Feature Extraction for Side-Channel Attacks

Eleonora Cagli

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PhD Supervisor : Emmanuel Prouff (ANSSI)
CEA Supervisor : Cécile Dumas (CEA-Leti Grenoble)

Contents

Secure Component and Embedded Cryptography

A piece of hardware with security properties.
It usually embeds cryptography to provide security services (authentication, signature, secure messaging with terminals...)

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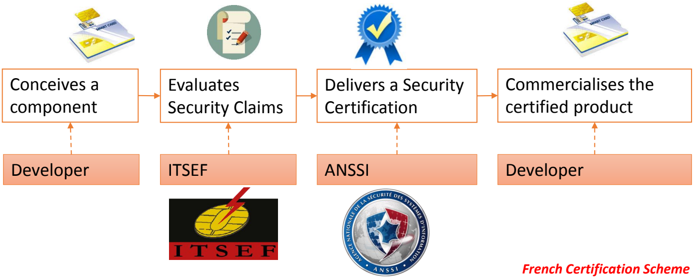
A piece of hardware with security properties.
It usually embeds cryptography to provide security services (authentication, signature, secure messaging with terminals...)

- Sensitive applications: ID cards, credit cards, transport cards, health cards, SIM
- Pervasive aspect: several billion smartcards sold par year
- Hard to update
- Hostile environment

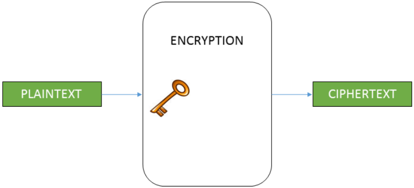
⇒ Requires protection against very high-level attacker

Security Certification



- Standardised Evaluation (*e.g.* ISO/IEC 15408 - Common Criteria)
- Many evaluation tasks
- AVA class: vulnerability assessment (penetration testing)

Side-Channel Vulnerability of Embedded Cryptography



	Classical Cryptanalysis	Side-Channel Cryptanalysis
	Mathematical vulnerability	
	Black Box	

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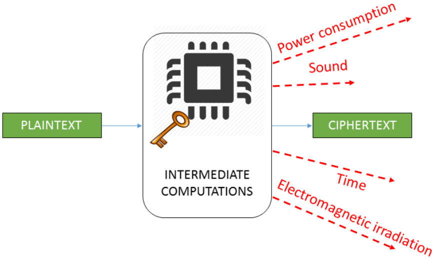

Classical Cryptanalysis	Side-Channel Cryptanalysis
Mathematical vulnerability	Physical vulnerability
Black Box	Grey Box / Divide-and-conquer

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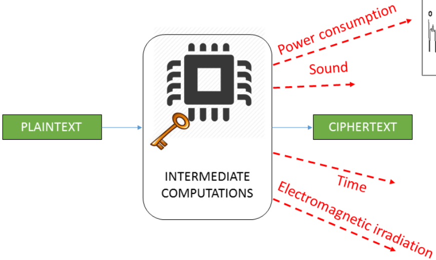

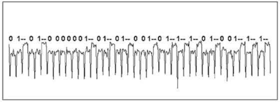

Source - Paul Kocher et al. "Introduction to differential power analysis".

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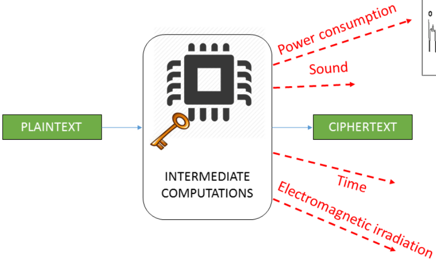

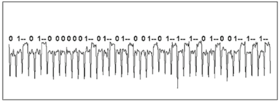

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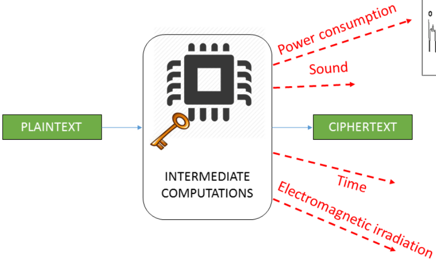

