1 Stuff relevant for us for now

The actuator disk simulation takes as input the loading on each blade and smears it out onto the disk. The following loading is prescribed on each blade:

$$\frac{\mathrm{d}T_B(r)}{\mathrm{d}r} = \frac{1}{B}\rho U_0^2 \pi r\, C_t(r) \quad [\mathrm{N/m}] \tag{1}$$

where B is the number of blades (3), and $C_t(r)$ is the local thrust coefficient. The total thrust force on the actuator disk is:

$$T = \int_0^R B\mathrm{d}T_B = \frac{1}{2}\rho U_0^2 2\pi \int_0^r rC_t(r)\mathrm{d}r \tag{2}$$

The total thrust coefficient is:

$$C_T \triangleq \frac{T}{\frac{1}{2}\rho\pi R^2 U_0^2} = \frac{2}{R^2}\int_0^R rC_t(r)\mathrm{d}r = 2\int_0^2 \overline{r}C_t(\overline{r})\mathrm{d}\overline{r} \tag{3}$$

2 Stuff from my book p117

The local thrust force $\mathrm{d}T_B$ and torque $\mathrm{d}Q_B$ from the section r of the blade B is defined as:

$$\mathrm{d}T_B(r) \triangleq \underline{dF}_B(r)\cdot \underline{e}_z\, \mathrm{d}r \tag{4}$$

$$\mathrm{d}Q_B(r) \triangleq [r\underline{e}_r \times \underline{dF}_B(r)\mathrm{d}r]\cdot \underline{e}_z \tag{5}$$

The term *local* is here used to highlight the fact that these quantities are defined for a given radius, as opposed to integrated quantities. The term *elementary* is also used in this book. The local thrust force $\mathrm{d}T(r)$ and torque $\mathrm{d}Q(r)$ on the rotor are obtained by summing the contribution from each blade, i.e. $\mathrm{d}T(r) = \sum_B \mathrm{d}T_B(r)$ and $\mathrm{d}Q(r) = \sum_B \mathrm{d}Q_B(r)$. The elementary power is defined as $\mathrm{d}P = \Omega\,\mathrm{d}Q$. The dimensionless local thrust, torque and power coefficients are defined using the reference speed U_{ref} and the area of the elementary annulus $2\pi r\mathrm{d}r$ as follows:

$$C_t(r) \triangleq \frac{\mathrm{d}T(r)}{\frac{1}{2}\rho U_{\mathrm{ref}}^2 2\pi r\mathrm{d}r}, \quad C_q(r) \triangleq \frac{\mathrm{d}Q(r)}{\frac{1}{2}\rho U_{\mathrm{ref}}^2 r 2\pi r\mathrm{d}r}, \quad C_p(r) \triangleq \frac{\mathrm{d}P(r)}{\frac{1}{2}\rho U_{\mathrm{ref}}^3 2\pi r\mathrm{d}r} \tag{6}$$

The total thrust, torque and power are obtained by integration of the elementary quantities along the span of the blade:

$$T \triangleq \int_{r_{\text{hub}}}^{R} \mathrm{d}T, \quad Q \triangleq \int_{r_{\text{hub}}}^{R} \mathrm{d}Q, \quad P \triangleq \int_{r_{\text{hub}}}^{R} \mathrm{d}P \tag{7}$$

The dimensionless coefficients associated to these integrated quantities are:

$$C_T \triangleq \frac{T}{\frac{1}{2}\rho A_{\text{ref}} U_{\text{ref}}^2}, \quad C_Q \triangleq \frac{Q}{\frac{1}{2}\rho A_{\text{ref}} U_{\text{ref}}^2 R}, \quad C_P \triangleq \frac{P}{\frac{1}{2}\rho A_{\text{ref}} U_{\text{ref}}^3} \tag{8}$$

where A_{ref} is commonly chosen as $A_{\text{ref}} = \pi R^2$ for wind turbines. The swept area $\pi(R^2 - r_{\text{hub}}^2)$ is a valid choice but not recommended since it is not as widely used. The above may be re-written as function of the local coefficients as follows:

$$C_T \triangleq \frac{2\pi}{A_{\text{ref}}} \int_{r_{\text{hub}}}^{R} r C_t(r) \mathrm{d}r, \quad C_Q \triangleq \frac{2\pi}{A_{\text{ref}} R} \int_{r_{\text{hub}}}^{R} r^2 C_q(r) \mathrm{d}r, \quad C_P \triangleq \frac{2\pi}{A_{\text{ref}}} \int_{r_{\text{hub}}}^{R} r C_p(r) \mathrm{d}r \tag{9}$$

It is noted that by definition the following relations always holds:

$$C_p(r) \equiv \lambda_r C_q(r) \tag{10}$$

$$C_P \equiv \lambda C_Q \tag{11}$$