Not ‘Won’t you be my neighbor?’ But ‘Should you be my neighbor?’

Optimal Information Gathering in Social Networks

Eleanor Brush

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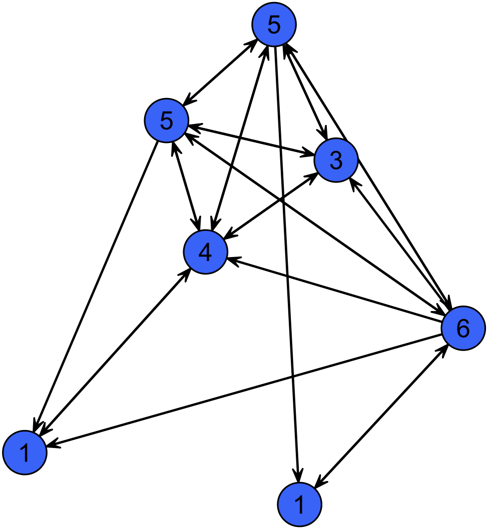

5
5
3
4
6
1
1

What's the optimal way to gather information from a social network?

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1. a criterion of performance

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1. a criterion of performance
2. a strategy to maximize that criterion

Overview

1. a criterion for the whole network
2. a criterion for individuals
3. consequences of and strategy for optimizing individual performance
4. a measure of informational burden
5. future directions

$$\dot{x}_i(t) = \sum_j w_{ij}\big(x_j(t) - x_i(t)\big) + \xi_i(t)$$

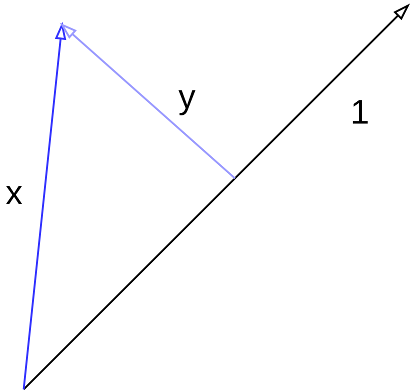
where $\sum_j w_{ij} = 1$ and $\xi_i(t) \sim N(0,1)$ i.i.d.

$\implies$ consensus with every $x_i = c$

$$\dot{x}(t) = -Lx(t) + \xi(t)$$

$$\text{where } L_{ij} = \begin{cases} 1 & \text{if } i = j \\ -w_{ij} & \text{if } i \neq j \end{cases}$$

$\Longrightarrow$ consensus with $x \propto \vec{1}$


x
y
1

Deviation from Consensus

Define the orthogonal transformation $Q \in \mathbb{R}^{n-1 \times n}$ with

1. $Q\vec{1} = 0$
2. $QQ^T = I_{n-1}$ and
3. $Q^T Q = I_n - \frac{1}{n}\vec{1} \times \vec{1}^T$

Define $y = Qx$ and $\overline{L} = QLQ^T$.

$$\implies \dot{y}(t) = -\overline{L}y(t) + Q\xi(t)$$

Starling flock networks manage uncertainty in consensus at low cost,
George F. Young, Luca Scardovi, Andrea Cavagna, Irene Giardina, and Naomi E. Leonard. 2012.

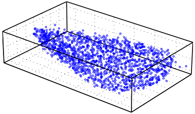
Network Optimization

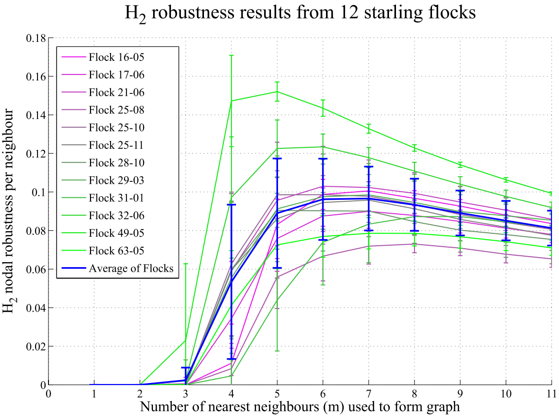
One measure of performance:

$$H(L) = \lim_{t \to \infty} E[||y(t)||]$$

Objective: choose the number of neighbors m that maximizes

$$\frac{1}{\left(\frac{H}{\sqrt{N}}\right) m}$$





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Learning about the world

One node gets an external signal so that

$$
\begin{aligned}
x_i(t) &= \text{signal} \;\; \text{for all } t \text{ and} \\
\dot{x}_j(t) &= \sum_k w_{jk}\big(x_k(t) - x_j(t)\big) + \xi_j(t) \text{ for } k \neq i \\
\delta_j(t) &:= \;\text{signal}\; - x_j(t) \\
\Rightarrow \dot{\delta}(t) &= -L^i\delta(t) + L^i\xi(t)
\end{aligned}
$$

where $L^i = L$ without the i^{th} row and column

We can approximate $\delta_j(t)$ with

$$\delta_j(t) \sim e^{-\lambda^i t} v_j^i$$

where λ^i is the smallest eigenvalue of L^i and v^i is its eigenvector.

Deviations

from consensus: $\dot{y}(t) = -\overline{L}y(t) + Q\xi(t)$

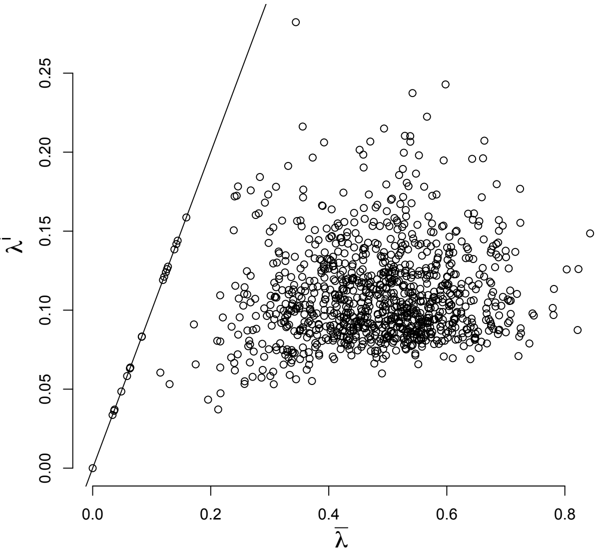
from external signal: $\dot{\delta}(t) = -L^i\delta(t) - L^i\xi(t)$

Minimizing

$$\delta_j(t) \sim e^{-\lambda^i t} v_j^i$$

usually optimizes consensus in the whole group because

1. the rate of convergence to consensus is given by $\overline{\lambda}$, the smallest eigenvalue of $\overline{L}$, and $\lambda^i \leq \overline{\lambda}$



0.00
0.05
0.10
0.15
0.20
0.25
λ^i
0.0
0.2
0.4
0.6
0.8
$\bar{\lambda}$

Deviations

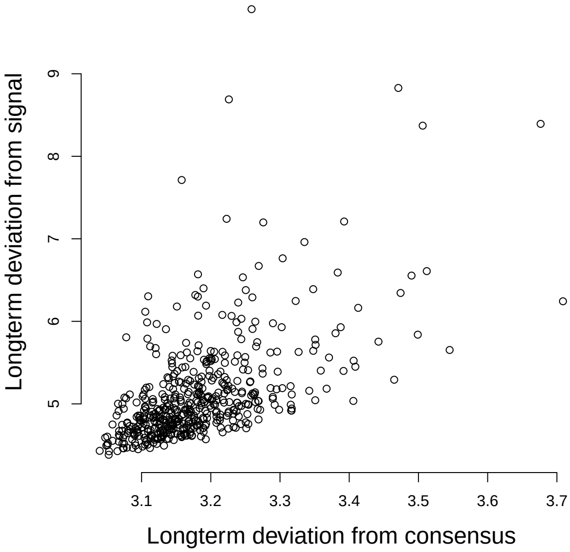
from consensus: $\dot{y}(t) = -\overline{L}y(t) + Q\xi(t)$
from external signal: $\dot{\delta}(t) = -L^i\delta(t) - L^i\xi(t)$

Minimizing

$$\delta_j(t) \sim e^{-\lambda^i t} v_j^i$$

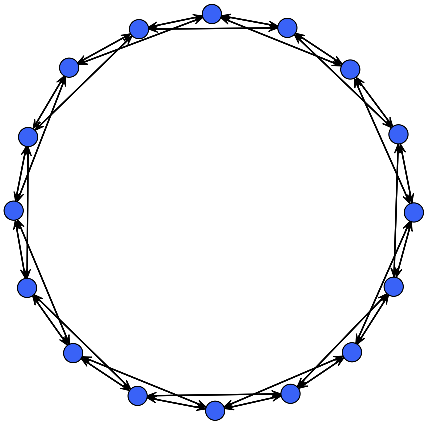
usually optimizes consensus in the whole group because