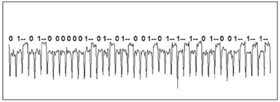

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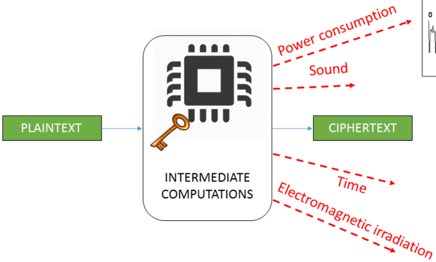

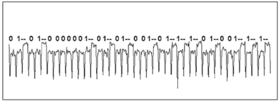

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Simple attack: only one plaintext

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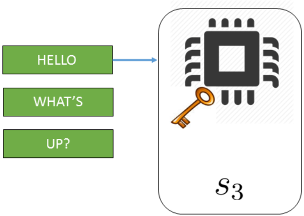
Advanced Side-Channel Attacks



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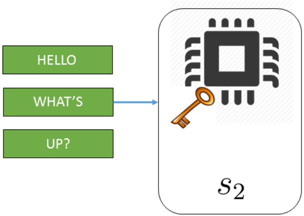

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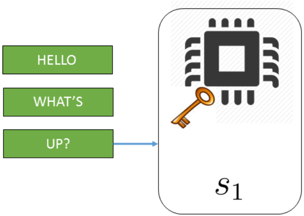

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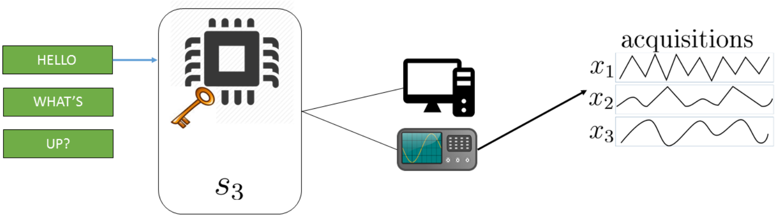

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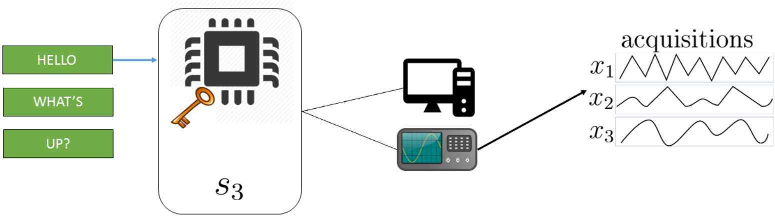

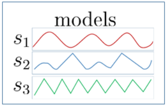

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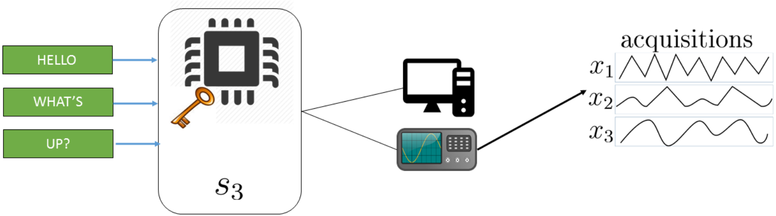

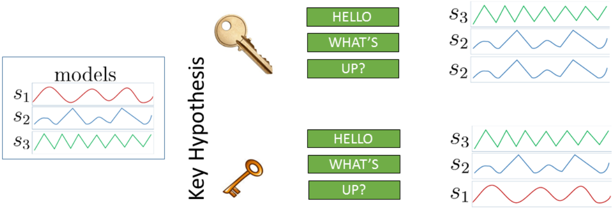

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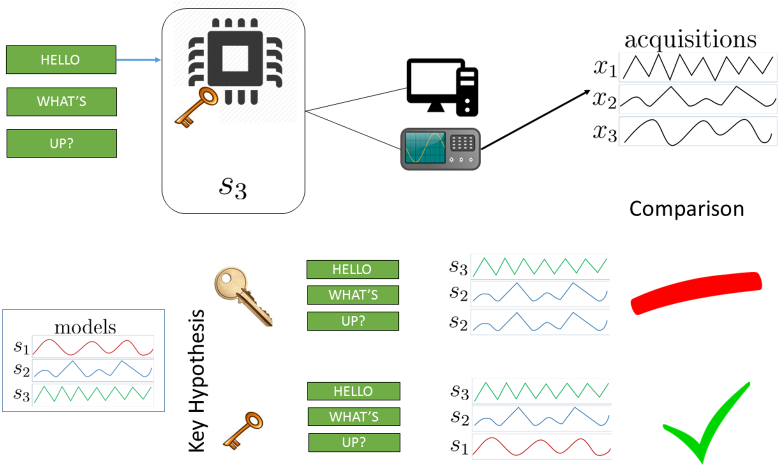

