**Original Ito & Clower (follows Raybaudi, M., M. Sola, and F. Spagnolo., Red Signals) MS-ADF Model:**

Notable features:

1. One model restricted to random walk regime (St = 0) and one mean-reverting regime.
2. Switching level and persistence (in some cases switching variance).
3. The same underlying random variable drives changes in level and persistence (and variance when it occurs).
4. Standard distribution for ADF test.

**Testing for “Bubbles” [Morita, Psaradakis, Sola, and Yunis]**

Notable features:

1. Switching level, persistency, and volatility.
2. No regime is restricted to random walk regime.
3. Three separate underlying random variables driving changes in level, persistence, and variance.
4. P-values must be bootstrapped for the unit-root test.

Underlying concepts:

1. When testing for bubbles two behaviors are assessed, non-stationarity and explosiveness.
2. They test for:
   1. The null hypothesis of a unit root (phi = 0) against the alternative of explosiveness (phi > 0).
   2. Hypothesis of a unit root is reject in favor of explosiveness at a given level of significance, alpha, if the bootstrap p-value does not exceed alpha.
3. In the context of inflation(prices):
   1. MS models are used to identify periods where **prices & their underlying fundamentals** are difference-stationary versus periods in which they are “explosive”.
   2. Cases:
      1. No sub-periods where prices are explosive:
         1. Implies fundamentals are always difference-stationary.
         2. Rules out existence of bubbles (explosiveness is a necessary but not sufficient condition for bubbles).
      2. Both fundamentals and prices are explosive in same sub-periods and difference-stationary in all others:
         1. Price explosiveness may be driven entirely by fundamentals, but bubbles cannot be ruled out.
      3. Fundamentals are always difference-stationary but prices are difference-stationary in some sub-periods and explosive in others:
      4. Sub-periods associated with explosive behavior prices evolve in a way that is potentially consistent with the presence of a (big enough) bubble.
4. **Main objective of paper:** “tackle the difficulties that arise when testing for the existence of bubbles in an environment in which changes in the growth rate and volatility of fundamentalsmay affect the evolution of prices in ways that mimic the explosive behavior of a bubble”
   1. “disentangle fundamentals-driven changes in the drift of prices, bubble-driven explosiveness, and volatility changes that may be fundamentals-driven and/or bubble-driven.”