Practical Shop Math
(Part I)

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Edition 1

Practical Shop Math (Part I)

Instructional Objectives

This module will teach you the basics of shop mathematics as they relate to machine technology. The first part of the module involves working with fractions and fractional parts of whole units. This is followed by decimals, powers of ten, squares and square roots, machine drive ratios, and the module ends with the practical application of the SI (commonly called metric) system of measurements.

The second module of this two-part lesson will present the fundamentals of geometry and trigonometry. Like this first module, the second lesson will teach you how geometry and trigonometry are applied in machine shop projects.

Upon completion of this module, you will be able to:

- *Add, subtract, multiply, and divide fractions and decimal values of whole units such as inches, feet, etc.*
- *Derive the square root of fractions and decimals.*
- *Compute a machine drive ratio for a given speed.*
- *Make calculations using the metric, or SI, system of measurement.*

Contents

Practical Shop Math (Part I)

1 Using Mathematics in Machining

A sound working knowledge of practical mathematics is vital to your work in a machine shop. You must be able to compute and measure the dimensions of parts to be made, and to calculate machine settings for various projects. You will also use mathematics when you inspect finished parts to be sure they are within specified tolerances and generally meet design requirements.

The machinist in **Fig. 1** is using an inside micrometer to measure the diameter of a hole in a workpiece. He must first compute the approximate settings for the instrument before inserting it into the hole for more accurate readings. Then, he will adjust the micrometer accordingly. The machinist could not perform the necessary calculations and measurements without a good working knowledge of practical shop mathematics.

In **Fig. 2**, the machine operator is setting the programmable numerical controls on a precision turning machine. He must be able to make accurate calculations of critical speeds and feeds before entering the data into the machine. This also requires a good background in shop mathematics.

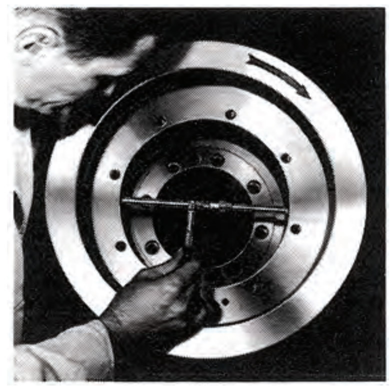

Fig. 1. Inside Micrometer

Fig. 2. Programmable numerical controls

2 Fractions

A fraction is a specific division of a whole number. A fraction is another way of stating a percentage using division. Any whole number can be written as a fraction. For example:

4 = 4/1
10 = 10/1

In a fraction, the number above the / symbol is called the numerator, and the number below the / symbol is called the denominator, where the / symbol indicates a division. A fraction is simplified by dividing the denominator into the numerator, hence:

4/1 = 4 ÷ 1 = 4
10/1 = 10 ÷ 1 = 10

This is the proof of the previous example.

Expanding on this proof, the equalities in the following example are also true.

4/4 = 1
10/10 = 1
6/2 = 3
25/5 = 5

A proper fraction is used to indicate a portion less than 100% of a quantity. For example:

$1/4 < 4/4$

The symbol $<$ indicates less than.

By simplifying a fraction, multiplying the result by 100, and adding the % symbol, you convert the fraction to a percent-age.

	1/4=1÷4	4/4=4÷4	125/100=125÷100
	1÷4=25	4÷4 = 1	125÷100=1.25
	.25×100=25%	1×100=100%	1.25×100=125%
therefore:	1/4=25%	4/4=100%	125/100=125%

Fractions are often used in machine shop work to specify dimensions of parts to be made. Simplification of fractions and conversions to percentages are often used when calculating part dimensions, and feeds and speeds. **Fig. 3** shows a typical drawing of a shaft. It has been dimensioned using a combination of fractions and whole numbers. The fractional dimensions in Fig. 3 are just as important to the machinist as those designated by the whole numbers.

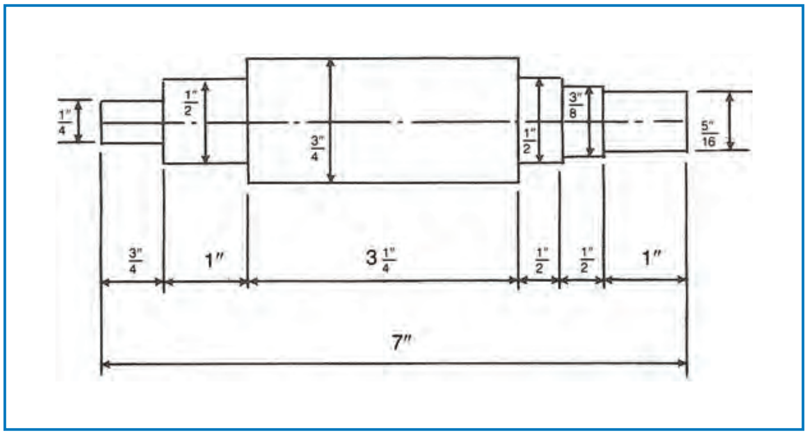

Fig. 3. Shaft dimensions use a combination of fractions and whole numbers.

3 Types of Fractions

You will be working with two basic kinds of fractions in a machine shop. A *proper* fraction has a numerator which is less than its denominator. For example, 1/2, 3/8, and 11/16 are all proper fractions. Fractions having a numerator greater than the denominator are called *improper* fractions. Examples of these are 10/3, 15/5, 20/4, etc. Whenever you encounter an improper fraction, you must convert it into a mixed number. Mixed numbers consist of a whole number and a proper fraction. You will learn more about mixed numbers further on in this module.

4 Converting Improper Fractions

Fig. 4(*a*) shows a steel disc which is to be machined to a diameter of 24/8″. It would be impractical to actually measure off 24 one-eighths of an inch to check the finished dimension. Instead we convert the improper fraction 24/8″ to a whole or mixed number. This is done by simply dividing the numerator of the improper fraction by its denominator. Thus the improper fraction 24/8″ is converted to a whole number dimension of 3″. This is much easier to step off on a scale or ruler. The correct dimensioning of the disc is shown in **Fig. 4**(*b*).

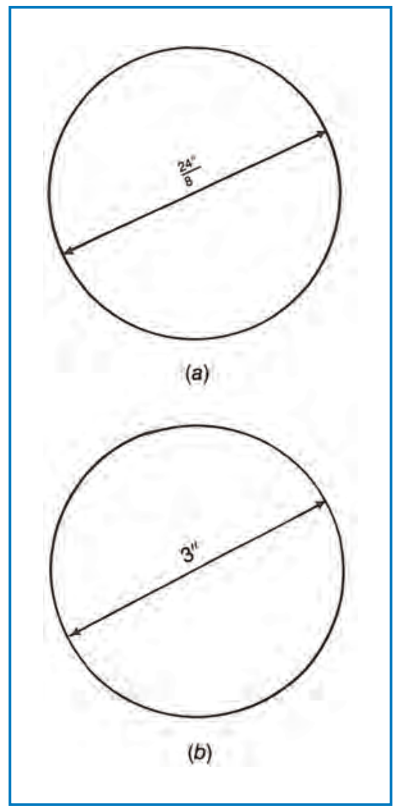

Fig. 4. Steel discs

5 Mixed Numbers

Suppose the improper fractional dimension in **Fig. 4**(*a*) was shown as 23/8″. To convert the improper fraction, divide the denominator as many times as it will go into the numerator evenly. The remainder becomes the new numerator while we retain the original denominator. So to convert 23/8 to a mixed number:

$$23/8 = 2 \text{ with a remainder of } 7.$$
$$\text{therefore: } 23/8 = 2\ 7/8$$

This remainder becomes the numerator of a fraction whose denominator is the original 8.

Mixed numbers are often used for dimensioning parts drawings. The dimensions of the pulley shown in **Fig. 5** are all designated by mixed numbers. Notice that the fractions of the various mixed number dimensions do not have the same denominators. This is merely a coincidence. The fractions could all have the same denominator, or only one or two could have like denominators. It has no effect on the pulley's measurements.

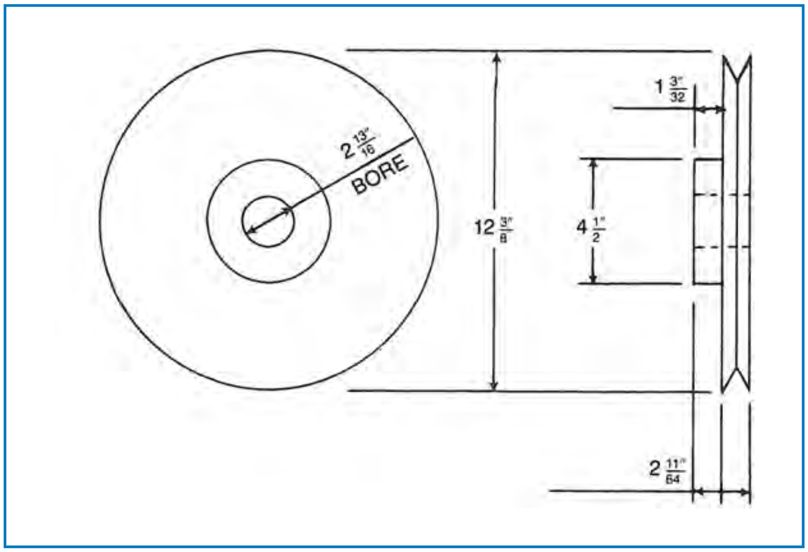

Fig. 5. Dimensions of a pulley

6 Converting Whole or Mixed Numbers to Improper Fractions

At times it may be desirable to convert whole or mixed numbers to improper fractions. This is particularly true when you must add or subtract fractions. These addition and subtraction operations will be demonstrated later in this module.

To convert a mixed number to an improper fraction, multiply the whole number part of the mixed number with the denominator part of the fraction and add the numerator part of the fraction to this result. This then becomes the numerator part of the improper fraction. To convert the mixed number to an improper fraction:

$$\begin{aligned} & 2\ 1/4 = ?/4 \\ & 2 \times 4 = 8 \\ & 8 + 1 = 9 \\ \text{therefore: } & 2\ 1/4 = 9/4 \end{aligned}$$

To convert a whole number to an improper fraction, choose your denominator and proceed as above:

$$\begin{aligned} & 9 = ?/4 & \quad & 9 = ?/6 \\ & 9 \times 4 = 36 & & 9 \times 6 = 54 \\ \text{therefore: } & 9 = 36/4 & & 9 = 54/6 \end{aligned}$$

The diameter of the sleeve bearing in **Fig. 6** is 3 1/2″. To determine how many one-half inches there are in this mixed number dimension, first multiply the whole number 3 by the denominator of the fraction. Thus, $3 \times 2 = 6$. Next add the numerator of the fraction to that product. The total of $6 + 1 = 7$ tells you how many one-half inches there are in a dimension of 3 1/2″. There are 7/2″ in 3 1/2″.

