Manifold Learning and Geometric Harmonics for fMRI Data Analysis

Shannon Hughes and Eugene Brevdo

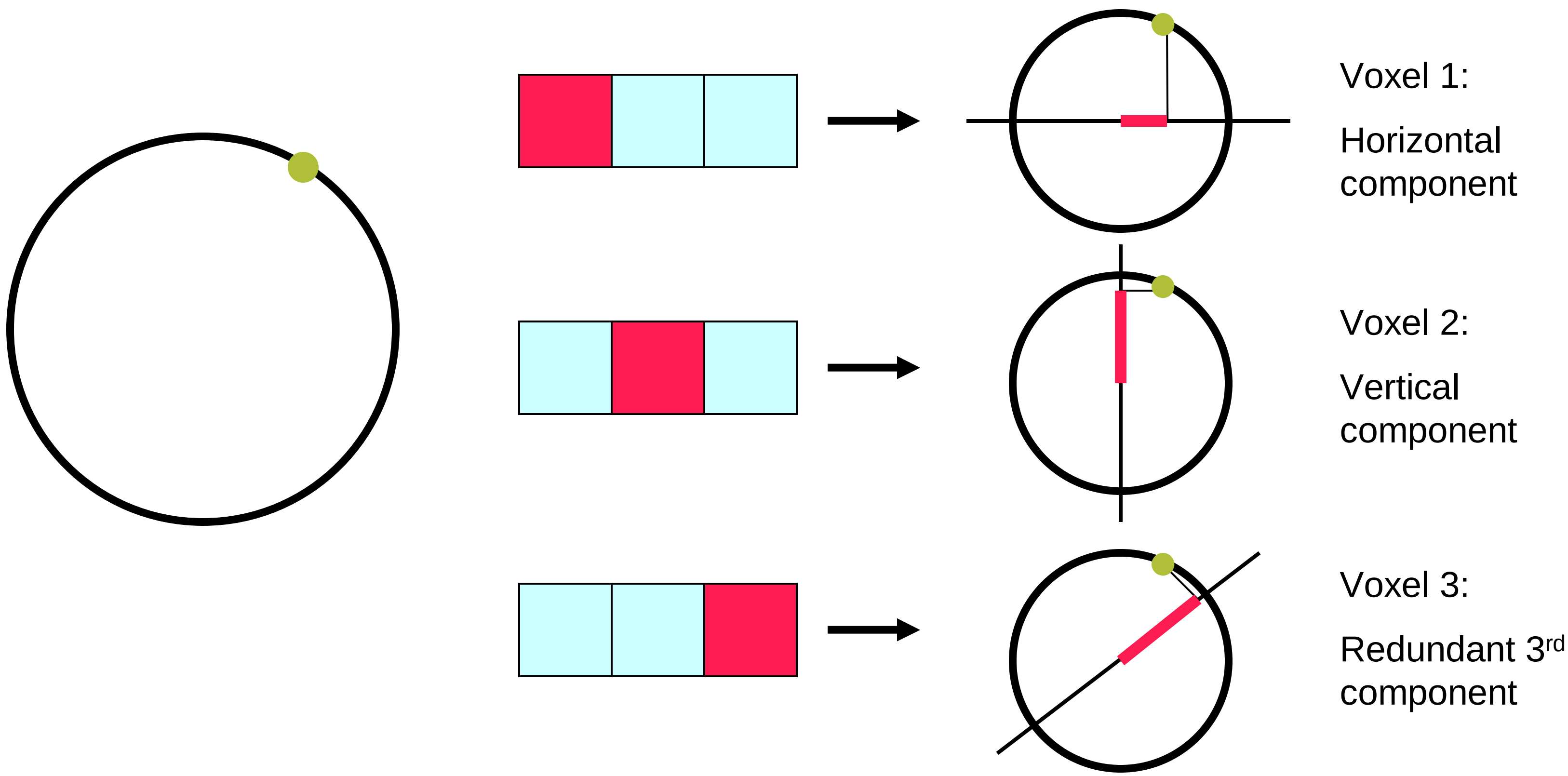
Overview

- We treat activation maps associated with particular stimuli as points in high-dimensional space.
- We hypothesis that patterns associated with simply-varying stimuli will lie on low-dimensional manifolds in this high-dimensional space.
- We then
  - Look for such low-dimensional representations of our patterns
  - Try to interpolate in order to predict stimuli associated with new unseen patterns
  - Try to reconstruct what the original low-dimensional manifold looked like in high-dimensional space.

A Simple Manifold Example

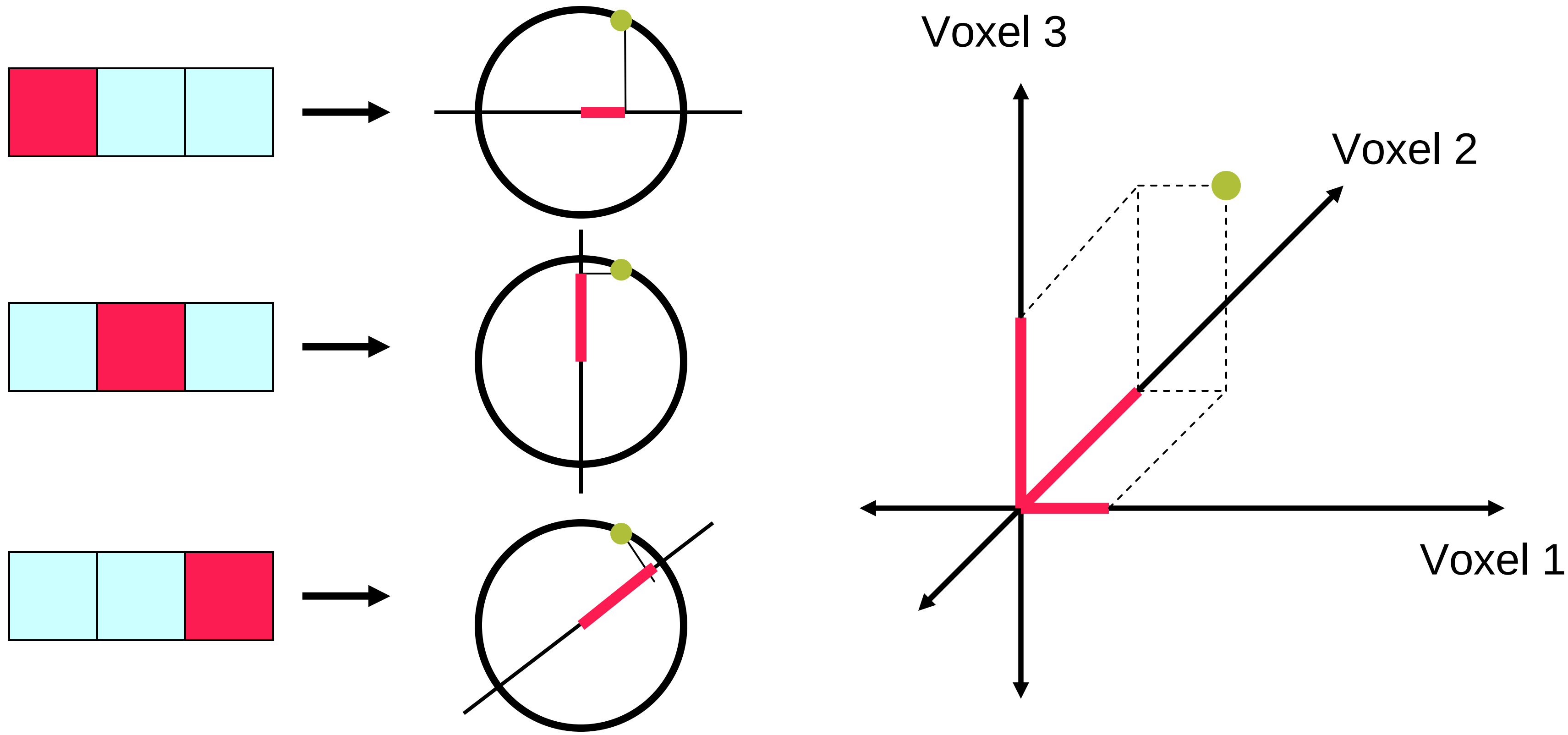
The stimulus: an angle

A three-voxel brain and its response to the stimulus



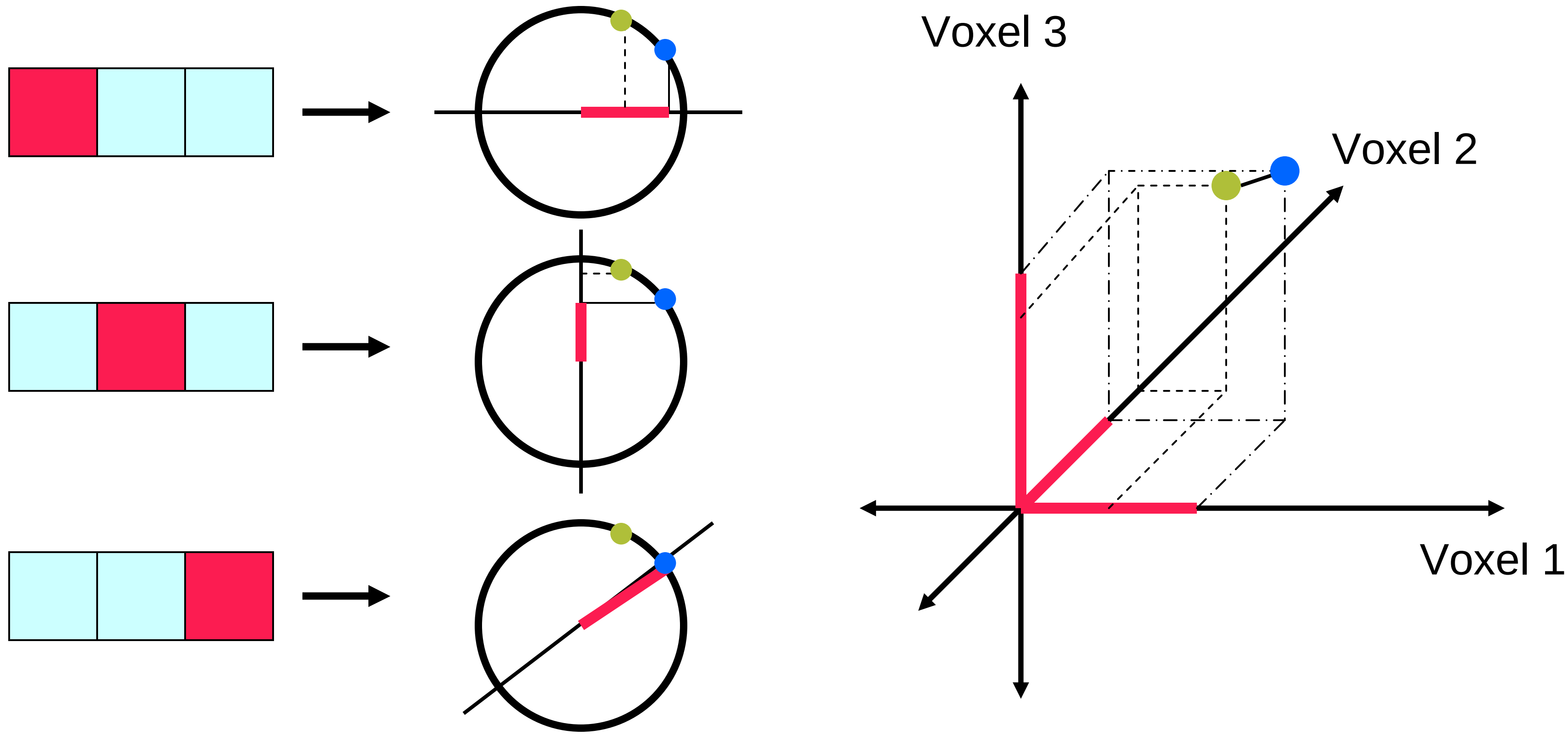
A Simple Manifold Example (cont.)

- Treat 3-voxel response pattern as a point in 3-dimensional space:

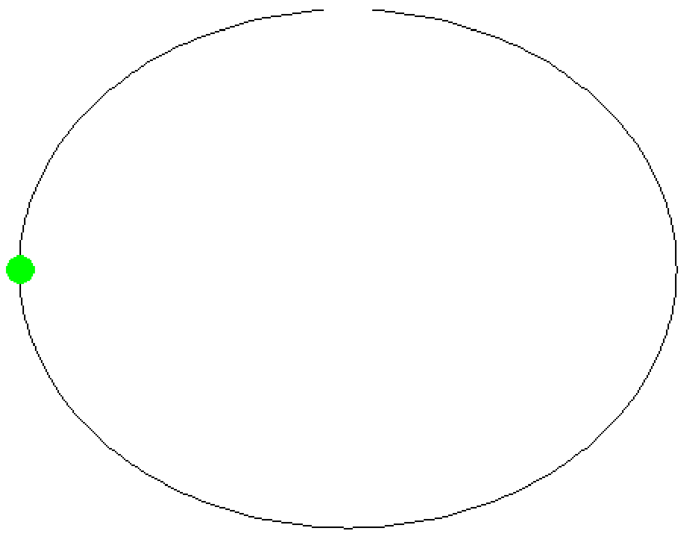


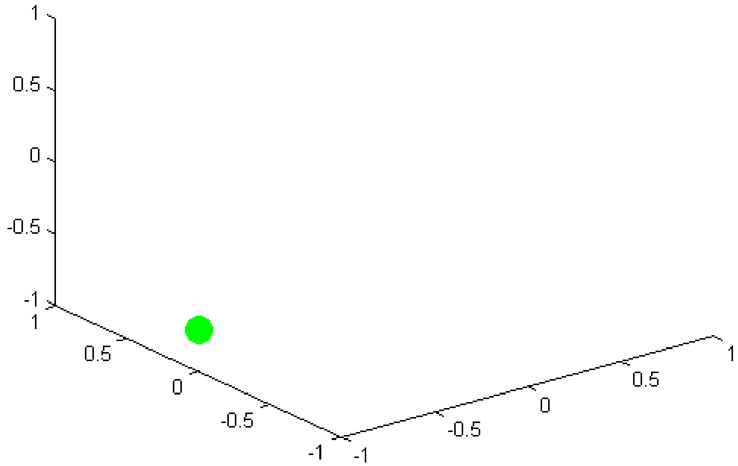
A Simple Manifold Example (cont.)

- What happens when we change our stimulus slightly?

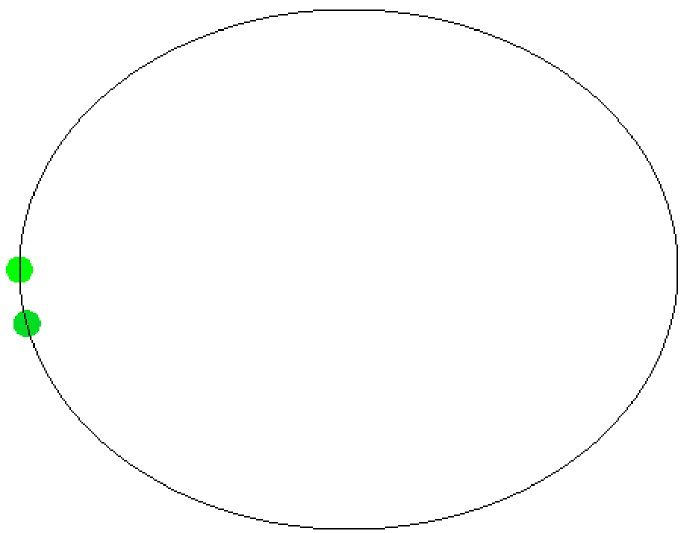


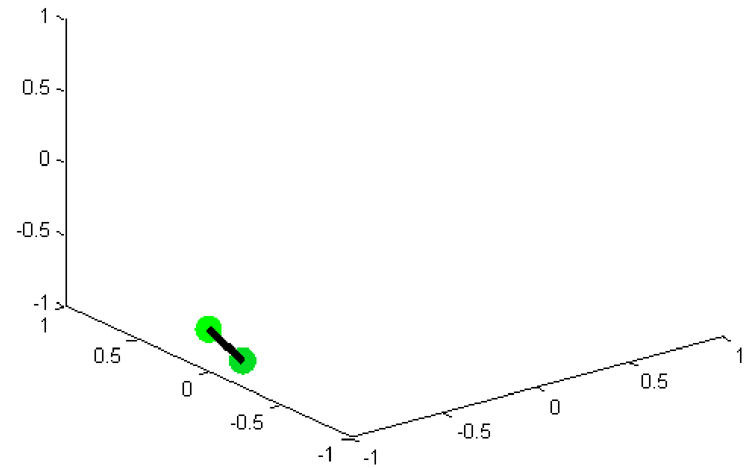
A Simple Manifold Example (cont.)



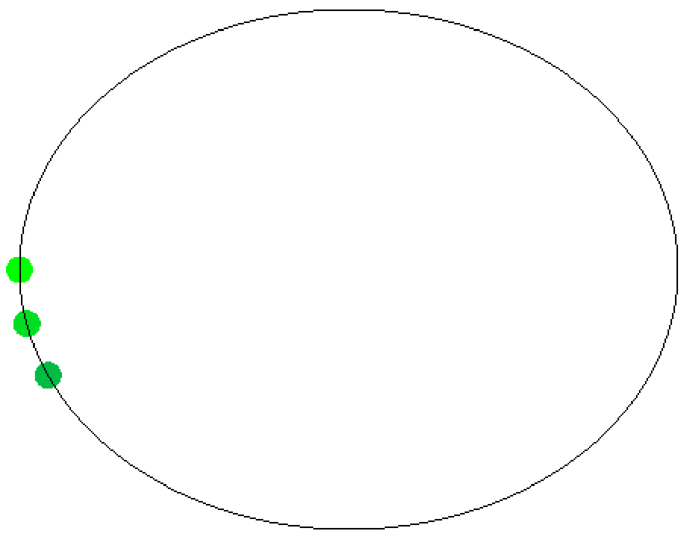


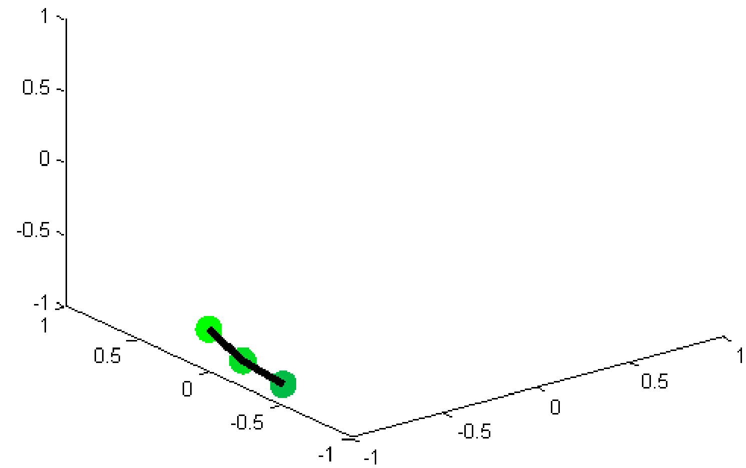
A Simple Manifold Example (cont.)



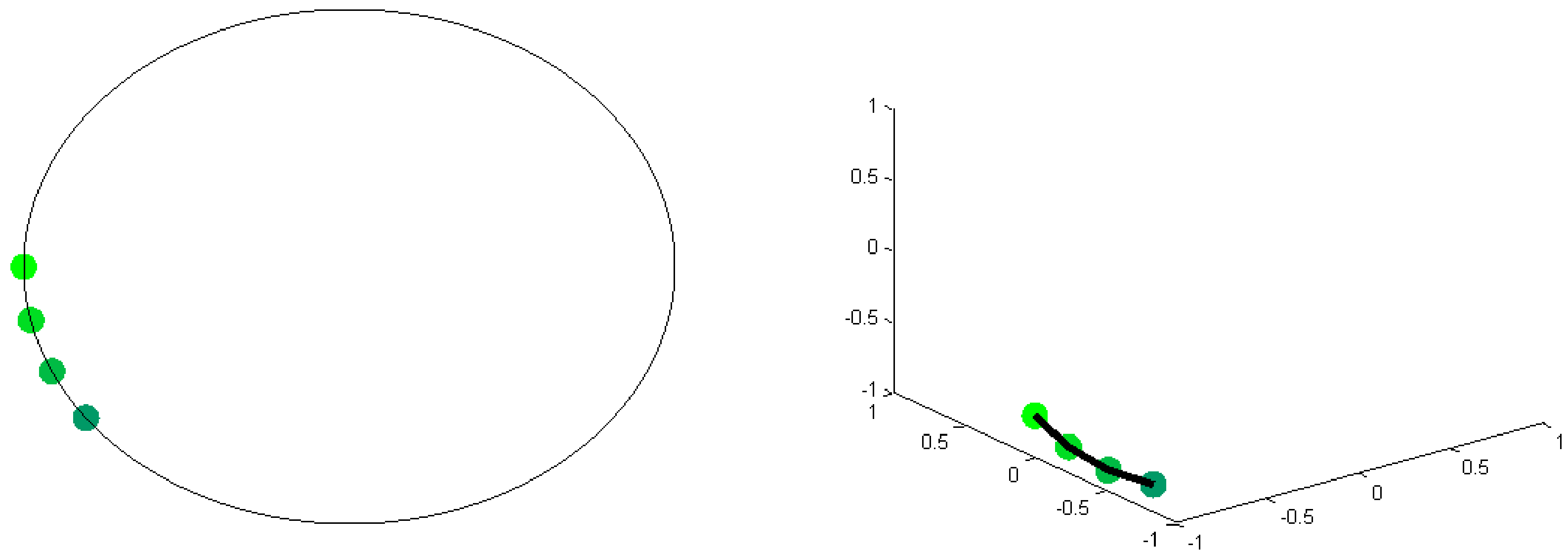


A Simple Manifold Example (cont.)

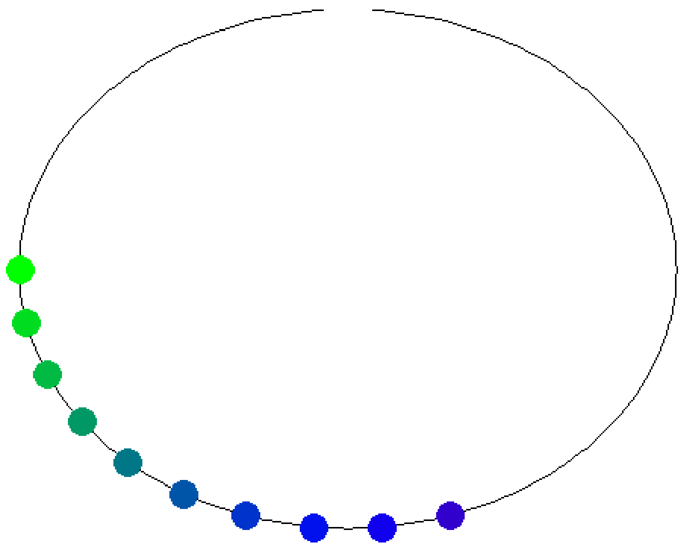


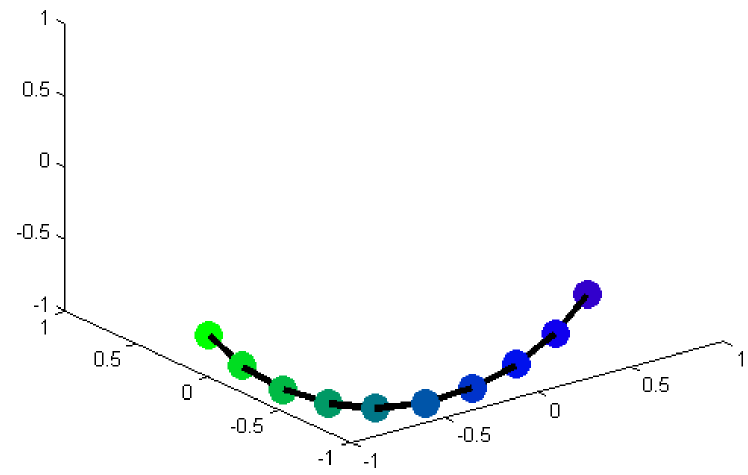


A Simple Manifold Example (cont.)

