

# Feature Extraction for Side-Channel Attacks

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05/12/2018, Paris

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## Contents

1. Context
2. State of the Art, Objectives, Contributions
3. Kernel Discriminant Analysis against Masking
  - 3.1 Linear Discriminant Analysis
  - 3.2 Kernel Discriminant Analysis
  - 3.3 Experimental Results
4. Deep Learning against Misalignment
  - 4.1 Data Augmentation
  - 4.2 Experimental Results
5. Conclusions

## Secure Component and Embedded Cryptography

**A piece of hardware with security properties.**

**It usually embeds cryptography to provide security services (authentication, signature, secure messaging with terminals...)**

## Secure Component and Embedded Cryptography

A piece of hardware with security properties.

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- ▶ Sensitive applications: ID cards, credit cards, transport cards, health cards, SIM
- ▶ Pervasive aspect: several billion smartcards sold per year
- ▶ Hard to update
- ▶ Hostile environment

⇒ Requires protection against very high-level attacker

## Security Certification



- ▶ Standardised Evaluation (e.g. ISO/IEC 15408 - Common Criteria)
- ▶ Assigns an Evaluation Assurance Level (EAL)
- ▶ The evaluator checks the Security Assurance Requirements (SAR), e.g. ADV, ALC, AVA, ...
- ▶ AVA: vulnerability assessment (penetration testing → attack potential rating)

## Side-Channel Vulnerability of Embedded Cryptography



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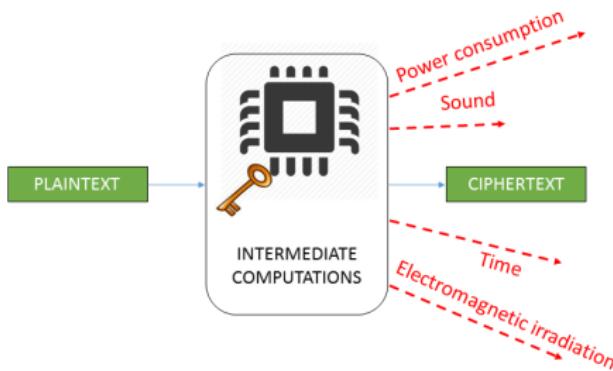
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Mathematical vulnerability  
Black Box

### Side-Channel Cryptanalysis

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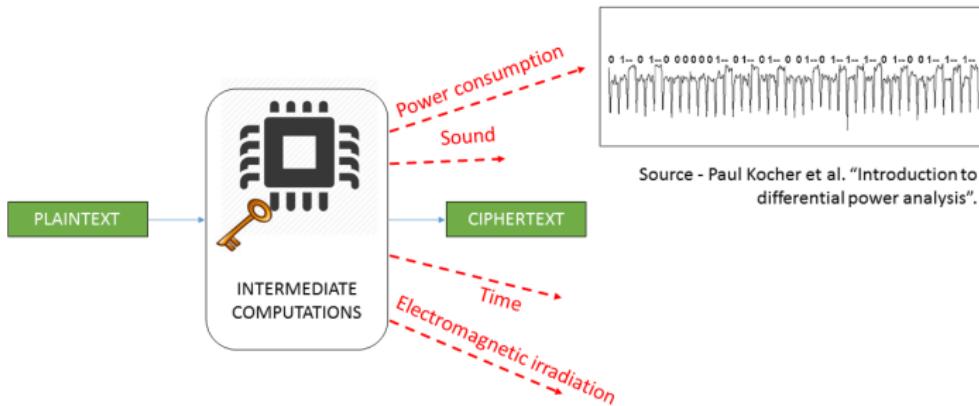
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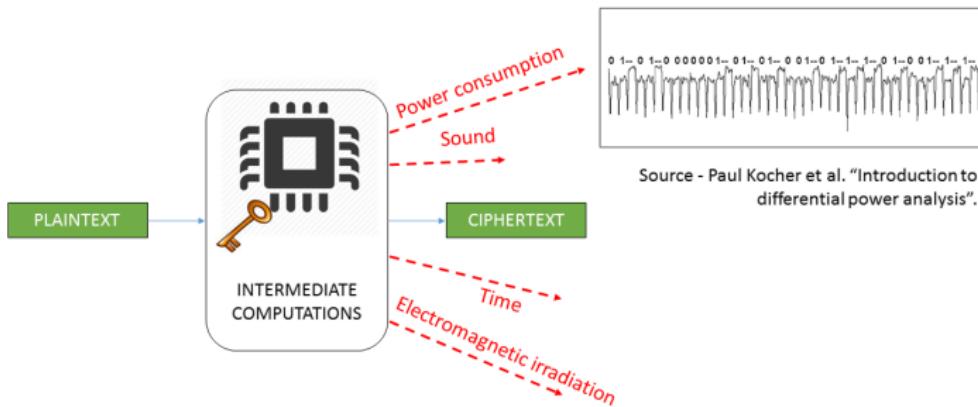
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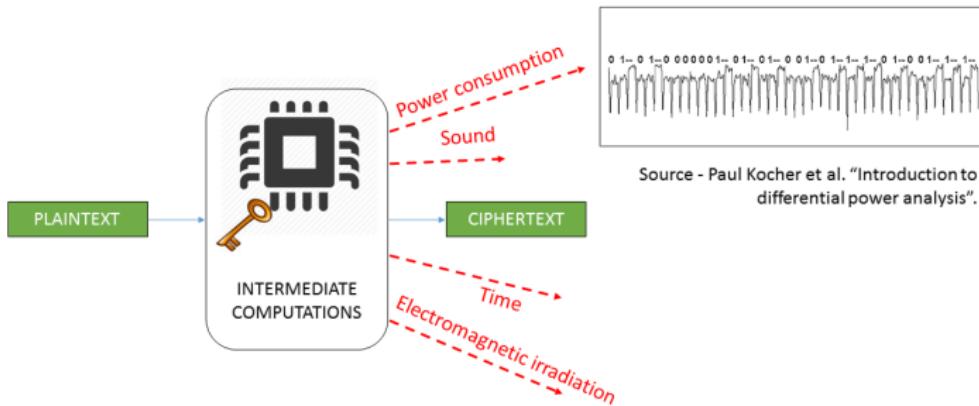
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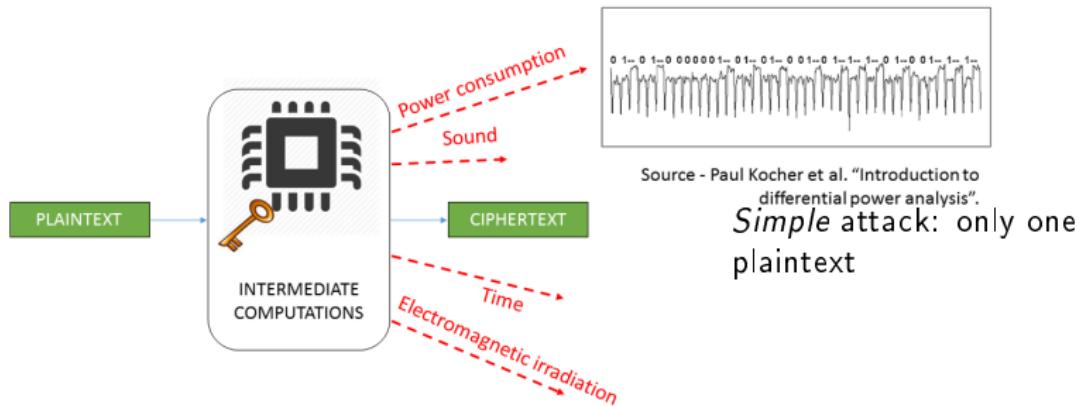
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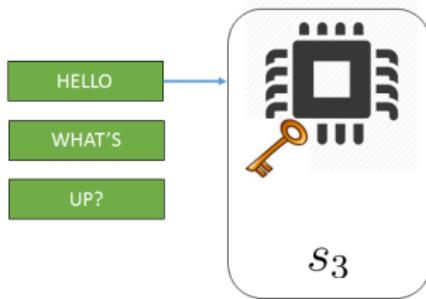
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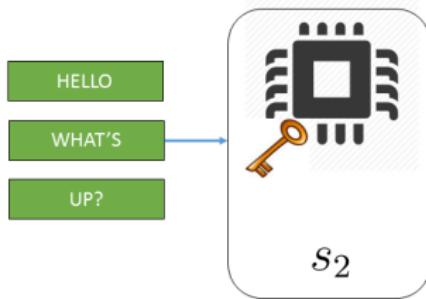
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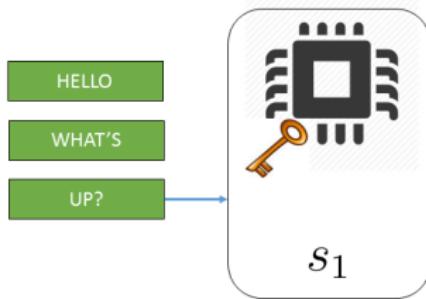
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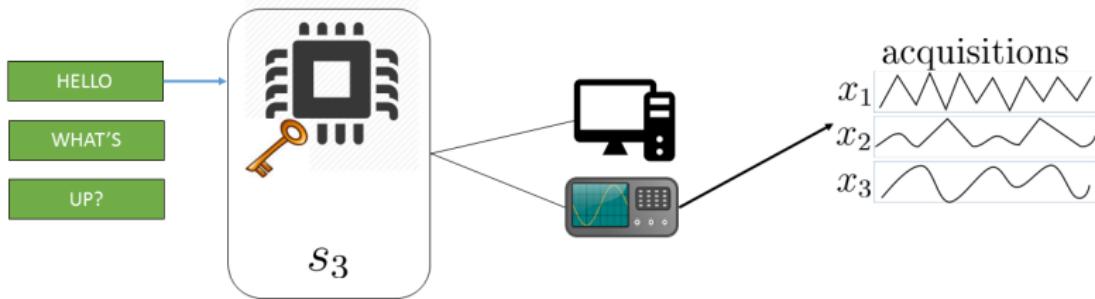
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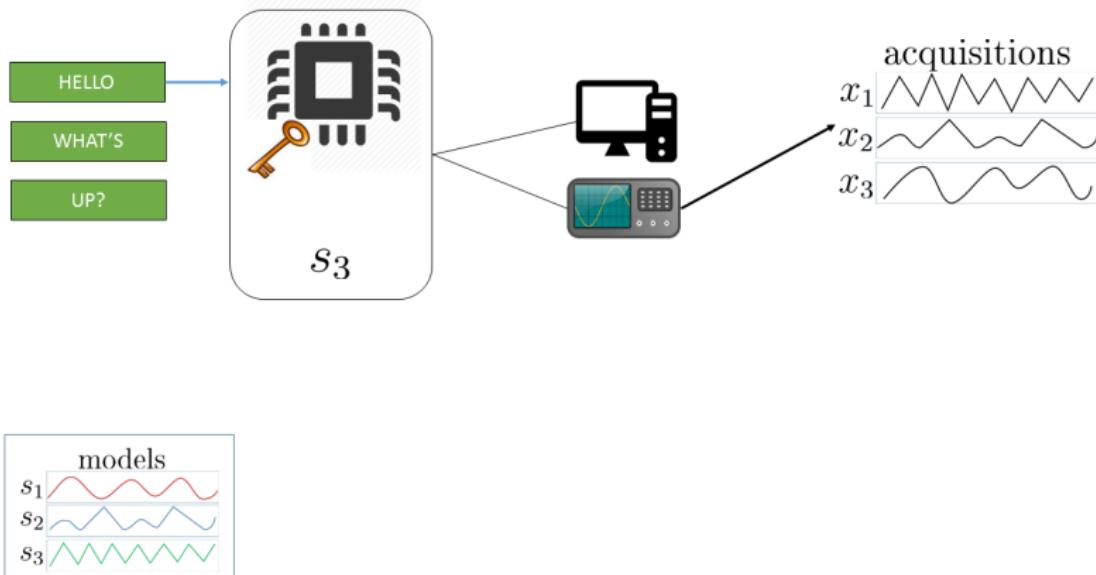
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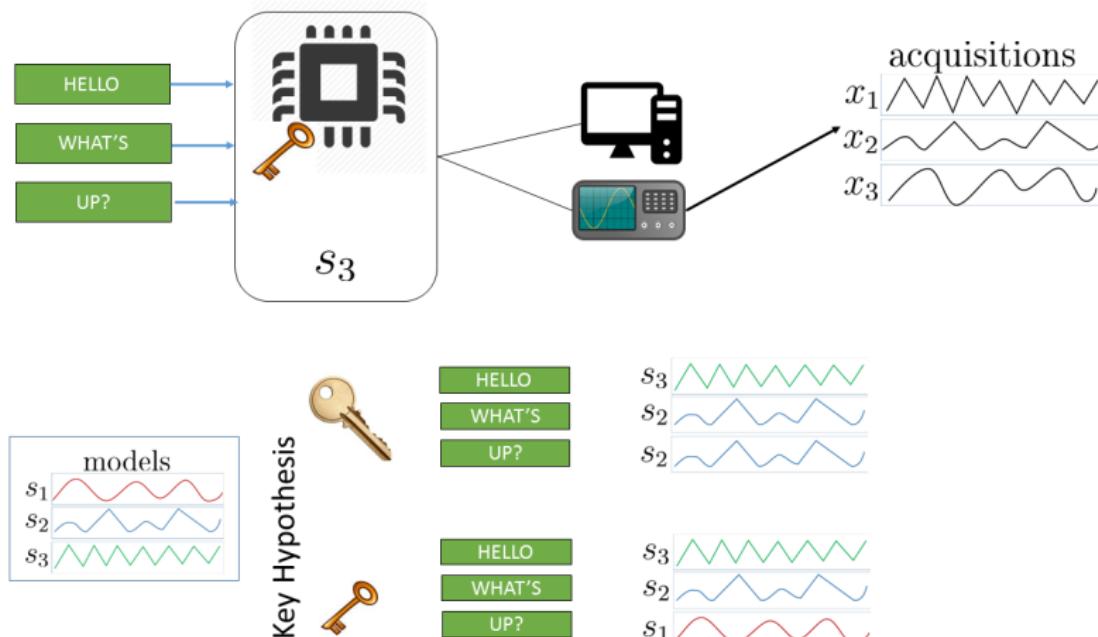
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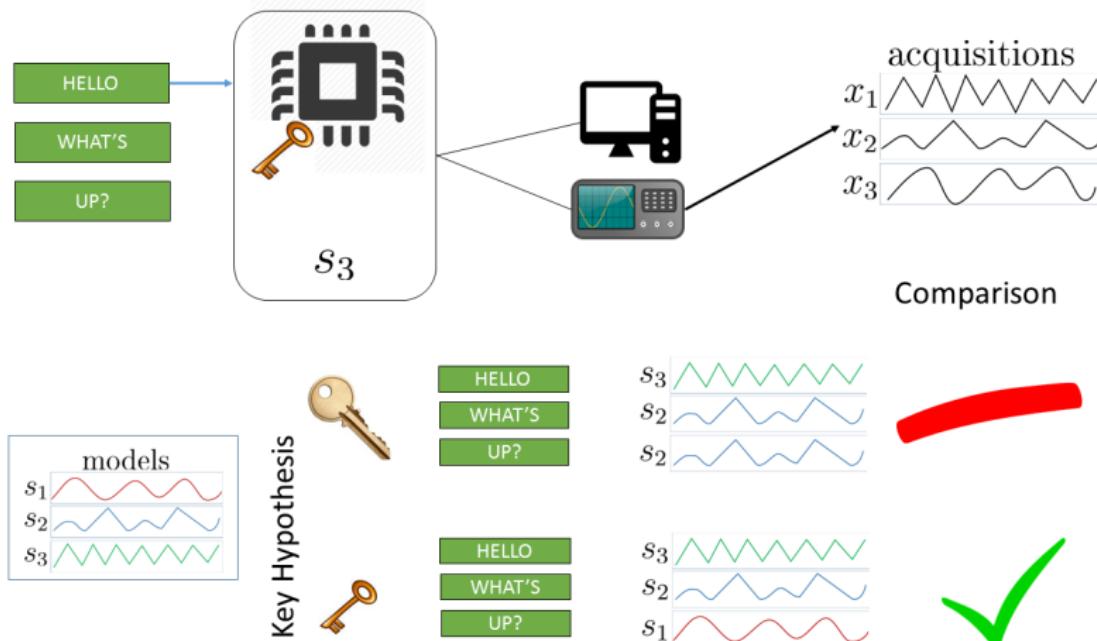
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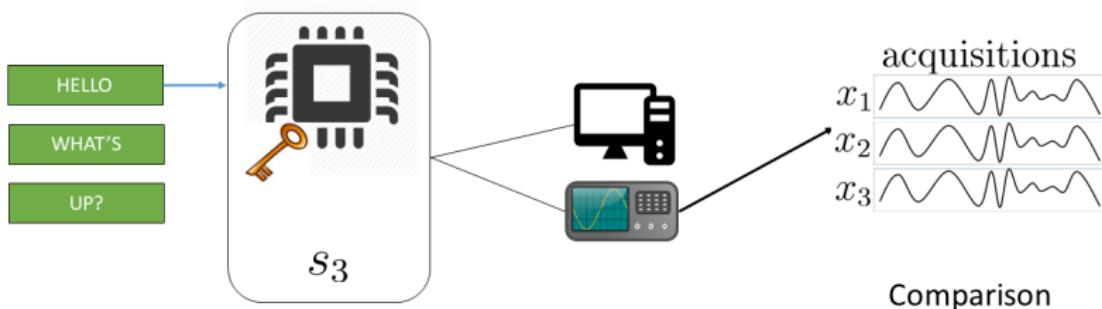