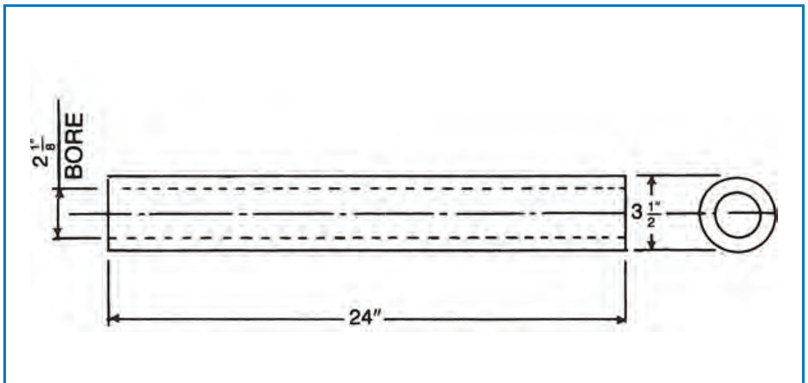

Fig. 6. Sleeve bearing

7 Reducing Fractions to Lowest Terms

In practical machine shop work fractions are expressed and used in their lowest terms. The numerator and the denominator of a fraction are called the *terms*. Fractions are reduced to their lowest possible terms by dividing both the numerator and the denominator by some number that produces a whole number in both terms that cannot be further divided evenly by any common number.

The shaft shown in **Fig. 7**(*a*) has several journals of various lengths. The lengths are expressed in fractions of an inch, however the fractions are not reduced to their lowest possible terms. If you were to measure the journal lengths using the fractions shown in Fig. 7(*a*), you would be doing a lot of extra, unnecessary counting of divisions on your scale or ruler.

Therefore, the fractions in Fig. 7(*a*) must be reduced to their lowest terms. This is done by dividing both terms by a common number to produce a whole number quotient. Thus,

$$\frac{2 \div 2}{4 \div 2} = \frac{1}{2} \quad , \quad \frac{10 \div 2}{16 \div 2} = \frac{5}{8} \quad \text{and} \quad \frac{14 \div 2}{16 \div 2} = \frac{7}{8}$$

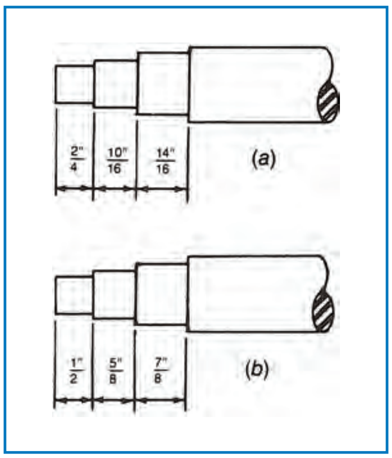

Fig. 7. Shaft

In the example it so happened that the number 2 was applicable as a divisor to reduce all of the fractions to their lowest terms. This may not always be the case. Three different divisors could have been required as will be seen later.

Proper dimensioning of the shaft journals is shown in **Fig. 7**(*b*). When reducing fractions to their lowest terms, you must find a number which, when divided into both terms, will produce a whole number.

You will learn to quickly recognize usable common divisors with practice. At first, it will be a trial and error procedure for you. Here is another example. Reduce 12/28 to its lowest possible terms.

Both 12 and 28 are evenly divisible by 2. Therefore,

$$\frac{12 \div 2}{28 \div 2} = \frac{6}{14}$$

Both 6 and 14 are also evenly divisible by 2, so:

$$\frac{6 \div 2}{14 \div 2} = \frac{3}{7}$$

Neither 3 nor 7 can be further divided evenly by any number to produce a whole number quotient. Thus, 3/7 is the final fraction reduced to its lowest terms.

Practice will show you that both terms of 12/28 could have been evenly divided by 4 or

$$\frac{12 \div 4}{28 \div 4} = \frac{3}{7}$$

thereby eliminating one step in the reduction process.

8 Determining Least Common Denominator

In order to add or subtract fractions, you must first change the fractions so that all have the same denominator. The *least common denominator* (LCD) of two or more fractions is the *smallest* denominator that is evenly divisible by the denominators of all the fractions involved. For example, to compute the LCD for 1/3, 1/6, and 1/12, first find a number that is evenly divisible by 3, 6, and 12. That number is 12.

The LCD for 1/4, 1/2, and 1/8 is 8 since 8 can be divided evenly by 4, 2, and 8. The number 16, while evenly divisible by 4, 2, and 8, is not the *least* common denominator for these values.

9 Adding Fractions

The flange in **Fig. 8** has three fractional dimensions of 3/8″, 3/4″, and 15/16″. To compute the *overall height* (OH) of the flange, you have to add these dimensions. However, this cannot be done until all dimensions have been converted to fractions having one least common denominator.

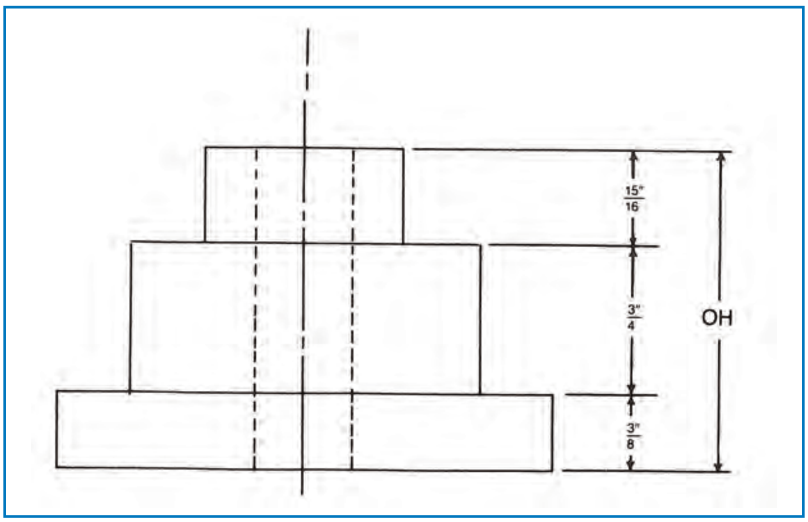

Fig. 8. Flange

By quick observation you see that 16 is the LCD, since it is evenly divisible by the denominators of all the fractions. Therefore, all three fractions must be changed to be expressed in 16ths of an inch. Since the 15/16″ dimension is already expressed properly, only the 3/8″ and the 3/4″ need to be changed.

Convert the 3/8″ dimension to 16ths by first dividing the denominator 8 into 16, producing 2 as a quotient. Next, multiply this 2 by the numerator of the fraction (3), producing 6. This number becomes the numerator of a fraction with an LCD of 16, or 6/16″.

Next, you must convert 3/4″ to 16ths. Using the same procedure, divide 4 into 16, producing 4. Multiply the numerator of 3/4″ by 4, making a numerator of 12 over a denominator of 16, or a fraction of 12/16″.

Now that you have converted all the dimensions of the flange in Fig. 8 to fractions having one common denominator, you can add the fractions to determine the overall height of the piece.

Addition of fractions is done by adding the *numerators only*. Thus,

$$\frac{3}{8} = \frac{6}{16}$$

$$\frac{3}{4} = \frac{12}{16}$$

$$\frac{15}{16} = \frac{15}{16}$$

$$\frac{33}{16} = \text{TOTAL}$$

The improper fraction 33/16″ must be converted to a mixed number by dividing 33 by 16. The result of 2-1/16″ is the overall height of the flange in Fig. 8.

Note: *Fractions cannot be added or subtracted until all are converted to fractions having a least common denominator.*

In many cases you will not be able to determine the LCD by quick observation. When this happens, use the following method to compute the LCD. Determine the LCD of 4/5, 3/8, and 13/48.

Step 1 — Select a number that divides evenly into most of the denominators. In this example the number is 8.
8) 5 * 8 * 48 = 5 * 1 * 6

Step 1 — Since 8 will not divide evenly into 5, divide each denominator by 5.
5) 5 * 1 * 6 = 1 * 1 * 6

Step 1 — The remaining 6 must now also be converted to a 1. Do this by dividing through by 6.
6) 1 * 1 * 6 = 1 * 1 * 1

Step 2 — You can now compute the LCD by multiplying all the divisors in Steps 1, 2, and 3 (8, 5, and 6) and all the quotients (1, 1, and 1) together. The product of 240 is the LCD of the fractions 4/5, 3/8, and 13/48.

To add these fractions, first change each to a fraction having 240 as a denominator. Thus,

$$\frac{4}{5} = \frac{192}{240}$$

$$\frac{3}{8} = \frac{90}{240}$$

$$\frac{13}{48} = \frac{65}{240}$$

$$\overline{\frac{347}{240}}$$

Converted to a mixed number = 1 107/240.

10 Subtracting Fractions

In order to subtract one fraction from another, both must be converted to fractions having one LCD. The lesser numerator is then subtracted from the greater. The steel bar in **Fig. 9** has a notch located 3/4″ in from its right-hand end. A machine shop project requires you to compute the remaining length of the bar, dimension “X”.

First find the LCD of the fractions 7/8″ and 3/4″. By inspection, this LCD is 8. Arrange the dimensions as follows:

$$6\ \frac{7''}{8}$$

$$\frac{6''}{8}$$

$$\overline{6\ \frac{1}{8}}\ \text{inches}$$

= remainder of the bar length.

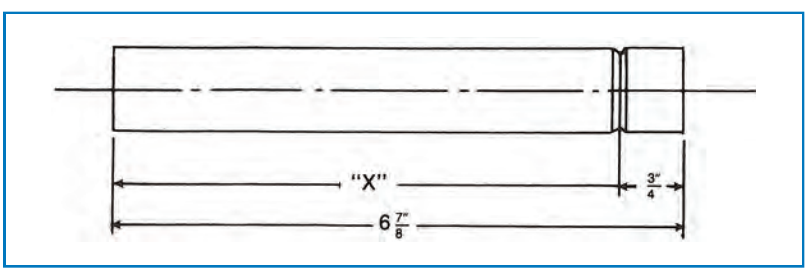

Fig. 9. Steel bar

11 Adding or Subtracting Mixed Numbers

When adding or subtracting mixed numbers, add or subtract the fractions and whole numbers separately. Convert any improper fractions into mixed numbers, then find the LCD of the fractional parts of the mixed numbers. For example, add:

$$\begin{array}{lcr} 2\ 7/16 & = & 2\ 7/16 \\ 3\ 3/8 & = & 3\ 6/16 \\ 5\ 3/4 & = & 5\ 12/16 \\ \underline{7\ 3/16} & \underline{=} & \underline{7\ 3/16} \\ & & 17\ 28/16 \end{array}$$

Convert the improper fraction 28/16 to a mixed number of 1 12/16. Reduce 12/16 to 3/4 making a new mixed number of 1 3/4. Add the whole number 1 of the mixed number to the 17 making a total sum of 18 3/4.

12 Multiplying and Dividing Fractions

Multiplication of fractions is quite simple. The numerators of the fractions are multiplied together, and the product becomes the numerator of the answer.