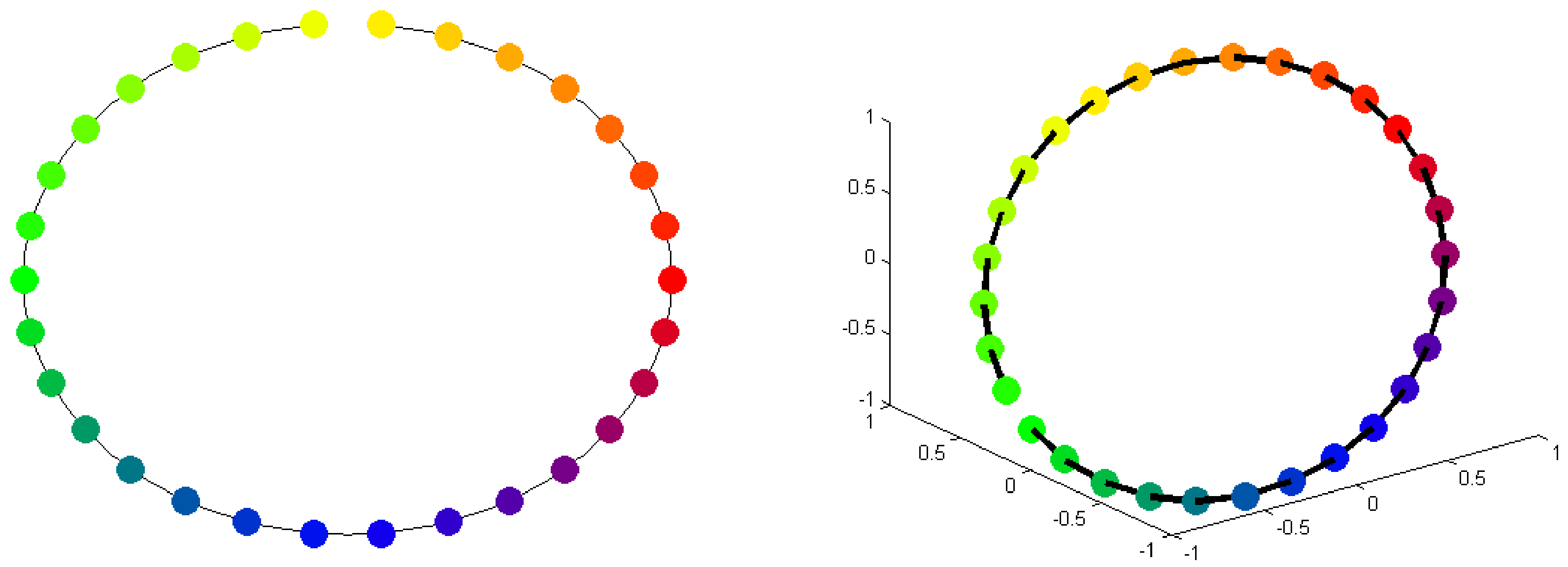


A Simple Manifold Example (cont.)

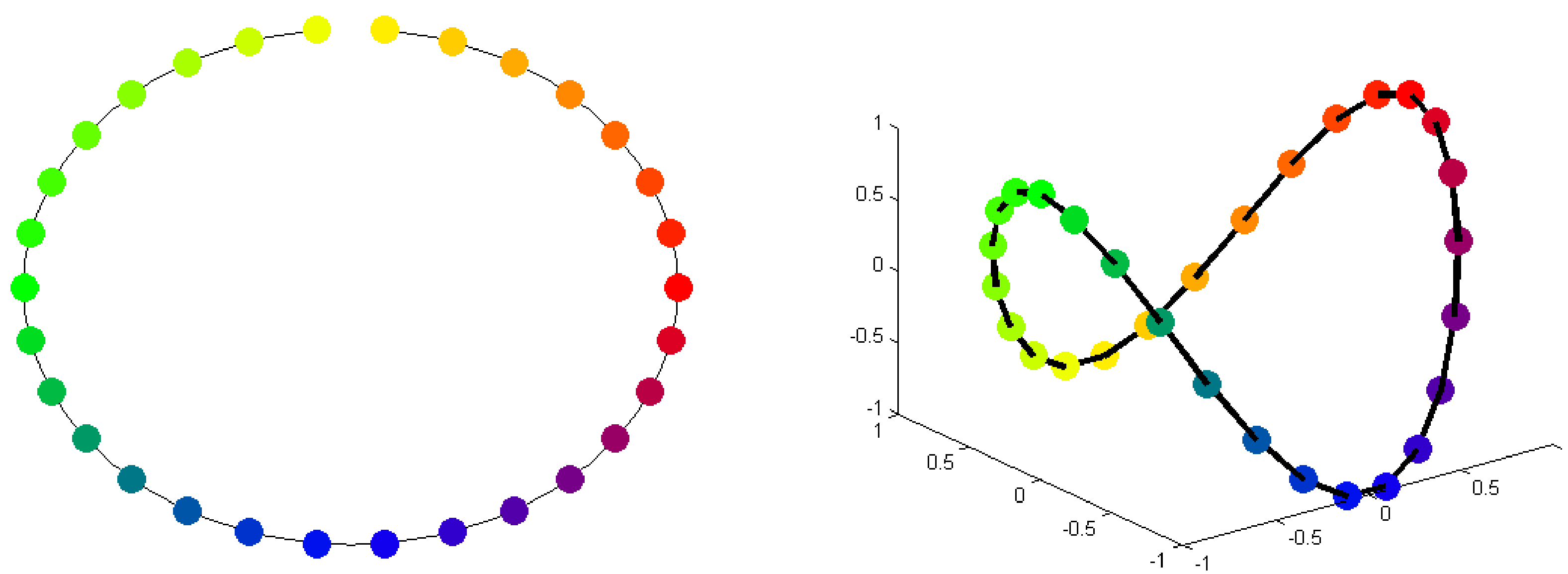




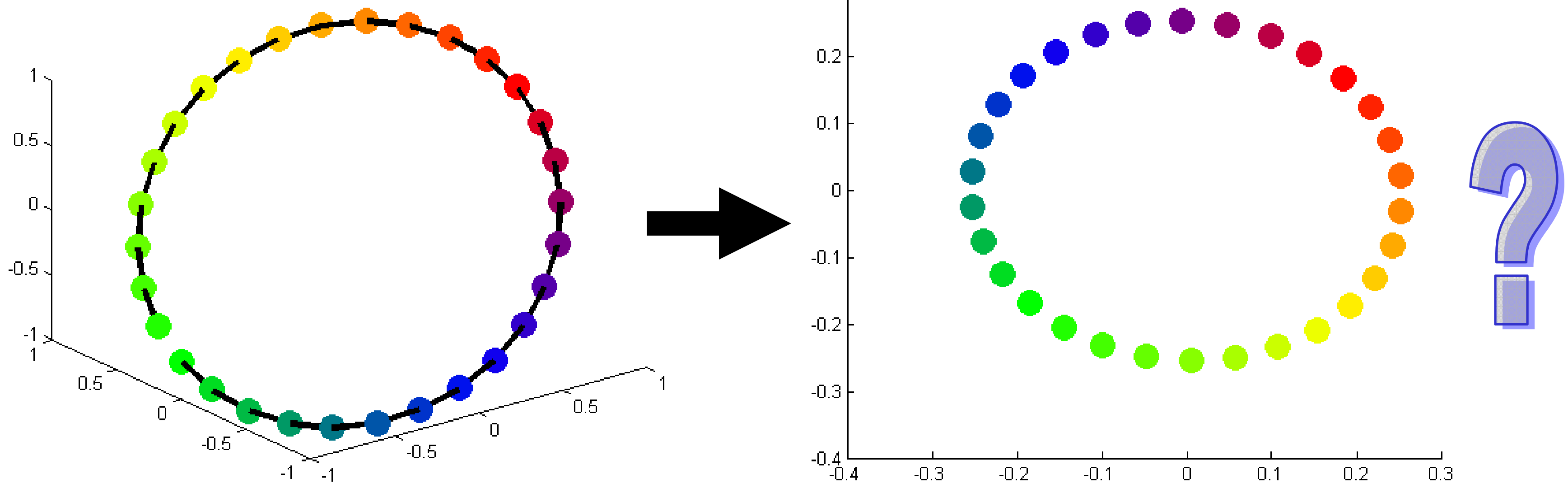
A Simple Manifold Example (cont.)



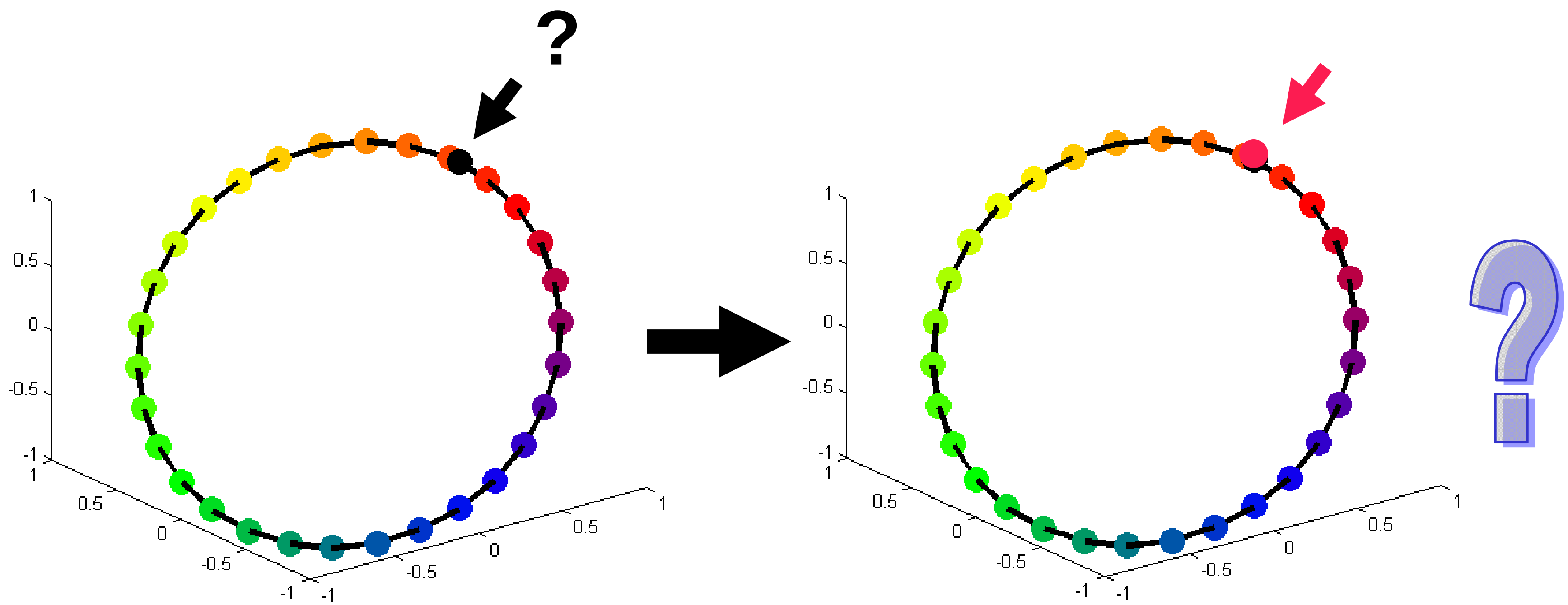
The manifold doesn’t have to be flat:



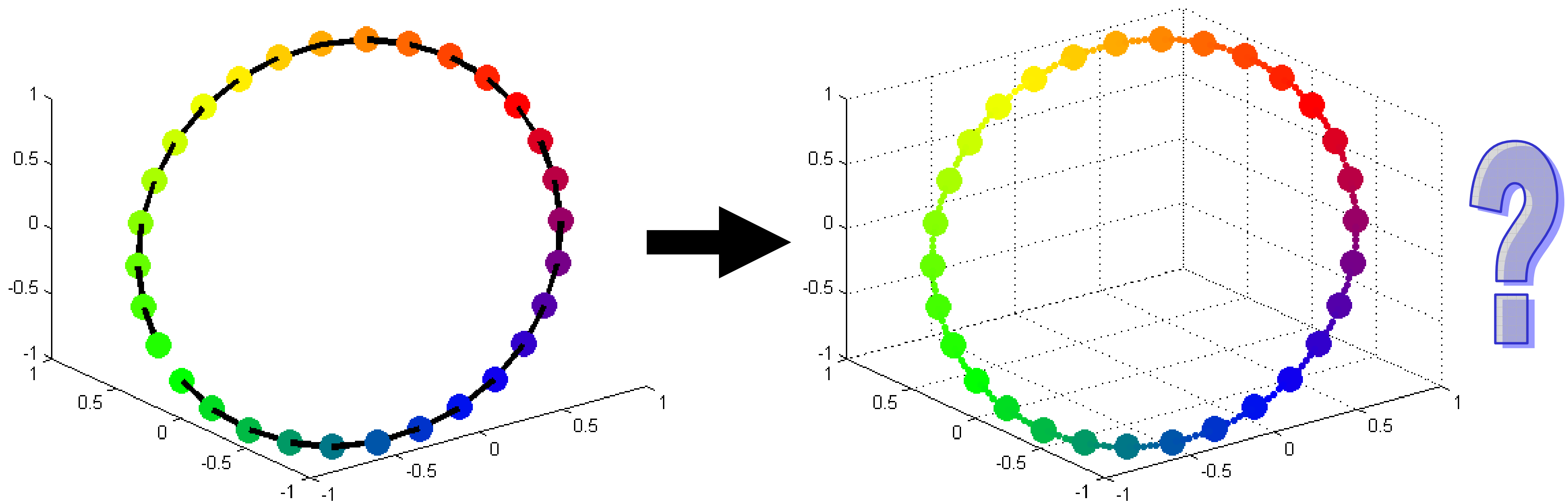
Problem 1: Can we learn an accurate low-dimensional representation of our data?



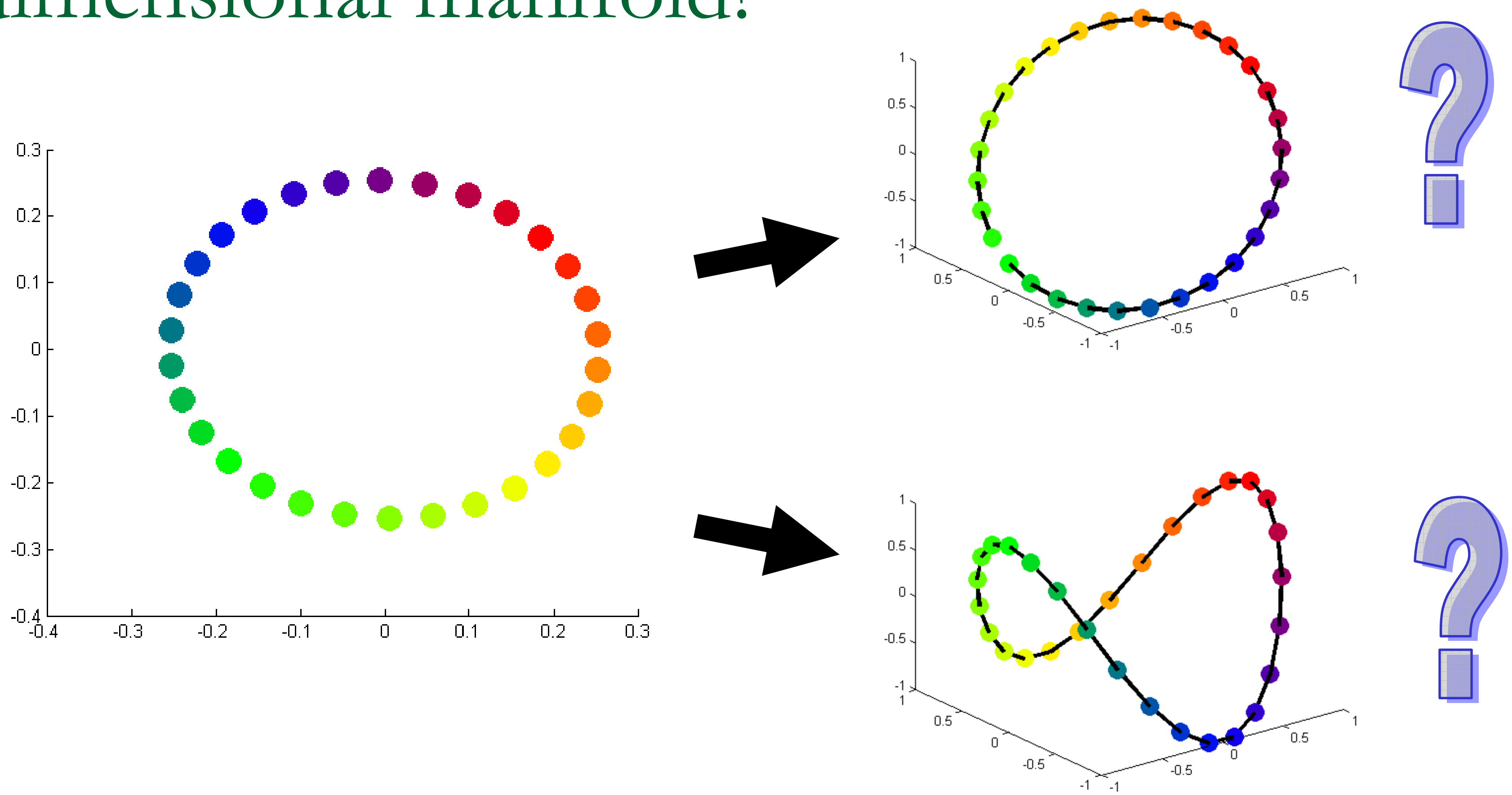
Problem 2: Given new activation maps, can we predict the associated stimulus?



Or more generally….



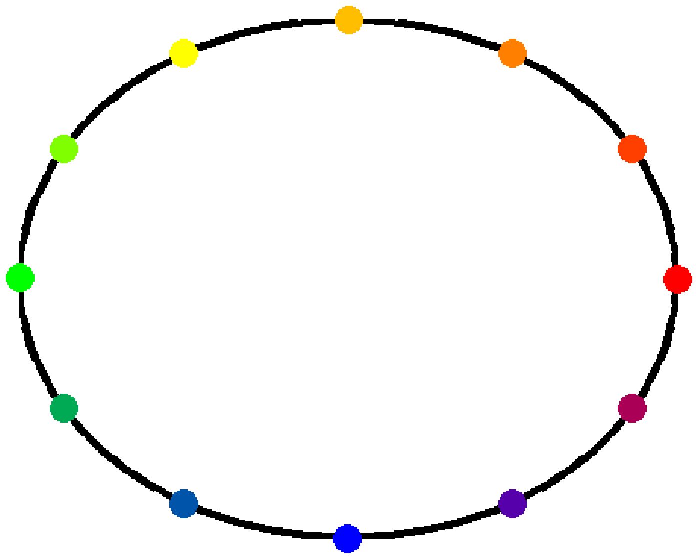
Problem 3: Given a low-dimensional representation, can we recover the high-dimensional manifold?



Outline of What We Will Cover Today

- Problem 1: Review of previous seqsac results and attempts to validate them on randsac data.
- Problem 2: A new method, geometric harmonics, with applications including EBC
- Problem 3: Some first attempts at using geometric harmonics to solve the problem.

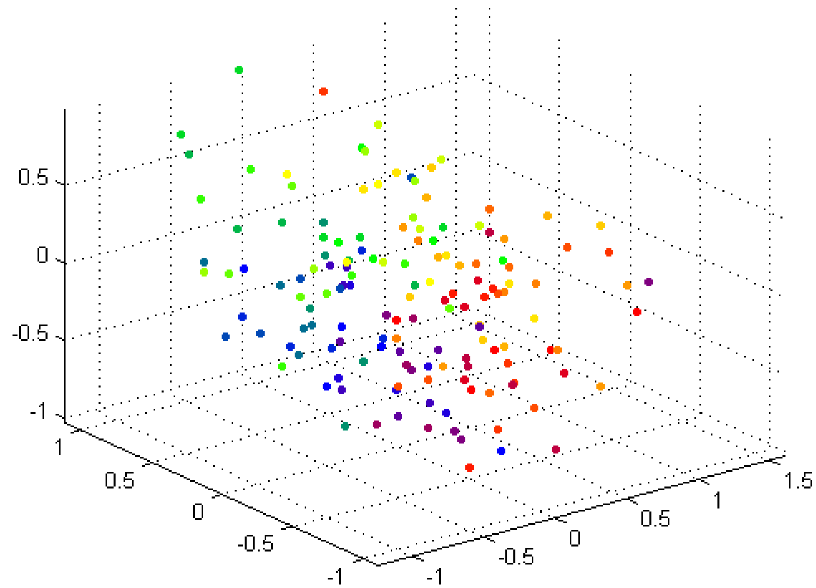
Review of Sequential Saccades Experiment



- Subjects are shown a dot at one of 12 clock positions.
- They are asked to remember the location of the dot after it disappears and to look in that direction.
  - 5 secs = 2.5 TRs at each position sequentially (no rest between)
  - 30 TRs/cycle
  - 8 cycles/run
  - 6 runs/subject

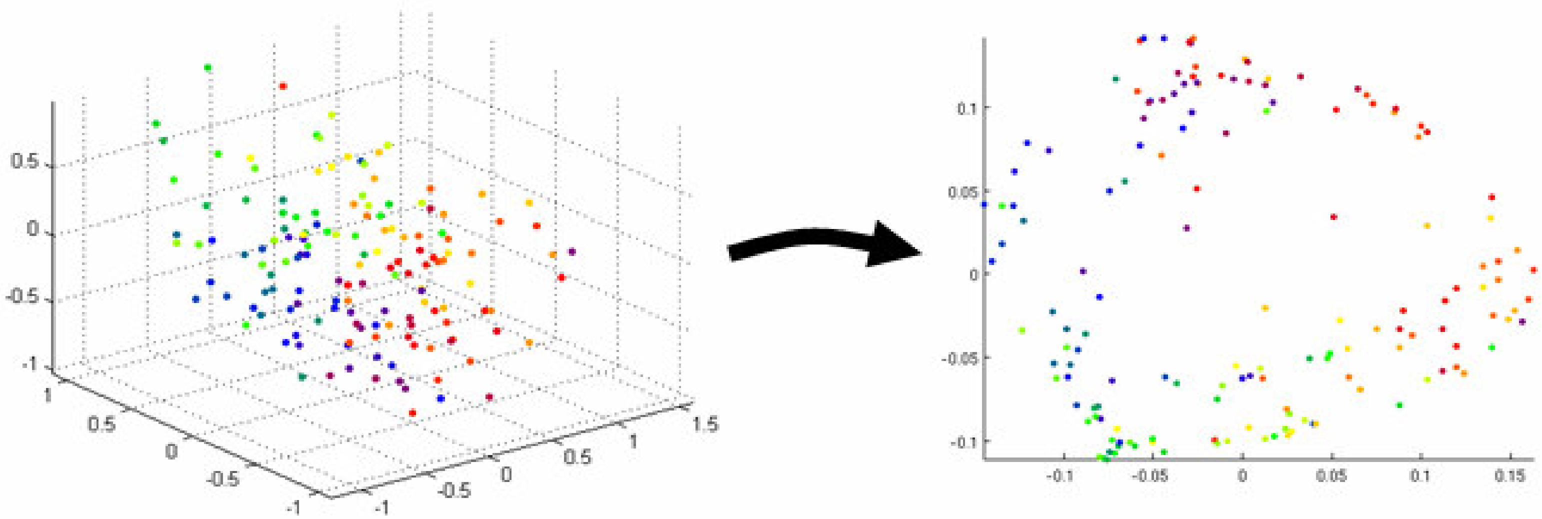
Our Original Sequential Saccades Results

- We look at the top n most significant voxels as judged by an ANOVA (top 3 shown below).
- Here we have averaged all TRs of the same condition within a run.



