4}{8}$ and $\frac{8}{16}$.

This time, let's start with a different fraction in the group, $\frac{8}{16}$.

$\frac{8 \times 4}{16 \times 4}$ — Multiply the numerator and the denominator of $\frac{8}{16}$ by 4.

$\frac{8 \times 4}{16 \times 4} = \frac{32}{64}$ — The result is $\frac{32}{64}$. Now we know that $\frac{4}{8}$, $\frac{8}{16}$, $\frac{16}{32}$, and $\frac{32}{64}$ are equivalent fractions.

WHY DOES IT WORK?

How can you multiply the numerator and denominator by the same number and still have the same value? It's easy to understand if you remember two rules:

- Any number divided by itself equals 1.
- Any number multiplied by 1 equals that same number.

When you multiply the numerator and denominator by the same number, say 5, you're really multiplying by $\frac{5}{5}$. The fraction $\frac{5}{5}$ means $5 \div 5$, or 1. Therefore, when you're multiplying by $\frac{5}{5}$, you're really multiplying by 1—and when you multiply by 1, you don't change the value of the number being multiplied.

This is true no matter what number you use. As long as you multiply both the numerator and denominator by the same number, it's just like multiplying by 1.

Reducing Fractions

You can also use division to determine equivalent fractions. When you use multiplication to determine equivalent fractions, the numbers in the numerator and denominator became larger. When you use division to determine equivalent fractions, the numbers become smaller. Using division to find equivalent fractions is called *reducing a fraction.*

Reducing fractions makes them easier to understand and to work with. Take the fraction $\frac{18}{54}$ as an example. Let's reduce this fraction.

$\dfrac{18}{54}$	Set up the fraction and determine what number will divide evenly into both the numerator and the denominator. (Usually it's easiest to start with 2.)
$\dfrac{18 \div 2}{54 \div 2} = \dfrac{9}{27}$	Since 2 divides evenly into both numbers, we now have a new equivalent fraction.
$\dfrac{9}{27}$	Look at the new fraction to see if you can reduce it further. Neither number can be divided evenly by 2, so let's try 3. Although 3 will work, there's an even larger number that goes into both, 9.
$\dfrac{9 \div 9}{27 \div 9} = \dfrac{1}{3}$	The resulting equivalent fraction is $\dfrac{1}{3}$.

The fraction $\frac{1}{3}$ is much easier to understand and work with than the fraction $\frac{18}{54}$ and yet they both stand for the same value.

Here are some more examples of how to reduce fractions:

$$\frac{9}{24} = \frac{9 \div 3}{24 \div 3} = \frac{3}{8} \qquad \frac{21}{77} = \frac{21 \div 7}{77 \div 7} = \frac{3}{11} \qquad \frac{20}{24} = \frac{20 \div 4}{24 \div 4} = \frac{5}{6}$$

When a fraction can't be reduced any further, it's said to be in its *lowest terms.*

When you solve fraction problems, you'll often need to find an equivalent fraction with a specific denominator. To do this, follow these two steps:

1. Divide the higher denominator by the lower denominator.
2. Multiply the quotient by the given numerator. The answer is the numerator of the second fraction.

The following examples illustrate these steps for you.

Example: What fraction with a denominator of 21 is equal to $\frac{2}{3}$?

$\frac{2}{3} = \frac{?}{21}$ Set up the problem as shown.

$21 \div 3 = 7$ Divide the higher denominator (21) by the lower one (3).

$7 \xrightarrow{\text{times}} \frac{2}{3} = \frac{14}{21}$ Multiply the quotient (7) by the given numerator (2). The answer (14) is the missing numerator of the second fraction.

Therefore, the fraction $\frac{2}{3}$ is equal to $\frac{14}{21}$.

Example: What fraction with a denominator of 24 is equal to $\frac{5}{12}$?

$\frac{5}{12} = \frac{?}{24}$ Set up the problem as shown.

$24 \div 12 = 2$ Divide the larger denominator (24) by the smaller one (12).

$2 \xrightarrow{\text{times}} \frac{5}{12} = \frac{10}{24}$ Multiply the quotient (2) by the given numerator (5). The answer (10) is the missing numerator of the second fraction.

Solving a Simple Industrial Problem

Now that you have some understanding of fractions, let's check out a problem that you may have on the job. Suppose you're to tear down one of the plant air compressors and inspect the connecting rods. The specs indicate that there must be a minimum $\frac{3}{64}$-in. fillet at all four corners where the rod and cap are milled for the bolt heads and lock plates. After checking all four corners with a fillet gage, as in Figure 6, you find the smallest fillet measures $\frac{1}{16}$ in. Here's the problem: Is $\frac{1}{16}$ smaller or larger than $\frac{3}{64}$ in.? How do you solve it?

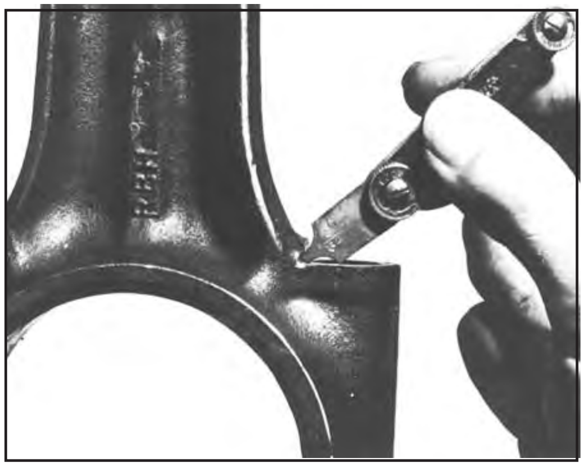


FIGURE 6—Checking details, such as connecting rod fillets during maintenance overhaul, can help prevent major breakdowns.

Now for the solution. To begin with, you can't compare $\frac{1}{16}$ and $\frac{3}{64}$, since these fractions don't have the same denominator. Since the fraction $\frac{3}{64}$ is already in its lowest terms, we can't reduce it to sixteenths. Therefore, we'll have to change the fraction $\frac{1}{16}$ to sixty-fourths.

What we're really saying is, find a fraction with a denominator of 64 that's equal to $\frac{1}{16}$.

$\frac{1}{16} = \frac{?}{64}$ Set up the problem as shown.

$64 \div 16 = 4$ Divide the larger denominator (64) by the smaller one (16).

$4 \xrightarrow{\text{times}} \frac{1}{16} = \frac{4}{64}$ Multiply the quotient (4) by the given numerator (1). The answer (4) is the missing numerator of the second fraction.

You can now compare the fractions $\frac{3}{64}$ and $\frac{4}{64}$. You can now say that the $\frac{1}{16}$-in. fillet is greater than the $\frac{3}{64}$-in. fillet specified. Therefore, the $\frac{1}{16}$-in. fillet is okay.

Changing Improper Fractions to Mixed Numbers

You'll recall that an improper fraction is one in which the numerator is larger than the denominator—for example, $\frac{64}{7}$. To change such a fraction into a mixed number, simply treat the fraction like a division problem. Remember, $\frac{64}{7}$ is the same as $64 \div 7$.

To change an improper fraction to a mixed number, follow these steps:

1. Divide the denominator of the fraction into the numerator. The quotient is the whole number part of your answer.

2. Write the remainder as the numerator of the fraction part of your answer.

3. Write the divisor as the denominator of the fraction part of your answer.

The following example illustrates this process.

Example: Change the improper fraction $\frac{64}{7}$ to a mixed number.

$7\overline{)64}$	Set up the fraction as a division problem.
$\begin{array}{r} 9 \\ 7\overline{)\ 64} \\ \underline{-63} \\ 1 \end{array}$	Carry out the division.
$9\frac{1}{7}$	Write the quotient (9) as the whole number part of your answer. Write the remainder (1) as the numerator of the fraction part of your answer. Write the divisor (7) as the denominator of the fraction part. Your answer is $9\frac{1}{7}$.

Example: Change the improper fraction $^{15}\!/_{2}$ to a mixed number.

$2\overline{)15}$	Set up the fraction as a division problem.
$\begin{array}{r} 7 \\ 2\overline{)\ 15} \\ \underline{-14} \\ 1 \end{array}$	Carry out the division.
$7\frac{1}{2}$	Write the quotient (7) as the whole number part of your answer. Write the remainder (1) as the numerator of the fraction part of your answer. Write the divisor (2) as the denominator of the fraction part. Your answer is $7\frac{1}{2}$.

Changing Mixed Numbers to Improper Fractions

You're now going to learn how to reverse the procedure you just used to change a mixed number to an improper fraction. To do so, you'll use multiplication, which makes sense since multiplication is the inverse, or opposite, of division.

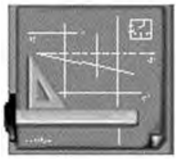
To see how it's done, let's use the examples from the previous section.

Example: Change $9\,^{1}\!/_{7}$ to an improper fraction.

$9\frac{1}{7}$ $7 \times 9 = 63$	Multiply the denominator of the fraction (7) by the whole number (9).
$63 + 1 = 64$	Add the numerator of the fraction to the product obtained in the preceding step.
$\frac{64}{7}$	Place the sum (64) over the denominator of the fraction (7). The answer is $\frac{64}{7}$.

Example: Change $7\frac{1}{2}$ to an improper fraction.

$7\frac{1}{2}$ $2 \times 7 = 14$	Multiply the denominator of the fraction (2) by the whole number (7).
$14 + 1 = 15$	Add the numerator of the fraction to the product obtained in the preceding step.
$\frac{15}{2}$	Place the sum (15) over the denominator of the fraction (2). The answer is $\frac{15}{2}$.



Check Your Learning 1

At various points in your *Fraction, Percents, Proportions, and Angles* text, you'll be asked to pause and check your understanding of what you've just read by completing a *Check Your Learning.* Writing the answers to these questions will help you review what you've studied so far. Please complete *Check Your Learning 1* now.

Match the fractions in the left-hand column with their equivalent fractions in the right-hand column.

1. $\frac{1}{2}$	a. $\frac{9}{12}$
2. $\frac{3}{4}$	b. $\frac{4}{12}$
3. $\frac{5}{6}$	c. $\frac{15}{24}$
4. $\frac{12}{20}$	d. $\frac{8}{16}$
5. $\frac{5}{8}$	e. $\frac{3}{5}$
6. $\frac{1}{3}$	f. $\frac{10}{12}$

7. Give the three common uses of fractions.

   a. ______________________________

   b. ______________________________

   c. ______________________________

8. Multiplying the numerator and the denominator of a fraction by the same number is the same as multiplying the fraction by the whole number _______.

9. Reduce the following fractions to their lowest terms.

   a. $^6/_8$ ______________________ d. $^9/_{18}$ ______________________

   b. $^8/_{12}$ ______________________ e. $^{24}/_{30}$ ______________________

   c. $^{15}/_{20}$ ______________________ f. $^{25}/_{40}$ ______________________

(Continued)

Check Your Learning 1

10. Indicate whether the following fractions are proper or improper.

a. $\frac{1}{10}$ ______________________ d. $\frac{9}{13}$ ______________________

b. $\frac{16}{14}$ ______________________ e. $\frac{6}{6}$ ______________________

c. $\frac{25}{20}$ ______________________ f. $\frac{14}{11}$ ______________________

11. Change the following improper fractions to mixed numbers and express your answer in the lowest terms.

a. $\frac{4}{3}$ ______________________ d. $\frac{24}{18}$ ______________________

b. $\frac{10}{4}$ ______________________ e. $\frac{29}{6}$ ______________________

c. $\frac{12}{5}$ ______________________ f. $\frac{55}{10}$ ______________________

12. Change the following mixed numbers to improper fractions.

a. $6\frac{1}{8}$ ______________________ d. $8\frac{3}{4}$ ______________________

b. $2\frac{3}{5}$ ______________________ e. $6\frac{5}{16}$ ______________________

c. $10\frac{1}{9}$ ______________________ f. $3\frac{3}{32}$ ______________________

Check your answers with those on page 53.

OPERATIONS WITH FRACTIONS

Adding and Subtracting Fractions

Like Fractions

Fractions that have the same denominators are *like fractions.* For example, the fractions $\frac{2}{9}$, $\frac{4}{9}$, $\frac{5}{9}$, and $\frac{7}{9}$ are like fractions because they all have a denominator of 9. Adding and subtracting like fractions is easy. Just follow these three steps:

1. Add or subtract the numerators. The result is the numerator of your answer. The denominator remains the same.
2. If the answer is an improper fraction, change it to a mixed number.
3. Reduce the fraction part of your answer to its lowest terms.