The same is done for the denominators. Example:

$$3/8 \times 3/4 = 9/32$$

Mixed numbers are first converted to improper fractions, and then multiplied as above. Example:

$$2\ 1/2 \times 3\ 7/8 = 5/2 \times 31/8 = 155/16 = 9\ 11/16$$

To divide one fraction into another, invert the *dividing* fraction. Then multiply the numerators and denominators as in multiplication. Example:

$$3/8 \div 1/4 = 3/8 \times 4/1 = 12/8 = 14/8 = 1\ 1/2$$

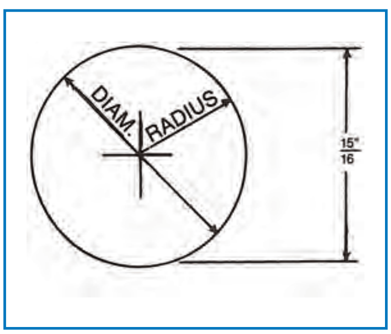

Fig. 10. Steel disc

13 Halving Fractions

Your work in the machine shop will often require you to divide fractional dimensions (single fractions or mixed numbers) in half.

Fig. 10 shows a steel disc whose *diameter* is 15/16″. To scribe this disc on a steel plate, you must first compute the *radius* of the disc. This will be one-half the diameter. A quick way to divide the 15/16″ diameter in half is to double the denominator (16) and put the existing numerator (15) over the new denominator. Thus: One-half of 15/16″ diameter = 15/32″ radius.

Mixed numbers may also be divided in half. If the mixed number contains an *even* whole number (2, 4, 6, 8, etc.), divide the even whole number in half. Then, divide the fraction as described previously.

Example: Compute one-half of 8 3/4.

One-half of 8 = 4

One-half of 3/4 = 3/8

Therefore, one-half of 8 3/4 = 4 3/8.

When the mixed number contains an *odd* whole number, (3, 5, 7, 9, etc.) the procedure is slightly more involved. In **Fig. 11** the centerline AB of the steel plate must be marked equidistant from each end. Compute dimension "X".

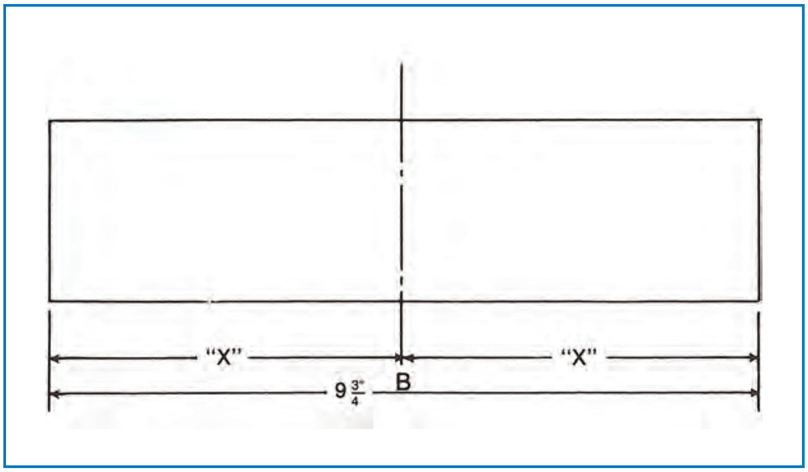

Fig. 11. Steel plate

Step 1 — Divide the odd whole number 9. 9 ÷ 2 = 4-1/2. Discard the 1/2″. Use only the 4.

Step 2 — Add the numerator and denominator of the fraction 3/4″ together. 3 + 4 = 7. This will be the numerator of a new fraction.

Step 3 — Multiply the original denominator (4) by 2 to get a new denominator of 8. This denominator used with the numerator 7 from Step 2 makes a new fraction of 7/8″. Combine this with the whole number 4 from Step 1 to get a dimension "X" of 4-7/8″ which is one-half of 9-3/4″.

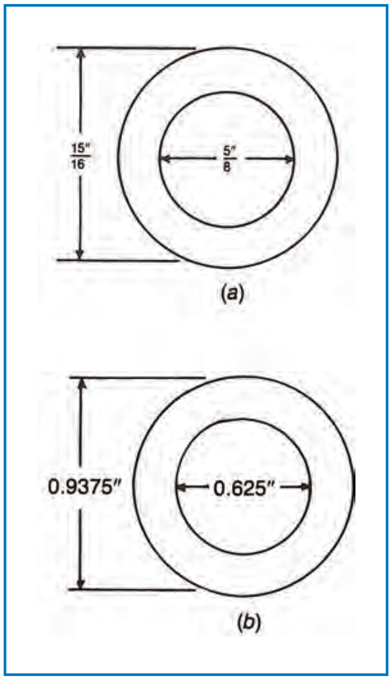

Fig. 12. Ring

14 Decimals

The outside diameter (OD) of the ring in **Fig. 12**(*a*) is 15/16″, and the inside diameter (ID) is 5/8″. The use of these fractional measurements tells an experienced machinist that the dimensions should be held as closely as possible, but there are no critical fits involved between the ring and other machine parts.

The line in a common fraction tells you that the numerator is divided by the denominator. If this division is carried out, a *decimal* will result. Thus, the dimension 15/16″ in Fig. 12(*a*) becomes 0.9375″, and the inside diameter of 5/8″ becomes 0.6250″. The ring would then be dimensioned as shown in Fig. 12(*b*).

If the two surfaces (OD and ID) of the ring had to be closely fitted with other machine parts, decimals would normally be used to designate the dimensions, because decimal dimensions can usually be held to closer tolerances than fractions on modern machine tools.

15 Reading Decimals

Decimal point numbers are based on powers often. Numbers on the left of the decimal point are multiplied by 1, 10, 100, 1000, – – –etc., numbers on the right of the decimal point are multiplied by .1, .01, .001, .0001, – – –etc. These parts are expressed as tenths, hundredths, thousandths, ten-thousandths, etc., of the whole, depending on how far to the right of the decimal point they appear.

For example:

0.1 = one-tenth
0.01 = one one-hundredth
0.001 = one one-thousandth
0.0001 = one ten-thousandth
0.00001 = one hundred-thousandth
0.000001 = one-millionth

As the value in the decimal proceeds to the right away from the decimal point, the value decreases in size.

1 place to the right = tenths
2 places to the right = hundredths
3 places to the right = thousandths
4 places to the right = ten-thousandths
5 places to the right = hundred-thousandths
6 places to the right = millionths

Values to the left of the decimal point are read as whole numbers.

Let's look at a few examples of converting fractions to decimals. When the common fraction 1/2 is divided out as a decimal (numerator ÷ denominator), it is written 0.5. Since the 5 is one place to the *right* of the decimal point, the value is expressed as 5 tenths.

A fraction of 3/4 is written decimally as 0.75 and is expressed as 75 hundredths because its last digit is two places to the right of the decimal point. The fraction 7/8 is expressed as 875 thousandths since its last digit is three places to the right of the point. It would be shown on a shop drawing as 0.875.

16 Adding Decimals

Addition, subtraction, multiplication, and division of decimals is relatively simple. The same basic methods are used as when working with whole numbers, except that there are certain additional rules to be followed for placing the decimal point in the answers.

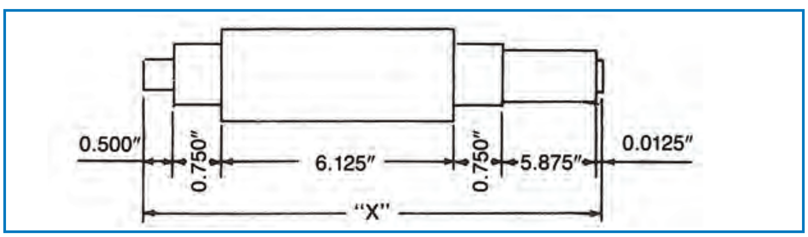

Fig. 13. Pump impeller shaft

The pump impeller shaft shown in **Fig. 13** has several journals of various lengths. The lengths are dimensioned in decimal form. To compute the overall length, "X", of the shaft, you must add all journal lengths together. Follow these steps for the addition:

Step 1 — Arrange the dimensions in a column so that the decimal points for each dimension are under one another.

$$\begin{array}{rl} & 0.500'' \\ & 0.750'' \\ & 6.125'' \\ & 0.750'' \\ & 5.875'' \\ + & \underline{\;0.0125''} \\ & 14.0125'' = \text{Total length of the shaft} \end{array}$$

Step 2 — Starting at the extreme right, add the column of decimal dimensions as you would a series of whole numbers.

Step 3 — The decimal point in the answer is placed under the points in the column.

The overall length of the shaft is 14.0125″.

17 Subtracting Decimals

Decimal values to be subtracted are arranged in the same manner used for addition, Suppose you know the overall length of the shaft in Fig. 13 to be 14.0125″. You also know the lengths of all the journals except the smallest one on the right-hand end. To compute this journal length:

1. Add the lengths of all the other journals.

$$\begin{array}{r} 0.050'' \\ 0.750'' \\ 6.125'' \\ 0.750'' \\ 5.875'' \\ \hline 14.000'' \end{array} = \text{Total length of the shaft}$$

2. Subtract this total from the overall length.

$$\begin{array}{r} 14.0125'' \\ -\ \underline{14.0000''} \\ 0.0125'' \end{array} = \text{length of the small journal.}$$

18 Multiplying Decimals

Decimals are multiplied in the same way that you would multiply whole numbers. After you obtain the product, count the total number of decimal places to the *right* of the points in both the multiplier and the multiplicand. Then, starting at the right-hand end of the answer, count off this total number of places and insert the decimal point in the answer. For example:

$$\begin{array}{r} 0.750 \\ \times\ \ 0.5 \\ \hline 0.3750 \end{array}$$

Since there is a total of four decimal places in the multiplier (0.5) and the multiplicand (0.75), you must count off four places from right to left in the answer.

If necessary, zeros may be added in the answer to provide the correct number of places. The zeros are placed *to the right of the decimal point.* For example: If you multiply 0.101 by 0.04, the answer is 404, but there are not enough decimal places to position the point five places left of the last digit 4. Therefore, add two zeros so that the answer reads 0.00404.

A machine shop project requires you to compute the area (length × width) of the plate shown in **Fig. 14.** The multiplication should be set up like this:

$$\begin{array}{r} 4.875'' \\ \times \quad 8.13'' \\ \hline 14625 \\ 4875 \\ 39000 \\ \hline 39.63375 \text{ square in.} \end{array}$$

There are five decimal places in the multiplied terms, so there must be five places in the answer.

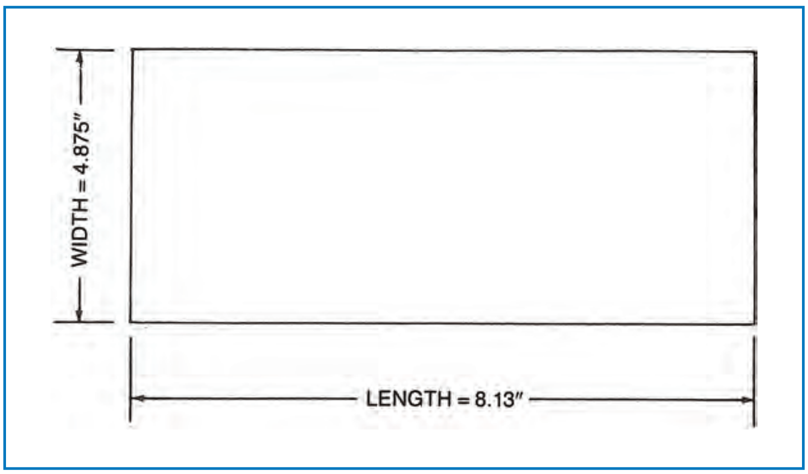

Fig. 14. Plate

19 Dividing Decimals by Whole Numbers

The ease by which decimals are divided depends largely upon what form the divisor takes. The simplest operations are those in which the divisor is a whole number. Then, you may use either short or long division, and position the decimal point of the quotient either below or above that of the dividend.

