Math 131 Fall 2018 Semester
Professor de Pillis

Homework Assignment #10
Introduction to Sequences.

Due: Friday, November 30, 2018, by 1pm.

Guidelines

Do the assigned reading. Details not covered in class can be found in the textbook.

You will only be submitting solutions to the "Submission Problems."

You should sincerely attempt to solve each of the Reading Problems that are also listed. Once you have made an attempt at answering an entire weekly batch of Reading Problems, feel free to discuss and share solutions with others.

Rewrites

Note: No rewrites.

Reading

- Read Rudin, chapter 3, pp.53–78 (Sequences [Upper and Lower Limits, Special Sequences], Series [Series of Nonnegative Terms (Cauchy Test), The Number e, Root and Ratio Tests, Power Series, Summation by Parts, Absolute Convergence, Addition and Multiplication of Series, Rearrangements]).
- Read Rudin, chapter 4, pp. 83–98 (Limits, Continuity, Compactness, Connectedness, Discontinuities, Monotonicity, Limits at Infinity).

Problems List

- Submission Problems. Submit solutions to Questions 1, 2, 3 below on Gradescope.
- Reading problems. Read, but do not submit:
  1. Rudin, chapter 3, problems 13 – 15, 19 (Cantor set).
  2. Rudin, chapter 4, problems 1 – 26.

Question 1 Let $a_n = \dfrac{n^n}{n!}$.

(a) Prove that $\displaystyle\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = e$.

(b) Determine (with justification) $\lim_{n\to\infty} \frac{n}{(n!)^{1/n}}$.

Hint: There are some very helpful theorems in our textbook. Be sure to cite them when you use them.

Question 2 Suppose $f(x) = x^2$. Is f is uniformly continuous on $\mathbb{R}$? Justify your conclusion.

Question 3 In $\mathbb{R}$, let f be a continuous function on the closed interval $[0, 1]$ with range also contained in $[0, 1]$. Prove that f must have a *fixed point*; that is, show $f(x) = x$ for at least one value of $x \in [0, 1]$. (Hint: use the Intermediate Value Theorem.)