On the variance of radio interferometric calibration solutions: Quality-based Weighting Schemes

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Abstract

SKA-era radio interferometric data volumes will need low-footprint, high-impact algorithms to improve the final science products. We investigate the possibility of improving radio interferometric images using an algorithm inspired by “lucky imaging" : selecting the best-calibrated points of a given observation. To do so, we investigate the statistical properties of residual visibilities associated with calibration. A fundamental relationship between residual visibility statistics and the noise-map is derived in this paper The noise-map can indeed be described as the Fourier transform of the residual visibilities from the **δuδv**-domain into the image-plane, which is further explored in our work. Based on this relationship, we then propose a means for estimating the covariance matrices for all baselines in a given observation in order to improve the images made with the calibration-corrected visibilities. This improvement should occur after calibration, but before imaging - it could also be performed between major iterations of self-calibration. Applying the weighting scheme to simulated data improves the noise level in the final image by a factor of 10, at negligible computational cost.

Fundamental Analytical Relationship

The key result of our work is a fundamental relationship between the variance in the image (henceforth noise-map) and the variance in the visibilities. In particular, the variance in the residual map can be directly linked to the covariance matrix of the residual visibilities for each baseline:

$$\boldsymbol{\mathcal{V}}_{pq,t\nu} = \sum_d \boldsymbol{K}^d_{p,t\nu}\boldsymbol{J}_{p,t\nu}\boldsymbol{B}^d_{\nu}\boldsymbol{J}^H_{q,t\nu}(\boldsymbol{K}^d_{q,t\nu})^H + n$$

$$= \sum_d s^{\nu}_d \boldsymbol{K}^d_{p,t\nu}\boldsymbol{J}_{p,t\nu}\boldsymbol{J}^H_{q,t\nu}(\boldsymbol{K}^d_{q,t\nu})^H + n$$

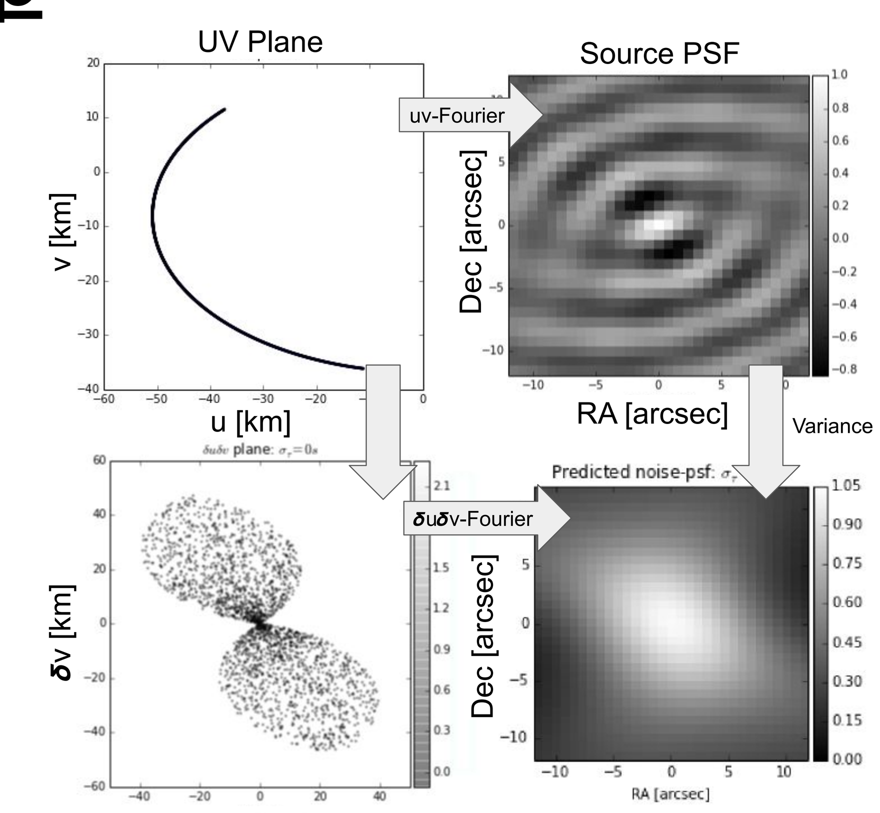
$$\mathrm{vec}\{\boldsymbol{\mathcal{V}}_{pq,t\nu}\} = \boldsymbol{\mathcal{S}}_{pq,t\nu}\boldsymbol{\mathcal{M}}_{pq,t\nu}\,\mathrm{vec}\{\boldsymbol{I}\} + n$$