The bar in **Fig. 15** has a total length of 18.36″, and must be marked off in six equal divisions. Calculating the length of each division may be done by short division, as when dividing whole numbers.

$$6\overline{)18.39}$$
$$3.06 \ = \text{length of each division}$$

Dividing larger decimals by larger whole numbers will require long division. However, the decimal point in the quotient will be placed *above* that in the divided number.

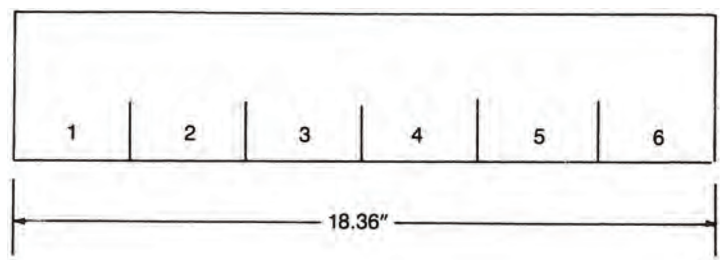

Fig. 15. Bar

20 Dividing Decimals by Decimals

When you must divide a decimal value by a decimal value, reposition the decimal place of the *divisor* to the right until the divisor is changed to a whole number. Next, move the decimal point in the dividend to the right the same number of places. Then, proceed as outlined for dividing by a whole number. Example: divide 0.375 by 0.25.

1. $0.25\overline{)0.375}$ = original expression
2. $025.\overline{)037.5}$ = divisor decimal point moved two places to the right causing same move in the dividend
3. $25\overline{)37.5}$ = final division
   $\quad 1.5$

At times it may be necessary to add zeros in the dividend in order to move the decimal point there for the proper number of places. Divide 0.3 by 0.125.

1. $0.125\overline{)0.3}$ = original expression

2. $125\overline{)\,300.0}$ = two zeros added in dividend to permit moving decimal point three places to right

3. $125\overline{)\,300.0}$ = answer
   2.4

21 Rounding Off Decimals

Many machine parts drawings have dimensions with three or four decimal places. These dimensions provide the accuracy necessary for most commercial fits of mating parts in a machine. Certain assemblies may require you to machine a part to more than four places, but these are rare applications.

Decimal dimensions are frequently "rounded off" to the desired number of places by using the number 5 as a controlling digit. If you are computing a decimal dimension to three places, carry out your division to four places. If the fourth place is less than 5, drop the last digit completely. On the other hand, if the fourth digit is 5 or more, add 1 to the third decimal digit.

Use the same process to round off a decimal dimension to four places. Carry out the division to five decimal places, then if it is less than 5, drop the fifth digit entirely. If the fifth-digit is 5 or more, add 1 to the fourth digit.

For example, to divide 451 by 7 and express the quotient accurately to three decimal places:

1. $7\overline{)\,451.0000}$
   64.4285

2. Rounding off to four places = 64.429

Divide 363 by 7 and round off to four decimal places.

1. $7\overline{)\,363.00000}$
   51.85714

2. Rounding off to four places = 51.8571

22 Converting Fractions and Decimals

You have learned that there is a certain relationship between fractions and decimals. For example, a decimal results when the numerator of a proper fraction is divided by the denominator. Also, a decimal may be expressed as a fraction by stating the value of the decimal as the numerator of a fraction, and the number of places to the right of the decimal point as the denominator. Thus, 0.5 may be expressed fractionally as 5/10ths or 1/2.

Fig. 16 is called a *Decimal Equivalent Chart.* This handy reference lists those fractions most commonly used in machine shop work *and* the decimal equivalents of those fractions. Decimal equivalent charts are used frequently by machinists. They save the time and trouble of physically computing and converting fractions to decimals and vice versa. The chart in Fig. 16 also provides the metric equivalent, in millimeters, of the most commonly used fractions.

Decimal equivalent charts are available from suppliers of machine tools, cutters, and machine shop accessories. They are usually free for the asking. The charts are also published in many machinists handbooks and manuals. You should obtain a copy of an equivalent chart for your personal use and keep it handy at your machine shop work station.

FRACTION	DECIMAL	MILLIMETERS	FRACTION	DECIMAL	MILLIMETERS
1/64	.01563	.3969		.51181	13.0000
1/32	.03125	.7938	33/64	.51563	13.0969
	.03937	1.0000	17/32	.53125	13.4938
3/64	.04688	1.1906	35/64	.54688	13.8906
1/16	**.06250**	**1.5875**		.55118	14.0000
5/64	.07813	1.9844	**9/16**	**.56250**	**14.2875**
	.07874	2.0000	37/64	.57813	14.6844
3/32	.09375	2.3813		.59055	15.0000
7/64	.10938	2.7781	19/32	.59375	15.0813
	.11811	3.0000	39/64	.60938	15.4781
1/8	**.12500**	**3.1750**	**5/8**	**.62500**	**15.8750**
9/64	.14063	3.5719		.62992	16.0000
5/32	.15625	3.9688	41/64	.64063	16.2719
	.15748	4.0000	21/32	.65625	16.6688
11/64	.17188	4.3656		.66929	17.0000
3/16	**.18750**	**4.7625**	43/64	.67188	17.0656
	.19685	5.0000	**11/16**	**.68750**	**17.4625**
13/64	.20313	5.1594	45/64	.70313	17.8594
7/32	.21875	5.5563		.70866	18.0000
15/64	.23438	5.9531	23/32	.71875	18.2563
	.23622	6.0000	47/64	.73438	18.6531
1/4	**.25000**	**6.3500**		.74803	19.0000
17/64	.26563	6.7469	**3/4**	**.75000**	**19.0500**
	.27559	7.0000	49/64	.76563	19.4469
9/32	.28125	7.1438	25/32	.78125	19.8438
19/64	.29688	7.5406		.78740	20.0000
5/16	**.31250**	**7.9375**	51/64	.79688	20.2406
	.31496	8.0000	**13/16**	**.81250**	**20.6375**
21/64	.32813	8.3344		.82677	21.0000
11/32	.34375	8.7313	53/64	.82813	21.0344
	.35433	9.0000	27/32	.84375	21.4313
23/64	.35938	9.1281	55/64	.85938	21.8281
3/8	**.37500**	**9.5250**		.86614	22.0000
25/64	.39063	9.9219	**7/8**	**.87500**	**22.2250**
	.39370	10.0000	57/64	.89063	22.6219
13/32	.40625	10.3188		.90551	23.0000
27/64	.42188	10.7156	29/32	.90625	23.0188
	.43307	11.0000	59/64	.92188	23.4156
7/16	**.43750**	**11.1125**	**15/16**	**.93750**	**23.8125**
29/64	.45313	11.5094		.94488	24.0000
15/32	.46875	11.9063	61/64	.95313	24.2094
	.47244	12.0000	31/32	.96875	24.6063
31/64	.48438	12.3031		.98425	25.0000
1/2	**.50000**	**12.7000**	63/64	.98438	25.0031
			1	**1.00000**	**25.4000**

Fig. 16. Measurement equivalents

Check Your Learning 1

Answer the following questions to check your understanding of the material you've read in the preceding articles. If you want to review the answer to a question, use the article reference included with the answer.

1. The number above the line in a fraction is called the __________.

2. The sum of 5/8″; 3/4″; 11/16″; and 1/2″ is __________.

3. Write the numerical value of one ten-thousandth.

4. What is the sum of 2.625; 4.125; 0.9375; and 0.025?

5. Find the product of 31.5783 × 2.671. Round off to three decimal places.

Your Answers

1. ____________________
2. ____________________
3. ____________________
4. ____________________
5. ____________________

Answers to Check Your Learning 1

1. numerator	Art. 2	4. 7.7125	Art. 16
2. 2 9/16″	Art. 9	5. 84.346	Art. 18
3. 0.0001	Art. 15		

23 Powers of Ten

You can use decimal points to conveniently multiply and divide numbers by 10, 100, 1000, 10000, or any power of 10. This is done by simply relocating the decimal point to the right or left depending on the operation involved. This saves you time required to physically multiply or divide the number.

The shaft shown in **Fig. 17** is used in a model of a machine which will be ten times the size of the model. You are required to specify the length of the real shaft and the maximum diameter of the stock from which it will be made. The requirements are to be made to three decimal places.

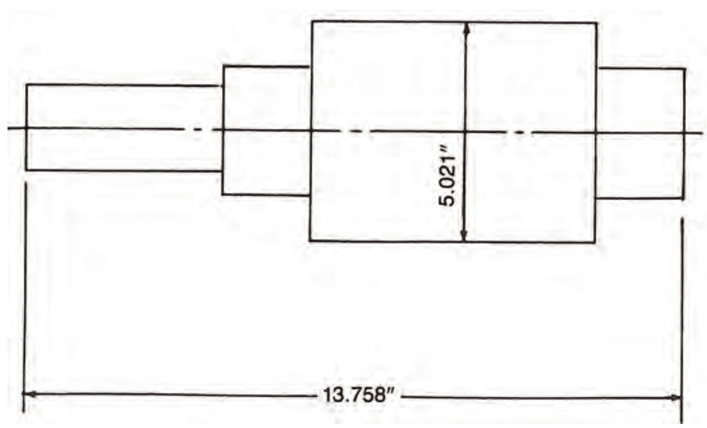

Fig. 17. Shaft

Since the dimensions are to be *increased* ten times, the decimal point will be moved to the *right* one place. Remember, one space to the right = tens, two spaces = one-hundredths, etc. Moving the decimal point one place to the right gives a real length for the shaft of 137.580″. A zero was added to provide three places as required. The maximum diameter of the stock will be 50.210″.

Dividing by numbers which are powers of ten (10, 100, 1000, etc.) is done in the reverse manner. The decimal point is then moved to the *left* for a number of places equal to the number of zeros in the divisor. To divide 72.40 by 100, move the decimal point two places to the *left* to produce the answer of 0.7240. If the 72.40 was divided by 1000, the answer would be 0.07240. You would have to add one zero to provide space for moving the decimal point.

24 Using Exponents for Powers of Ten

Powers often are most often designated by exponents. In the expression 10^2, the small 2 to the upper right of the 10 tells you that this value is 10 squared. Likewise, a designation written 10^3 tells you that the 10 is cubed, or $10 \times 10 \times 10 = 1000$. Using exponents to designate a power of 10, eliminates writing all the zeros, and makes the expression of large and clumsy numbers easier.

