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Metaheuristic Lesson: Genetic Algorithm – VRP (Vehicle Routing Problem)

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Valdecy Pereira

Universidade Federal Fluminense

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Lesson: Genetic Algorithm – VRP (Vehicle Routing Problem)

Professor: Valdecy Pereira, D. Sc.

email: valdecy.pereira@gmail.com

Outline

1. Genetic Algorithm

2. Initialization

3. Selection

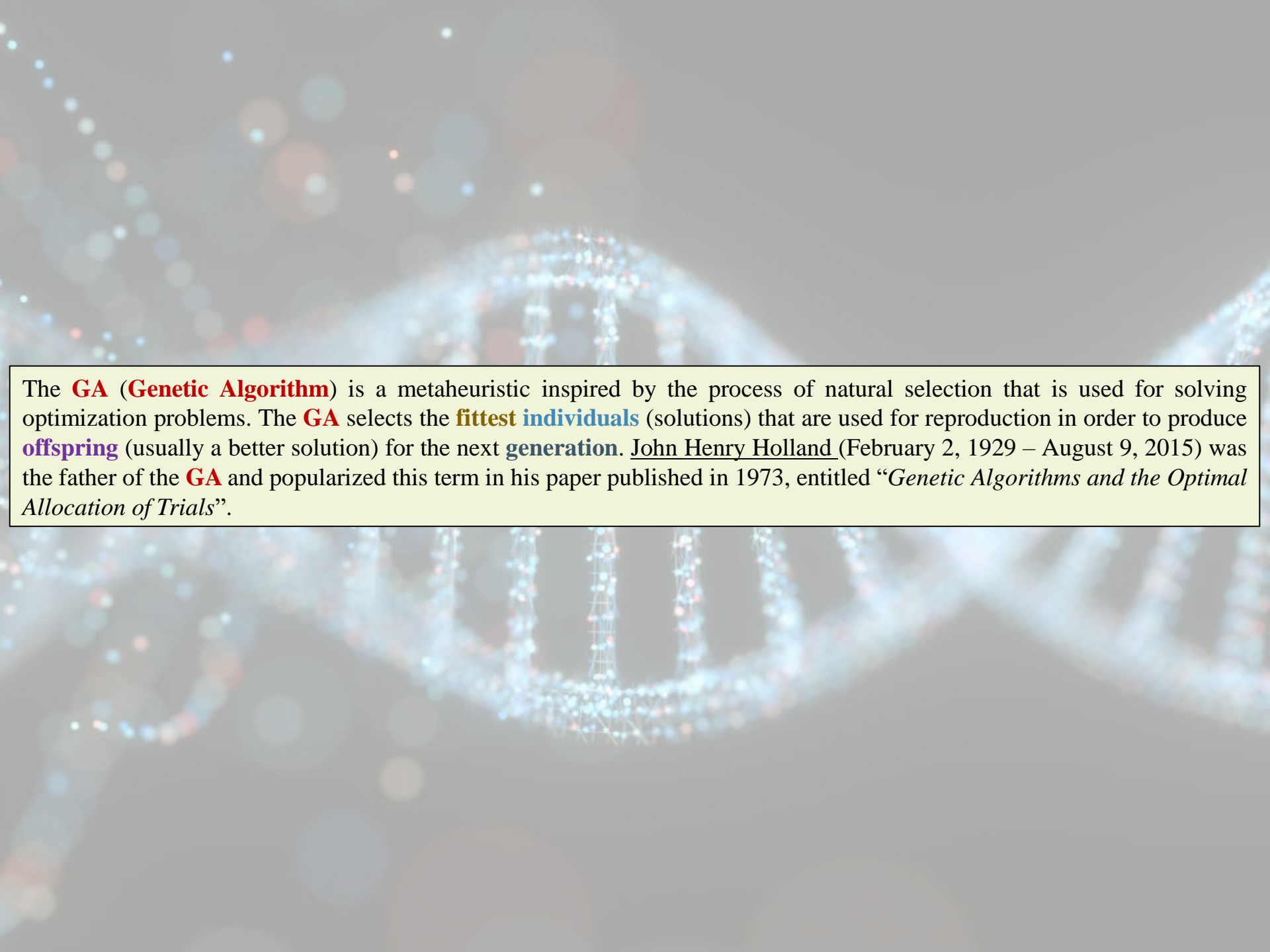
4. Crossover

5. Mutation

6. Elitism

7. Github





The **GA** (**Genetic Algorithm**) is a metaheuristic inspired by the process of natural selection that is used for solving optimization problems. The **GA** selects the **fittest individuals** (solutions) that are used for reproduction in order to produce **offspring** (usually a better solution) for the next **generation**. John Henry Holland (February 2, 1929 – August 9, 2015) was the father of the **GA** and popularized this term in his paper published in 1973, entitled “*Genetic Algorithms and the Optimal Allocation of Trials*”.

MH – *Genetic Algorithm*

The **GA** (**Genetic Algorithm**) can be understood as a search and optimization technique, inspired by the Darwinian principle of the evolution of species and genetics. It can find the global optimum solution or local optimum solutions depending on the initial setup and parameter calibration.

The **GA** creates an abstract representations (**chromosome** or **genome**) of a candidate solution (**individual**). Traditionally, solutions are encoded in a binary form, but other encodings are also possible. To solve an optimization problem, initially a random set of candidate solutions are created (**population**), and then it evolves towards a better solution using genetic operators called **crossover** and **mutation** to create a **new population**.

The **GA** represents an iterative process, where each iteration is called a **generation**. Then, evaluating the **fitness** of each **individual**, we can select a pair for breeding that will generate an **offspring**. To create the **offspring**, the **crossover** operator exchanges parts of the pair of **chromosomes**, and the **mutation** operator changes the **gene** value in some randomly chosen location of the **offspring chromosome**. This process is repeated until the new **population** has enough members, ending the current **generation**. A common practice is to terminate a **GA** after a specified number of **generations**.

The best solution (**elite**) of a **generation** can be stored and then given to next **generation**, this optional process is called **elitism**.

MH – Genetic Algorithm

GA Pseudocode

*Randomly initialize the first **Population** ($i = 0$)*

*Set η (**Population** size) and k (number of **generations**)*

for ($i = 1 ; i = k ; i++$) **do**

*Add the **Elite** as a member of the **Population** i (Optional)*

while $\text{card}(\text{Population } i) < \eta$ **do**

*Evaluate the **Fitness** of all **Individuals** the **Population** $i - 1$*

*Using the **Fitness** from the **Population** $i - 1$, select any two **Individuals** to breed*

*Create one **Offspring** using the genetic operators **Crossover** and **Mutation***

*Add the **Offspring** as a member of the **Population** i*

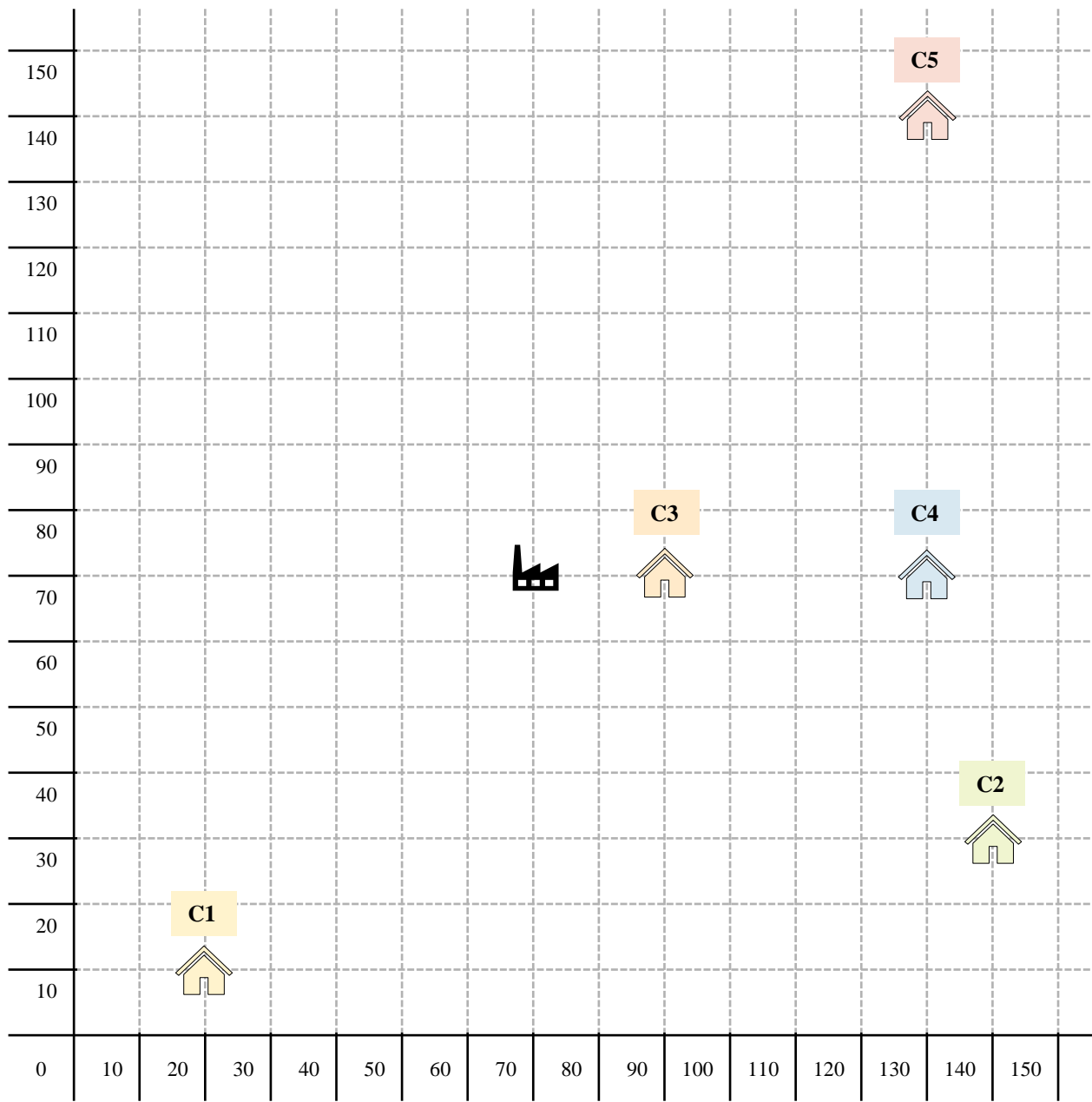
next i

MH – Genetic Algorithm

Suppose that we want to minimize the following **CVRP** (Capacitated Vehicle Routing Problem), where each vehicle capacity is equal to 250. Infeasible solutions are penalized with a value equal to 1000 for each violation.

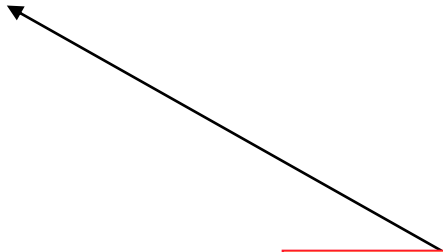
Id	x	y	Demand
Depot	70	70	-/-
Client 1	20	10	50
Client 2	140	30	60
Client 3	90	70	50
Client 4	130	70	40
Client 5	130	140	250

	Depot	Client 1	Client 2	Client 3	Client 4	Client 5
Depot	0	78.10	80.62	20.00	60.00	92.20
Client 1	78.10	0	121.66	92.20	125.30	170.29
Client 2	80.62	121.66	0	64.03	41.23	110.45
Client 3	20.00	92.20	64.03	0	40.00	80.62
Client 4	60.00	125.30	41.23	40.00	0	70.00
Client 5	92.20	170.29	110.45	80.62	70.00	0