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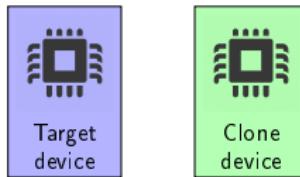


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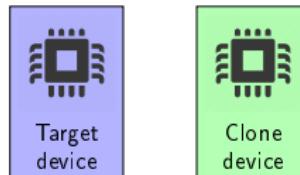
Don't panic!



## Profiling Attacks...Supervised Learning



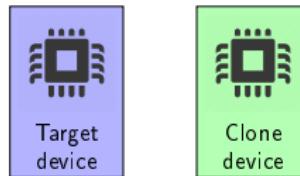
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Machine Learning

Supervised Learning

## Profiling Attacks...Supervised Learning

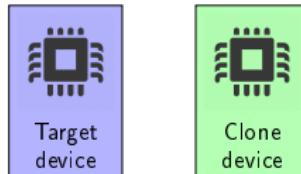


### Machine Learning

"A computer program is said to learn from experience E with respect to some task T and performance measure P, if its performance on T, as measured by P, improves with experience E. "[TM97]

### Supervised Learning

## Profiling Attacks...Supervised Learning



### Machine Learning

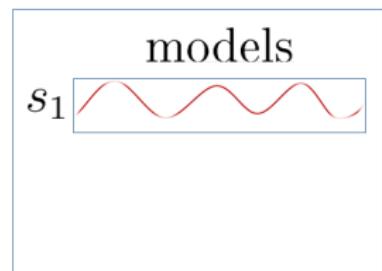
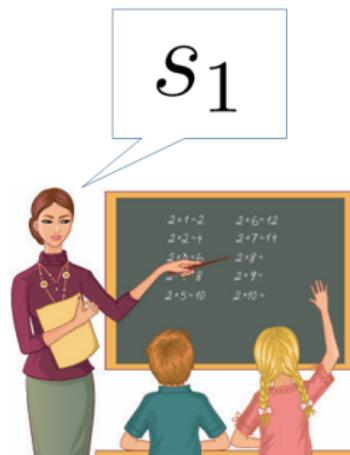
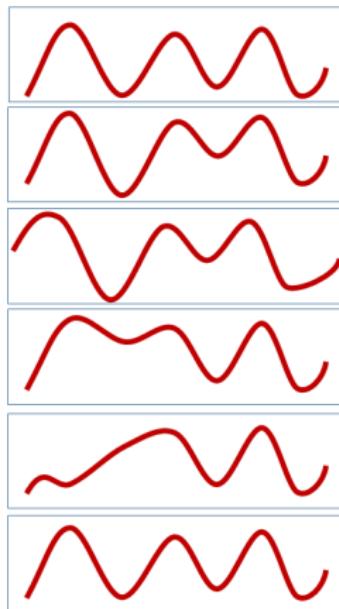
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### Supervised Learning

The *supervised* learning algorithms access to a dataset of examples, each associated in general to a *target* or *label*.



## Classroom Side-Channel Attacks



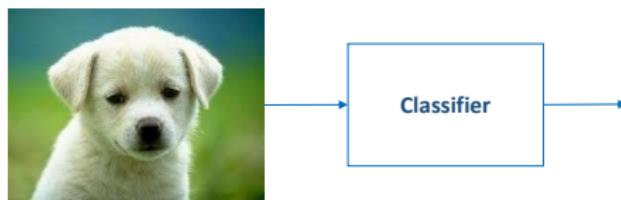
## Classroom Side-Channel Attacks



## Classification

### Classification problem

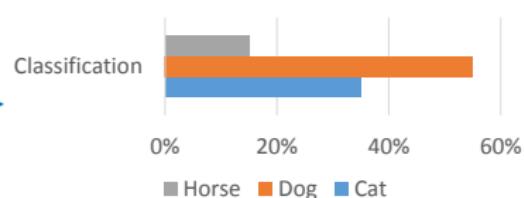
Assign to a datum  $\vec{X}$  (e.g. an image) a label  $Z$  among a set of possible labels  
 $\mathcal{Z} = \{\text{Cat, Dog, Horse}\}$



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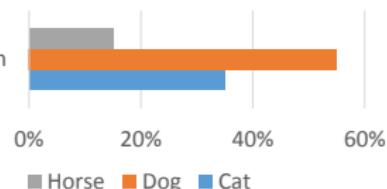
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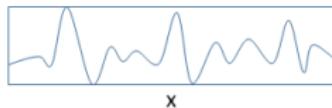


Classifier

Classification



### Simple Attack as a Classification Problem



Classifier

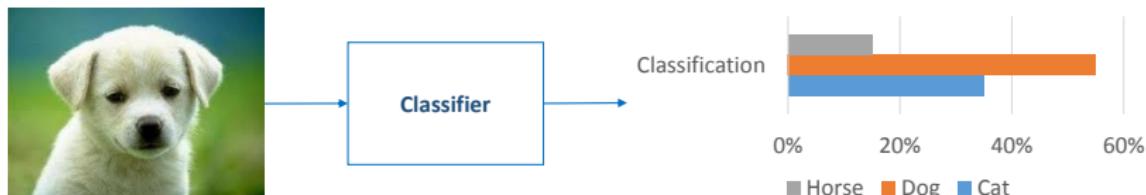
$P(K|x)$



## Classification

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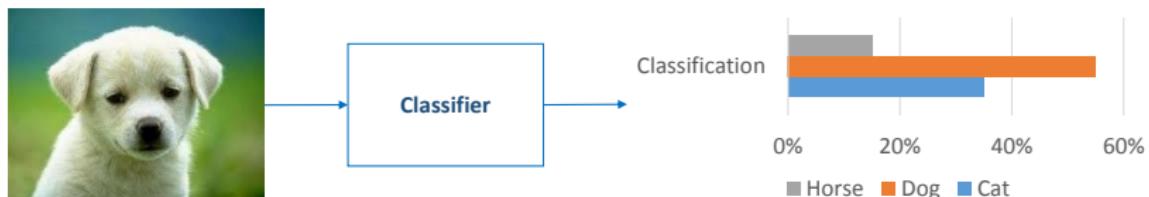
### Advanced Attack as a Classification Problem



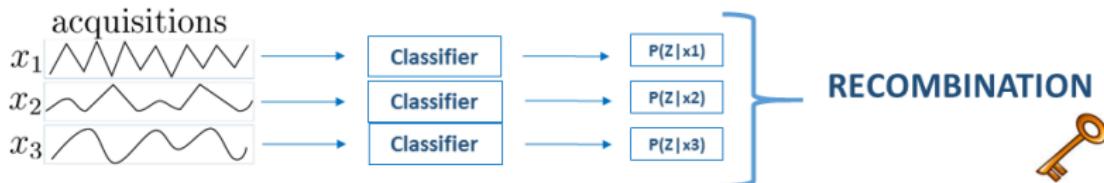
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### Advanced Attack as Multiple Classification Problems



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## Notations

### Notations and generalities

- ▶ Side-channel traces: realizations of a random vector  $\vec{X} \in \mathbb{R}^D$
- ▶  $D$  is the number of time samples (or features)
- ▶ Target: a *sensitive* variable  $Z = f(\text{plaintext}, \text{key})$  in  $\mathcal{Z} = \{s_1, \dots, s_{|\mathcal{Z}|}\}$
- ▶ each label  $s_i$  identifies a *class*

### Profiling attack scenario

- ▶ labelled traces  $\mathcal{D}_{\text{train}} = (\vec{x}_i, z_i)_{i=1}^N$ , acquired under known  $Z$ , to characterise the signals
- ▶ attack traces  $\mathcal{D}_{\text{attack}} = (\vec{x}_i, p_i)_{i=1}^{N_a}$  acquired under known plaintext

## Profiling Attack

### Profiling phase

- ▶ estimate
  - ▶  $p_{\vec{X}} | Z=z$

### Attack phase

- ▶ Log-likelihood score for each key hypothesis  $k$

$$d_k = \prod_{i=1}^{N_a} p_{\vec{X} | Z=f(p_i, k)}(\vec{x}_i)$$

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$$d_k = \prod_{i=1}^{N_a} p_{\vec{X} \mid Z=f(p_i, k)}(\vec{x}_i)$$

- ▶ A-posteriori probability score for each key hypothesis  $k$

$$p_{Z \mid \vec{X}=\vec{x}}(z) = \frac{p_{\vec{X} \mid Z=z}(\vec{x}) p_Z(z)}{p_{\vec{X}}(\vec{x})} \text{ Bayes' theorem}$$

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$$\vec{X} \in \mathbb{R}^D$$

Curse of dimensionality!

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**Curse of dimensionality!**

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- ▶ mandatory dimensionality reduction  $[\mathcal{D}_{\text{train}} \longrightarrow \epsilon: \mathbb{R}^D \rightarrow \mathbb{R}^C]$
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### Objectives

- ▶ Ameliorate the template attack routine by proposing efficient dimensionality reduction techniques
- ▶ Consider the presence of most-commonly-implemented SCA countermeasures (masking, hiding)

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## Dimensionality Reduction: State of the Art

### Dimensionality Reduction

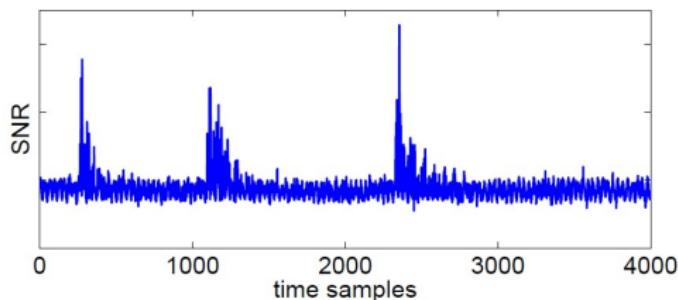
$$\begin{aligned}\epsilon: \mathbb{R}^D &\rightarrow \mathbb{R}^C \\ \vec{x} &\mapsto \epsilon(\vec{x})\end{aligned}$$

- ▶ Feature selection (Points of Interest selection)
- ▶ Feature extraction

### Feature selection

$\epsilon$  performs a sub-sampling

- ▶ SOD [CRR03]
- ▶ SOST [BDP10]
- ▶ SNR [MOP08]/ NICV [Bha+14]
- ▶  $t$ -test,  $F$ -test,... [GLRP06; CK14]



## Dimensionality Reduction: State of the Art

### Dimensionality Reduction

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### Linear feature extraction

$$\epsilon(\vec{x}) = A\vec{x} \text{ with } A \in M_{\mathbb{R}}(C, D)$$

- ▶ Principal Component Analysis (PCA) [Arc+06; BHW12]
- ▶ Linear Discriminant Analysis (LDA) [SA08; Bru+15]
- ▶ Projection Pursuits (PP) [Dur+15]

## Contributions

- ▶ **Linear Dimensionality Reduction**([CARDIS 2015]):
  - ▶ PCA, choise of components ELV
  - ▶ LDA in case of undersampling
- ▶ **Kernel Discriminant Analysis**([CARDIS 2016]): application of an appropriate kernel trick to LDA, in order to manage masking countermeasure
- ▶ **Convolutional Neural Networks**([CHES 2017]) :
  - ▶ discriminative model by means of neural network classifiers
  - ▶ convolutional layers to manage desyncronisation (a form of hiding)
  - ▶ Data Augmentation techniques to reduce overfitting