$$\boldsymbol{\mathcal{S}}_{pq,t\nu} = \sum_d s^{\nu}_d \boldsymbol{K}^d_{q,t\nu} \otimes \boldsymbol{K}^d_{p,t\nu}$$

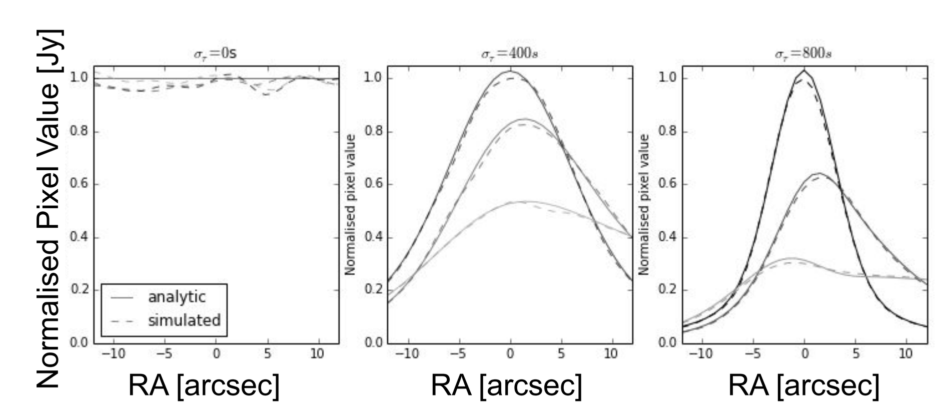
$$\boldsymbol{\mathcal{M}}_{pq,t\nu} = \boldsymbol{J}_{q,t\nu} \otimes \boldsymbol{J}_{p,t\nu}$$

$$\mathrm{vec}\{\mathrm{Cov}\{\tilde{\boldsymbol{y}}\}\} = (\boldsymbol{\mathcal{F}} \otimes \boldsymbol{\mathcal{F}}^*)(\boldsymbol{\mathcal{W}} \otimes \boldsymbol{\mathcal{W}}^*)(\boldsymbol{\mathcal{S}} \otimes \boldsymbol{\mathcal{S}}^*)\mathrm{vec}\{\mathrm{Cov}\{\tilde{\boldsymbol{\mathcal{M}}}\}\} + (\boldsymbol{\mathcal{F}} \otimes \boldsymbol{\mathcal{F}}^*)(\boldsymbol{\mathcal{W}} \otimes \boldsymbol{\mathcal{W}}^*)\mathrm{vec}\{\sigma^2\boldsymbol{I}\}$$

Where a tilde denotes a residual quantity (measured - modeled). This result flows directly (indeed, nearly trivally) from the form of the RIME (Radio Interferometer’s Measurement Equation), and allows us to understand the variance measured at each individual pixel - the noise-map - as the Fourier transform of the *vectorised* covariance matrix of each baseline. In the absence of time-correlation in the residual visibilities, no structure is expected to appear - similarly, strongly time-correlated residuals (whether due to unsubstracted sources or to solver-induced errors) will introduce a noise-pattern at given positions. In the absence of direction-dependent effects, assuming a complete sky model and unbiased calibration, what we find is then a *noise-PSF* - a pattern convoluted to each source in the field, describing how the uncertainty in pixel flux count will vary as a function of position. It is this quantity that our weighting scheme flattens.



