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Do Recessions and Bear Markets Occur Concurrently across Countries? A Multinomial Logistic Approach*

Aubrey Poon[1,2] and Dan Zhu [3]

[1]School of Economics, University of Kent, United Kingdom, [2]School of Economics, Örebro University, Sweden and [3]Department of Econometrics and Business Statistics, Monash University, Australia

Address correspondence to Dan Zhu, Department of Econometrics and Business Statistics, Monash University, Wellington Road, Clayton VIC 3800, Australia, or e-mail: dan.zhu@monash.edu

Abstract

We introduce a novel multinomial logistic model for detecting and forecasting concurrent recessions and bear markets across multiple countries. Our framework leverages cross-country panel features and provides additional information for robust analysis. Through a comprehensive simulation study, we demonstrate the computational efficiency and accuracy of our model, even when handling multiple binary indicators. Applying our framework to empirical data from the United States, the UK, and Euro Area, we find that the multinomial logistic model produces superior medium-term forecasting of concurrent recession and bear market events across countries compared to multiple independent single logistic models. Additionally, our counterfactual analysis reveals that specific events, such as a recession and bear market in the United States, along with the tightening of financial conditions and a negative interest rate spread in the United States, increase the probability of concurrent and individual recession and bear market occurrences in the UK and Euro Area.

Keywords: recession prediction, bear markets, multinomial logistic, cross-country, mixed frequency, Bayesian estimation

JEL classifications: E32; E37; C22; C25

In the event that the United States experiences a recession or enters a bear market (or both), what are the chances that the UK and the Euro Area will follow suit? This question should be highly relevant today in an ever-increasing globalized world. Since the rapid technological progress made through the early 21st century, countries worldwide are more interconnected than ever. In particular, financial integration has allowed investors and firms to invest and operate in multiple countries simultaneously with relative ease. Consequently, global economies are now highly dependent on each other. As the largest open economy globally, the United States has been the subject of numerous studies documenting the spillover effects of its economic performance and policy changes on countries worldwide (see Bagliano and Morana 2012; Kose et al. 2017; Fadejeva et al. 2017; Bhattarai et al. 2020). Furthermore, extensive research has highlighted linkages and

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spillover effects between the U.S. stock market and stock markets in other countries (see Becker et al. 1995; Masih and Masih 2001; Ehrmann et al. 2011). Therefore, it is highly plausible that recessions and bear markets are interconnected across multiple countries.

Forecasting the probability of a simultaneous recession and bear market across countries should be a top priority for policymakers and investors worldwide. Specifically, a recession and a bear market would signify a period of substantial decline in economic activity and stock market prices from their local previous peaks in their corresponding cycles, respectively.[1] Obtaining this probability measure would serve as an early warning signal regarding a country's economic and financial market status, enabling swift decision-making for policymakers and investors alike. Surprisingly, a considerable portion of the extensive previous literature has predominantly focused on modeling recessions or bear markets exclusively within the U.S. economy. For example, studies by Chauvet and Potter (2002), Kauppi and Saikkonen (2008), and Rudebusch and Williams (2009) employ various econometric strategies to forecast recession probabilities in the United States, while studies by Chen (2009), Nyberg (2013), and Kole and Van Dijk (2017) concentrate on forecasting bear markets exclusively within US stock returns. To address this literature gap, we present a novel multinomial logistic model that extracts the probability of concurrent recessions and bear markets across multiple countries. Additionally, our proposed framework can forecast the probability of a recession and bear market occurring concurrently or independently, exclusively in a specific country.

Our article is closely related to the research conducted by Kauppi and Saikkonen (2008) and Nyberg (2013), wherein they, respectively, forecasted U.S. recession and bear market probabilities using binary time-series models. However, within the framework we present, both of these models can be viewed as restricted versions tailored to a single binary (country) case within our more general multinomial logistic model. The key strength of our proposed framework lies in its ability to capitalize on the cross-country panel structure of the data. This unique feature enhances our capability to detect recessions and bear markets at both individual and collective country levels. In contrast, the aforementioned binary time-series models are confined to individual-specific country cases and do not account for interdependencies between countries.

Another alternative approach that could be used to forecast recession probabilities is the qualitative VAR framework of McCracken et al. (2022). The advantage of using a qualitative VAR is that it captures the endogenous relationship between the binary events and other observed variables. However, estimating a qualitative VAR model can be computationally challenging. In particular, sampling the latent variables associated with the binary events typically employs a single-move sampler, which generates a single latent variable at a time. This would be even more computationally costly when modeling multiple binary indicators concurrently. In contrast, our proposed model focuses on the reduced-form structure and allows multiple binary indicators concurrently in a simple and efficient manner.

Our proposed model can also be formulated within a general Markov-switching framework. Numerous studies in the literature, including works by Kim and Nelson (1999), Chauvet and Potter (2002), Nalewaik (2012), and Guérin and Marcellino (2013), utilize Markov-switching models to detect recession probabilities in the United States. However, all these previous studies assume homogeneous time-invariant transition probabilities,

[1] According to Chauvet and Potter (2000), a bear market is characterized by periods of generally decreasing stock market prices, a concept widely accepted by financial commentators. However, despite this conceptual agreement, there lacks a formal definition of a bear market in the literature, as noted by Lunde and Timmermann (2004). Consequently, various econometric methods have been employed to identify bear market phases within stock markets. In our study, we adhere to the approach outlined by Lunde and Timmermann (2004) and utilize their filtering algorithm to construct a binary bear market indicator for each of the three countries under investigation.

representing the likelihood that the current regime stays the same or changes. In contrast, our model offers a more flexible alternative within the Markov-switching framework. It can be interpreted as incorporating non-homogeneous time-varying transition probabilities, drawing inspiration from Filardo (1994), and includes the additional dimension of cross-country dependence.

Empirically, our article extends the existing literature on predicting recessions and bear markets using financial variables in two significant ways. First, many prior studies have predominantly focused on predicting these economic conditions using their respective interest rate spreads or yields (see Estrella and Hardouvelis 1991; Estrella and Mishkin 1998; Estrella et al. 2003; Haubrich 2006; Chen 2009; Nyberg 2013). In contrast, our proposed model specification considers a set of interest rate spreads or yields across multiple countries to predict recessions and bear markets across countries. Additionally, our model enables us to directly assess whether another country's interest rate spread can predict the occurrence of a specific country's recession and bear market. Secondly, we build upon the recent work by Adrian et al. (2019) by incorporating financial condition indices from multiple countries into our model. The key insight from Adrian et al. (2019) is that deteriorating financial conditions are associated with a substantial increase in downside risk for U.S. Real GDP. By including multi-country financial condition indices in our proposed framework, our study becomes the first in the literature to explicitly test whether each country's financial condition is a robust predictor of recessions and bear markets, both domestically and across countries.

From a methodological perspective, we provide three important contributions. First, we extend the Polson et al. (2013) state-of-the-art Bayesian multinomial logistic model to the dynamic case that jointly models recession and bear market occurrence between the United States, the UK, and Euro Area. To capture the dynamic interdependencies between countries, we follow Canova and Ciccarelli (2013) by including the lagged terms of each country's recession and bear market indicators in the model specification. Second, we extend the multinomial logistic model to a mixed frequency setting where we incorporate monthly and weekly frequency variables into our proposed model. More specifically, we modeled the joint recession and bear market probabilities across countries at a monthly frequency and in certain specifications, we included weekly financial condition indices as exogenous predictors in the model. Therefore, two of the four model specifications we proposed in the empirical application can be considered a multinomial logistic mixed-data sampling (MIDAS) model. Moreover, including the weekly variables in the model could provide additional information that can be used to strengthen the predictability of recessions and bear market outcomes across countries. Finally, we extend the multinomial logistic model to a big data context by considering a large set of economic activity exogenous predictors in one of our four proposed model specifications. To overcome the overparameterization problem in this big data model, we implement Alhamzawi and Ali's (2018) Bayesian adaptive lasso shrinkage prior to the coefficients of the exogenous predictors, which implicitly selects the most important exogenous regressors in the model.