## Contributions

- ▶ **Linear Dimensionality Reduction**([CARDIS 2015]):
  - ▶ PCA, choise of components ELV
  - ▶ LDA in case of undersampling
- ▶ **Kernel Discriminant Analysis**([CARDIS 2016]): application of an appropriate kernel trick to LDA, in order to manage masking countermeasure
- ▶ **Convolutional Neural Networks**([CHES 2017]) :
  - ▶ discriminative model by means of neural network classifiers
  - ▶ convolutional layers to manage desyncronisation (a form of hiding)
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## Contents

1. Context
2. State of the Art, Objectives, Contributions
3. Kernel Discriminant Analysis against Masking
  - 3.1 Linear Discriminant Analysis
  - 3.2 Kernel Discriminant Analysis
  - 3.3 Experimental Results
4. Deep Learning against Misalignment
  - 4.1 Data Augmentation
  - 4.2 Experimental Results
5. Conclusions

## Model-based SNR and Fisher's Criterion

### Signal-to-Noise Ratio (SNR)

- ▶ Independent Noise Assumption:  $\vec{X} = \varphi(Z) + \vec{B}$
- ▶  $\vec{\mu}_s = \hat{\mathbb{E}}[\vec{X} | Z = s] = \frac{1}{N_s} \sum_{i: z_i=s} \vec{x}_i$  sample mean per class ( $\approx \varphi(Z)$ )
- ▶  $\vec{\varrho}_s = \hat{\text{Var}}(\vec{X} | Z = s) = \frac{1}{N_s - 1} \sum_{i: z_i=s} (\vec{x}_i - \vec{\mu}_s)^2$  sample variance per class ( $\approx \text{var}(\vec{B})$ )
- ▶

$$\text{SNR}(t) = \frac{\hat{\text{Var}}(\vec{\mu}_Z(t))}{\hat{\mathbb{E}}[\vec{\varrho}_Z(t)]} \quad \frac{\text{variance inter-class}}{\text{variance intra-class}}$$

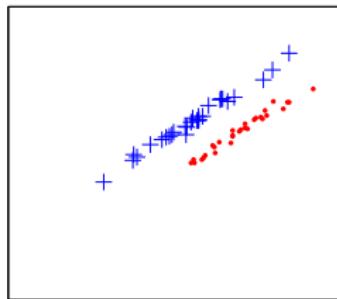
### Fisher's Criterion for Linear Dimensionality Reduction

- ▶  $\mathbf{S}_B = \sum_{s \in \mathcal{Z}} N_s (\vec{\mu}_s - \bar{\vec{x}})(\vec{\mu}_s - \bar{\vec{x}})^\top$  (inter-class scatter matrix)
- ▶  $\mathbf{S}_W = \sum_{s \in \mathcal{Z}} \sum_{i=1: z_i=s} (\vec{x}_i - \vec{\mu}_s)(\vec{x}_i - \vec{\mu}_s)^\top$  (intra-class scatter matrix)
- ▶ Fisher's criterion

$$\hat{\vec{\alpha}} = \operatorname{argmax}_{\vec{\alpha}} \frac{\vec{\alpha}^\top \mathbf{S}_B \vec{\alpha}}{\vec{\alpha}^\top \mathbf{S}_W \vec{\alpha}}, \quad (1)$$

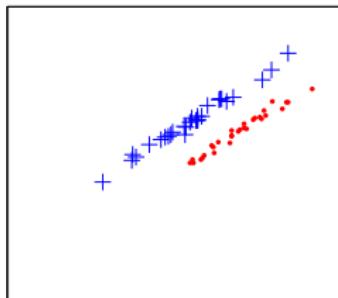
$\epsilon(\vec{x}) = A\vec{x}$  where  $A = [\vec{\alpha}_1, \dots, \vec{\alpha}_{|\mathcal{Z}|-1}]$  eigenvectors of  $\mathbf{S}_W^{-1} \mathbf{S}_B$

## LDA: an optimal binary linear classifier



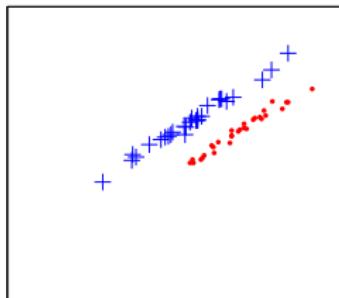
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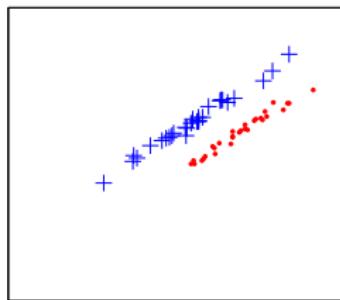
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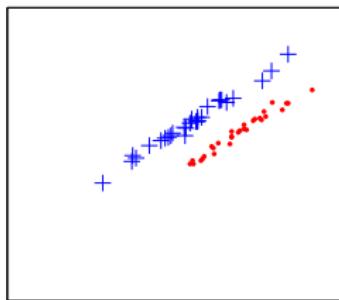
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(boundary surface  $a = 0$ )

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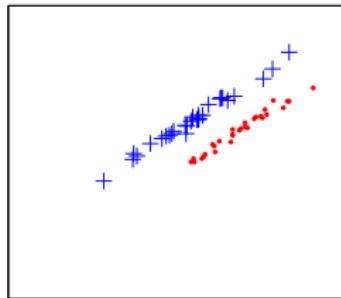
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## LDA: an optimal binary linear classifier



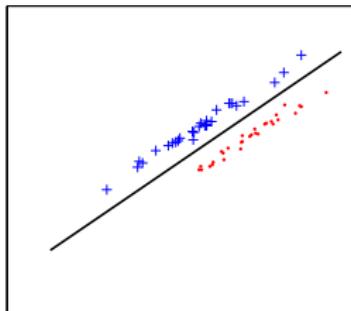
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### Generalised linear discriminative model

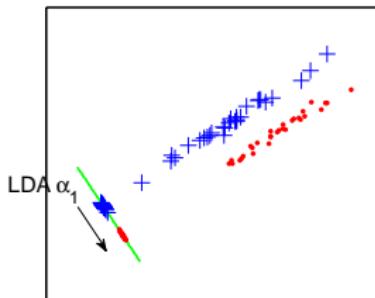
$$\Pr(s_1 | \vec{x}) = \sigma(\vec{w}^\top \vec{x} + w_0) \text{ , where } \sigma(a) = \frac{1}{1 + e^{-a}} \text{ logistic sigmoid} \quad (2)$$

## LDA and Fisher's Discriminant



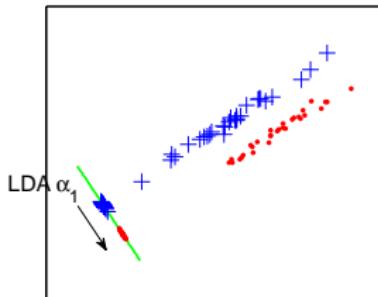
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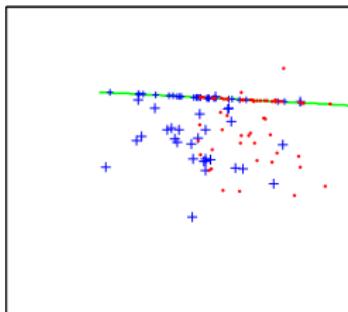


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### Fact, abuse and preference for the dimensionality reduction formulation

- ▶ When LDA assumptions are met, the solution  $\vec{\alpha}_1$  of the Fisher's criterion is orthogonal to  $\vec{w}$ .
- ▶ assumption not required
- ▶ naturally multi-class

## Linear separability



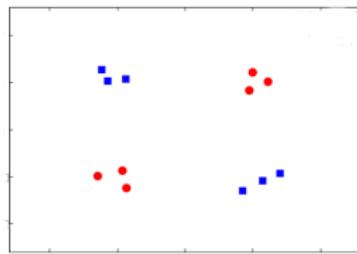
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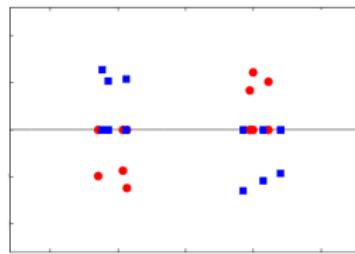
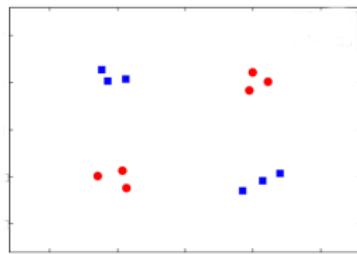


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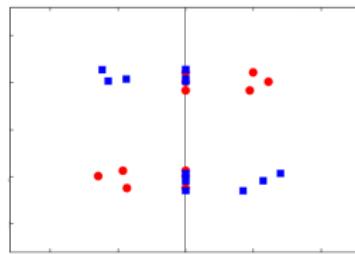
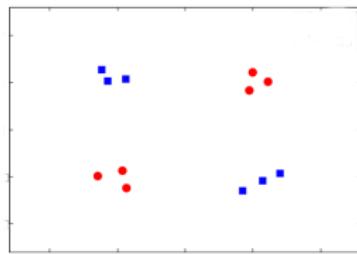
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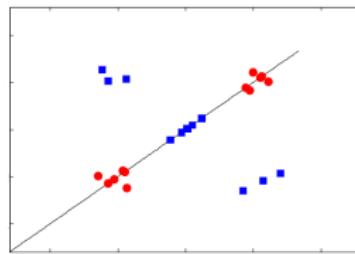
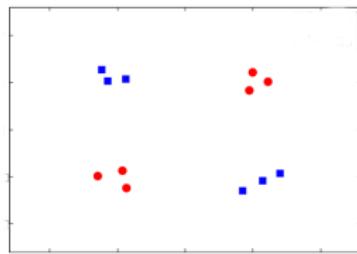
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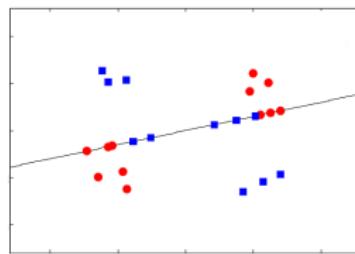
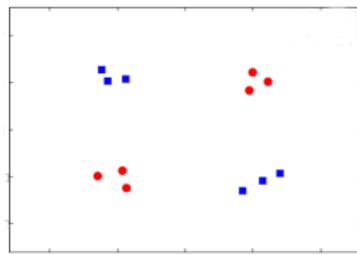
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## Dimensionality reduction in presence of masking

### $(d - 1)$ th-order Sharing (or Masking)

Split each sensitive  $Z$  into shares  $Z = M_1 \star \cdots \star M_d$

with  $M_1, \dots, M_{d-1}$  random shares (or masks)

and  $M_d = Z \star M_1^{-1} \star \cdots \star M_{d-1}^{-1}$

Software implementations: shares are handled at different time samples

$$t_1, \dots, t_d$$

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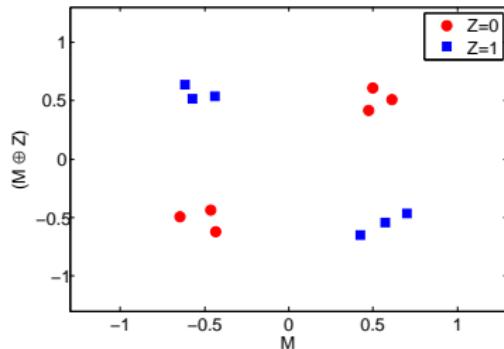
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Toy example: 2 time samples, 1-bit data

$t_1: M + n, n \sim \mathcal{N}(0, 0.1)$

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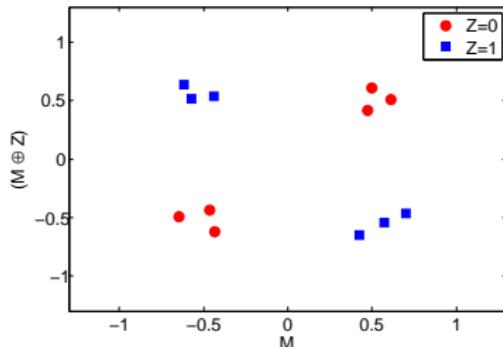
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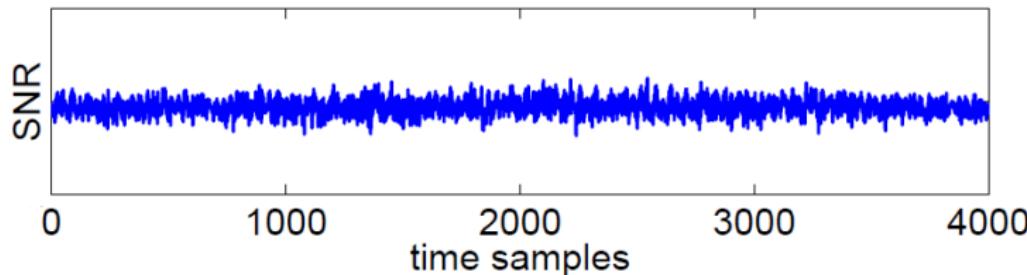
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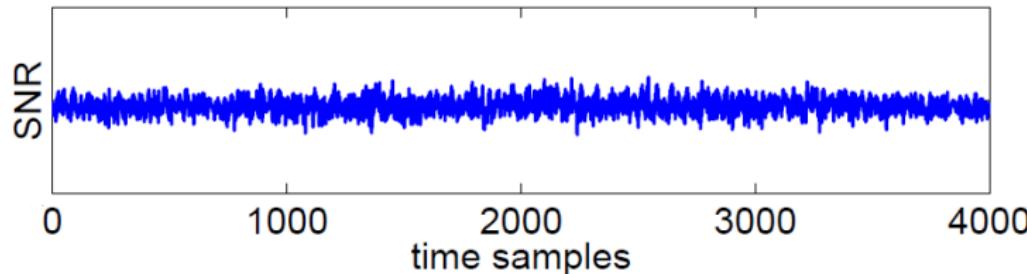
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### Higher-Order Side-Channel Attacks

Exploit  $f(z) = \mathbb{E} \left[ \vec{X}[t_1] \vec{X}[t_2] \dots \vec{X}[t_d] | Z = z \right]$  non-constant.