Powers of ten are rarely, if ever, used for dimensioning machine parts drawings. You are more likely to encounter them in written specifications, formulae, and in certain instructions for setting up computer controlled machines.

The value five million must be written numerically as 5,000,000 unless it is specified using a power of ten. Using an exponent allows this value to be written as 5×10^6. In effect, the exponent tells you how many zeros to add to the number multiplied by a power of ten.

25 Raising Numbers by Powers of Ten

Two factors are involved when using the power of ten method, or scientific notation, of raising the value of a number. One is a whole number from 1 to 9 with a decimal point to its right. The other factors is some power of 10. The whole number represents the multiplied value, or the multiplicand, in the operation. The other factor is the 10 with its exponent, or the multiplier.

The chart in Fig. 18 shows powers of 10 up to 10^9. You can also see the number of zeros to be added when a whole

$10 = 10 \times 1 = 10$

$10^2 = 10 \times 10 = 100$

$10^3 = 10 \times 10 \times 10 = 1000$

$10^4 = 10 \times 10 \times 10 \times 10 = 10000$

$10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100000$

$10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1000000$

$10^7 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10000000$

$10^8 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 100000000$

$10^9 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1000000000$

Fig. 18. Powers of ten

number is raised by a certain power. In the case of a decimal, the number of zeros tells you how many places to the right to move the decimal point.

Assume that you are instructed to set a numerically controlled lathe to a mode specified as "dimension A $\times$ 10^3." On your project at hand, dimension A = 0.0260. Referring to Fig. 18, you see that multiplying a whole number by 10^3 requires adding three zeros. In the example involving a decimal, the decimal point is moved three places to the right. Therefore, the mode of the lathe will be $0.0260 \times 10^3 = 26$.

26 Decreasing Numbers by Powers of Ten

A power of ten may also be used to denote a *decrease* in a number. Negative exponents are written as 10^{-2}, 10^{-3}, 10^{-4}, etc. A negative exponent tells you that the decimal point must be moved to the *left* for a specific number of places.

For example, in the equation $A = 24.31 \times 10^{-4} = 0.002431$, the decimal point was moved four places to the left. Two zeros were added to make room for the decimal point. The use of a negative exponent eliminated the manual division of 24.31 by 10,000.

27 Squares and Square Roots

A number produced by multiplying two identical numbers together is called the *square* of those numbers. Multiplying 5×5 produces 25, which is said to be the square of 5. Another way of stating this is that any number multiplied once by itself produces the square of that number. The square of 6 is 36, since $6 \times 6 = 36$.

The number multiplied by itself one time is said to be the *square root* of the product. Thus, since $12 \times 12 = 144$, the square root of 144 is 12. Likewise, the square root of 9 is 3. The symbol $\sqrt{}$ is used to indicate a *root* of a number. This symbol is called a *radical sign*. Any number appearing under the radical sign shown above tells you that the square root of that number is to be used in your calculations. For example, if a shop formula appears as $3 \times \sqrt{64}$, it means that you are to multiply 3×8 (since $8 \times 8 = 64$) to get a product of 24.

If the radical sign is used without a small number inserted in its "V" portion, it is assumed that the *square* root is to be taken of the number under the sign. Square root could also be indicated by $\sqrt[2]{}$. The small number in a radical sign is called the *root index*. It is used to designate which root is to be taken of the number under the sign.

For example, a designation of $\sqrt[3]{27}$ asks for the *cube* root of 27, or a number which, when multiplied three times, will produce 27. In this case, the $\sqrt[3]{27}$ is 3, since $3 \times 3 \times 3 = 27$. Root indexes higher than 3 are rarely used in machine shop work or calculations. Higher roots are found mostly in more scientific formulae and computations. Of the two, square root and cube root, you will encounter square root work more frequently.

The next module of this course will teach you how to calculate area, volume, etc., of various geometric shapes. However, to show you how squares and square root calculations may be encountered in machine shop work, we will extract a very small portion of that module for illustration purposes.

Fig. 19 shows a circle whose area (A) is 50.24 square inches. The formula for computing the area of a circle is $A = \pi \times r^2$, wherein, $\pi = 3.14$ and r = radius of the circle.

A shop project requires you to scribe this circle on a sheet of steel. To do this, you must first determine the radius of the circle. If $A = \pi \times r^2$, then

$$\frac{A}{\pi} = r^2. \text{ Therefore, } \frac{50.24}{3.14} = r^2 = 16.$$

What number, when multiplied by itself once, equals 16? You can see by quick inspection that $4 \times 4 = 16$. Thus, the square root of 16 = 4, and this is the radius of the circle in Fig. 19.

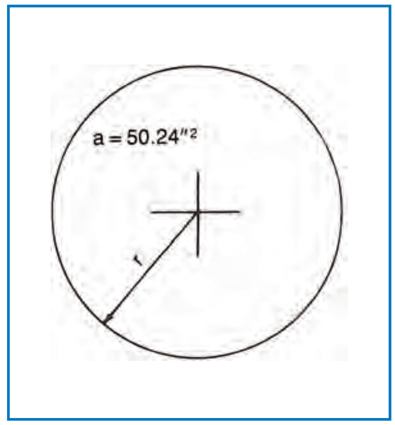

Fig. 19. Circle

28 Computing Square Root

Various handbooks and machine shop manuals contain precalculated square and cube roots of a wide range of numbers. These are in table form and are very valuable as a quick reference. However, if a table is not available, it is important to know how the square root of a number is manually extracted.

A notation on a shop drawing may specify that a certain dimension is equal to the square root of another variable dimension on the drawing. **Fig. 20** illustrates such a case in which dimension A, as a variable dimension, controls the location of the hole in the workpiece. Assume that, in Fig. 20, dimension A = 26.9102 inches. In order to find the vertical location of the hole, you must compute the square root of

26.9102″. The square root is to be expressed to only two decimal places.

Follow these steps:

Step 1 — Mark off the number 26.9102 into blocks of two digits each. Begin with the whole number to the left of the decimal point and work to the right. Thus, $26'.91'02$

Step 2 — Arrange the problem like this, $26'.91'02\sqrt{\quad}$

Step 3 — **Fig. 21** shows the problem as solved.

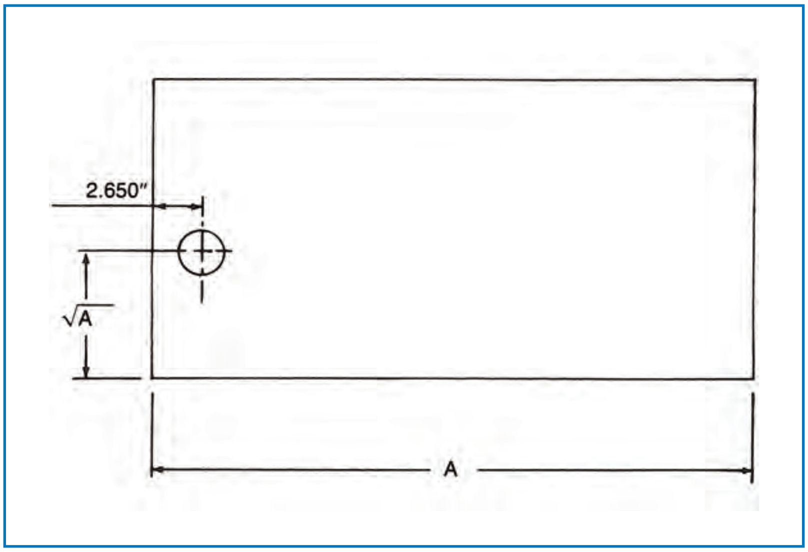

Fig. 20. Workpiece

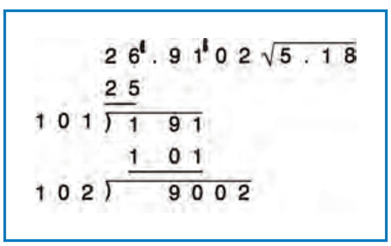

Fig. 21. Problem solved

Follow these remaining steps for instructions on how the problem was done.

Step 1 — Determine the number whose square is *less* than the first left-hand block of two digits. This is 5 in our example. Place the 5 to the right of the given number: $26'.91'02\sqrt{5}$. This is the first digit of the square root.

Step 2 — Subtract the square of this number from the first left-hand digit block:

$$\begin{array}{l} 26'.91'02\sqrt{5.} \\ \underline{25} \\ \ \ 1 \end{array}$$

Step 3 — Bring down the next block of two digits:

$$\begin{array}{l} 26'.91'02\sqrt{5.} \\ \underline{25} \\ 191 \end{array}$$

Step 4 — Double the root figure just found (5) and place it as a *partial* divisor to the dividend 191:

$$\begin{array}{rl} & 26'.91'02\ \sqrt{5.}\\ & \underline{25}\\ 10 & \overline{)\,191} \end{array}$$

Step 5 — Disregard the last right-hand digit of the dividend (1). Find how many times the partial divisor will go into 19. Place this number as the second root number, and also as the third digit in the divisor:

$$\begin{array}{rl} & 26'.91'02\ \sqrt{5.1}\\ & \underline{25}\\ 10 & \overline{)\,191} \end{array}$$

Step 6 — Multiply the divisor 101 by the second root figure (1) and subtract it from the dividend 191. Bring down the next block of two digits (02):

$$\begin{array}{rl} & 26'.91'02\ \sqrt{5.1}\\ & \underline{25}\\ 101 & \overline{)\,191}\\ & \ \ \underline{101}\\ & \ \ 9002 \end{array}$$

Step 7 — Double the root numbers found so far (51; disregard the decimal point for this operation). Place this figure as a partial divisor to the dividend 9002:

$$\begin{array}{rl} & 26'.91'02\ \sqrt{5.1}\\ & \underline{25}\\ 101 & \overline{)\,191}\\ & \ \ \underline{101}\\ 102 & \overline{)\,9002} \end{array}$$

Step 8 — Disregard the last digit (2) in the dividend. Divide 102 into 900 to get a quotient of 8. Place this as the next digit in the root:

$$\begin{array}{rl} & 26'.91'02\ \sqrt{5.18}\\ & \underline{25}\\ 101 & \overline{)\,191}\\ & \ \ \underline{101}\\ 102 & \overline{)\,9002} \end{array}$$

If dimension A in Fig. 20 is 26.9102″, then the height of the hole centerline is 5.18″.

29 Finding the Square Root of Fractions

The square root of a fraction is equal to the square root of its numerator over the square root of its denominator. This is expressed graphically as:

$$\sqrt{\frac{9}{64}} = \frac{\sqrt{9}}{\sqrt{64}} = \frac{3}{8}$$

If neither the numerator nor the denominator are perfect squares of some number, you can use a convenient method to convert one or the other to a square. Multiply one or the other by itself, thereby making it a perfect square of some number. *Both terms must be multiplied by that number.*

Example: Find the square root of the fraction 5/8.

$$\sqrt{\frac{5}{8}} = \sqrt{\frac{5 \times 5}{8 \times 5}} = \sqrt{\frac{25}{40}} = \frac{\sqrt{25}}{\sqrt{40}} = \frac{5}{6.325}$$

30 Machine Tool Drive Ratios

Most machine tools, such as lathes, milling machines, drill presses, etc., are driven by an electric motor. Some use a direct current (DC) motor whose speed is adjustable over a certain range. However, many machines have alternating current (AC) motors which operate only at a fixed speed.