To validate the efficacy of our proposed framework, we conducted a simulation study across diverse settings. Specifically, we applied our multinomial logistic model to various data-generating processes (DGPs) with differing sample sizes and numbers of binary indicators specified in the model. Our findings indicate that the accuracy of our proposed model improves with larger sample sizes, and this accuracy remains consistent as the number of binary indicators considered in the model increases. Furthermore, our proposed model demonstrates computational efficiency even with an increase in the number of binary indicators. In a simplified scenario involving only two binary indicators, our results reveal a close alignment between all estimated posterior probabilities and their corresponding true DGP probabilities. Consequently, our proposed multinomial logistic model exhibits robustness, enabling the simultaneous detection of probabilities across all potential outcomes, even with a substantial number of binary indicators.

We applied our proposed framework to detect and forecast recessions and bear market outcomes across the U.S., UK, and Euro Area economies. Specifically, we estimated four variants of the multinomial logistic model, each distinguished by its set of exogenous predictors. In assessing the in-sample fit, we observed that all four models exhibited comparable performance, generating similar model fits. Additionally, the estimated posterior probabilities from each model accurately identified the corresponding true categorical outcomes.

In the context of the out-of-sample forecasting exercise, our analysis reveals that our proposed multinomial logistic model demonstrates superior medium-term forecasting efficacy in predicting concurrent recession and bear market events across countries, especially in the United States and the Euro Area. This superiority is evident when compared to a forecast derived from aggregating predictions obtained through multiple independently modeled single logistic models. Notably, the multinomial logistic model, incorporating the interest rate spreads and monthly financial condition indices of all countries as exogenous predictors, emerges as the best forecasting model across the twelve forecast horizons. This outcome aligns with the findings of Adrian et al. (2019), indicating that deteriorating financial conditions reliably predict recessions. Furthermore, our analysis suggests that a parsimonious variant of the multinomial logistic model is sufficient for accurately predicting recession and bear market outcomes across multiple countries.

Finally, we undertake a counterfactual exercise where we assess the potential impact of a U.S. recession or negative financial shocks on the UK and Euro Area. Specifically, our focus centers on four counterfactual events: a U.S. recession, a U.S. bear market, a tightening of the U.S. National Financial Conditions Index (NFCI), and a negative U.S. interest rate spread. Our findings reveal that all four counterfactual events heighten the probabilities of both a recession and a bear market occurring simultaneously and individually for each country. Significantly, the analysis indicates that both a U.S. bear market and the tightening of the U.S. NFCI increase the likelihood of a simultaneous recession and bear market in both the United States and the Euro Area, particularly in the aftermath of the coronavirus disease 2019 (COVID-19) pandemic. This extends the findings of Adrian et al. (2019), emphasizing that the tightening of the U.S. NFCI not only predicts U.S. recessions but also serves as a predictive indicator for similar economic contractions in the UK and the Euro Area. Furthermore, our results are consistent with Ehrmann et al. (2011), highlighting the substantial role played by U.S. financial markets in explaining approximately 30 percent of fluctuations in Euro Area financial markets.

The rest of the article is organized as follows. Section 1 presents the proposed multinomial logistic model in a general framework and an empirical application context. Section 2 reports and discusses the results of the simulation study. Section 3 presents the empirical application for detecting and forecasting recessions and bear markets jointly across the U. S., UK, and Euro Area economies. Finally, Section 4 concludes.

1 Methodology

This section introduces the multinomial logistic model for detecting concurrent recessions and bear markets across multi-countries. Section 1.1 describes the multinomial logistic model framework for a general case with multiple discrete choices or binary variables. The following subsection illustrates how the general multinomial logistic model framework can be applied to a simple two countries U.S.–UK example. Finally, Sections 1.3 and 1.4 provide details on the data gathered for the empirical application and the Bayesian estimation of the multinomial logistic model.

1.1 A General Multinomial Logistic Model

To detect the dependence of recessions and bear markets across multi-countries, we propose a multinomial logistic model. A multinomial logistic model is a flexible approach to model multiple discrete choices or binary variables jointly. Specifically, we can define a categorical random variable for n binary indicators (recession or bear market indicators) as

$$Y_t = \sum_{j=1}^{2^n} j\left(\prod_{i\in P_j} I_{it}\right)\left(\prod_{i\in P_j^c}(1-I_{it})\right), \tag{1}$$

where $I_{it} = 1\{\text{country } i\, in\, recession/bear\, market\}$ is each country i indicator function that denotes if the country is in a recession (expansion) or bear (bull) market at a particular time period. Furthermore, $\mathcal{P}$ can denote the set of all non-empty subsets of $\{1,\ldots,n\}$, which intuitively can be interpreted as all possible combinations of n binary outcomes.[2] Thus, the $dim(\mathcal{P}) = 2^n$, which signifies the exponential growth in possible combinations as the number of binary indicators n increases. In the context of our application, each $P_j \in \mathcal{P}$ represents a set of countries experiencing recessions and bear markets simultaneously. Conversely, the complement of P_j, denoted as P_j^c, corresponds to the case where expansions and bull markets occur concurrently in a particular set of countries.

Therefore, using Equation (1), the probability of the vector of n binary variables falling within the jth category can be written as a logistic function

$$\mathbb{P}(Y_t = j|X_t = x) = \frac{\exp(x'\beta_j)}{1+\sum_{k=1}^{2^n-1}\exp(x'\beta_k)}\forall j = 1,\ldots,2^n-1., \tag{2}$$

where x is a vector of $d\times 1$ explanatory variables and the last reference category can be defined as

$$\mathbb{P}(Y_t = 2^n|X_t = x) = \frac{1}{1+\sum_{k=1}^{2^n-1}\exp(x'\beta_k)}, \tag{3}$$

The main intuition behind the multinomial logistic model is that it allows the researcher to infer the direct probability (or dependence) of recessions and bear markets occurring across countries concurrently, which is extracted from Equations (2) and (3). In contrast, the study by Kauppi and Saikkonen (2008) is only able to infer the probability of a recession or bear market of a specific country, which is a restricted case where $n = 1$ under our proposed specification. In essence, the approach in Kauppi and Saikkonen (2008) is tailored to understanding the economic dynamics of individual countries, whereas our multinomial logistic model extends its capability to capture the interdependencies of economic states across a broader spectrum of countries.

Our primary focus is to comprehend the occurrence of recessions, with indicators compiled through the analysis of hundreds of economic indicators. These indicators collectively offer an overarching view of economic activity in the current state of the economy. While one could opt for a large-dimensional VAR, potentially involving hundreds of variables for each country, and consider a shrinkage prior for parsimonious considerations, such VARs are expensive to estimate and can be sensitive to prior specifications. Moreover, forecasting these large variables would still necessitate analysis and convergence into indicators. The

[2] For example, in a $n=2$ case, we have $\mathcal{P} = \{P_1, P_2, P_3, P_4\}$ where $P_1 = \{1,2\}, P_2 = \phi, P_3 = \{1\}$, and $P_4 = \{2\}$; in a $n=3$ case, we have $\mathcal{P} = \{P_j\}_{j=1}^8$ where $P_1 = \{1,2,3\}, P_2 = \phi, P_3 = \{1,2\}$, $P_4 = \{2,3\}, P_5 = \{1,3\}, P_6 = \{1\}, P_7 = \{2\}, P_8 = \{3\}$.

ultimate goal is to distill this information into a single binary variable that provides a clear signal regarding the future overall state of the economy.

In terms of our empirical application, we apply our proposed framework to six binary indicators $n = 6$, encompassing recession and bear market indicators for the United States, the UK, and Euro Area, resulting in $Y_t = 2^n = 64$ possible outcomes.