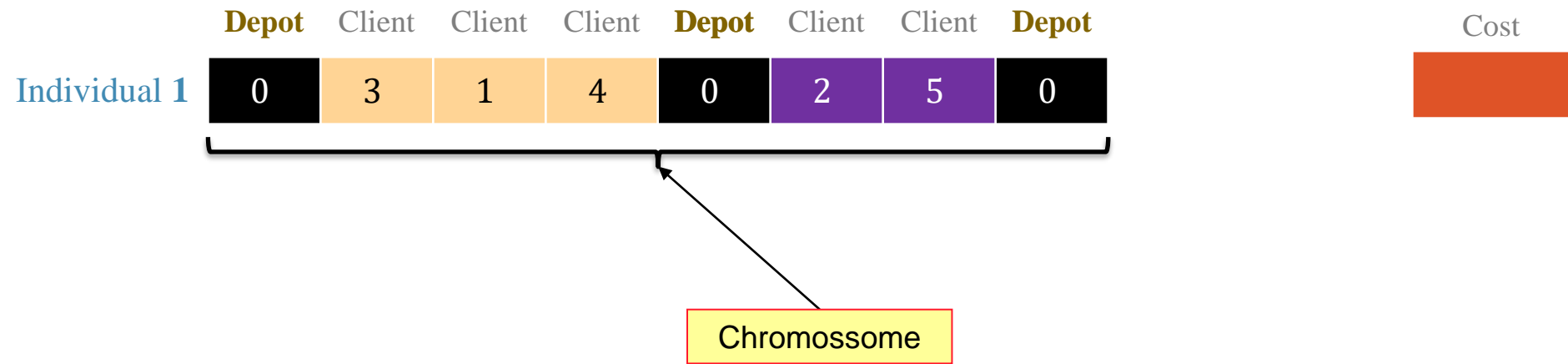
MH – *Genetic Algorithm*

Individual 1

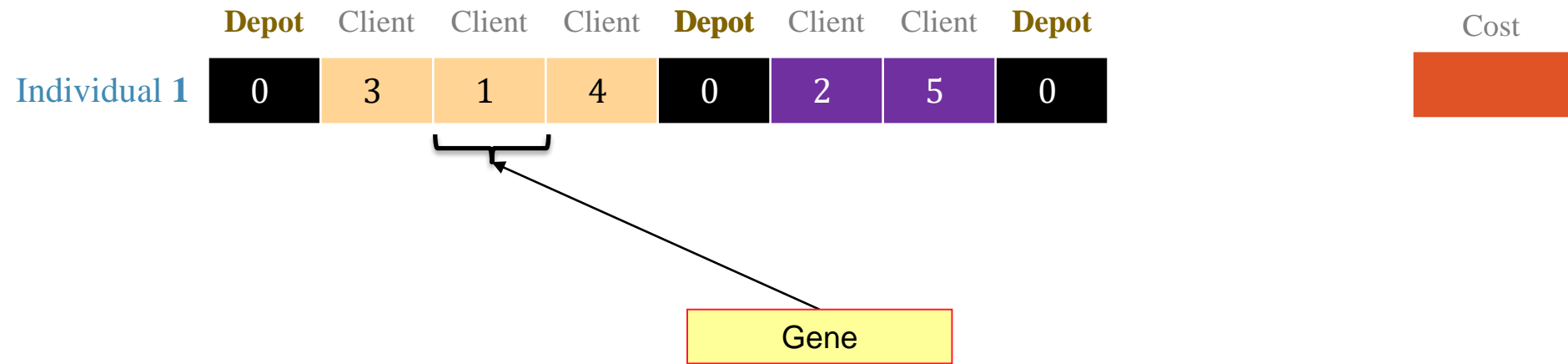


Encoding a
Solution

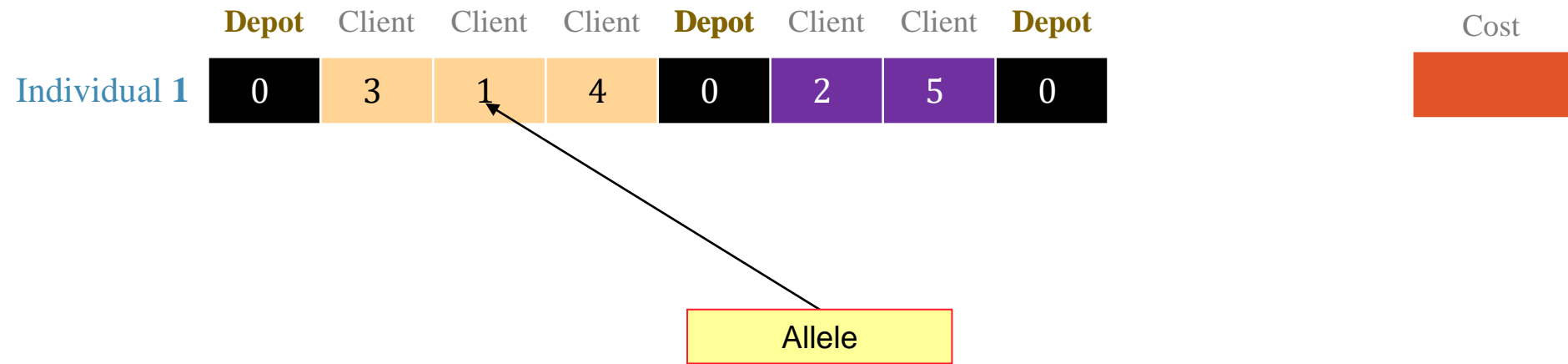
MH – *Genetic Algorithm*



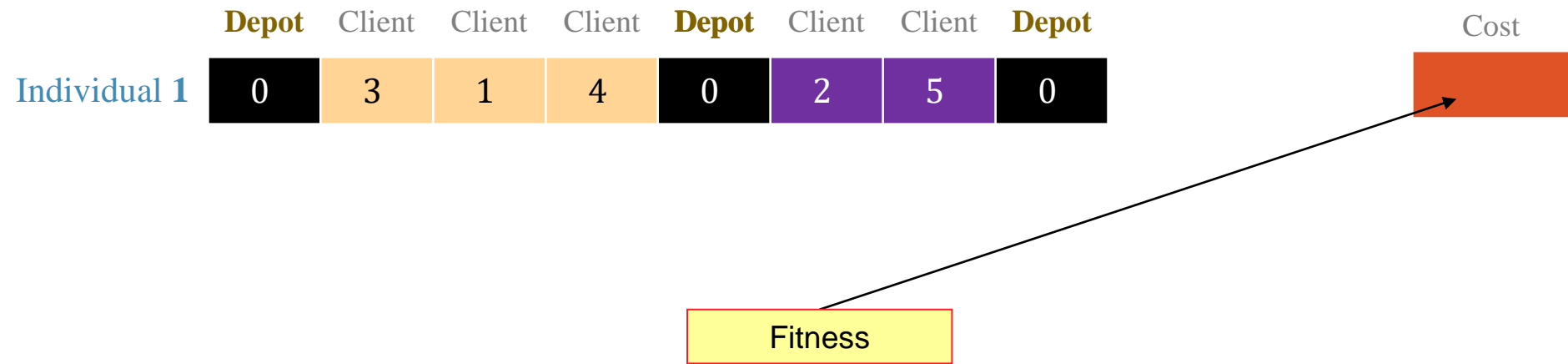
MH – *Genetic Algorithm*



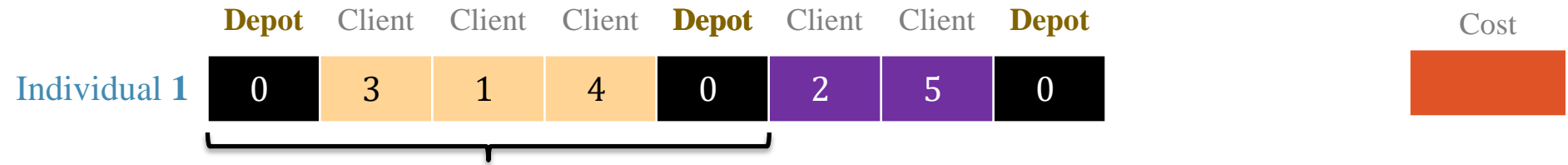
MH – *Genetic Algorithm*



MH – *Genetic Algorithm*

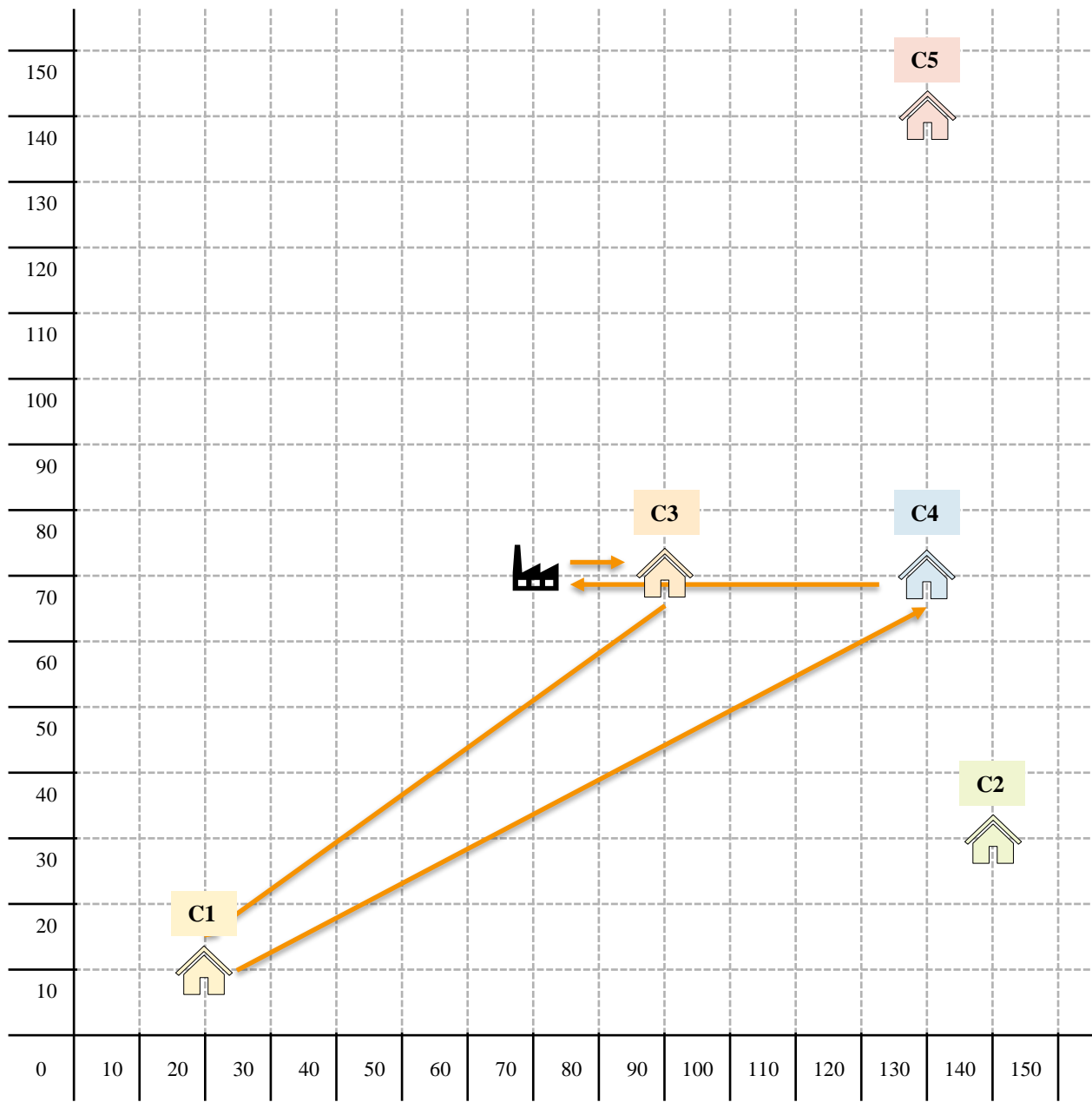


MH – *Genetic Algorithm*

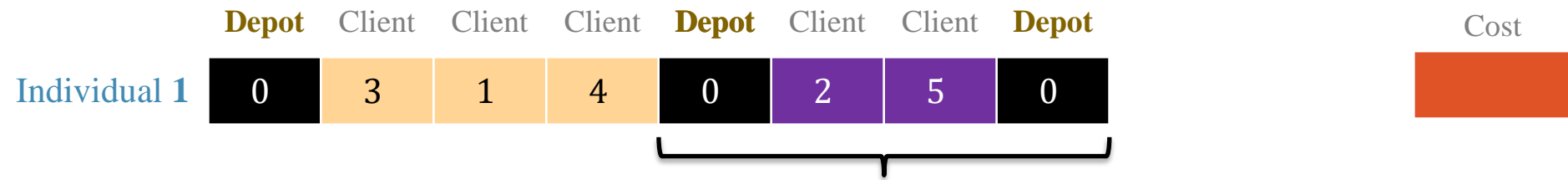


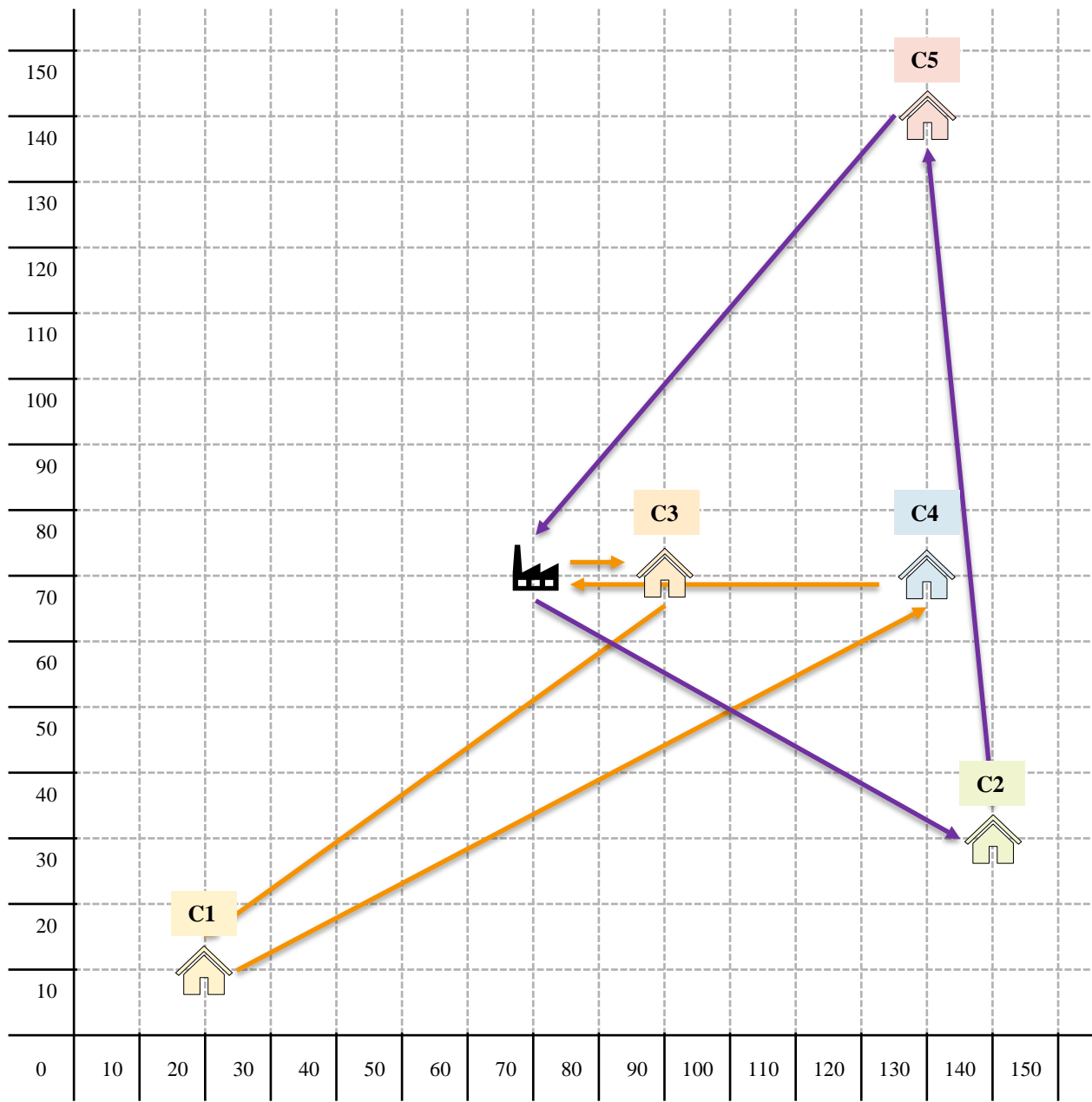
Route 1:

0 – 3 – 1 – 4 – 0



MH – *Genetic Algorithm*





Route 1: 0 – 3 – 1 – 4 – 0

Route 2: 0 – 2 – 5 – 0

	Depot	Client 1	Client 2	Client 3	Client 4	Client 5
Depot	0	78.10	80.62	20.00	60.00	92.20
Client 1	78.10	0	121.66	92.20	125.30	170.29
Client 2	80.62	121.66	0	64.03	41.23	110.45
Client 3	20.00	92.20	64.03	0	40.00	80.62
Client 4	60.00	125.30	41.23	40.00	0	70.00
Client 5	92.20	170.29	110.45	80.62	70.00	0

$$\text{Load} = 50 + 50 + 40 = 140$$

Route 1: 0 – **3** – **1** – **4** – 0

Route 2: 0 – 2 – 5 – 0

Id	Demand
Client 1	50
Client 2	60
Client 3	50
Client 4	40
Client 5	250

	Depot	Client 1	Client 2	Client 3	Client 4	Client 5
Depot	0	78.10	80.62	20.00	60.00	92.20
Client 1	78.10	0	121.66	92.20	125.30	170.29
Client 2	80.62	121.66	0	64.03	41.23	110.45
Client 3	20.00	92.20	64.03	0	40.00	80.62
Client 4	60.00	125.30	41.23	40.00	0	70.00
Client 5	92.20	170.29	110.45	80.62	70.00	0

Load = 140

Route 1: **0 – 3** – 1 – 4 – 0

Cost = 20

Route 2: 0 – 2 – 5 – 0

	Depot	Client 1	Client 2	Client 3	Client 4	Client 5
Depot	0	78.10	80.62	20.00	60.00	92.20
Client 1	78.10	0	121.66	92.20	125.30	170.29
Client 2	80.62	121.66	0	64.03	41.23	110.45
Client 3	20.00	92.20	64.03	0	40.00	80.62
Client 4	60.00	125.30	41.23	40.00	0	70.00
Client 5	92.20	170.29	110.45	80.62	70.00	0

Load = 140

Route 1: 0 – 3 – 1 – 4 – 0

Cost = 20 + 92.20

Route 2: 0 – 2 – 5 – 0

	Depot	Client 1	Client 2	Client 3	Client 4	Client 5
Depot	0	78.10	80.62	20.00	60.00	92.20
Client 1	78.10	0	121.66	92.20	125.30	170.29
Client 2	80.62	121.66	0	64.03	41.23	110.45
Client 3	20.00	92.20	64.03	0	40.00	80.62
Client 4	60.00	125.30	41.23	40.00	0	70.00
Client 5	92.20	170.29	110.45	80.62	70.00	0

Load = 140

Route 1: 0 – 3 – **1** – **4** – 0

Cost = 20 + 92.20 + 125.30

Route 2: 0 – 2 – 5 – 0

	Depot	Client 1	Client 2	Client 3	Client 4	Client 5
Depot	0	78.10	80.62	20.00	60.00	92.20
Client 1	78.10	0	121.66	92.20	125.30	170.29
Client 2	80.62	121.66	0	64.03	41.23	110.45
Client 3	20.00	92.20	64.03	0	40.00	80.62
Client 4	60.00	125.30	41.23	40.00	0	70.00
Client 5	92.20	170.29	110.45	80.62	70.00	0

Load = 140

Route 1: 0 – 3 – 1 – **4** – **0**

Cost = 20 + 92.20 + 125.30 + 60

Route 2: 0 – 2 – 5 – 0

	Depot	Client 1	Client 2	Client 3	Client 4	Client 5
Depot	0	78.10	80.62	20.00	60.00	92.20
Client 1	78.10	0	121.66	92.20	125.30	170.29
Client 2	80.62	121.66	0	64.03	41.23	110.45
Client 3	20.00	92.20	64.03	0	40.00	80.62
Client 4	60.00	125.30	41.23	40.00	0	70.00
Client 5	92.20	170.29	110.45	80.62	70.00	0

Load = 140

Route 1: 0 – 3 – 1 – 4 – 0

Cost = 20 + 92.20 + 125.30 + 60 = 297.50

Route 2: 0 – 2 – 5 – 0

	Depot	Client 1	Client 2	Client 3	Client 4	Client 5
Depot	0	78.10	80.62	20.00	60.00	92.20
Client 1	78.10	0	121.66	92.20	125.30	170.29
Client 2	80.62	121.66	0	64.03	41.23	110.45
Client 3	20.00	92.20	64.03	0	40.00	80.62
Client 4	60.00	125.30	41.23	40.00	0	70.00
Client 5	92.20	170.29	110.45	80.62	70.00	0

Load = 140

Route 1: 0 – 3 – 1 – 4 – 0

Cost = 297.50

Route 2: 0 – **2** – **5** – 0

Load = 60 + 250 = 310

Id	Demand
Client 1	50
Client 2	60
Client 3	50
Client 4	40
Client 5	250

	Depot	Client 1	Client 2	Client 3	Client 4	Client 5
Depot	0	78.10	80.62	20.00	60.00	92.20
Client 1	78.10	0	121.66	92.20	125.30	170.29
Client 2	80.62	121.66	0	64.03	41.23	110.45
Client 3	20.00	92.20	64.03	0	40.00	80.62
Client 4	60.00	125.30	41.23	40.00	0	70.00
Client 5	92.20	170.29	110.45	80.62	70.00	0

Load = 140

Route 1: 0 – 3 – 1 – 4 – 0

Cost = 297.50

Route 2: 0 – 2 – 5 – 0

Cost = 1000

Load = 310

Penalty Value for violating the vehicle
maximum capacity constraint (250)

	Depot	Client 1	Client 2	Client 3	Client 4	Client 5
Depot	0	78.10	80.62	20.00	60.00	92.20
Client 1	78.10	0	121.66	92.20	125.30	170.29
Client 2	80.62	121.66	0	64.03	41.23	110.45
Client 3	20.00	92.20	64.03	0	40.00	80.62
Client 4	60.00	125.30	41.23	40.00	0	70.00
Client 5	92.20	170.29	110.45	80.62	70.00	0

Load = 140

Route 1: 0 – 3 – 1 – 4 – 0

Cost = 297.50

Route 2: 0 – 2 – 5 – 0

Cost = 1000

Load = 310

	Depot	Client 1	Client 2	Client 3	Client 4	Client 5
Depot	0	78.10	80.62	20.00	60.00	92.20
Client 1	78.10	0	121.66	92.20	125.30	170.29
Client 2	80.62	121.66	0	64.03	41.23	110.45
Client 3	20.00	92.20	64.03	0	40.00	80.62
Client 4	60.00	125.30	41.23	40.00	0	70.00
Client 5	92.20	170.29	110.45	80.62	70.00	0

Load = 140

Route 1: 0 – 3 – 1 – 4 – 0

Cost = 297.50

Route 2: 0 – 2 – 5 – 0

Cost = 1000 + 80.62

Load = 310

	Depot	Client 1	Client 2	Client 3	Client 4	Client 5
Depot	0	78.10	80.62	20.00	60.00	92.20
Client 1	78.10	0	121.66	92.20	125.30	170.29
Client 2	80.62	121.66	0	64.03	41.23	110.45
Client 3	20.00	92.20	64.03	0	40.00	80.62
Client 4	60.00	125.30	41.23	40.00	0	70.00
Client 5	92.20	170.29	110.45	80.62	70.00	0

Load = 140

Route 1: 0 – 3 – 1 – 4 – 0

Cost = 297.50

Route 2: 0 – 2 – 5 – 0

Cost = 1000 + 80.62 + 110.42

Load = 310

	Depot	Client 1	Client 2	Client 3	Client 4	Client 5
Depot	0	78.10	80.62	20.00	60.00	92.20
Client 1	78.10	0	121.66	92.20	125.30	170.29
Client 2	80.62	121.66	0	64.03	41.23	110.45
Client 3	20.00	92.20	64.03	0	40.00	80.62
Client 4	60.00	125.30	41.23	40.00	0	70.00
Client 5	92.20	170.29	110.45	80.62	70.00	0

Load = 140

Route 1: 0 – 3 – 1 – 4 – 0

Cost = 297.50

Route 2: 0 – 2 – 5 – 0

Cost = 1000 + 80.62 + 110.42 + 92.20

Load = 310

	Depot	Client 1	Client 2	Client 3	Client 4	Client 5
Depot	0	78.10	80.62	20.00	60.00	92.20
Client 1	78.10	0	121.66	92.20	125.30	170.29
Client 2	80.62	121.66	0	64.03	41.23	110.45
Client 3	20.00	92.20	64.03	0	40.00	80.62
Client 4	60.00	125.30	41.23	40.00	0	70.00
Client 5	92.20	170.29	110.45	80.62	70.00	0

Load = 140

Route 1: 0 – 3 – 1 – 4 – 0

Cost = 297.50

Route 2: 0 – 2 – 5 – 0

Cost = 1000 + 80.62 + 110.42 + 92.20 = 1283.24

Load = 310

	Depot	Client 1	Client 2	Client 3	Client 4	Client 5
Depot	0	78.10	80.62	20.00	60.00	92.20
Client 1	78.10	0	121.66	92.20	125.30	170.29
Client 2	80.62	121.66	0	64.03	41.23	110.45
Client 3	20.00	92.20	64.03	0	40.00	80.62
Client 4	60.00	125.30	41.23	40.00	0	70.00
Client 5	92.20	170.29	110.45	80.62	70.00	0

Route 1: 0 – 3 – 1 – 4 – 0

Cost = 297.50

Route 2: 0 – 2 – 5 – 0

Cost = 1283.24

Total Cost = 297.50 + 1283.24 = 1580.74 km

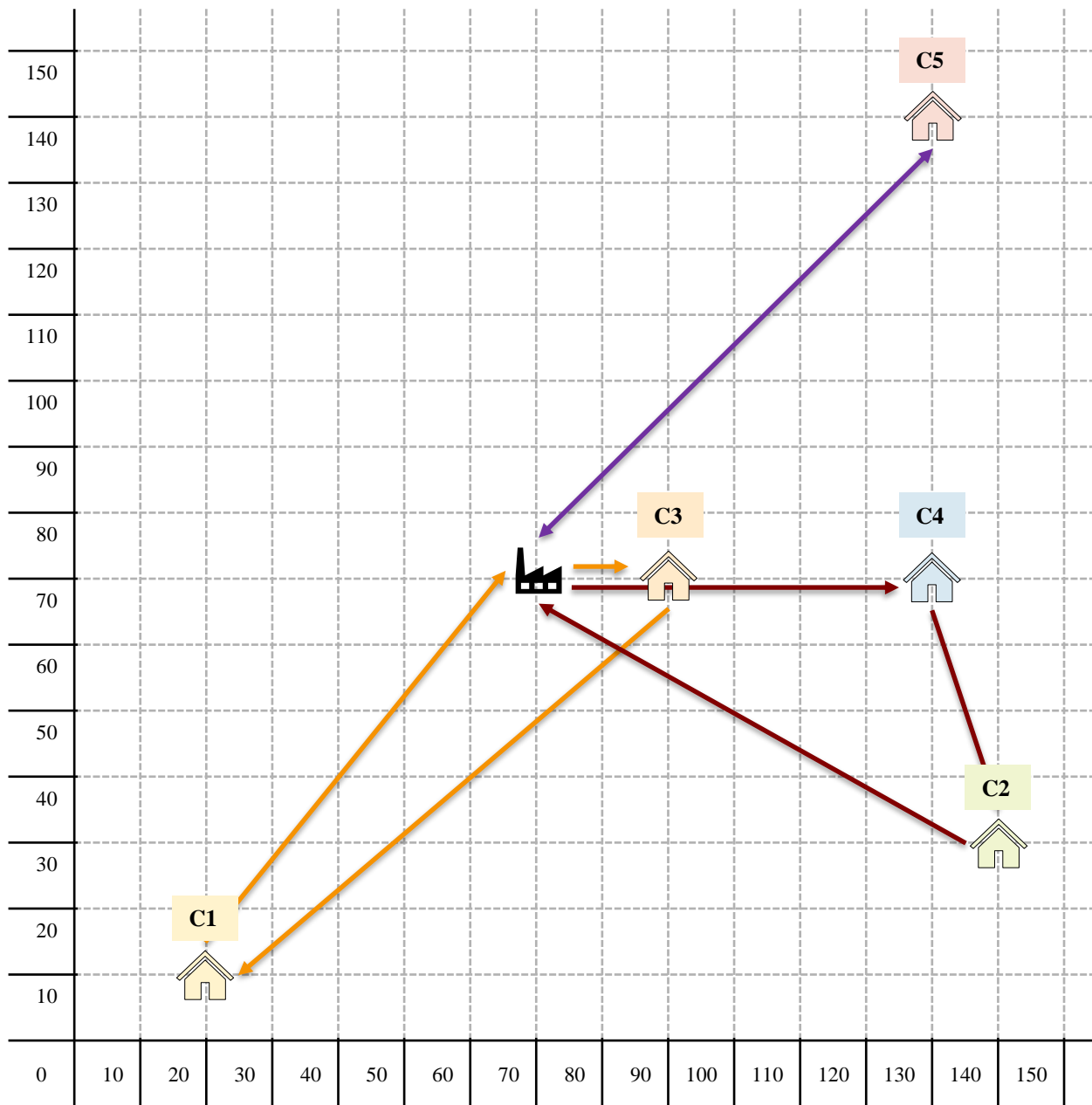
	Depot	Client 1	Client 2	Client 3	Client 4	Client 5
Depot	0	78.10	80.62	20.00	60.00	92.20
Client 1	78.10	0	121.66	92.20	125.30	170.29
Client 2	80.62	121.66	0	64.03	41.23	110.45
Client 3	20.00	92.20	64.03	0	40.00	80.62
Client 4	60.00	125.30	41.23	40.00	0	70.00
Client 5	92.20	170.29	110.45	80.62	70.00	0