### Necessary condition

[Car+14] The statistic extracted from measurements must contain

$$\vec{X}[t_1] \vec{X}[t_2] \dots \vec{X}[t_d]$$

## Pols Research

How to detect the  $d$ -tuple  $t_1, \dots, t_d$ ?

### A lacking literature

- ▶ many HO attacks papers assume the knowledge of  $t_1, \dots, t_d$
- ▶ Pol research exploiting the random shares knowledge (back to unprotected case using  $M_1, \dots, M_d$  instead of  $Z$ )
- ▶ naive strategy: infer over all possible  $d$ -tuples
- ▶ Hand selection via educated guess [Osw+06]

### Generalizing extractors for higher-order context

- ▶ Selecting extractors → Projection Pursuits [Dur+15]
- ▶ Projecting extractors → Kernel Discriminant Analysis [CARDIS '16]

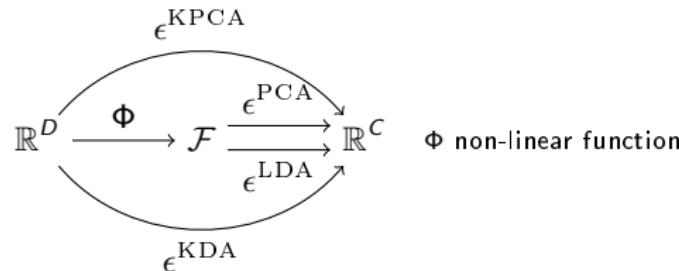
## KDA: the purpose

### Problem

Useful statistics lie in a high-dimensional *feature* space:

$$\mathcal{F} = \mathbb{R}^{\binom{D+d-1}{d}}$$

(all  $d$ th-degree monomials in the trace coordinates)



### What KDA provides

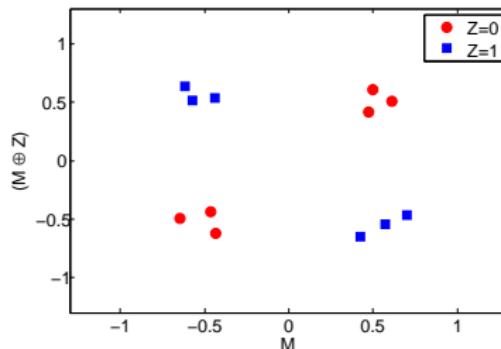
KDA allows performing LDA in  $\mathcal{F}$ , remaining in  $\mathbb{R}^D$ .

## KDA: an intuition

Toy example: 2 time samples, 1-bit data

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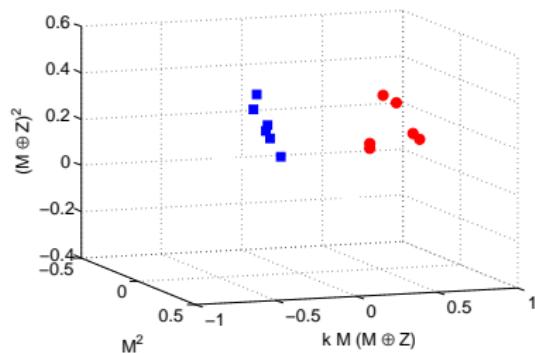
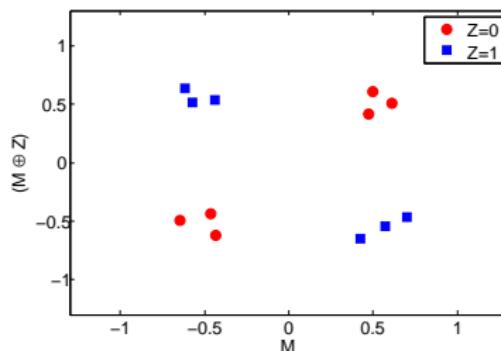


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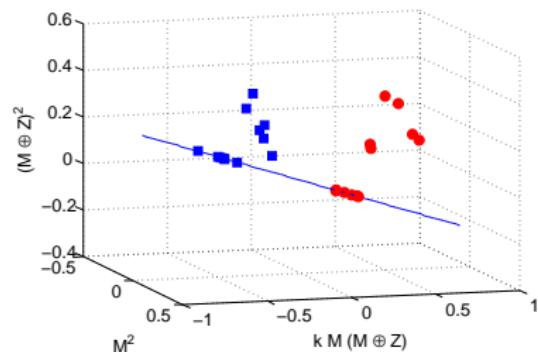
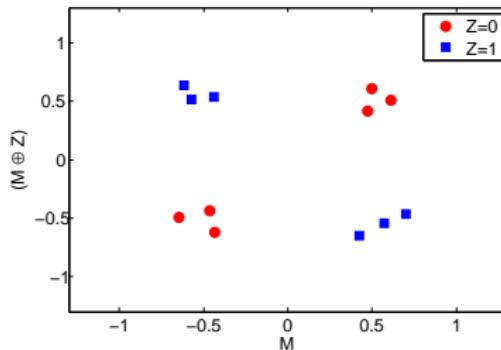
$$\Phi: \mathbb{R}^D \rightarrow \mathbb{R}^{\binom{D+d-1}{d}}$$
$$\Phi(t_1, t_2) = (t_1^2, t_2^2, k t_1 t_2)$$

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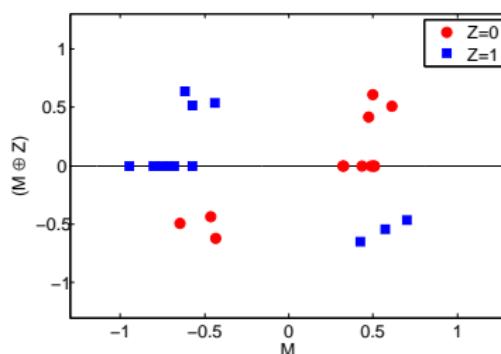
$\Phi \rightarrow \text{LDA}$

## KDA: an intuition

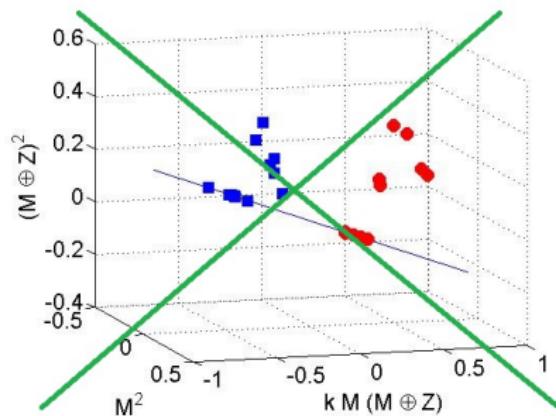
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KDA  
remains in  $\mathbb{R}^D$



## KDA: an intuition

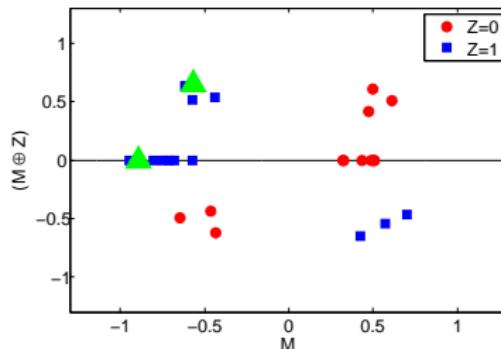
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### KDA procedure

- ▶ Training  $\rightarrow \nu_\ell$ ,  $C$  vectors of coefficients ( $\ell = 1, \dots, C$ )

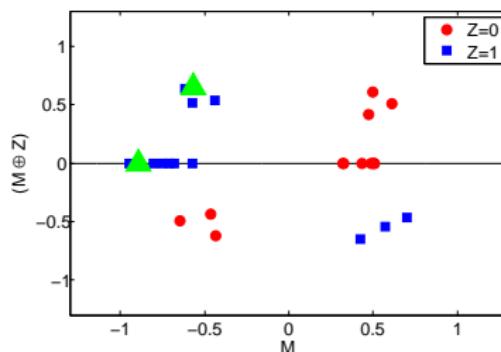


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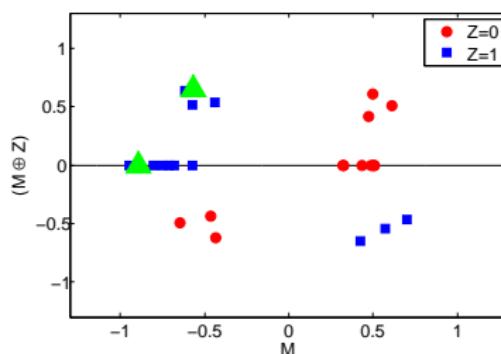
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- ▶ Compare the new trace with all training traces  $K(\mathbf{x}_i^{z_i}, \mathbf{x})$ ,  $K(\mathbf{x}_i^{z_i}, \mathbf{x})$

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- ▶ KDA projection

$$\epsilon_\ell^{\text{KDA}}(\vec{x}) = \sum_{i=1}^N \nu_\ell[i] K(\mathbf{x}_i^{z_i}, \mathbf{x}) . \quad (3)$$

## Kernel Function

### Kernel Function

$K: \mathbb{R}^D \times \mathbb{R}^D \rightarrow \mathbb{R}$

$$K(\mathbf{x}_i, \mathbf{x}_j) = \Phi(\mathbf{x}_i) \cdot \Phi(\mathbf{x}_j) \quad (4)$$

### Polynomial Kernel Function

$$K(\mathbf{x}_i, \mathbf{x}_j) = (\mathbf{x}_i \cdot \mathbf{x}_j)^d \quad \leftrightarrow \quad \Phi: \mathbb{R}^D \rightarrow \mathcal{F} \subset \mathbb{R}^{\binom{D+d-1}{d}} \text{ all } d\text{th-degree monomials}$$

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### Example: 2nd Degree Polynomial Kernel Function

Toy example:  $D = 2 \rightarrow \mathbf{x}_i = [a, b], \mathbf{x}_j = [c, d]$

$$K(\mathbf{x}_i, \mathbf{x}_j) = (ac + bd)^2 = a^2 c^2 + 2abcd + b^2 d^2$$

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## Kernel Function

### Kernel Function

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## KDA - the training

### Between-class (inter-class) Scatter Matrix

#### LDA

$$\blacktriangleright \mathbf{S_B} = \sum_{z \in \mathcal{Z}} N_z (\vec{\mu}_s - \bar{\vec{x}})(\vec{\mu}_s - \bar{\vec{x}})^T$$

#### KDA

$$\blacktriangleright \mathbf{M} = \sum_{z \in \mathcal{Z}} N_z (\vec{M}_s - \vec{M}_T)(\vec{M}_s - \vec{M}_T)^T$$



<sup>1</sup>  $\vec{M}_s$  and  $\vec{M}_T$  are two  $N$ -sized column vectors whose entries are given by:

$$\vec{M}_z[j] = \frac{1}{N_z} \sum_{i:z_i=z}^{N_z} K(\mathbf{x}_j^{z_j}, \mathbf{x}_i^{z_i}), \quad \vec{M}_T[j] = \frac{1}{N} \sum_{i=1}^N K(\mathbf{x}_j^{z_j}, \mathbf{x}_i^{z_i}).$$

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## KDA - the training

### Within-class (inter-class) Scatter Matrix

#### LDA

- ▶  $\mathbf{S_B} = \sum_{z \in \mathcal{Z}} N_z (\vec{\mu}_s - \bar{\vec{x}})(\vec{\mu}_s - \bar{\vec{x}})^T$
- ▶  $\mathbf{S_W} = \sum_{z \in \mathcal{Z}} \sum_{i=1}^{N_z} (\mathbf{x}_i^z - \vec{\mu}_s)(\mathbf{x}_i^z - \vec{\mu}_s)^T$

#### KDA

- ▶  $\mathbf{M} = \sum_{z \in \mathcal{Z}} N_z (\vec{M}_s - \vec{M}_T)(\vec{M}_s - \vec{M}_T)^T$  <sup>1</sup>
- ▶  $\mathbf{N} = \sum_{z \in \mathcal{Z}} \mathbf{K}_z (\mathbf{I} - \mathbf{I}_{N_z}) \mathbf{K}_z^T$  <sup>2</sup>
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---

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## KDA - the training

### Eigenvector problem

Computational Complexity  $O(D^3)$

#### LDA

- ▶  $\mathbf{S}_B = \sum_{z \in \mathcal{Z}} N_z (\vec{\mu}_s - \bar{\vec{x}})(\vec{\mu}_s - \bar{\vec{x}})^\top$
- ▶  $\mathbf{S}_W = \sum_{z \in \mathcal{Z}} \sum_{i=1}^{N_z} (\mathbf{x}_i^z - \vec{\mu}_s)(\mathbf{x}_i^z - \vec{\mu}_s)^\top$
- ▶  $\vec{\alpha}_i$  eigenvectors of  $\mathbf{S}_W^{-1} \mathbf{S}_B$   $[D \times D]$

Computational Complexity  $O(N^3)$

#### KDA

- ▶  $\mathbf{M} = \sum_{z \in \mathcal{Z}} N_z (\vec{M}_s - \vec{M}_T)(\vec{M}_s - \vec{M}_T)^\top$ <sup>1</sup>
- ▶  $\mathbf{N} = \sum_{z \in \mathcal{Z}} \mathbf{K}_z (\mathbf{I} - \mathbf{I}_{N_z}) \mathbf{K}_z^\top$ <sup>2</sup>
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- ▶

---

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## KDA - the training

New trace projection

Computational Complexity  $O(D^3)$

### LDA

- ▶  $\mathbf{S_B} = \sum_{z \in \mathcal{Z}} N_z (\vec{\mu}_s - \bar{\vec{x}})(\vec{\mu}_s - \bar{\vec{x}})^\top$
- ▶  $\mathbf{S_W} = \sum_{z \in \mathcal{Z}} \sum_{i=1}^{N_z} (\mathbf{x}_i^z - \vec{\mu}_s)(\mathbf{x}_i^z - \vec{\mu}_s)^\top$
- ▶  $\vec{\alpha}_i$  eigenvectors of  $\mathbf{S_W}^{-1} \mathbf{S_B}$   $[D \times D]$
- ▶  $\epsilon_\ell^{LDA}(\vec{x}) = \sum_{i=1}^D \vec{\alpha}_\ell[i] \vec{x}[i]$

Computational Complexity  $O(N^3)$

### KDA

- ▶  $\mathbf{M} = \sum_{z \in \mathcal{Z}} N_z (\vec{M}_s - \vec{M}_T)(\vec{M}_s - \vec{M}_T)^\top$ <sup>1</sup>
- ▶  $\mathbf{N} = \sum_{z \in \mathcal{Z}} \mathbf{K}_z (\mathbf{I} - \mathbf{I}_{N_z}) \mathbf{K}_z^\top$ <sup>2</sup>
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---

