Centennial Mu Alpha Theta

April 6, 2024

Combinatorics Round

Do not begin until instructed to do so.

This is the Combinatorics Round test for the 2024 DECAGON Math Tournament. You will have 50 minutes to complete 15 problems. All problems are weighted equally, but ties will be broken based on the hardest question solved (not necessarily highest numbered question). Express all answers in simplest form. Only answers recorded on the answer sheet below will be scored. Only writing tools and plain scratch paper are allowed. Assume all questions are in base 10 unless otherwise indicated. We designed this test so that most people will not be able to finish all the questions in time, so don't worry if you are struggling! Feel free to skip questions and come back to them later.

Name: _____ Competitor ID: _____ Team ID: _____

1. _____	2. _____	3. _____
4. _____	5. _____	6. _____
7. _____	8. _____	9. _____
10. _____	11. _____	12. _____
13. _____	14. _____	15. _____

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1. Caleb writes down five distinct digits from 0 to 9 on a board and realizes that the numbers can be arranged to form a sequence of five consecutive numbers. How many different ways could Caleb have chosen his five numbers, where order does not matter?

2. Anurag the Ant climbs up a tree subject to the following rules:

   - On the first day, he climbs up three meters
   - If he traveled x meters on the nth day then he travels up either $2x$ meters or $x-1$ meters on the next day with equal probability for both cases.

   Suppose Anurag climbs for 3 days. What is the expected number of meters he climbed?

3. 10 coins are placed in a row on a table, all facing heads up. Then two people randomly choose a coin and flip it over. What is the probability that at the end of the process, all coins are still heads up?

4. Kevin is playing a game with the set of numbers $S = \{1, 2, 3, 4, 5\}$. There is an answer which is a subset of S of four numbers, and Kevin must try to guess the answer by choosing four numbers from S.

   If Kevin's guess shares exactly three elements with the answer, then he is told he is one off. Suppose that after guessing $\{1, 2, 3, 4\}$ he is one off, and after guessing $\{1, 2, 3, 5\}$ he is one off again. What are the number of possibilities for the answer?

5. Find the number of ways to color a 1×5 rectangle with 3 colors such that no two consecutive unit squares are the same color.

6. Find the minimum N such that for any set of size N of positive integers less than 100, one can find a number that is odd.

7. A spinner is colored with 5 red sectors and 5 black sectors, with each sector of equal area. Two sectors are then randomly labeled W. I then spin the spinner twice. What is the probability that in both spins, I land on a sector labeled W that is red?

8. What is the minimum N to guarantee that in a room of N people, either three all know each other or three people all mutually do not know each other?

9. Count the number of ways we can partition 12 coins into three baskets with pairwise different number of coins such that at least one basket has a prime number of coins.

10. Ana and Banana play the following game:

    - At the beginning of the game, there is an integer N satisfying $N \leq 100$.
    - Ana goes first, and the two alternate turns.
    - On someone's move, they can subtract x from N as long as x is a palindrome and $x \leq N$.
    - If N is zero at the start of a player's turn, then that player loses.

    What is the largest value of N for which Banana can win?

11. Five distinct letters are chosen from CENTENNIAL, where each distinct letter has equal probability, and then are arranged in a row. What is the probability that the word LANE is spelled out consecutively somewhere in the row?

12. Count the number of permutations of 12345 such that a prime number is adjacent to another prime number in the permutation.

13. There are 2024 lilypads, and a frog is hopping from from the first lilypad to the last. A hop can cover any number of lilypads but can only be in the forwards direction. What is the probability the frog lands on 1989 at some point in time?

14. An ant is traveling on the vertices of a unit cube such that each second, it chooses an edge connected to the vertex it is currently located at and travels down that edge.

    Suppose that the ant starts at the top of the unit cube. What is the probability that after 3 moves, it is at the top again?

15. Compute the number of permutations of 12345 such that there do not exist three numbers a, b, c in that order in the permutation with $a < b < c$.