MH – *Genetic Algorithm*

	Depot	Client	Client	Client	Depot	Client	Client	Depot	Cost
Individual 1	0	3	1	4	0	2	5	0	1580.74

MH – *Genetic Algorithm*

	Depot	Client	Client	Client	Depot	Client	Client	Depot	Cost
Individual 1	0	3	1	4	0	2	5	0	1580.74
Individual 2	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot
	0	3	1	0	5	0	4	2	0



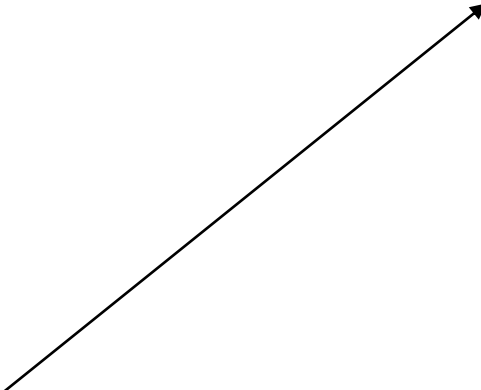
MH – Genetic Algorithm

Individual 1	Depot	Client	Client	Client	Depot	Client	Client	Depot	Cost	
	0	3	1	4	0	2	5	0	1580.74	
Individual 2	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot	Cost
	0	3	1	0	5	0	4	2	0	556.54

MH – *Genetic Algorithm*

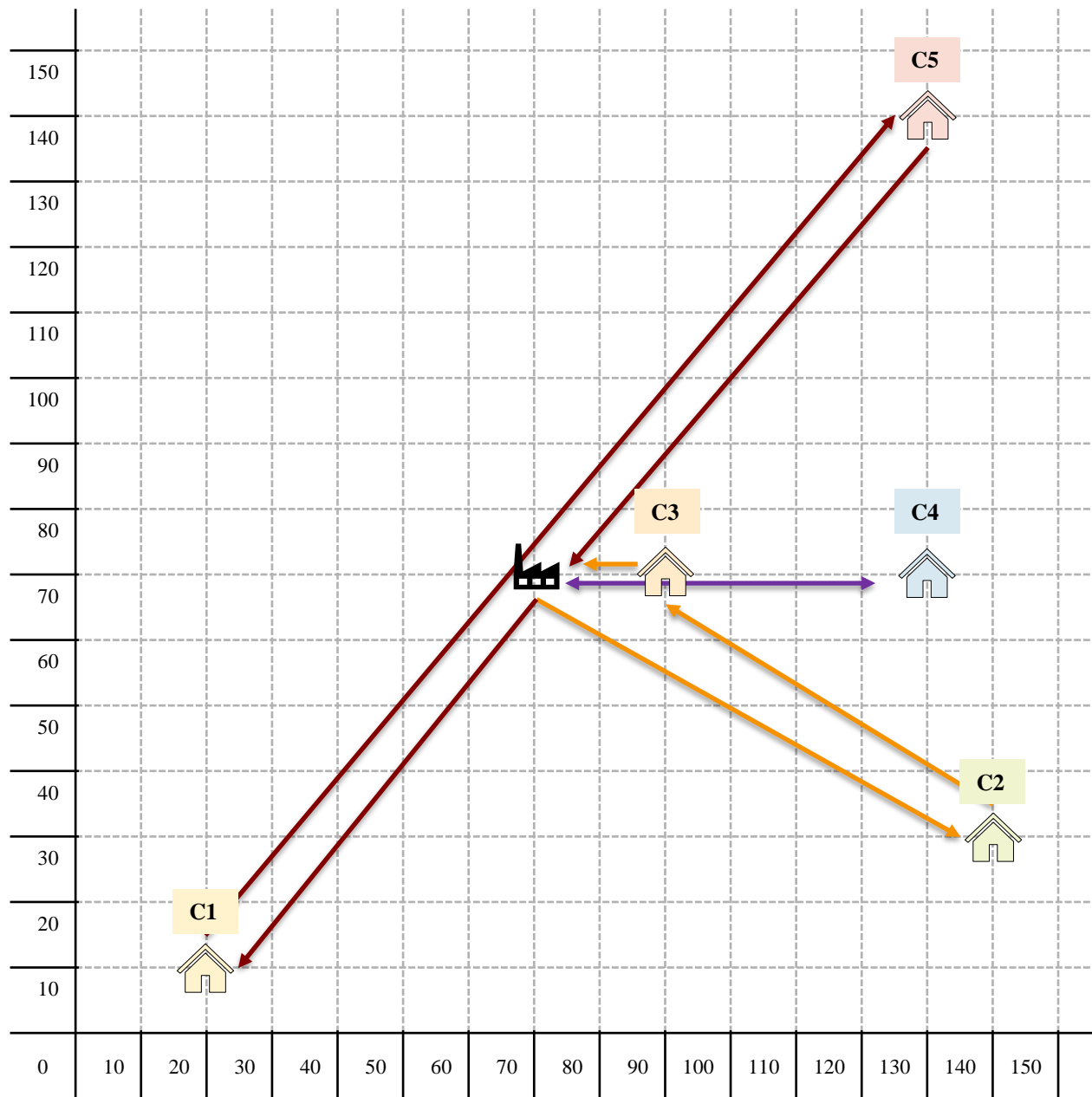
	Depot	Client	Client	Client	Depot	Client	Client	Depot	Cost
Individual 1	0	3	1	4	0	2	5	0	1580.74
Individual 2	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot
	0	3	1	0	5	0	4	2	0
									Cost
									556.54

No Penalty



MH – *Genetic Algorithm*

	Depot	Client	Client	Client	Depot	Client	Client	Depot	Cost
Individual 1	0	3	1	4	0	2	5	0	1580.74
Individual 2	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot
	0	3	1	0	5	0	4	2	0
Individual 3	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot
	0	2	3	0	4	0	1	5	0



MH – *Genetic Algorithm*

	Depot	Client	Client	Client	Depot	Client	Client	Depot	Cost
Individual 1	0	3	1	4	0	2	5	0	1580.74
Individual 2	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot
	0	3	1	0	5	0	4	2	0
Individual 3	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot
	0	2	3	0	4	0	1	5	0

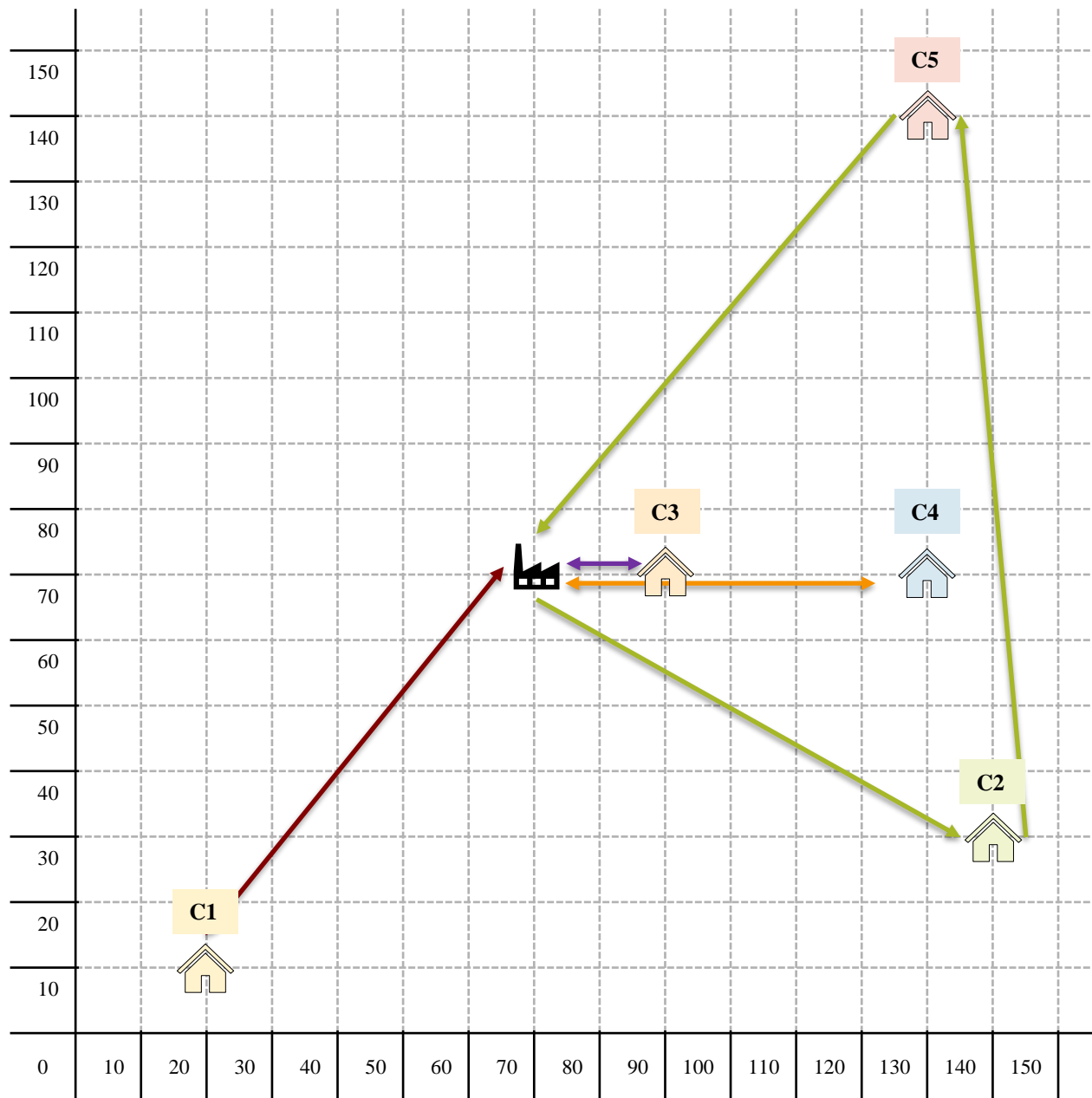
MH – Genetic Algorithm

	Depot	Client	Client	Client	Depot	Client	Client	Depot	Cost
Individual 1	0	3	1	4	0	2	5	0	1580.74
Individual 2	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot
	0	3	1	0	5	0	4	2	0
Individual 3	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot
	0	2	3	0	4	0	1	5	0

Penalty Value → 1625.25

MH – *Genetic Algorithm*

Individual 1	Depot	Client	Client	Client	Depot	Client	Client	Depot	Cost		
	0	3	1	4	0	2	5	0			
Individual 2	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot	Cost	
	0	3	1	0	5	0	4	2	0		
Individual 3	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot	Cost	
	0	2	3	0	4	0	1	5	0		
Individual 4	Depot	Client	Depot	Client	Depot	Client	Depot	Client	Client	Depot	Cost
	0	4	0	3	0	1	0	2	5	0	



MH – *Genetic Algorithm*

Individual 1	Depot	Client	Client	Client	Depot	Client	Client	Depot	Cost		
	0	3	1	4	0	2	5	0			
Individual 2	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot	Cost	
	0	3	1	0	5	0	4	2	0		
Individual 3	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot	Cost	
	0	2	3	0	4	0	1	5	0		
Individual 4	Depot	Client	Depot	Client	Depot	Client	Depot	Client	Client	Depot	Cost
	0	4	0	3	0	1	0	2	5	0	

MH – Genetic Algorithm

	Depot	Client	Client	Client	Depot	Client	Client	Depot		Cost	
Individual 1	0	3	1	4	0	2	5	0		1580.74	
	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot	Cost	
Individual 2	0	3	1	0	5	0	4	2	0	556.54	
	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot	Cost	
Individual 3	0	2	3	0	4	0	1	5	0	1625.25	
	Depot	Client	Depot	Client	Depot	Client	Depot	Client	Client	Depot	Cost
Individual 4	0	4	0	3	0	1	0	2	5	0	1599.48

Penalty Value



Initialization

MH – *Genetic Algorithm*

In the first **generation**, the **population** size depends on the nature of the problem, but a common approach is to have a **population** size with hundreds of **individuals**. Traditionally, this **population** is generated randomly, but it can start with any set of solutions.

MH – Genetic Algorithm

	Depot	Client	Client	Client	Depot	Client	Client	Depot		
Individual 1	0	3	1	4	0	2	5	0	Cost 1580.74	
	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot	
Individual 2	0	3	1	0	5	0	4	2	0	
	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot	
Individual 3	0	2	3	0	4	0	1	5	0	
	Depot	Client	Depot	Client	Depot	Client	Depot	Client	Client	Depot
Individual 4	0	4	0	3	0	1	0	2	5	0
	Depot	Client	Depot	Client	Depot	Client	Depot	Client	Client	Depot

Population



Selection

MH – *Genetic Algorithm*

For each successive **generation**, the fitter **individuals** (solutions with the best **fitness** values) are typically more likely to be selected to breed to form the **new population**. A small proportion of less fit **individuals** can also be selected to breed, helping keep the diversity of the **population**, preventing premature convergence on poor solutions.

A popular and well-studied selection method, called **Roulette Wheel Selection**, can be performed to make selections of **individuals** in a way that is directly proportionate to their **fitness**.

Each pair can generate one or more **offspring**, however as a good practice, different pairs should be used to generate the **new population**.

MH – *Genetic Algorithm*

Roulette Wheel Selection – Step 1. Take from each **individual** the **fitness** value. For maximization problems, use the **fitness** values without modifications, for minimization problems go to Step 1b.