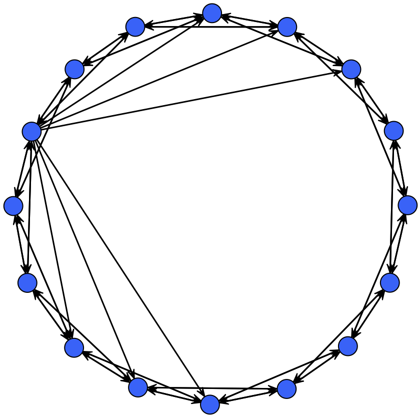
1. the rate of convergence to consensus is given by $\overline{\lambda}$, the smallest eigenvalue of $\overline{L}$, and $\lambda^i \leq \overline{\lambda}$
2. $\lim_{t\to\infty} E[\|\delta(t)\|] \geq \lim_{t\to\infty} E[\|y(t)\|]$


Longterm deviation from signal
5
6
7
8
9
3.1
3.2
3.3
3.4
3.5
3.6
3.7
Longterm deviation from consensus

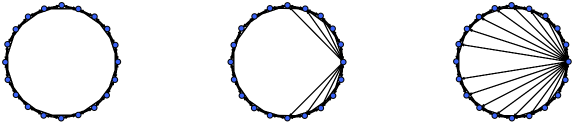
Individual Optimization

Objective: each node j wants to minimize $s_j = \langle e^{-\lambda^i} v_j^i \rangle_i$

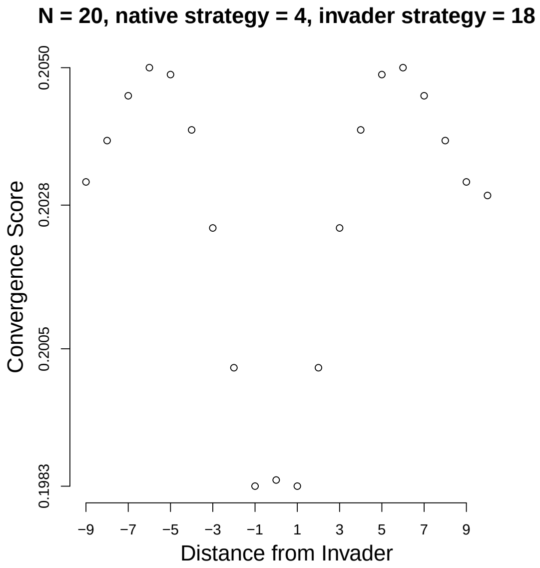


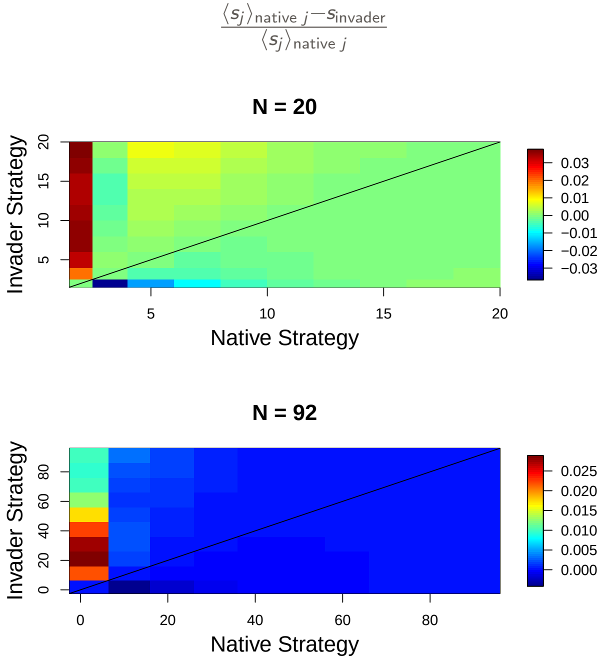


Optimizing s_j affects everyone.

