An investor has 20,000 pounds to invest in a combination of the following:
There are three scenarios for the price of stock XYZ six months from today: the price will be the same as today, the price will go up to £40, or drop to £12. The investor's best estimate is that each of these scenarios is equally likely.

1. Buying n number of Stock XYZ today at £20 per share

item	stock price (6 months later)	stock price 6 months later – price of stock today	Loss or gain
1	20	20-20	0
2	12	12-20	-8
3	40	40-20	20

2. Buying n number of European call options that each one gives right the investor to buy 100 shares of stock XYZ at £15 per share exactly six months from today for a cost of £1,000.

item	stock price	Cost of buy	Grass profit	net profit (Cost – profit)	Execute Status	Loss or gain
1	20	-1000	(100*20) - (100*15) = 500	-500	Yes	-500
2	12	-1000	(100*12) - (100*15) = -300	-1300	No	-1000
3	40	-1000	(100*40) - (100*15) = 2500	1500	Yes	1500

Note: item1) If the market price of stock XYZ be £20 six months from now, and the holder of the call option exercised it, they would be able to buy 100 shares of the stock at the strike price of £15 per share, for a total cost of £1,500. They could then immediately sell those 100 shares at the market price of £20 per share, for a total revenue of £2,000, resulting in a profit of £500.

However, the holder of the call option needs to subtract the cost of the premium they paid to purchase the option, which is £1,000, from their profit, resulting in a net profit of £500 - £1,000 = -£500. Therefore, if the market price stays at same price of today, £20, by exercising the option investor would make a loss of £500 if not he will lose £ 1000. It is reasonable to execute the option to minimize the loss.

Item2) investor will not execute the option because stock price in market is cheaper than the option, so there is no reason to execute the option.

The investor can buy maximum 50 call options.

3. Selling n number of European call options that each one gives right the investor to sell 100 shares of stock XYZ at £15 per share exactly six months from today for a cost of £1,000 and immediately invested.

item	stock price	gain	Grass profit	net profit	Execute Status	Loss or gain
1	20	1000	(100*15) - (100*20) = -500	500	Yes	500
2	12	1000	(100*15) - (100*12) = 300	1300	No	1000
3	40	1000	(100*15) - (100*40) = -2500	-1500	Yes	-1500

The investor can sell maximum 50 call options.

4. Buying a 6-month riskless zero-coupon bond with £100 face value sells for a cost of £90.

An investor may buy or sell a maximum of 50 call options.

Assign the variables:

S = Stock
CB = Call_buy
CS = Call_sell
B = Bond

Constraints:

S*£20 + CB*£1000 - CS*£1000 + B*£90 <= £20,000
Or
S*£20 + CB*£1000 + B*£90 <= £20,000+ CS*£1000

** Due to terms that 0 <= CS <= 50 and £20,000 initial fund, investor maximum can invest £70,000.

For Part I
-8*S - 1000*CB + 1000*CS + 10*B >= 0 # minimum profit with stock price 12
0*S - 500*CB +500*CS + 10*B >= 0 # minimum profit with stock price 20
20*S + 1500*CB - 1500*CS + 10*B >= 0 # minimum profit with stock price 40

For Part II
-8*S - 1000*CB + 1000*CS + 10*B >= 2000 # minimum profit with stock price 12
0*S - 500*CB +500*CS + 10*B >= 2000 # minimum profit with stock price 20
20*S + 1500*CB - 1500*CS + 10*B >= 2000 # minimum profit with stock price 40

0 <= CS <= 50
0 <= CB <= 50
CS + CB <= 50
B >= 0
S >= 0

Objective function:

N(S, CB, CS, B) = 0.33 * (-8*S - 1000*CB + 1000*CS + 10*B)
+ 0.33 * (0*S - 500*CB + 500*CS + 10*B)
+ 0.33 * (20*S + 1500*CB - 1500*CS + 10*B)