Our Original Sequential Saccades Results

- Result after using Laplacian eigenmaps with 5 nearest neighbors:



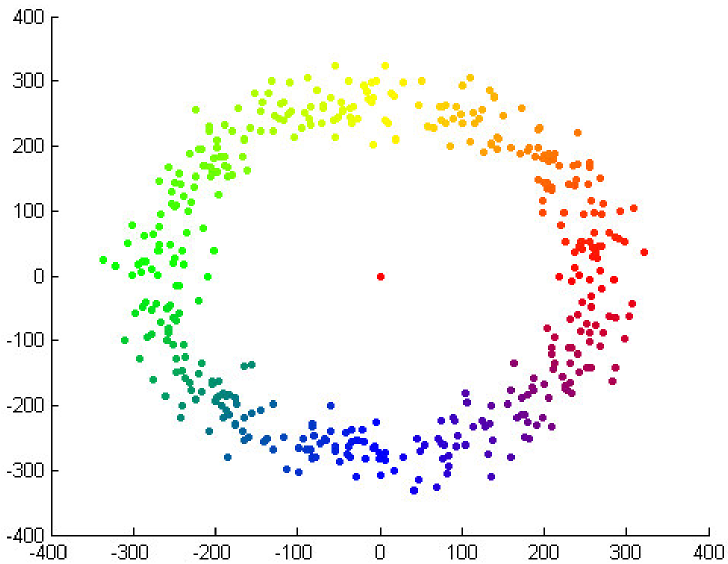
Before Manifold Learning

After Manifold Learning

Concerns about Original Seqsac Results

- At the NIAM meeting in which we originally presented these results, some wondered if we were getting too much “help” from the hemodynamic blur.
- To test this, we have attempted to validate our results on the fullrandsac experiment.

Randomized Saccades Experimental Setup

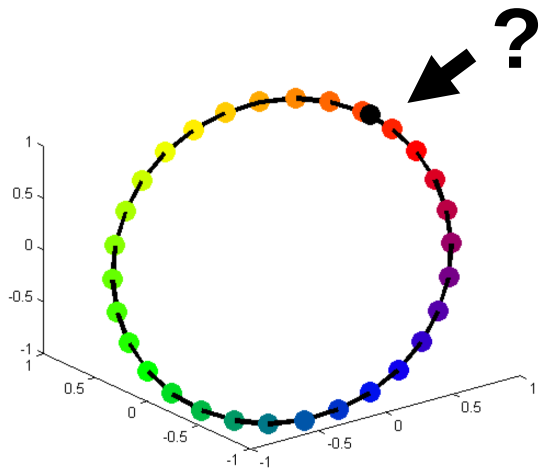


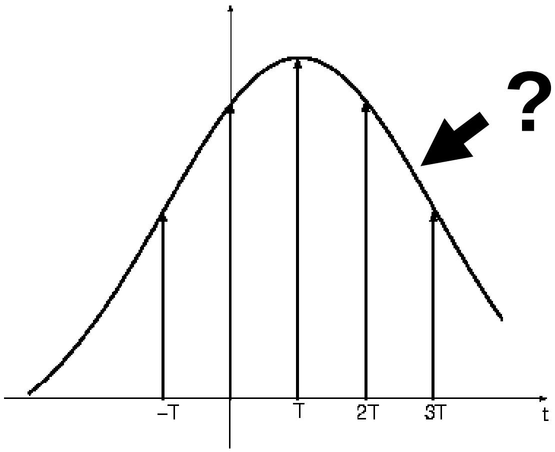
- Task similar to sequential saccades:
  - a subject is shown a dot at some spatial location and asked to look in the direction of the dot a few seconds after it disappears
- Key differences:
  - Dots may be anywhere on the unit circle; we are not restricted to only the 12 clock positions.
  - There is noise corrupting the distances of the dots from the origin.
  - Order is randomized.
- Still no rest between trials.

Results of Manifold Learning on Randomized Saccades

- Are not very good.
- We think this is because we are being unfairly disadvantaged by the hemodynamic blur.
- Things to try:
  - The semirandsac experiment data, in which there are breaks between trials.
  - Telling the algorithm that we don’t wish to preserve local distances between patterns if those patterns are also close together in time.

The Question of Interpolation





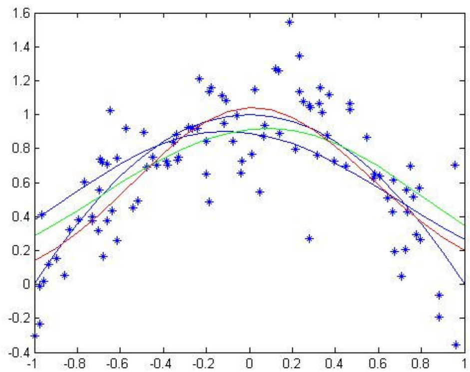
- Trying to predict the stimuli associated with new patterns, from the sample stimulus-pattern pairs we have, is analogous to trying to interpolate a function on the manifold.

Geometric Harmonics

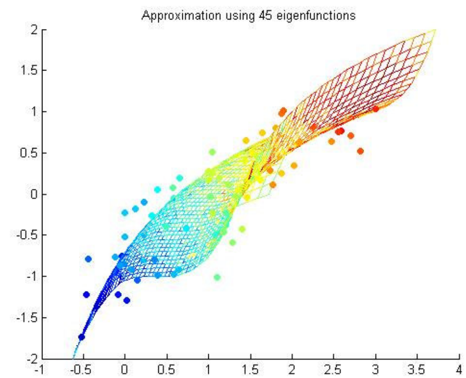
- Geometric Harmonics is a way of reconstructing *continuous* functions from a set of function samples on a surface. In defining a *kernel* (a similarity measure between pairs of points); we control the properties of the reconstruction.
- Using geometric harmonics, we can quickly and easily find an extension from our samples to the surrounding space that
  - Matches the function exactly on our samples
  - Has the properties of the kernels

Four Examples of Geometric Harmonics for Interpolation

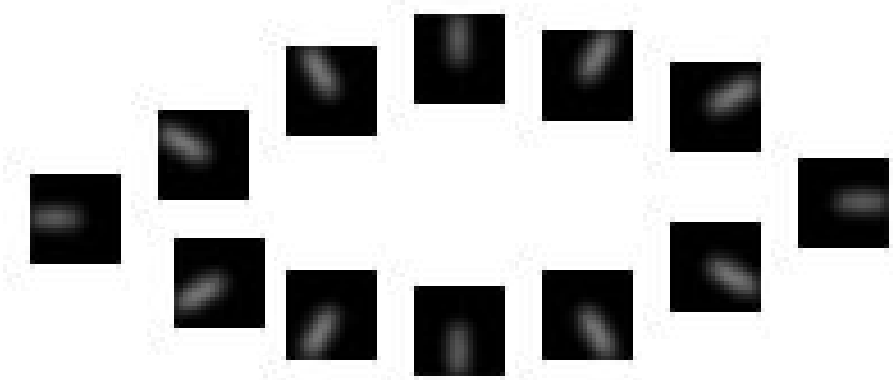
Functions on:



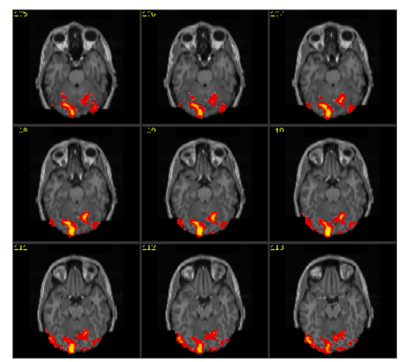
The 1-D line



A 2-D manifold in 3-D space

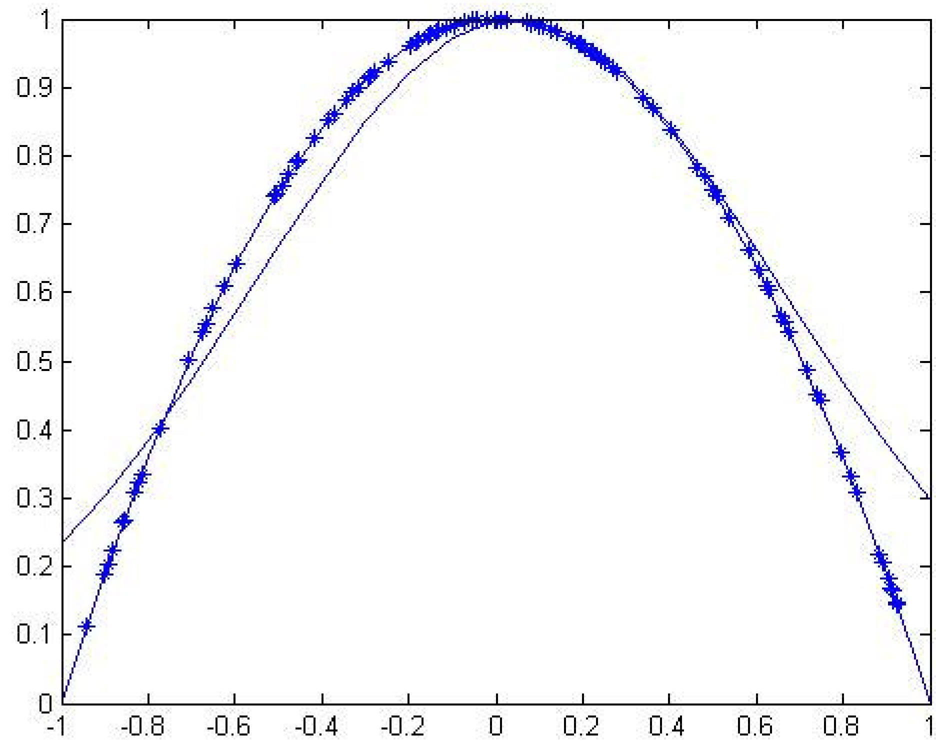


Synthetic Brain Patterns

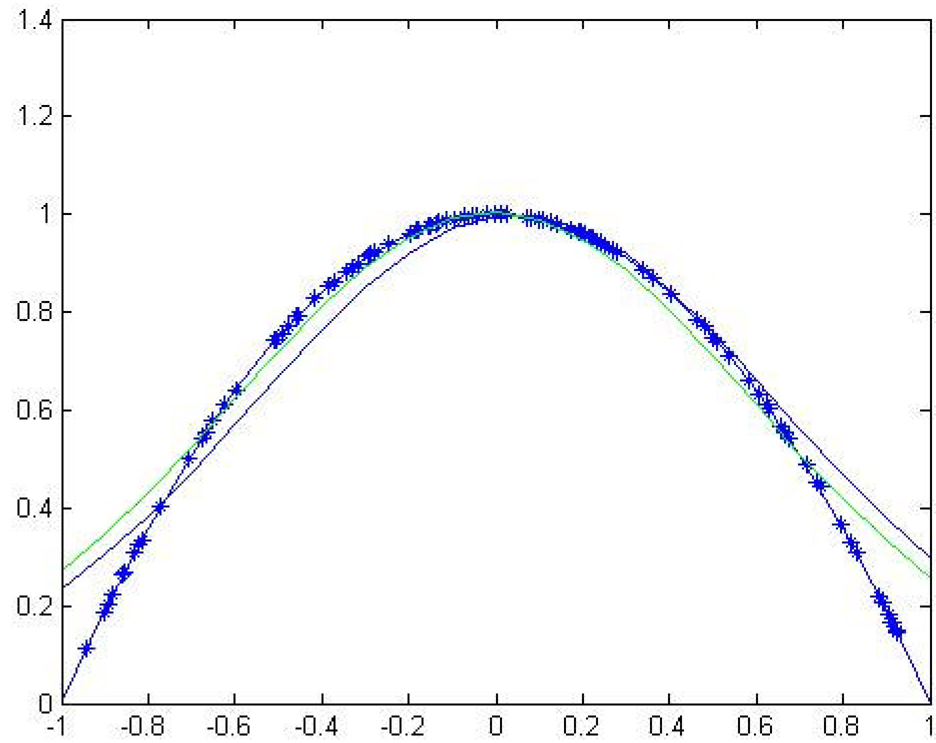


The EBC Competition Data

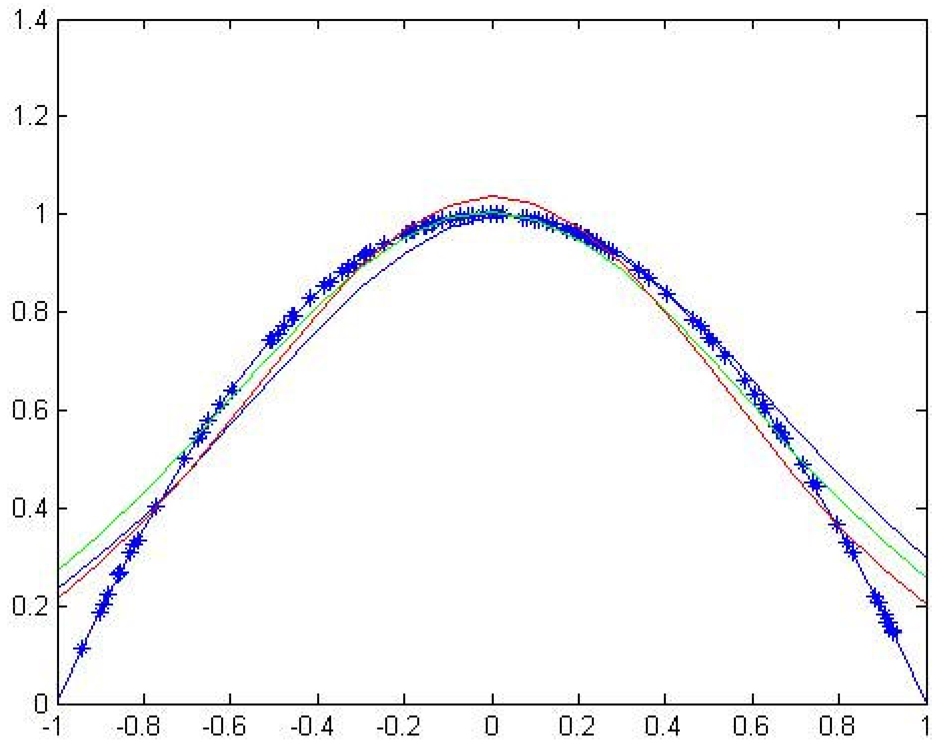
1-D example



1-D example



1-D example



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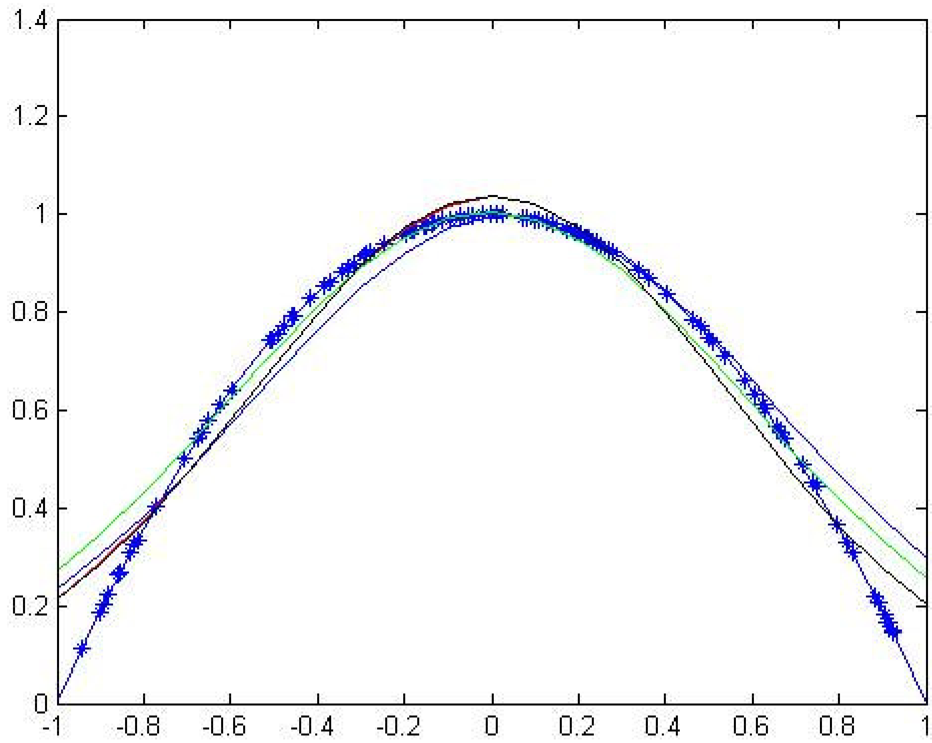
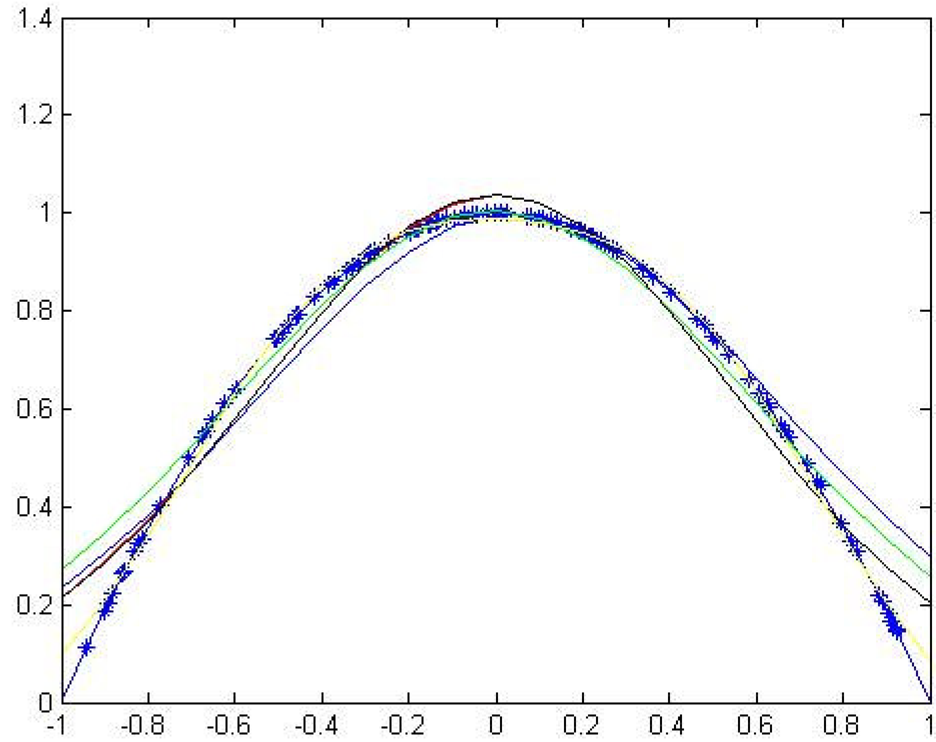
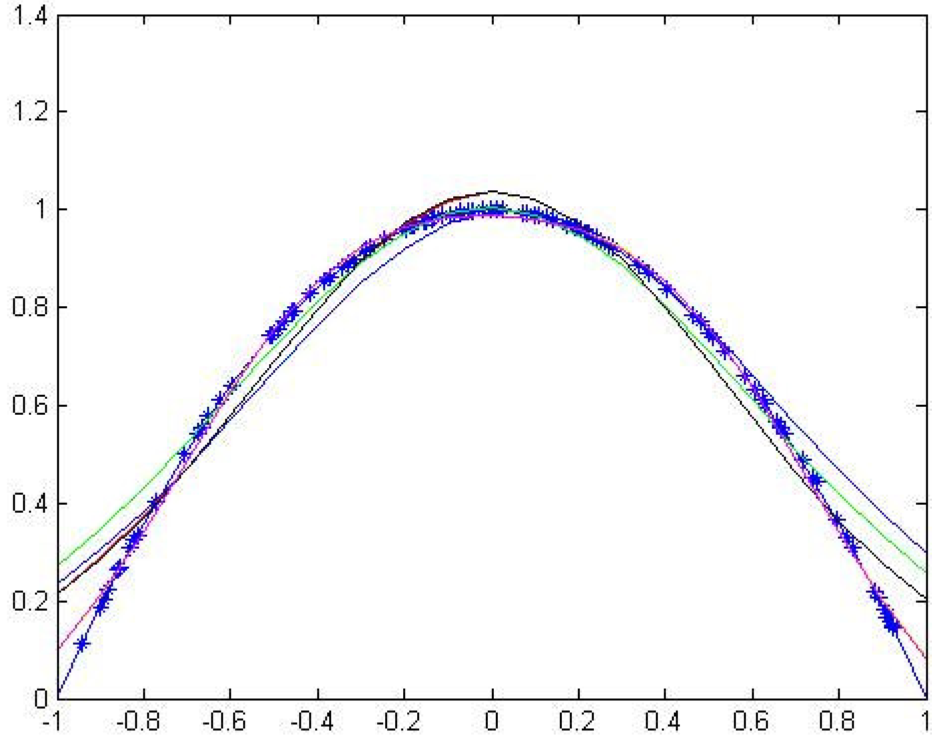


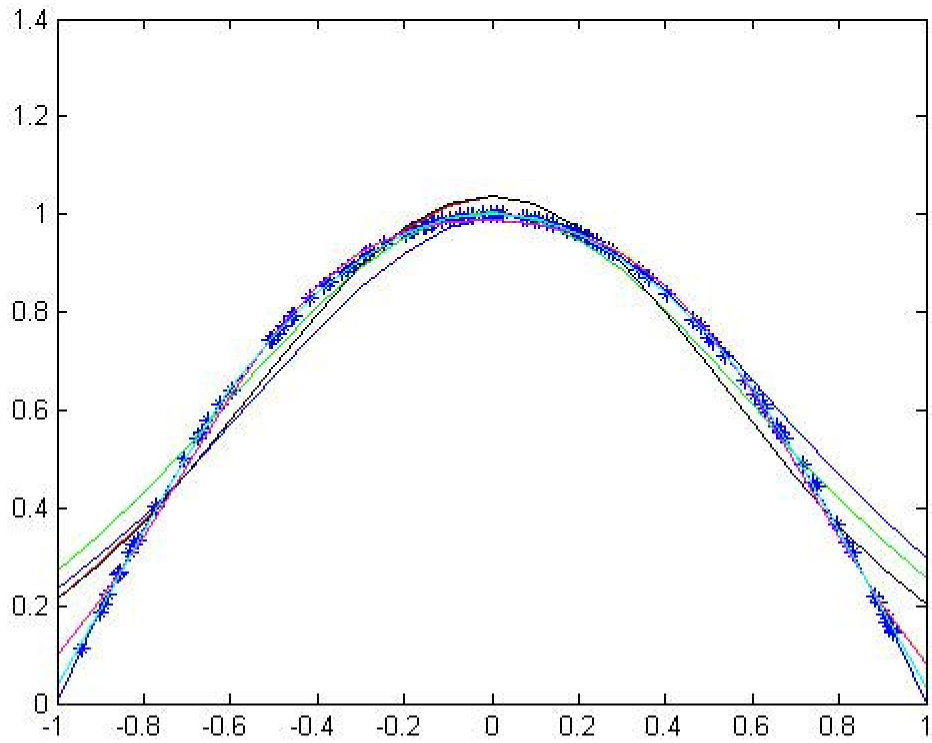
1-D example



1-D example



1-D example



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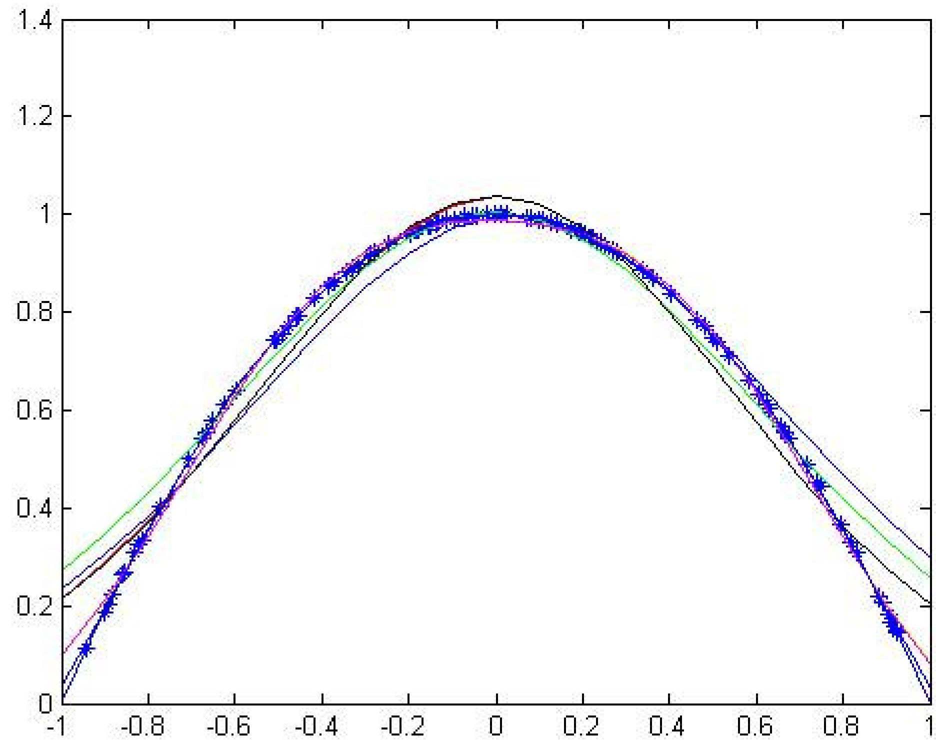

