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The Persian calendar for 3000 years

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Abstract. Using the analytical theory of the motion of the Earth around the Sun the times of the vernal (Spring) equinox has been calculated over the period from the Hijra (AD 622) to AD 3800. These data alone allow to decide whether a particular Persian (or Jalaali) calendar year is common or leap. Presented analysis shows that an algorithm implemented in the so called Khayam program is valid for the years 1799 to 2256 (1178 to 1634 Jalaali). A concise algorithm has been worked out that reconstructs the pattern of leap years over time span of about 3000 years. FORTRAN routines for conversion between the Jalaali, Gregorian and Julian calendars and the Julian Day Number are presented.

1. Rules of the Persian calendar

The Persian or Jalaali calendar is officially used in Iran and surrounding areas. It is a solar calendar closely following the astronomical seasons, thus requires the knowledge of exact times of the vernal (Spring) equinox.

The rules of the Jalaali calendar are quite simple. Years of 12 months are reckoned from the era of the Hijra, commemorating the migration of the Prophet and his followers from Mecca to Medina in AD 622. A Jalaali year begins on the first day of astronomically determined spring or on the day following it according to whether the exact moment of the equinox occurs before or after, respectively, 12:00 of the Teheran mean time. The first 6 months all have 31 days and the last 6 manths all have 30 days in leap years. In common years the last month has 29 days. Thus every season is associated with three consecutive months.

As expected, roughly every fourth year in the Persian calendar is the leap one. This is the well known regularity in solar calendars. However, normally after every 32 (sometimes after 28 or 36) years follows an extra common year making four consecutive 365-day years instead of the usual three. Currently the leap years go smoothly in the 33-year cycles and specifically they are those years that after dividing by 33 leave a remainder of 1, 5, 9, 13, 17, 22, 26 and 30. E.g., the Jalaali year 1375 that begun on March 20, 1996 has the remainder of 22 and thus is the leap year. These rules are implemented in the so called Khayam program of Ali Moayedian and Mash Cheragh-Ali, which is available on the Internet. Unfortunately this simple behaviour will not last indefinitly and the 33-year cycle is sure to break sometimes. Hossein Bagherzadeh Rafsanjani in his introduction to the program suggests that a break may occur `early in the next century Hijra' and that the program should be correct until early 2050's of Gregorian calendar. It is the purpose of this paper to determine exactly how long the above rule of remainders will be valid and to establish the rules for the Jalaali calendar as far into future as is presently practical.

2. Astronomical background

In general, to determine if a Persian year is leap or common it is necessary to find the date and time of the vernal equinox at the beginning of that year and at the beginning of the next year. To this end I have employed an analytical theory of the motion of the Earth around the Sun (Bretagnon et al.1986), which is accurate to about 2″ in the angular coordinates of the Sun as viewed from the Earth. The theory has been used in a converging process to find the Terrestrial Time (TT or Ephemeris Time) of the epoch at which the celestial longitude of the Sun equalled 0. The TT of the equinox has been then converted to the Universal Time (UT1) using past estimates, measurements, and future predictions of the difference $\Delta T = TT - UT1 = 32.184 - (UT1 - TAI)$, where TAI is the atomic time scale of BIPM. For past years and far future predictions I have used essentially tabular data and formulae of Stephenson and Morrison (1984):

$$\Delta T = \begin{cases} (44.3t + 320)t + 1360 & \text{for } \ y < 948 \\ 25.5t^2 & \text{for } \ 948 \le y < 1637 \\ 25.5t^2 - 36 & \text{for } \ y > 2005 \end{cases}$$

(here the last expression has an extra –36 s to ensure a smooth transition from tabular data), where $t = (y - 1800)/100$, y is the Julian year, and ΔT is obtained in seconds. The most recent tabular data and predictions until 2006 were taken from *IERS Annual Reports* and the USNO archives available on Internet.

In Table I we present partial list of vernal equinoxes given in the Universal Time that might be of general interest. As a final step, the mean time of Teheran has been calculated by adding its geographic longitude, 3.425 hours, to the UT1 of the equinoxes.