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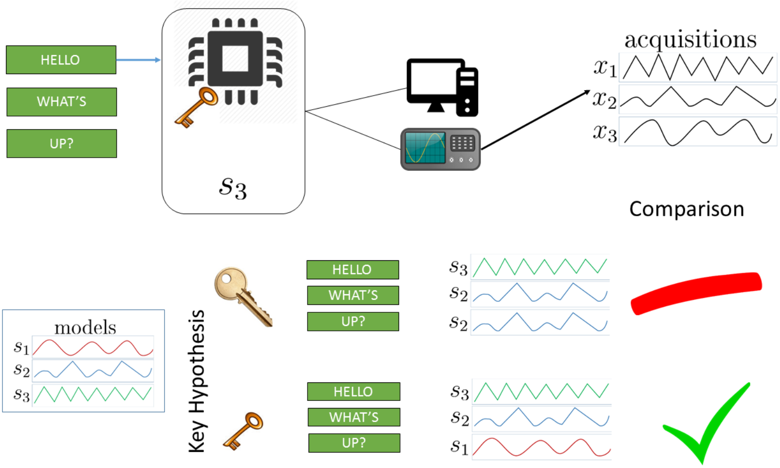
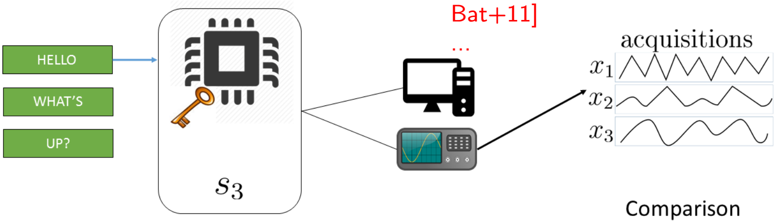


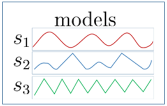
Advanced Side-Channel Attacks

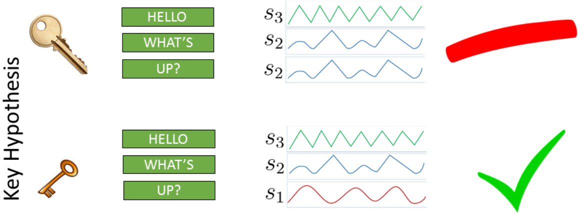
Differential Power Analysis [KJJ99]
Correlation Power Analysis [BCO04]
Mutual Information Analysis [Gie+08; Bat+11]
...



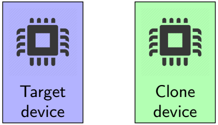
Comparison

Non-profiling attacks
Profiling attacks ...





Profiling Attacks...Supervised Learning



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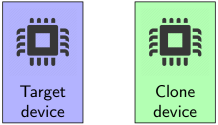

Machine Learning

Supervised Learning

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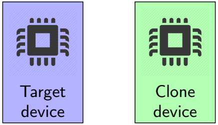
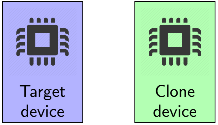


Machine Learning

"A computer program is said to learn from experience E with respect to some task T and performance measure P, if its performance on T, as measured by P, improves with experience E. "[TM97]