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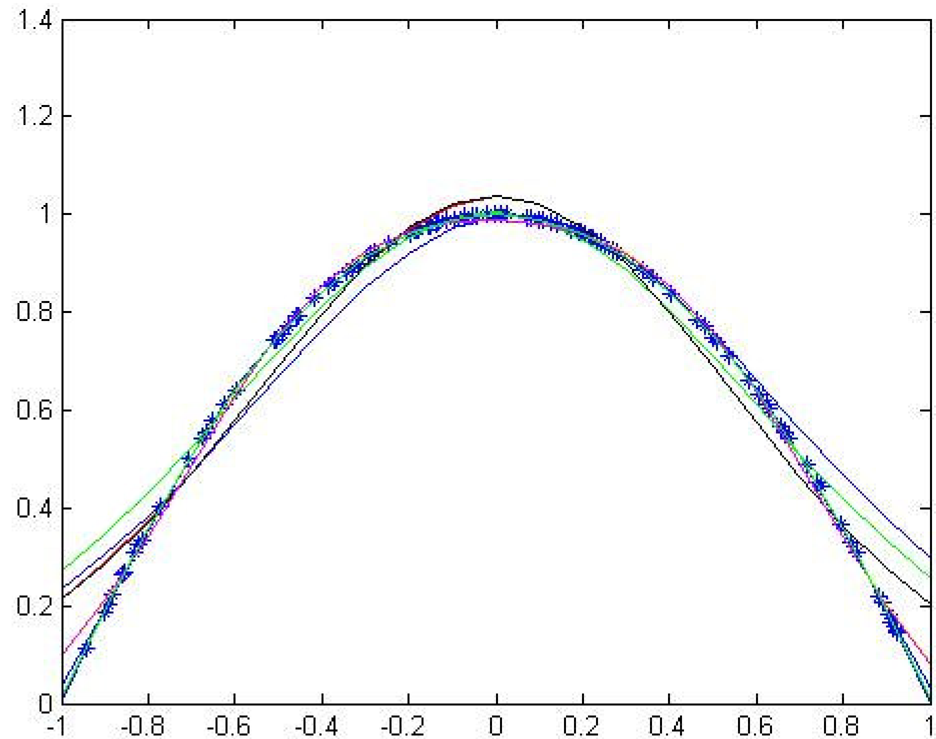

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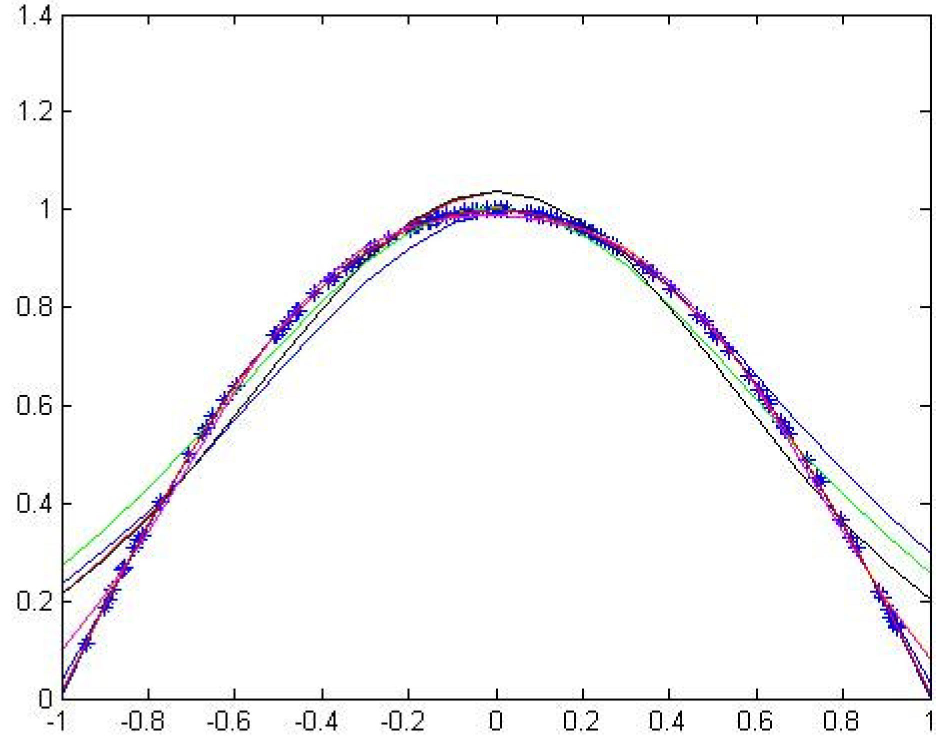
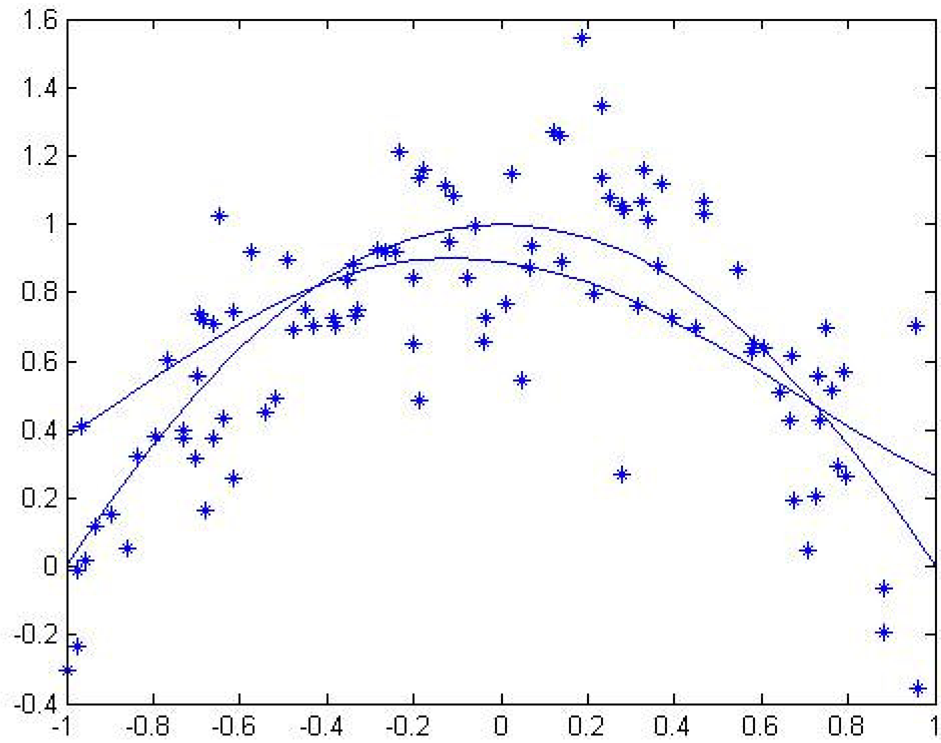


1-D example (noise)



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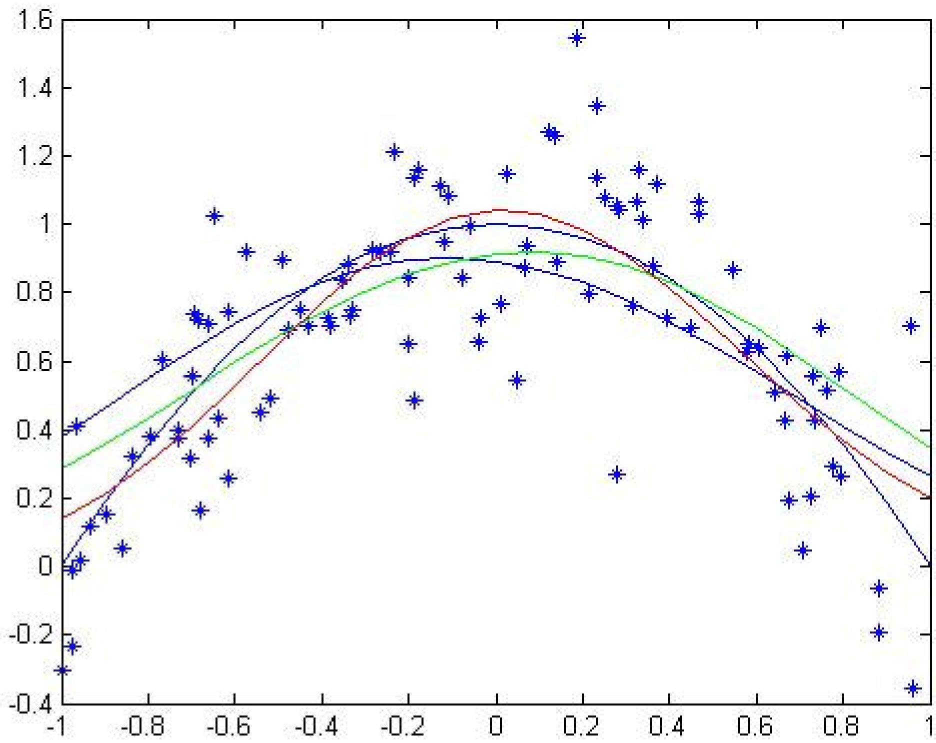

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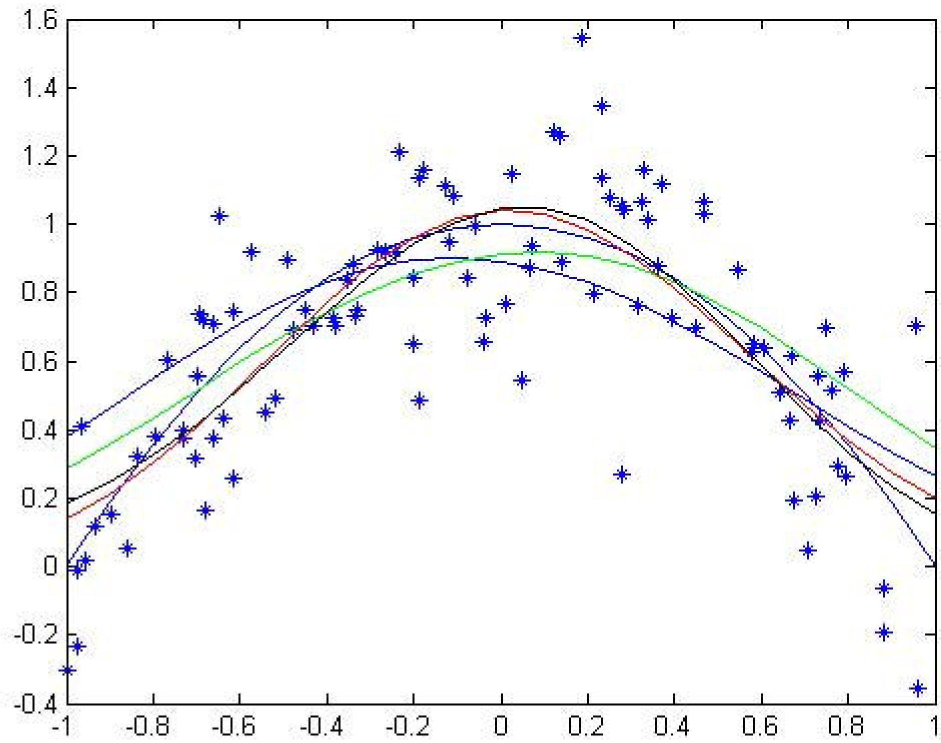

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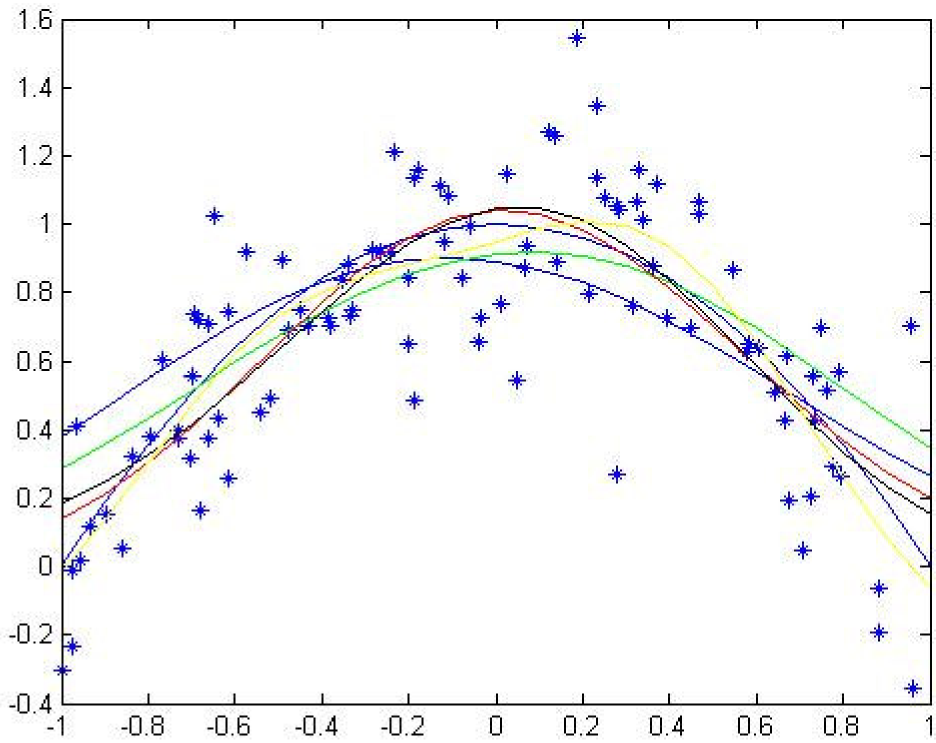

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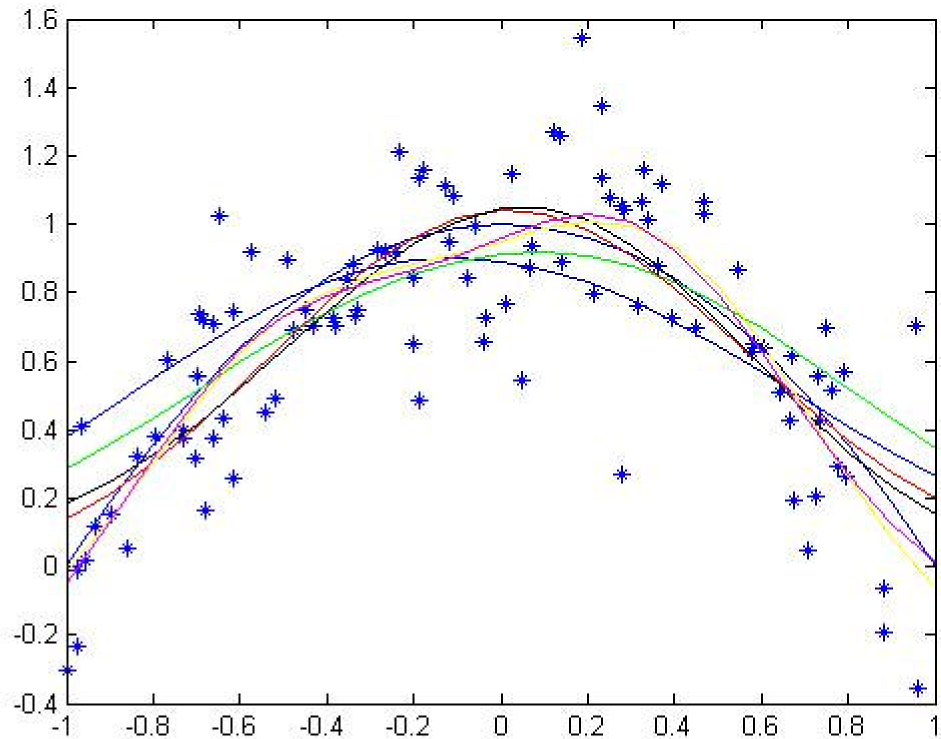

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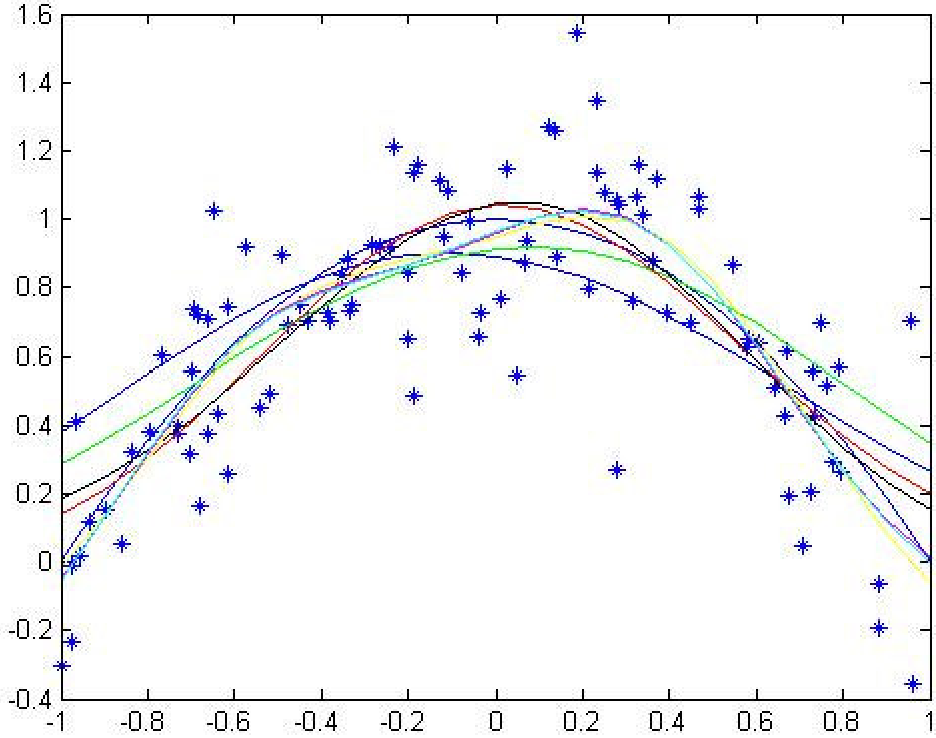

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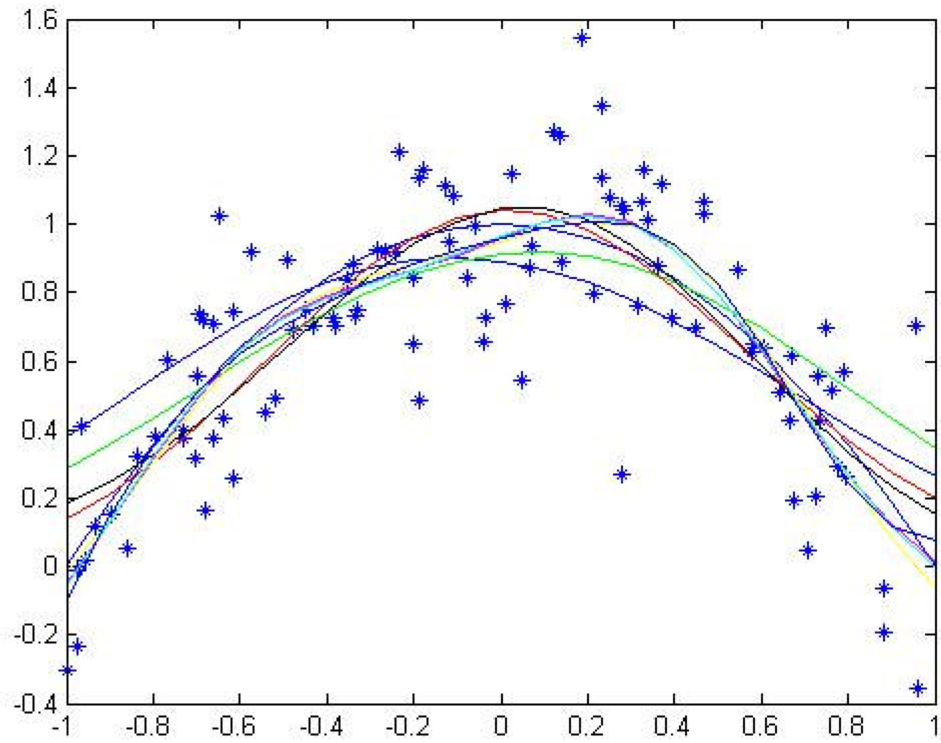

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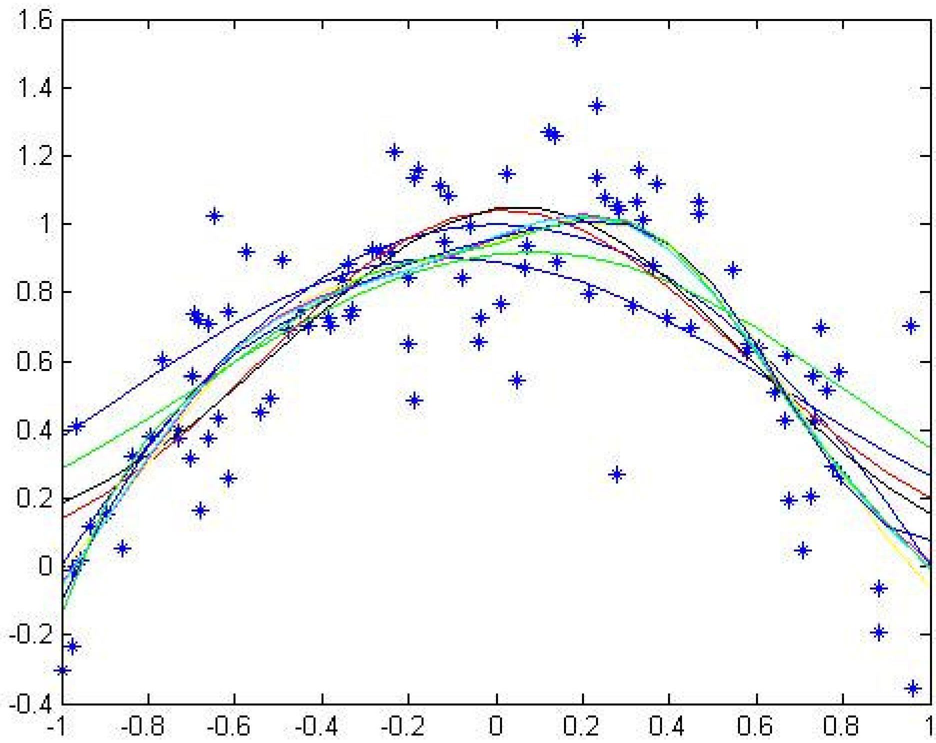