Table I: Vernal equinoxes 1900 – 2099

Year AD	Day Mar	UT1 [h:m]	Year AD	Day Mar	UT1 [h:m]	Year AD	Day Mar	UT1 [h:m]	Year AD	Day Mar	UT1 [h:m]
1900	21	1:39	1950	21	4:35	2000	20	7:35	2050	20	10:19
1901	21	7:24	1951	21	10:26	2001	20	13:31	2051	20	15:58
1902	21	13:16	1952	20	16:13	2002	20	19:16	2052	19	21:55
1903	21	19:15	1953	20	22:01	2003	21	1:00	2053	20	3:46
1904	21	0:58	1954	21	3:53	2004	20	6:48	2054	20	9:33
1905	21	6:58	1955	21	9:35	2005	20	12:33	2055	20	15:28
1906	21	12:53	1956	20	15:21	2006	20	18:25	2056	19	21:10
1907	21	18:33	1957	20	21:16	2007	21	0:07	2057	20	3:07
1908	21	0:28	1958	21	3:05	2008	20	5:48	2058	20	9:04
1909	21	6:13	1959	21	8:55	2009	20	11:44	2059	20	14:44
1910	21	12:03	1960	20	14:43	2010	20	17:32	2060	19	20:38
1911	21	17:54	1961	20	20:32	2011	20	23:21	2061	20	2:25
1912	20	23:29	1962	21	2:30	2012	20	5:14	2062	20	8:07
1913	21	5:18	1963	21	8:20	2013	20	11:01	2063	20	13:59
1914	21	11:11	1964	20	14:10	2014	20	16:57	2064	19	19:38
1915	21	16:51	1965	20	20:05	2015	20	22:45	2065	20	1:27
1916	20	22:47	1966	21	1:52	2016	20	4:30	2066	20	7:19
1917	21	4:37	1967	21	7:37	2017	20	10:28	2067	20	12:53
1918	21	10:26	1968	20	13:22	2018	20	16:15	2068	19	18:48
1919	21	16:19	1969	20	19:08	2019	20	21:58	2069	20	0:44
1920	20	21:59	1970	21	0:56	2020	20	3:49	2070	20	6:34
1921	21	3:51	1971	21	6:38	2021	20	9:37	2071	20	12:34
1922	21	9:48	1972	20	12:21	2022	20	15:33	2072	19	18:20
1923	21	15:29	1973	20	18:13	2023	20	21:24	2073	20	0:13
1924	20	21:21	1974	21	0:06	2024	20	3:06	2074	20	6:08
1925	21	3:12	1975	21	5:57	2025	20	9:01	2075	20	11:46
1926	21	9:01	1976	20	11:50	2026	20	14:45	2076	19	17:38
1927	21	14:59	1977	20	17:42	2027	20	20:24	2077	19	23:30
1928	20	20:44	1978	20	23:33	2028	20	2:16	2078	20	5:10
1929	21	2:35	1979	21	5:22	2029	20	8:01	2079	20	11:00
1930	21	8:30	1980	20	11:10	2030	20	13:51	2080	19	16:43
1931	21	14:06	1981	20	17:03	2031	20	19:40	2081	19	22:34
1932	20	19:54	1982	20	22:55	2032	20	1:21	2082	20	4:31

1933	21	1:43	1983	21	4:39	2033	20	7:22	2083	20	10:10
1934	21	7:27	1984	20	10:24	2034	20	13:17	2084	19	15:59
1935	21	13:18	1985	20	16:14	2035	20	19:03	2085	19	21:53
1936	20	18:57	1986	20	22:03	2036	20	1:02	2086	20	3:35
1937	21	0:45	1987	21	3:52	2037	20	6:49	2087	20	9:28
1938	21	6:43	1988	20	9:39	2038	20	12:40	2088	19	15:16
1939	21	12:28	1989	20	15:28	2039	20	18:31	2089	19	21:06
1940	20	18:24	1990	20	21:19	2040	20	0:11	2090	20	3:02
1941	21	0:21	1991	21	3:02	2041	20	6:06	2091	20	8:41
1942	21	6:10	1992	20	8:48	2042	20	11:52	2092	19	14:33
1943	21	12:03	1993	20	14:41	2043	20	17:27	2093	19	20:34
1944	20	17:48	1994	20	20:28	2044	19	23:20	2094	20	2:21
1945	20	23:37	1995	21	2:15	2045	20	5:07	2095	20	8:15
1946	21	5:33	1996	20	8:03	2046	20	10:57	2096	19	14:02
1947	21	11:13	1997	20	13:55	2047	20	16:52	2097	19	19:48
1948	20	16:57	1998	20	19:54	2048	19	22:33	2098	20	1:40
1949	20	22:48	1999	21	1:46	2049	20	4:28	2099	20	7:17

