PERIOD DOUBLING AND CHAOTIC BEHAVIOR IN RLD CIRCUITS

Ebram Youssef

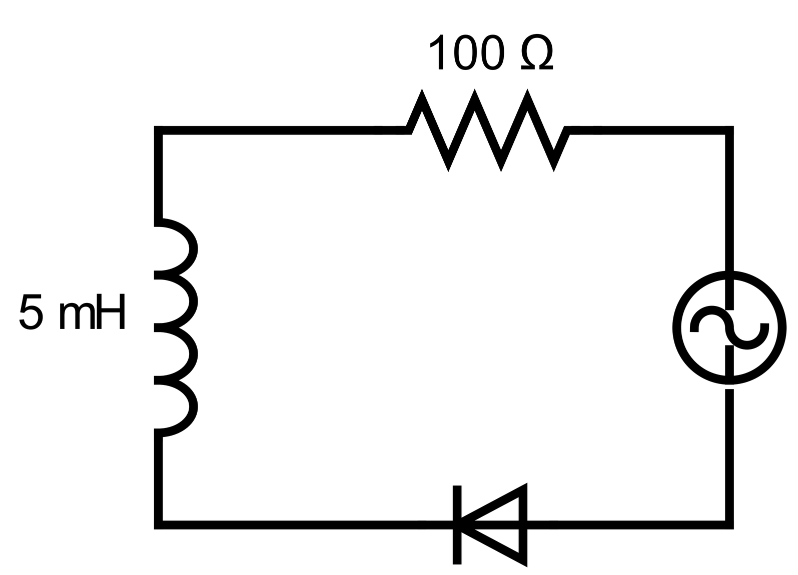
INTRODUCTION

Chaos is the non-periodic behavior of nonlinear dynamic systems that is highly sensitive to initial conditions. A simple RLD circuit is used to study period doubling and eventual evolution to chaos in diodes. A diode has two phases: a conducting (forward-bias) phase where it acts like a negative emf source with voltage equal to its forward-bias voltage, and a non-conducting (reverse-bias) phase where it acts like a conductor with capacitance C and a charging time when driven by a periodic source. Due to the nonlinearity of diodes, they could be made to act chaotically and exhibit bifurcations.

EXPERIMENT

– SETUP

The circuit used consists of a 100 Ω resistor, a 5 mH (Murata G2505C) inductor, a 1N4007 diode, and a signal generator all connected in series. The sinusoidal signal from the generator, as well as the voltage measured across the resistor, are shown on a computer through a Picoscope.