Supervised Learning

Profiling Attacks...Supervised Learning



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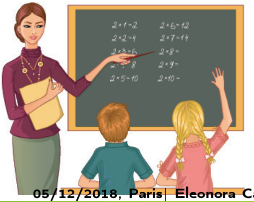
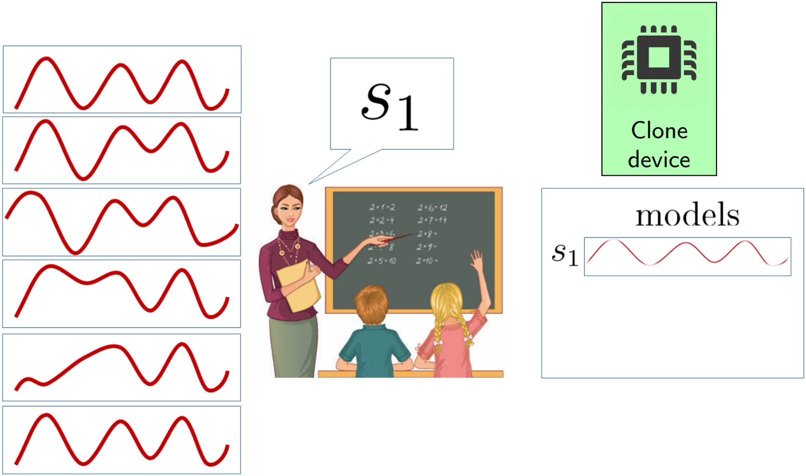
Supervised Learning

The *supervised* learning algorithms access to a dataset of examples, each associated in general to a *target* or *label*.



Classroom Side-Channel Attacks



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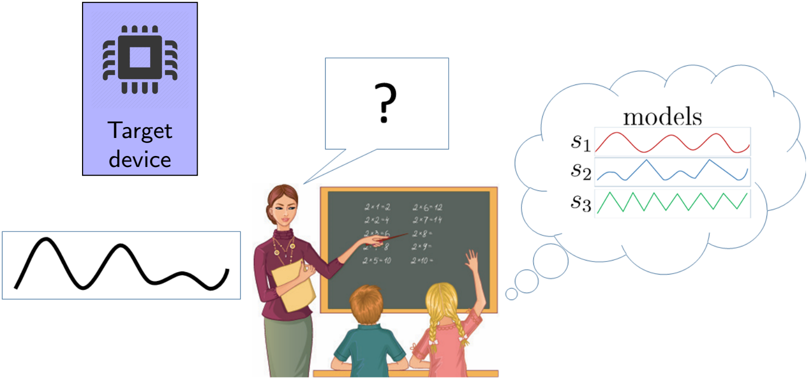
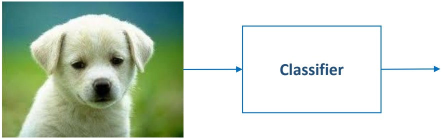


Classification

Classification problem

Assign to a datum $\vec{X}$ (e.g. an image) a label Z among a set of possible labels $\mathcal{Z} = \{\mathrm{Cat}, \mathrm{Dog}, \mathrm{Horse}\}$



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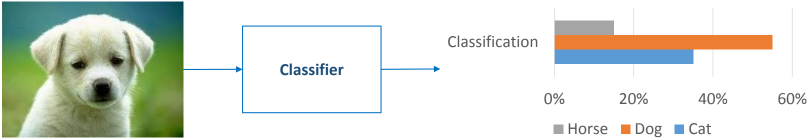

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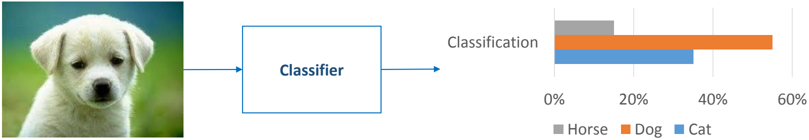

Advanced Attack as a Classification Problem

Classification

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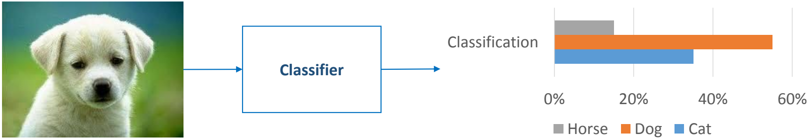
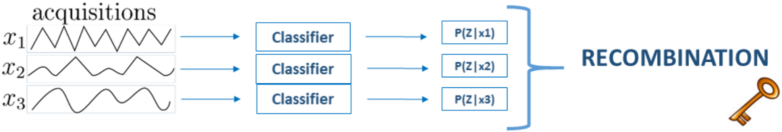


Advanced Attack as Multiple Classification Problems



Contents

Notations

Notations and generalities

- Side-channel traces: realizations of a random vector $\vec{X} \in \mathbb{R}^D$
- D is the number of time samples (or features)
- Target: a *sensitive* variable $Z = f(\mathrm{plaintext}, \mathrm{key})$ in $\mathcal{Z} = \{s_1, \ldots, s_{|\mathcal{Z}|}\}$