Example: A board $\frac{3}{8}$ in. thick has a $\frac{1}{8}$-in. layer of paint. What is the total thickness of the board?

To find the answer, you add the fractions $\frac{3}{8}$ and $\frac{1}{8}$, which are like fractions (Figure 7).

$\frac{3}{8}+\frac{1}{8}=\frac{4}{8}$ Add the numerators. The sum is the numerator of your answer. The denominator remains the same. The answer isn't an improper fraction so it can't be changed to a mixed number.

$\frac{4\div 4}{8\div 4}=\frac{1}{2}$ Reduce the fraction to its lowest form.

The answer is $\frac{1}{2}$. The board with the coat of paint is $\frac{1}{2}$ in. thick.

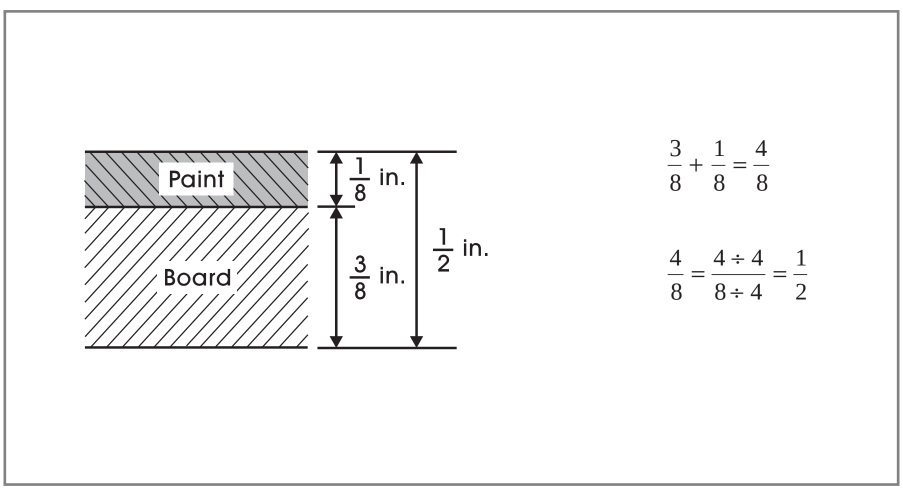
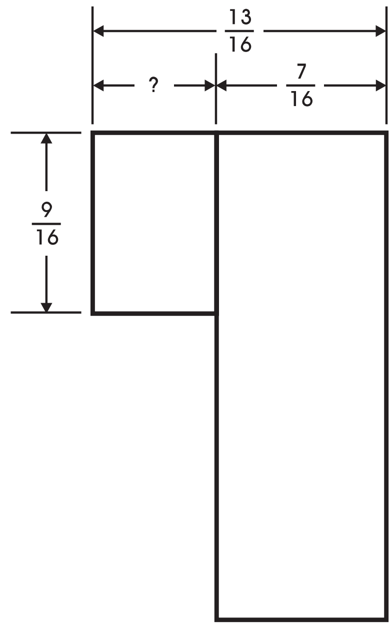


FIGURE 7—As long as the fractions have the same denominator, it's easy to find their sum. Remember, you should always reduce the answer to its lowest terms.

Example:



Look at the drawing above. The dimensions of the part are given in inches. What's the dimension of the part marked with a question mark (?)?

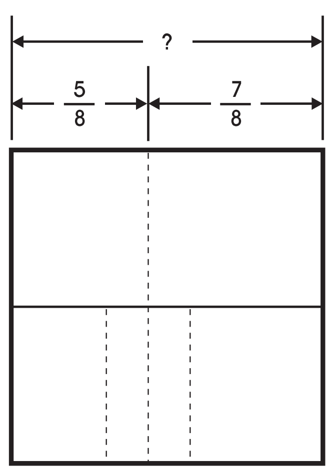
To find this dimension, you must subtract $\frac{7}{16}$ from $\frac{13}{16}$, which again, are like fractions.

$\frac{13}{16} - \frac{7}{16} = \frac{6}{16}$ Subtract the numerators. The difference is the numerator of your answer. The denominator remains the same.

$\frac{6 \div 2}{16 \div 2} = \frac{3}{8}$ Reduce the fraction to its lowest terms.

The answer is $\frac{3}{8}$. The missing dimension marked by the ? is $\frac{3}{8}$ in.

Example: The drawing shows two dimensions (in inches). Find the total length of the side.



To find this dimension, you must add the fractions $\frac{7}{8}$ and $\frac{5}{8}$.

$\frac{7}{8}+\frac{5}{8}=\frac{12}{8}$	Add the numerators. The sum is the numerator of your answer. The denominator remains the same.
$\frac{12}{8}=1\frac{4}{8}$	Since the answer is an improper fraction, change it to a mixed number.
$1\frac{4 \div 4}{8 \div 4}=1\frac{1}{2}$	Reduce the fraction to its lowest terms.

The answer is $1\frac{1}{2}$. The total length of the side is $1\frac{1}{2}$ in.

Unlike Fractions

Fractions with different denominators are *unlike fractions*. For example, the fractions $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{7}{10}$ are unlike fractions because their denominators are different. To add and subtract unlike fractions, you must first change the fractions so that all have the same denominator. The denominator should be the *lowest common denominator* (LCD), which is the smallest number that can be divided (without a remainder) by all of the denominators.

One way to find the lowest common denominator is to consider using the denominator of the fraction with the largest denominator.

Example: Find the lowest common denominator for the fractions $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{8}$.

$\frac{1}{2}\quad\frac{1}{4}\quad\frac{3}{8}$	Look at the fraction with the largest denominator (8). Ask yourself if the other denominators (2 and 4) can be evenly divided into that denominator.
$\frac{1}{2}\quad\frac{1}{4}\quad\frac{3}{8}$	Since both 2 and 4 can be evenly divided into 8, the lowest common denominator is 8.
$\frac{1}{2}=\frac{4}{8}\qquad\frac{1}{4}=\frac{2}{8}$	Change the other fractions into equivalent fractions with a denominator of 8. (You learned how to do this on pages 4–6.) Now all the fractions have the same denominator.

You may find that the largest denominator in the group isn't the lowest common denominator.

Example: Find the lowest common denominator for the fractions $\frac{3}{4}$, $\frac{2}{5}$, and $\frac{1}{10}$.

$\frac{3}{4}\quad\frac{2}{5}\quad\frac{1}{10}$	Look at the fraction with the largest denominator (10). Ask yourself if the other denominators (4 and 5) can be evenly divided into that denominator.
$\frac{3}{4}\quad\frac{2}{5}\quad\frac{1}{10}$	Although 5 can be divided evenly into 10, the denominator 4 can't be. Therefore, we must go to a higher number to find the lowest common denominator.

$2 \times 10 = 20$	Multiply the largest denominator by 2 and ask yourself again if the other denominators (4 and 5) can be evenly divided into the denominator. Since both 4 and 5 can be evenly divided into 20, you know that the lowest common denominator is 20.
$\frac{3 \times 5}{4 \times 5} = \frac{15}{20}$ $\frac{2 \times 4}{5 \times 4} = \frac{8}{20}$ $\frac{1 \times 2}{10 \times 2} = \frac{2}{20}$	Change all fractions to equivalent fractions with a denominator of 20.

If this process doesn't work, try multiplying the largest denominator by 3, then 4, and so on until you reach a number that can be evenly divided by all denominators in the group.

Example: If you worked $\frac{1}{2}$ hr (hour) overtime on Monday and $\frac{3}{4}$ hr overtime on Tuesday, how many hours overtime did you work in the two days together?

To find the answer, you add the fractions $\frac{1}{2}$ and $\frac{3}{4}$.

$\frac{1}{2} + \frac{3}{4}$	Since the fractions are unlike fractions, you must find the lowest common denominator. In this case the lowest common denominator is 4 because 2 can be divided evenly into 4.
$\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$	Change the fraction $\frac{1}{2}$ to an equivalent fraction with a denominator of 4.
$\frac{2}{4} + \frac{3}{4} = \frac{5}{4}$	Add the fractions as you did for like fractions.
$\frac{5}{4} = 1\frac{1}{4}$	Since the answer is an improper fraction, change it to a mixed number. The fraction is already in its lowest form.

The answer is $1\frac{1}{4}$. You worked $1\frac{1}{4}$ hr in the two days.

Example: Here's a simple problem that you might run into. Suppose you have a $\frac{7}{8}$-in. piece of rod that you're going to use as a spacer. It must fit flush with the face of the coupling. When you put it in place, you find that it sticks out $\frac{3}{32}$ in. Thus, you'll have to cut it to fit. Your problem is to find what the actual length of the spacer should be.

To do this, you must subtract $\frac{3}{32}$ from $\frac{7}{8}$.

$\frac{7}{8} - \frac{3}{32}$	Since the fractions are unlike fractions, you must find the lowest common denominator. In this case it's 32.
$\frac{7}{8} = \frac{28}{32}$	Change the fraction $\frac{7}{8}$ to an equivalent fraction with a denominator of 32.
$\frac{28}{32} - \frac{3}{32} = \frac{25}{32}$	Subtract the fractions as you did for like fractions.

The answer is $\frac{25}{32}$. The spacer must be $\frac{25}{32}$ in. long to fit properly.

Adding and Subtracting Mixed Numbers and Fractions

To add or subtract mixed numbers and fractions, follow these steps:

1. If necessary, find the lowest common denominator and change the unlike fractions to equivalent fractions with the lowest common denominator as the denominator.
2. Add or subtract the fractions.
3. Add or subtract the whole numbers.
4. Reduce your answer if necessary.

Example: Two pieces of pipe, one $3\frac{1}{4}$ in. long and one $2\frac{1}{4}$ in. long, have been welded together. What's the total length of the pipe?

To find the answer, you add $3\frac{1}{4}$ and $2\frac{1}{4}$.

$$\begin{array}{r} 3\frac{1}{4} \\ +2\frac{1}{4} \\ \hline \frac{2}{4} \end{array}$$

Since the fractions are like fractions, you can add them. Add the fraction parts.

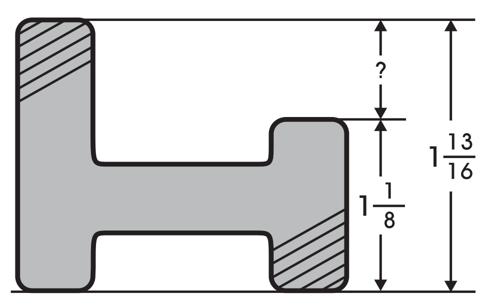
$$\begin{array}{r} 3\frac{1}{4} \\ +2\frac{1}{4} \\ \hline 5\frac{2}{4} \end{array}$$

Add the whole numbers.

$$5\frac{2}{4} = 5\frac{1}{2}$$

Reduce your answer.

Example: The drawing below is a simplified drawing of a pulley with the dimensions given in inches. Find the dimension marked by the question mark.



To find this dimension, you must subtract $1\frac{1}{8}$ from $1\frac{13}{16}$.

$$\begin{array}{r} 1\frac{13}{16} \\ -\,1\frac{1}{8} \\ \hline \end{array} = \begin{array}{r} 1\frac{13}{16} \\ -\,1\frac{2}{16} \\ \hline \end{array}$$

Change the fractions to equivalent fractions with the lowest common denominator. In this case the lowest common denominator is 16.

$$\begin{array}{r} 1\frac{13}{16} \\ -1\frac{2}{16} \\ \hline \frac{11}{16} \end{array}$$

Subtract the fractions and then the whole numbers.

The answer is $^{11}\!/_{16}$. The dimension is $^{11}\!/_{16}$ in.

Sometimes you have to do some extra work to reduce the fraction in your answer. For example, when you add fractions, the sum may be an improper fraction. Here's what you would do:

Example:

$$\begin{array}{r} 1\frac{3}{4} \\ +3\frac{3}{4} \\ \hline 4\frac{6}{4} \end{array}$$

Add the fractions and then the whole numbers.

$$\frac{6}{4} = 1\frac{2}{4} = 1\frac{1}{2}$$

Change the fraction in the answer to a mixed number and reduce if necessary.

$$\begin{array}{r} 4 \\ +1\frac{1}{2} \\ \hline 5\frac{1}{2} \end{array}$$

Add this mixed number to the whole number in the original answer.