1.2 Illustrative Example for a Two-Country Recession Case

To further demonstrate our proposed framework, we provide an illustrative example of detecting dependence of recessions across two countries. For example, if the United States is in a recession, it is highly likely that the UK will be in recession, too, given that the United States is a top trading partner of the UK. Using our proposed multinomial logistic model, we can derive a probability measure of recessions occurring concurrently across the United States and the UK. Formally, we can define a categorical random variable for both the United States and the UK ($n = 2$) as

$$Y_t = I_{1t}I_{2t} + 2(1 - I_{1t})(1 - I_{2t}) + 3I_{1t}(1 - I_{2t}) + 4(1 - I_{1t})I_{2t}, \tag{4}$$

where $I_{1t} = 1\{\text{US}\,\textit{recession}\}$ and $I_{2t} = 1\{\text{UK}\,\textit{recession}\}$ are the corresponding recession indicators for both the United States and the UK, respectively. From Equation (4), we can derive four possible outcomes from the categorical random variable $Y_t \in \{1, 2, 3, 4\}$, they are:

$$Y_t = \begin{cases} Y_t = 1 & \text{Both US and UK are in a concurrent recession,} \\ Y_t = 2 & \text{Both US and UK are in a concurrent expansion,} \\ Y_t = 3 & \text{US is in a exclusive recession,} \\ Y_t = 4 & \text{UK is in a exclusive recession,} \end{cases} \tag{5}$$

For each point of time, we also consider a $d \times 1$ vector of exogenous predictors that includes a constant term

$$X_t = [1, I_{1t-1}, I_{2t-1}, Z_t']' \in \mathbb{R}^d, \tag{6}$$

where I_{1t-1} and I_{2t-1} are the lagged recession indicators for the United States and the UK, respectively. We included these lagged indicators as it captures the dynamic interdependencies between both countries according to Canova and Ciccarelli (2013). Here, Z_t is the $z \times 1$ vector of exogenous predictors that potentially could contain both countries' economic activity predictors or interest rate spreads. Therefore, the four probability measures extracted from the multinomial logistic model are

$$P(Y_t = j | X_t = x) = \frac{\exp(x'\beta_j)}{1 + \sum_{k=1}^{3} \exp(x'\beta_k)} \forall j = 1, \ldots, 3., \tag{7}$$

and the last reference category can be defined as

$$P(Y_t = 4 | X_t = x) = \frac{1}{1 + \sum_{k=1}^{3} \exp(x'\beta_k)}. \tag{8}$$

Note here the β is a $d \times 1$ vector of parameters that is estimated in the model.

In addition, we can reformulate our proposed multinomial logistic model into a parsimonious Markov-switching model, where the transition probability matrix is of time-varying nature, that is non-homogeneous Markov Chains. For example, in our two countries' case, we have a four-state Markov chain, where a non-homogeneous time-varying 4×4 transition probability matrix $\mathbf{P}(t)$ describing the dynamic evolution of the Markov chain depends on the vector of covariates. Specifically, we have

$$P_{1j}(t) = \begin{cases} \dfrac{\exp([1,1,1,z_t']\beta_j)}{1+\sum_{k=1}^{3} \exp([1,1,1,z_t']\beta_k)}, & j = 1,2,3., \\ \dfrac{1}{1+\sum_{k=1}^{3} \exp([1,1,1,z_t']\beta_k)}, & j = 4., \end{cases}$$

$$P_{2j}(t) = \begin{cases} \dfrac{\exp([1,0,0,z_t']\beta_j)}{1+\sum_{k=1}^{3} \exp([1,0,0,z_t']\beta_k)}, & j = 1,2,3., \\ \dfrac{1}{1+\sum_{k=1}^{3} \exp([1,0,0,z_t']\beta_k)}, & j = 4., \end{cases}$$

$$P_{3j}(t) = \begin{cases} \dfrac{\exp([1,1,0,z_t']\beta_j)}{1+\sum_{k=1}^{3} \exp([1,1,0,z_t']\beta_k)}, & j = 1,2,3., \\ \dfrac{1}{1+\sum_{k=1}^{3} \exp([1,1,0,z_t']\beta_k)}, & j = 4., \end{cases}$$

$$P_{4j}(t) = \begin{cases} \dfrac{\exp([1,0,1,z_t']\beta_j)}{1+\sum_{k=1}^{3} \exp([1,0,1,z_t']\beta_k)}, & j = 1,2,3., \\ \dfrac{1}{1+\sum_{k=1}^{3} \exp([1,0,1,z_t']\beta_k)}, & j = 4. \end{cases}$$

1.3 Data

In our empirical analysis, we employ the U.S. NBER recession indicator as a reference for identifying recessions. However, for the UK and Euro Area, where no business cycle dating committee is in place, we rely on the Organisation for Economic Co-operation and Development (OECD) recession indicator as an alternative. To establish binary indicators for bear markets, we implement the filtering algorithm proposed by Lunde and Timmermann (2004) and apply it to the S&P500 (United States), FTSE 100 (UK), and Euro STOXX (Euro Area) price indices. Another approach to derive these binary indicators is the Pagan and Sossounov (2003) dating method. Nonetheless, a notable drawback of the Pagan and Sossounov (2003) method is the necessity to specify a predetermined window of data points to identify bull/bear markets, subject to a set of censoring rules that dictate both the minimum length of the bull/bear market (4 months) and the minimum duration of a full cycle (16 months). In contrast, the filtering algorithm proposed by Lunde and Timmermann (2004) does not impose such restrictions.[3] Additionally, we consider various sets of economic activity predictors for Z_t across the three countries, as detailed in Table 1. All data are sourced from the U.S. FRED database and the European Central Bank data portal. The sample period extends from January 1999 to August 2022, with data collected at both monthly and weekly frequencies.

[3] For the sake of robustness, we present in-sample empirical results using the binary bear market indicators derived from the Pagan and Sossounov (2003) dating method in appendix of the article.

Table 1. Data information

Data variable	M1	M2	M3	Transformation	Frequency
U.S. NBER recession indicator	×	×	×	Level	Monthly
UK OECD recession indicator	×	×	×	Level	Monthly
Euro Area OECD recession indicator	×	×	×	Level	Monthly
U.S. bear market indicator (S&P 500)	×	×	×	Level	Monthly
UK bear market indicator (FSTE 100)	×	×	×	Level	Monthly
Euro Area bear market indicator (Euro STOXX)	×	×	×	Level	Monthly
U.S. interest rate spread	×	×	×	Level	Monthly
UK interest rate spread	×	×	×	Level	Monthly
Euro Area interest rate spread	×	×	×	Level	Monthly
U.S. National Financial Condition Index		×	×	Level	Weekly
UK composite indicator of systemic stress		×	×	Level	Weekly
Euro Area composite indicator of systemic stress		×	×	Level	Weekly
U.S. industrial production			×	$\triangle \ln x_t$	Monthly
U.S. housing starts			×	$\triangle \ln x_t$	Monthly
U.S. all employees, total nonfarm			×	$\triangle \ln x_t$	Monthly
U.S. retail sales			×	$\triangle \ln x_t$	Monthly
U.S. real manufacturing and trade industries sales			×	$\triangle \ln x_t$	Monthly
UK industrial production			×	$\triangle \ln x_t$	Monthly
UK total manufacturing			×	$\triangle \ln x_t$	Monthly
UK retail sales			×	$\triangle \ln x_t$	Monthly
UK total employment			×	$\triangle \ln x_t$	Monthly
Euro Area industrial production			×	$\triangle \ln x_t$	Monthly
Euro area retail sales			×	$\triangle \ln x_t$	Monthly
Euro area constructions			×	$\triangle \ln x_t$	Monthly
Euro area unemployment rate			×	Level	Monthly

Notes: The interest rate spread for three countries is computed by taking the difference between the 10-year treasury government bond rate and the 3-month treasury bill rate.