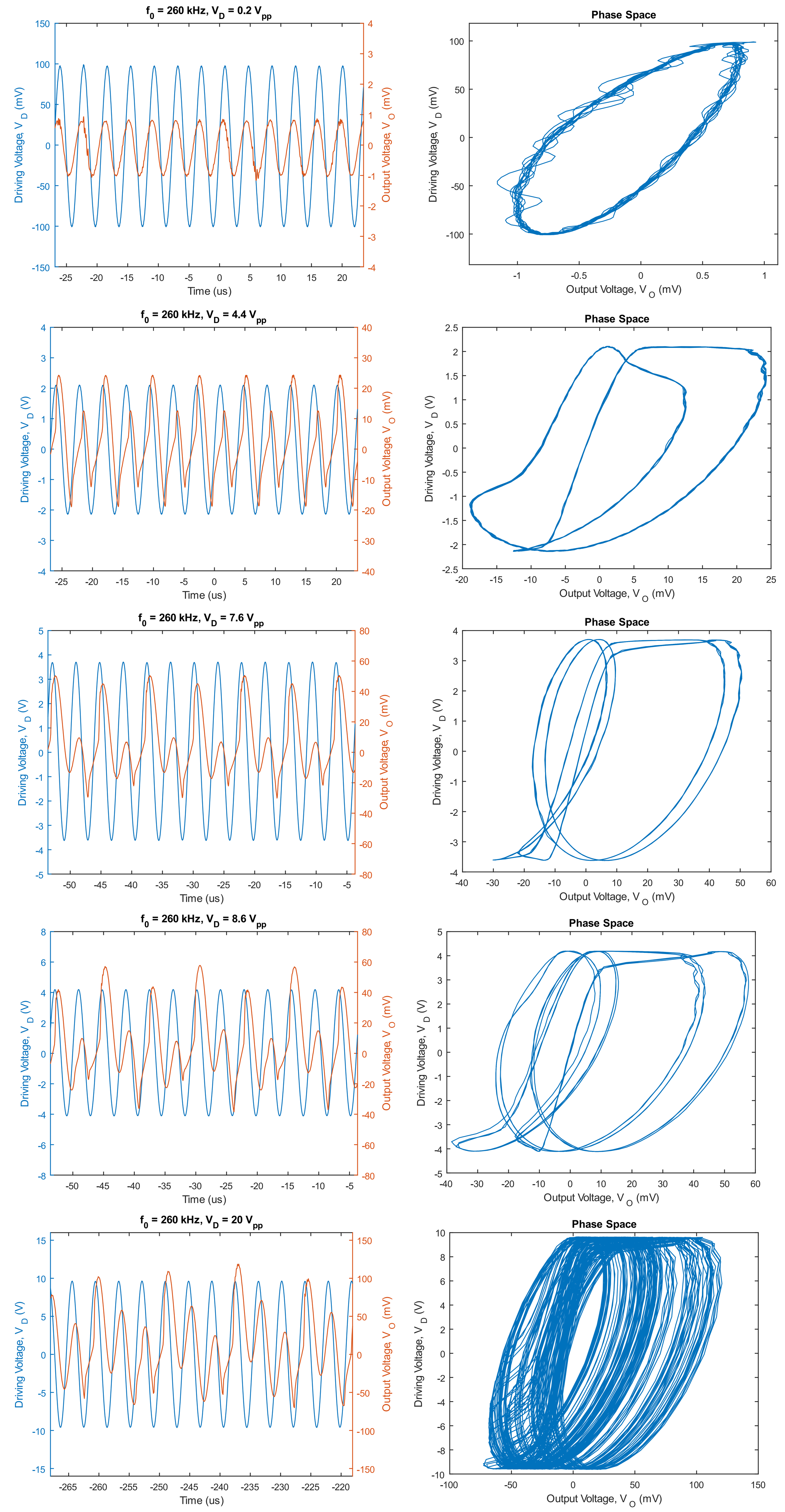


– PROCEDURE

- First, the resonance frequency f_0 of the circuit is found. That is, the signal frequency that corresponds to the maximum amplitude of the output voltage for a given driving voltage. The initial driving voltage used here is 0.2 V_{pp}.
- After finding the resonant frequency of the circuit at 0.2 V_{pp}, the driving voltage is increased in increments of 0.2 V_{pp} up to 20 V_{pp}. At each value, the system's response is recorded and saved.
- A numerical algorithm is developed to find the values of distinct peaks in the output voltage for each driving voltage. These values are then used to plot a bifurcation diagram and calculate the Feigenbaum constants of the system.