	Cost
Individual 1	1580.74
Individual 2	556.54
Individual 3	1625.25
Individual 4	1599.48

MH – *Genetic Algorithm*

Roulette Wheel Selection – Step 1b. Use the inverse formula to transform a minimization problem into a maximization problem.

$$inverse = \frac{1}{1 + Cost + |\min(Cost)|}$$

	Cost
Individual 1	1580.74
Individual 2	556.54
Individual 3	1625.25
Individual 4	1599.48

MH – *Genetic Algorithm*

Roulette Wheel Selection – Step 1b. Use the inverse formula to transform a minimization problem into a maximization problem.

$$inverse = \frac{1}{1 + Cost + |\min(Cost)|}$$

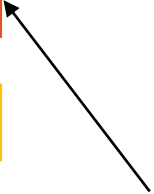
	Cost	<i>inverse</i>
Individual 1	1580.74	
Individual 2	556.54	
Individual 3	1625.25	
Individual 4	1599.48	

MH – Genetic Algorithm

Roulette Wheel Selection – Step 1b. Use the inverse formula to transform a minimization problem into a maximization problem.

$$inverse = \frac{1}{1 + Cost + |\min(Cost)|}$$

	Cost	<i>inverse</i>
Individual 1	1580.74	4.68
Individual 2	556.54	
Individual 3	1625.25	
Individual 4	1599.48	

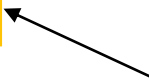
$$\frac{1}{1 + 1580.74 + |556.54|}$$


MH – *Genetic Algorithm*

Roulette Wheel Selection – Step 1b. Use the inverse formula to transform a minimization problem into a maximization problem.

$$inverse = \frac{1}{1 + Cost + |\min(Cost)|}$$

	Cost	<i>inverse</i>
Individual 1	1580.74	4.68
Individual 2	556.54	8.98
Individual 3	1625.25	
Individual 4	1599.48	

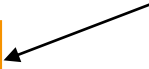
$$\frac{1}{1 + 556.54 + |556.54|}$$


MH – Genetic Algorithm

Roulette Wheel Selection – Step 1b. Use the inverse formula to transform a minimization problem into a maximization problem.

$$inverse = \frac{1}{1 + Cost + |\min(Cost)|}$$

	Cost	<i>inverse</i>
Individual 1	1580.74	4.68
Individual 2	556.54	8.98
Individual 3	1625.25	4.58
Individual 4	1599.48	

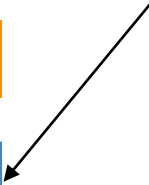
$$\frac{1}{1 + 1625.25 + |556.54|}$$


MH – Genetic Algorithm

Roulette Wheel Selection – Step 1b. Use the inverse formula to transform a minimization problem into a maximization problem.

$$inverse = \frac{1}{1 + Cost + |\min(Cost)|}$$

	Cost	<i>inverse</i>
Individual 1	1580.74	4.68
Individual 2	556.54	8.98
Individual 3	1625.25	4.58
Individual 4	1599.48	4.64

$$\frac{1}{1 + 1599.48 + |556.54|}$$


MH – *Genetic Algorithm*

Roulette Wheel Selection – Step 1b. Use the inverse formula to transform a minimization problem into a maximization problem.

$$inverse = \frac{1}{1 + Cost + |\min(Cost)|}$$

	Cost	<i>inverse</i>
Individual 1	1580.74	4.68
Individual 2	556.54	8.98
Individual 3	1625.25	4.58
Individual 4	1599.48	4.64

MH – *Genetic Algorithm*

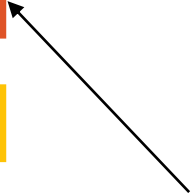
Roulette Wheel Selection – Step 2. Accumulate the values sequentially.

	Cost	<i>inverse</i>	<i>acc.</i>
Individual 1	1580.74	4.68	
Individual 2	556.54	8.98	
Individual 3	1625.25	4.58	
Individual 4	1599.48	4.64	

MH – *Genetic Algorithm*

Roulette Wheel Selection – Step 2. Accumulate the values sequentially.

	Cost	<i>inverse</i>	<i>acc.</i>
Individual 1	1580.74	4.68	
Individual 2	556.54	8.98	
Individual 3	1625.25	4.58	
Individual 4	1599.48	4.64	

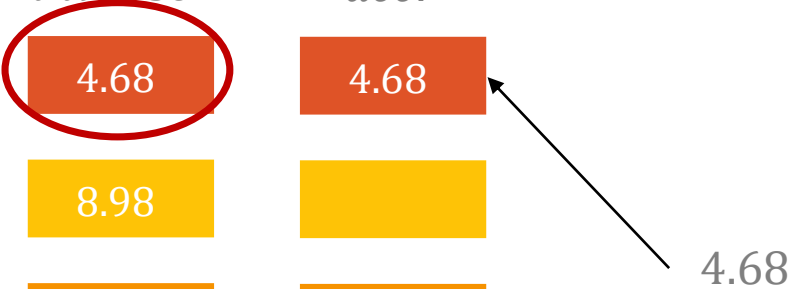


4.68

MH – *Genetic Algorithm*

Roulette Wheel Selection – Step 2. Accumulate the values sequentially.

	Cost	<i>inverse</i>	<i>acc.</i>
Individual 1	1580.74	4.68	4.68
Individual 2	556.54	8.98	
Individual 3	1625.25	4.58	
Individual 4	1599.48	4.64	



MH – Genetic Algorithm

Roulette Wheel Selection – Step 2. Accumulate the values sequentially.

	Cost	<i>inverse</i>	<i>acc.</i>
Individual 1	1580.74	4.68	4.68
Individual 2	556.54	8.98	13.66
Individual 3	1625.25	4.58	
Individual 4	1599.48	4.64	

4.68 + 8.98

MH – Genetic Algorithm

Roulette Wheel Selection – Step 2. Accumulate the values sequentially.

	Cost	<i>inverse</i>	<i>acc.</i>
Individual 1	1580.74	4.68	4.68
Individual 2	556.54	8.98	13.66
Individual 3	1625.25	4.58	18.24
Individual 4	1599.48	4.64	

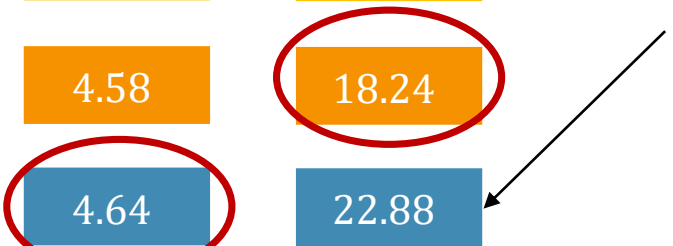
4.58 + 13.66

MH – Genetic Algorithm

Roulette Wheel Selection – Step 2. Accumulate the values sequentially.

	Cost	<i>inverse</i>	<i>acc.</i>
Individual 1	1580.74	4.68	4.68
Individual 2	556.54	8.98	13.66
Individual 3	1625.25	4.58	18.24
Individual 4	1599.48	4.64	22.88

4.64 + 18.24



MH – *Genetic Algorithm*

Roulette Wheel Selection – Step 2. Accumulate the values sequentially.

	Cost	<i>inverse</i>	<i>acc.</i>
Individual 1	1580.74	4.68	4.68
Individual 2	556.54	8.98	13.66
Individual 3	1625.25	4.58	18.24
Individual 4	1599.48	4.64	22.88

MH – *Genetic Algorithm*

Roulette Wheel Selection – Step 3. Divide each value of the *acc.* column by the sum of the *inverse* column.

	Cost	<i>inverse</i>	<i>acc.</i>
Individual 1	1580.74	4.68	4.68
Individual 2	556.54	8.98	13.66
Individual 3	1625.25	4.58	18.24
Individual 4	1599.48	4.64	22.88
	<i>Sum</i>	22.88	

MH – *Genetic Algorithm*

Roulette Wheel Selection – Step 3. Divide each value of the *acc.* column by the sum of the *inverse* column.

	Cost	<i>inverse</i>	<i>acc.</i>	%
Individual 1	1580.74	4.68	4.68	
Individual 2	556.54	8.98	13.66	
Individual 3	1625.25	4.58	18.24	
Individual 4	1599.48	4.64	22.88	
	<i>Sum</i>	22.88		

MH – Genetic Algorithm

Roulette Wheel Selection – Step 3. Divide each value of the acc. column by the sum of the inverse column.

	Cost	<i>inverse</i>	<i>acc.</i>	%
Individual 1	1580.74	4.68	4.68	20.45%
Individual 2	556.54	8.98	13.66	
Individual 3	1625.25	4.58	18.24	
Individual 4	1599.48	4.64	22.88	
<i>Sum</i>		22.88		

$\frac{4.68}{22.88}$

MH – Genetic Algorithm

Roulette Wheel Selection – Step 3. Divide each value of the acc. column by the sum of the inverse column.

	Cost	<i>inverse</i>	<i>acc.</i>	%
Individual 1	1580.74	4.68	4.68	20.45%
Individual 2	556.54	8.98	13.66	59.70%
Individual 3	1625.25	4.58	18.24	
Individual 4	1599.48	4.64	22.88	
<i>Sum</i>		22.88		

$\frac{13.66}{22.88}$

MH – Genetic Algorithm

Roulette Wheel Selection – Step 3. Divide each value of the acc. column by the sum of the inverse column.

	Cost	<i>inverse</i>	<i>acc.</i>	%
Individual 1	1580.74	4.68	4.68	20.45%
Individual 2	556.54	8.98	13.66	59.70%
Individual 3	1625.25	4.58	18.24	79.72%
Individual 4	1599.48	4.64	22.88	
<i>Sum</i>		22.88		

$\frac{18.24}{22.88}$

MH – Genetic Algorithm

Roulette Wheel Selection – Step 3. Divide each value of the *acc.* column by the sum of the *inverse* column.

	Cost	<i>inverse</i>	<i>acc.</i>	%
Individual 1	1580.74	4.68	4.68	20.45%
Individual 2	556.54	8.98	13.66	59.70%
Individual 3	1625.25	4.58	18.24	79.72%
Individual 4	1599.48	4.64	22.88	100%
<i>Sum</i>		22.88		