The answer is $5\,^{1}\!/_{2}$.

Sometimes you'll have to borrow when subtracting fractions. Here's the procedure for borrowing with fractions:

Example: Suppose you have a pipe $4\,^{1}\!/_{4}$ ft (feet) long. From this pipe you cut a piece $2\,^{3}\!/_{4}$ ft long. How much pipe is remaining?

$$\begin{array}{r} {}^{3}\cancel{4}\frac{1}{4} \\ -\quad 2\frac{3}{4} \\ \hline \end{array}$$

Although $\frac{1}{4}$ and $\frac{3}{4}$ are like fractions, you can't subtract 3 from 1. Therefore, you must borrow 1, or in this case $\frac{4}{4}$, from the whole number 4. Cross out the 4 and write 3 above it.

$$\begin{array}{r} {}^{3}\cancel{4}\,\frac{\cancel{1}}{\cancel{4}}\,\frac{5}{4} \\ -\;2\frac{3}{4} \\ \hline \end{array}$$

Add the $\frac{4}{4}$ to the $\frac{1}{4}$ in the fractions column. Cross out the $\frac{1}{4}$ and write $\frac{5}{4}$ above it $\frac{1}{4}+\frac{4}{4}=\frac{5}{4}$..

$$\begin{array}{r} {}^{3}\cancel{4}\,\frac{\cancel{1}}{\cancel{4}}\,\frac{5}{4} \\ -\;2\frac{3}{4} \\ \hline \end{array}$$

Now you can subtract—first the fractions, then the whole numbers.

$$1\frac{2}{4}=1\frac{1}{2}$$

Reduce the fraction to its lowest terms.

The answer is $1\frac{1}{2}$. You have $1\frac{1}{2}$ ft of pipe remaining.

> **BORROWING HINT**
>
> If you must borrow when subtracting fractions, be sure to change the borrowed number into a fraction with the proper denominator. For example, if the fractions you're subtracting are fifths, the borrowed number will become $\frac{5}{5}$. If the fractions are sixths, the borrowed number will be $\frac{6}{6}$, and so on.

Multiplying Fractions

You'll be pleased to discover that multiplying fractions is much easier and faster than adding or subtracting them. This is because you can multiply both like and unlike fractions without changing their denominators.

To multiply one fraction by another, follow these steps:

1. Multiply the numerators and make the product the numerator of the answer.
2. Multiply the denominators and make the product the denominator of the answer.
3. Simplify the answer if necessary.

Example: Multiply $\frac{3}{4}$ by $\frac{2}{3}$.

$$\frac{3}{4}\times\frac{2}{3}=\frac{6}{}$$

Multiply the numerators and make the product the numerator of the answer.

$$\frac{3}{4}\times\frac{2}{3}=\frac{6}{12}$$

Multiply the denominators and make the product the denominator of the answer.

$\dfrac{6 \div 6}{12 \div 6} = \dfrac{1}{2}$ Reduce the answer.

That was easy, wasn't it? And the same procedure is followed no matter what kinds of fractions are involved.

CANCELLATION: A MULTIPLICATION SHORTCUT

When you multiply fractions, you can use the cancellation method—a shortcut way to arrive at the correct answer. Remember that cancellation works only for multiplication. Never try cancellation when you add or subtract fractions. Here's how it works:

Example:

What is the product of $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{2}{3}$?

***Solution 1*:** Let's find the answer the way you've already learned.

$\dfrac{3}{4} \times \dfrac{7}{8} \times \dfrac{2}{3} = \dfrac{42}{}$ Multiply the numerators and make the product the numerator of the answer.

$\dfrac{3}{4} \times \dfrac{7}{8} \times \dfrac{2}{3} = \dfrac{42}{96}$ Multiply the denominators and make the product the denominator of the answer.

$\dfrac{42 \div 6}{96 \div 6} = \dfrac{7}{16}$ Reduce the answer to its lowest terms.

Solution 2: Now let's find the answer by cancellation.

$\dfrac{{}^{1}\cancel{3}}{4} \times \dfrac{7}{8} \times \dfrac{2}{\cancel{3}_{1}} =$ First cross off any pairs of identical numbers that appear in both the numerator and denominator. Replace these with 1's.

$\dfrac{{}^{1}\cancel{3}}{4} \times \dfrac{7}{\cancel{8}_{4}} \times \dfrac{\cancel{2}^{1}}{\cancel{3}_{1}} =$ Divide any numerator and denominator by the same number. In this case we'll divide the 2 in the numerator and the 8 in the denominator by 2.

$\dfrac{{}^{1}\cancel{3}}{4} \times \dfrac{7}{\cancel{8}_{4}} \times \dfrac{\cancel{2}^{1}}{\cancel{3}_{1}} = \dfrac{7}{16}$ Now multiply as usual. You come up with the same answer, but now you don't have to reduce it.

Important: When you use cancellation, remember to cancel in pairs. Cancel one *numerator* and then one *denominator*. You can't cancel numerator to numerator or denominator to denominator.

Multiplying Fractions by Whole Numbers

Every whole number can be considered a fraction by placing it over the number 1. For example, $5 = \frac{5}{1}$, $12 = \frac{12}{1}$, $432 = \frac{432}{1}$. Therefore, in order to multiply any fraction by a whole number, simply convert the whole number to a fraction and use the multiplication rules you just learned.

Example: Suppose you're going to place three spacers on a stud bolt. Each spacer is $\frac{3}{8}$ in. thick. How much space will the three spacers take up?

To find the answer, you multiply 3 by $\frac{3}{8}$.

$\frac{3}{8} \times \frac{3}{1}$	Convert the whole number to a fraction by placing it over 1.
$\frac{3}{8} \times \frac{3}{1} = \frac{9}{8}$	Multiply as usual.
$\frac{9}{8} = 1\frac{1}{8}$	Since the answer is an improper fraction, change it to a mixed number.

The answer is $1\frac{1}{8}$. The three spacers will take up $1\frac{1}{8}$ in. on the stud bolt.

Dividing Fractions

Dividing fractions is almost as easy as multiplying fractions. Here are the two steps for dividing fractions.

1. Invert (turn over) the divisor and change the division sign to a multiplication sign.
2. Multiply as usual.

Example: Determine how many $\frac{1}{16}$-in. spacers you would need to fill a $\frac{5}{8}$-in. gap.

To find out, divide the size of the gap ($\frac{5}{8}$) by the size of the spacers ($\frac{1}{16}$).

$\frac{5}{8} \div \frac{1}{16}$	Set up the problem.
$\frac{5}{8} \times \frac{16}{1}$	Invert the divisor and change it to a multiplication problem.
$\frac{5}{\underset{1}{\cancel{8}}} \times \frac{\overset{2}{\cancel{16}}}{1} = \frac{10}{1} = 10$	Use cancellation and then multiply.

The answer is 10. You'll need 10 spacers to fill the gap.

Dividing Fractions by Whole Numbers

Suppose you must divide a measurement of $\frac{15}{16}$ in. into three equal sections. What will be the length of each section?

To find the answer, you divide $\frac{15}{16}$ by 3.

$\frac{15}{16} \div \frac{3}{1}$	Change the whole number to an improper fraction by placing it over 1.
$\frac{15}{16} \times \frac{1}{3}$	Invert the divisor and change it to a multiplication problem.
$\frac{\overset{5}{\cancel{15}}}{16} \times \frac{1}{\underset{1}{\cancel{3}}}$	Use cancellation. Divide 3 into one numerator and one denominator.
$\frac{5}{16} \times \frac{1}{1} = \frac{5}{16}$	Multiply as usual.

The answer is $\frac{5}{16}$. Each section will be $\frac{5}{16}$ in. long.

Dividing Whole Numbers by Fractions

The following practical problem will show you how to divide a fraction into a whole number.

Example: Suppose you have 10 containers of oil. Each container holds 1 gallon. One of your jobs is to fill the machines in your department with this oil. Each machine holds $\frac{2}{5}$ gallon of oil. How many machines can you fill?

To solve this problem, you determine the number of times $\frac{2}{5}$ goes into 10.

$\frac{10}{1} \div \frac{2}{5}$	Change the whole number 10 to an improper fraction by placing it over 1.
$\frac{10}{1} \times \frac{5}{2}$	Invert the divisor and change it to a multiplication problem.
$\frac{\overset{5}{\cancel{10}}}{1} \times \frac{5}{\underset{1}{\cancel{2}}}$	Use cancellation.
$\frac{5}{1} \times \frac{5}{1} = \frac{25}{1}$	Multiply as usual.
$\frac{25}{1} = 25$	Change the improper fraction in the answer to a whole number.

The answer is 25. You can fill 25 machines with the oil.

Check Your Learning 2

Match the terms in the left-hand column with the examples given in the right-hand column.

1. numerator	a. $\frac{1}{9}, \frac{5}{9}, \frac{11}{9}$
2. denominator	b. $\rightarrow \frac{1}{3}$
3. like fractions	c. $\frac{3}{2}, \frac{7}{3}, \frac{12}{9}$
4. unlike proper fractions	d. $1\frac{3}{4}$
5. improper fractions	e. $\frac{1}{3}, \frac{1}{4}, \frac{2}{5}$
6. equivalent fractions	f. $\frac{5}{6} \leftarrow$
7. mixed number	g. $\frac{1}{2}, \frac{4}{8}, \frac{8}{16}$

8. Solve the following problems involving fractions.

   a. $\frac{3}{6} + \frac{2}{6}$ ____________________

   b. $1\frac{1}{5} + 2\frac{3}{10}$ ____________________

   c. $\frac{9}{11} - \frac{7}{11}$ ____________________

   d. $3\frac{11}{20} - 1\frac{4}{5}$ ____________________

   e. $\frac{1}{2} \times \frac{1}{3}$ ____________________

   f. $3 \times \frac{5}{6}$ ____________________

   g. $\frac{3}{10} \div \frac{6}{10}$ ____________________

   h. $\frac{5}{9} \div 3$ ____________________

9. Your shop employs 7 people. Today 2 people are sick. What fraction represents the employees who are sick?

10. Three layers of paint have been applied to a surface. The first layer is $\frac{3}{64}$ in. thick, the second layer is $\frac{1}{32}$ in. thick, and the third layer is $\frac{1}{16}$ in. thick. What is the total thickness of all three layers?

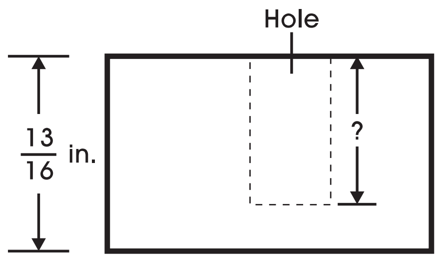
11. An electrician takes a piece of wire $\frac{7}{8}$ ft long and cuts off $\frac{1}{4}$ ft. How much wire is left?

(Continued)

Check Your Learning 2

12. You weld two pieces of steel together. The total thickness is $\frac{33}{64}$ in. If one piece of steel is $\frac{9}{32}$ in. thick, how thick is the other piece?

13. A hole is drilled in a block of wood. The hole is $\frac{3}{4}$ of the total depth of the wood. The total depth of the block of wood is $\frac{13}{16}$ in. How deep is the hole?



14. Here is a challenge for you. Read the question carefully. To find the answer, you'll have to perform three separate operations.

    A metal rod is $\frac{51}{64}$ in. long. The rod needs to be trimmed. Cut $\frac{1}{64}$ in. from one end and $\frac{1}{32}$ in. from the other end. Now cut the rod into six equal pieces. What will be the length of each piece? (Be sure to reduce your answer to its lowest terms.)

Check your answers with those on page 53.

DECIMALS AND PERCENTS

As you learned in the previous section, a proper fraction is a way to express an amount that's less than a whole. The fractions $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{2}$ are less than 1. A decimal is another way of writing a number that's less than 1. In this section, you'll be learning about decimals and how to use them to solve problems involving percents.

Reading Decimals

To read a decimal, follow these steps:

1. Ignore the decimal point and any zeros immediately to the right of the decimal point. For example, in the decimal .003, ignore the decimal point and the two zeros. Instead, go immediately to the 3.