Profiling attack scenario

- labelled traces $\mathcal{D}_{\text{train}} = (\vec{x}_i, z_i)_{i=1}^{N}$, acquired under known Z, to characterise the signals
- attack traces $\mathcal{D}_{\text{attack}} = (\vec{x}_i, p_i)_{i=1}^{N_a}$ acquired under known plaintext

Profiling Attack

Profiling phase

- estimate
  - $p_{\vec{X} \mid Z=z}$

Attack phase

- Likelihood score for each key hypothesis k

$$d_k = \prod_{i=1}^{N_a} p_{\vec{X} \mid Z=f(p_i,k)}(\vec{x}_i)$$

Profiling Attack

Profiling phase

- estimate
  - $p_{\vec{X} \mid Z=z}$, $p_{\vec{X}}$, p_Z

Attack phase

- Likelihood score for each key hypothesis k

$$d_k = \prod_{i=1}^{N_a} p_{\vec{X} \mid Z=f(p_i,k)}(\vec{x}_i)$$

- A-posteriori probability score for each key hypothesis k

$$p_{Z \mid \vec{X}=\vec{x}}(z) = \frac{p_{\vec{X} \mid Z=z}(\vec{x}) p_Z(z)}{p_{\vec{X}}(\vec{x})} \text{Bayes' theorem}$$

$$d_k = \prod_{i=1}^{N_a} p_{Z \mid \vec{X}=\vec{x}_i}(f(k, e_i)) \ ,$$

Profiling Attack

Profiling phase

- estimate
  - $p_{\vec{X} \mid Z=z}$, $p_{\vec{X}}$, p_Z (generative model)
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Profiling Attack

$\vec{X} \in \mathbb{R}^D$

Curse of dimensionality!

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Profiling phase

- estimate
  - $p_{\vec{X} \mid Z=z}$, $p_{\vec{X}}$, p_Z (generative model)
    - Gaussian hypothesis (**Template Attack**) [CRR03]
    - Variants: *pooled* version [CK14], linear regression [SLP05]
  - $p_{Z \mid \vec{X}=\vec{x}}$ (discriminative model)

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Profiling Attack

$\vec{X} \in \mathbb{R}^D$
Curse of dimensionality!

Profiling phase

- mandatory dimensionality reduction $[\mathcal{D}_{\text{train}} \longrightarrow \epsilon\colon \mathbb{R}^D \to \mathbb{R}^C]$
- estimate
  - $p_{\epsilon(\vec{X}) \mid Z=z}$, $p_{\epsilon(\vec{X})}$, p_Z (generative model)
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Profiling Attack

$\vec{X} \in \mathbb{R}^D$

Curse of dimensionality!

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Objectives

Profiling phase

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- Ameliorate the template attack routine by proposing efficient dimensionality reduction techniques

Objectives

Profiling phase

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Objectives

- Ameliorate the template attack routine by proposing efficient dimensionality reduction techniques
- Consider the presence of most-commonly-implemented SCA countermeasures (masking, hiding)

Objectives

Profiling phase

- manage de-synchronization problem [$\mathcal{D}_{\text{train}} \longrightarrow \rho \colon \mathbb{R}^D \to \mathbb{R}^D$]
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  - $Z \mid \epsilon(\vec{X}) = \epsilon(\vec{x})$ (discriminative model)

Objectives

- Ameliorate the template attack routine by proposing efficient dimensionality reduction techniques
- More generally, ameliorate the profiling attack strategy
- Consider the presence of most-commonly-implemented SCA countermeasures (masking, hiding)

Objectives

Profiling phase

- manage de-synchronization problem $[\mathcal{D}_{\text{train}} \longrightarrow \rho\colon \mathbb{R}^D \to \mathbb{R}^D]$
- mandatory dimensionality reduction $[\mathcal{D}_{\text{train}} \longrightarrow \epsilon\colon \mathbb{R}^D \to \mathbb{R}^C]$

DEEP LEARNING

- estimate
  - $p_{\epsilon(\vec{X}) \mid Z=z}$, $p_{\epsilon(\vec{X})}$, p_Z (generative model)
    - Gaussian hypothesis (**Template Attack**) [CRR03]
    - Variants: *pooled* version [CK14], linear regression [SLP05]
  - $Z \mid \vec{X} = \vec{x}$ (discriminative model)