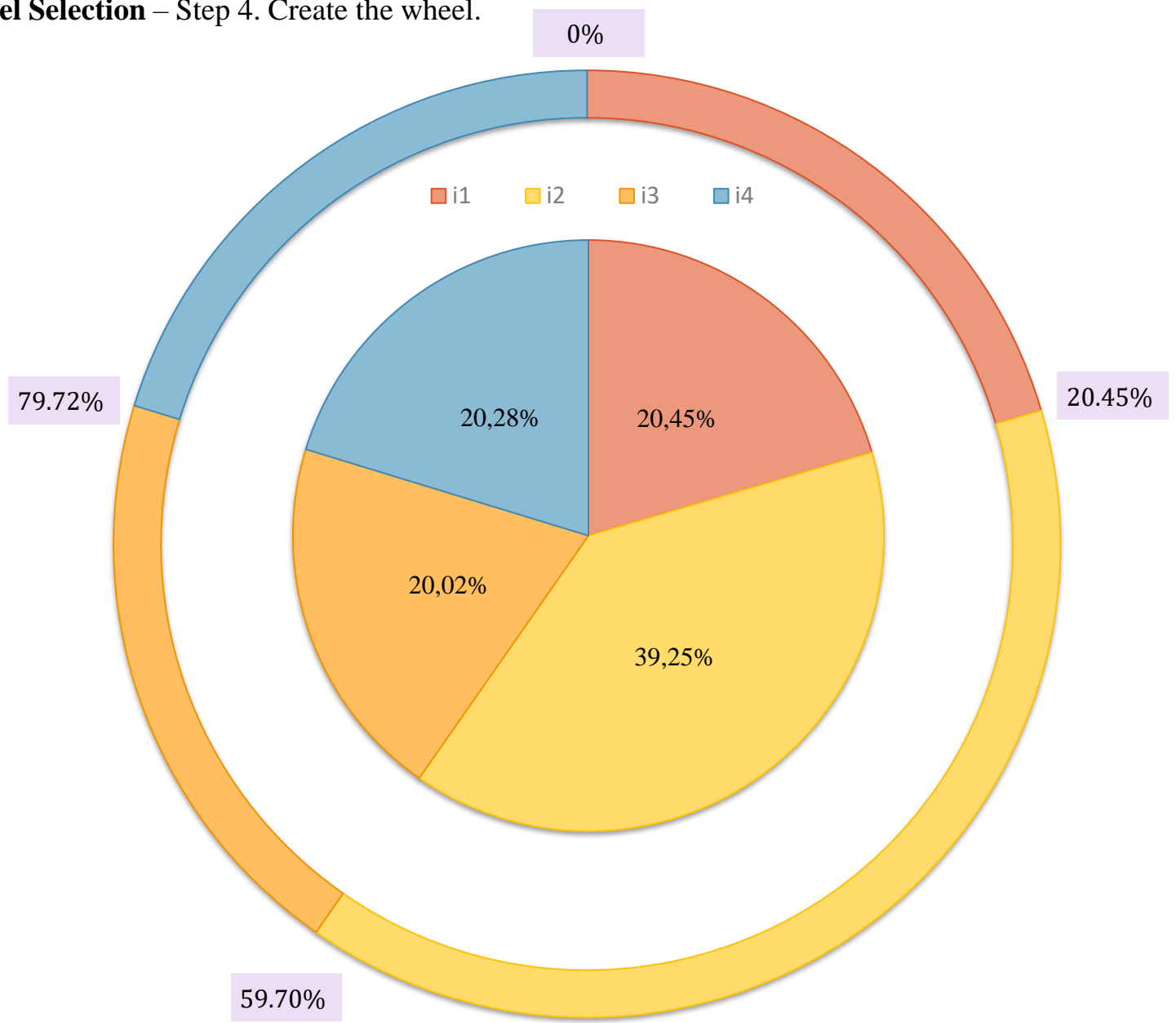
$\frac{22.88}{22.88}$

MH – *Genetic Algorithm*

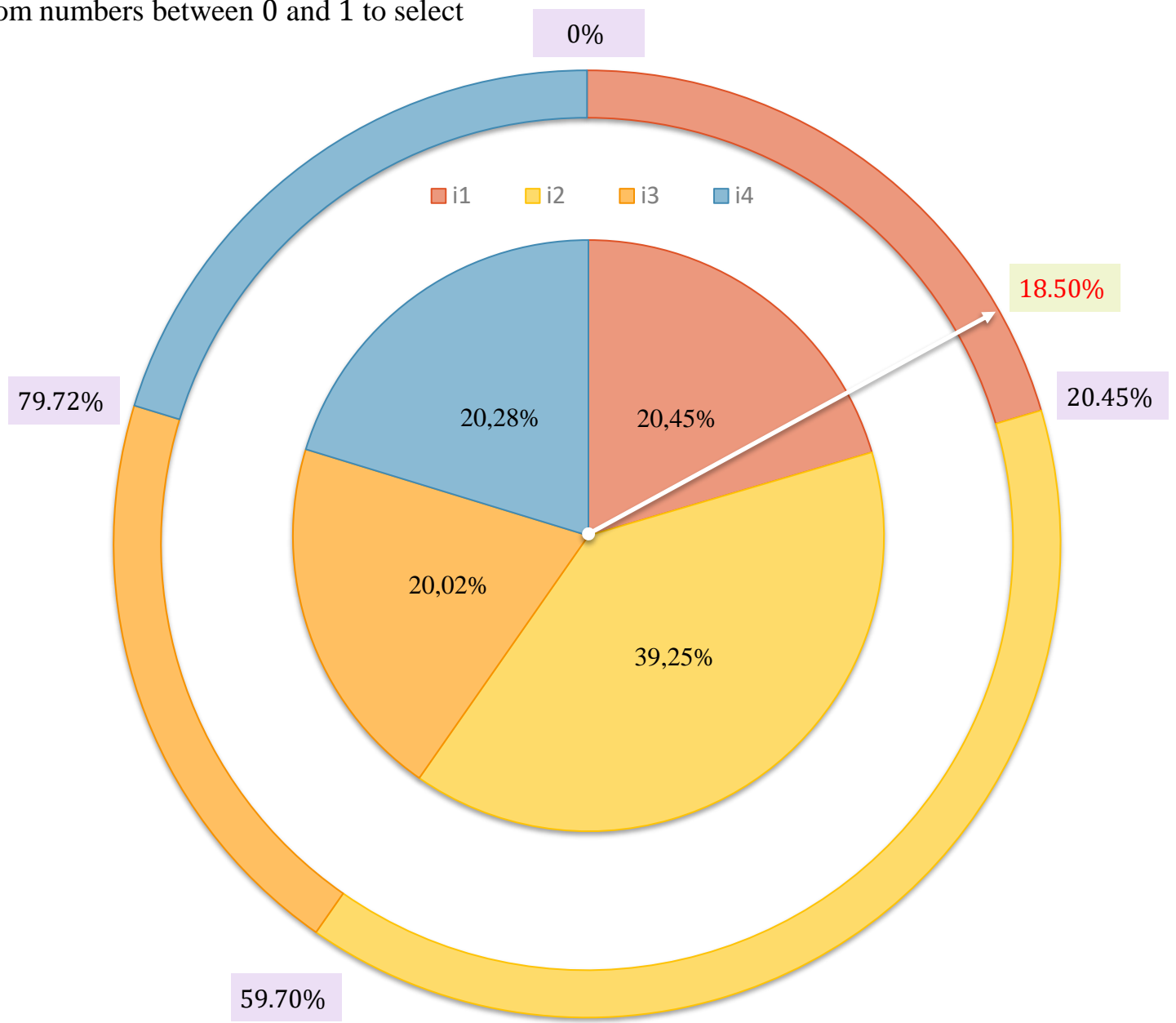
Roulette Wheel Selection – Step 3. Divide each value of the *acc.* column by the sum of the *inverse* column.

	Cost	<i>inverse</i>	<i>acc.</i>	%
Individual 1	1580.74	4.68	4.68	20.45%
Individual 2	556.54	8.98	13.66	59.70%
Individual 3	1625.25	4.58	18.24	79.72%
Individual 4	1599.48	4.64	22.88	100%
	<i>Sum</i>	22.88		

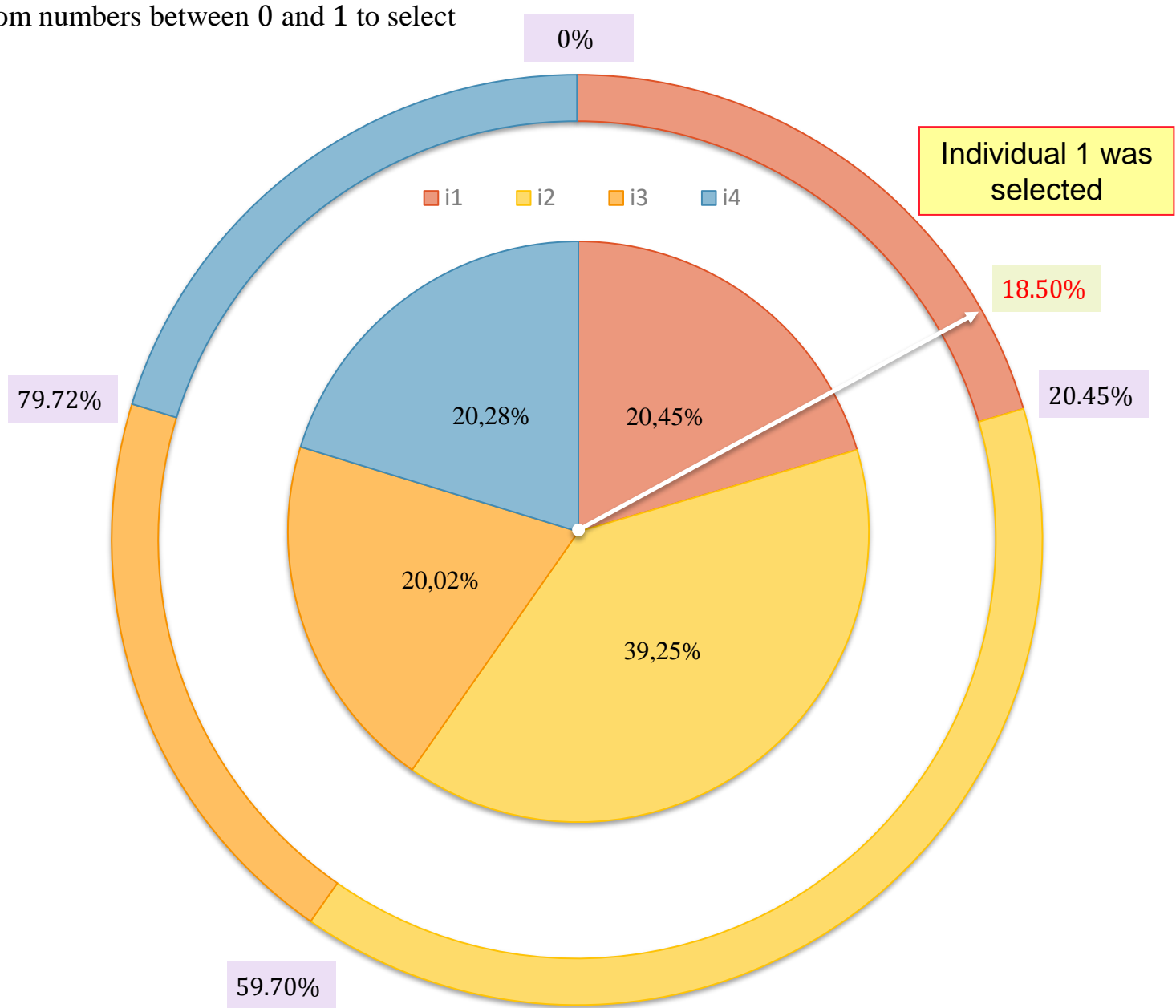
Roulette Wheel Selection – Step 4. Create the wheel.



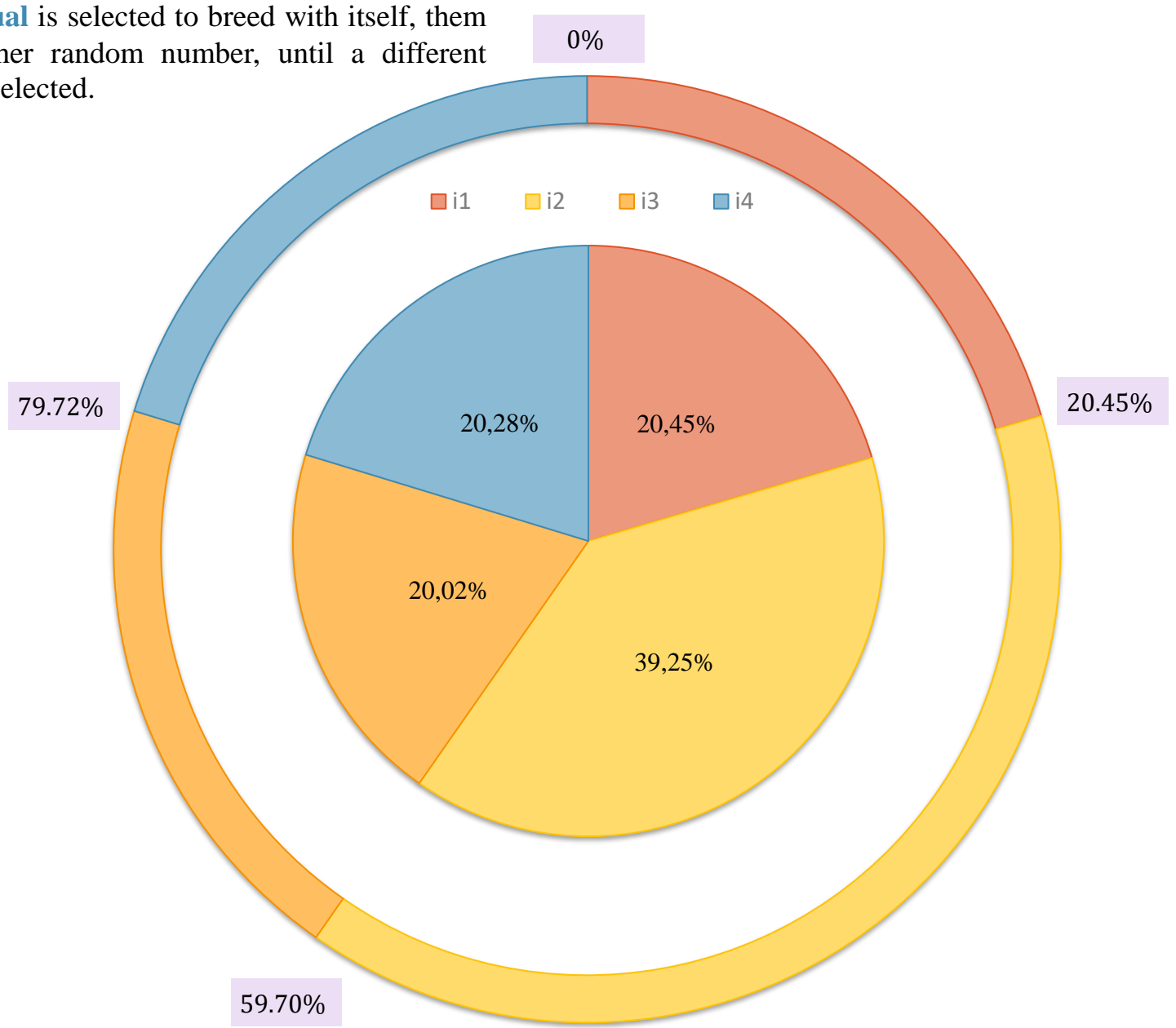
Generate random numbers between 0 and 1 to select an **individual**.



Generate random numbers between 0 and 1 to select an **individual**.



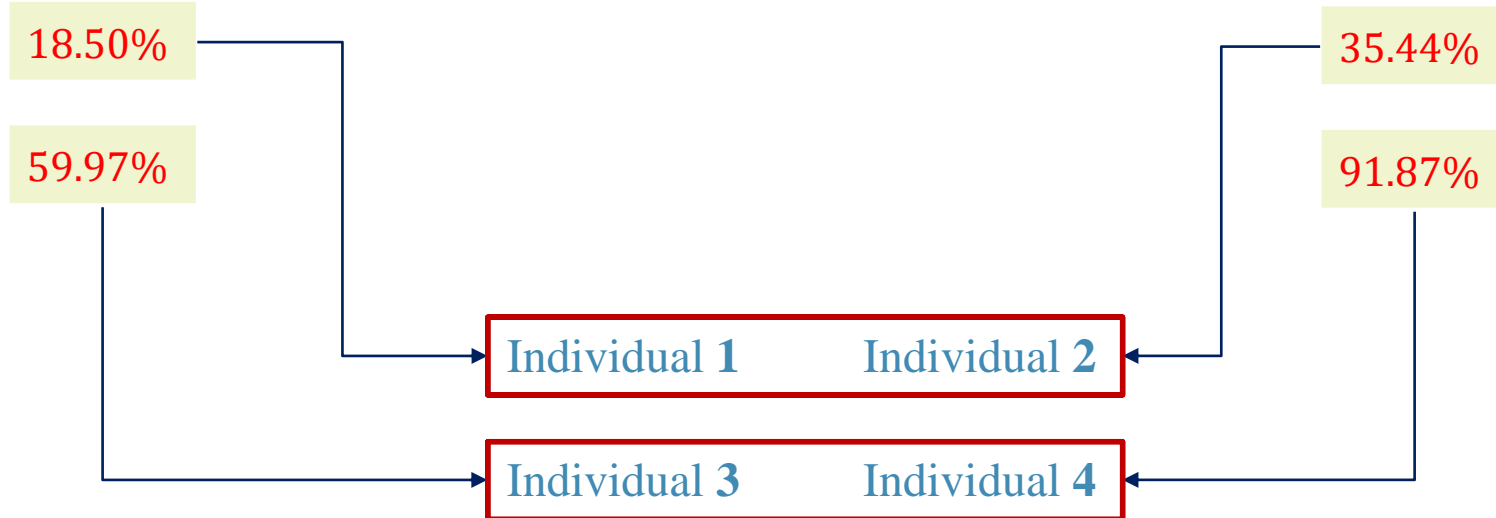
If the **individual** is selected to breed with itself, them generate another random number, until a different **individual** is selected.



MH – *Genetic Algorithm*

Roulette Wheel Selection – Step 5. Select the pairs.

Random Number





Crossover

MH – *Genetic Algorithm*

The **crossover** may happen always (100% chance) or have a probability to occur, for example, a **crossover** for a selected pair may have a chance of 70% to breed, and if no **crossover** occurs then the selected pair is copied directly to the **new population**. The most common type of **crossover** operator, called **OX (Order Based Crossover)**, is performed by copying a route from the first element of a pair (**parent₁** & **parent₂**) and the remaining routes from the second element of the same pair. Adjusts may be needed to avoid repeated nodes in the **offspring**.