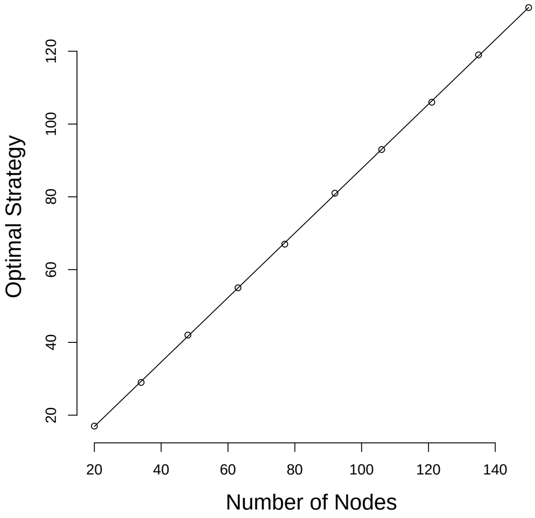


s_{invader}	0.2018	0.1993	0.1984
$\langle s_j \rangle_{\text{native } j}$	0.2018	0.2025	0.2029
$\lim_{t\to\infty} E[\lVert y(t) \rVert]$	4.10	3.99	3.89
$\lim_{t\to\infty} E[\lVert \delta(t) \rVert]_{i=10}$	5.98	5.48	5.09


N = 20, native strategy = 4, invader strategy = 18
Convergence Score
0.2050
0.2028
0.2005
0.1983
−9
−7
−5
−3
−1
1
3
5
7
9
Distance from Invader



⟨s_j⟩native j − s_invader / ⟨s_j⟩native j
N = 20
Invader Strategy
Native Strategy
5
10
15
20
0.03
0.02
0.01
0.00
−0.01
−0.02
−0.03
N = 92
Invader Strategy
Native Strategy
0
20
40
60
80
0.025
0.020
0.015
0.010
0.005
0.000



Optimal Strategy
20
40
60
80
100
120
20
40
60
80
100
120
140
Number of Nodes

Future Directions

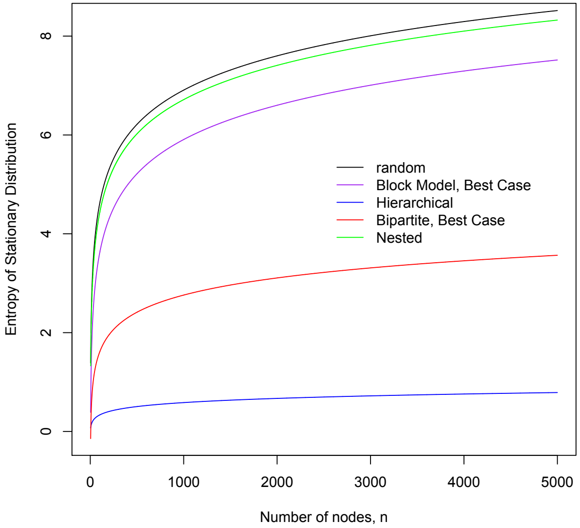
- allow for more variation among strategies across nodes and allow the strategies to evolve
- presence of multiple signals with different values
- incorporate a principled measure of cognitive costs / informational burden of paying attention to more neighbors
- extend to other systems
- real network data

Measure of Informational Burden

$w_{ij} =$ weight of edge from $i \rightarrow j$, $\sum_j w_{ij} = 1$

$\implies$ the vector p such that $pW = p$ is the stationary distribution

$\beta(W) :=$ the entropy of the distribution p , $-\sum_i p_i \log(p_i)$



Entropy of Stationary Distribution
0
2
4
6
8
0
1000
2000
3000
4000
5000
Number of nodes, n
random
Block Model, Best Case
Hierarchical
Bipartite, Best Case
Nested

Future Directions

- real network data
- how to minimize informational burden while maintaining network structure
- how informational constraints affects network inference

Acknowledgements

Simon Levin
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George Young

H_2 norm

Consider a dynamical system $\dot{v}(t) = Av + Bw$ and $z = Cv$. Let Σ be the solution to the Lyapunov equation

$$A^T\Sigma + \Sigma A + BB^T = 0.$$

Then the H_2 norm[1] of the system is

$$\sqrt{\mathrm{Tr}(C\Sigma C^T)}.$$

For y, this means that Σ solves

$$-\overline{L}^T\Sigma - \Sigma\overline{L} + I = 0 \text{ and } H_2 = \sqrt{\mathrm{Tr}(\Sigma)}.$$

It can be shown[1] that $H(L) = \lim_{t\to\infty} E[\|y(t)\|] = H_2(L)$.

[1] Robustness of Noisy Consensus Dynamics with Directed Communication, Proceedings of the American Control Conference, George Forrest Young, Luca Scardovi, and Naomi Ehrich Leonard, 2010.