Objectives

- Ameliorate the template attack routine by proposing efficient dimensionality reduction techniques
- More generally, ameliorate the profiling attack strategy
- Consider the presence of most-commonly-implemented SCA countermeasures (masking, hiding)

Dimensionality Reduction: State of the Art

Dimensionality Reduction

$$\epsilon \colon \mathbb{R}^D \to \mathbb{R}^C$$
$$\vec{x} \mapsto \epsilon(\vec{x})$$

- Feature selection (Points of Interest selection)
- Feature extraction

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Feature selection

ϵ performs a sub-sampling

- SOD [CRR03]
- SOST [BDP10]
- SNR [MOP08]/ NICV [Bha+14]
- t-test, F-test,... [GLRP06; CK14]

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Linear feature extraction

$\epsilon(\vec{x}) = A\vec{x}$ with $A \in M_{\mathbb{R}}(C, D)$

- Principal Component Analysis (PCA) [Arc+06; BHW12]
- Linear Discriminant Analysis (LDA) [SA08; Bru+15]
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SNR, Fisher's criterion and LDA classifier

Feature selection - SNR

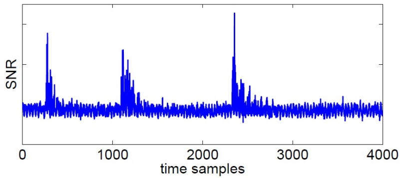

Figure: $SNR(t) = \frac{\textit{variance inter-class}}{\textit{variance intra-class}}$

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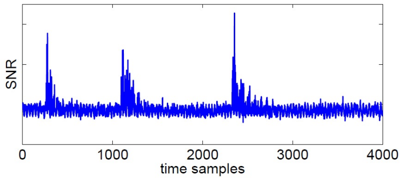

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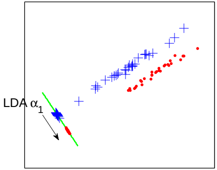

Figure: Fisher's Criterion: project into a subspace in which $\frac{\textit{inter-class covariance matrix}}{\textit{intra-class covariance matrix}}$ is maximised

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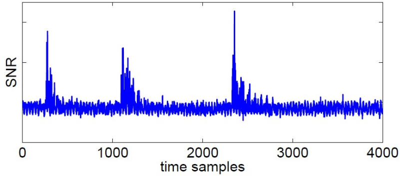

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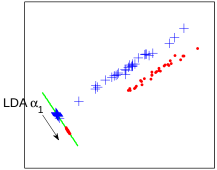

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LDA: optimal **classifier** under following hypothesis

- Gaussian distributions with parameters μ_j, Σ_j
- Homoscedasticity: $\Sigma_j = \Sigma$ for all j

Fisher's criterion $\Leftrightarrow$ LDA

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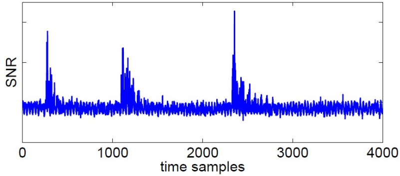

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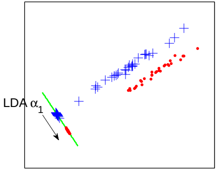

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SNR and LDA most suitable selector and extractor for classification purposes

Contributions

- **Linear Dimensionality Reduction**([CARDIS 2015]):
  - PCA, choise of components ELV
  - LDA in case of undersampling
- **Kernel Discriminant Analysis**([CARDIS 2016]): application of an appropriate kernel trick to LDA, in order to manage masking countermeasure
- **Convolutional Neural Networks**([CHES 2017]) :
  - discriminative model by means of neural network classifiers
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Contents

Dimensionality reduction in presence of masking

$(d-1)$th-order Sharing (or Masking)

Split each sensitive Z into shares $Z = M_1 \star \cdots \star M_d$
with $M_1, \ldots, M_{d-1}$ random shares (or masks)
and $M_d = Z \star M_1^{-1} \star \cdots \star M_{d-1}^{-1}$
Shares are handled at time samples

$$t_1, \ldots, t_d \qquad \text{(in general different if software countermeasure)}$$