RESULTS

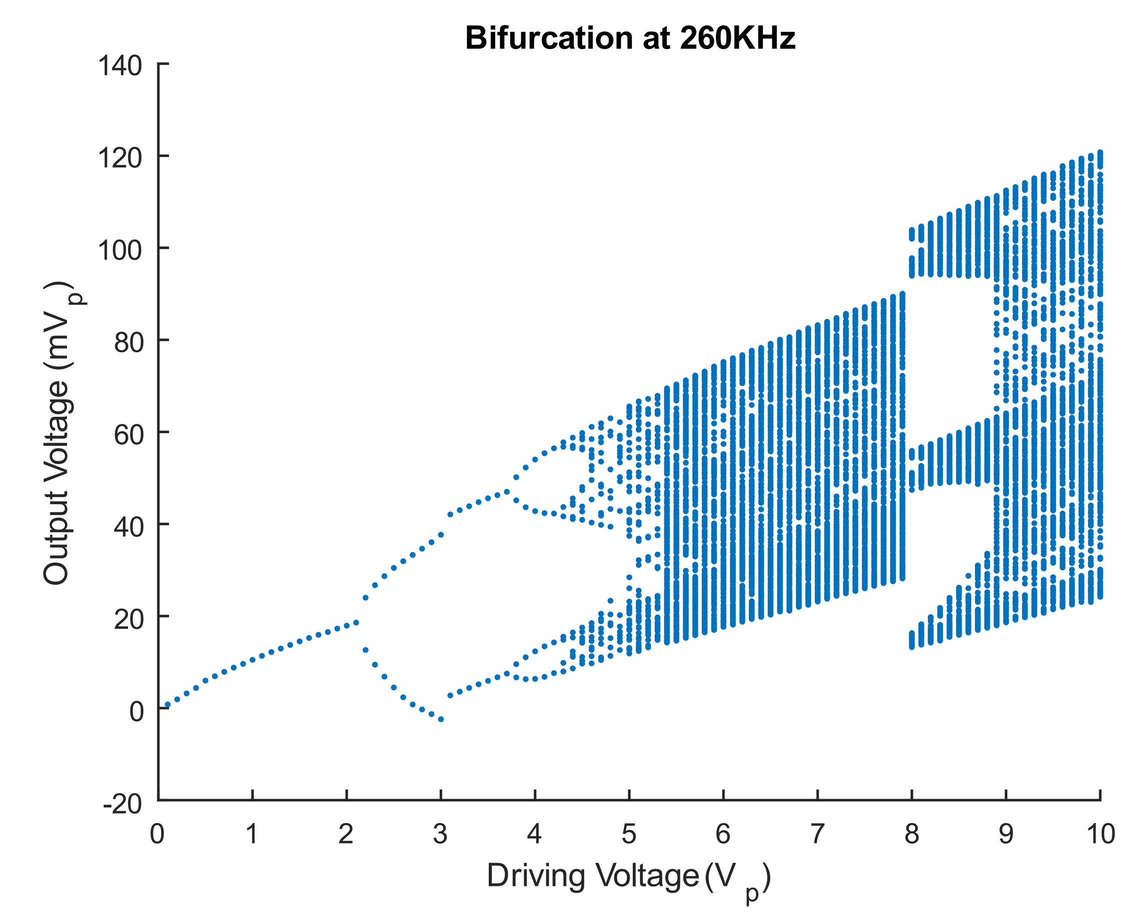
- The resonance frequency of the circuit was found to be $f_0 = 260$ **kHz**, at which the peak-to-peak output voltage was maximum ($V_o = 2.236 \pm 0.121\ mV_{pp}$). The Q factor of the circuit at this resonant frequency is $Q = 81.68$. Below are the system's responses.



ANALYSIS

– BIFURCATION AND CHAOS

- The first bifurcation happens at $4.4\ V_{pp}$ $(2.2\ V_p)$, as evident by the 2 stable periods in the output voltage and the phase space diagram that crosses itself once.
- The second bifurcation happens at $7.6\ V_{pp}$, while the third happens at $8.6\ V_{pp}$. Chaos starts at around $9.6\ V_{pp}$.
- A bifurcation diagram was produced using computational techniques. The diagram confirms the bifurcation and chaos values found from the plots, and it shows other interesting phenomena.
- Two relatively large periodic windows (islands of stability) could be found between $15.8\ V_{pp}$ and $17.8\ V_{pp}$, as typical in chaotic systems.
- A discontinuous jump could be seen at $6.2\ V_{pp}$ $(3.1\ V_p)$ due to sudden behavioral change in the system that isn't caused by period doubling. This effect couldn't be explained but was found in other studies [3].
- Another large discontinuity occurs when the islands of stability start forming ($15.8\ V_{pp}$).