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## KDA - the training

Computational Complexity  $O(D^3)$

### LDA

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- ▶  $\vec{\alpha}_i$  eigenvectors of  $\mathbf{S_W^{-1} S_B}$  [ $D \times D$ ]
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Computational Complexity  $O(N^3)$

### KDA

- ▶  $\mathbf{M} = \sum_{z \in \mathcal{Z}} N_z (\vec{M}_s - \vec{M}_T)(\vec{M}_s - \vec{M}_T)^\top$ <sup>1</sup>
- ▶  $\mathbf{N} = \sum_{z \in \mathcal{Z}} \mathbf{K}_z (\mathbf{I} - \mathbf{I}_{N_z}) \mathbf{K}_z^\top$  + **μI**
- ▶  $\vec{\nu}_i$  eigenvectors of  $\mathbf{N}^{-1} \mathbf{M}$  [ $N \times N$ ]
- ▶  $\epsilon_\ell^{KDA}(\vec{x}) = \sum_{i=1}^N \vec{\nu}_\ell[i] K(\mathbf{x}_i^{z_i}, \mathbf{x})$

$\mu$  regularization parameter

Memory-based machine

<sup>1</sup>  $\vec{M}_s$  and  $\vec{M}_T$  are two  $N$ -sized column vectors whose entries are given by:

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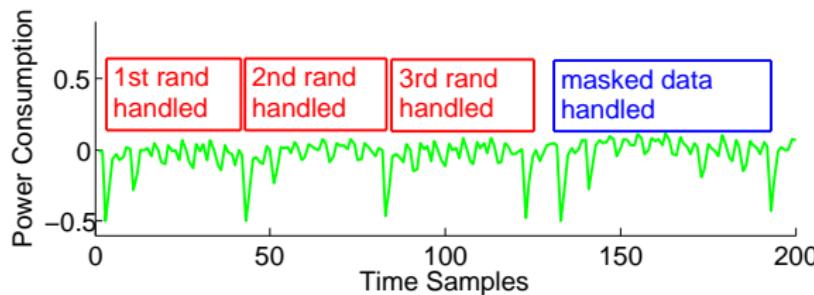
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## Experimental setup

Target device and acquisitions:

- ▶ 8-bit AVR microprocessor Atmega328P
- ▶ power-consumption acquired via the ChipWhisperer [OC14] platform
- ▶  $D = 200$ , 4 clock-cycles are selected

Sensitive variable:  $Z = \text{Sbox}_{\text{AES}}(P \oplus K^*)$



## Efficiency/Accuracy trade-off

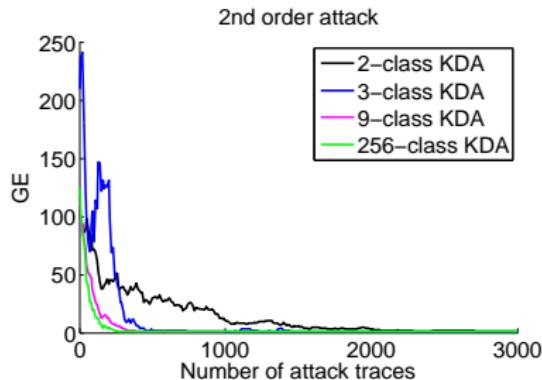
KDA training set size fixed ( $\sim 9000$  traces) to control efficiency

Adjust the number of classes to gain in accuracy

$Z = 0, \dots, 255$

Model	number of classes	traces per class
Value	256	35
HW	9	1000
HW $<, >, = 4$	3	3000
HW $\leq, > 4$	2	4500

## Attack of second order

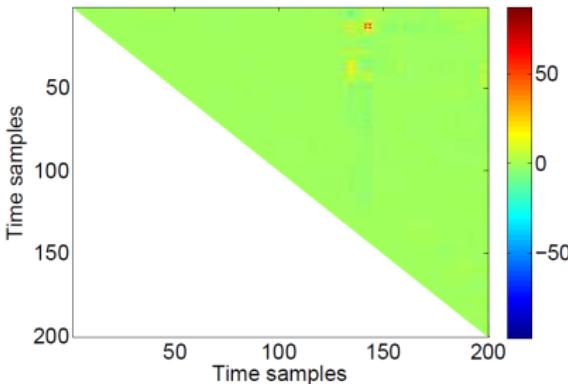
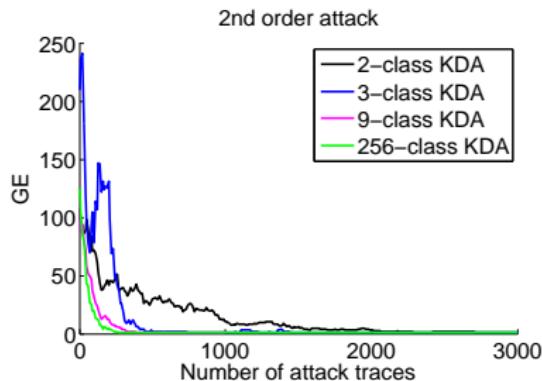


### Implicit coefficients

$$\epsilon_{\ell}^{\text{KDA}}(\mathbf{x}) = \sum_{i=1}^N \vec{\nu}_{\ell}[i] K(\mathbf{x}_i^{z_i}, \mathbf{x}) = \sum_{j=1}^D \sum_{k=1}^D [ \underbrace{(\mathbf{x}[j]\mathbf{x}[k])}_{\text{multiplied time samples}} \underbrace{(\sum_{i=1}^N \nu_{\ell}[i] \mathbf{x}_i[j] \mathbf{x}_i[k])}_{\text{implicit coefficients}} ] \quad (5)$$

$$d = 2 \rightarrow \binom{200+2-1}{2} = 20.100 \text{ implicit coefficients}$$

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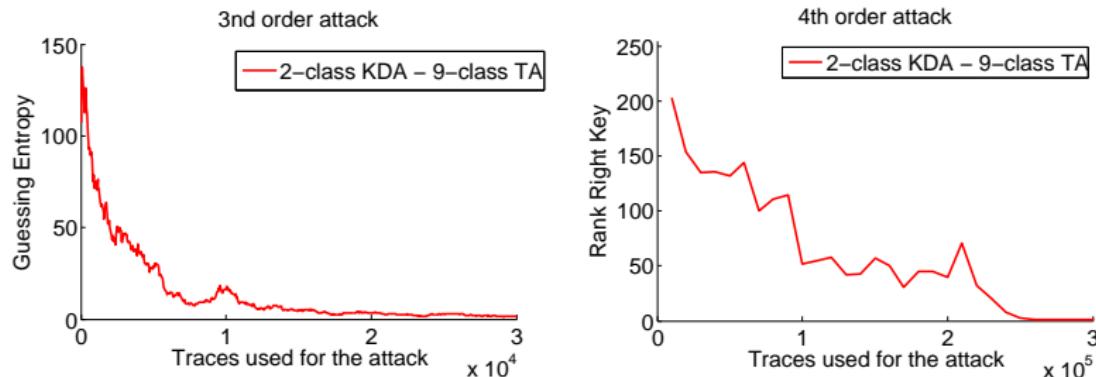


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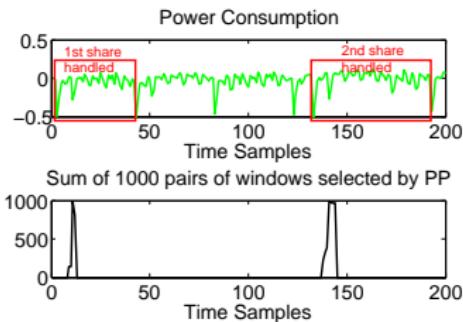
## Third and Fourth Order



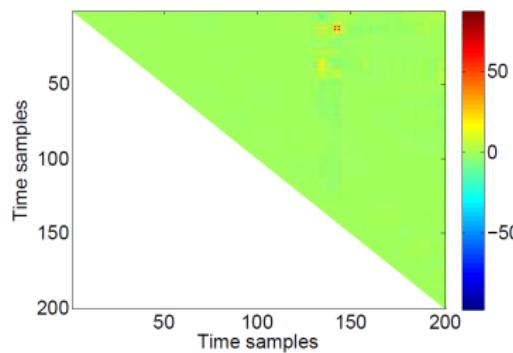
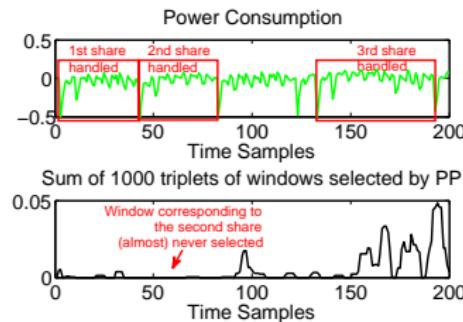
- $d = 3 \rightarrow \binom{200+3-1}{3} = 1.353.400$  implicit coefficients
- $d = 4 \rightarrow \binom{200+4-1}{4} = 68.685.050$  implicit coefficients

Same time of execution of the KDA algorithm!

## Qualitative comparison with PP Second Order



## Third Order



## Conclusions on KDA

### Strong points

- ▶ KDA is suitable to attack  $(d - 1)$ th-order masking
- ▶ Choice of the  $d$ -th degree polynomial kernel function
- ▶ KDA computational complexity is independent from the order  $d$
- ▶ Tested and effective on a real case, positively compared to PP

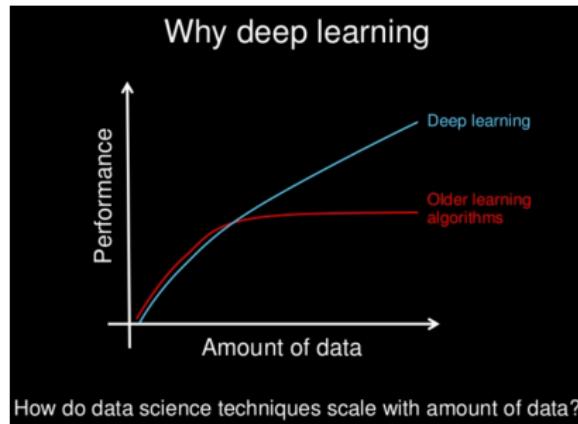
### Limits and drawbacks

- ▶ Memory-based +  $O(N^3)$  complexity → Non-scalability to big training set  
→ not suitable for highly-noisy higher-order masked signals
- ▶ Regularization hyper-parameter  $\mu$  (and slow validation)

## Contents

1. Context
2. State of the Art, Objectives, Contributions
3. Kernel Discriminant Analysis against Masking
  - 3.1 Linear Discriminant Analysis
  - 3.2 Kernel Discriminant Analysis
  - 3.3 Experimental Results
4. Deep Learning against Misalignment
  - 4.1 Data Augmentation
  - 4.2 Experimental Results
5. Conclusions

## Motivations (1)



- ▶ not memory-based
- ▶ parallelizable computation (GPU optimizations)
- ▶ many hyper-parameters but faster validation

## Motivations (2)

### Profiling phase

- ▶ manage de-synchronization problem  $[\mathcal{D}_{\text{train}} \longrightarrow \rho: \mathbb{R}^D \rightarrow \mathbb{R}^D]$
- ▶ mandatory dimensionality reduction  $[\mathcal{D}_{\text{train}} \longrightarrow \epsilon: \mathbb{R}^D \rightarrow \mathbb{R}^C]$
- ▶ estimate
  - ▶  $p_{\epsilon(\rho(\vec{x})) \mid Z=z}$ ,  $P_{\epsilon(\rho(\vec{x}))}$ ,  $p_Z$  (generative model)
    - ▶ Gaussian hypothesis [CRR03]
    - ▶ Variants: *pooled* version [CK14], linear regression [SLP05]
  - ▶  $p_Z \mid \epsilon(\rho(\vec{x})$  (discriminative model)

Many independent preprocessing steps and assumptions

## Motivations (2)

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    - ▶ Variants: pooled version [CK14], linear regression [SLP05]
  - ▶  $p_{Z \mid \vec{X}}$  (discriminative model)  
by means of a neural network  $F(\vec{X}) \approx p_{Z \mid \vec{X}}$  (Universal approximation theorem)

Many independent preprocessing steps and assumptions

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Many independent preprocessing steps and assumptions  
↔ integrated and agnostic approach

## Multi-Layer Perceptron

### Multi-Layer Perceptron (MLP)

$$F(\vec{x}, W) = s \circ \lambda_n \circ \sigma_{n-1} \circ \lambda_{n-1} \circ \dots \circ \lambda_1(\vec{x}) = \vec{y} \approx \Pr[Z | \vec{X} = \vec{x}]$$

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$\lambda_i$  linear functions (linear combinations of time samples) depending on some **trainable weights**  $W$

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$\textcolor{red}{s}$  normalizing *softmax* function

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$\lambda_i$  linear functions (linear combinations of time samples) depending on some **trainable weights**  $W$

$\sigma_i$  non-linear *activation* functions

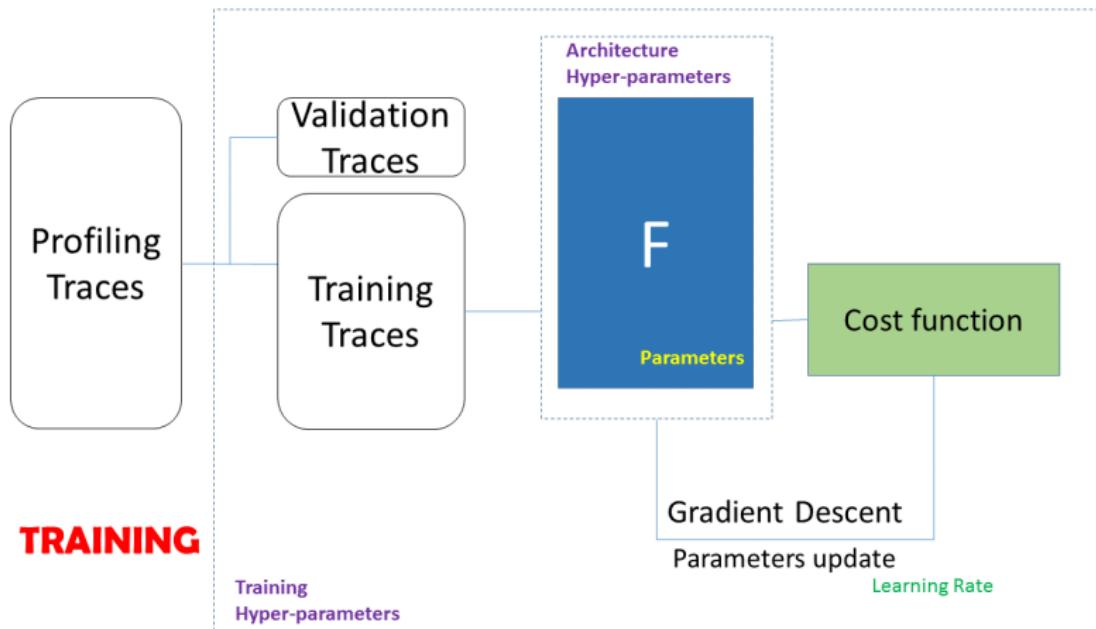
$\textcolor{red}{s}$  normalizing *softmax* function