Optionally a local search method (**2-opt**, **k-opt**, **best insertion**, etc.) can be performed to improve the **offspring fitness**.

MH – Genetic Algorithm

Let's suppose that the pair (**individual 1** & **individual 2**) will breed, the pair (**individual 3** & **individual 4**) will be copied to directly to the **new population**. Therefore:

	Depot	Client	Client	Client	Depot	Client	Client	Depot		Cost
Individual 1	0	3	1	4	0	2	5	0		1580.74
	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot	Cost
Individual 2	0	3	1	0	5	0	4	2	0	556.54

	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot	Cost
Individual 2	0	3	1	0	5	0	4	2	0	556.54

MH – Genetic Algorithm

Let's suppose that the pair (**individual 1** & **individual 2**) will breed, the pair (**individual 3** & **individual 4**) will be copied to directly to the **new population**. Therefore:

	Depot	Client	Client	Client	Depot	Client	Client	Depot	Cost
Individual 1	0	3	1	4	0	2	5	0	1580.74

	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot	Cost
Individual 2	0	3	1	0	5	0	4	2	0	556.54

Route Randomly
Selected

MH – Genetic Algorithm

Let's suppose that the pair (**individual 1** & **individual 2**) will breed, the pair (**individual 3** & **individual 4**) will be copied to directly to the **new population**. Therefore:

	Depot	Client	Client	Client	Depot	Client	Client	Depot	Cost
Individual 1	0	3	1	4	0	2	5	0	1580.74

	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot	Cost
Individual 2	0	3	1	0	5	0	4	2	0	556.54

Offspring	Cost

MH – Genetic Algorithm

Let's suppose that the pair (**individual 1** & **individual 2**) will breed, the pair (**individual 3** & **individual 4**) will be copied to directly to the **new population**. Therefore:

Individual 1	Depot	Client	Client	Client	Depot	Client	Client	Depot	Cost	
	0	3	1	4	0	2	5	0	1580.74	
Individual 2	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot	Cost
	0	3	1	0	5	0	4	2	0	556.54
Offspring	Depot	Client	Client	Client	Depot					Cost
	0	3	1	4	0					

MH – Genetic Algorithm

Let's suppose that the pair (**individual 1** & **individual 2**) will breed, the pair (**individual 3** & **individual 4**) will be copied to directly to the **new population**. Therefore:

	Depot	Client	Client	Client	Depot	Client	Client	Depot	Cost
Individual 1	0	3	1	4	0	2	5	0	1580.74
	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot
Individual 2	0	3	1	0	5	0	4	2	0
	Depot	Client	Client	Client	Depot				Cost
Offspring	0	3	1	4	0				

MH – Genetic Algorithm

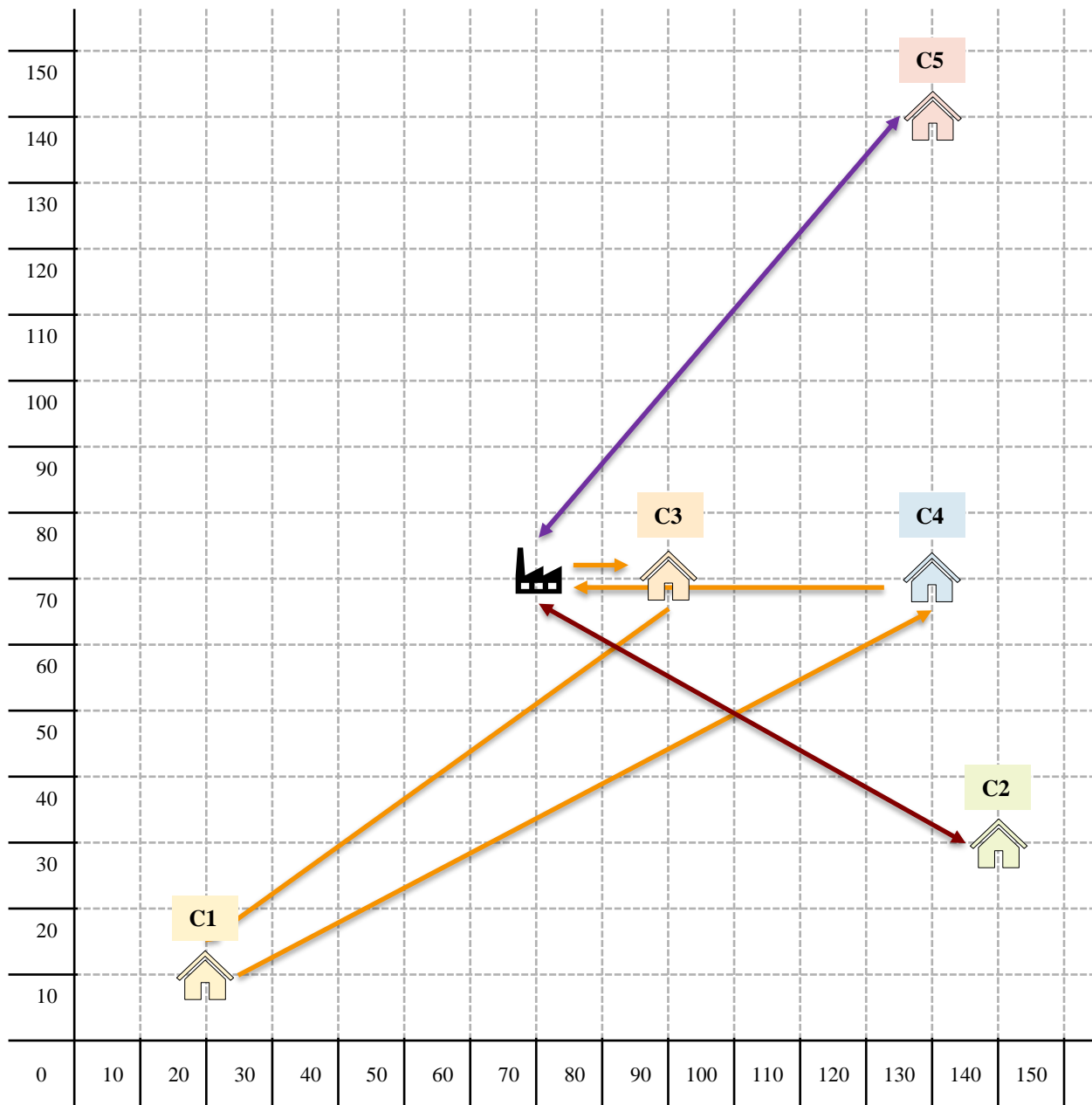
Let's suppose that the pair (**individual 1** & **individual 2**) will breed, the pair (**individual 3** & **individual 4**) will be copied directly to the **new population**. Therefore:

	Depot	Client	Client	Client	Depot	Client	Client	Depot	Cost
Individual 1	0	3	1	4	0	2	5	0	1580.74
Individual 2	0	3	1	0	5	0	4	2	0
Offspring	0	3	1	4	0	5	0	2	

MH – Genetic Algorithm

Let's suppose that the pair (**individual 1** & **individual 2**) will breed, the pair (**individual 3** & **individual 4**) will be copied to directly to the **new population**. Therefore:

	Depot	Client	Client	Client	Depot	Client	Client	Depot		Cost
Individual 1	0	3	1	4	0	2	5	0		1580.74
	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot	Cost
Individual 2	0	3	1	0	5	0	4	2	0	556.54
	Depot	Client	Client	Client	Depot	Client	Depot	Client	Depot	Cost
Offspring	0	3	1	4	0	5	0	2	0	



MH – Genetic Algorithm

Let's suppose that the pair (**individual 1** & **individual 2**) will breed, the pair (**individual 3** & **individual 4**) will be copied to directly to the **new population**. Therefore:

Individual 1	Depot	Client	Client	Client	Depot	Client	Client	Depot	Cost	
	0	3	1	4	0	2	5	0		
Individual 2	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot	Cost
	0	3	1	0	5	0	4	2	0	
Offspring	Depot	Client	Client	Client	Depot	Client	Depot	Client	Depot	Cost
	0	3	1	4	0	5	0	2	0	

Mutation

MH – *Genetic Algorithm*

The **mutation** is a genetic operator that, for a small chance (usually between 10% and 15%), can modify a **gene** of the **offspring**. This process ensures the genetic diversity and avoids solutions to be trapped in local maxima/minima. The most common type of **mutation** operator is called **Insertion** and it randomly selects a **gene** and inserts it in another randomly selected place of the same **individual**.

MH – *Genetic Algorithm*

Let's suppose that the following **gene** is mutated:

	Depot	Client	Client	Client	Depot	Client	Depot	Client	Depot	Cost
Offspring	0	3	1	4	0	5	0	2	0	643.13

MH – *Genetic Algorithm*

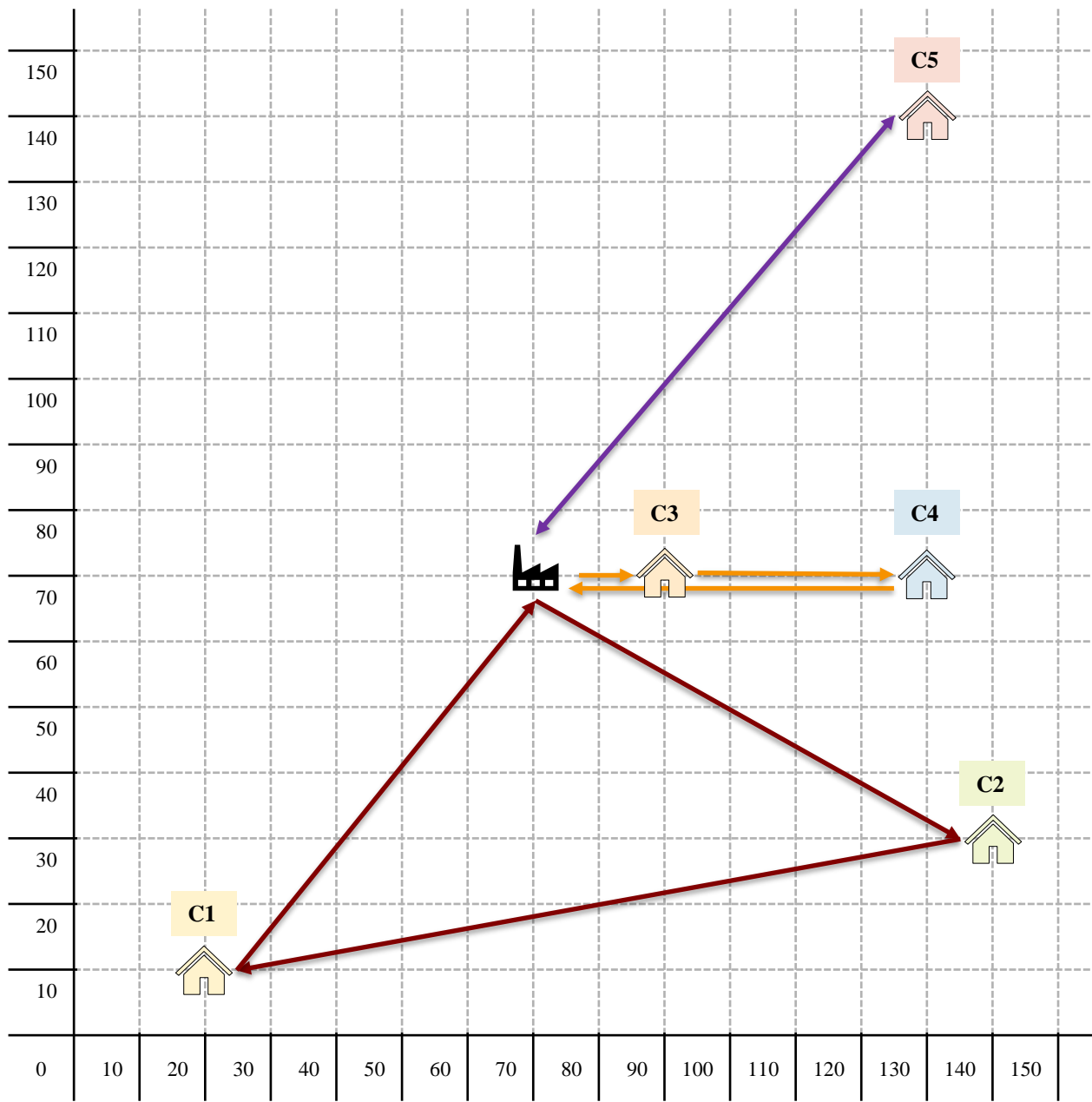
Let's suppose that the following **gene** is mutated:

	Depot	Client	Client	Client	Depot	Client	Depot	Client	Depot	Cost
Offspring	0	3	1	4	0	5	0	2	0	643.13

MH – *Genetic Algorithm*

Let's suppose that the following **gene** is mutated:

	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot	Cost
Offspring	0	3	4	0	5	0	2	1	0	



MH – *Genetic Algorithm*

Let's suppose that the following **gene** is mutated:

	Depot	Client	Client	Depot	Client	Depot	Client	Client	Depot	Cost
Offspring	0	3	4	0	5	0	2	1	0	584.77



Elitism

The **elitism** is an optional process, where the **best individual** of the last the **population** is included in the **new population**. Hence:

Individual 1	0	3	1	4	0	2	5	0		1580.74	
Individual 2	0	3	1	0	5	0	4	2	0	556.54	
Individual 3	0	2	3	0	4	0	1	5	0	1625.25	
Individual 4	0	4	0	3	0	1	0	2	5	0	1599.48

The **elitism** is an optional process, where the **best individual** of the last the **population** is included in the **new population**. Hence:

Individual 1	0	3	1	4	0	2	5	0		1580.74	
Individual 2	0	3	1	0	5	0	4	2	0	556.54	
Individual 3	0	2	3	0	4	0	1	5	0	1625.25	
Individual 4	0	4	0	3	0	1	0	2	5	0	1599.48

The **elitism** is an optional process, where the **best individual** of the last the **population** is included in the **new population**. Hence:

Individual 1	0	3	1	4	0	2	5	0		1580.74	
Individual 2	0	3	1	0	5	0	4	2	0	556.54	
Individual 3	0	2	3	0	4	0	1	5	0	1625.25	
Individual 4	0	4	0	3	0	1	0	2	5	0	1599.48
Elite											
New Ind. 1	0	3	1	0	5	0	4	2	0	556.54	

The **elitism** is an optional process, where the **best individual** of the last the **population** is included in the **new population**. Hence:

Individual 1	0	3	1	4	0	2	5	0		1580.74	
Individual 2	0	3	1	0	5	0	4	2	0	556.54	
Individual 3	0	2	3	0	4	0	1	5	0	1625.25	
Individual 4	0	4	0	3	0	1	0	2	5	0	1599.48

Copied

New Ind. 1	0	3	1	0	5	0	4	2	0		556.54
New Ind. 2	0	2	3	0	4	0	1	5	0		1625.25
New Ind. 3	0	4	0	3	0	1	0	2	5	0	1599.48

The **elitism** is an optional process, where the **best individual** of the last the **population** is included in the **new population**. Hence:

Individual 1	0	3	1	4	0	2	5	0		1580.74	
Individual 2	0	3	1	0	5	0	4	2	0	556.54	
Individual 3	0	2	3	0	4	0	1	5	0	1625.25	
Individual 4	0	4	0	3	0	1	0	2	5	0	1599.48

Crossover & Mutation

New Ind. 1	0	3	1	0	5	0	4	2	0		556.54
New Ind. 2	0	2	3	0	4	0	1	5	0		1625.25
New Ind. 3	0	4	0	3	0	1	0	2	5	0	1599.48
New Ind. 4	0	3	4	0	5	0	2	1	0		584.77

We discard the **old population** and use the **new population** to repeat this process all over again, or in another words, to create a new **generation**.