The speeds at which machine tools cut metal must be adjustable, because some metals are machined at a slower rate than others due to their hardness and other factors. Likewise, the rate at which a cutter may be fed into a workpiece will vary with the workpiece materials, cutter hardness, and other elements. Therefore, the normal running speed of an AC motor must be adjusted to the speed required to properly machine the workpiece. This is usually accomplished by a belt drive system, a gear drive system, or a combination of both.

Fig. 22 shows one type of quick change gearing arrangement for a modern precision turning machine. The self-contained gear housing is mounted on the main frame of the machine. Speeds are changed by shifting a lever actuating the yoke. This system allows the operator to select and shift gear combinations to adjust the speed of the spindle to suit the requirements of the machining operation. The gear transmission in Fig. 22 is used in conjunction with a motor and silicon rectifier. Many older machine tools still require the operator to calculate the gear ratios needed for a job, and to then select the proper gear combinations. You should be able to calculate the speeds and feeds for a given project.

A ratio is best defined as a proportion of one thing to another. The ratio of 12 to 6 is 2 to 1. The ratio of 100 rpm (revolutions per minute) to 10 rpm is 10 to 1.

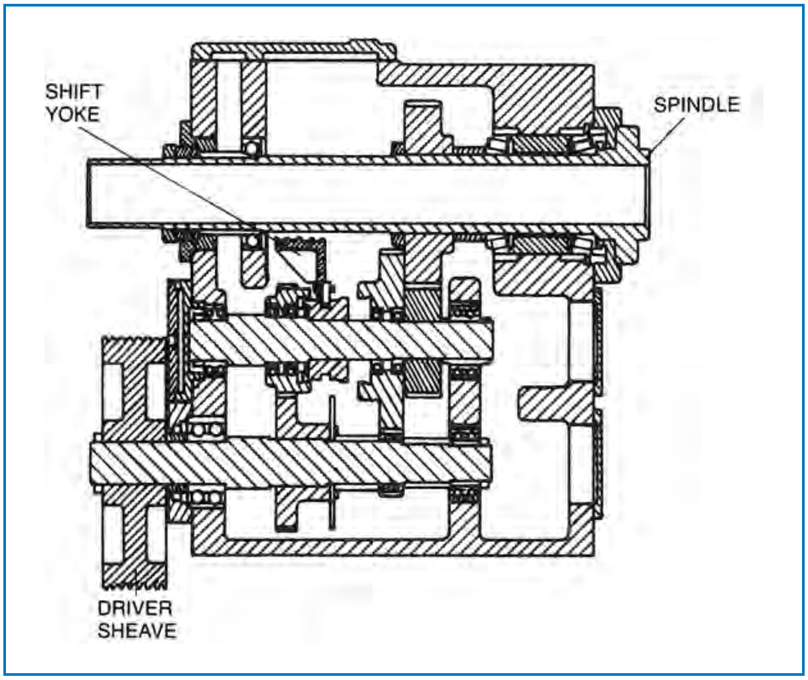

Fig. 22. Quick change gearing arrangement

31 Reduction Ratios

The belt drive in **Fig. 23** has a pulley mounted on the driving motor shaft which turns at a speed of 1000 rpm. The belt connects this pulley to another on the machine's driven shaft. The *driven* pulley is twice as large as the *driver* pulley. This means that the driven pulley will make only one complete revolution while the driver pulley makes two turns. The ratio of the belt drive in Fig. 23 is 2 to 1, sometimes written as 2:1.

If the motor pulley in Fig. 23 turns at 1000 rpm, and the belt drive ratio is 2:1, then the speed of the driven pulley can be computed:

$$\text{Speed Dn} = \frac{\text{Speed Dr}}{\text{Ratio}} = \frac{1000\,\text{rpm}}{2} = 500\ \text{rpm}$$

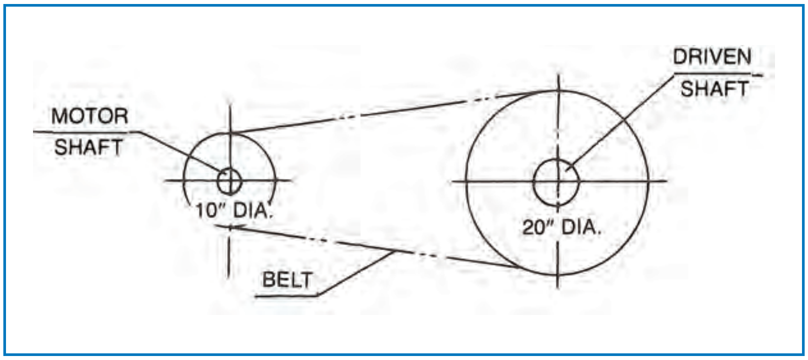

Fig. 23. Belt drive

This drive is said to have a *reduction ratio* because the speed of the driver pulley has been reduced from 1000 rpm to 500 rpm by means of the belt drive. To find the ratio of reduction of a single stage belt or gear drive, use the following formulae:

$$\text{Ratio} = \frac{\text{Diameter of Driven Pulley}}{\text{Diameter of Driver Pulley}}$$

$$\text{Ratio} = \frac{\text{No. of Teeth in Driven Gear}}{\text{No. of Teeth in Driver Gear}}$$

$$\text{Ratio} = \frac{\text{Driver rpm}}{\text{Driven rpm}}$$

32 Increaser Ratios

To *increase* the speed of a driving motor or shaft to obtain the required speed for a cutter or machine spindle, you can install the larger pulley or gear on the driver shaft as shown in **Fig. 24**. The ratio of a speed increasing drive system is written as 1:2, 1:3, 1:4, etc. This means that the driving shaft is turning once for every 2, 3, or 4 times that the driven shaft turns.

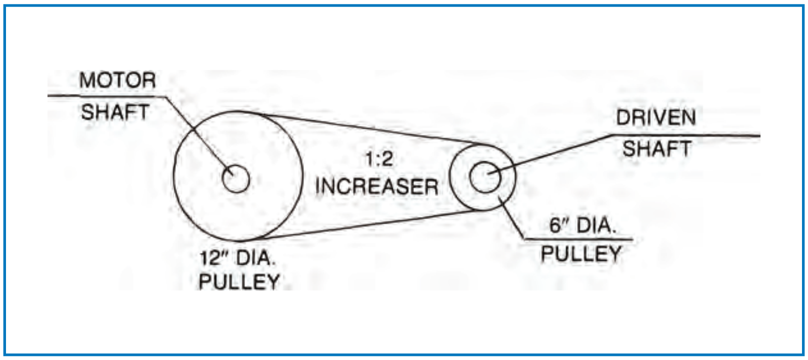

Fig. 24. Driven shaft

33 Compound Ratios

Machine tool gear reductions from the motor to the driven shafts are seldom done in a single stage or set of gears. They are most often designed in two or more stages, each with its own driver and driven gear. **Fig. 25** shows a three-stage gear drive system.

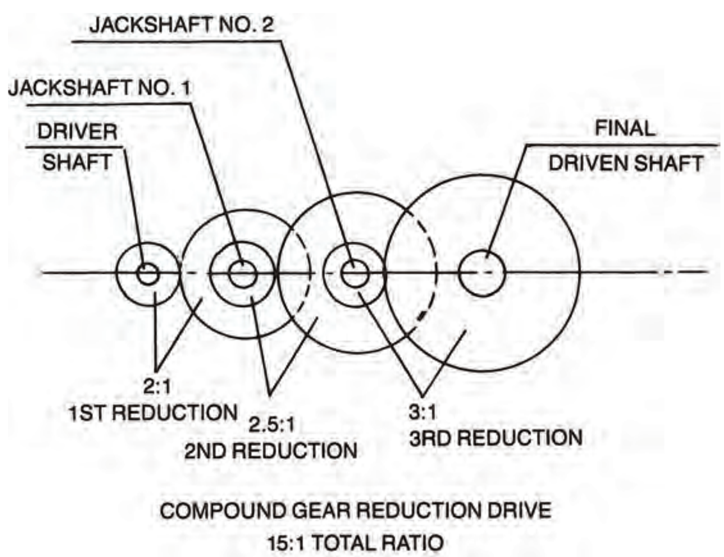

Fig. 25. Three-stage gear drive system

The *total* reduction ratio of the system in Fig. 25 is 15:1. The *total* ratio is found by multiplying all individual ratios in the system together. Thus, $2 \times 2.5 \times 3 = 15$. Compound ratios are necessary since it is not possible to absorb high ratios in a single stage at higher motor speeds.

In addition, compound ratios permit changing of gear ratios within the system, thereby adjusting the cutter and spindle speeds of the machine to suit various operations. This is done by interchanging gears with various combinations of tooth numbers. For example, the first stage reduction in Fig. 25 (2:1) could be obtained by using a 21-tooth driving gear and a 42-tooth driven gear. If a combination of a 20-tooth driver and a 43-tooth driven gear is substituted, the new first stage ratio would be 2.15:1, and the final output speed of the system would be changed to $2.15 \times 2.5 \times 3 = 16.13:1$.

The gear systems in manually adjustable machine tools are often called *change gears* because they can be interchanged to provide different operating speeds. Most machines have a metal plate attached detailing the necessary change gear ratios to be used to obtain various spindle or cutter speeds. The operating manuals accompanying the machines usually indicate the specific locations for the various sizes of gears to produce a desired ratio.

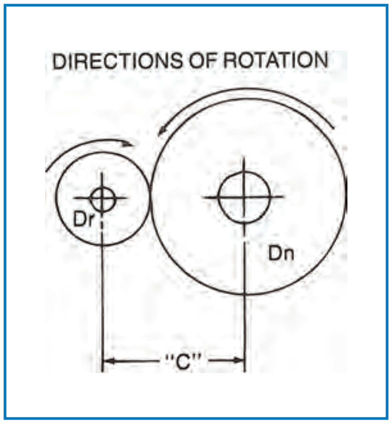

Fig. 26. Center distance

34 Use and Effects of Idler Gears

The distance between the axes of the driver (Dr) and driven (Dn) shafts is called the *center distance,* shown as dimension *C* in **Fig. 26**. On some machine tools the center distance becomes too great to permit using a single gear system like the one in Fig. 26. The gears would have to be too large in diameter, requiring a bigger housing and shaft sizes.