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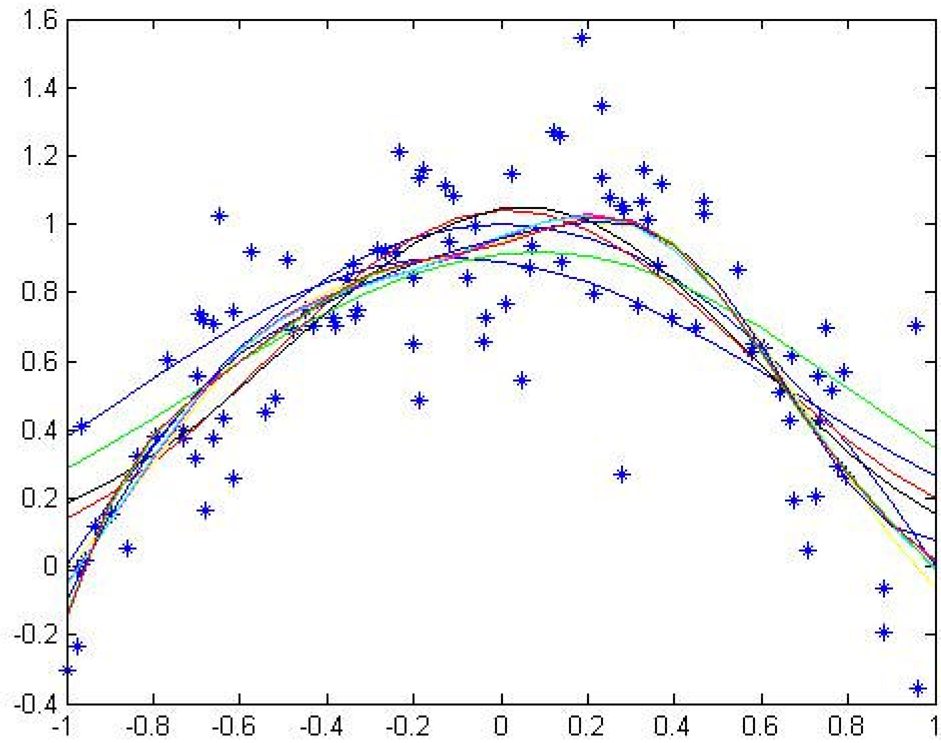
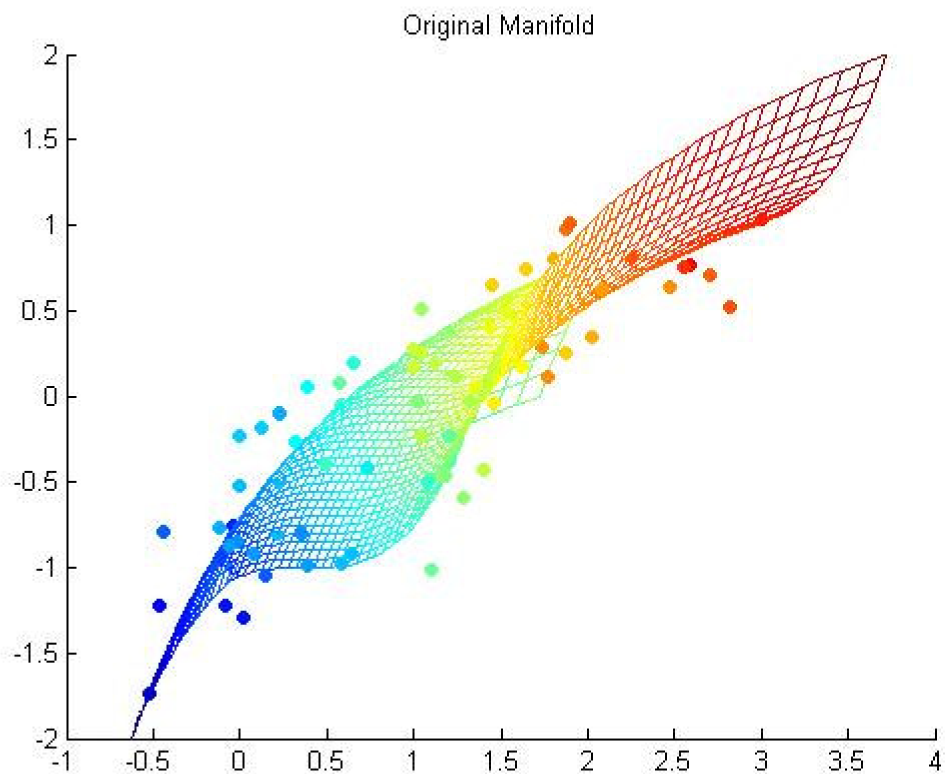


2-D example



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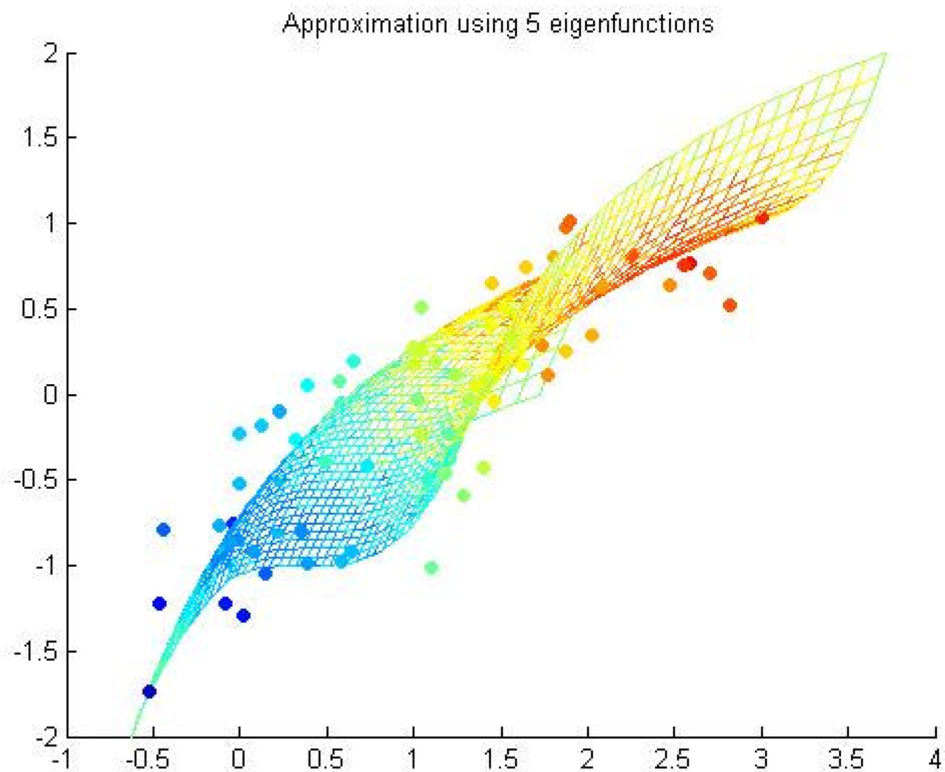
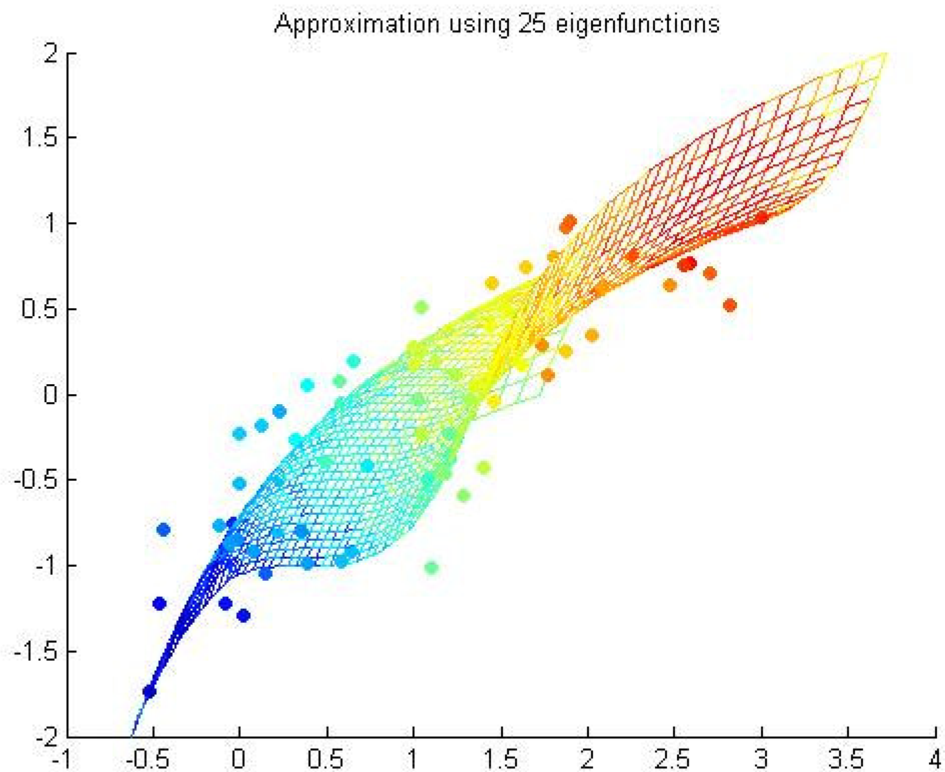
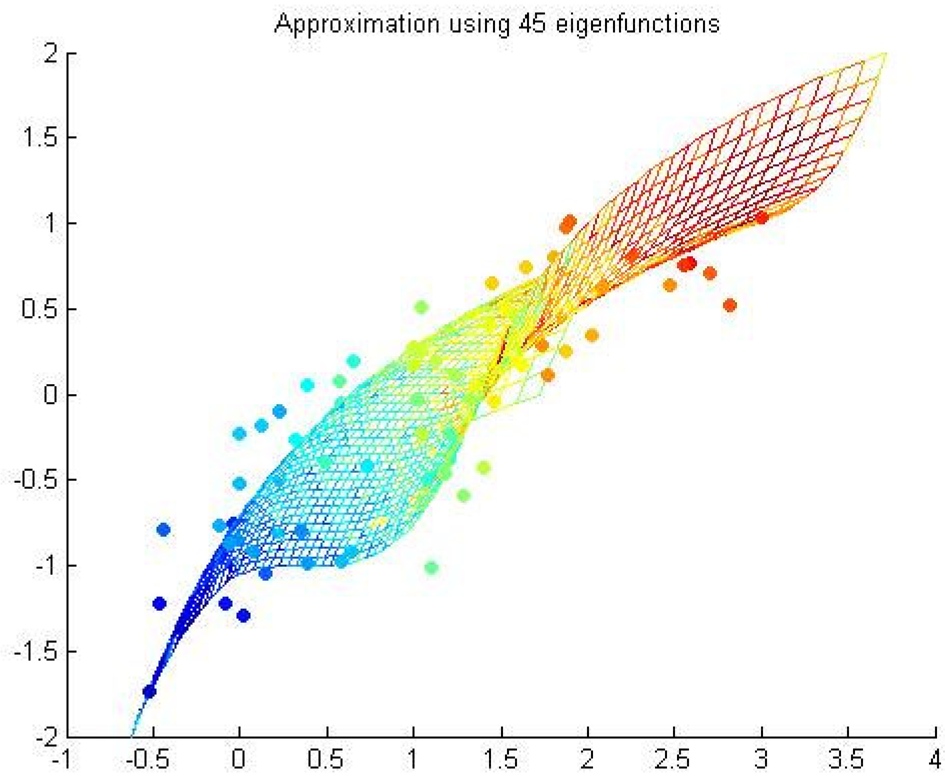

Approximation using 5 eigenfunctions
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4

2-D example



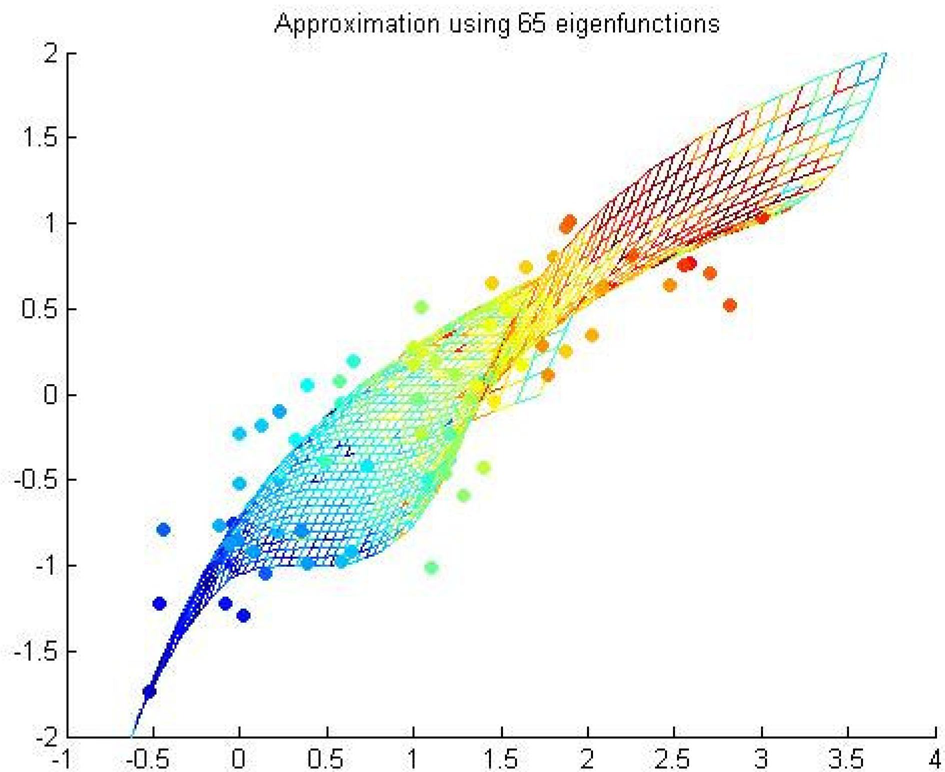
Approximation using 25 eigenfunctions
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4

2-D example



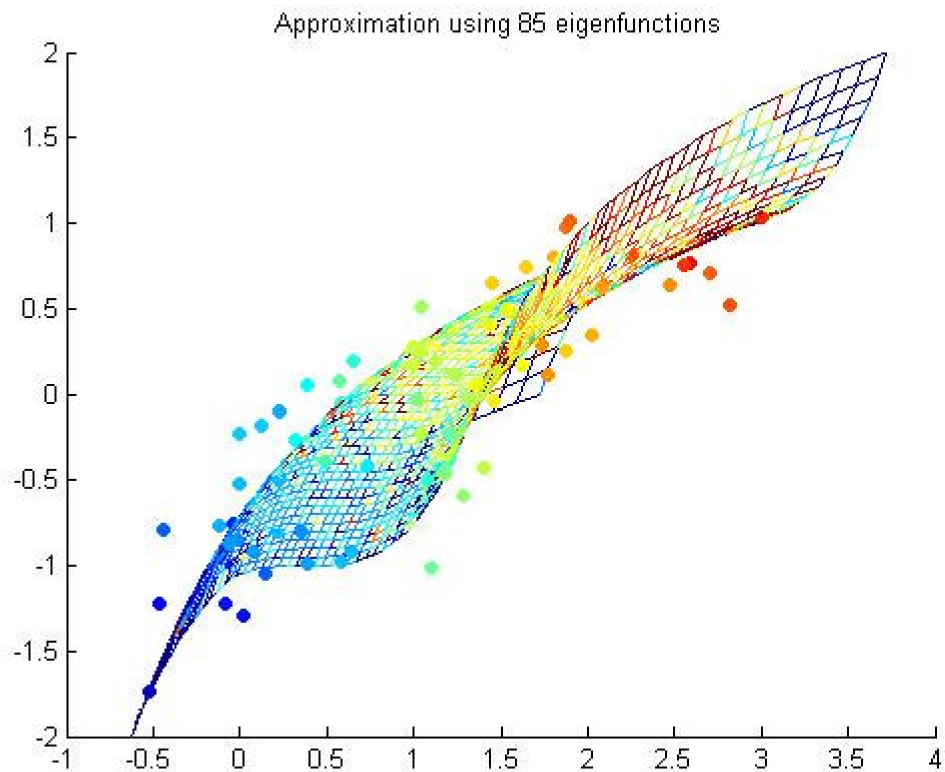
Approximation using 45 eigenfunctions
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4

2-D example



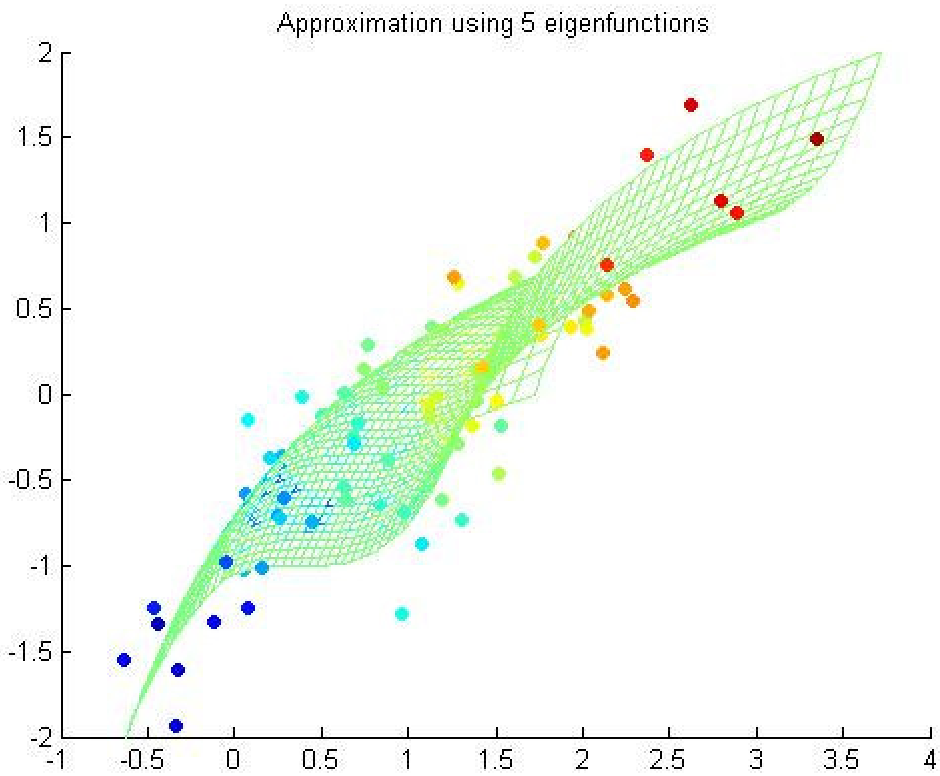
Approximation using 65 eigenfunctions
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4

2-D example



Approximation using 85 eigenfunctions
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4

Effect of kernel? – Sigma too small



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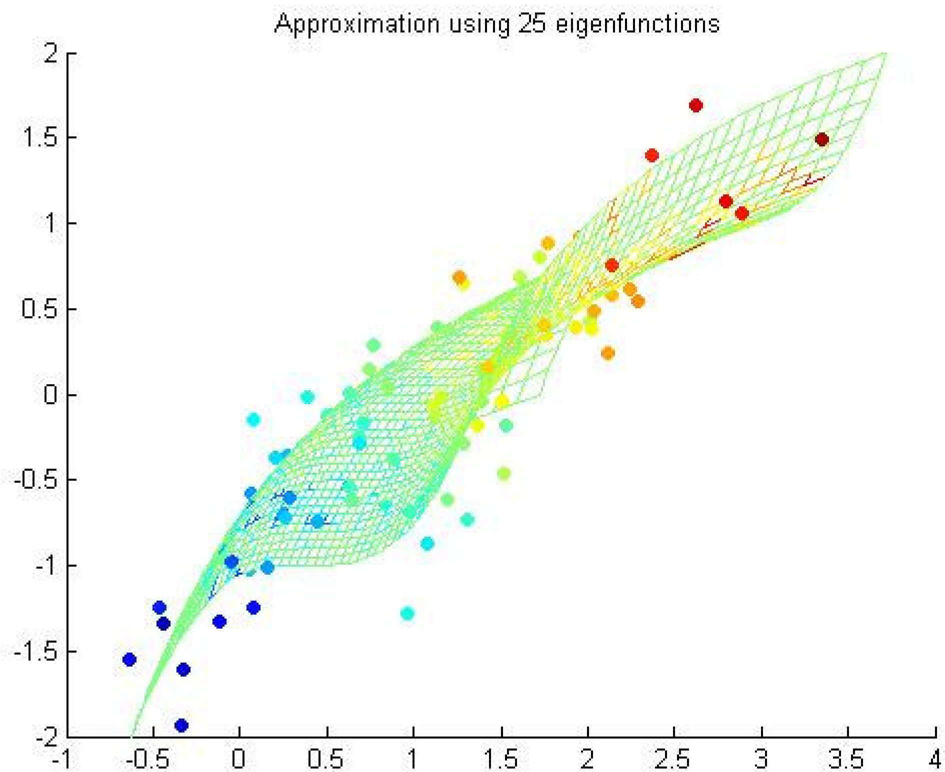