2. Read the remaining digits as if they were a whole number. In the decimal .003, read the 3 as "three."

3. Say the place value of the digit on the far right. In the decimal .003, the digit on the far right is 3, which is in the thousandths place. Therefore, .003 would be read as "three thousandths."

Example: Read each of the following decimals.

Decimal	Decimal as Read
.7	seven tenths
.02	two hundredths
.004	four thousandths
.56	fifty-six hundredths
.754	seven hundred fifty-four thousandths

To read a mixed decimal, such as 2.75, follow these steps:

1. Read the whole number (the number to the left of the decimal point). For the number 2.75, say "two."

2. Read the decimal point as "and."

3. Read the decimal as you've just learned. For the number 2.75, say "seventy-five hundredths."

Therefore, the mixed decimal 2.75 is read as "two and seventy-five hundredths."

Example: Read each of the following mixed decimals.

Mixed Decimal	**Decimal as Read**
21.07	twenty-one and seven hundredths
5.13	five and thirteen hundredths
14.26	fourteen and twenty-six hundredths
300.007	three hundred and seven thousandths
35.22	thirty-five and twenty-two hundredths
496.325	four hundred ninety-six and three hundred twenty-five thousandths

Fractions and Decimals: A Comparison

As you know, fractions and decimals are similar in that they both represent part of a whole. In fact, every decimal can be written as a fraction, and every fraction can be converted to decimal form. Let's look at how these procedures are done.

Changing Decimals to Fractions

To write a decimal as a fraction, follow these steps:

1. Delete the decimal point and use the number in the decimal as the numerator, ignoring any zeros that immediately follow the decimal point.
2. Determine the place value of the decimal (tenths, hundredths, thousandths, and so on). (You learned how to do this in the section Reading Decimals.)
3. Use the place value of the decimal as the denominator.

Example: Write the decimal .23 as a fraction.

$\frac{23}{\ }$ Delete the decimal point and use the number in the decimal as the numerator.

.23 Determine the place value of the decimal. The place value of the decimal .23 is hundredths.

$\frac{23}{100}$ Use the place value of the decimal as the denominator.

The decimal .23 can be written as the fraction $\frac{23}{100}$.

Example: Write the decimal .016 as a fraction.

$\frac{16}{\ }$ Delete the decimal point and use the number in the decimal as the numerator, ignoring any zeros that immediately follow the decimal point.

.016	Determine the place value of the decimal. The place value of the decimal .016 is thousandths.
$\frac{16}{1000}$	Use the place value of the decimal as the denominator.

The decimal .016 can be written as the fraction $\frac{16}{1000}$.

Changing Fractions to Decimals

The previous example shows that decimals can represent fractions with denominators such as 1, 10, 100, 1,000, 10,000, and so on. So, if the denominator of a fraction is 10, 100, 1,000, etc., the fraction can easily be changed to a decimal because decimals are expressed in tenths, hundredths, thousandths, ten thousandths, and so on. Table 1 gives you an idea of how this works.

Table 1

EXPRESSING FRACTIONS AS DECIMALS

Fraction	Equivalent Decimal
$\frac{1}{10}$	.1
$\frac{1}{100}$	.01
$\frac{1}{1000}$	.001
$\frac{1}{10,000}$	.0001
$\frac{1}{100,000}$	.00001
$\frac{1}{1,000,000}$	.000001

However, if the denominator isn't a power of 10, you have to do a little more work. You should remember that a fraction is nothing more than a division problem. The fraction $\frac{5}{8}$, for example, is the same as $5 \div 8$. To change any proper fraction to its decimal equivalent, you must simply carry out this division. The following examples show you how this is done.

Example: Change the fraction $\frac{5}{8}$ to a decimal.

$8\overline{)5}$	Write the fraction as a division problem.
$8\overline{)5.}$	Add the decimal point to both the dividend and the quotient. (*Remember:* The decimal point in a whole number is at the far right.)

$$\begin{array}{r} .625 \\ 8\overline{)\,5.000} \\ \underline{-4.8} \\ 20 \\ \underline{-16} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

Divide as you would for whole numbers. Add as many zeros as you need to make the division come out evenly.

The fraction $\frac{5}{8}$ is the same as .625 in decimal form.

Example: Change $\frac{3}{4}$ to a decimal.

$$\begin{array}{r} .75 \\ 4\overline{)\,3.00} \\ \underline{-2\,8} \\ 20 \\ \underline{-20} \\ 0 \end{array}$$

Write the fraction as a division problem. Divide, adding as many zeros as needed to make the division come out evenly.

The fraction $\frac{3}{4}$ is the same as .75 in decimal form.

In both these examples, the division ends evenly. That is, there's no remainder. In such cases, you call the quotient a *terminating decimal*. Sometimes, however, the division doesn't end evenly. In such cases you should carry out the division one place beyond the desired number of decimal places and then round off.

Example: Change the fraction $\frac{1}{7}$ to a decimal with three decimal places.

$$\begin{array}{r} .1428 \\ 7\overline{)\,1.0000} \\ \underline{-7} \\ 30 \\ \underline{-28} \\ 20 \\ \underline{-14} \\ 60 \\ \underline{-56} \\ 4 \end{array}$$

Set up the fraction as a division problem, add the decimal point, and divide out to four decimal places.

.1428 = .143 Round off your answer to three decimal places.

Therefore, the fraction $\frac{1}{7}$ is approximately the same as .143 in decimal form.

Here's a summary of the steps for rounding off a number:

Step 1: Find the digit you want to round to. (It may help if you circle this digit.)

Step 2: Look at the digit immediately to the right of the circled digit.

Step 3: If the digit to the right is 5 or more, then *round up* by increasing the circled digit by 1. If the digit to the right is less than 5, you *round down*—you don't change the circled digit.

Step 4: Drop all digits (including zeros) to the right of the rounded digit.

Table 2 lists fractions commonly used in the industrial field, along with their decimal equivalents. This table is included for your easy reference.

Table 2

FRACTIONS AND DECIMAL EQUIVALENTS

Fraction	Decimal	Fraction	Decimal
$\frac{1}{64}$	0.015625	$\frac{33}{64}$	0.515625
$\frac{1}{32}$	.03125	$\frac{17}{32}$	.53125
$\frac{3}{64}$	.046875	$\frac{35}{64}$	.546875
$\frac{1}{16}$	.0625	$\frac{9}{16}$	.5625
$\frac{5}{64}$	.078125	$\frac{37}{64}$	.578125
$\frac{3}{32}$	.09375	$\frac{19}{32}$	.59375
$\frac{7}{64}$	.109375	$\frac{39}{64}$	.609375
$\frac{1}{8}$	.125	$\frac{5}{8}$	.625
$\frac{9}{64}$	.140625	$\frac{41}{64}$	.640625
$\frac{5}{32}$	.15625	$\frac{21}{32}$	.65625
$\frac{11}{64}$	.171875	$\frac{43}{64}$	.671875
$\frac{3}{16}$	.1875	$\frac{11}{16}$	.6875
$\frac{13}{64}$	.203125	$\frac{45}{64}$	.703125

(Continued)

Table 2

FRACTIONS AND DECIMAL EQUIVALENTS

$\frac{7}{32}$	.21875	$\frac{23}{32}$	.71875
$\frac{15}{64}$	.234375	$\frac{47}{64}$	.734375
$\frac{1}{4}$	.25	$\frac{3}{4}$	.75
$\frac{17}{64}$	.265625	$\frac{49}{64}$	.765625
$\frac{9}{32}$	.28125	$\frac{25}{32}$	.78125
$\frac{19}{64}$	.296875	$\frac{51}{64}$	.796875
$\frac{5}{16}$	.3125	$\frac{13}{16}$	.8125
$\frac{21}{64}$	.328125	$\frac{53}{64}$	.828125
$\frac{11}{32}$	.34375	$\frac{27}{32}$	.84375
$\frac{23}{64}$	.359375	$\frac{55}{64}$	.859375
$\frac{3}{8}$	.375	$\frac{7}{8}$	.875
$\frac{25}{64}$	.390625	$\frac{57}{64}$	.890625
$\frac{13}{32}$	.40625	$\frac{29}{32}$	.90625
$\frac{27}{64}$	.421875	$\frac{59}{64}$	.921875
$\frac{7}{16}$	.4375	$\frac{15}{16}$	.9375
$\frac{29}{64}$	.453125	$\frac{61}{64}$	.953125
$\frac{15}{32}$	.46875	$\frac{31}{32}$	.96875
$\frac{31}{64}$	.484375	$\frac{63}{64}$	0.984375
$\frac{1}{2}$	0.500	1.000	1.000

Percent and Its Application

As an industrial worker, you probably deal with percents quite often. For example, if you're checking to determine if a pump, compressor, or motor is working properly, you may have to reduce its rated capacity by a certain percent to allow for load conditions, altitude, and temperature. You may be asked to remove a certain percent of the material loaded on a storage pallet, or you may have to check the operation of a control valve at a percent of its full-open position.

The Vocabulary of Percent

The word *percent* means by the hundreds. Thus, a number expressing percent is the numerator of a fraction which has a denominator of 100. This denominator isn't actually written, since the word *percent* or the percent symbol (%) is generally used. Therefore, 23% means $\frac{23}{100}$, or 0.23.

Most percent problems involve three numbers—the *rate*, the *base*, and the *percentage.* The connection between these numbers can be expressed in an equation, or formula, as

$$R \times B = P$$

The R in this equation stands for the rate, the B for the base, and the P for the percentage. Let's take a closer look at each of these terms.

Rate. In percent problems, the rate is expressed either by the word *percent* or by the symbol %. Therefore, the rate is usually very easy to identify in a problem. For example, suppose you're told that 6% sales tax is charged on a purchase. The rate is 6%.

Base. The term *base* refers to the whole amount to which the rate is applied. The base most often follows the word "of." For example, if a problem asks you to find 25% of 60, the number 60 is the base.

Percentage. The percentage is the part of the base, or part of the whole, that you're often asked to calculate. It's the number that results from multiplying the base by the rate.

In almost all percent problems, you'll be given *two* of the three numbers (rate, base, or percentage) and asked to find the *unknown third* number. For example, if you're given the rate and the base, you must find the percentage. If you're given the percentage and the base, you must find the rate.

Depending on what number you need to find, you'll use a certain arrangement of the formula $R \times B = P$. Figure 8 contains the three arrangements of the formula you'll need to solve percent problems.

To Find:	Use the Formula:
Percentage (P)	$P = R \times B$
Rate (R)	$R = P \div B$
Base (B)	$B = P \div R$

FIGURE 8—Percent Formulas

Solving Percent Problems

Let's take a look at some actual problems so that you can see how percent works.

Example: Suppose you have to reduce the horsepower of a standby diesel engine by 15%. If the rated horsepower of the engine is 320 hp (horsepower), how much of a horsepower decrease is required? Also, what would be the new horsepower that the engine should deliver?

To solve this problem, you must use the percent formula. You're given the rate (15%) and the base (320). Therefore, you know that you're looking for the percentage.

$P = R \times B$ **Select the correct formula.**

$P = .15 \times 320$ **Substitute the known values into the formula. That is, put the values for *R* and *B* into the formula in place of the letters. Change the rate to a decimal (15% = .15).**

$$
\begin{array}{r}
320 \\
\times \quad .15 \\
\hline
48.00
\end{array}
$$

Perform the calculation.

The answer is 48. You must reduce the horsepower by 48. Therefore, the new horsepower will be 320 – 48 = 272.

Example: Suppose you have to find the percent of power loss on a standby diesel engine that's delivering 225 hp instead of its rated 330 hp.

In this problem, you're given the base (330) and the percentage (225). You must find the rate. (Carry your answer out to four decimal places.)

$R = P \div B$ **Select the correct formula.**

$R = 225 \div 330$ **Substitute the known values into the formula.**

$$
\begin{array}{r}
.6818 \\
330\overline{)\;225.0000} \\
-198\,0 \\
\hline
27\,00 \\
-26\,40 \\
\hline
600 \\
-330 \\
\hline
2700 \\
-2640 \\
\hline
60
\end{array}
$$

Perform the calculation.

The answer is 0.6818. As you can see, however, this isn't a rate (a percent). It's simply a decimal. To change this decimal to a percent, move the decimal point two places to the right and add the percent sign. Thus, 0.6818 becomes 68.18%.