Machine tool manufacturers usually use idler gears to span the larger center distances as shown in **Fig. 27**. Using an idler gear *does not* affect the total ratio of the system. Suppose the driver in Fig. 27 is turning at 200 rpm. The gear drive ratio is 2:1, since

$$R = \frac{\text{Teeth in Dn}}{\text{Teeth in Dr}} = \frac{30}{15} = 2$$

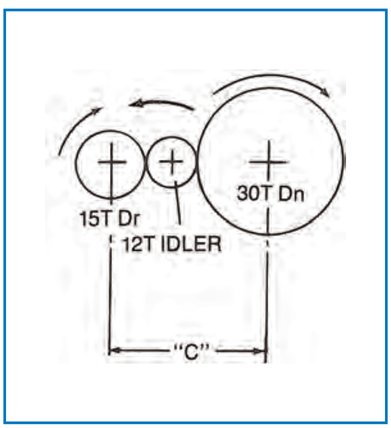

Fig. 27. Idler gears

The final speed of the driven gear is 100 rpm. Tracing the speed through the system in Fig. 27, it is increased by a ratio of 1:1.25 from the Dr shaft to the idler shaft

$$(\frac{15T}{12T} = 1.25 \text{ increase ratio}),$$

causing the idler shaft to turn at 250 rpm. The *reduction* ratio from the idler shaft to the final driven shaft is 2.5:1

$$(\frac{30T}{12T} = 2.5:1),$$

reducing the final shaft's speed to 100 rpm. This is the same speed that would be produced using a 15T Dr and a 30T Dn without the idler gear in the system.

Using an idler gear *does* have an effect on the relative direction of rotation of the driver and driven shafts, however. The two gear arrangement in Fig. 26 causes the driver and driven shafts to turn in opposite directions. Installing an idler gear in the system, as in Fig. 27, causes the driver and driven shafts to turn in the same direction. A general rule is that *when a gear system has an even number of gears*, including any idlers, the first and last shafts will rotate in *opposite* directions. In systems having an *odd* number of gears, the first and last shafts will turn in the *same* direction.

You may find that on some machine tools, idler gears are used to adjust the relative directions of rotation between the driving motor's shaft and that of the machine spindle. Remember that the use of an idler gear does not affect the gear drive ratio.

35 Computing Machine Tool Speeds and Feeds

The cutting speeds of machine tools are usually expressed in surface feet per minute (sfpm). While there is a certain relationship between rpm and sfpm, in machine shop technology the term rpm is not used to define cutting speed. The recommended cutting speed for a given machining operation will vary with the metallurgy of the workpiece, the type of cutter to be used, the rate at which the tool is fed into the work or vice versa, the desired finish of the workpiece surface, and the general condition of the machine itself.

Machine tool manufacturers and publishers of machinists' handbooks provide tables of recommended cutting speeds for a wide range of materials and machine types. You should adhere to these recommendations to avoid ruining the cutter, spoiling the workpiece, or damaging the machine itself.

Cutting speed in sfpm refers to a point on the circumference of a cutter or the workpiece, depending on the machine tool to be used. For example, the cutting speed of a milling machine refers to a point on the periphery of the cutter, while the cutting speed of a lathe describes the sfpm of a point on the circumference of the workpiece. If a milling cutter speed is expressed as 50 sfpm, it means that if the cutter was placed on the floor and allowed to roll, it would cover a distance of 50 feet in 1 minute.

If a lathe tool is to run at 200 sfpm, that means that at any given diameter of workpiece, 200 feet of material will pass the edge of the cutting tool in one minute. As the diameter of the workpiece gets smaller, the lathe spindle must rotate faster to maintain the 200 sfpm and vice versa.

When you know the diameter in inches of a milling cutter, a drill, or the workpiece diameter exposed to a lathe tool, you can compute cutting speed in sfpm as follows:

$$\text{SFPM} = 0.262 \times \text{rpm} \times \text{d}$$

wherein,

- 0.262 = a constant derived by dividing 3.14 by 12 to express speed in ft./min.
- rpm = rotational speed of the cutter or work
- d = diameter of cutter or workpiece *in inches*

If you know the recommended cutting speed for a project, you can determine the rpm setting for the machine by transposing the above formula:

$$\text{rpm} = \frac{\text{sfpm}}{0.262 \times \text{d}}$$

Suppose you want to drill a 5/8″ diameter hole in a workpiece made of machine steel. The size of the hole dictates the size of the drill to be used. The recommended cutting speed for a drill made of high speed steel cutting through machine steel is 80 to 110 sfpm. The rpm of the drill press spindle will be set to produce a cutting speed of 95 rpm, the middle of the range.

$$\text{rpm} = \frac{\text{sfpm}}{0.262 \times \text{d}} = \frac{95}{0.262 \times 0.625} = 579.3. \text{ Use } 580 \text{ rpm}$$

36 Machine Tool Feeds

The definition of feed of a machine tool will vary with the type of machine involved. However, feed is always expressed in fractions or decimals of an inch. When turning work in a lathe, a non-rotating cutting tool contacts a rotating workpiece. The cutting *feed* of the machine refers to the distance that the cutter is moved parallel to the workpiece axis for every one revolution of the work. If the cutting tool is moved 1/32″ as the work makes one revolution, the feed will be stated as 1/32″, or 0.03125″. Sometimes whole numbers alone are used to describe lathe feeds using the denominator of the fraction such as 32; 64; 16; etc.

Drill press feed is the distance that the drill is advanced into the workpiece for one revolution of the tool. The formula is: feed = distance ÷ rpm. If a drill advances a distance of 2″ into a workpiece, and makes 200 rpm as it advances, the feed is 2 ÷ 200 = 1/100th″, or 0.010″.

Milling machine feeds are usually represented in inches per minute (IPM) as opposed to IPR as used for the lathe work. Inches per minute describes the rate of table feed. To convert IPR to IPM, multiply IPR with RPM. For example:

RPM = 2000
.005 IPR = ? IPM
.005 × 2000 = 10 IPM

37 Computing Tapers

Many machine shop projects required tapered profiles to be turned on the outside diameters or in the bores of workpieces. Simply stated, a *taper* is the rate of change from one diameter to another over a specific length. A taper is usually specified as a change in diameter per inch or per foot of length. The piece in **Fig. 28** is reduced in diameter from 3″ at its larger end to 1″ at the smaller end. This reduction takes place over a length of 12″. The amount of taper is computed:

$$\text{Taper Per Inch (TPI)} = \frac{D - d}{L^T}$$

where:

- D = larger diameter
- d = smaller diameter
- L^T = length of taper *in inches*

In the example of Fig. 28:

$$\text{TPI} = \frac{3'' - 1''}{12''} = \frac{2''}{12} = \frac{1''}{6}$$

The taper in Fig. 28 could be expressed 2″ per 12″; 2″ per foot or 1/6″ per inch.

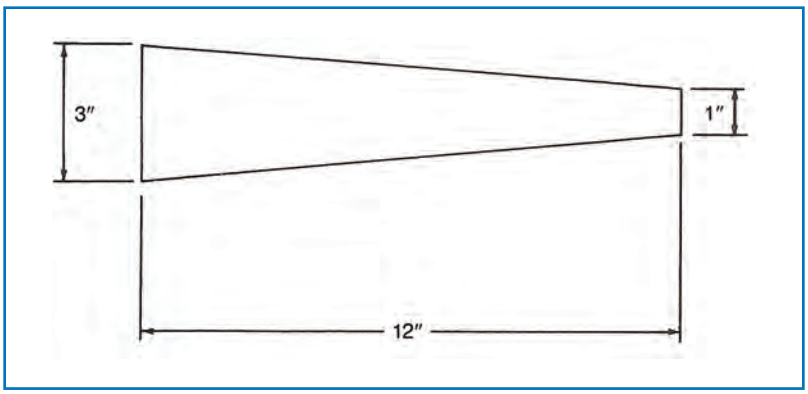

Fig. 28. Taper

The spindle sleeve shown in **Fig. 29** has a tapered inside diameter designed to accept the tapered shank of another part. The taper is computed and expressed as:

$$\text{TPI} = \frac{D - d}{L^T} = \frac{1.00'' - 0.75''}{8.5''} = \frac{0.25''}{8.5} = 0.029''\ \text{TPI}$$

Fig. 30 is a drawing of a lathe center. This part has a double taper. To determine the TPI of each, treat each tapered portion as a separate workpiece. Tapered section A in Fig. 30 reduces a diameter of 1″ to 0″ at the point. Therefore its TPI is:

$$\text{TPI} = \frac{1.000" - 0"}{1.500"} = \frac{1.000"}{1.500} = 0.667" \text{ TPI}$$

The TPI for section B of the lathe center is:

$$\text{TPI} = \frac{1.000" - 0.625"}{4.750"} = \frac{0.375"}{4.750} = 0.079" \text{ TPI}$$

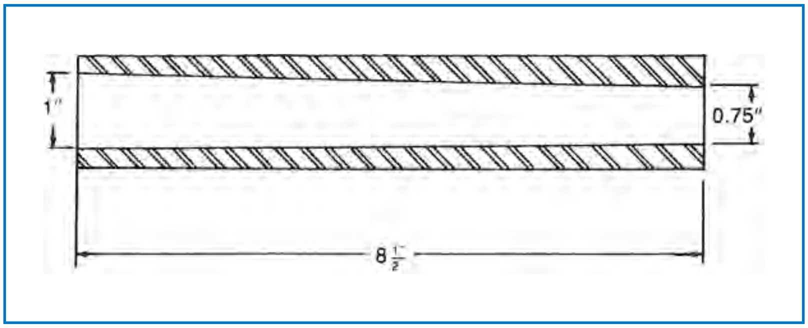

Fig. 29. Spindle sleeve

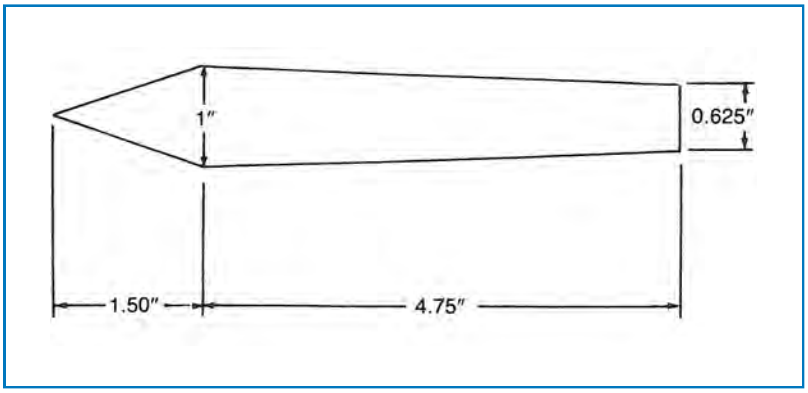

Fig. 30. Lathe center

Longer parts often have their tapered surfaces described in inches per foot. Examples of these are the outside diameters of some naval gun barrels and the diameters of large conveyor pulleys. The pulley in **Fig. 31** is *crowned* at its center to provide better tracking of the conveyor belt. The crowning creates a tapered surface on each side of the pulley centerline. To express and measure this large an area would require using very small values of an inch. The accuracy required for this type of application permits using a taper expression in inches per foot. Compute the TPI and multiply by 12 to get the taper in TPF. The taper of each side of the pulley centerline in Fig. 31 is 0.01″ TPI. Expressing this as 0.01″ × 12 produces 0.12″/ft., or approximately 1/8″ per foot.