Representation of the relationships between uv-plane, image-plane, **δuδv**-plane, and noise map



Simulation of noise-map at different correlation lengths, calculated with both **δuδv**-Fourier transform and by taking variance at each pixel over 2000 realisations

Weighting Scheme

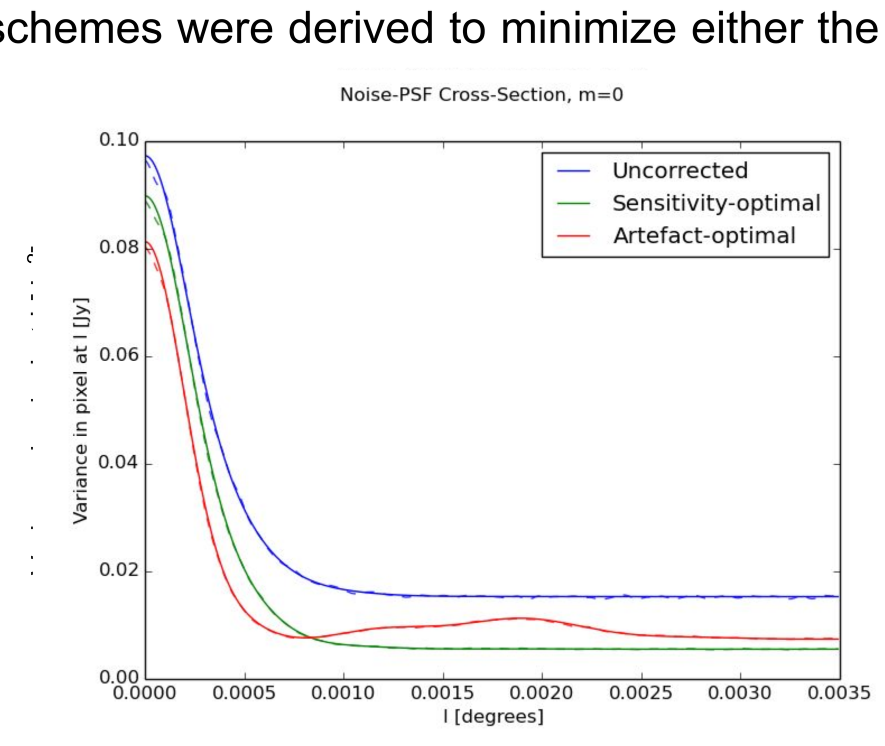
Simulations

Based on the simulation work done to test the fundamental analytical relationship that was found above, the following weighting schemes were derived to minimize either the peak of the noise-PSF, or its wings:

$w_i = 1/\mathrm{Var}(\mathbf{y}_i)$ (diag)

$\mathbf{w} = \mathrm{Cov}(\mathbf{y})^{-1}\,\mathbf{1}$ (full)