If you don't need your answer to be extremely accurate, you may round off 68.18% to 68.2%, or even 68%. So, the engine when operating at 225 hp is actually performing at 68% of its capacity. The power loss would therefore be 100% – 68% = 32%.

Example: You and your department are working on a large contract for an important client company. The job calls for the production of 2,500 units. At the end of the week you find that you've completed 1,200 units. Your supervisor wants to know your percent of completion. In other words, you must determine what percent of the job is completed.

This is another rate problem. You know the base (2,500) and the percentage (1,200). You must find the rate.

$R = P \div B$ **Select the formula.**

$R = 1200 \div 2500$ **Substitute the values in the formula.**

Perform the calculation.

$$\begin{array}{r} .48 \\ 2500\overline{)\,1200.00} \\ -1000\,0 \\ \hline 200\,00 \\ -\,200\,00 \\ \hline 0 \end{array}$$

$.48 = 48\%$ **Change the decimal to a percent.**

The answer is 48%. You've completed 48% of the job so far.

Fractional Equivalents for Percents

Sometimes it's necessary for you to change a percent to a fraction. To do this, you simply remove the percent sign from the given percent and write the number remaining over a denominator of 100. For example, 37% equals the fraction $\frac{37}{100}$.

Here's an example of what you may come across in your work:

Example: Suppose you receive three quarters of a drum of fuel oil for the plant's standby diesel engine. When you check the drum, you find that 5% of it is water. Your job is to find out exactly what part of the contents of the barrel is water, so that it can be reported to the supplier. Therefore, you must find 5% of $\frac{3}{4}$.

Although you can't multiply a fraction by a decimal, you can solve this problem by changing the percent to a fraction and then multiplying the two fractions.

$5\% = \frac{5}{100}$ **Change the percent to a fraction.**

$\frac{5}{100} = \frac{1}{20}$ **Reduce the fraction to its lowest terms.**

$\frac{1}{20} \times \frac{3}{4} = \frac{3}{80}$ **Perform the multiplication.**

The answer is $\frac{3}{80}$. Therefore, $\frac{3}{80}$ of the drum is water.

Very often you can make a percent problem simpler by going directly to a fractional equivalent. For example, if you were asked to remove $33\frac{1}{3}\%$ of a 90-lb (pound) load of bricks, it's quicker and easier to convert $33\frac{1}{3}\%$ to its fractional equivalent, $\frac{1}{3}$, and then multiply 90 by $\frac{1}{3}$ to get the answer. Table 3 provides some of the commonly used percents and their fractional equivalents.

Table 3

FRACTIONAL EQUIVALENTS OF COMMON PERCENTS

Fraction	Percent	Fraction	Percent
$\frac{1}{20}$	5%	$\frac{1}{3}$	$33\frac{1}{3}\%$, or 33.3%
$\frac{1}{16}$	$6\frac{1}{4}\%$, or 6.25%	$\frac{1}{2}$	50%
$\frac{1}{8}$	$12\frac{1}{2}\%$, or 12.5%	$\frac{2}{3}$	$66\frac{2}{3}\%$, or 66.7%
$\frac{1}{5}$	20%	$\frac{3}{4}$	75%
$\frac{1}{4}$	25%	$\frac{7}{8}$	$87\frac{1}{2}\%$, or 87.5%

RATIOS AND PROPORTIONS

Knowing the principles of ratio and proportion is very helpful to those who work in the industrial field. These principles are used in finding such things as gear speeds and gear reductions, compression ratios, parts costs, and in solving many other maintenance problems.

Definition of Terms

A ratio is a mathematical comparison of two numbers by division. In ratios, however, the division is simply implied; it's not actually completed. By using a ratio, you can compare two numbers or quantities. For example, a toolbox 6 in. wide and 18 in. long has a width-to-length ratio of 6 to 18 (or 1 to 3). Ratios can be expressed in many ways: as a fraction ($\frac{6}{18}$), as a division problem ($6 \div 18$), or with a colon (6 : 18). To read a ratio, you say the word *to* for the fraction line, the division sign, or the colon. Therefore, the ratio 6 : 18 is read "6 to 18," no matter how it's written.

A *proportion* is a mathematical statement showing that one ratio is equal to another ratio. For example, since we know that the ratio 1 : 3 equals the ratio 6 : 18, we can write these two ratios as the following proportion:

$$1 : 3 = 6 : 18$$

You read this proportion as "one is to three as six is to eighteen."

You may have noticed that a proportion is really a pair of equivalent fractions. Knowing this should make solving proportion problems easier for you.

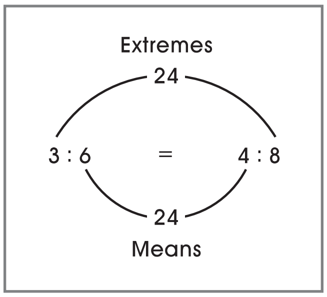


FIGURE 9—When the product of the means and the product of the extremes are equal, the two ratios are in exact proportion.

When two equal ratios are written as a proportion, the product of the two outside numbers, called *extremes,* equals the product of the two inside numbers, called *means.* Look at Figure 9. In the proportion 3 : 6 = 4 : 8, the extremes (the outside numbers) are 3 and 8; the means (the inside numbers) are 6 and 4. Notice that both products equal 24.

Direct Proportions

Solving Direct Proportion Problems

To help explain what a direct proportion is, let's use an example.

Example: Suppose you know that it takes 32 gal (gallons) of diesel fuel for the maintenance truck to travel 200 miles, and you want to find out how many miles the truck can go on 45 gal. The proportion you would use is $32 : 45 = 200 : x$. The letter x is used as the unknown quantity—in this case, the number of miles the truck can go on 45 gal.

Here are the steps in solving a proportion problem:

1. Set up the proportion, using the letter x as the unknown term.

2. Multiply the extremes and then multiply the means. Set up the products as equal.

3. Divide both sides of the equation by the remaining original term. The result is the value of the unknown term.

Let's use this procedure to solve the fuel problem just given.

$32 : 45 = 200 : x$ **Set up the proportion.**

$32 : 45 = 200 : x$ **Multiply the extremes and then multiply the means. Set up the products as equal.**
$32x = 9000$

$\frac{\cancel{32}x}{\cancel{32}} = \frac{9000}{32}$ **Divide both sides of the equation by the remaining original term (32). The result is the value of the unknown term.**
$x = 281.25$

The answer is 281.25. The maintenance truck will go 281.25 miles on 45 gal of fuel.

Setting Up a Direct Proportion Problem

How did we know how to set up the numbers in the preceding example? Actually, they can be set up in several ways and still be correct, but the best way is to make sure that the units used in the first ratio are alike and the units in the second ratio are alike. In the fuel example, both units in the first ratio were gallons, and both units in the second ratio were miles.

Notice something about this example. If the terms on one side of the equal sign increase, the terms on the other side will also increase. If the terms on one side of the equal sign decrease, the terms on the other side will also decrease. Such a proportion is known as a *direct proportion.*

In a direct proportion, you must also be sure that you set up both ratios in the same manner. In the example, the first ratio had the smaller amount of fuel over the larger amount of fuel. Therefore, the second ratio should have the smaller mileage over the larger mileage.

RULES FOR SETTING UP DIRECT PROPORTIONS

1. The units used in the first ratio must be alike, and the units used in the second ratio must be alike.
2. Both ratios must be set up in a similar manner (larger to smaller or smaller to larger).

Indirect Proportions

As you might have guessed, an indirect proportion is the opposite of a direct proportion and therefore must be treated differently. In an indirect proportion, as one term increases, the other decreases.

For example, if 2 maintenance workers can overhaul all of the pumps in a plant in 12 work days, how many days would it take 4 maintenance workers to do the same job? It should take 4 workers less time to do the job than it did the 2 workers. In other words, as the number of workers increases, the number of days required to do the job decreases; and conversely, as the number of workers decreases, the number of days increases.

The terms in an indirect proportion must be written in a somewhat different manner than those in a direct proportion, but once written, the proportion is solved in the same manner.

Example: Let's find out how many days it would take the 4 maintenance workers to do the job that 2 workers did in 12 days.

$2 : 4$	**Set up the first proportion. Just as it was in a direct proportion, the terms of the first ratio should be alike. In this ratio, both the 2 and the 4 refer to the number of workers. We're comparing workers to workers.**
$2 : 4 = x : 12$	**Let x stand for the unknown number of days. Write the second ratio in reverse order of the first. (Notice, however, that the second ratio contains like terms—we're comparing days to days.)**
$2 : 4 = x : 12$ $4x = 24$ $x = 6$	**Solve the proportion as you would solve a direct proportion.**

The answer is 6. It will take six days for 4 workers to do the job. The more workers there are, the less time it will take to do the same job.

Since indirect proportions may be a bit confusing, let's take a look at another one.

Example: A factory has contracted to complete a particular job in 7 working days. The manager knows that the 4 machines in the plant can do the job in 14 days. How many machines does the plant need to complete the job on time? (This is an indirect proportion because you know that the more machines you have, the less time it will take. Therefore, as one term increases, the other decreases.)

$7 : 14$	**Set up the first ratio. (Notice that the units in the ratio are alike—days.)**
$7 : 14 = 4 : x$	**Set up the second ratio in reverse order.**
$7 : 14 = 4 : x$	
$7x = 56$ $x = 8$	**Solve the proportion.**

The answer is 8. The plant needs 8 machines to do the job in the allotted time. (It already has 4 machines. Therefore, it needs an additional 4 machines to do the job.)

ANGLES

Kinds of Angles

An *angle* is formed by two straight lines drawn from a common point. The point is called the *vertex* of the angle, and the lines are called the *sides.* An angle really represents a section of a circle. The vertex of the angle is the center point of the circle (Figure 10).

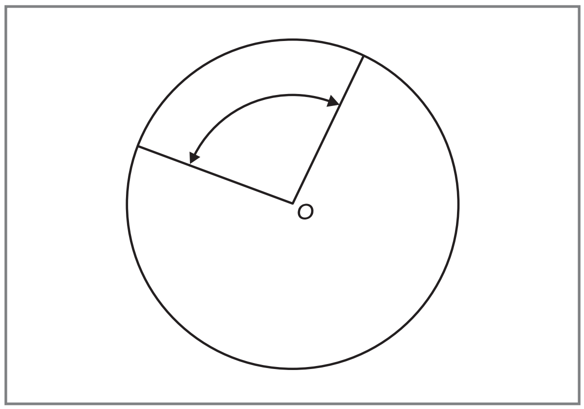


FIGURE 10—The arrows in this figure indicate one type of angle. Point O is both the center of the circle and the vertex of the angle.

A circle is divided into 360 equal parts, called degrees. The symbol for degrees is °; therefore, 90° is read as "ninety degrees."

Let's use a circular clock to illustrate the kinds of angles. Each time the minute hand on a clock completes one whole turn around the face, it passes through 360°. At six o'clock the hands of a clock divide the face in half. Therefore, the angle is 180° ($\frac{1}{2} \times \frac{360}{1} = 180$). Such an angle is called a *straight angle,* as shown in Figure 11A. At three o'clock the hands of a clock encompass $\frac{1}{4}$ of the face. The angle represented is 90° ($\frac{1}{4} \times \frac{360}{1} = 90$) and is called a *right angle,* as shown in Figure 11B. Finally, at nine o'clock the hands represent an angle of 270° because it covers $\frac{3}{4}$ of the face of the clock ($\frac{3}{4} \times \frac{360}{1} = 270$) Figure 11C.

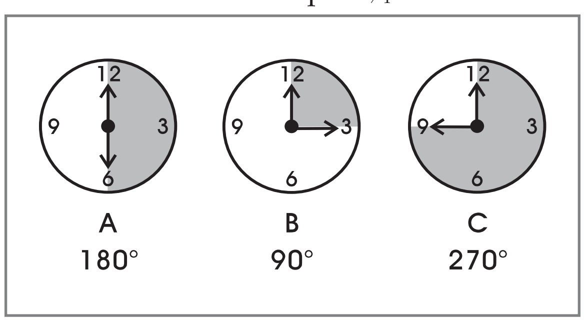


FIGURE 11—The shaded areas in each of the clocks indicate the angles. The hands of the clock in A show a straight angle (180°); in B, a right angle (90°); and in C, a 270° angle.