A full list of equinoxes comprising the years AD 550 to 3800, referred to the Teheran time has been used to identify the leap years of the Jalaali calendar. The pattern of leap years is quite regular: they come every 4 years in groups of 28, 32 or 36 years with each group supplemented with an extra common year. Usually these are 32+1 year groups and the few occasional exceptions, called here breaks, can be used in a practical procedure to reconstruct the entire sequence of leap years. In Table II the Jalaali years represent the first year (leap) of a new cycle (mostly the 33-year cycle) after a break (i.e. after the 4-th common year closing the 28 or 36 year group).

Table II: The years of the Gregorian calendar (Gy) that mark the end of a 29- or 37-year period which breaks the 33-year rule validity. Around 20 March Gy ends the 4th common year and begins the Jalaali leap year Jy

Gy	Jy	Gy	Jy	Gy	Jy	Gy	Jy
560	–61	1307	686	1831	1210	2883	2262
630	9	1377	756	2256	1635	2945	2324

659	38	1439	818	2681	2060	3015	2394
820	199	1732	1111	2718	2097	3077	2456
1047	426	1802	1181	2813	2192	3799	3178

As can be seen from Table II, the 33-year cycle operates continuously between 1831 and 2256. Closer examination shows that in fact it operated from 1799, since 1831 marks just the end of the 29-year period, and would not appear here if the previous break would fall four years earlier. This is thus the time span of validity of the Khayam algorithm. There is however a possibility that it will not be as good as that and the upper limit may fall on AD 2124. I shall return to this question when discussing errors.

3. Algorithm for the Jalaali calendar

The proposed algorithm is based on the described list of the breaking years in which the fourth common Jalaali year occurs after 28 or 36 (and not 32) years of 4-year periods since the previous occurence of four common years in succession. During 3000 years there are only about 20 such breaking years, and they allow for an easy reconstruction of a complete sequence of Jalaali leap years.

To determine whether a Jalaali year is common or leap, one finds the number, say N, of years that have passed since the last breaking year of Table II. With one exception, the year of interest is leap if –1 plus the remainder of (N + 1)/33 is evenly divisible by 4, or

$$l_p = \mathrm{MOD}[\mathrm{MOD}(N + 1,33) - 1,4]$$

equals 0, where MOD is the operation of finding the remainder when the first argument is divided by the second. The exception concerns the case when the year of interest lies within 5 years of the following breaking year. In this case instead of N + 1 above the value of N + 1 ± 4 must be used, where the + sign is chosen if the cycle, where the considered year belongs, is 29-year long, and the – sign otherwise (i.e. with the 37-year long cycle).

Using the integer arithmetic this algorithm reduces to just two FORTRAN lines (apart of searching the list for the required breaking years). For the common years the remainder of division by 4, i.e. l_p, tells how many years have passed since the last leap year. E.g. the remainder of 1 (2 or 3) means that the leap year was the previous one (2 or 3 years ego). Obviously, it is also possible to tell whether the considered year is the 4-th common one. The case occurs when N + 1 (or N + 1 ± 4 if it is the exceptional situation), is evenly divisible by 33.

To relate the Jalaali calendar to the Gregorian one it is necessary to find the number of leap years since an initial epoch in both these calendars. In the Jalaali calendar, the number of leap years between two adjacent break years of Table II, say between y_i and y_j is

$$l_j = 8\ \mathrm{INT}(N_j/33) + \mathrm{INT}[\mathrm{MOD}(N_j,33)/4]$$

where $N_j = y_j - y_i$ and INT is the function taking the integral part of the evaluated argument. The numbers l_j must be summed until y_j remains smaller than the year in question. Then the sum must be increased by

$$8\ \mathrm{INT}(N/33) + \mathrm{INT}\{[\mathrm{MOD}(N,33) + 3]/4\} + k,$$

the number of leap years since the last break year. Here k is 1 only when the considered year lies 4 years before the following breaking year (y_j) and the current group is 37 year long; otherwise it is 0.