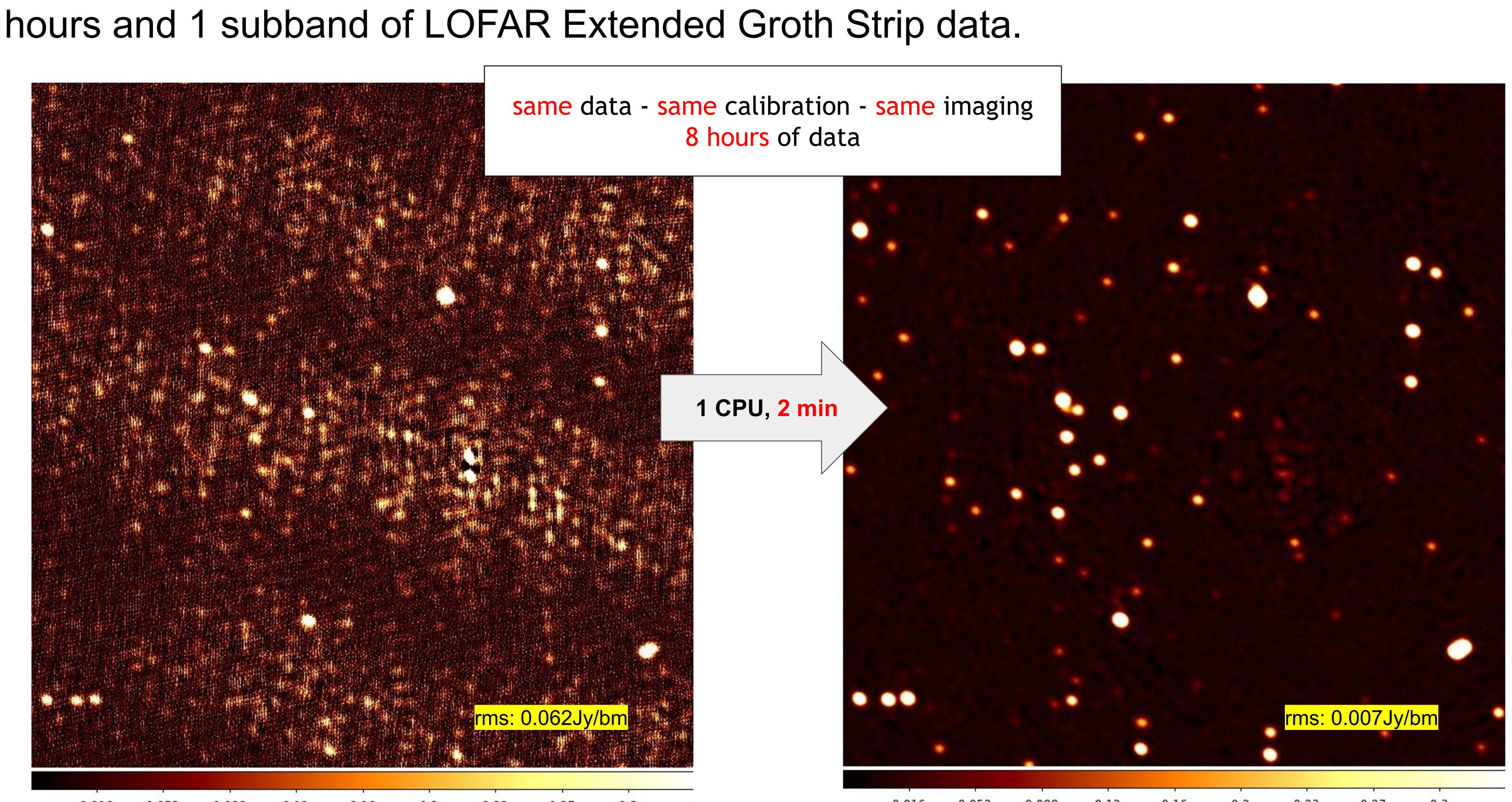
Here, **y** are the residual visibilities, **1** a vector of ones, and w_i are the components of **w**. The effect of the weights are shown on the diagram to the right. Diag weights optimize far-field sensitivity; full-weights optimize near-source sensitivity.



One key point to note is that we are looking at the noise-PSF of a single baseline, using simulated noise: the underlying covariance matrix is perfectly known, and so the weighting is optimal. In practice, one such set of weights would be found per baseline, and each would be based on imperfect estimation: the net effect is thus challenging to predict. In spite of these caveats, however, the results speak for themselves.

Application to Data

Based on the work outlined to the left, an algorithm to estimate the covariance matrix for each baseline (and subsequently derive the appropriate weighting) was written: using an antenna-based approach to improve matrix conditioning, weights can be determined from residual visibilities swiftly and effectively. The result is shown below, applied on 8 hours and 1 subband of LOFAR Extended Groth Strip data.



Work remains: notably, the weights normalisation in this very simple case decreases resolution sightly in the final image, as the normalisation effectively downweights the international baselines. However, the net improvement is undeniable.