Indistinguishability up to order $d-1$

$f(z) = \mathbb{E}[\vec{X}|Z = z]$ constant
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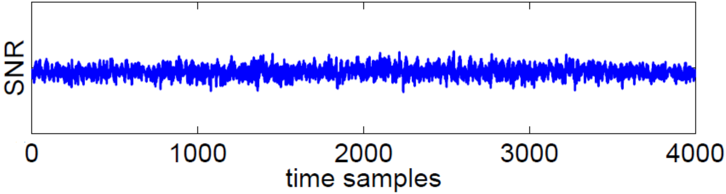
$\Rightarrow$ extract features containing $\vec{X}[t_1]\vec{X}[t_2]\dots\vec{X}[t_d]$ **(Necessary condition)**

How to detect the d-tuple $t_1, \ldots, t_d$?

$f(z) = \mathbb{E}[\vec{X}|Z = z]$ is constant $\Rightarrow$ Variance inter-class null! $\Rightarrow$ SNR, SOD, t-statistic null!



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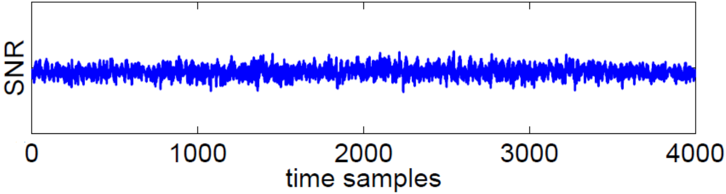

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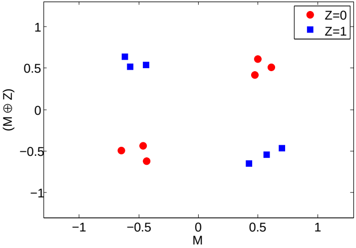

Toy example: 2 time samples, 1-bit data

t_1: $M + n$, $n \sim \mathcal{N}(0, 0.1)$

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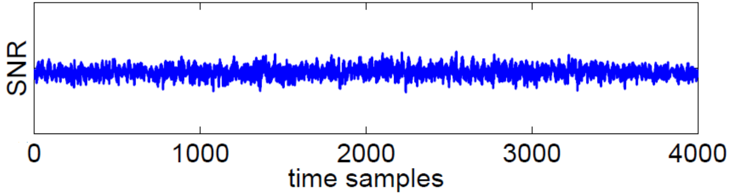

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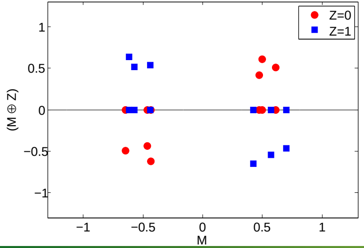

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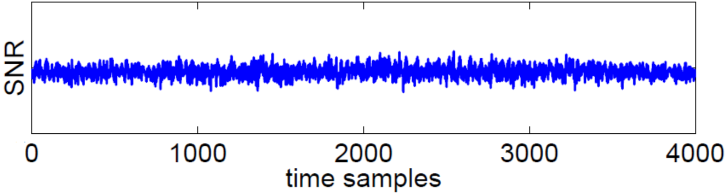

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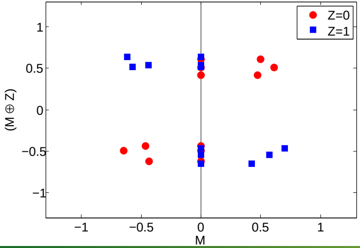

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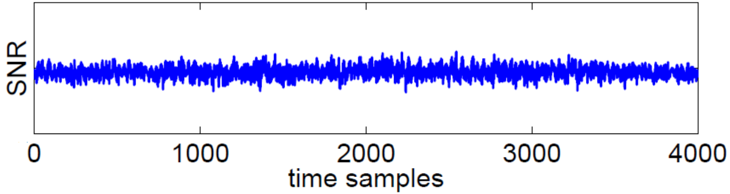

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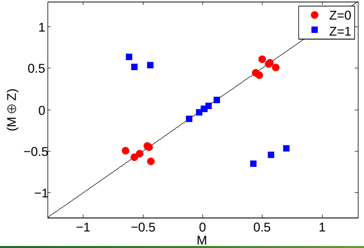