Any angle less than 90° is called an *acute angle*. Any angle more than 90° is called an *obtuse angle*. Examples of the types of angles are given in Figure 12.

Division of Angles

You may think that one degree is a rather small part of a circle. Although you're right, each degree is further divided into 60 equal parts called *minutes.* Then, each minute is divided into 60 equal parts called

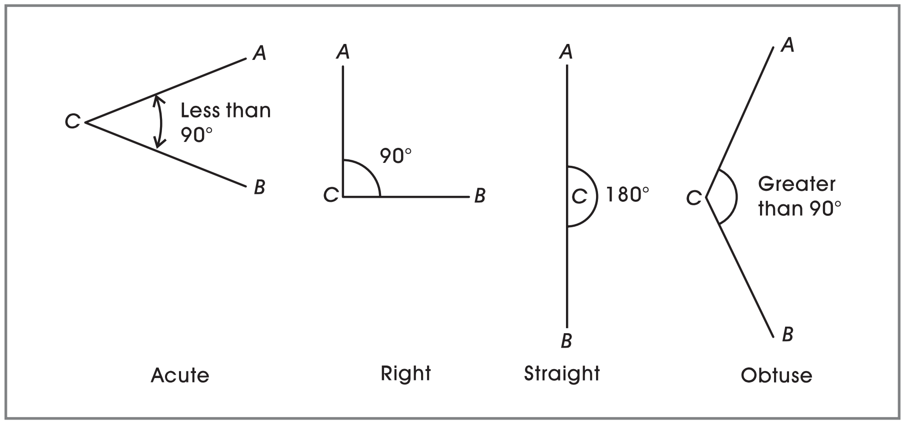


FIGURE 12—In each of the angles here, the point C at which the two lines meet is called the vertex of the angle. Lines AC and BC are the sides of the angles.

seconds. These very small divisions allow for measurements of extreme accuracy.

The symbols for these divisions are ′ (minutes) and ″ (seconds). Therefore, an angle of 35°19′12″ is read as "thirty-five degrees nineteen minutes and twelve seconds."

Protractors

A Protractor

Look at the protractor in Figure 13. A protractor is a half circle, usually made of clear plastic. The outer edge of the curved area is marked off in half degrees. Notice that the scale on this arc goes from 0° to 180° in *each* direction so you can measure an angle from either side. Each half degree, of course, represents 30 minutes ($\frac{1}{2} \times \frac{60}{1} = 30$).

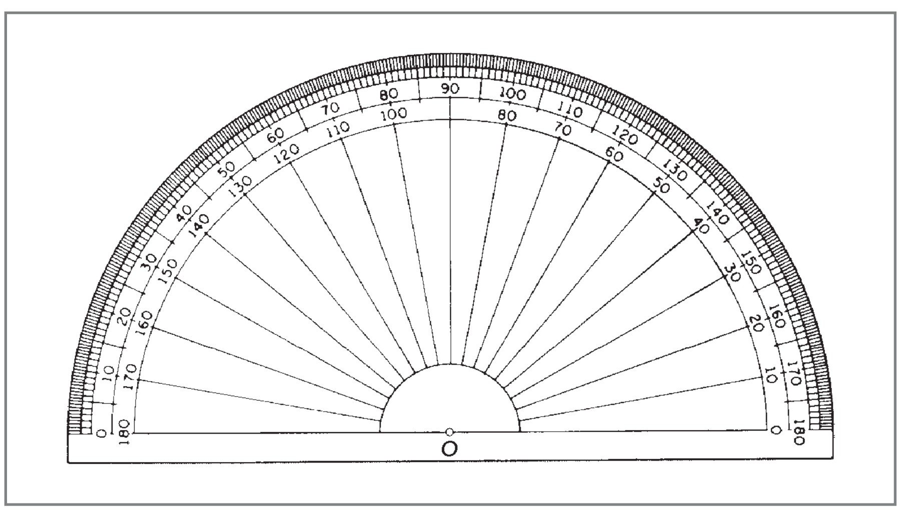


FIGURE 13—A Protractor

You need a protractor to complete some exercises in this section. You can ask your supervisor if he or she has one you can borrow. Or, you can purchase one at a discount store or a school supply store.

A protractor can be used either to measure an angle or to lay out an angle. Refer to Figure 14 on page 41. To measure an angle, follow this procedure:

1. Place the protractor on the angle to be measured. Position the center point of the protractor (*O*) at the vertex of the angle. Line up one side of the angle with the line along the straight edge of the protractor, as shown in Figure 14A.

2. Check to see where the other side of the angle crosses the protractor scale, as shown in Figure 14B.

3. To determine the angle, read the scale that shows zero at the first side, as shown in Figure 14C.

Angle *BOC* in Figure 14C is 54°, because the second side crosses the appropriate scale at 54.

You'll notice in Figure 14 that there's a second angle (*AOC*). Although you can measure this angle with the protractor, there's another way to calculate the angle. Since the line *AB* is a straight line, angle *AOC* is a straight angle and therefore has 180°. If angle *BOC* is 54°, then angle *AOC* is 180° – 54° = 126°. Any time a straight angle is divided into two angles, the sum of the two angles is 180°.

Now let's say you want to draw an angle of 60°. To lay out an angle with a protractor, follow these steps:

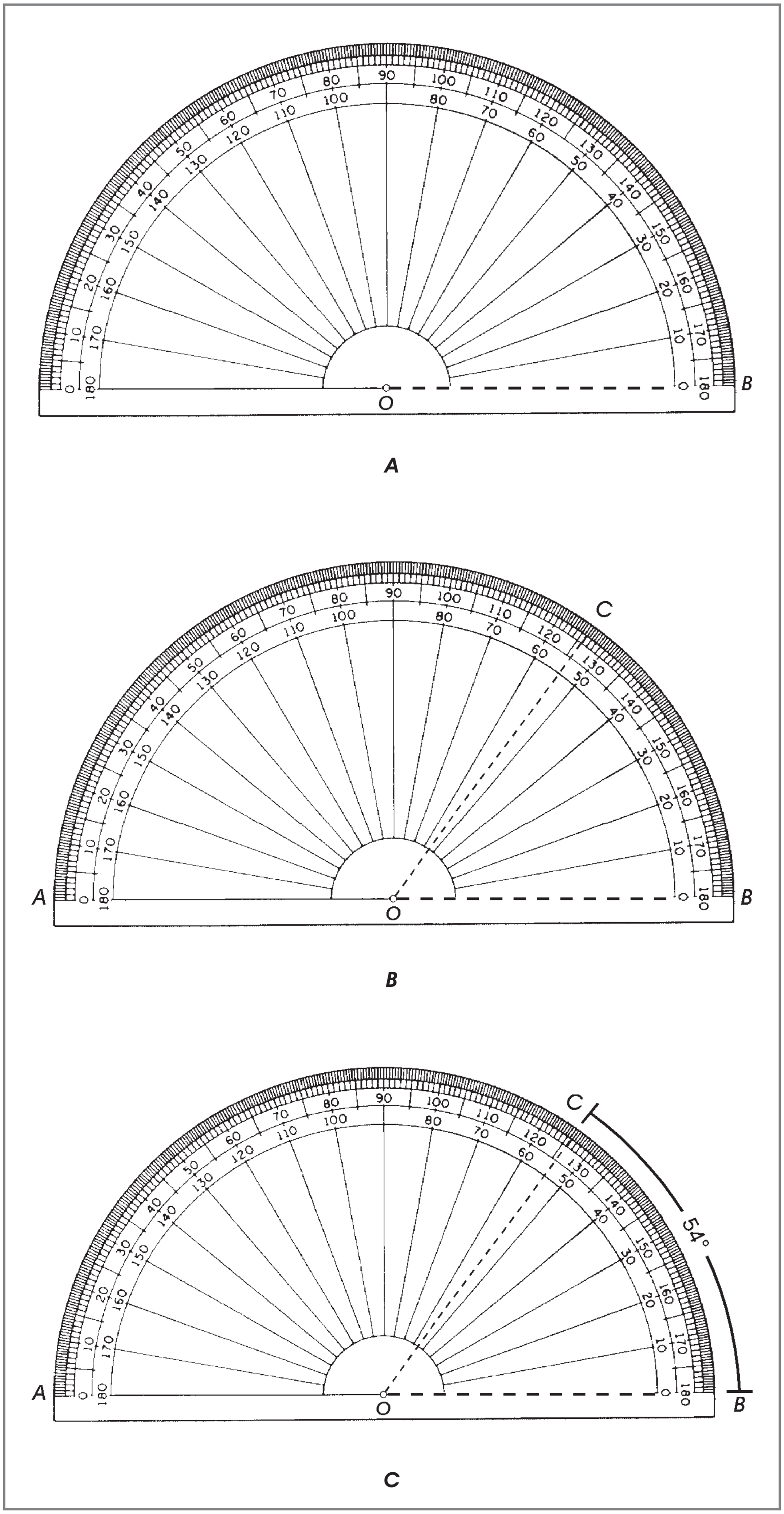
1. Use a straightedge to draw a straight line to represent one side of the angle, as shown in Figure 15A on page 42. (You can use the flat side of the protractor.)

2. Place the 180° (straight) line of the protractor directly over the line you've drawn. Insert your pencil point through the hole at *O* on the protractor. Make a dot on the line, as shown in Figure 15B. (This dot marks the vertex of the angle you're drawing.)

3. Use the scale that begins with zero by the first arm. Follow this scale until you reach 60. Make a dot along the curved edge of the protractor opposite the 60. See Figure 15C.

4. Use the straightedge of the protractor to connect the vertex dot with the dot you just made, as shown in Figure 15D. The angle is 60°.

FIGURE 14—Shown here is the procedure for measuring an angle with a protractor. In A, one side of the angle is lined up along the straight edge of the protractor. In B, you can see where the second side of the angle crosses the scale on the protractor. In C, the scale shows that the angle is 54°.



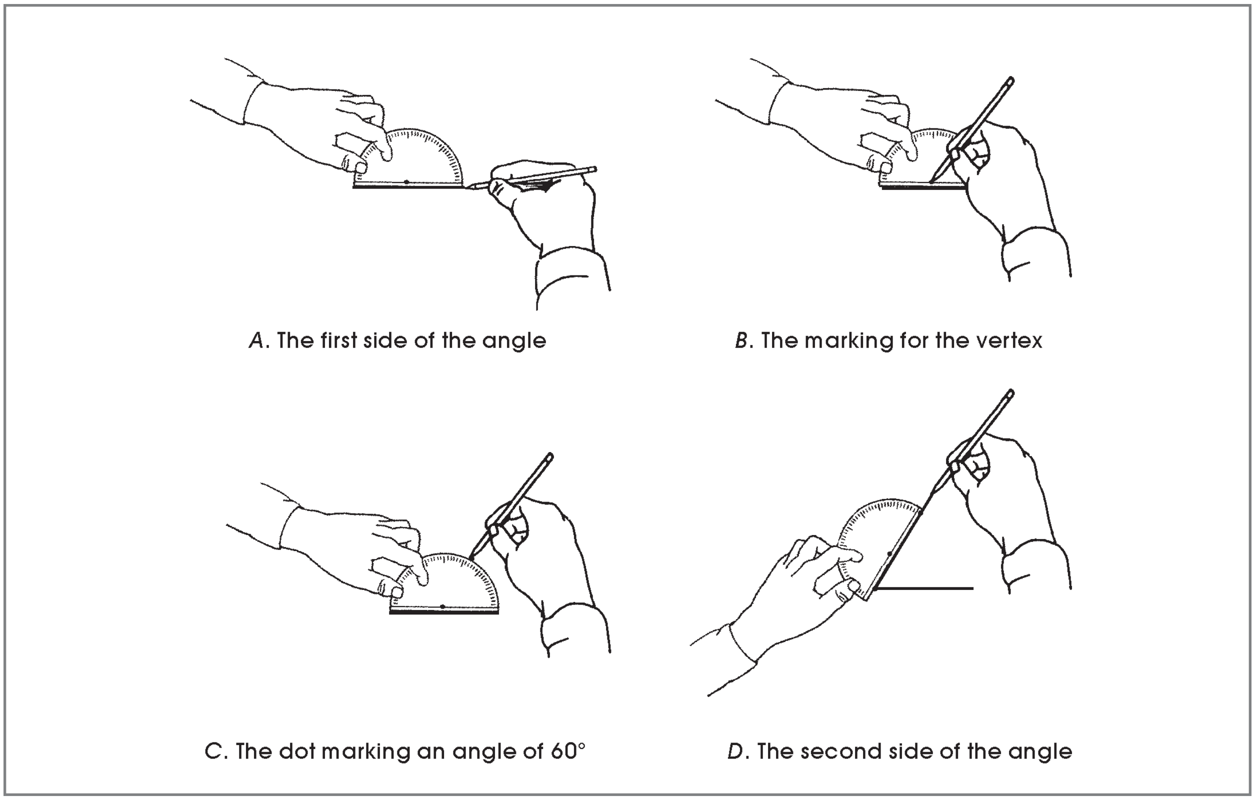


FIGURE 15—This figure shows the steps for drawing an angle.

