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# CQF Exam One

June 2025 Cohort

## Instructions

All questions must be attempted. Requested mathematical and full computational workings must be provided to obtain maximum credit. CQF material and books may be referred to but you should answer the questions in your own words. You are not allowed to discuss the exam work with anyone else.

Making operational sense of exam questions and coding is part of your individual exam work. Please make a good use of lectures, solutions and labs/tutorials. Tutor is unable to confirm formulae, discuss your numerical answers or provide hints beyond ones given.

## Submission

You must prepare PDF REPORT that integrates workings, numerical answers and plots in the order of questions. ‘Refer to Python code’ statements will not be accepted as answers.

1. If you draft the report from Python notebook – remove unnecessary output and unused code, add maths with Markdown TeX. Save IPYNB file to HTML first, then print PDF from HTML.
2. If you draft from Word/LaTeX – add maths requested, numerical solutions and plots. Here you can insert code at your discretion, eg full/part code after each question or in an appendix.
3. Exam tasks give specific instructions, eg to organise results into a table or specific plot. Handwritten & scanned math workings are acceptable without loss in marks.

Upload two files: E1.YOURNAME.REPORT.PDF and E1.YOURNAME.CODE.ZIP (not .rar) with source code and declaration if not in PDF. If experiencing technical difficulties, you can exceptionally upload report as HTML or IPYNB, however that will delay the processing.

**Marking Scheme: Q1 30% Q2 30% Q3 40%. Total is 100%.**

## E1 2025-June. Exploring Optimal Portfolios

**Task 1.** Tangency Portfolio invested in risky assets but on the risk-adjusted basis.

$$\begin{aligned} \underset{\mathbf{w}}{\operatorname{argmax}} \frac{\mathbf{w}'\boldsymbol{\mu} - r_f}{\sqrt{\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}}} &= \frac{\mu_{\Pi} - r_f}{\sigma_{\Pi}} \\ \text{s.t. } \mathbf{w}'\mathbf{1} &= 1 \end{aligned}$$

The analytical solution is possible by the method of Lagrangian and we recommend the form,

$$\mathbf{w}_T^* = \frac{\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - r\mathbf{1})}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - r\mathbf{1})}.$$

- For the range of tangency portfolios for  $r_f = 25bps, 125bps, 200bps$  compute allocations and  $\sigma_{\Pi}$ . Present results in a table. Asset and correlation data – at the end of this question.
- Plot the specific form of the Efficient Frontier for each of these portfolios. Hint: check Solutions for relevant lectures, tutor will not confirm how the plot should look like.
- Consider the portfolio with  $r_f = 200bps$ . For each additional unit of risk taken, does the investor reduce or increase their excess return?

Asset	$\mu$	$\sigma$	$w$
A	0.02	0.05	$w_1$
B	0.07	0.12	$w_2$
C	0.15	0.17	$w_3$
D	0.20	0.25	$w_4$

$$\mathbf{R} = \begin{pmatrix} 1 & 0.3 & 0.3 & 0.3 \\ 0.3 & 1 & 0.6 & 0.6 \\ 0.3 & 0.6 & 1 & 0.6 \\ 0.3 & 0.6 & 0.6 & 1 \end{pmatrix}$$

**Task 2.** Consider the optimization with a target return  $m$ . There is no risk-free asset.

$$\underset{\mathbf{w}}{\operatorname{argmin}} \frac{1}{2} \mathbf{w}' \boldsymbol{\Sigma} \mathbf{w}$$

$$\mathbf{w}'\mathbf{1} = 1$$

$$\mathbf{w}'\boldsymbol{\mu} = m$$

Compute optimal allocations and portfolio risk with  $m = 7\%$  for the three levels of correlation matrix  $\times 1$ ,  $\times 1.3$ ,  $\times 1.8$ . The latter two are stressed subject to the upper limit 0.99 for each cross-asset correlation. Diagonal elements remain equal to 1.

Asset	$\mu$	$\sigma$	$w$
A	0.05	0.07	$w_1$
B	0.07	0.28	$w_2$
C	0.15	0.25	$w_3$
D	0.22	0.31	$w_4$

$$\mathbf{Corr} = \begin{pmatrix} 1 & 0.4 & 0.3 & 0.3 \\ 0.4 & 1 & 0.27 & 0.42 \\ 0.3 & 0.27 & 1 & 0.5 \\ 0.3 & 0.42 & 0.5 & 1 \end{pmatrix}$$

Hint: in this question, the analytical formula for optimal allocation is for you to identify. Negative and outsized allocations ( $> \pm 100\%$ ) are possible, particularly for the increased correlation levels. Please do not reconfirm your numerical results via support.

## E1 2025-June. Understanding Value at Risk

Assume you are an analyst concerned with how risky **NASDAQ-100** became over **S&P 500**. Perform the backtesting of Analytical VaR (99%/10day) on the data provided in .csv files.

**Task 3.** The quick guide is given below, but please refer to the tutorial and CQF material.

$$\text{VaR}_{10D,t} = \text{Factor} \times \sigma_t \times \sqrt{10}$$

- Compute the rolling standard deviation  $\sigma_t$  from 21 daily returns. Timescale of  $\sigma_t$  remains ‘daily’ regardless of how many returns are in the sample.
- To make a projection over 10 days, we use the additivity of variance  $\sigma_{10D} = \sqrt{\sigma_t^2 \times 10}$ .
- A breach occurs when the forward realised 10-day return is below the  $\text{VaR}_t$  quantity.

$$r_{10D,t+10} < \text{VaR}_{10D,t} \quad \text{given both numbers are negative.}$$

VaR is fixed at time  $t$  and compared to the return from  $t$  to  $t + 10$ , computed  $\ln(S_{t+10}/S_t)$ . A puristic approach can compare to  $\ln(S_{t+11}/S_{t+1})$  but state this assumption in your report in bold.

Present the following deliverables in your report:

- (a) The count and percentage of VaR breaches.
- (b) Provide a plot which identifies the breaches with crosses or other marks.
- (c) Provide a list of breaches with columns [Date, ClosingPrice, LogReturn, VaR\_10D, Ret\_10D].
- (d) In your own words describe, was NASDAQ-100 more risky than S&P 500 during COVID pandemic news 2020-Feb to 2020-Mar? What about the subsequent market correction period in 2021-2022?

**END OF EXAM**