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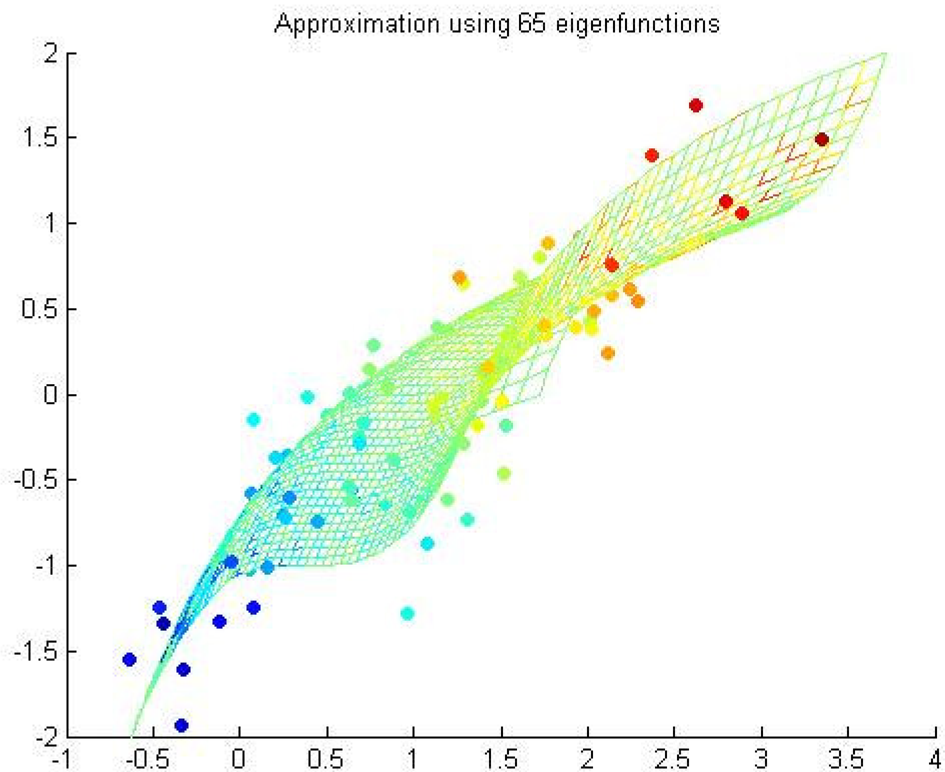
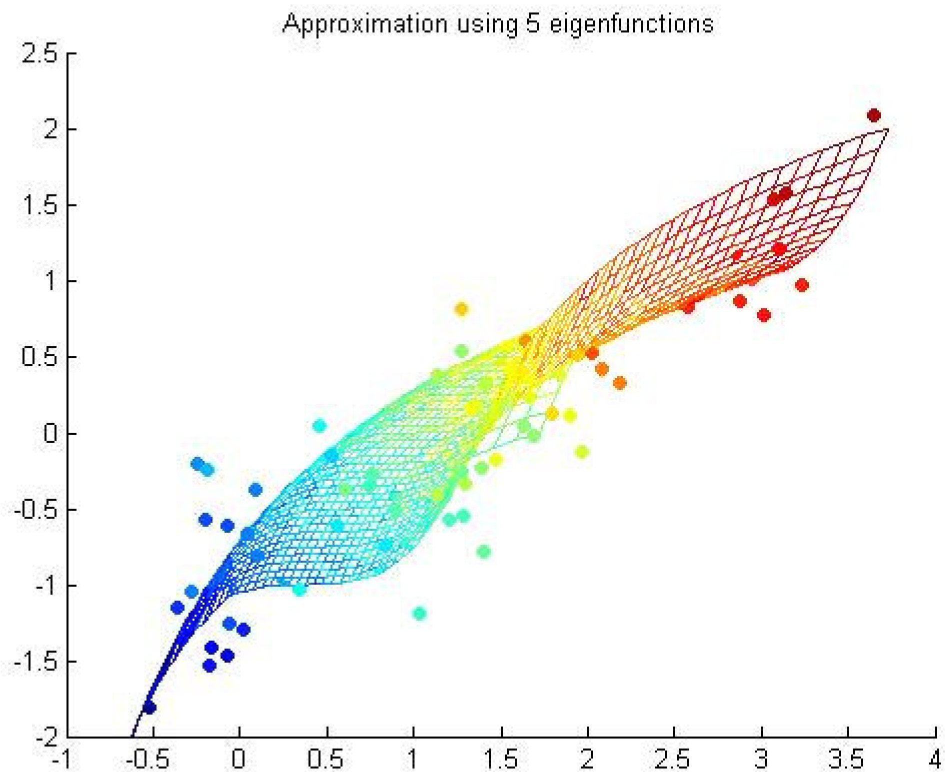


Effect of kernel? – Sigma too big



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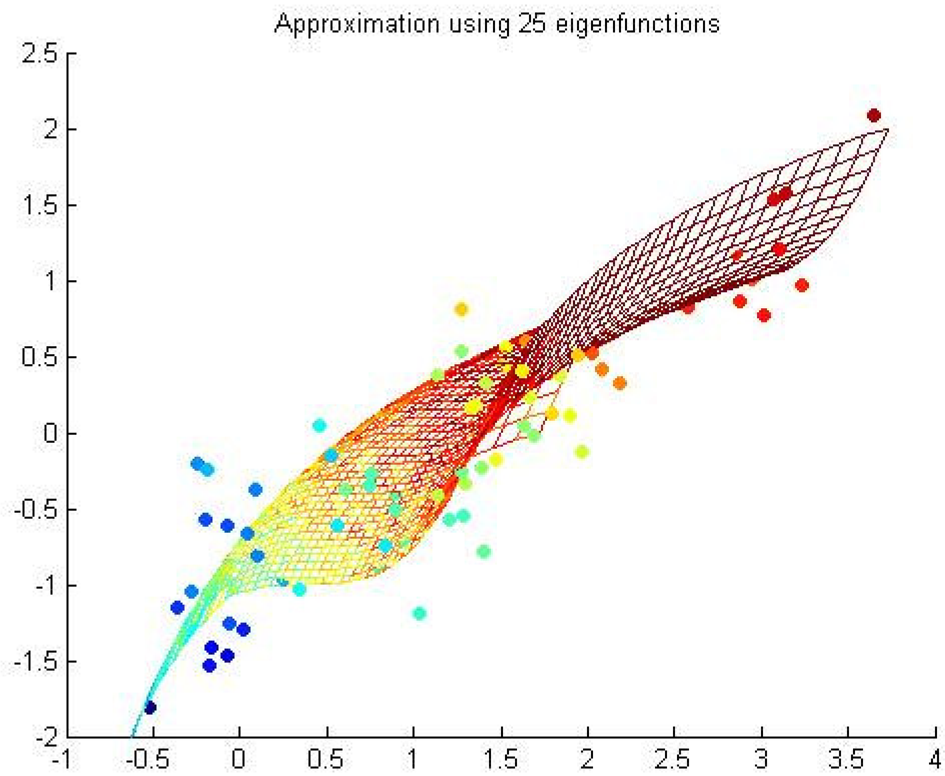

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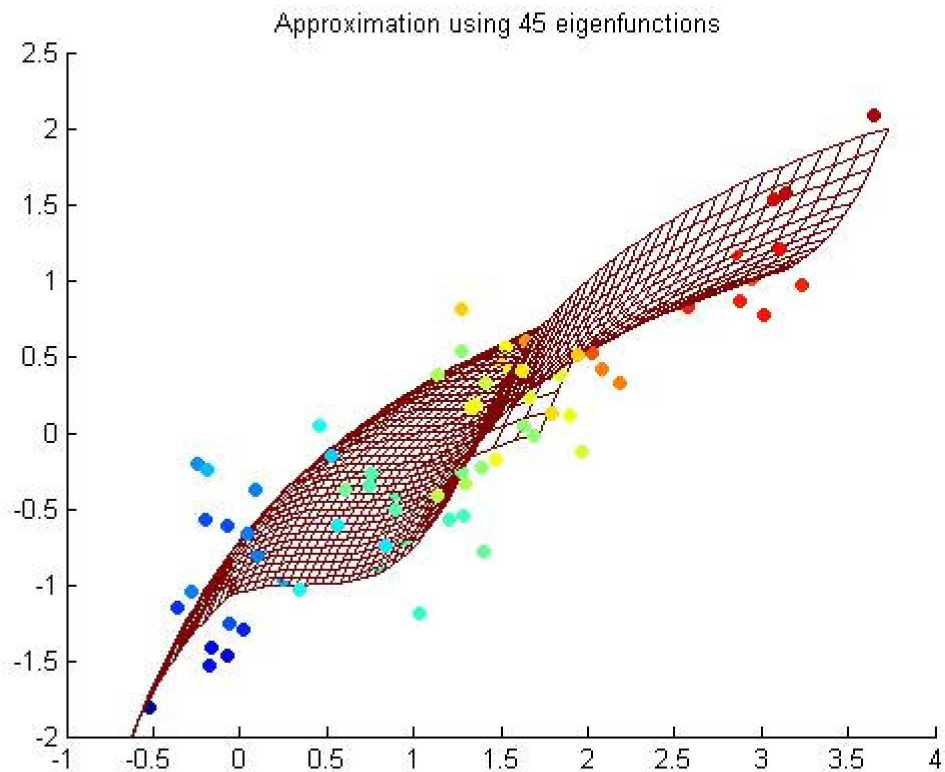
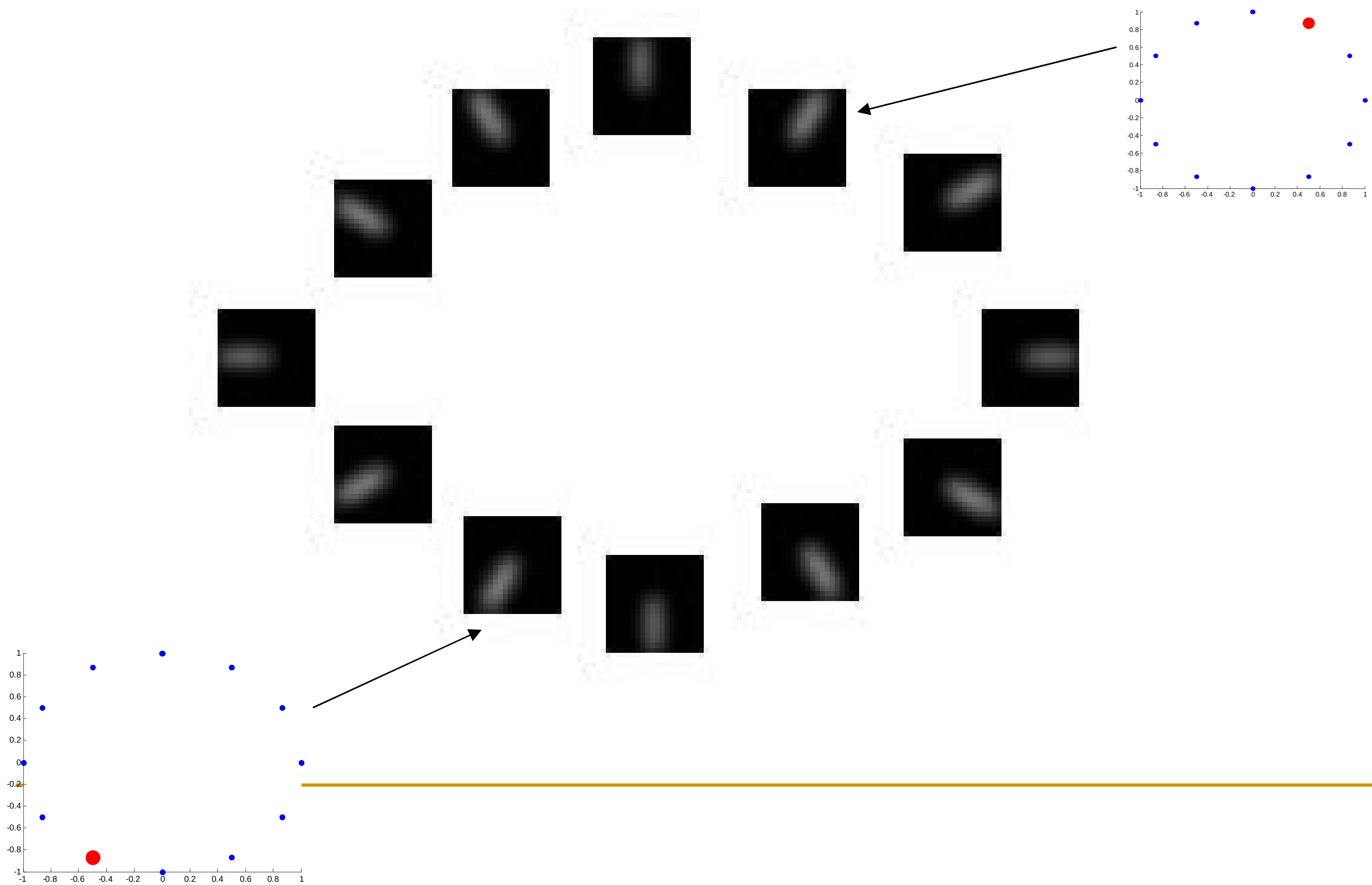


A higher-dimensional synthetic manifold

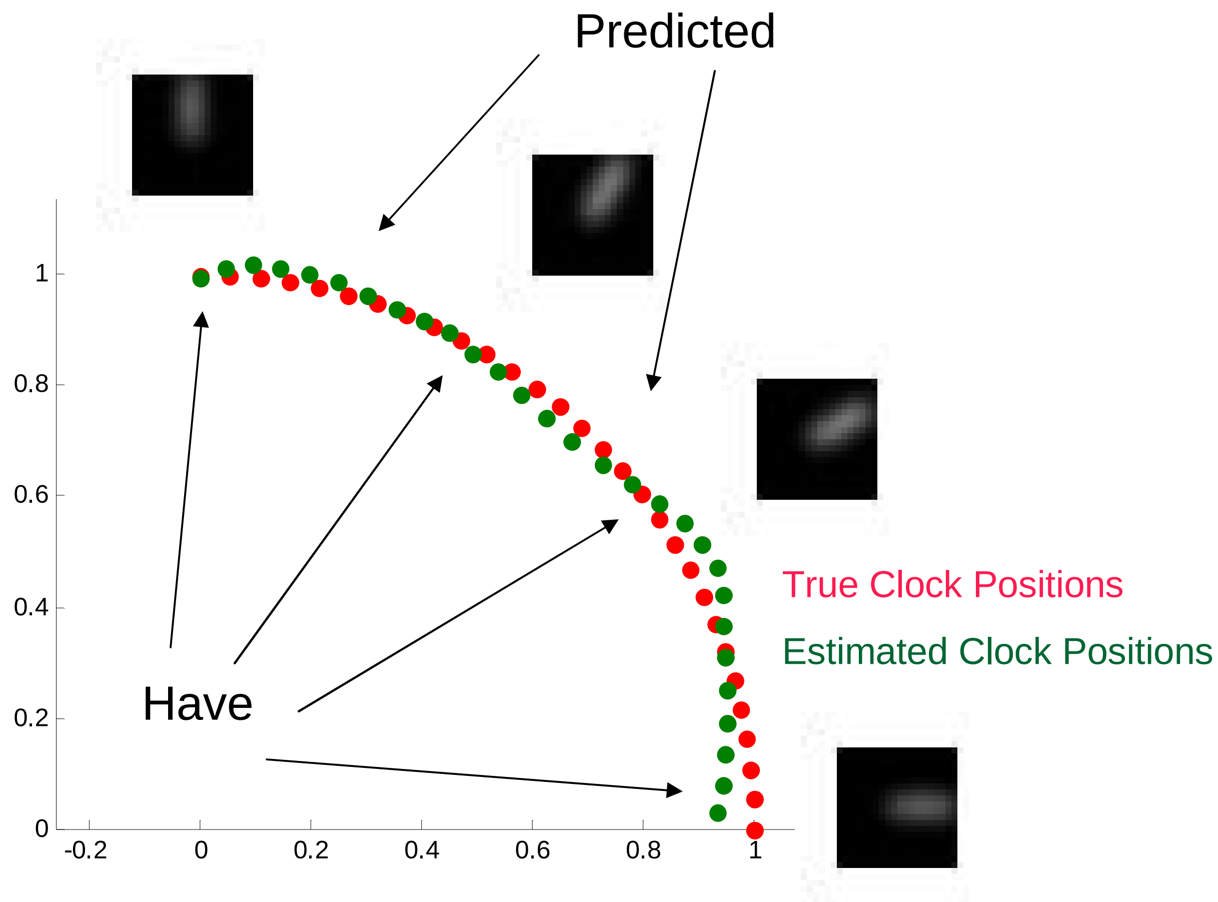
For each of 12 clock positions, we create a synthetic brain response:



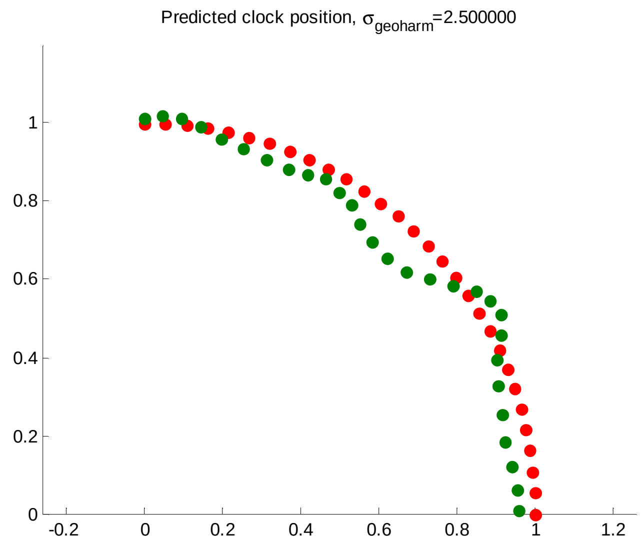
Predicting Clock-hands Associated with New Brain Patterns

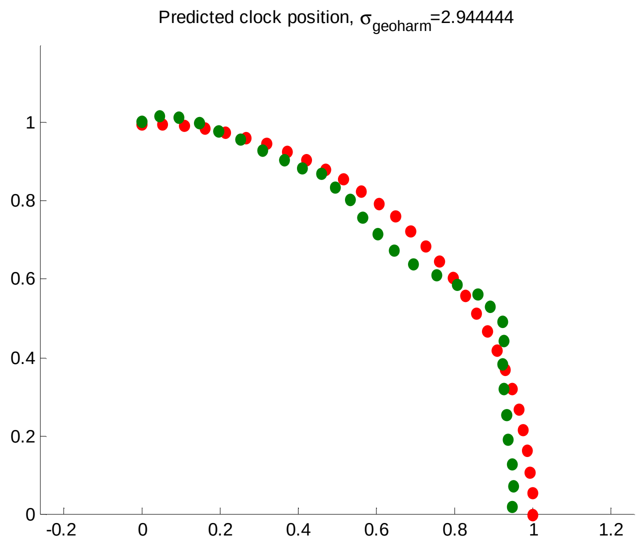
- Train on 12 original clock hand patterns (12 o'clock, 1 o'clock, 2 o'clock,…)
- For each of several new clock hand patterns (evenly spaced between 12 o'clock and 3 o'clock):
  - Predict associated clock hand position
- This is a simple synthetic set-up of what we would like to do on real brain patterns.