Here's another way that a protractor can be used. Suppose you have to mount a turbocharger assembly on an engine. Suppose also that the oil drain outlet must point downward at no more than 30° from the vertical, as shown in Figure 16. You could locate the posi-

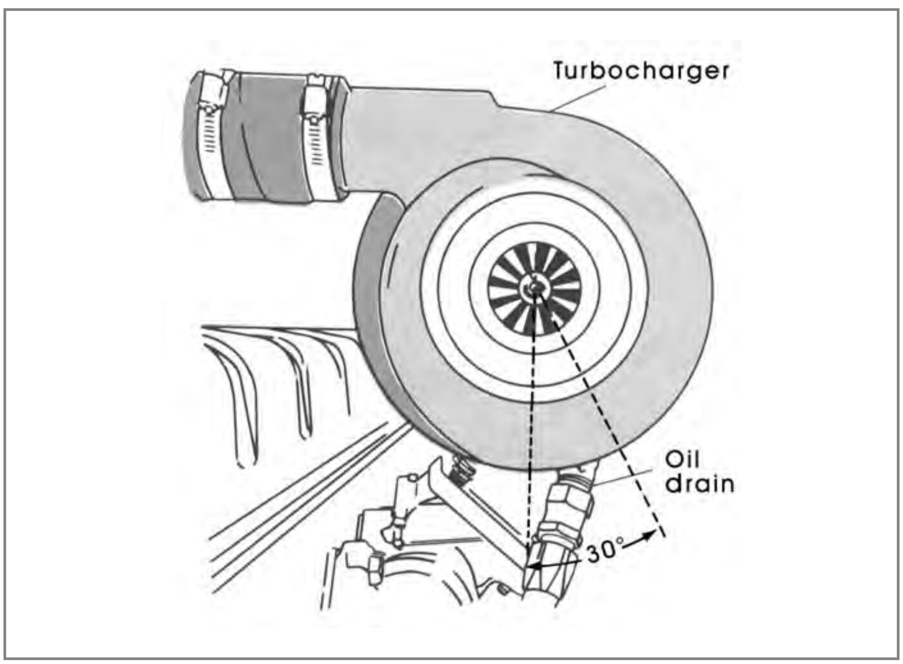


FIGURE 16—The oil drain position on the turbocharger must fall within the 30° angle shown here.

tion of the drain by eye, but as a good industrial worker, you should make sure its position is accurate. To do this, make a cardboard wedge or template cut at 30°. Place the template against the surface of the turbocharger at a vertical level to see if the oil drain position is correct.

To construct such a template, simply lay out an angle of 30° on a piece of cardboard. (Be sure to make the sides long enough to do the measurement on the engine.) Cut out the angle from the cardboard and you have your template for checking the position of the drain outlet.

Other Protractors and Angle Measuring Tools

When more accurate angle measurements are needed, a protractor with a vernier scale (Figure 17) may be used. Although the instrument shown here can measure an angle to the nearest minute, others are made which can measure in seconds.

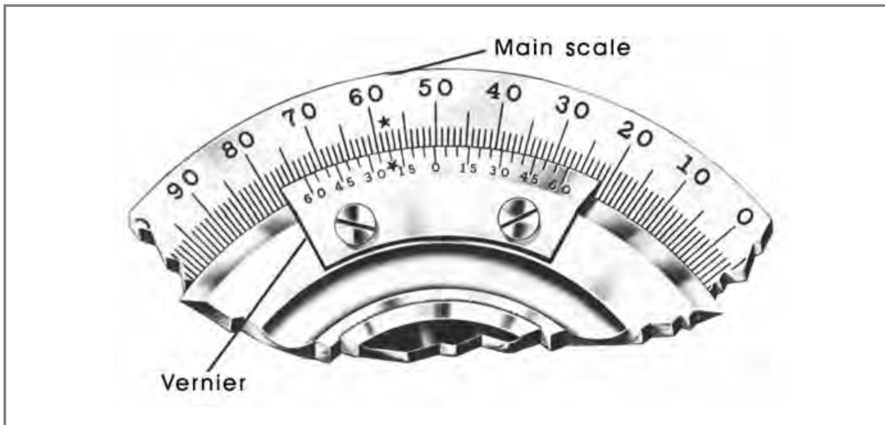


FIGURE 17—A vernier protractor allows you to measure an angle with a certain amount of precision. Here the basic angle is 50° (on the main scale) plus 20′, as indicated by the vernier. The two stars show you which 5-minute mark on the vernier lines up perfectly with a degree mark so that the measured angle is 49° 40′.

In Figure 18, on the next page a maintenance worker is shown using a steel protractor combination set to check the angle of a lock-release mechanism on a machine.

The accuracy and depth of a machined block is being checked in Figure 19. The tool is a combination square. Note that the square head and steel rule form a 90°, or right, angle. The measured depth of the machine piece here appears to be 3 ¼ in.

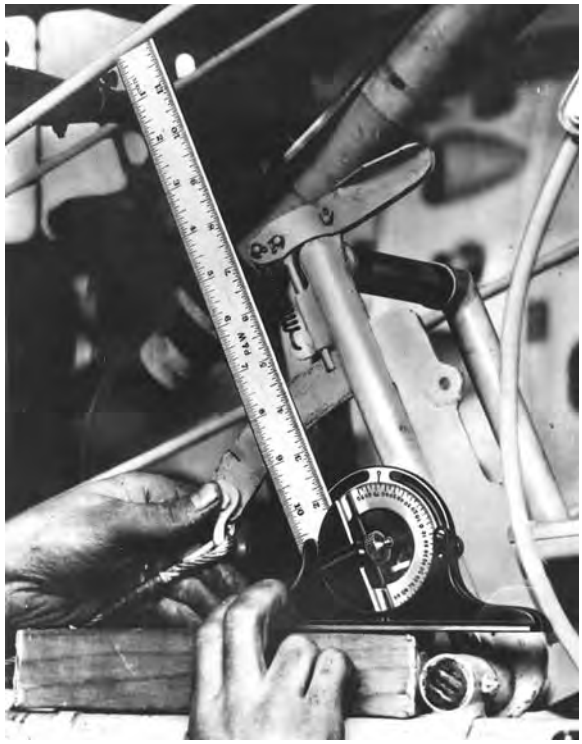


FIGURE 18—By placing the base of the protractor on a horizontal flat board, the angle of the mechanism can easily be checked. (Courtesy of Pratt & Whitney Machine Tool, Division of Colt Industries)

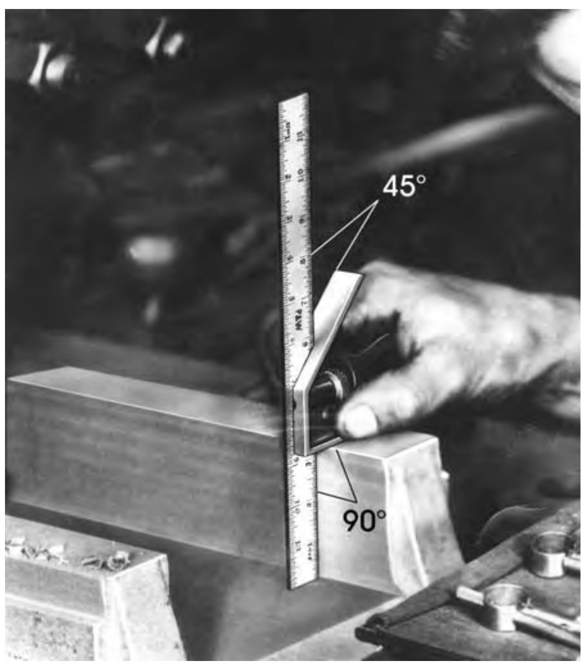


FIGURE 19—It's a simple task to measure an angle and a dimension when you have the proper tool. Since this combination square does NOT have an adjustment for varying the angle, only a 90° angle and a 45° angle can be checked for trueness.

Using Your Calculator

You may have been thinking that your calculator would come in handy for some of the calculations you've been doing in this study unit. Of course, you're right. We're going to look at three separate applications for your calculator.

Changing Fractions into Decimals

You already know how to change a fraction into a decimal (by dividing) and you know how to use your calculator to divide. Therefore, you should be able to use your calculator to change any fraction into a decimal.

Let's take a look at two examples.

Examples: To change the fraction $\frac{3}{4}$ into a decimal, follow these steps:

1. Turn on the calculator.
2. Enter the numerator (3).
3. Press the division key.
4. Enter the denominator (4) and press the equal key. The window on your calculator should display the decimal 0.75.

That's a lot easier than performing the division on paper, isn't it? But that was a rather easy one. Let's try another.

To change the fraction $\frac{62}{109}$ to a decimal, follow these steps:

1. Turn on the calculator.
2. Enter the numerator (62).
3. Press the division key.
4. Enter the denominator (109) and press the equal key.

Now look at the window of your calculator. It should display the decimal 0.5688073. Your calculator will carry out the division to as many place values that will fit in the window. You can then round off the decimal to the place value you desire.

You can see that a calculator makes an easy job of changing complex fractions into decimals. Remember, however, to do each calculation twice to make sure you've entered the numbers correctly.

Solving Percent Problems

Any percent problem can be easily and quickly solved on a calculator. Suppose you purchase a CD player for $89.99. If the sales tax is 6%, how much will you pay for the player?

Here are two methods you can use to solve this problem.

Method 1: Using multiplication

1. Enter the price of the CD player (89.99).

2. Press the multiplication key.
3. Enter the tax rate (.06). (You must first change the percent to a decimal.)
4. Press the equal key. The window will display the amount of tax (5.3994). To this tax amount you must now add the price of the CD player.
5. Press the addition key.
6. Enter the price again (89.99) and press the equal key. The window will display the amount of 95.3894, which you'll round off to $95.39.

Method 2: Using the percent key. If your calculator has a key with a percent sign (%) on it, you can use this key to solve percent problems. Let's do the same problem we did in Method 1.

1. Enter the price of the CD player (89.99).
2. Press the multiplication key.
3. Enter the rate (6) and then press the percent key. The window should display the same amount it did in method 1 (5.3994). To this amount you must add the price of the CD player.
4. Press the addition key.
6. Enter the price again (89.99) and press the equal key. The window will display the amount of 95.3894, which you'll round off to $95.39.

Some calculators with percent keys allow you to add (or subtract) the percentage with only one calculation. In other words, you don't have to enter the base twice. Check the instructions that came with your calculator to see if it has this capability. The instructions should explain how to do such calculations.

Solving Proportions

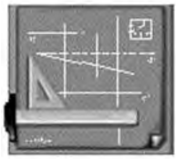
Here's an example of how to use a calculator to solve a proportion problem:

Example: Suppose you need to find x in this proportion:

$$3 : 4 = x : 20$$

1. Multiply the extremes (enter 3, press the multiplication key, and enter 20).
2. Divide by the known mean (4). (Press the division key, enter 4, and press the equal key.) The window should display the answer 15.

That was easy wasn't it? Remember, however, that your calculator will only perform the calculations you tell it to. If you enter the numbers incorrectly or press the wrong operation, the calculator can't correct you. Therefore, even though it can speed up the calculation process and assure accuracy, you still must understand *how* to solve the problems you're presented with.