We employ our proposed multinomial logistic model to estimate three distinct specifications of Z_t as outlined in Table 1. In the first specification (referred to as M1), Z_t solely comprises the interest rate spread, denoting the difference between the 10-year treasury government bond rate and the 3-month treasury bill rate across the three countries. This interest rate spread is interpreted by Estrella and Hardouvelis (1991) and Estrella and Mishkin (1998) as the implicit slope of the yield curve, demonstrating its significance as a pivotal predictor of recessions in the U.S. economy. Moving to the second specification (M2), Z_t includes both the interest rate spread and the corresponding financial condition index for each country. The rationale for incorporating these financial condition indices stems from findings by Adrian et al. (2019), who observe that worsening financial conditions correlate with a substantial increase in downside risk for GDP. Intuitively, financial conditions are anticipated to be robust predictors of recession across the three countries. For the United States, the Chicago Fed's weekly U.S. NFCI is utilized. However, due to limited data availability, we adopt the ECB Composite Indicator of Systemic Stress (CISS) measure, as suggested by Figueres and Jarociński (2020), to gage financial conditions in both the UK and Euro Area. Figueres and Jarociński (2020) demonstrate that employing CISS effectively reproduces tail risk stylized facts similar to those identified by Adrian et al. (2019) in the U.S. economy, but applied to the Euro Area. The third specification (M3) introduces a big data variant of Z_t, encompassing the interest rate spread, financial condition indices, and an extensive set of economic activity predictors across all countries. We categorize these three Z_t specifications outlined in Table 1 as the M1, M2, and M3 models, respectively.

In the M2 and M3 models, all financial condition indices utilized are of a weekly frequency. Despite the fact that we are modeling the probability of recessions and bear markets at a monthly frequency, we have chosen to leverage the high-frequency nature of these indicators by consolidating them into four vectors, w_t^1, w_t^2, w_t^3, and w_t^4, which are then incorporated into the composite variable Z_t. To elaborate, w_t^2 represents the financial conditions indices of the second week within a given month t, and a similar interpretation applies to the other three vectors. Consequently, both the M2 and M3 models can be characterized as MIDAS models, as weekly information from all financial conditions indices is included. Additionally, we explore a model variant of M2, denoted as M4, where all weekly financial indices are transformed to a monthly frequency. This adjustment ensures uniformity in the frequency of the data in M4, eliminating mixed-frequency characteristics.

1.4 Bayesian Estimation

We estimate our multinomial logistic model via Bayesian inference. More specifically, we follow Polson et al. (2013) and implement their proposed data-augmentation strategy for Bayesian estimation of logistic models. Polson et al. (2013) show that draws from conditional posterior of β_j can be simulated using the Polya-Gamma method, which results in a simple and efficient Gibbs sampler. In particular, we assume the prior of β_j follows

$$\beta_j \sim N(0_d, B_{0j}), \tag{9}$$

where 0_d is a $d \times 1$ vector of zeros of the prior mean and B_{0j} is a $d \times d$ matrix of the prior covariance. Next, Polson et al. (2013) show that using the Polya-Gamma method, the conditional posterior of β_j simplifies to a Gaussian distribution for each $j = 1, 2, 3.$,

$$\beta_j | \boldsymbol{\omega_j}, \boldsymbol{\beta}_{-j} \sim N\Big(\mu(\boldsymbol{\omega}_j), K_j(\boldsymbol{\omega}_j, B_{oj})^{-1}\Big), \tag{10}$$

where $\boldsymbol{\omega_j}$ is a $d \times 1$ vector of Polya-Gamma latent variables, $\boldsymbol{\beta}_{-j}$ is a vector of $\beta's$ that excludes β_j, and the precision matrix and the conditional posterior mean are, respectively, given as

$$\begin{aligned} K_j(\boldsymbol{\omega}, B_{oj}) &= B_{oj}^{-1} + \sum_{t=1}^{T} X_t \omega_{tj} X_t', \\ \mu(\boldsymbol{\omega}_j) &= K_j(\boldsymbol{\omega}_j, B_{oj})^{-1} \left(\sum_{t=1}^{T} X_t \left(1\{Y_t = j\} - 0.5 + \omega_{tj} \log \left(1 + \sum_{k \neq j}^{3} \exp\left(X_t' \beta_k\right) \right) \right) \right). \end{aligned} \tag{11}$$

We defer the reader to Polson et al. (2013) for more details on the implementation of the above Gibbs sampler.

Lastly, we impose a relatively non-informative prior for M1 and M4, where the prior covariance for β_j equals $B_{0j} = 10\mathbf{I}_d$. Regarding the M2 and big data variant of M3, since we have a large set of mixed-frequency and economic activity predictors in the model, we need to have some form of shrinkage in the model to ensure parsimony and we impose Alhamzawi and Ali (2018) adaptive lasso prior

$$\tau_{ij} \sim \text{EXP}\left(\frac{\lambda_{ij}}{2}\right), \text{ for } i = 1, \ldots d., \quad (12)$$
$$\lambda_{ij} \sim IG(a_0, b_0),$$

where in (9), $B_{0j} = \text{diag}(\tau_{1j}, \ldots, \tau_{dj})$ and we set $a_0 = b_0 = 0.01$. Note EXP and IG are denoted here as the exponential and inverse-gamma distribution, respectively. Finally, we estimate all three models using 10000 MCMC draws with a burn-in period of 5000 draws[4].

2 Simulation Study

To evaluate the accuracy of probability detection in our proposed specification, we estimate our multinomial logistic model across various DGPs of different T time periods and n countries. Specifically, we consider a DGP of the model structure to be

$$Y_t = W_t\gamma + \epsilon_t, \epsilon_t \sim N(0, \Sigma), \quad (13)$$

where $W_t = [1, I_{t-1}, X_t']$, and set each element of X_t and γ to be standard normal and $\Sigma \sim IW(0.1\mathbf{I}_n, n + 5)$. Furthermore, we simulate the probability of the indicators that fall within the jth combination by

$$\Phi(0_n, W_t\gamma S_j, \Sigma), \quad (14)$$

where $\Phi(.)$ is a multivariate Gaussian cumulative density function (CDF) and S_j is a diagonal matrix of dimension n with its ith diagonal equal to -1 if the $i \in P_j$ and 1 otherwise. Thus, I_{tj} is set to be 1 when the restriction $I_{tj} = 1\{Y_{tj} > 0\}$ is satisfied.

We estimate the multinomial logistic model on each DGP using the above-described Gibbs sampler and impose very non-informative normal priors on β_j predictors. Our estimation procedure involves 10,000 MCMC draws, preceded by a burn-in period of 5,000 draws and implemented across 10 parallel chains. The results are summarized in Table 2, which illustrates the average mean absolute deviation (MAD) between the true probabilities inherent in the DGPs and the estimated posterior mean probabilities. Notably, as the sample size of the data increases, the average MAD between the actual and estimated probabilities diminishes. Additionally, our multinomial logistic model exhibits sustained accuracy even with an increasing number of binary indicators. Table 2, also provides insights into the associated computation times. Importantly, the estimation algorithm for our proposed framework is computationally efficient. For instance, in a model featuring $n = 6$ binary indicators and $T = 1000$ observations, the total computation time remains below 10 min. While theoretically the model complexity increases geometrically with n, the actual computational time does not manifest in our Monte Carlo exercise. For large n cases, we can potentially explore parallelizing the sampling of coefficients.

We have also plotted the estimated posterior probabilities against the true probabilities generated by the DGP for a binary indicator case ($n = 2$) in Figure 1 of a single parallel chain. The graph illustrates a close correspondence between the estimated posterior probabilities and the true DGP probabilities across all four possible outcomes. In the majority of cases, the estimated probabilities successfully capture the peaks observed in the true probabilities. This observation implies that, given a sufficiently large sample size, our proposed multinomial logistic model exhibits accuracy in simultaneously detecting probabilities across all potential outcomes, even when considering multiple binary indicators. This

[4] In Section 6.1 of the appendix, we present the average inefficiency factors for the β parameters in Table 6 across the four model specifications. The average inefficiency factors for all models are either around or below 20, indicating satisfactory convergence diagnostics.