### Softmax $\sim$ multi-class logistic sigmoid

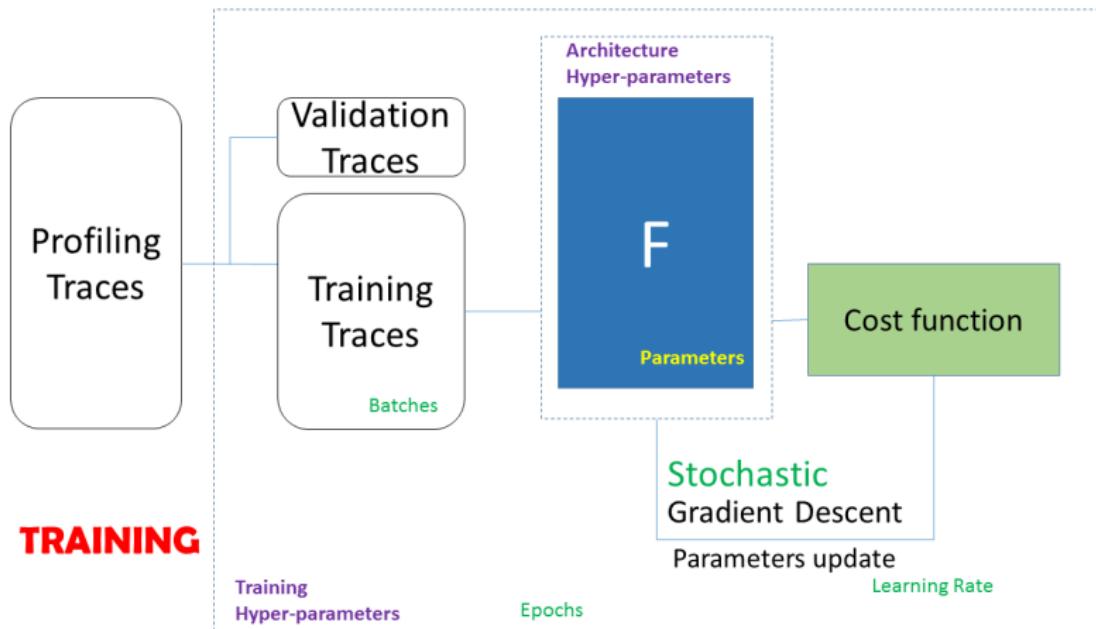
$$s(\mathbf{a})[k] = \frac{e^{\mathbf{a}[k]}}{\sum_{j=1}^{|\mathcal{Z}|} e^{\mathbf{a}[j]}} . \quad (6)$$

$$\Pr(s_j | \vec{x}) = \frac{\Pr(\vec{x} | s_j)\Pr(s_j)}{\Pr(\vec{x})} = \frac{\Pr(\vec{x} | s_j)\Pr(s_j)}{\sum_{k=1}^{|\mathcal{Z}|} \Pr(\vec{x} | s_k)\Pr(s_k)} = s(\mathbf{a})[j] , \quad (7)$$

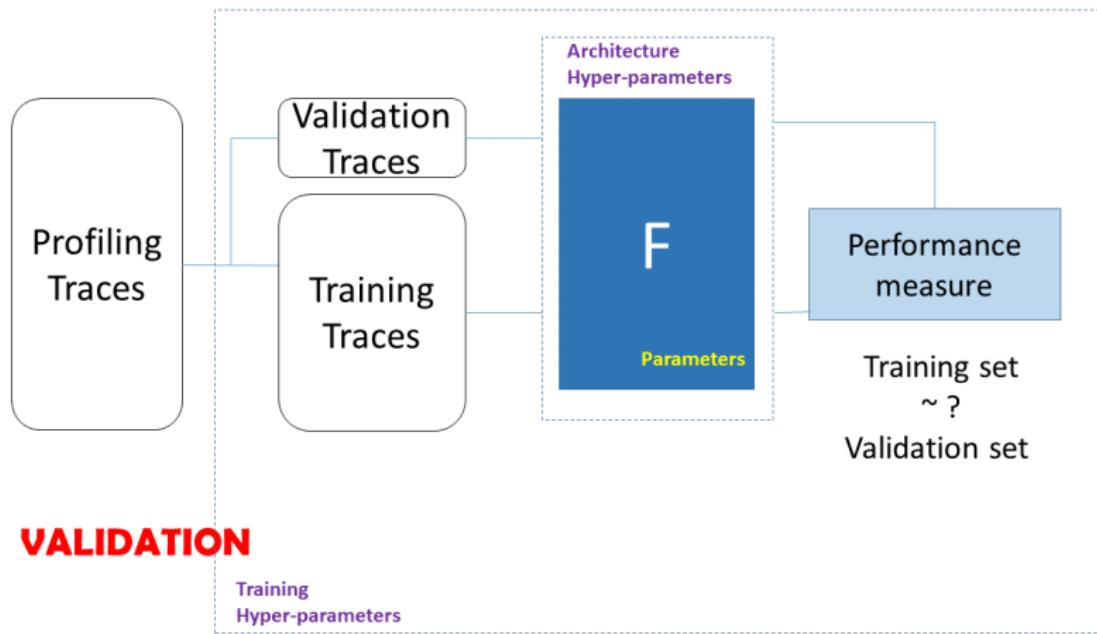
## Training-Validation-Test



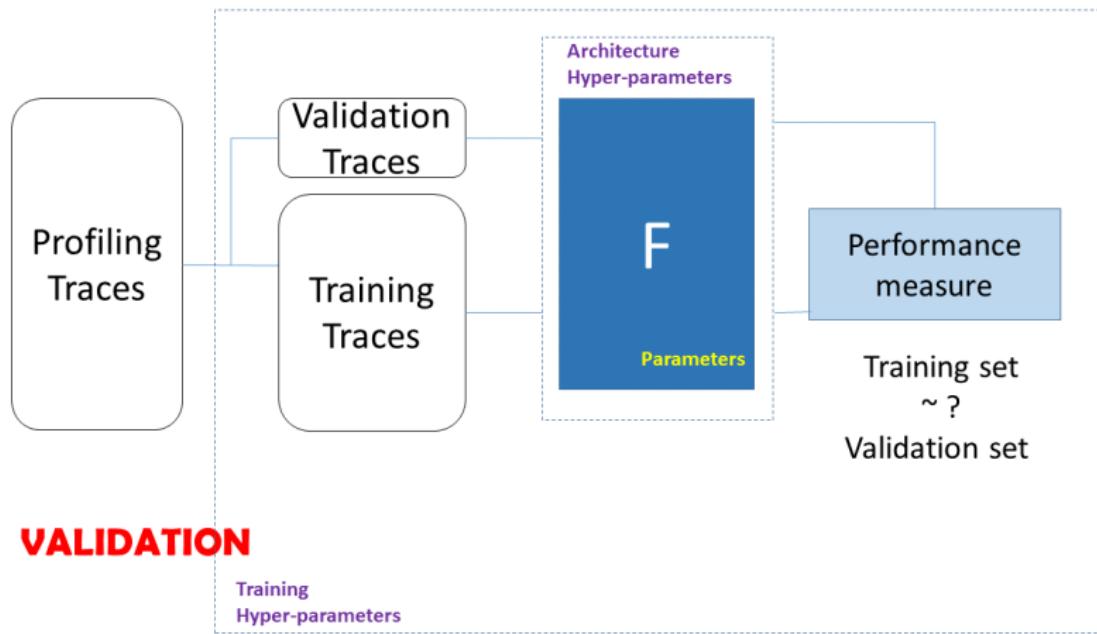
## Training-Validation-Test



## Training-Validation-Test



## Training-Validation-Test



## VALIDATION

Training  
Hyper-parameters

## Cost function - Cross-entropy

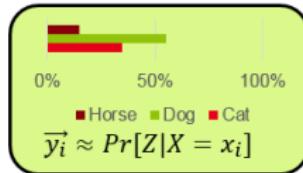
- ▶ batch of training data  $(\vec{x}_i, z_i)_{i \in I}$ , outputs of the current model  $(\vec{y}_i)_{i \in I}$
- ▶ labels  $z_i = s_j$  are *one-hot encoded*:  $\vec{z}_i = \vec{s}_j = (0, \dots, 0, \underbrace{1}_j, 0, \dots, 0)$

## Loss function

$$\mathcal{L} = -\frac{1}{|I|} \sum_{i \in I} \sum_{t=1}^{|Z|} \vec{z}_i[t] \log \vec{y}_i[t] \quad (8)$$

## Maximum-*a-posteriori* or Cross-entropy

- ▶  $\vec{y}_i \approx \Pr[Z \mid \vec{X} = \vec{x}_i]$



## Cost function - Cross-entropy

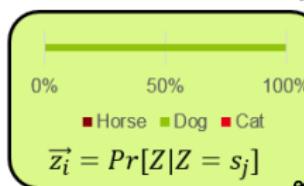
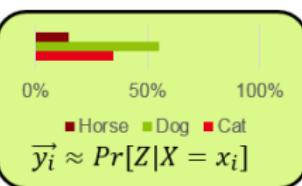
- ▶ batch of training data  $(\vec{x}_i, z_i)_{i \in I}$ , outputs of the current model  $(\vec{y}_i)_{i \in I}$
- ▶ labels  $z_i = s_j$  are *one-hot encoded*:  $\vec{z}_i = \vec{s}_j = (0, \dots, 0, \underbrace{1}_{j}, 0, \dots, 0)$

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## Maximum-a-posteriori or Cross-entropy

- ▶  $\vec{y}_i \approx \Pr[Z | \vec{X} = \vec{x}_i]$
- ▶  $\vec{z}_i \approx \Pr[Z | Z = \vec{s}_j]$
- ▶  $\mathbb{H}(\vec{z}_i, \vec{y}_i) = \mathbb{H}(\vec{z}_i) + D_{KL}(\vec{z}_i || \vec{y}_i) = \mathbb{E}_{\vec{z}_i}[-\log \vec{y}_i] = -\sum_{t=1}^{|Z|} \vec{z}_i[t] \log \vec{y}_i[t]$



## Capacity-Overfitting-Regularization

### Regression example

Performance metric: Mean Square Error (MSE)

MSE\_train=44.228280, MSE\_test=330.984916

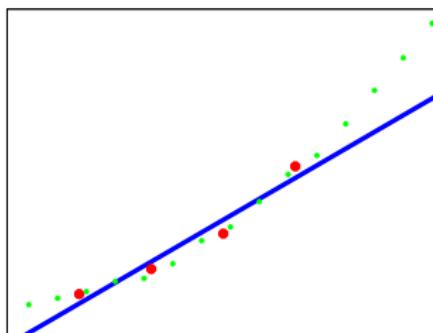


Figure: Linear regression → underfitting

## Capacity-Overfitting-Regularization

### Regression example

Performance metric: Mean Square Error (MSE)

MSE\_train=44.228280, MSE\_test=330.984916

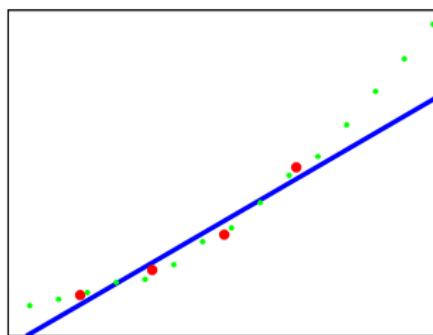


Figure: Linear regression → underfitting

MSE\_train=2.243097, MSE\_test=61.891672

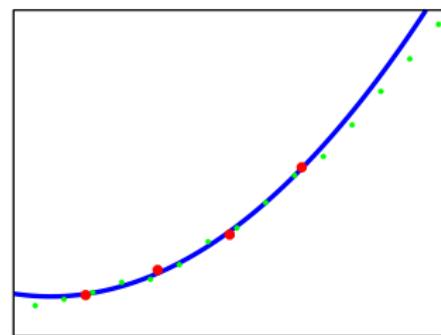


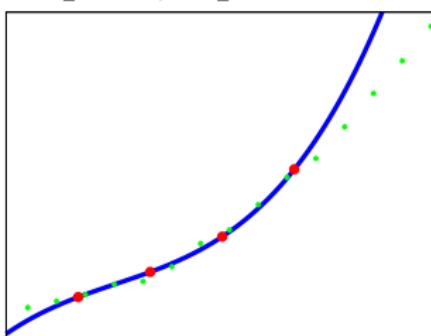
Figure: Quadratic regression → fits

## Capacity-Overfitting-Regularization

### Regression example

Performance metric: Mean Square Error (MSE)

MSE\_train=0, MSE\_test=970.081580



MSE\_train=2.243097, MSE\_test=61.891672

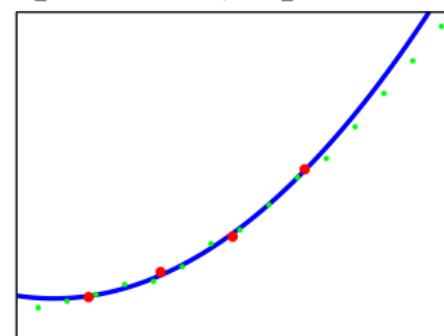


Figure: Cubic regression → overfitting

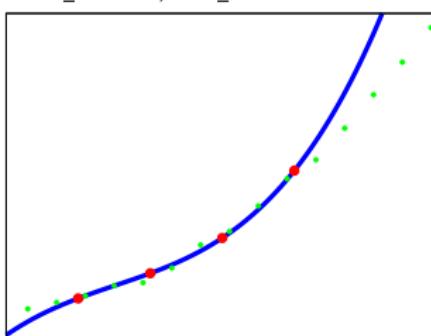
Figure: Quadratic regression → fits

## Capacity-Overfitting-Regularization

### Regression example

Performance metric: Mean Square Error (MSE)