- Suppose that the investor wants a profit of at least **£0** in any of the three scenarios for the price of XYZ six months from today. **(Risk less portfolio: all scenarios profit > 0)**

```
Status: Optimal
Maximum Profit: £ 11999.999999999998
Investment in Stock: £ 60000.0
Investment in Call Options (Buy): £ 0.0
Investment in Call Options (Sell): £ 40000.0
Investment in Bond: £ 0.0
S:  3000.0 CB:  0.0 CS:  40.0 B:  0.0
```

1/3	= prob of each scenario		Scenarios			
Invest	**number**	**cost/gain**	12	20	40	
stock XYZ	3000	20	-8	0	20	
buy call option	0	1000	-1000	-500	1500	
sell call option	40	-1000	1000	500	-1500	
bond	0	90	10	10	10	
	Total:	20000	16000	20000	0	12000 **: Max Profit**
			Profit of each Scenario			

- Suppose that the investor just wants to maximize profit and no matter if lose in some scenarios.

```
Status: Optimal
Maximum Profit: £ 13999.999999999998
Investment in Stock: £ 70000.0
Investment in Call Options (Buy): £ 0.0
Investment in Call Options (Sell): £ 50000.0
Investment in Bond: £ 0.0
S:  3500.0 CB:  0.0 CS:  50.0 B:  0.0
```

1/3	= prob of each scenario		Scenarios			
Invest	**number**	**cost/gain**	12	20	40	
stock XYZ	3500	20	-8	0	20	
buy call option	0	1000	-1000	-500	1500	
sell call option	50	-1000	1000	500	-1500	
bond	0	90	10	10	10	
	Total:	20000	22000	25000	-5000	14000 **: Max Profit**
			Profit of each Scenario			

- Suppose that the investor wants a **profit of at least £2,000** in any of the three scenarios for the price of XYZ six months from today.

```
Status: Optimal
Maximum Profit: £ 11199.999999999998
Investment in Stock: £ 56000.0
Investment in Call Options (Buy): £ 0.0
Investment in Call Options (Sell): £ 36000.0
Investment in Bond: £ 0.0
S:  2800.0 CB:  0.0 CS:  36.0 B:  0.0
```

1/3	= prob of each scenario		Scenarios			
Invest	**number**	**cost/gain**	12	20	40	
stock XYZ	2800	20	-8	0	20	
buy call option	0	1000	-1000	-500	1500	
sell call option	36	-1000	1000	500	-1500	
bond	0	90	10	10	10	
	Total:	20000	13600	18000	2000	11200 **: Max Profit**
			Profit of each Scenario			

Sample of Code

```
import pulp

# Create the problem object
prob = pulp.LpProblem("Investment Optimization", pulp.LpMaximize)

# Define the decision variables
S = pulp.LpVariable("S", lowBound=0, cat='Integer')
CB = pulp.LpVariable("CB", lowBound=0, upBound=50, cat='Integer')
CS = pulp.LpVariable("CS", lowBound=0, upBound=50, cat='Integer')
B = pulp.LpVariable("B", lowBound=0, cat='Integer')

# Define the objective function
prob += 1/3 * (-8*S - 1000*CB + 1000*CS + 10*B) \
        + 1/3 * (0*S - 500*CB + 500*CS + 10*B) \
        + 1/3 * (20*S + 1500*CB - 1500*CS + 10*B)

# Define the constraints
prob += S*20 + CB*1000 - CS*1000 + B*90 <= 20000
prob += CS + CB <= 50
prob += -8*S - 1000*CB + 1000*CS + 10*B >= 2000
prob += 0*S - 500*CB + 500*CS + 10*B >= 2000
prob += 20*S + 1500*CB - 1500*CS + 10*B >= 2000

# Solve the problem
prob.solve()

# Print the results
print("Status:", pulp.LpStatus[prob.status])
print("Maximum Profit: £", pulp.value(prob.objective))
print("Investment in Stock: £", pulp.value(S)*20)
print("Investment in Call Options (Buy): £", pulp.value(CB)*1000)
print("Investment in Call Options (Sell): £", pulp.value(CS)*1000)
print("Investment in Bond: £", pulp.value(B)*90)
print("S: ", pulp.value(S), "CB: ", pulp.value(CB), "CS: ", pulp.value(CS),"B: ", pulp.value(B))
```