Table 2. Average MAD between true probabilities and estimated posterior probabilities, along with corresponding computation time (in seconds)

	No. of time periods		
No. of binary indicators	T = 300	T = 500	T = 1000
$n = 2$	0.03	0.03	0.02
$n = 3$	0.03	0.02	0.01
$n = 4$	0.02	0.02	0.01
$n = 5$	0.01	0.01	0.01
$n = 6$	0.01	0.01	0.01
Computation time (in seconds)			
$n = 2$	14	17	48
$n = 3$	35	52	95
$n = 4$	60	93	153
$n = 5$	105	149	273
$n = 6$	178	268	468

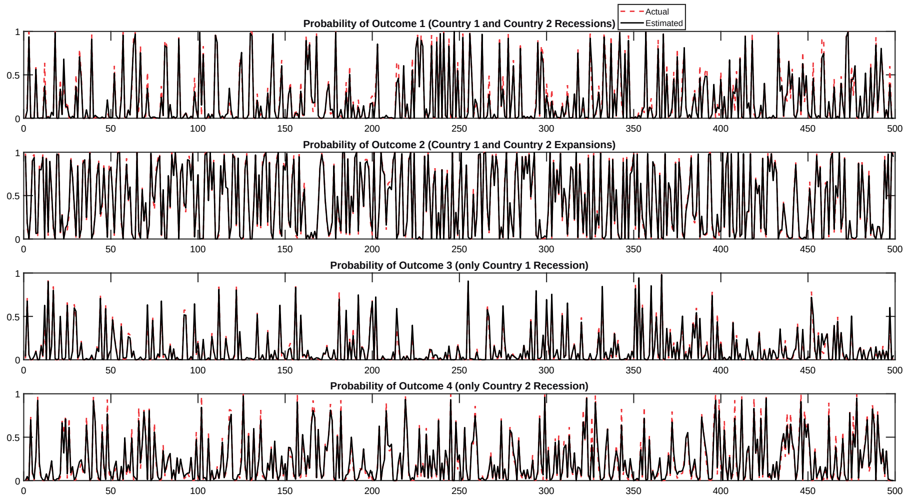

Figure 1. Plot of the estimated posterior probabilities against the true posteriors for $n = 2$ and $T = 500$.

Notes: The red dotted line is the actual DGP true probabilities and the black line is the estimated posterior probabilities from the multinomial logistic model. These plots are generated from a single parallel chain.

capacity is achieved in a computationally efficient manner, highlighting the model's robustness in capturing intricate probabilistic patterns.

3 Empirical Results

In this section, we showcase the empirical results of modeling the interdependence between recessions and bear markets across the United States, the UK, and Euro Area. Sections 3.1 and 3.2 present the in-sample and out-of-sample results, providing a comprehensive analysis of the model's performance. Section 3.3 focuses on the outcomes derived from the counterfactual study, offering additional insights into the hypothetical scenarios.

Table 3. The in-sample pseudo R^2 measures across the four models

Model	Estrella and Mishkin (1998)	McFadden (1973)
M1	0.99	0.68
M2	0.98	0.64
M3	0.98	0.64
M4[a]	0.98	0.64

[a] This model is the same M2, but all the financial condition indices are converted to monthly frequency. There is no mixed-frequency in this model.

3.1 In-Sample Analysis

We assess the in-sample fit of the four models by employing both the Estrella and Mishkin (1998) and McFadden (1973) Pseudo R^2 measures. In accordance with Kauppi and Saikkonen (2008), the Pseudo R^2 values range from 0 to 1 and can be interpreted similarly to the coefficient of determination in a standard linear regression. Table 3 presents the Pseudo R^2 values for all four models, revealing consistently comparable measures under both evaluation methods. Notably, Model M1 exhibits a marginally superior in-sample fit across the four models when assessed using the McFadden (1973) Pseudo R^2 measure. This outcome implies that the inclusion of financial condition indices and an extensive set of economic activity predictors from the three countries does not yield considerable improvements in model fit. Consequently, the parsimonious M1 model, featuring only interest rate spreads, proves sufficient in capturing recessionary and bear market events across the three countries. Our findings align with those of Estrella and Hardouvelis (1991) and Faria and Verona (2020), supporting the assertion that the slope of the yield curve effectively predicts a country's economic activity and stock market returns. Additionally, we acknowledge studies by Wu and Xia (2016); Wu and Xia (2020), which discuss the impact of the zero lower bound on the slope of the yield curve. Specifically, Wu and Xia (2020) illustrate how the zero lower bound can alter the movement and shape of the yield curve in the Euro Area. Given that our sample period includes zero lower bound periods, for robustness, we also estimate a version of the M1 model where the interest rate spread is computed via the shadow rate proposed by Wu and Xia (2016). We report the in-sample posterior results of this version of the model in section 6.1 appendix and find these results are virtually identical to those of our existing M1 specification.[5]

In light of the findings presented in Table 3, for the sake of clarity and coherence, we have chosen to exclusively depict the posterior mean probabilities derived from Model M1 in Figures 2 and 3. Furthermore, the posterior mean probabilities for the corresponding figures exhibit comparable patterns across the other three model variations. Figure 2 illustrates the posterior probabilities of recessions (expansions) and bear (bull) markets occurring concurrently in all three countries over time, set against their true categorical outcomes represented by shaded areas. The graphical representation highlights three distinct periods characterized by simultaneous recessions and bear markets in all three countries: the early 2000s, the Great Recession period of 2008–2009, and the recent COVID-19 pandemic period. The estimated posterior probabilities, denoted by the black line,

[5] We calculate the interest rate spread by subtracting the shadow rate from the 10-year treasury government bond rate for the US, UK, and the Euro Area. The shadow rate data for US, UK, and the Euro Area were gathered from Cythnia Wu's webpage https://sites.google.com/view/jingcynthiawu/shadow-rates?authuser=0. In Figures 7 and 8 of Section 6.1 in the appendix, we present the posterior probabilities of two versions of M1: our current M1 specification utilizing the interest rate spread as defined in Table 6 of the section 6.1, and an alternative M1 version using the interest rate spread computed from the shadow rate. Both figures indicate virtually identical posterior probabilities for the two models, accurately capturing their respective true categorical outcomes. Furthermore, the M1 model utilizing the interest rate spread derived from the shadow rate yields Pseudo R^2 measures identical to those of the existing M1 specification, as shown in Table 3. Therefore, our results remain robust even when employing an alternative interest rate spread computed using the shadow rate.

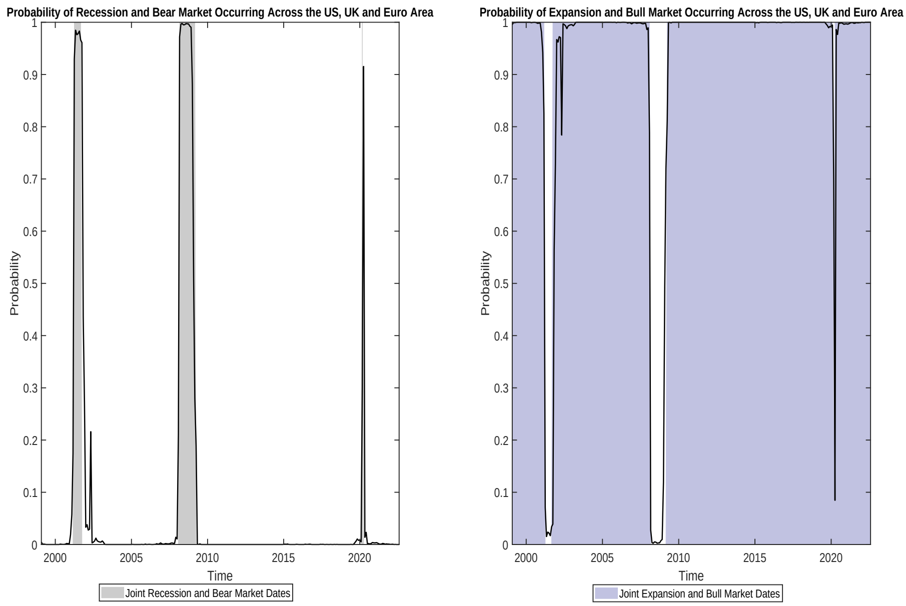

Figure 2. Posterior probabilities of recessions (expansions) and bear (bull) markets occurring concurrently in the United States, the UK and Euro Area from M1.

