Student Name: _____
Math 131
Homework 10
Due Date: November 30, 2018

# Homework 10

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**Problem 1.**
Let $a_n = \frac{n^n}{n!}$

1. Prove that $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = e$
2. Determine (with justification) $\lim_{n\to\infty} \frac{n}{(n!)^{\frac{1}{n}}}$

   Hint: There are some very helpful theorems in our textbook. Be sure to cite them when you use them.

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**Solution.**

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**Problem 2.**
Suppose $f(x) = x^2$. Is $f$ is uniformly continuous on $\mathbb{R}$? Justify your conclusion.

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**Solution.**

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**Problem 3.**

In $\mathbb{R}$, let $f$ be a continuous function on the closed interval $[0,1]$ with range also contained in $[0,1]$. Prove that $f$ must have a fixed point; that is, show $f(x) = x$ for at least one value of $x \in [0,1]$. (Hint: use the Intermediate Value Theorem.)

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**Solution.**