This algorithm has been encoded in the FORTRAN integer arithmetic subroutine (Table III) `JalCal(Jy,leap,Gy,March)`, which for a given Jalaali year `Jy` returns information on the leap year in the `leap` variable, which assumes values 0 to 4, as described above. The routine returns also the Gregorian date of the first day of the Jalaali year in the variables `Gy` (Gregorian year) and `March` (day of March). Thus it can be directly used to relate the beginning of the Persian calendar for any year within about 3000 years to the Gregorian calendar.

Table III:

```
      subroutine JalCal(Jy,leap,Gy,March)
c This procedure determines if the Jalaali (Persian) year is
c leap (366-day long) or is the common year (365 days), and
c finds the day in March (Gregorian calendar) of the first
c day of the Jalaali year (Jy)
c Input:  Jy - Jalaali calendar year (-61 to 3177)
c Output:
c   leap  - number of years since the last leap year (0 to 4)
c   Gy    - Gregorian year of the beginning of Jalaali year
c   March - the March day of Farvardin the 1st (1st day of Jy)
      integer breaks(20),Gy
c Jalaali years starting the 33-year rule
      data breaks/-61,9,38,199,426,686,756,818,1111,1181,
     *   1210,1635,2060,2097,2192,2262,2324,2394,2456,3178/
      Gy=Jy+621
      leapJ=-14
      jp=breaks(1)
      if(Jy.lt.jp.or.Jy.ge.breaks(20)) print'(a,i5,a,i5,a)',
     *' Bad year number:',Gy,' Gregorian   =',Jy,' Jalaali'
c Find the limiting years for the Jalaali year Jy
      do 1 j=2,20
      jm=breaks(j)
```

```
      jump=jm-jp
       if(Jy.lt.jm) go to 2
      leapJ=leapJ+jump/33*8+MOD(jump,33)/4
1     jp=jm
2     N=Jy-jp
c Find the number of leap years from AD 621 to the beginning
c of the current Jalaali year in the Persian calendar
      leapJ=leapJ+N/33*8+(MOD(N,33)+3)/4
       if(MOD(jump,33).eq.4.and.jump-N.eq.4) leapJ=leapJ+1
c and the same in the Gregorian calendar (until the year Gy)
      leapG=Gy/4-(Gy/100+1)*3/4-150
c Determine the Gregorian date of Farvardin the 1st
      March=20+leapJ-leapG
c Find how many years have passed since the last leap year
      if(jump-N.lt.6) N=N-jump+(jump+4)/33*33
      leap=MOD(MOD(N+1,33)-1,4)
      if(leap.eq.-1) leap=4
         end
```

Such a procedure can be employed for full date conversion from the Persian calendar if we note that until the Jalaali date m (month) and d (day)

$$31(m - 1) - (m - 7)\,\mathrm{INT}(m/7) + d$$

days have elapsed since the beginning of any Jalaali year.

Practical programs were written and tested for convertion of Jalaali calendar, first into the Julian Day number (JD), and then into the Gregorian or Julian calendar, and also for reverse conversion of these calendars to the Persian one. The following procedures in FORTRAN serving this purpose are available from the author (see this complete coversion program listing and the DOS executable):

- `function Jal2JD(Jy,m,d)` — converts the Jalaali calendar date to the Julian Day number at the mean Greenwich noon (12:00 UT1),
- `function JG2JD(JGy,m,d,1/0)` — converts the Julian/Gregorian date to the Julian Day number,
- `subroutine JD2Jal(JDN,Jy,m,d)` — converts the Julian Day number to the Jalaali calendar date,
- `subroutine JD2JG(JDN,JGy,m,d,1/0)` — converts the Julian Day number to the Julian/Gregorian date.