Notes: The shaded gray (purple) bars denote the joint recession (expansion) and bear (bull) market outcomes given by the recession (expansion) and bear (bull) market binary indicators across three the countries. The black line is the posterior probabilities for each specific outcome from model M1.

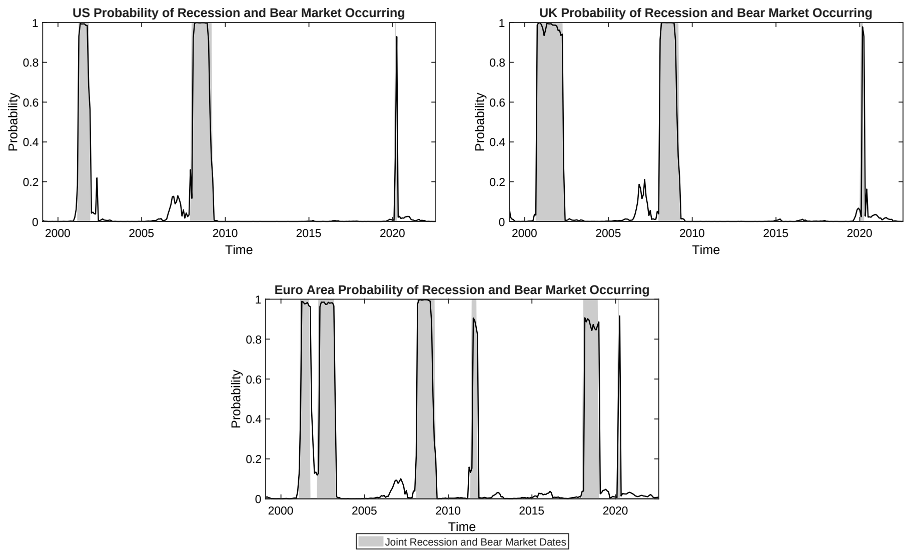

Figure 3. Posterior probabilities of recessions and bear markets occurring concurrently exclusively in each country from M1.

Notes: The shaded gray bars denote the joint recession and bear market outcomes given by the recession and bear market binary indicators for each country. The black line is the posterior probabilities for each specific outcome from model M1.

effectively capture all three categorical outcomes. Notably, during the Great Recession period, the co-occurrence of a bear market across the three countries is unsurprising, given that the Great Recession originated from a financial crisis in the United States. Similarly, during the COVID-19 pandemic period, the implementation of widespread social distancing restrictions by governments worldwide resulted in contractions in economic activity and stock markets across the majority of countries. Finally, in the early 2000s, the dot-com crash and the September 11 terrorist attacks led to a sharp decline in real economic activity and stock markets in the United States, UK, and the Euro Area. Importantly, our model effectively captures all these significant recessions and bear market events, affirming its robustness in perceiving key patterns in the data.

Figure 3 illustrates the posterior probabilities of a recession and bear market co-occurring exclusively in each country over time. The estimated posterior probabilities for the United States and the UK align precisely with the three distinct recession and bear market periods depicted in Figure 2. This observation implies a high degree of synchronization between the economies and financial markets of the United States and the UK. Consequently, it suggests that adverse real economic or financial developments in the United States may likely result in spillover effects on the UK economy. In contrast, the Euro Area exhibited an additional three periods (2002–2003, 2011–2012, and 2018–2019) where it experienced a simultaneous recession and bear market. This divergence from the patterns observed in the United States and the UK may be attributed to the Euro Area consisting of a group of countries. A plausible explanation is that a negative economic shock originating in one of these countries could potentially have a substantial spillover effect across all European countries. For instance, the European debt crisis, primarily originating from Greece, may have contributed to the recession and bear market observed in 2011–2012. This suggests the possibility of interdependence among the countries in the Euro Area, illustrating how shocks in one member state might potentially trigger a ripple effect throughout the entire region.

In summary, our in-sample analysis shows that all four models produce the exact model fit and posterior probabilities. We find that our model specification can accurately detect all the respective true categorical outcomes.[6]

3.2 Out-of-Sample Analysis

To assess the predictive accuracy of our proposed model, we conduct a pseudo-out-of-sample forecasting exercise using four distinct models. Our focus is on predicting concurrent recessions and bear market events across three countries individually and collectively. Comparing our model's forecasting performance to a single binary indicator model proves challenging. The latter aims to predict either a recession or a bear market, unlike our model, which simultaneously predicts these events across countries.

To establish a comparable benchmark for our model specifications, we estimate a standard logistic model for each of the six binary indicators. For instance, the first logistic model focuses solely on modeling U.S. recessions, using the U.S. interest rate spread as the exogenous predictor. A similar approach is applied to the remaining logistic models, each representing one of the six binary indicators. Note that with these six binary models, we can still estimate the probability of events, such as simultaneous recessions and bear markets, by treating the occurrence of these events as independent, as opposed to our joint estimation approach. This combined measure, referred to as "Joint logistic," is presented in Table 4.

[6] In section 6.2 of the appendix, we present the Pseudo R^2 measures and in-sample posterior probabilities where the binary bear market indicators are generated using the method proposed by Pagan and Sossounov (2003). Our findings indicate a high degree of similarity between the results obtained with these binary bear market indicators and the results reported in Section 4.1.

Table 4. The out-of-sample pseudo Estrella and Mishkin (1998) R^2 measures across the four models for different forecast horizons

	Forecast horizon												
Model	$h=1$	$h=2$	$h=3$	$h=4$	$h=5$	$h=6$	$h=7$	$h=8$	$h=9$	$h=10$	$h=11$	$h=12$	Average
(a) Probability of recession and bear market concurrently occurring across all countries													
M1	0.94	0.66	0.65	0.64	0.63	0.64	0.65	0.62	0.65	0.68	0.67	0.69	0.68
M2	0.96	0.66	0.67	0.66	0.65	0.65	0.66	0.65	0.66	0.65	0.66	0.67	0.68
M3	0.86	0.60	0.60	0.53	0.60	0.67	0.61	0.72	0.69	0.69	0.63	0.63	0.65
M4	0.96	0.73	0.71	0.69	0.68	0.66	0.66	0.66	0.66	0.67	0.69	0.70	**0.71**
Joint logistic	0.94	0.87	0.81	0.76	0.71	0.64	0.57	0.51	0.48	0.42	0.38	0.35	0.62
(b) Probability of recession and bear market concurrently occurring only in the United States													
M1	0.92	0.80	0.79	0.79	0.79	0.79	0.78	0.79	0.79	0.79	0.79	0.79	0.80
M2	0.89	0.69	0.68	0.68	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.70
M3	0.77	0.63	0.62	0.54	0.62	0.67	0.60	0.69	0.67	0.67	0.60	0.60	0.64
M4	0.92	0.83	0.82	0.81	0.81	0.81	0.81	0.81	0.81	0.81	0.81	0.81	**0.82**
Joint logistic	0.94	0.89	0.85	0.80	0.75	0.71	0.66	0.62	0.58	0.54	0.52	0.50	0.70
(c) Probability of recession and bear market concurrently occurring only in the UK													
M1	0.91	0.77	0.77	0.76	0.76	0.76	0.76	0.76	0.77	0.78	0.78	0.79	0.78
M2	0.89	0.68	0.67	0.67	0.67	0.68	0.68	0.68	0.69	0.69	0.69	0.70	0.70
M3	0.78	0.61	0.61	0.53	0.61	0.68	0.60	0.71	0.70	0.70	0.64	0.64	0.65
M4	0.90	0.79	0.79	0.79	0.78	0.78	0.78	0.78	0.79	0.79	0.80	0.80	0.80
Joint logistic	0.96	0.92	0.91	0.91	0.90	0.90	0.89	0.87	0.87	0.87	0.87	0.87	**0.90**
(d) Probability of recession and bear market concurrently occurring only in the Euro Area													
M1	0.82	0.44	0.43	0.41	0.42	0.42	0.42	0.43	0.43	0.44	0.46	0.46	0.47
M2	0.81	0.44	0.43	0.43	0.44	0.44	0.44	0.45	0.46	0.46	0.46	0.47	**0.48**
M3	0.66	0.37	0.37	0.28	0.37	0.43	0.36	0.45	0.44	0.46	0.49	0.39	0.42
M4	0.82	0.41	0.39	0.38	0.38	0.39	0.39	0.40	0.40	0.42	0.42	0.44	0.44
Joint logistic	0.87	0.78	0.66	0.52	0.43	0.35	0.29	0.26	0.21	0.17	0.15	0.13	0.40