MSE\_train=0, MSE\_test=970.081580



MSE\_train=3.040333, MSE\_test=58.377719

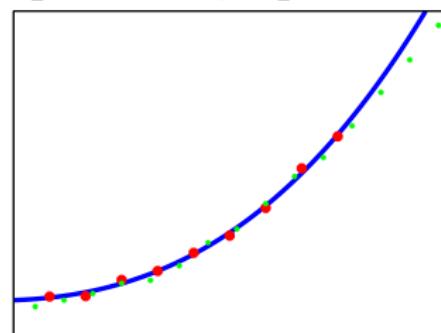


Figure: Cubic regression → overfitting

Figure: Cubic regression with more training data

## Capacity-Overfitting-Regularization

### Regression example

Performance metric: Mean Square Error (MSE)

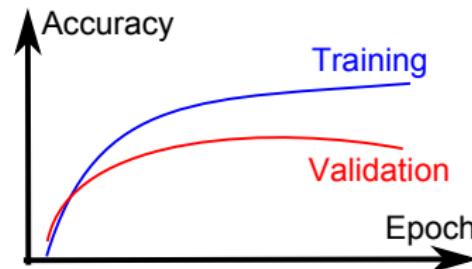
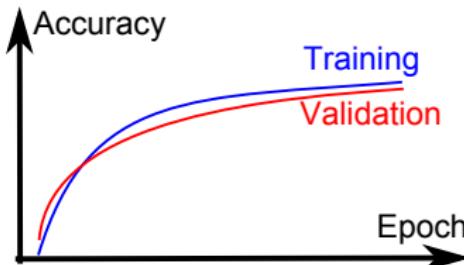
### Classification via Neural Network

Performance measure: Accuracy (Classification rate)

Evaluate and compare training and validation accuracy

Understand significant features

Learn by heart (**OVERTFITTING**)



## Capacity-Overfitting-Regularization

### Regression example

Performance metric: Mean Square Error (MSE)

### Classification via Neural Network

Performance measure: Accuracy (Classification rate)

Evaluate and compare training and validation accuracy

Learn by heart (**OVERTFITTING**)

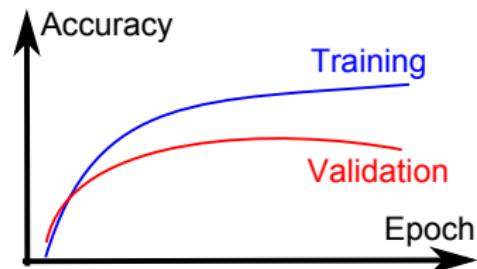
Why?

Too complex model

Not enough training data

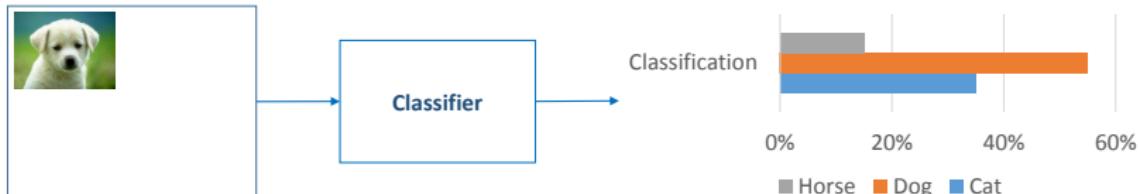
Solution?

Data augmentation



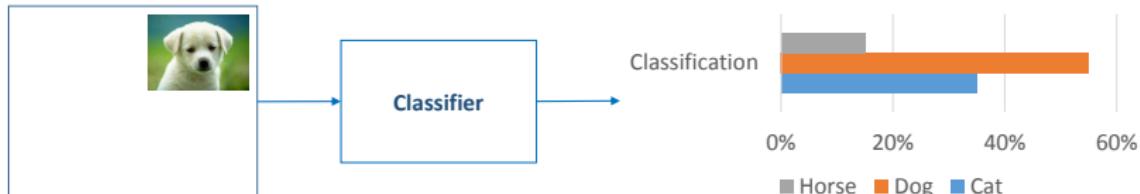
## Convolutional Neural Networks

### Translation-invariance



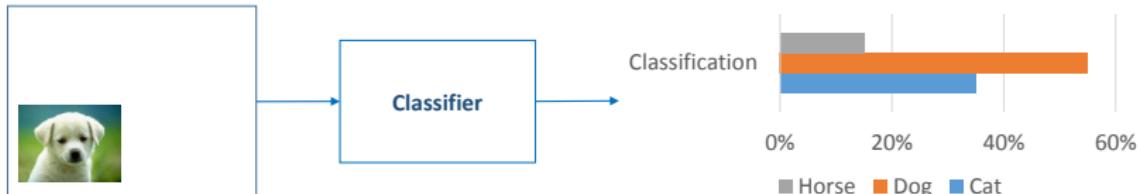
## Convolutional Neural Networks

### Translation-invariance



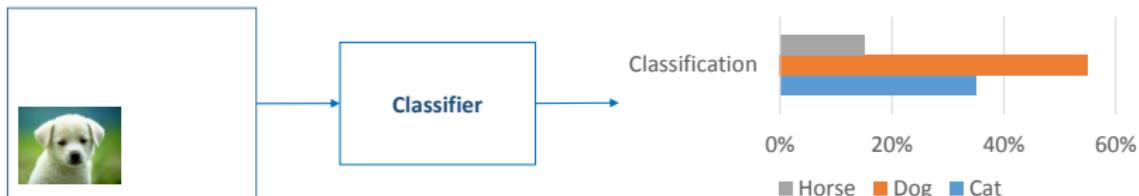
## Convolutional Neural Networks

### Translation-invariance



## Convolutional Neural Networks

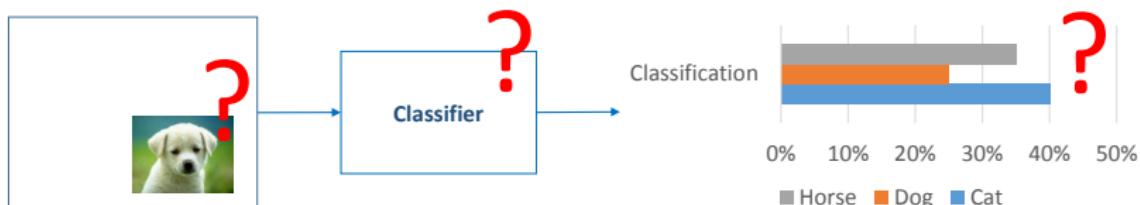
### Translation-invariance



It is important to explicit the data translation-invariance

## Convolutional Neural Networks

### Translation-invariance



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## Convolutional Neural Networks

### Translation-invariance



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## Convolutional Neural Networks

### Translation-invariance



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## Convolutional Neural Networks

### Translation-invariance



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## Convolutional Neural Networks

### Translation-invariance



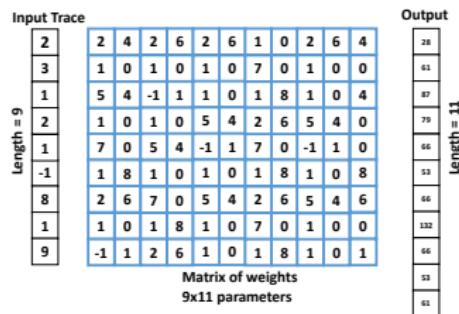
It is important to explicit the data translation-invariance  
Convolutional Neural Networks: share weights across space

## Convolutional Neural Networks

### Translation-invariance



It is important to explicit the data translation-invariance  
Convolutional Neural Networks: share weights across space

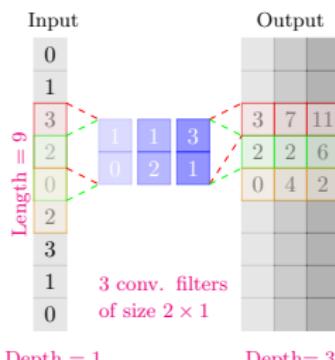


## Convolutional Neural Networks

### Translation-invariance



It is important to explicit the data translation-invariance  
 Convolutional Neural Networks: share weights across space



Input Trace	Output
2	28
3	61
1	87
2	79
1	66
-1	53
8	46
2	32
1	44
9	53
	61

Length = 9                              Length = 11

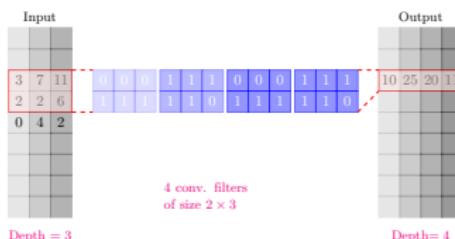
Matrix of weights  
9x11 parameters

## Convolutional Neural Networks

### Translation-invariance



It is important to explicit the data translation-invariance  
 Convolutional Neural Networks: share weights across space



**Figure:** Linear layer in a ConvNet (*Convolutional Layer*)

	Input Trace									Output												
	2	4	2	6	2	6	1	0	2	6	4	28	61	87	79	66	53	66	112	66	53	61
2	1	0	1	0	1	0	7	0	1	0	0	28	61	87	79	66	53	66	112	66	53	61
3	5	4	-1	1	1	0	1	8	1	0	4	28	61	87	79	66	53	66	112	66	53	61
1	1	0	1	0	5	4	2	6	5	4	0	28	61	87	79	66	53	66	112	66	53	61
2	7	0	5	4	-1	1	7	0	-1	1	0	28	61	87	79	66	53	66	112	66	53	61
1	1	8	1	0	1	0	1	8	1	0	8	28	61	87	79	66	53	66	112	66	53	61
-1	2	6	7	0	5	4	2	6	5	4	6	28	61	87	79	66	53	66	112	66	53	61
8	1	0	1	8	1	0	7	0	1	0	0	28	61	87	79	66	53	66	112	66	53	61
9	-1	1	2	6	1	0	1	8	1	0	1	28	61	87	79	66	53	66	112	66	53	61

Matrix of weights  
9x11 parameters

## Convolutional Neural Networks

### Translation-invariance



It is important to explicit the data translation-invariance  
Convolutional Neural Networks: share weights across space

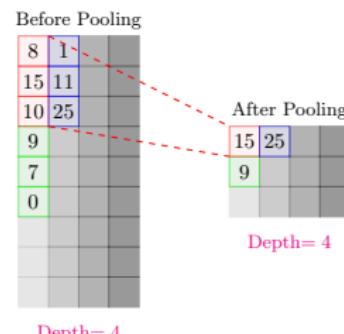
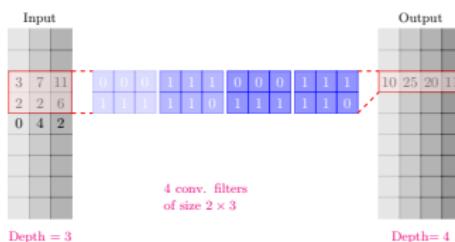
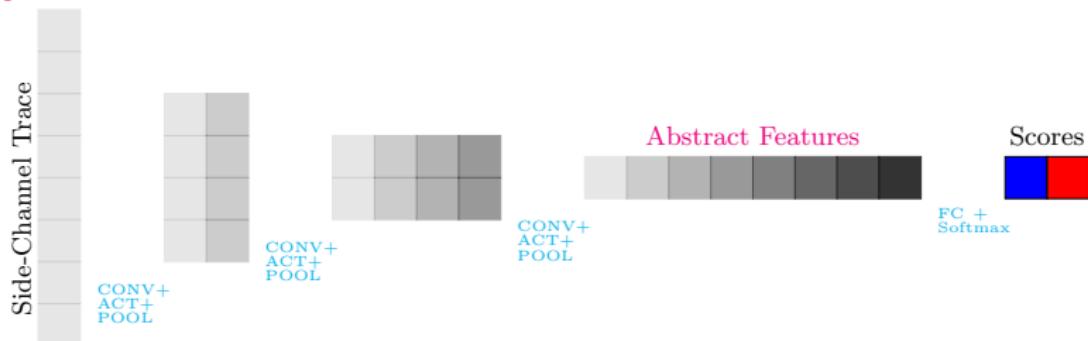


Figure: Linear layer in a ConvNet (*Convolutional Layer*)

## A kind of CNN architecture

Temporal Features



VGG-like [simonyan2014very]:

- ▶ Reduce temporal features to only one
- ▶ Maintain time complexity of each layer (one-half pooling when number of feature maps are doubled)
- ▶ Small filters

Model used in our experiments

$$s \circ [\lambda]^1 \circ [\delta \circ [\sigma \circ \gamma]^1]^4$$

## Data Augmentation

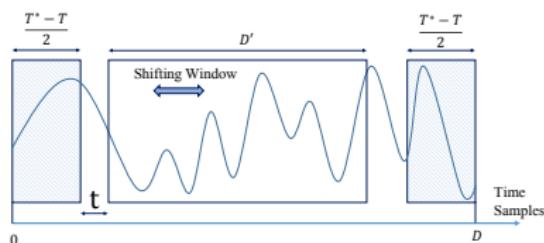
### Data Augmentation

Artificially generate new training data by deforming those previously acquired,  
Applying transformations that preserve the label  $Z$

### Countermeasure Emulation Idea

Emulate the effects of misaligning countermeasures to generate new traces

#### SHIFTING



#### ADD-REMOVE

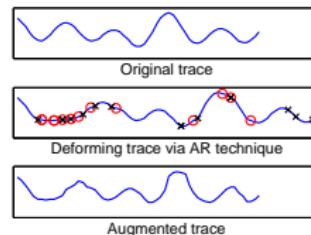


Figure:  $SH_T$

Figure:  $AR_R$

Parameter  $T$ : # of possible positions

Parameter  $R$ : # of added and removed points

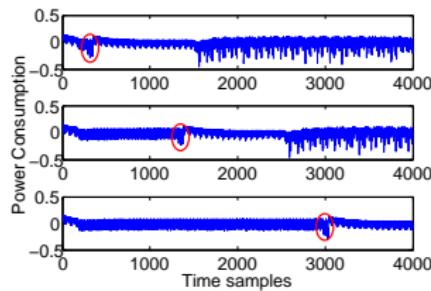
Data Augmentation techniques are applied online during the training phase.