4. Error discussion

Due to approximate nature of the ephemeris the results of the equinox time may be in error of about 1 minute of time. Uncertainty of ΔT adds to the error of the final result. This parameter is very well known from direct observations only back to about AD 1630.

Table IV: Critical years in the Jalaali calendar most likely to alter the sequence of breaking years. The first four columns give Gregorian date in March, the Teheran mean time (in hours and minutes) of the vernal equinox, and the value of ΔT = TT – UT1. Then there is the day in March of the first day of the Jalaali year (Farvardin the 1st) and the year itself. The last column shows possible additions (+) to and subtractions (–) from the breaking years list of Table II.

Year AD	Day of March	Teheran time	ΔT [min]	1st Farv.	Jalaali year	Effect of actual	error
626	21	12:00.4	61.8	22	5	–9	–38
659	21	11:56.6	57.9	21	38	–38	+71
886	20	11:57.7	35.6	20	265	+269	+298
1113	21	11:58.5	20.0	21	492	+496	+525
1373	20	12:01.3	7.7	21	752	–756	+789
2124	20	12:00.3	3.9	21	1503	+1503	+1540
2322	21	11:58.0	11.0	21	1701	+1705	+1734
2681	20	11:59.6	32.4	20	2060	–2060	–2097
2780	20	12:04.8	40.2	21	2159	+2159	–2192
2813	20	11:56.3	43.0	20	2192	–2192	+2225
2846	20	11:59.1	45.9	20	2225	+2229	–2262
2879	20	12:03.5	48.9	21	2258	–2262	+2295
2912	20	12:03.3	52.0	21	2291	+2291	–2324
3011	21	12:00.9	61.7	22	2390	–2394	+2427
3044	20	12:02.2	65.2	21	2423	+2423	–2456
3176	20	11:56.7	79.9	20	2555	+2559	+2588
3209	20	11:59.9	83.8	20	2588	+2592	+2621
3370	20	12:11.0	104.2	21	2749	+2749	+2786
3473	20	11:48.2	118.4	20	2852	+2856	+2885
3502	21	12:10.4	122.5	22	2881	+2881	+2918
3634	20	12:04.5	142.4	21	3013	+3013	+3050
3667	20	12:03.8	147.6	21	3046	+3046	+3083

The past breaking years, as being purely astronomical and based on contemporary knowledge, may not coincide with the actual Jalaali calendar, used then for astronomical purposes only. Also, the future breaking years of Table II may disappear altogether or new ones will have to be introduced depending on real rotation of the Earth (the behaviour of the ΔT value). The most critical years are listed in Table IV, where the date in March (Gregorian calendar) and the predicted Teheran mean time are given along with the assumed ΔT value. The table contains all the cases that the equinox time is removed from 12:00 Teheran time by less than $(1 + \Delta T/10)$ minutes, where the 1 minute is due to uncertainty of the equinox determination and the other term is intended to account for 10% error in ΔT (an ad hoc error estimate). The Jalaali years indicated in Table IV are leap years if the equinox time is less than 12:00, otherwise the previous year is leap. The expected change to be introduced to the list of breaking years in case the actual equinox time shifts across the noon point (12:00) due to combined effect of the ephemeris error and a change in ΔT, is shown in the last column of this table.

It will be noted, that the discussed change affects only the year in question (shifting it by one day relative to e.g. Gregorian calendar) and the last day of the previous Jalaali year. Thus the algorithm in the form presented may lose its validity only for some of the future Jalaali years given in Table IV. In the worst case, if my estimates of ΔT are systematically low, the 10 critical years with equinox greater than 12:00 of that table may drop to less then 12:00 and thus the corresponding Jalaali years would shift by one day into the previous year, themeselves becoming the leap years. Should the ΔT be systematically high instead, there would be only 7 future years of Table IV endangered by an opposite shift.

2124 is the nearest doubtful year. The Teheran times of the vernal equinox indicate that in this year the 1503 Jalaali year starts on 21 March and not on 20 March just because the calculated equinox time was only about one third of a minute past 12:00 (see Table IV). Thus if ΔT will be greater by some 20 seconds than predicted in this analysis, this year will become an additional breaking point to be added to the presented algorithm along with another one 37 years later (1540 Jalaali).

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