The bold number represents the best-performing model, determined by its average performance across the twelve forecast horizons.

Additionally, we adopt an expanding window approach for the out-of-sample evaluation, where our initial holdout period spans from January 1999 to December 2005, with the subsequent forecasting evaluation covering the period from January 2006 to August 2022. Following the methodology of Kauppi and Saikkonen (2008) and Estrella and Mishkin (1998), we assume a 12-month lag for knowledge of recession and bear market dates across all countries. This temporal lag aligns with the average announcement release delay observed by the NBER business cycle committee. Simultaneously, we assume real-time data availability for the interest rate spread and other weekly and monthly economic activity predictors. For instance, in the initial estimation window of our out-of-sample exercise, our information set encompasses recession and bear market indicators up to December 2005, with assumed access to data on interest rate spreads and economic activity predictors until December 2006. Utilizing this information set, we generate 12 iterative forecasts using promptly released data. This process is repeated for subsequent estimation windows in our forecast evaluation period. As we progress through each estimation window, we update our information set with an additional observation of recession and bear market indicators, as well as interest rate spreads and economic activity predictors.

In the evaluation of out-of-sample forecasts across the four model specifications and the benchmark, we employ Pseudo R^2 measures following the methodology proposed by Estrella and Mishkin (1998), as outlined in Table 4.[7] The table provides Pseudo R^2

[7] In this article's appendix, we also provide Pseudo R^2 measures as proposed by McFadden (1973). However, similar inferences can be drawn as those presented in Table 4.

measures for each model under four distinct outcomes. Outcome (a) assesses the forecast performance for predicting simultaneous recession and bear market events across all three countries, while outcomes (b), (c), and (d) pertain to the same events but for each individual country. Across the twelve forecast horizons, M4 consistently emerges as the superior model among the five specifications in predicting recession and bear market events simultaneously across the three countries. This aligns with prior findings by Adrian et al. (2019), suggesting that deteriorating financial conditions serve as a reliable predictor of recessions across multiple countries. Notably, the incorporation of weekly indicators and a set of large economic predictors (M3) across the three countries does not yield additional forecasting benefits on average.[8] Thus, suggesting that the parsimonious models of M1 and M4 are sufficient for forecasting the occurrence of recession and bear market events across countries.

It is worth highlighting that all five models exhibit similar Pseudo R^2 measures for one-step (month)-ahead forecasts in outcome (a). However, beyond the 6-month forecast horizon, the Joint logistic Pseudo R^2 measures experience a notable deterioration compared to our proposed four model specifications. Turning to the individual country forecasts for recession and bear market events, M4 consistently proves to be the best-performing model on average for the United States, corroborating the findings of Adrian et al. (2019). Conversely, for the UK, the Joint logistic emerges as the best-performing model on average, implying that recession and bear market outcomes in the UK may be more influenced by domestic factors than global ones. In the Euro Area, our proposed model specification yields slightly better forecasts on average than the Joint logistic model. Similarly, in both the United States and Euro Area, the Joint logistic Pseudo R^2 measures experience significant deterioration after the 6-month forecast horizon relative to our four model specifications. These results suggest that considering cross-country dependence in our multinomial logistic framework indeed leads to superior medium-term forecasting of recession and bear market events occurring concurrently across countries, particularly in the United States and Euro Area, compared to independently modeling multiple single logistic models.

Subsequently, we present the one-step (month)-ahead forecast posterior probabilities over time for the best-performing model, M4, in Figures 4 and 5. Consistent with the in-sample analysis, the majority of the estimated out-of-sample posterior probabilities effectively align with their actual respective categorical outcomes, as indicated by the shaded areas. Specifically, in both figures, a notable proportion of the estimated posterior probabilities consistently exceeds the 80% threshold for the corresponding joint recession and bear market outcomes. Furthermore, both figures depict a discernible increase in estimated posterior probabilities just before the onset of the preceding recession and bear market events. This observation suggests that our proposed framework holds the potential to identify concurrent recession and bear market outcomes at an early stage.

3.3 Counterfactual Analysis

We demonstrate the applicability of our proposed multinomial logistic model through the examination of four counterfactual events designed to assess the potential impact of a U.S. recession or negative financial shock on the UK and Euro Area. Specifically, we concentrate on the best-performing forecasting model, M4, generating the one-step-ahead forecasts for three distinct time periods: March 2010, November 2017, and September 2021. In these periods, no actual recession or bear market events occur simultaneously. We produce one-

[8] We also assessed a modified version of M3 by incorporating the horseshoe (global-local) prior instead of the adaptive lasso prior. The out-of-sample forecasting outcomes for this modified model are presented in Table 8 in section 6.2 of the appendix. The forecasting results obtained with the horseshoe prior closely resemble those obtained with the adaptive lasso, as shown in Table 4. Consequently, employing an alternative shrinkage prior does not provide any additional forecasting advantages.

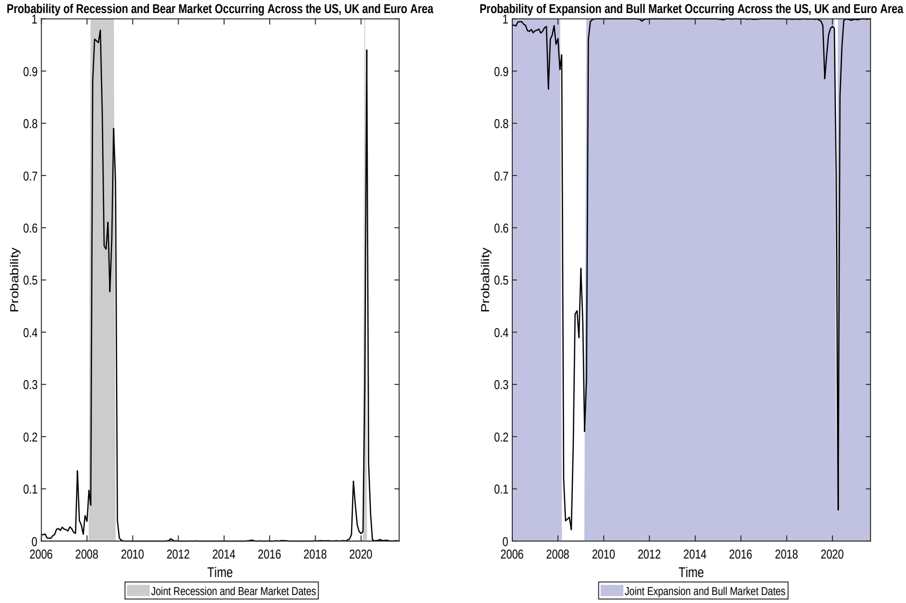

Figure 4. The one-step (month)-ahead posterior probabilities of recessions (expansions) and bear (bull) markets occurring concurrently in the United States, the UK, and Euro Area from M4.

