

# A Multigrid Solver for Graph Laplacian Linear Systems on Power-Law Graphs

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## 1 Abstract

The Laplacian matrix,  $L$ , of a graph,  $G$ , contains degree and edge information of a given network. Solving a Laplacian linear system  $Lx = b$  provides information about flow through the network, and in specific cases, how that information orders the nodes in the network. I propose a novel way to solve this linear system by first partitioning  $G$  into its maximum locally-connected subgraph and a small subgraph of the remaining teleportation edges. I then apply optimal multigrid solves to the locally-connected subgraph, and linear algebra and a solve on the so-called "teleportation" subgraph to solve the original linear system. I show results for this method on real-world graphs from the biological systems of the *C. Elegans* worm, Facebook friend networks, and the power grid of the Western United States.