– FEIGENBAUM'S CONSTANTS

- Feigenbaum's constants are universal theoretical constants that apply to all nonlinear systems that exhibit multistable solutions and chaos.
- The first constant, δ, is defined as

$$\delta = \frac{V_{D,n+1} - V_{D,n}}{V_{D,n+2} - V_{D,n+1}} \rightarrow \delta_{n=\infty} = 4.6692 \ldots$$

Where $V_{D,n}$ is the voltage of one bifurcation. It was calculated as follows:

$$\delta_1 = \frac{V_2 - V_1}{V_3 - V_2} = \frac{3.7 - 2.1}{4.2 - 3.7} = 3.2 \pm 0.3\,,$$

$$\delta_2 = \frac{V_3 - V_2}{V_4 - V_3} = \frac{4.2 - 3.7}{3.8 - 3.7} = 5 \pm 0.3$$

$$\rightarrow \bar{\delta} = 4.10 \pm 0.95$$

Which agrees with the theoretical values within uncertainty.

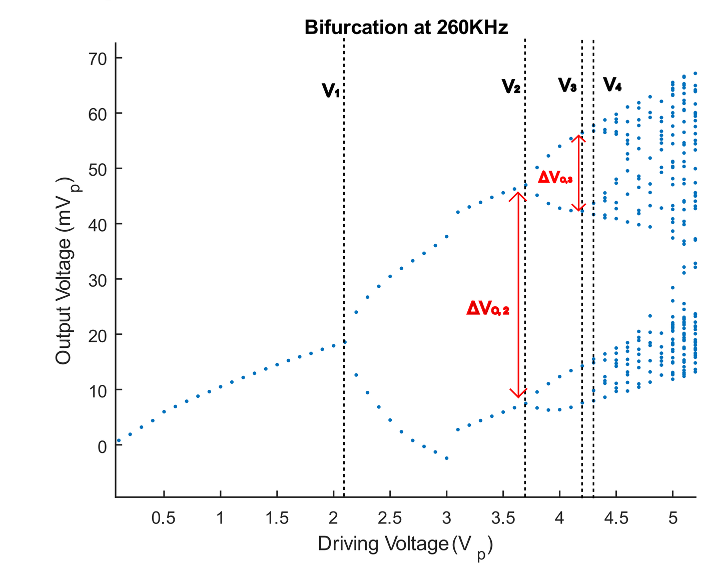
- The second constant, α, is defined as

$$\alpha = \frac{\Delta V_{o,n}}{\Delta V_{o,n+1}} \rightarrow \alpha_{n=\infty} = 2.5029 \ldots$$

where $\Delta V_{o,n}$ is the difference between output voltages for one bifurcation (starting at the second bifurcation). It was calculated as follows:

$$\alpha = \frac{\Delta V_{o,2}}{\Delta V_{o,3}} = \frac{46.99 - 7.49}{56.44 - 42.27} = 2.79 \pm 0.30$$

Which agrees with the theoretical values within uncertainty.



– RECOMMENDATIONS

- For future studies, it's recommended to run the computational analyses on stronger computers in order to process more data points in a time efficient manner, which is important especially after chaos is reached. It's also important to increase the discretization (number of steps) of the driving voltage in order to increase the resolution of the bifurcation diagram and produce more accurate results.
- Other visuals, such as logistic maps, Lyapunov exponent plots, and output voltage in spectral space, could be produced for better understanding of the system.
- Some behaviors need to be better explained, such as the discontinuities in the bifurcation diagram.

REFERENCES AND ACKNOWLEDGEMENTS

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UNIVERSITY OF ROCHESTER
Department of Physics and Astronomy