Notes: The shaded gray (purple) bars denote the joint recession (expansion) and bear (bull) market outcomes given by the recession (expansion) and bear (bull) market binary indicators across three the countries. The black line is the one-step (month)-ahead posterior probabilities for each specific outcome from model M4.

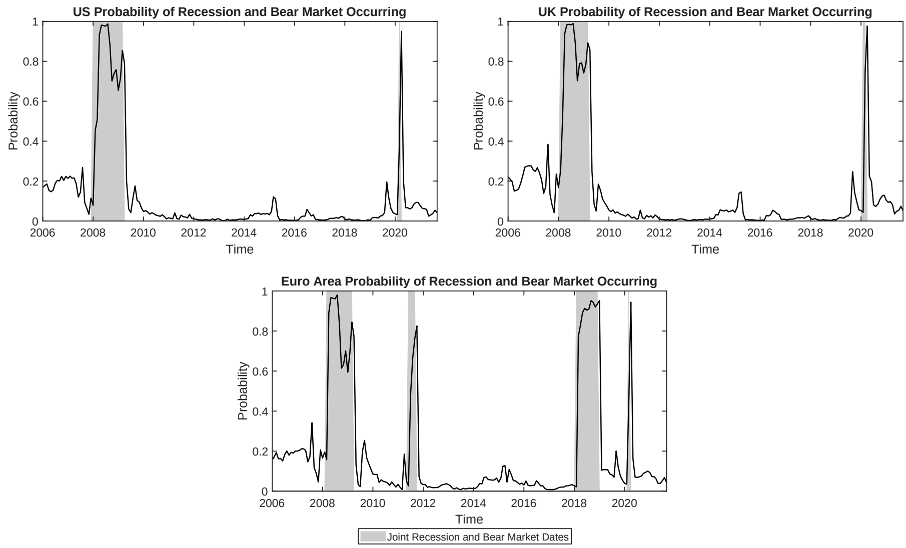

Figure 5. The one-step (month)-ahead posterior probabilities of recessions and bear markets occurring concurrently exclusively in each country from M4.

Notes: The shaded gray bars denotes the joint recession and bear market outcomes given by the recession and bear market binary indicators for each country. The black line is the one-step (month)-ahead posterior probabilities for each specific outcome from model M4.

Table 5. The difference in the probabilities between the counterfactual and the unconditioned out-of-sample one-step-ahead forecasts for the six outcomes

Time	1. UK recession and bear	2. UK recession	3. UK bear	4. Euro Area recession and bear	5. Euro Area recession	6. Euro Area bear
(a) A U.S. recession, %						
March 2010	14	27	19	10	28	9
November 2017	22	38	31	8	30	20
September 2021	27	47	33	8	37	19
(b) A U.S. bear market, %						
March 2010	13	19	33	24	33	24
November 2017	28	34	45	29	44	36
September 2021	33	38	51	35	51	41
(c) A tightening of U.S. NFCI, %						
March 2010	10	53	23	12	50	27
November 2017	14	54	31	14	44	39
September 2021	21	44	42	20	33	50
(d) A negative U.S. interest rate spread, %						
March 2010	17	41	31	16	33	37
November 2017	24	49	42	20	38	44
September 2021	16	44	29	19	40	39

step-ahead forecasts conditioned on the counterfactual event and compare them with unconditional one-step-ahead forecasts.

The four counterfactual events are detailed in Table 5. The first event (denoted as event (a)) assumes the occurrence of a recession exclusively in the United States in the previous period. The second event (denoted as event (b)) assumes the occurrence of a bear market exclusively in the United States in the previous period. The third event (denoted as event (c)) involves an increase or tightening of the U.S. NFCI, where the corresponding NFCI predictor is set to 2.5 at $T+1$. During the Great Recession, the U.S. NFCI peaked at around 2.8 in November 2008. Consequently, our third counterfactual event can be interpreted as an indicator of a financial crisis in the United States. Finally, for the fourth event (denoted as event (d)), we assume a negative 2% U.S. interest rate spread at time $T+1$. Although this negative 2 percent interest rate spread may seem extreme within the context of our sample, it is worth noting that more recently, during the period between April 2023 and July 2023, the difference between the U.S. 10-year and 3-month treasury yields became increasingly negative, averaging around −1.7 percent.

Table 5 presents the difference in probabilities between each counterfactual scenario and the unconditional one-step-ahead forecast, encompassing three distinct periods, with a focus on six categorical outcomes. These outcomes are the joint occurrence of recession and bear market outcomes for each country, alongside their individual recessionary and bear market outcomes. Across all four counterfactual events, a consistent pattern emerges, indicating a positive increase in probabilities for all six categorical outcomes. Specifically, our findings suggest that both a U.S. bear market and tightening of the U.S. NFCI increases the likelihood of a simultaneous recession and bear market in both the United States and the Euro Area, particularly in the aftermath of the COVID-19 pandemic. This builds upon the findings of Adrian et al. (2019), highlighting that the tightening of U.S. NFCI not only predicts U.S. recessions but also serves as a predictive indicator for similar economic contractions in the UK and the Euro Area. Furthermore, our results are consistent with Ehrmann et al. (2011), who assert that U.S. financial markets play a substantial role in explaining approximately 30 percent of fluctuations in Euro Area financial markets. This underscores the predominant influence of U.S. financial markets as a primary driver of global financial dynamics.

Concerning U.S. recessionary events, the likelihood of an exclusive recession in the UK and Euro Area also rises post-COVID-19 pandemic period. However, for the joint occurrence of a recession and bear market, the probability increases exclusively for the UK, whereas the Euro Area's probability remains constant compared to the other two periods. Finally, in the context of the last counterfactual event, a negative U.S. interest rate spread (or slope of the yield curve) exerts a positive influence on all six categorical outcomes. In most instances, the probabilities of these events remain relatively stable across the three selected periods. This implies that an inversion of the U.S. yield curve slope also serves as a robust indicator for recessions and bear markets in both the UK and the Euro Area. Consequently, our findings indicate potential evidence of dynamic spillover effects stemming from U.S. recessionary and financial market events on the economies of the UK and Euro Area. This underscores the significance for policymakers in these countries to not only be attentive to domestic factors but also to consider global and U.S. factors, given the interconnected nature revealed by our analysis.

4 Conclusion

We have developed a novel multinomial logistic model designed to detect and forecast concurrent recessions and bear markets across multiple countries. The primary advantage of our proposed approach lies in its capacity to leverage the additional informational content in the cross-country panel feature of the data for the detection of recessions and bear markets across countries and in their respective economies. Additionally, we extend our novel multinomial logistic model to a mixed-frequency setting by incorporating weekly financial condition indices as exogenous predictors in the model specification. Lastly, we explore a big data variant of the multinomial logistic model, including a substantial set of economic activity exogenous predictors from all three countries.

In a series of simulated experiments within our proposed framework, we observed an increase in our model's accuracy as the dataset's sample size expanded. Importantly, we demonstrated that the accuracy of our model maintained consistency and computational efficiency even as the number of binary indicators (representing recession or bear market occurrences) increased. In the empirical application, all four model specifications exhibited similar in-sample fit measures, accurately identifying the respective true categorical outcomes. In the out-of-sample forecasting exercise, our multinomial logistic framework demonstrated superior medium-term forecasting accuracy for concurrent recession and bear market events across countries, notably outperforming the independent modeling of multiple single logistic models. Finally, in a counterfactual exercise, our analysis indicated that both a U.S. recession and negative financial shock positively increased the probability of recession and bear market outcomes for the UK and the Euro Area.

A potential extension of our multinomial logistic framework is to incorporate time-varying parameters and volatility. However, introducing time-varying volatility into the model specification is non-trivial and is deferred to a future avenue of research.

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