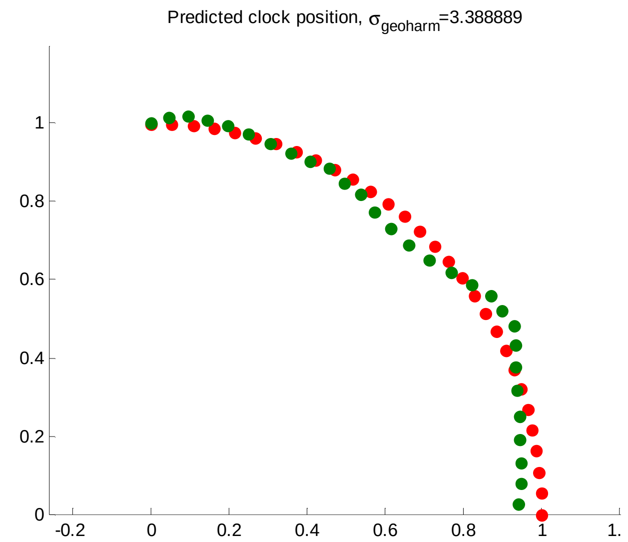
Learning Clock Hand Positions

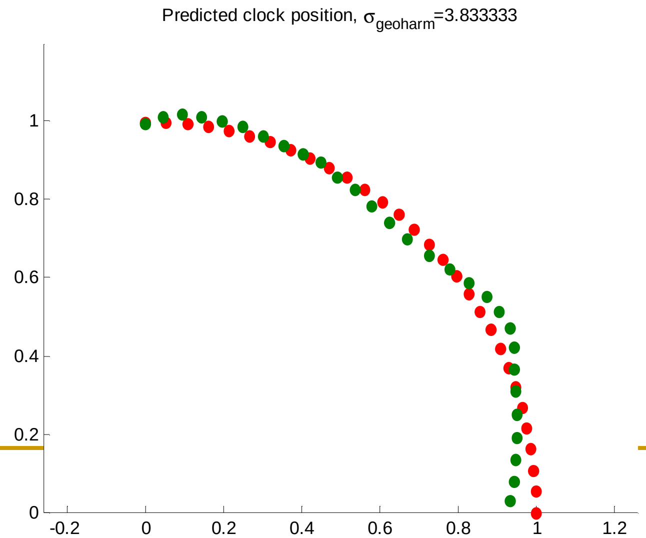


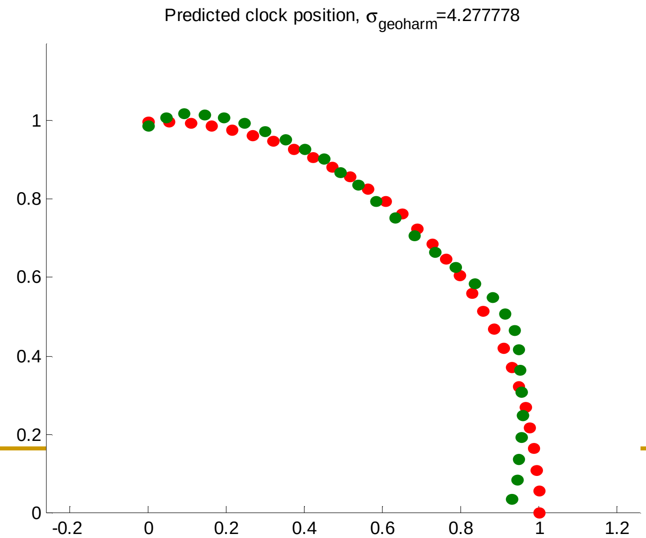
Clock Hand Positions – Effect of Params



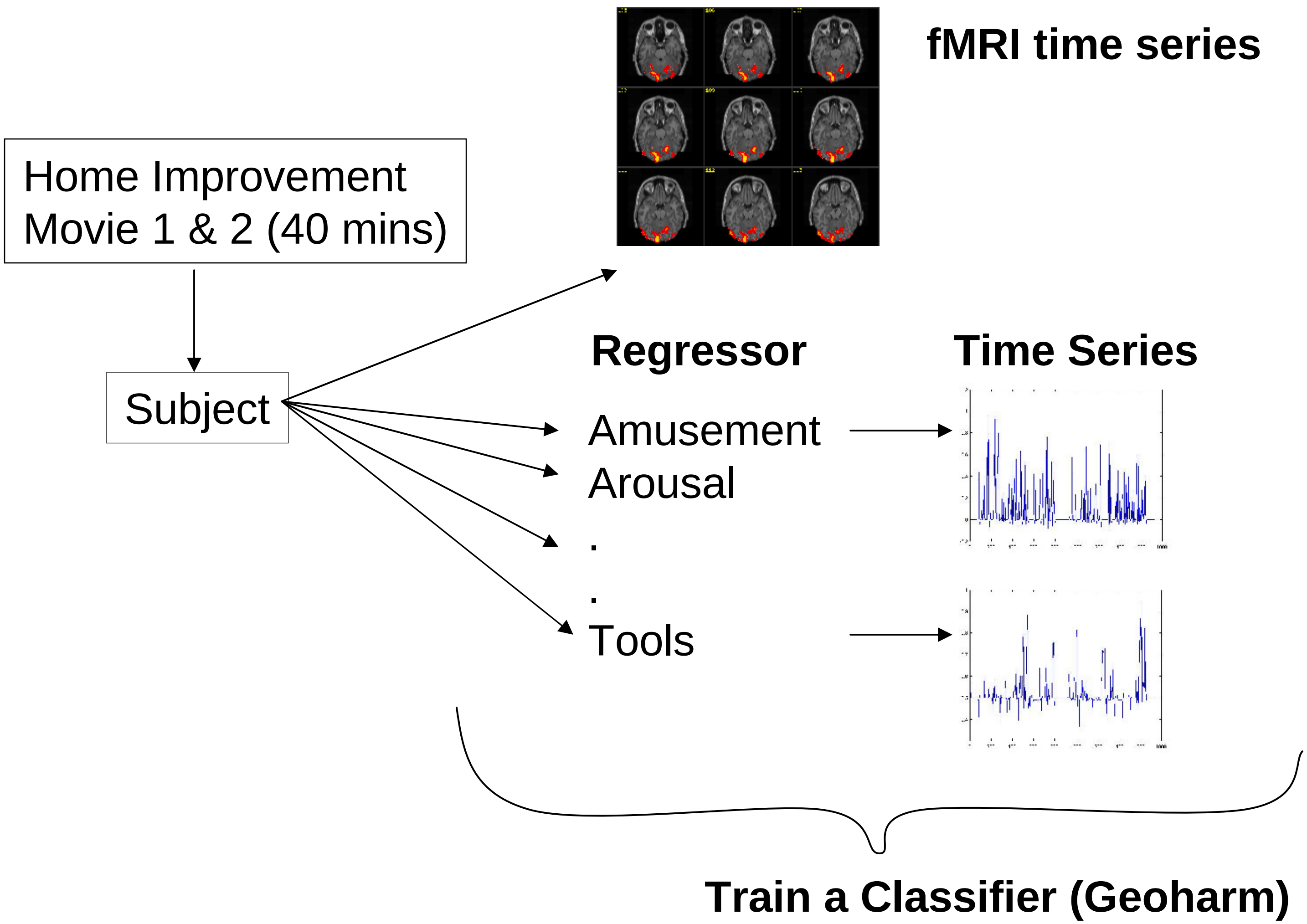






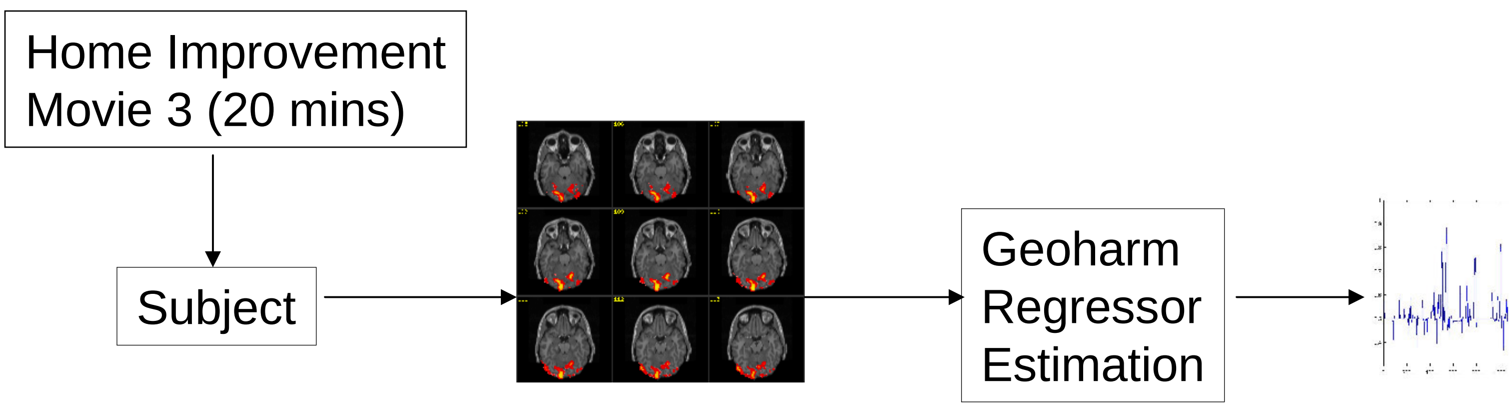


Review of the EBC competition



Geoharmonics on EBC data

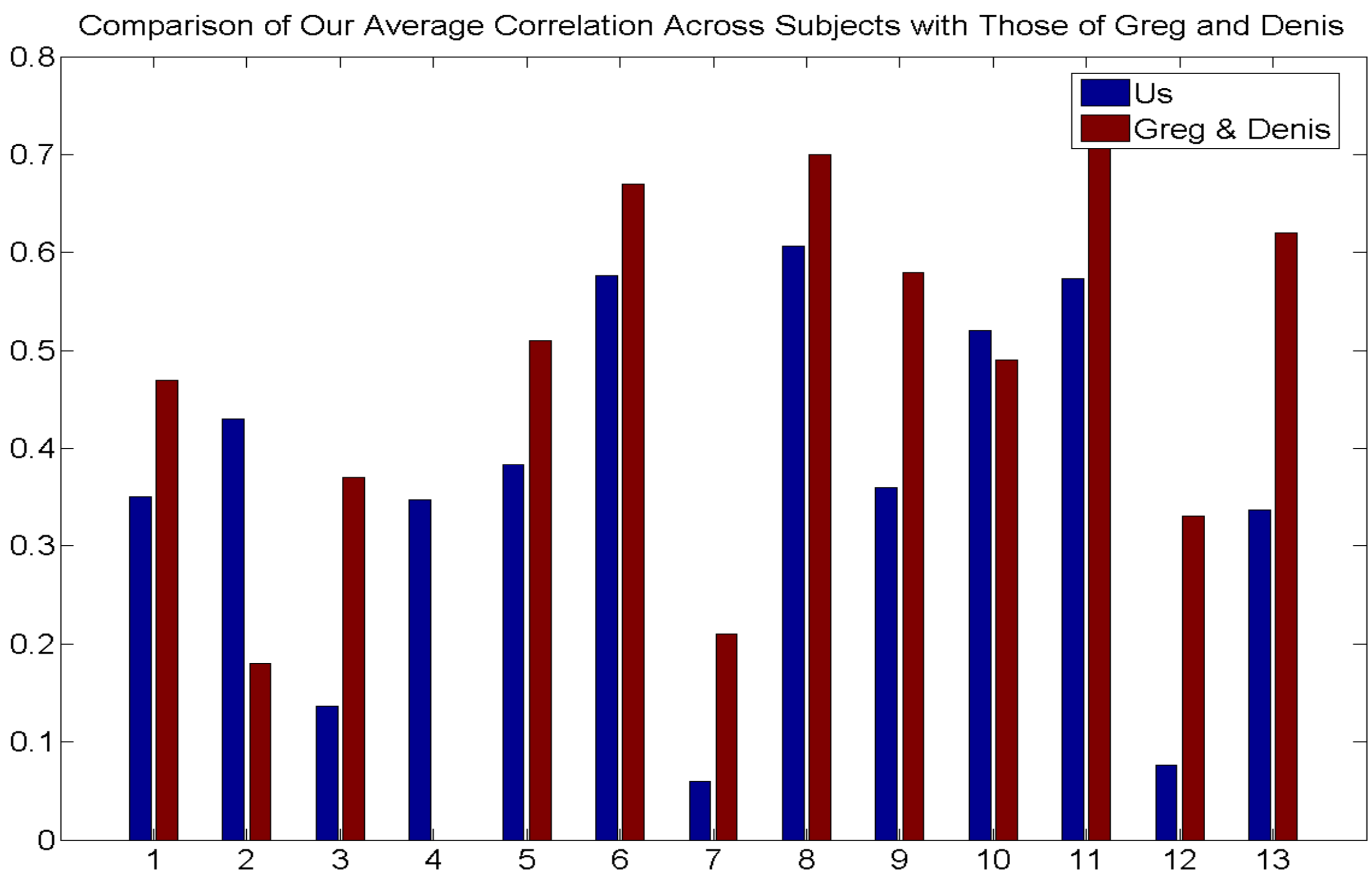
- Movie 1 and Movie 2 data combined to 1726 TRs
- Used Geometric Harmonics interpolation to predict movie 3 data.



EBC Geoharmonics Methods

- Used Greg and Denis' patterns, preprocessing, spatial and temporal averaging (for each regressor, a different set of 'optimal parameters')
- Replaced ridge regression with Geometric Harmonics
- Ridge regression has set of optimal regularization parameters for each regressor (Amusement, Arousal, …)
- Geometric Harmonics has two parameters to optimize over for each regressor: (sigma, eigenvalue count)
- Basic grid search to find optimal parameters for each regressor:
  - **Break up Movie1, Movie2 data into 10 contiguous 10% chunks**
  - **Train on 90%, test on remaining 10%**

Results



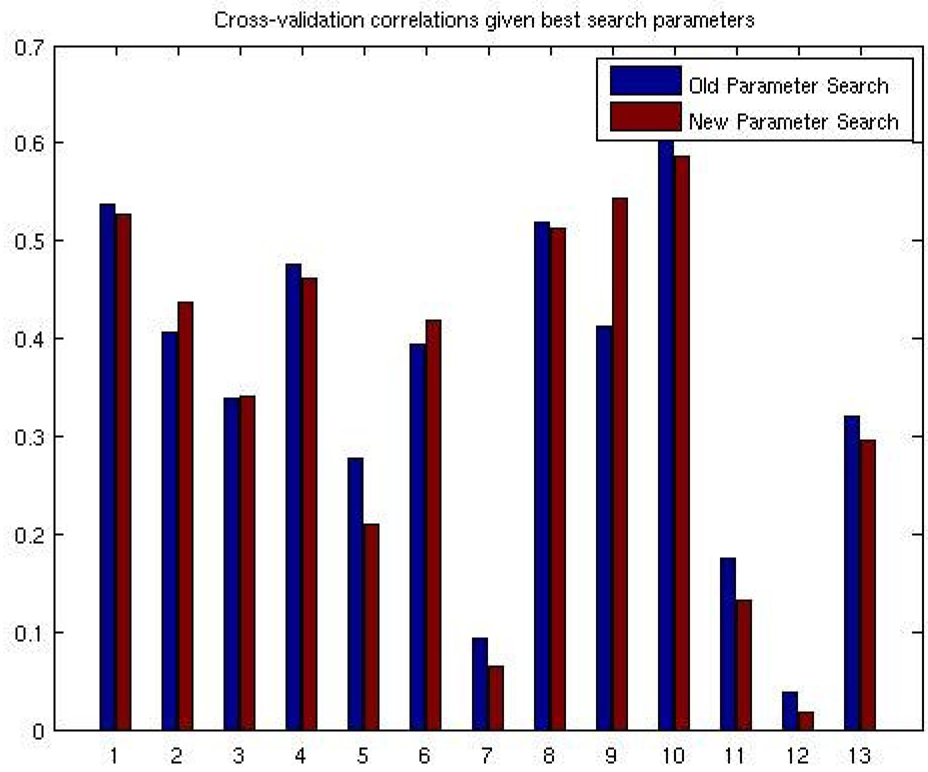
Comparison of Our Average Correlation Across Subjects with Those of Greg and Denis
Us
Greg & Denis
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
1
2
3
4
5
6
7
8
9
10
11
12
13

Commentary on the EBC results

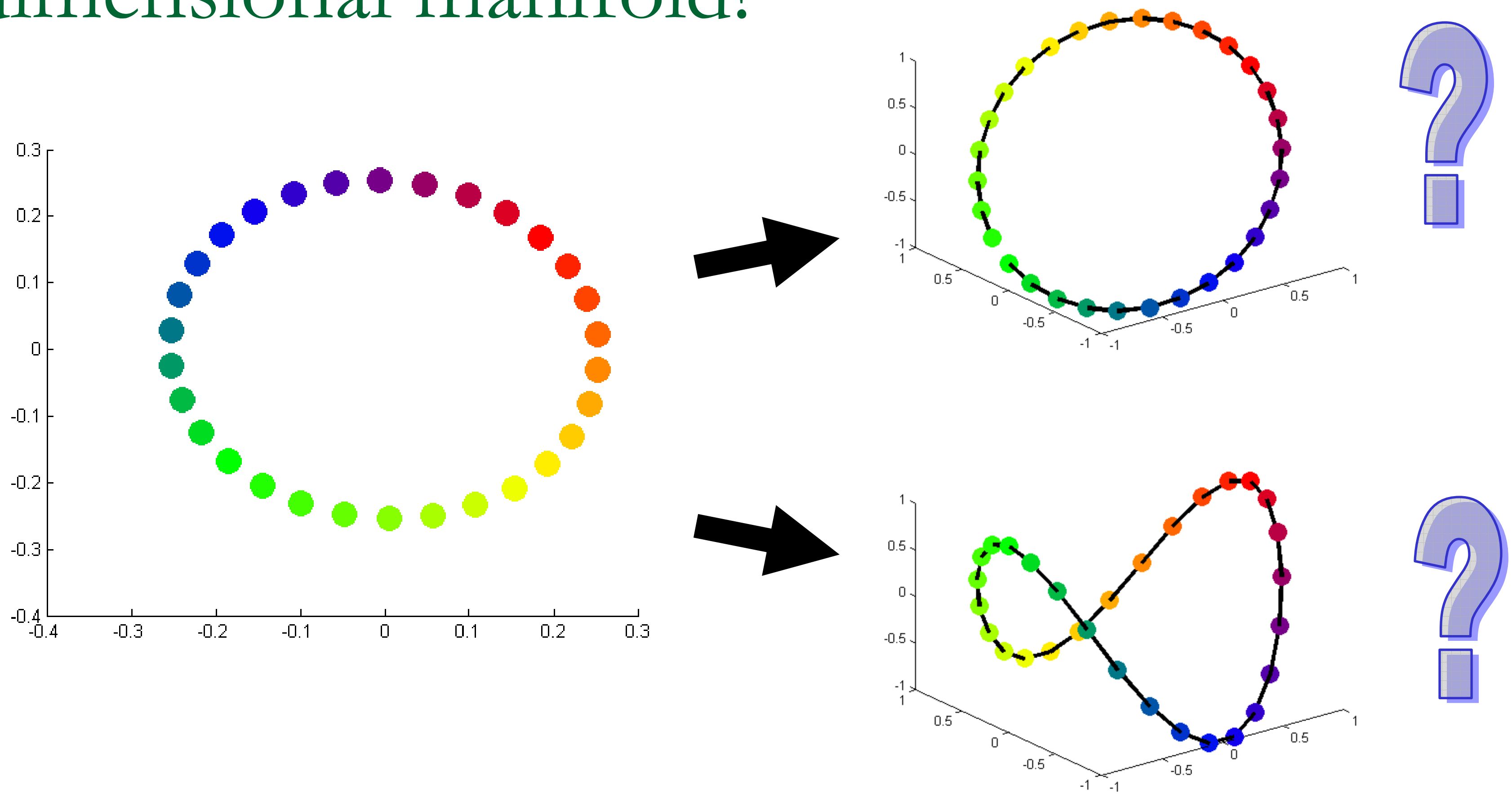
- Not doing as well as Greg and Denis
- Fair Performance
  - Validates a form of continuity assumption on the brain
  - Created a model for response
- Couple of other things we can still try
  - Temporal smoothing and wavelet denoising on output regressor time series

Commentary on the EBC results

- Could we do better with better parameters?



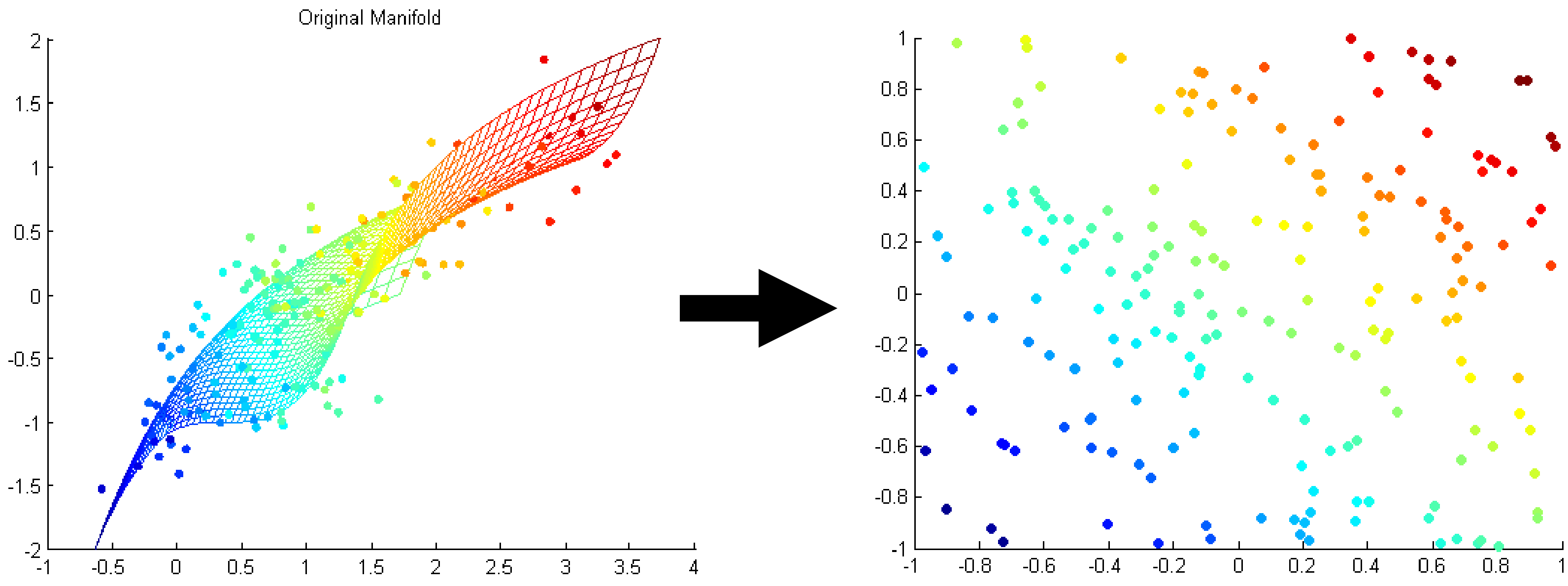
Problem 3: Given a low-dimensional representation, can we recover the high-dimensional manifold?



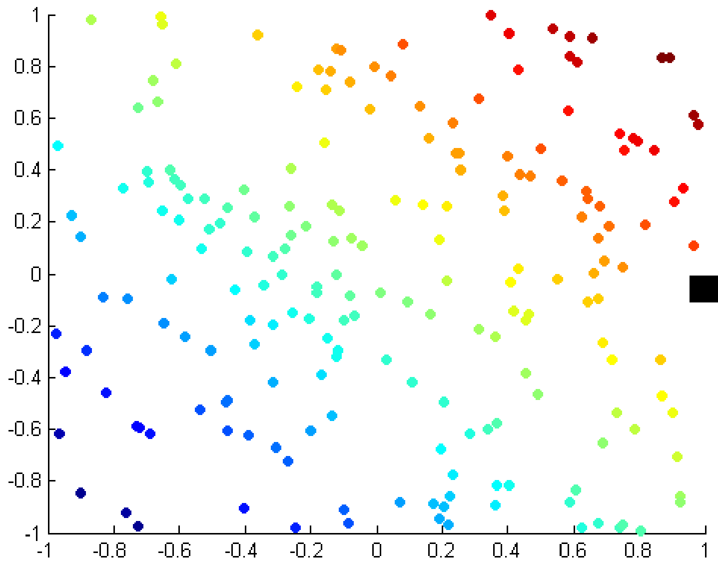
Recovery of Original Manifold: One Attempt by Way of Geometric Harmonics

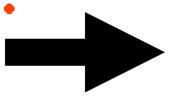
- Can think of the original position of each point as a function on the low-dimensional space.
- Want to interpolate.

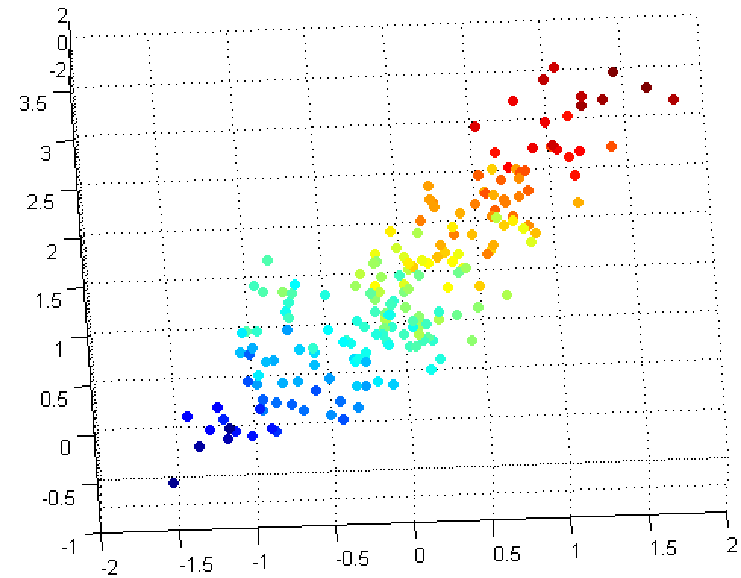
2D Example: Recovery of Original Manifold



2D Example: Recovery of Original Manifold

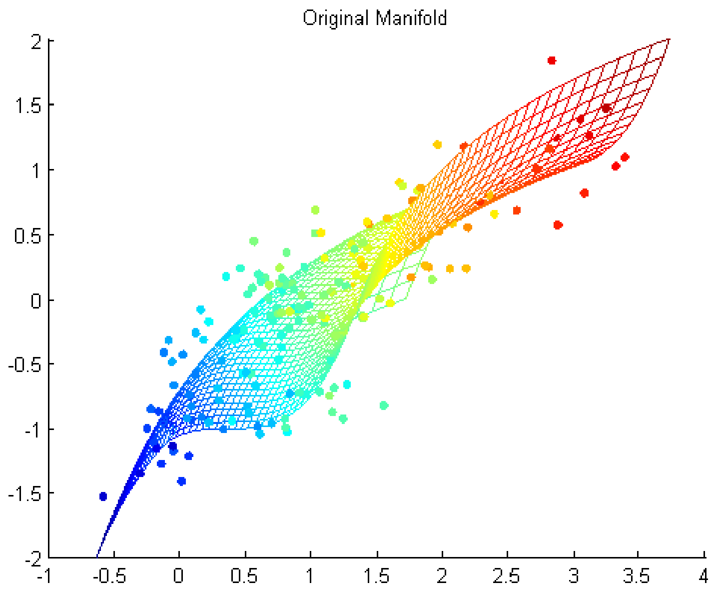




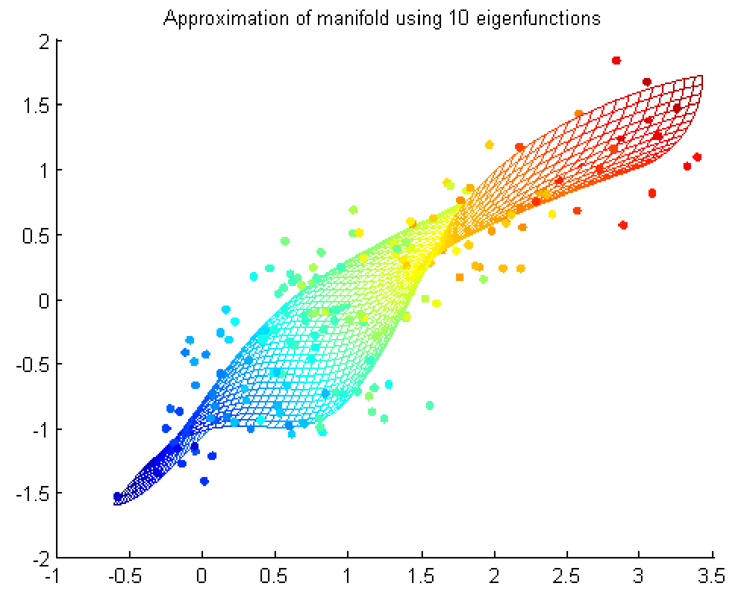


Where is the manifold?