Check Your Learning 3

1. Change the following decimals to fractions. Reduce your answer to its lowest terms.

   a. .07 ____________

   b. .3 ____________

   c. .168 ____________

   d. .0012 ____________

   e. .14 ____________

2. Change the following fractions to decimals and round off to two decimal places.

   a. $\frac{1}{5}$ ____________

   b. $\frac{3}{8}$ ____________

   c. $\frac{9}{40}$ ____________

   d. $\frac{6}{100}$ ____________

   e. $\frac{1}{9}$ ____________

3. Change the following percents to decimals.

   a. 76% ____________

   b. 23% ____________

   c. 59% ____________

   d. 2% ____________

   e. 9% ____________

4. Change the following decimals to percents.

   a. .13 ____________

   b. .25 ____________

   c. .163 ____________

   d. .9 ____________

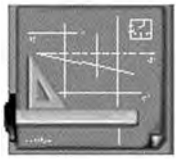
   e. .8312 ____________

5. Justus electronics rates its final testers according to the percent of a required lot they complete each week. In one week, Jim tested 33 of the 94 units in his lot. In the same week, Cheryl received a lot of 121 and tested 42 of them. According to company policy, who rates higher? (*Hint:* You must find the percent tested by each employee and then compare to determine which percent is higher.)

6. A 1000-ohm resistor has a tolerance factor of 20%. This means that the resistor's actual value can acceptably vary by ±20%. What is the acceptable range of values for this resistor?

7. The employees of a production department have been promised a 7% bonus at Christmas if they exceed their production quota for the year. What would be the bonus for an employee earning $23,500 each year?

(Continued)



Check Your Learning 3

8. Solve the following proportions for x. Round your answer to two decimal places.

   a. $2 : 3 = 6 : x$ ____________________

   b. $9 : 10 = 15 : x$ __________________

   c. $14 : 18 = x : 90$__________________

   d. $3 : 4 = x : 288$ _____________________

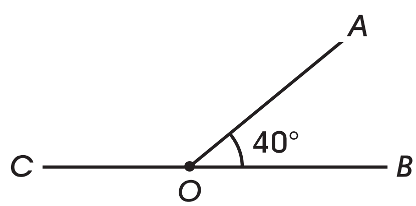
   e. $.6 : 1.8 = 1.8 : x$_____________________

9. The dimensions on a blueprint indicate that 1″ (inch) equals 3.5′ (feet). If one wall of a room on the blueprint is 6″, what is the actual length of the wall?

10. The building in which you work is to be cleaned inside from top to bottom. You're responsible for preparing the cleaning solution. The instructions call for ½ gal (gallon) of solution for every 6 gal of water. If you have 4 gal of solution, how much water will you need? (*Hint:* You may change the fraction to a decimal first if it will help in your calculation.)

11. It takes the 6 employees in your department 8 days to conduct the annual inventory of stock. If you hire 4 extra people, how long will the job take? (*Hint:* Careful on this one—there's a little catch.)

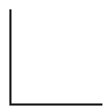
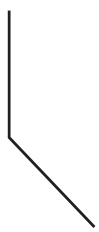
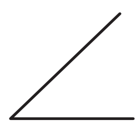
12. If angle *AOB* in the figure below is 40°, calculate the degrees in angle *AOC*.



(Continued)

Check Your Learning 3

13. Match the angles in the left-hand column below with their names in the right-hand column.

1.
2. 
3. 
4. 

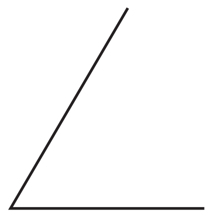
a. obtuse
b. acute
c. straight
d. right

(Continued)

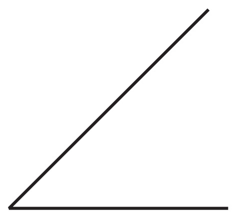
Check Your Learning 3

14. Use a protractor to measure the following angles.

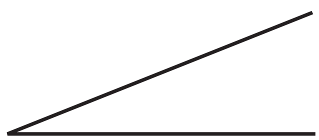
a.



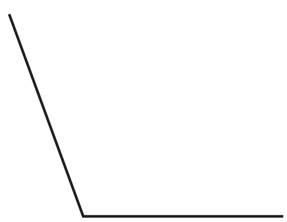
b.



c.



d.



e.

15. Use your calculator to check your answers to questions 2 and 5–11.

Check your answers with those on page 54.

CONCLUSION

You covered a lot of material in this study unit. You can be proud of your hard work. You discovered that fractions express part of a whole number. You learned how fractions are related to decimals. You can now solve problems involving percents and proportions.

You also learned how to compare numbers using ratios and equivalent ratios. And you can now measure and draw angles using a protractor. You should be able to see the value of what you studied in relation to your work and everyday life. Good luck in your future study of math!

Check Your Learning Answers

1

1. d
2. a
3. f
4. e
5. c
6. b
7. a. To stand for part of one whole thing
   b. To stand for part of a group
   c. To show division
8. 1
9. a. $\frac{3}{4}$
   b. $\frac{2}{3}$
   c. $\frac{3}{4}$
   d. $\frac{1}{2}$
   e. $\frac{4}{5}$
   f. $\frac{5}{8}$
10. a. proper
    b. improper
    c. improper
    d. proper
    e. improper
    f. improper
11. a. $1\frac{1}{3}$
    b. $2\frac{1}{2}$
    c. $2\frac{2}{5}$
    d. $1\frac{1}{3}$
    e. $4\frac{5}{6}$
    f. $5\frac{1}{2}$
12. a. $\frac{49}{8}$
    b. $\frac{13}{5}$
    c. $\frac{91}{9}$
    d. $\frac{35}{4}$
    e. $\frac{101}{16}$
    f. $\frac{99}{32}$

2

1. b
2. f
3. a
4. e
5. c
6. g
7. d
8. a. $\frac{5}{6}$ $\qquad \frac{3}{6}+\frac{2}{6}=\frac{5}{6}$
   b. $3\frac{1}{2}$

$$\begin{array}{r} 1\frac{1}{5} \\ +2\frac{3}{10} \end{array} = \begin{array}{r} 1\frac{2}{10} \\ +2\frac{3}{10} \\ \hline 3\frac{5}{10} \end{array} = 3\frac{1}{2}$$

c. $\frac{2}{11}$ $\quad \frac{9}{11} - \frac{7}{11} = \frac{2}{11}$

D. $1\frac{3}{4}$

$$\begin{array}{rcr} 3\frac{11}{20} & = & {}^{2}\cancel{3}\,\frac{\cancel{11}^{\frac{31}{20}}}{20} \\ -\;1\frac{4}{5} & = & -\;1\frac{16}{20} \\ \hline & & 1\frac{15}{20} = 1\frac{3}{4} \end{array}$$

e. $\frac{1}{6}$ $\quad \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

f. $2\frac{1}{2}$ $\quad 3 \times \frac{5}{6} = \frac{3}{1} \times \frac{5}{6} = \frac{15}{6} = 2\frac{3}{6} = 2\frac{1}{2}$

or

$$3 \times \frac{5}{6} = \frac{{}^{1}\cancel{3}}{1} \times \frac{5}{\cancel{6}_{2}} = \frac{5}{2} = 2\frac{1}{2}$$

g. $\frac{1}{2}$ $\quad \frac{3}{10} \div \frac{6}{10} = \frac{{}^{1}\cancel{3}}{{}_{1}\cancel{10}} \times \frac{{}^{1}\cancel{10}}{\cancel{6}_{2}} = \frac{1}{2}$

h. $\frac{5}{27}$ $\quad \frac{5}{9} \div 3 = \frac{5}{9} \times \frac{1}{3} = \frac{5}{27}$

9. $\frac{2}{7}$

10. $\frac{9}{64}$ in.

$$\begin{array}{rcr} \frac{3}{64} & = & \frac{3}{64} \\ \frac{1}{32} & = & \frac{2}{64} \\ +\frac{1}{16} & = & +\frac{4}{64} \\ \hline & & \frac{9}{64} \end{array}$$

11. $\frac{5}{8}$ ft.

$$\begin{array}{rcr} \frac{7}{8} & = & \frac{7}{8} \\ -\frac{1}{4} & = & -\frac{2}{8} \\ \hline & & \frac{5}{8} \end{array}$$

12. $\frac{15}{64}$ in.

$$\begin{array}{rcr} \frac{33}{64} & = & \frac{33}{64} \\ -\frac{9}{32} & = & -\frac{18}{64} \\ \hline & & \frac{15}{64} \end{array}$$

13. $\frac{39}{64}$ in. $\quad \frac{3}{4} \times \frac{13}{16} = \frac{39}{64}$ in.

14. $\frac{1}{8}$ in.

First you must find the total amount cut from the rod.

$$\begin{array}{rcr} \frac{1}{64} & = & \frac{1}{64} \\ +\frac{1}{32} & = & +\frac{2}{64} \\ \hline & & \frac{3}{64} \end{array}$$

Next, subtract this amount from the length of the rod.

$$\begin{array}{r} \frac{51}{64} \\ -\frac{3}{64} \\ \hline \frac{48}{64} = \frac{3}{4} \end{array}$$

Finally, divide the new length of the rod by 6.

$$\frac{3}{4} \div 6 = \frac{{}^{1}\cancel{3}}{4} \times \frac{1}{\cancel{6}_{2}} = \frac{1}{8}$$

(*Note:* If you prefer, you can subtract separately each amount cut from the rod and then divide by 6. The answer should be the same.)

3

1. a. $\frac{7}{100}$

b. $\frac{3}{10}$

c. $\frac{21}{125}$

d. $\frac{3}{2{,}500}$

e. $\frac{7}{50}$

2. a. 0.2
   b. 0.375 or 0.38
   c. 0.225 or 0.23
   d. .06
   e. 0.111 or 0.11
3. a. .76
   b. .23
   c. .59
   d. .02
   e. .09
4. a. 13%
   b. 25%
   c. 16.3%
   d. 90%
   e. 83.12%
5. First calculate Jim's percent.
   35.1% = Jim's percent

$$\begin{array}{r} .351 \\ 94\overline{)\ 33.000} \\ -28\ 2 \\ \hline 4\ 80 \\ -4\ 70 \\ \hline 100 \\ -94 \\ \hline 6 \end{array}$$

   Then calculate Cheryl's percent.
   34.7% = Cheryl's percent

$$\begin{array}{r} .347 \\ 121\overline{)\ 42.000} \\ -36\ 3 \\ \hline 5\ 70 \\ -4\ 74 \\ \hline 860 \\ -847 \\ \hline 13 \end{array}$$

   Finally, compare the two percents.
   Jim's rate is higher.

6. The acceptable range of values for this resistor is 800–1200 ohms.

   First, calculate the percentage of tolerance allowable.

$$\begin{array}{r} 1000 \\ \times \quad .20 \\ \hline 200.00 \end{array} = \text{allowable tolerance}$$

   Then, add and subtract this amount from the base amount to determine the range.

$$\begin{array}{r} 1000 \\ -\ \ 200 \\ \hline 800 \end{array} \qquad \begin{array}{r} 1000 \\ +\ \ 200 \\ \hline 1200 \end{array}$$

7. $1,645.00

   $\$23{,}500 \times 7\% = \$23{,}500 \times .07 = \$1{,}645.00$

8. a. $2:3 = 6:x$
      $2x = 18$
      $x = 9$
   b. $9:10 = 15:x$
      $9x = 150$
      $x = 16.666 = 16.67$
   c. $14:18 = x:90$
      Reduce 14 : 18 = 7 : 9
      $7:9 = x:90$
      $9x = 630$
      $x = 70$
   d. $3:4 = x:288$
      $4x = 864$
      $x = 216$
   e. $.6:1.8 = 1.8:x$
      $.6x = 3.24$
      $x = 5.4$

9. 1 in. = 3.5 ft = 6 in.: x ft
   $x = 21$ ft

10. $.5:6 = 4:x$
    $.5x = 24$
    $x = 48$ gal

11. x = 4.8 days

    If you hire 4 extra people, you'll have 10 people working on the inventory. The proportion, which is an indirect proportion, is

$$\begin{aligned} 6:10 &= x:8 \\ 10x &= 48 \\ x &= 4.8 \end{aligned}$$

12. The straight angle *COB* is 180°. Angle *AOB* is 40°. Therefore, angle *AOC* is 180° – 40° = 140°.

13. 1. c

    2. d

    3. a

    4. b

14. a. 60°

    b. 45°

    c. 23°

    d. 110°

    e. 86°

15. After you complete each problem on the calculator, check your answer against the one you got and against the one in this answer key.