## Experimental Results

- ▶ Random delays
- ▶ Artificial Jitter
- ▶ Real Jitter

Keras 1.2.1 library with Tensorflow backend [Cho+15] (open source, today 2.2.4)

## Random delays



(a) One leaking operation

## Setup

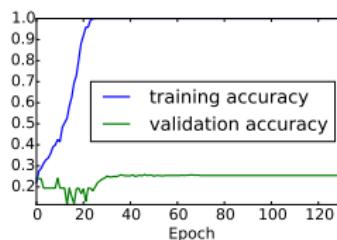
- ▶ Target Chip: Atmega328P
- ▶ Target Variable:  $Z = \text{HW}(\text{Sbox}(P \oplus K))$
- ▶ Acquisition: through *ChipWhisperer®* platform,  $\approx 4,000$  time samples
- ▶ Countermeasure: Random Delays - insertion of  $r$  *nop* operations,  $r \in [0, 127]$  uniform random
- ▶ 1,000 training traces

## Random delays

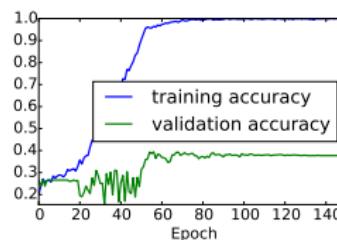
Data augmentation vs overfitting

### Metrics

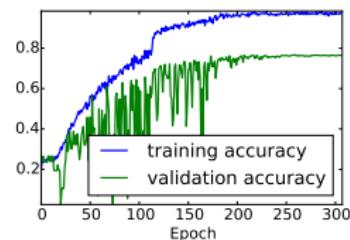
- ▶ Test accuracy: classification accuracy over the attack traces
- ▶  $N^*$ : minimum number of attack traces to make *guessing entropy* of the right key permanently equal to one ( $N^*$  estimated over 10 independent attacks)



$SH_0$



$SH_{100}$



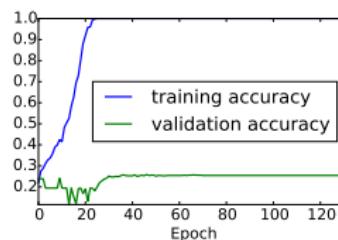
$SH_{500}$

## Random delays

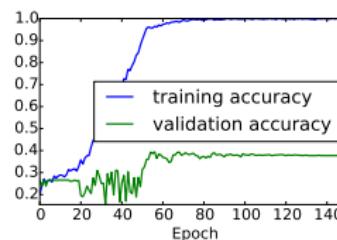
Data augmentation vs overfitting

### Metrics

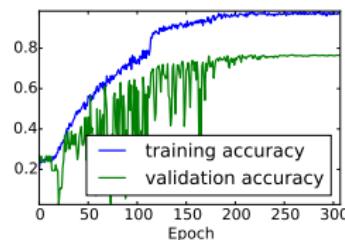
- ▶ Test accuracy: classification accuracy over the attack traces
- ▶  $N^*$ : minimum number of attack traces to make *guessing entropy* of the right key permanently equal to one ( $N^*$  estimated over 10 independent attacks)



$SH_0$



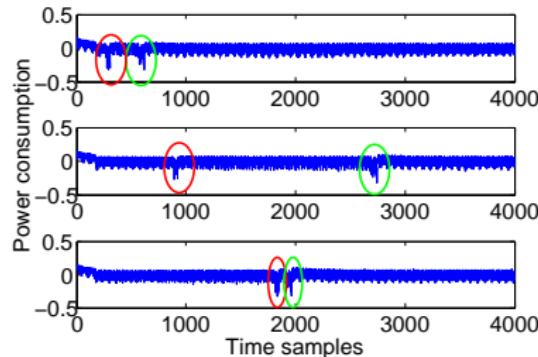
$SH_{100}$



$SH_{500}$

		$SH_0$		$SH_{100}$		$SH_{500}$	
Acc	$N^*$	27.0%	> 1,000	31.8%	101	78%	7

## Random Delays - Two Leaking Operations

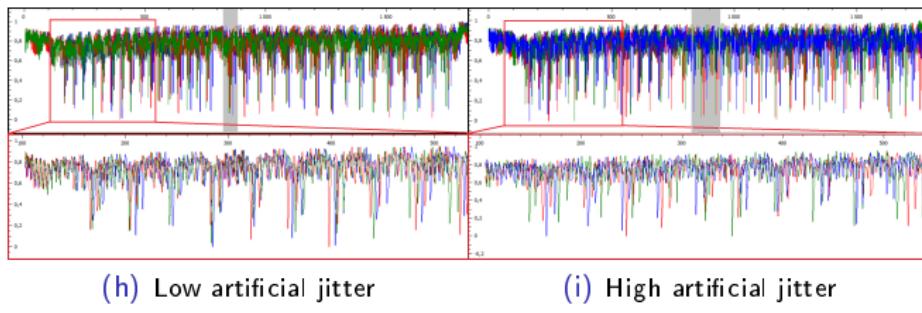


### Two leaking operations

First operation - Test acc: 76.8%,  $N^* = 7$

Second operation - Test acc: 82.5%,  $N^* = 6$

## Artificial Jitter



### Target

- ▶ Target Variable:  $Z = \text{HW}(\text{Sbox}(P \oplus K))$
- ▶  $\approx 2000$  time samples
- ▶ Countermeasure: artificial signal treatment simulating clock jitter
- ▶ 10000 training traces

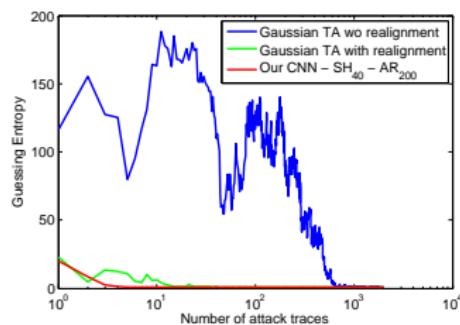
## Artificial Jitter (2)

*Low\_jitter*

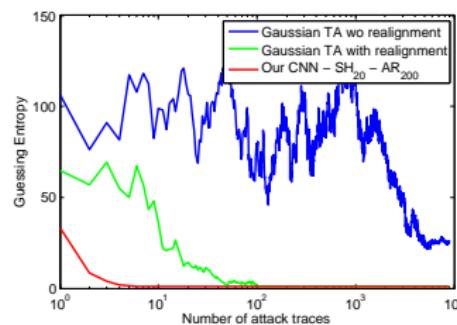
Acc	$N^*$	$SH_0$	$SH_{20}$	$SH_{40}$	
AR <sub>0</sub>		57.4%	14	82.5%	6
AR <sub>100</sub>		86.0%	6	87.0%	5
AR <sub>200</sub>		86.6%	6	85.7%	6
				83.6%	6
				87.5%	6
				87.7%	5

*High\_jitter*

Acc	$N^*$	$SH_0$	$SH_{20}$	$SH_{40}$	
AR <sub>0</sub>		40.6%	35	51.1%	9
AR <sub>100</sub>		50.2%	15	72.4%	11
AR <sub>200</sub>		64.0%	11	75.5%	8
				62.4%	11
				73.5%	9
				74.4%	8



(j) Low Jitter



(k) High Jitter

## Artificial Jitter

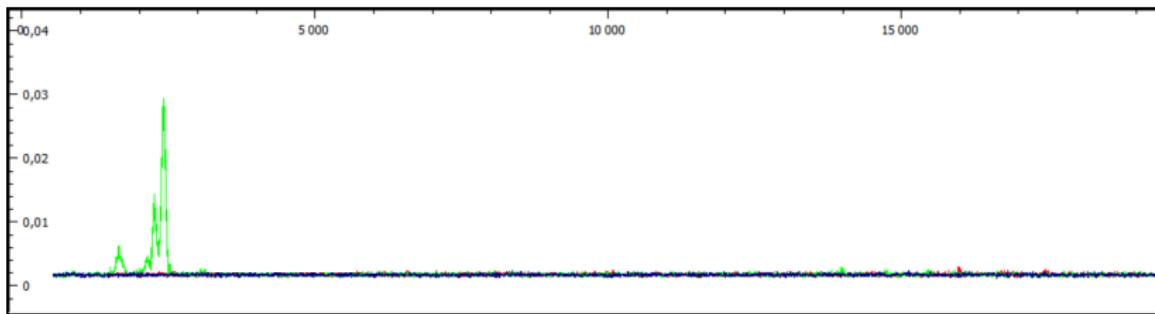
<i>DS_low_jitter</i>		SH <sub>0</sub>		SH <sub>20</sub>		SH <sub>40</sub>		SH <sub>200</sub>	
<i>a</i>	<i>b</i>								
<i>c</i>	<i>d</i>								
AR <sub>0</sub>	100.0%	68.7%	99.8%	86.1%	98.9%	84.1%			
	57.4%	14	82.5%	6	83.6%	6			
AR <sub>100</sub>	87.7%	88.2%	82.4%	88.4%	81.9%	89.6%			
	86.0%	6	87.0%	5	87.5%	6			
AR <sub>200</sub>	83.2%	88.6%	81.4%	86.9%	80.6%	88.9%			
	86.6%	6	85.7%	6	87.7%	5			
AR <sub>500</sub>							85.0%	88.6%	
							86.2%	5	
<i>DS_high_jitter</i>		SH <sub>0</sub>		SH <sub>20</sub>		SH <sub>40</sub>		SH <sub>200</sub>	
<i>a</i>	<i>b</i>								
<i>c</i>	<i>d</i>								
AR <sub>0</sub>	100%	45.0%	100%	60.0%	98.5%	67.6%			
	40.6%	35	51.1%	9	62.4%	11			
AR <sub>100</sub>	90.4%	57.3%	76.6%	73.6%	78.5%	76.4%			
	50.2%	15	72.4%	11	73.5%	9			
AR <sub>200</sub>	83.1%	67.7%	82.0%	77.1%	82.6%	77.0%			
	64.0%	11	75.5%	8	74.4%	8			
AR <sub>500</sub>							83.6%	73.4%	
							68.2%	11	

## Real Jitter (1)

### Target

- ▶ AES hardware implementation
- ▶ strong jitter effect
- ▶ Target Variable:  $Z = \text{Sbox}(P \oplus K)$
- ▶ 2,500 selected time samples
- ▶ 99,000 training traces

SNR first Sbox

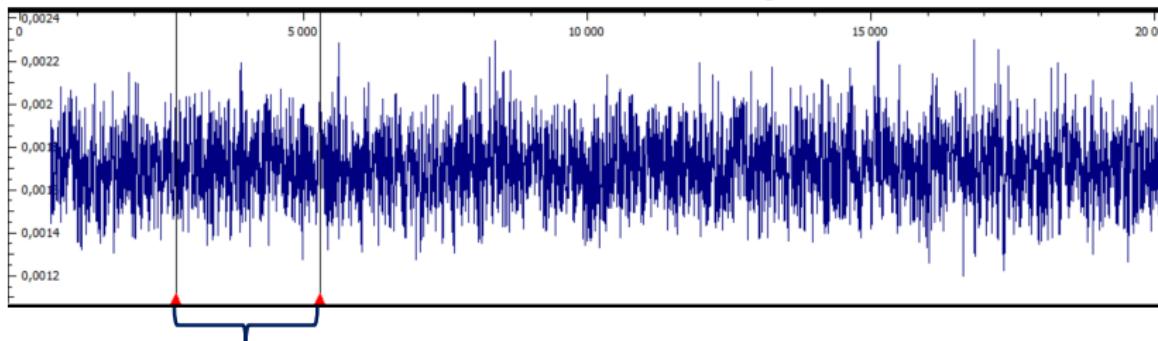


## Real Jitter (1)

### Target

- ▶ AES hardware implementation
- ▶ strong jitter effect
- ▶ Target Variable:  $Z = \text{Sbox}(P \oplus K)$
- ▶ 2,500 selected time samples
- ▶ 99,000 training traces

SNR second Sbox without realignment



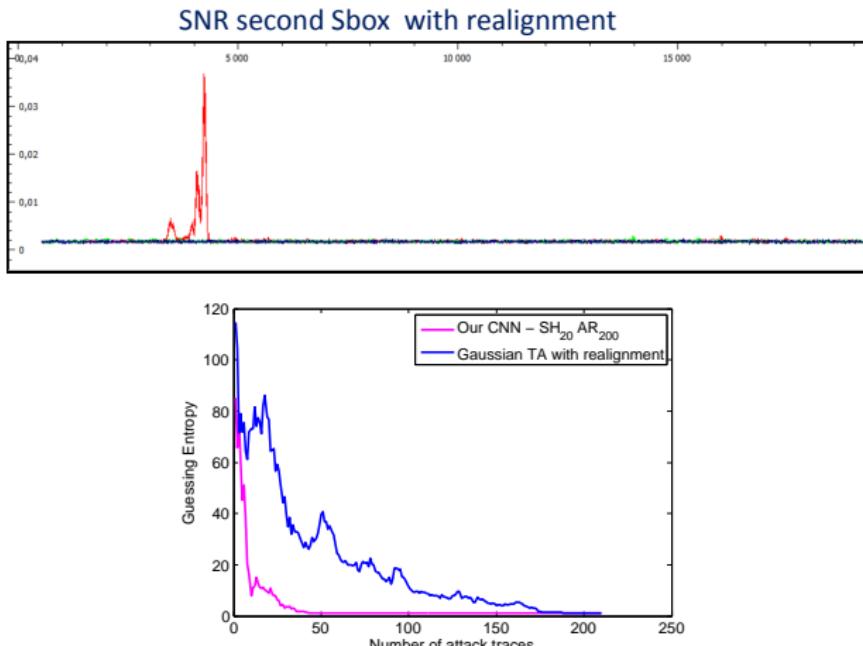
Entry region for CNN (2,500 pts)

## Real Jitter (2)

		$SH_0 AR_0$	$SH_{10} AR_{100}$	$SH_{20} AR_{200}$		
Acc	$N^*$	1.2%	137	1.3%	89	1.8%
						54

## Real Jitter (2)

		SH <sub>0</sub> AR <sub>0</sub>	SH <sub>10</sub> AR <sub>100</sub>	SH <sub>20</sub> AR <sub>200</sub>
Acc	N*	1.2%	137	1.3%
			89	1.8% 54



## Conclusions about CNN

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1. Context
2. State of the Art, Objectives, Contributions
3. Kernel Discriminant Analysis against Masking
  - 3.1 Linear Discriminant Analysis
  - 3.2 Kernel Discriminant Analysis
  - 3.3 Experimental Results
4. Deep Learning against Misalignment
  - 4.1 Data Augmentation
  - 4.2 Experimental Results
5. Conclusions

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