Results on 2D Example:

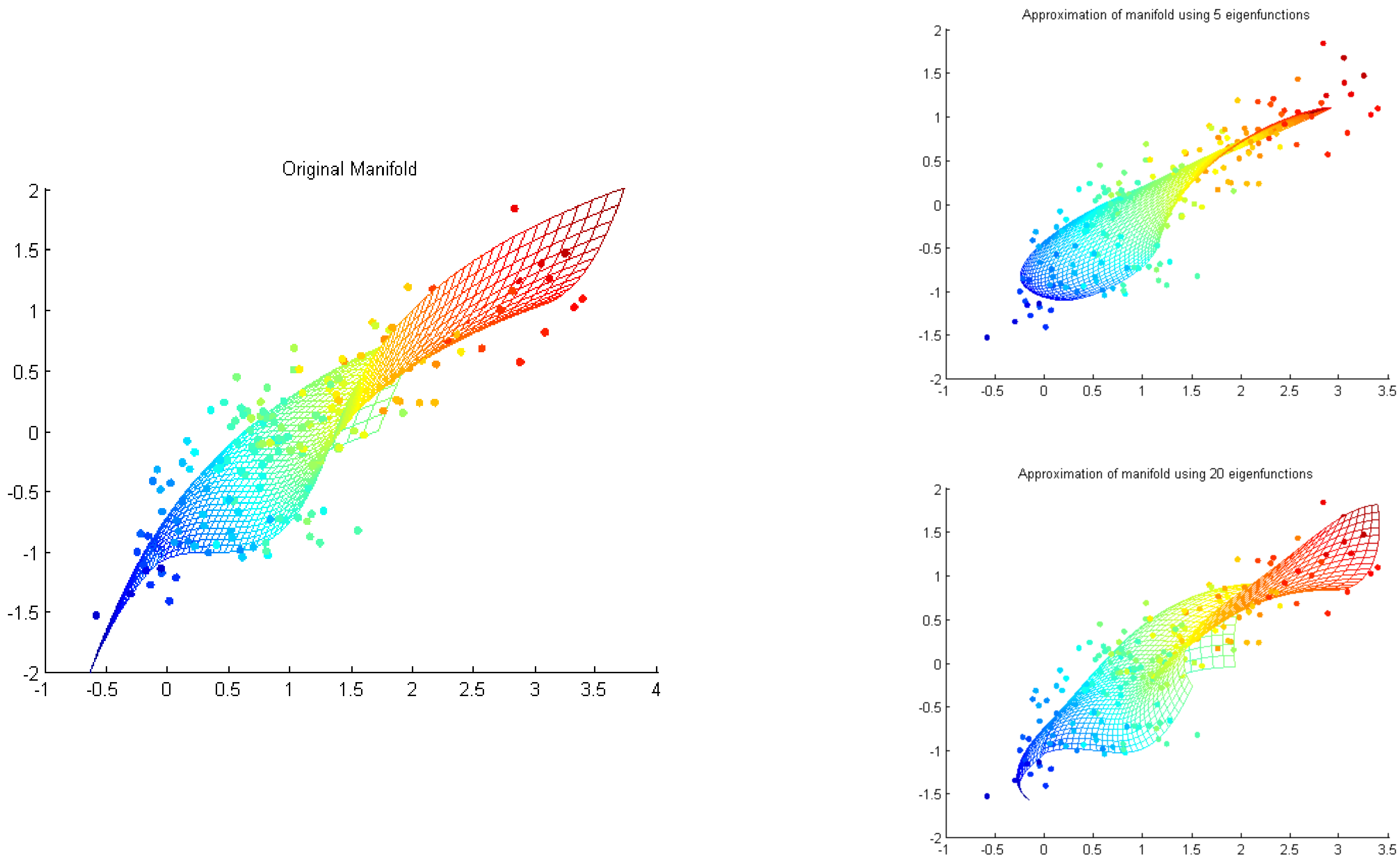


Real Manifold

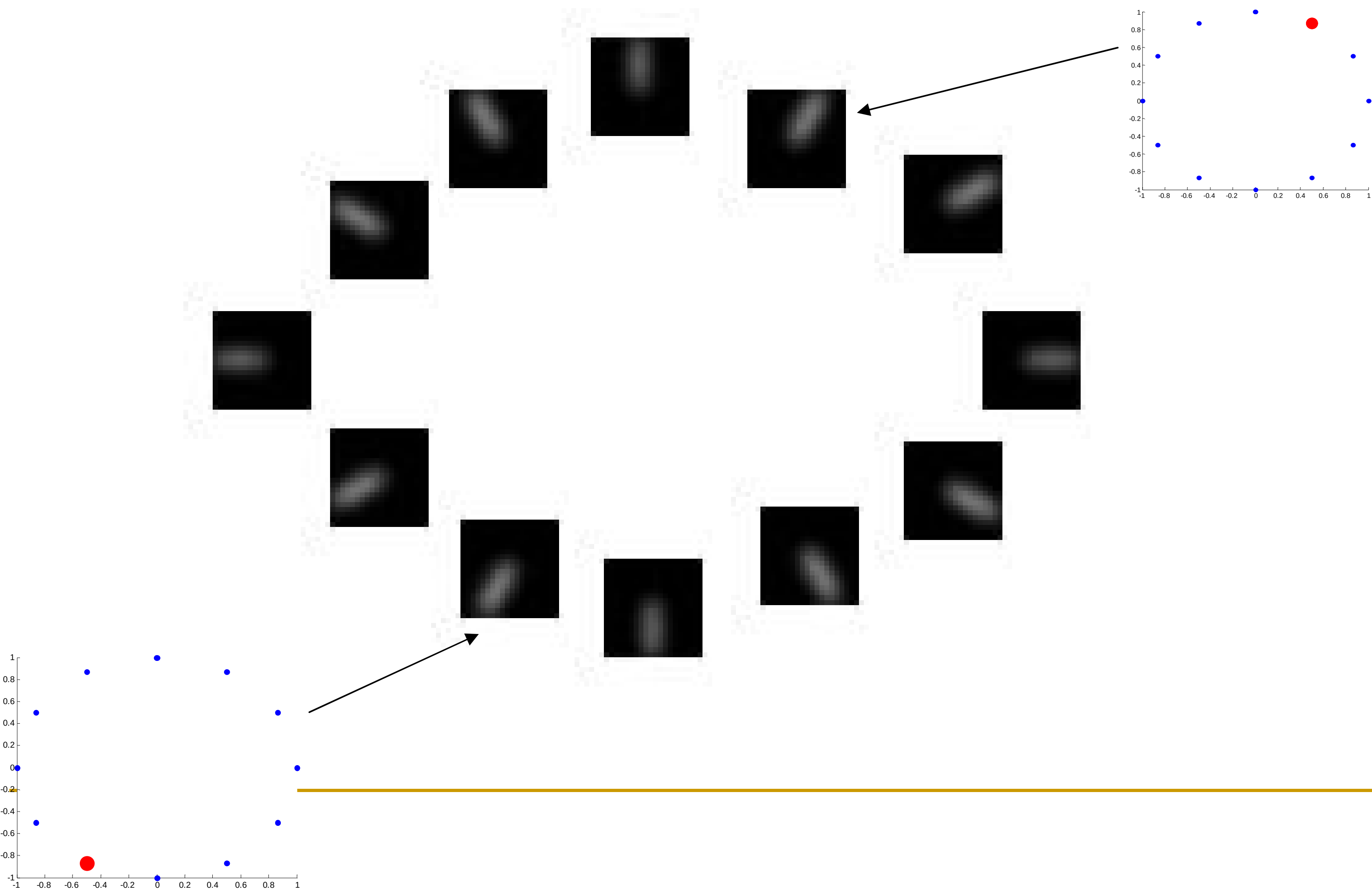


Manifold Approximation Found
With Geometric Harmonics

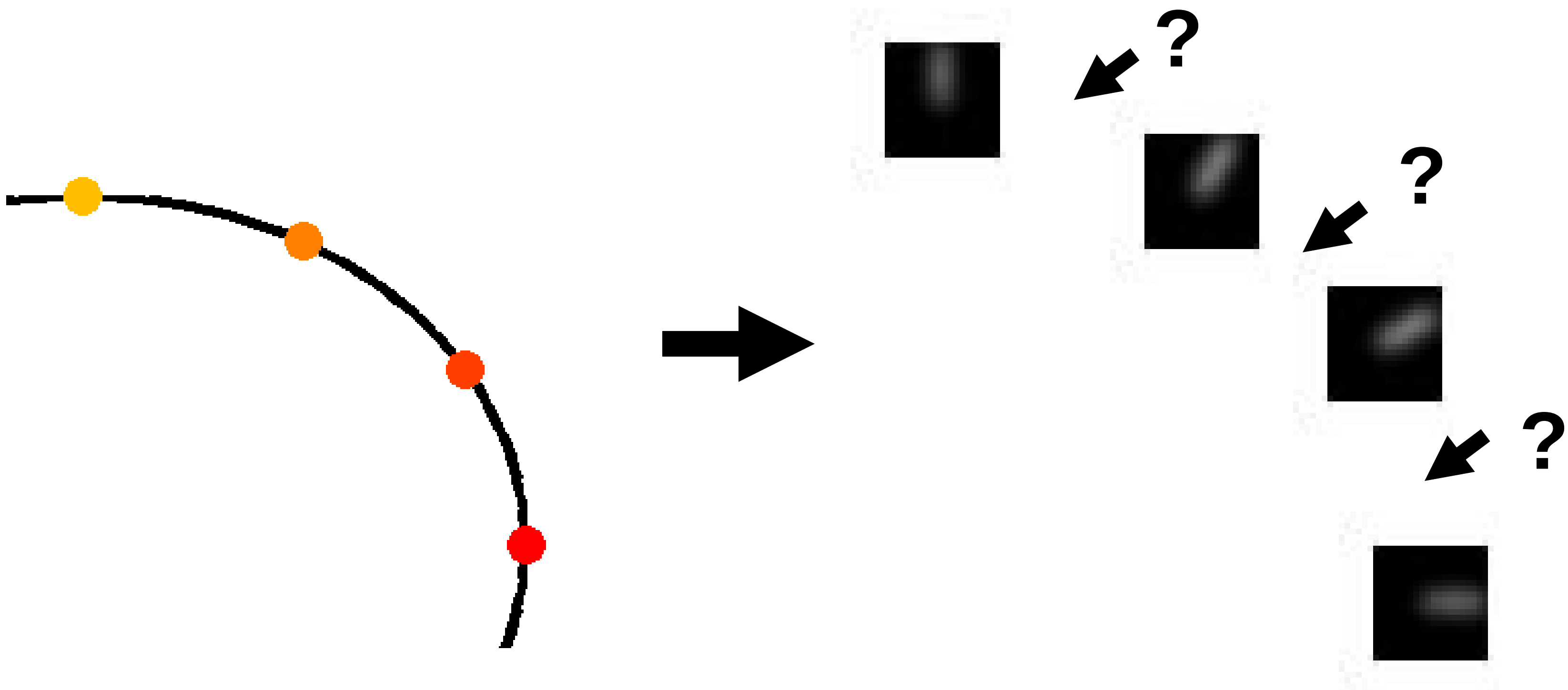
Potential Pitfalls: Results From Geometric Harmonics with Bad Parameters



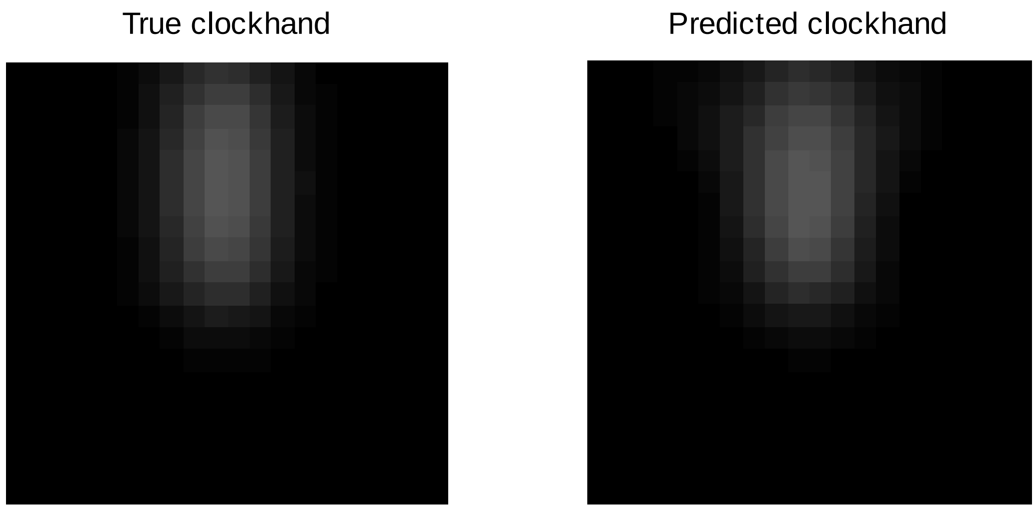
Our Higher-Dimensional Synthetic Manifold



Predicting Brain Responses Between Sample Points on the Stimulus Manifold



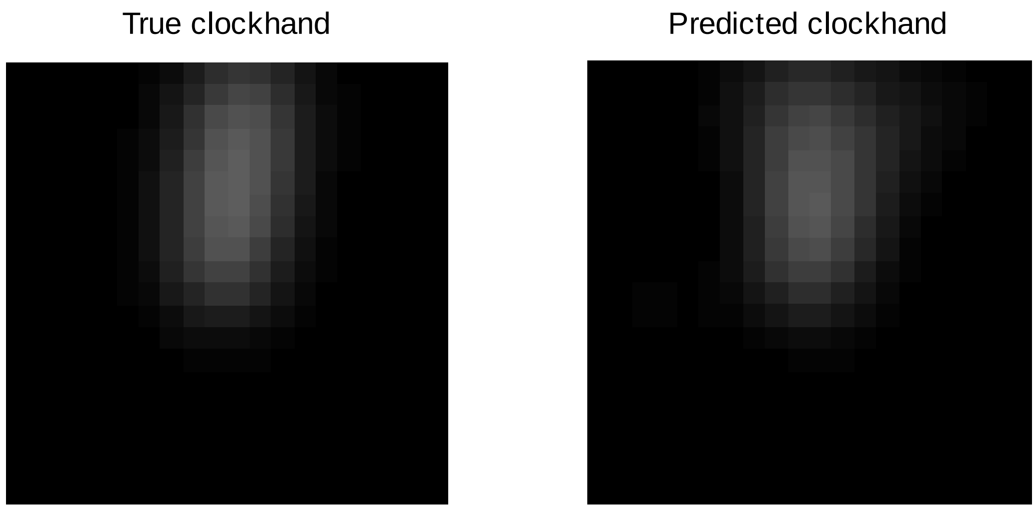
Learning clock hand images



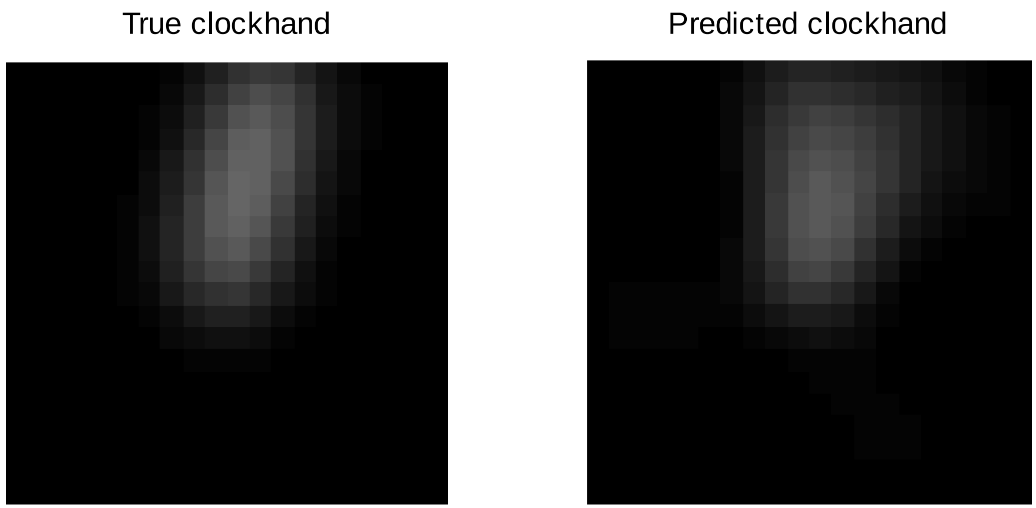
Learning clock hand images

True clockhand Predicted clockhand

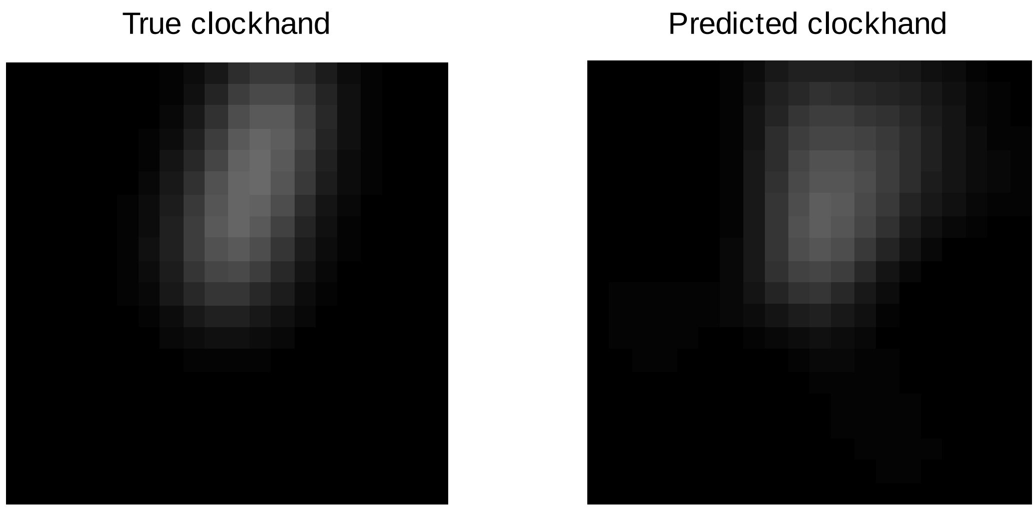
Learning clock hand images



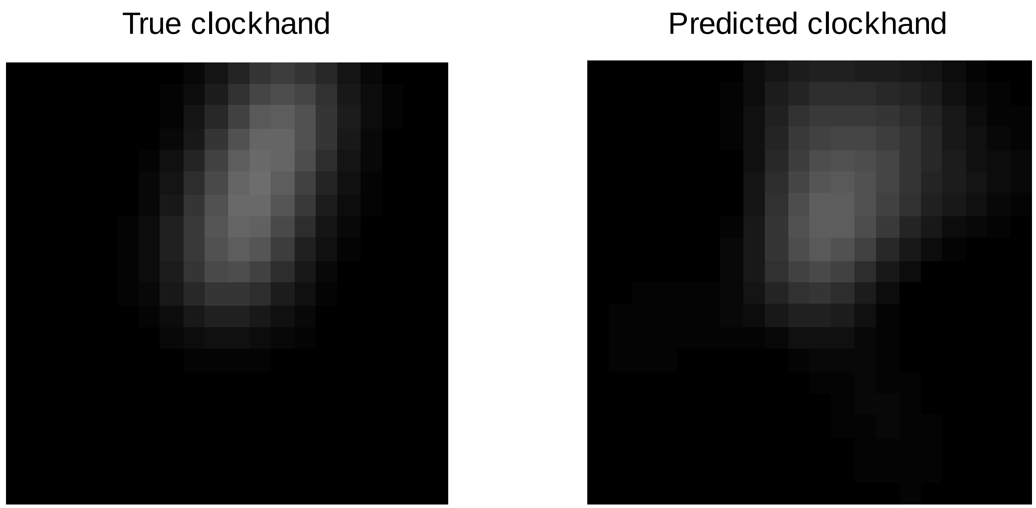
Learning clock hand images



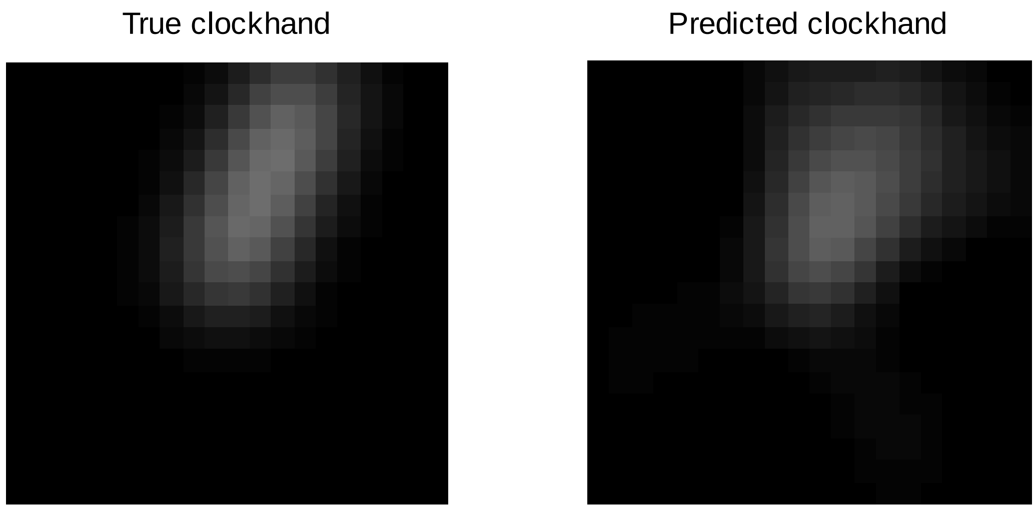
Learning clock hand images



Learning clock hand images

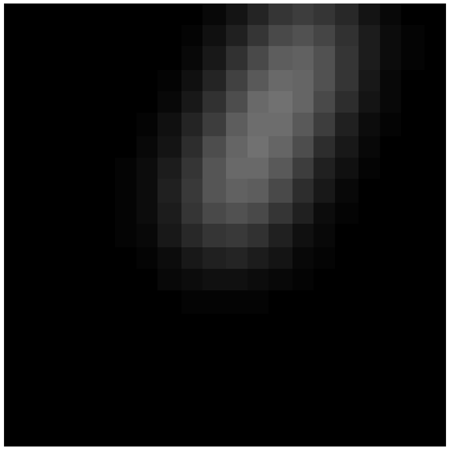


Learning clock hand images

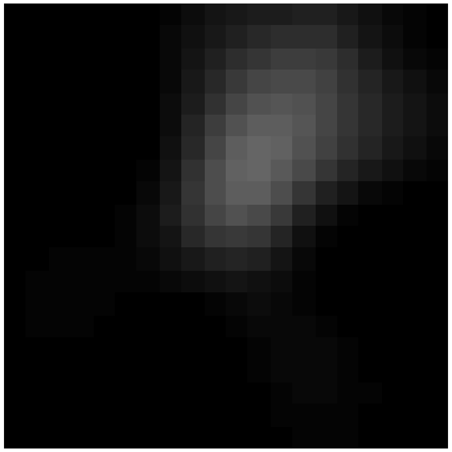


Learning clock hand images

True clockhand

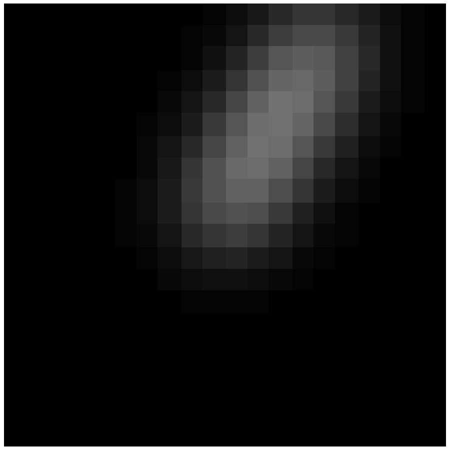


Predicted clockhand

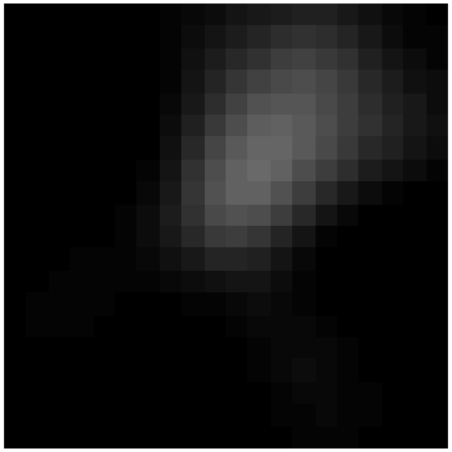


Learning clock hand images

True clockhand



Predicted clockhand



Future Directions

- Three limits to geometric harmonics:
  - Not robust to noise
    - Regularization?
  - Needs many samples
    - Compressive Dimensionality Reduction?
  - Parameter specificity
    - Finding a consistent way to choose parameters
    - Trade-offs between voxel count, voxel variation, regressor variation, number of TRs, and optimal parameters.

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