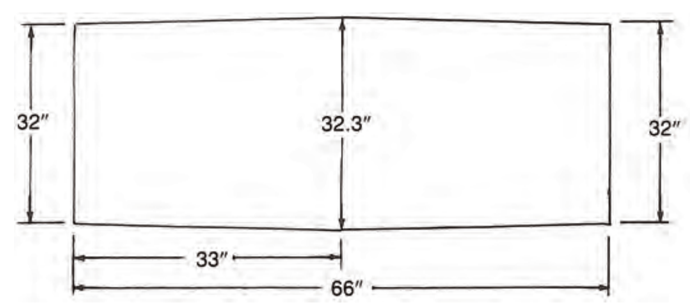

Fig. 31. Pulley

38 Using Metric Values

Metrics are used as a measuring system on almost a worldwide basis. The system is widely used in machine shop work, even in the United States where it has not yet been adopted as a national standard. Our present method of measurement (inches, feet, ounces, pounds, etc.) is commonly called the English, or Customary System, of measurement.

You will work with shop drawings and specifications using our present customary system, and the metric system. At times, you will work with a combination of both, referred to as *dual dimensioning.* Dual dimensioning is used where parts are made in more than one location and where both systems may be used.

There are published reference tables and charts available for converting from customary measurements to metric and vice versa. However, certain interchangeability problems may arise from time to time. Most machine tools made in Europe have metric dimensions, even down to the fasteners which hold the machine together and those which must be adjusted when setting the machine up for a job. This frequently requires a machinist to have at hand and use two completely different sets of hand tools.

39 SI Units of Measurement

The initials *SI* stand for *Systeme International d'Unites,* a French term literally translating to *International System of Units.* The abbreviation *SI* is most often used instead of the full name. We will use SI throughout our discussions in lieu of the word *metric.* The initials are the more modern description of the system.

There are seven *base units* used in SI to measure the seven *major units* of length, mass, time, electric current, thermodynamic temperature, amount of substance, and luminous intensity. The corresponding base units are shown in **Fig. 32**. In practical machine shop work, you will normally be involved with only a few of these major units on a day-to-day basis. These are length, mass, time, and temperature.

SI Major Unit	Base Unit	Base Unit Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric current	Ampere	A
Thermodynamic Temperature	Degree Kelvin	K
Amount of Substance	Mole	mol
Luminous Intensity	Candela	cd

Fig. 32. Corresponding base units

40 Using SI Prefixes

The SI system of measurement uses decimal arithmetic as its foundation. It is quite simple in theory and actual operation. Each SI base unit may be increased or decreased in size by moving the decimal point, which is the same as multiplying by a power of 10. A prefix is used in conjunction with the base unit's name or symbol to designate the increase or decrease.

Fig. 33 shows the established SI prefixes, the symbol for each prefix, the power of 10 used to define the value of the base unit, and the position of the decimal point for that value. Study Fig. 33 carefully. It represents the basis of the SI system, and will be a handy reference in your progress through this module.

	Prefix	Symbol	Power of 10	Decimal Places
I	Tera	T	10^{12}	1,000,000,000,000.
N	Giga	G	10^{9}	1,000,000,000.
C	Mega	M	10^{6}	1,000,000.
R	Kilo	k	10^{3}	1,000.
E	Hecto	h	10^{2}	100.
A	Deka	da	10	10.
S				
E				
	1 Base Unit			1.
D	Deci	d	10^{-1}	.1
E	Centi	c	10^{-2}	.01
C	Milli	m	10^{-3}	.001
R	Micro	μ	10^{-6}	.000001
E	Nano	n	10^{-9}	.000000001
A	Pico	p	10^{-12}	.000000000001
S	Femto	f	10^{-15}	.000000000000001
E	Atto	a	10^{-18}	.000000000000000001

Fig. 33. SI prefixes and symbols

The meter (m) is the SI base unit for measuring length. To specify a length of one one-hundredth of a meter, you write 1/100 m. However, there is no need to write this fraction as such. Instead, use the prefix *centi* to designate one one-hundredth, and its symbol is placed in front of the meter symbol m. Thus one one-hundredth m is written as 1 cm. Since this represents a *decrease* in the value of 1 m, the decimal point is moved to the left as shown in Fig. 33. The same prefixes are used to denote an increase or decrease in value regardless of the base unit involved. A road sign indicating a distance of 10 km tells you that the distance is 10,000 m. The designation 5 dkg specifies 5/10 or 0.5 kg. You read a time of 3 ms as 0.003 seconds.

41 Adding and Subtracting Metric Values

You perform addition and subtraction of metric or SI values in generally the same manner as when working with decimals. *There is, however, one additional consideration. You* *must first convert all the values so that all are expressed terms.* The lengths of the roller journals in **Fig. 34** are specified in different multiples of a meter. To determine dimension *L,* you must add the individual journal lengths.

First, convert all the dimensions to one common expression. In this example we will use the meter as a common expression. Lay out the problem as follows, starting with the 5 dm dimension at the far right of Fig. 34.

$$
\begin{array}{rcl}
5\ \text{dm} & = & 0.500\ \text{m} \\
30\ \text{mm} & = & 0.030\ \text{m} \\
1\ \text{m} & = & 1.000\ \text{m} \\
10\ \text{cm} & = & 0.100\ \text{m} \\
\underline{2\ \text{dm}} & \underline{=} & \underline{0.200\ \text{m}} \\
 & & 1.830\ \text{m} = \text{dimension L}
\end{array}
$$

Subtraction of SI values is done following the same procedures of first converting all measurements to one common expression, and then carrying out the operations.

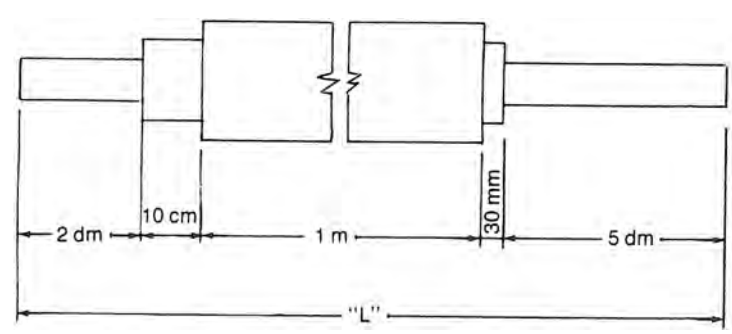

Fig. 34. Roller Journals

42 Mass

Many people tend to confuse the mass of an object with its weight. The weight of a body is the attraction of gravity on the body. This is why weight decreases as the body gets farther way from the Earth's gravitational pull. Upon reaching a certain altitude, astronauts experience *weightlessness* because they are farther away from the Earth's center.

Mass is an unchanging quantity; it remains the same no matter where the object is located in respect to the Earth's center. Mass is unaffected by any attraction of gravity. Mass, not weight, is the SI major unit for the *amount of body.* Its base unit is the kilogram (kg).

Under the old metric system, the base unit for mass was the gram (g). The prefix *kilo* was later applied because the gram is so small that it requires the use of a prefix most of the time. Therefore, the kilogram is the only SI base unit that has a prefix already applied.

The same rule applies when adding or subtracting quantities of mass as for other SI units. All must first be converted to a common expression. The total mass of the blocks in **Fig. 35** is computed:

1.7	mkg	=	0.0017 kg
2.0	ckg	=	0.0200 kg
3.3	kg	=	3.3000 kg
4.13	dkg	=	41.3000 kg
			44.6217 kg = total mass

Mass of metals and other workpiece materials is often a critical factor when selecting a machine or its cutters for a machine shop project. This is particularly true in the case of equipment built in most foreign countries.

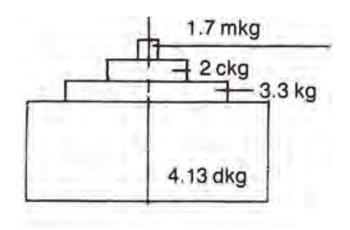

Fig. 35. Blocks

43 SI Scale Systems

Machine shop working drawings are not always made to the full size of the part shown therein. Most of the time a certain *scale* will be shown in the title block of the drawing. This scale designates the proportion the drawing shows compared to the real dimensions of the part.

In the customary system of dimensioning a shop drawing, a scale may be noted as 3″ = 1′–0″; 1″ = 1′–0″; etc. This tells you that a distance of 3″ in the drawing is equal to 1′–0″ on the real part, or that 1″ equals 1′–0″ on the full size piece. Dimensioning scales are used to keep the size of the drawing to a minimum, particularly when the drawing is of a relatively large part.

The SI method of designating the scale used in a drawing is similar to that used for ratios or proportions. For example, an SI scale of 1 : 1 designates that the drawing shows the part at its full size. It has not been reduced for drawing purposes.

On the other hand, a scale of 1 : 2 tells you that one actual measured value (m, cm, mm, etc.) on the drawing represents two times that measurement on the real part. In other words, the actual measured size of the part on the drawing is only one-half of the part's real size.

The chart in **Fig. 36** shows the most commonly used SI drawing scales for engineering and shop working drawings. The chart also shows the *nearest* corresponding customary scale for each.

Drawing Class	SI Decimal Scale	Nearest Customary Scale
Engineering	1:1	Full size
Drawings	1:2	6″ = 1 ft.
	1:5	3″ = 1 ft.
	1:10	1″ = 1 ft.
	1:20	1/2″ = 1 ft.
Working	1:50	1/4″ = 1 ft.
Drawings	1:100	1/8″ = 1 ft.
	1:200	1/16″ = 1 ft.

Fig. 36. Most commonly used SI drawing scales

44 SI Derived Units

An SI *derived unit* results when two or more base units are multiplied or divided by one another. For example, multiplying 4 m by 2 m = 8 m^2. The answer, 8 m^2, is a derived unit. The speed of a car traveling at the rate of 5 meters per second is expressed as 5 m/s, and the symbol m/s, meaning meters per second, is a derived unit. A few of the more commonly used SI derived units are listed below. Some have been assigned special names and symbols, while others use their common names.

REQUIRED QUANTITY	NAME/SYMBOL	FORMULA
Acceleration		m/s^2
Velocity		m/s
Area		m^2
Volume		m^3
Work	joule/J	N • m
Power	watt/W	J/s
Force	newton/N	kg • m/s^2
Voltage	volt/V	W/A
Electrical resistance	ohm/ Ω	V/A
Pressure	pascal/Pa	N/m^2
Density		kg/m^3

Check Your Learning 2

Answer the following questions to check your understanding of the material you've read in the preceding articles. If you want to review the answer to a question, use the article reference included with the answer.

1. What is the value of $10^2 \times 10^3$?
2. Compute the following: $\sqrt[3]{64}$; $\sqrt{81}$; $\sqrt[3]{8}$.
3. A driver pulley turns at a speed of 850 rpm. It drives another pulley turning at 320 rpm. The ratio of reduction is __________.
4. Using a power of 10 with the proper exponent, show the value of 1 km.
5. What is the sum of 6 dm + 40 mm + 2 m + 10 cm?

Your Answers

1. ____________________
2. ____________________
   ____
   ____
3. ____________________
4. ____________________
5. ____________________

Answers to Check Your Learning 2

1. 100,000 Art. 24
2. 4; 9; 2 Art. 27
3. 2.66 to 1 Art. 31
4. 10^3 Art. 40
5. 2.740 m Art. 41