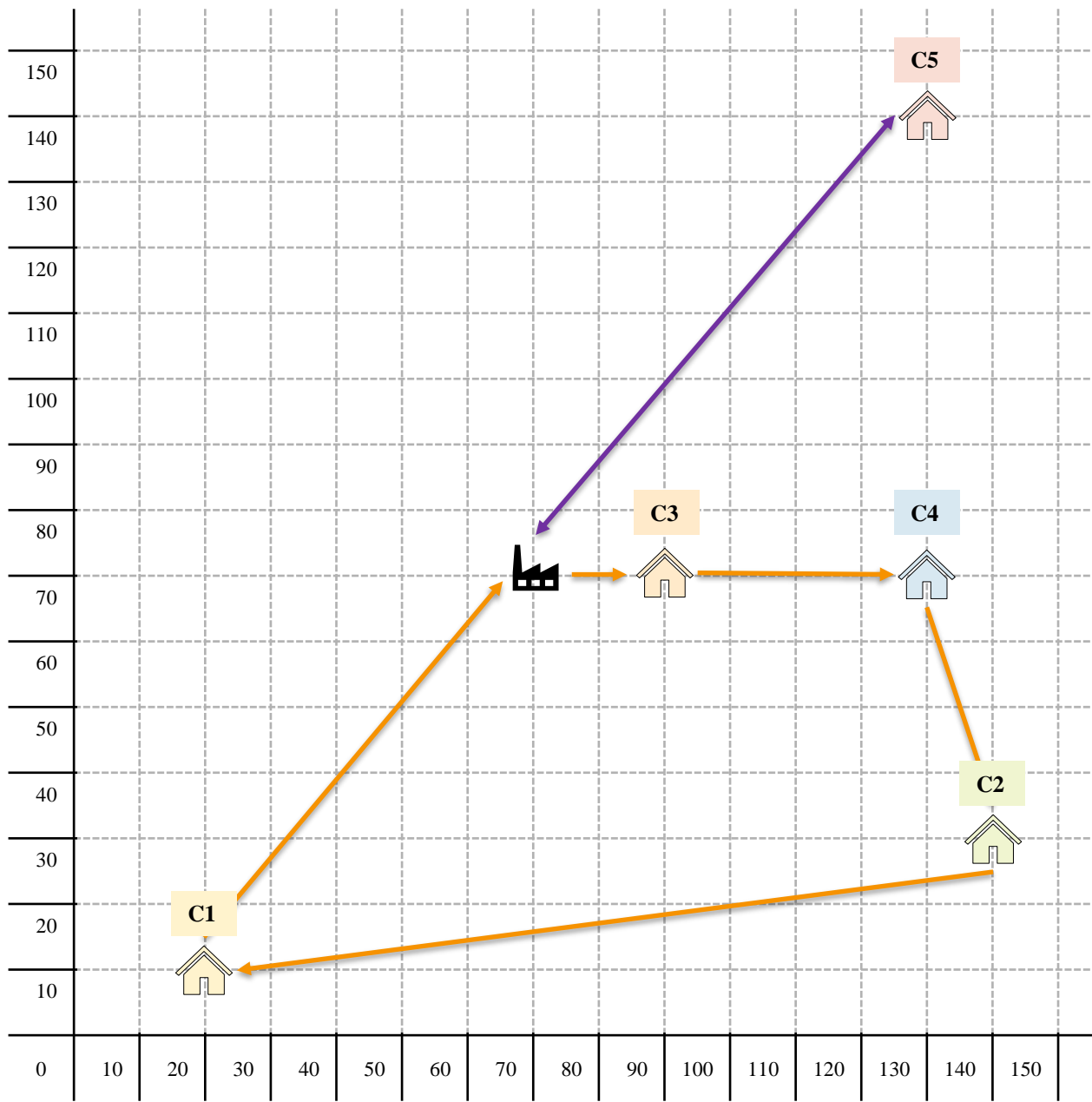
New Ind. 1	0	3	1	0	5	0	4	2	0	556.54	
New Ind. 2	0	2	3	0	4	0	1	5	0	1625.25	
New Ind. 3	0	4	0	3	0	1	0	2	5	0	1599.48
New Ind. 4	0	3	4	0	5	0	2	1	0	584.77	

After 2,500 generations

MH – *Genetic Algorithm*

The **best individual** found was:

								<i>Cost</i>
0	3	4	2	1	0	5	0	485.38



MH – *Genetic Algorithm*

<https://github.com/Valdecy/pyVRP>

The **pyVRP** is python library that solves (using **Genetic Algorithms**): Capacitated VRP, Multiple Depot VRP, VRP with Time Windows, VRP with Homogeneous or Heterogeneous Fleet, VRP with Finite or Infinite Fleet, Open or Closed Routes, TSP, mTSP and various combination of these types.

Follow the link to try it online.

MH – *Genetic Algorithm*

<https://github.com/Valdecy/J-Horizon>

The **J-Horizon** is java based vehicle problem software that uses the jsprit library to solve: Capacitated VRP, Multiple Depot VRP, VRP with Time Windows, VRP with Backhauls, VRP with Pickups and Deliveries, VRP with Homogeneous or Heterogeneous Fleet, Finite or Infinite Fleet, TSP, mTSP and various combination of these types.

Accepted inputs: Cartesian Coordinates, Latitude and Longitude, Distance Matrix, Time Matrix, Distance and Time Matrices.

J-Horizon: CVRP with LatLong Example

Model Selection <input type="radio"/> TSP (Travelling Salesman Problem) <input type="radio"/> MTSP (Multiple TSP) <input checked="" type="radio"/> VRP (Vehicle Routing Problem)	Route Type <input checked="" type="radio"/> Closed <input type="radio"/> Open Fleet Size (Hmg or Htg) <input type="radio"/> Finite <input checked="" type="radio"/> Infinite Depots <input checked="" type="radio"/> Single <input type="radio"/> Multiple Time Window <input checked="" type="radio"/> Without <input type="radio"/> With	Vehicle Capacity <input type="radio"/> Unlimited <input checked="" type="radio"/> Limited Map Preferences <input checked="" type="radio"/> Street Map (Latitude and Longitude) <input type="radio"/> Cartesian Plane (xy Coordinates) Show Routes <input type="radio"/> Urban Waypoints <input checked="" type="radio"/> Line Waypoints	Route Animation <input checked="" type="radio"/> Static <input type="radio"/> Animated Distances <input type="radio"/> Euclidean <input type="radio"/> Manhattan <input checked="" type="radio"/> Haversine <input type="radio"/> My Dist/Time Matrix
Pickups and Deliveries <input checked="" type="radio"/> Without <input type="radio"/> With			
Backhauls <input checked="" type="radio"/> Without <input type="radio"/> With			
Build Problem			



Model Input Dist/Time Output

Clients (CT): 24 Depots (DT): 1 Vehicle Types (VT): 1 Velocity (km/h): 50 Build Map Dist/Time Solve Map-Routes

Id	Demand	Latitude	Longitude	VT_1: QT	VT_1: CT	VT_1: FC	VT_1: VC	Break (early)	Break (late)	Duration
# DT-0 #		32.8243085	-17.1110778	Infinite	1000	0	1	Optional	Optional	Optional
CT-1	94	32.7937899	-17.0469557	-/-	-/-	-/-	-/-	-/-	-/-	-/-
CT-2	87	32.817304	-16.9775394	-/-	-/-	-/-	-/-	-/-	-/-	-/-
CT-3	76	32.8256602	-16.9204501	-/-	-/-	-/-	-/-	-/-	-/-	-/-
CT-4	56	32.8035571	-16.8964856	-/-	-/-	-/-	-/-	-/-	-/-	-/-
CT-5	63	32.7822902	-16.8690048	-/-	-/-	-/-	-/-	-/-	-/-	-/-
CT-6	63	32.7641901	-16.8370148	-/-	-/-	-/-	-/-	-/-	-/-	-/-
CT-7	82	32.7287655	-16.8089019	-/-	-/-	-/-	-/-	-/-	-/-	-/-
CT-8	75	32.7405861	-16.7391154	-/-	-/-	-/-	-/-	-/-	-/-	-/-
CT-9	72	32.70294	-16.803803	-/-	-/-	-/-	-/-	-/-	-/-	-/-
CT-10	90	32.6540457	-16.8527448	-/-	-/-	-/-	-/-	-/-	-/-	-/-
CT-11	97	32.6554867	-16.8827067	-/-	-/-	-/-	-/-	-/-	-/-	-/-
CT-12	91	32.6727226	-16.9153369	-/-	-/-	-/-	-/-	-/-	-/-	-/-
CT-13	98	32.6615632	-16.9440078	-/-	-/-	-/-	-/-	-/-	-/-	-/-
CT-14	78	32.6601177	-17.0062457	-/-	-/-	-/-	-/-	-/-	-/-	-/-
CT-15	76	32.6712399	-17.0428638	-/-	-/-	-/-	-/-	-/-	-/-	-/-
CT-16	51	32.6895283	-17.0986065	-/-	-/-	-/-	-/-	-/-	-/-	-/-
CT-17	80	32.7014374	-17.1346762	-/-	-/-	-/-	-/-	-/-	-/-	-/-
CT-18	100	32.751613	-17.2168406	-/-	-/-	-/-	-/-	-/-	-/-	-/-
CT-19	97	32.7766829	-17.2335843	-/-	-/-	-/-	-/-	-/-	-/-	-/-
CT-20	57	32.8418269	-17.21424	-/-	-/-	-/-	-/-	-/-	-/-	-/-
CT-21	53	32.8551822	-17.1821677	-/-	-/-	-/-	-/-	-/-	-/-	-/-
CT-22	67	32.7270353	-17.1659397	-/-	-/-	-/-	-/-	-/-	-/-	-/-
CT-23	52	32.7262798	-17.0392339	-/-	-/-	-/-	-/-	-/-	-/-	-/-
CT-24	54	32.7333322	-16.8920878	-/-	-/-	-/-	-/-	-/-	-/-	-/-

Problem						
Indicator		Value				
Clients	24					
Services	24					
Shipments	0					
Breaks	0					
Fleet Size	INFINITE					
Solution						
Indicator		Value				
Costs	164.59					
Vehicles	2					
Unassign. Jobs	0					
Detailed Solution						
Route	Vehicle	Activity	Job	ArrTime (minutes)	EndTime (minutes)	Costs
1	DT_0_VT-1#1	start	-	***	0	0
1	DT_0_VT-1#1	service	2	15.008	15.008	12.506
1	DT_0_VT-1#1	service	3	21.507	21.507	17.923
1	DT_0_VT-1#1	service	4	25.499	25.499	21.249
1	DT_0_VT-1#1	service	5	29.69	29.69	24.741
1	DT_0_VT-1#1	service	6	34.017	34.017	28.347
1	DT_0_VT-1#1	service	7	39.701	39.701	33.085
1	DT_0_VT-1#1	service	8	47.694	47.694	39.745
1	DT_0_VT-1#1	service	9	56.434	56.434	47.028
1	DT_0_VT-1#1	service	10	65.04	65.04	54.2
1	DT_0_VT-1#1	service	11	68.413	68.413	57.011
1	DT_0_VT-1#1	service	12	72.741	72.741	60.618
1	DT_0_VT-1#1	service	24	81.242	81.242	67.702
1	DT_0_VT-1#1	service	1	100.406	100.406	83.671
1	DT_0_VT-1#1	end	-	108.672	***	90.56
2	DT_0_VT-1#1	start	-	***	0	0
2	DT_0_VT-1#1	service	23	15.369	15.369	12.807
2	DT_0_VT-1#1	service	13	29.117	29.117	24.264
2	DT_0_VT-1#1	service	14	36.113	36.113	30.095
2	DT_0_VT-1#1	service	15	40.487	40.487	33.74
2	DT_0_VT-1#1	service	16	47.209	47.209	39.341
2	DT_0_VT-1#1	service	17	51.561	51.561	42.967
2	DT_0_VT-1#1	service	22	56.46	56.46	47.05
2	DT_0_VT-1#1	service	18	63.049	63.049	52.541
2	DT_0_VT-1#1	service	19	66.887	66.887	55.739
2	DT_0_VT-1#1	service	20	75.848	75.848	63.207
2	DT_0_VT-1#1	service	21	79.862	79.862	66.552
2	DT_0_VT-1#1	end	-	88.836	***	74.03