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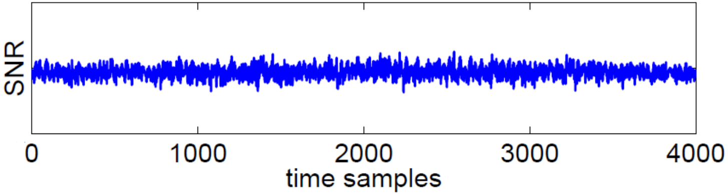

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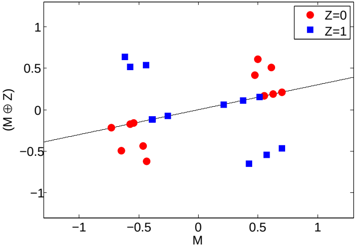

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PoIs Research

A lacking literature

- many HO attacks papers assume the knowledge of $t_1, \ldots, t_d$
- PoI research exploiting the random shares knowledge (back to unprotected case using $M_1, \ldots, M_d$ instead of Z)
- Hand selection via educated guess [Osw+06]
- Feature Selection for Higher-Order Attacks $\longrightarrow$ Projection Pursuits [Dur+15]

Kernel Discriminant Analysis starting point

Naive strategy: infer over all possible d-tuples

KDA: the purpose

Problem

Naive strategy: useful statistics lie in a high-dimensional *feature* space:

$$\mathcal{F} = \mathbb{R}^{\binom{D+d-1}{d}}$$

(all dth-degree monomials in the trace coordinates)

$$\begin{tikzcd}
\mathbb{R}^D \arrow[r, "\Phi"] \arrow[rr, bend left=60, "\epsilon^{\mathrm{KPCA}}"] \arrow[rr, bend right=60, "\epsilon^{\mathrm{KDA}}"'] & \mathcal{F} \arrow[r, shift left, "\epsilon^{\mathrm{PCA}}"] \arrow[r, shift right, "\epsilon^{\mathrm{LDA}}"'] & \mathbb{R}^C
\end{tikzcd}$$

Φ non-linear function

What KDA provides

KDA allows performing LDA in $\mathcal{F}$, remaining in $\mathbb{R}^D$.

KDA: an intuition

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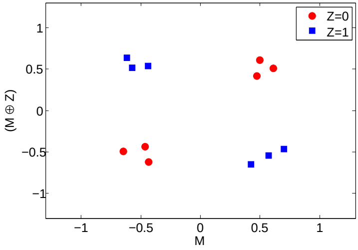

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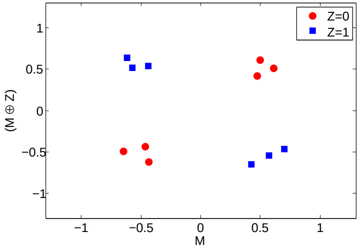

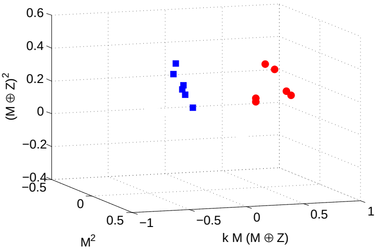

$$\Phi : \mathbb{R}^D \rightarrow \mathbb{R}^{\binom{D+d-1}{d}}$$
$$\Phi(t_1, t2) = (t_1^2, t_2^2, k t_1 t_2)$$

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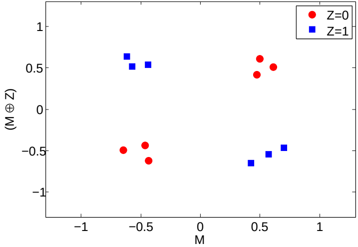

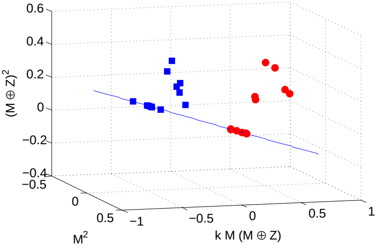

$\Phi \longrightarrow$ LDA

KDA: an intuition

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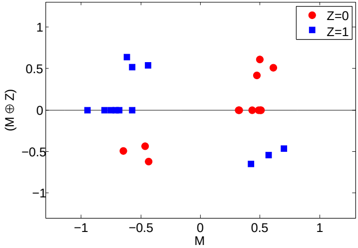

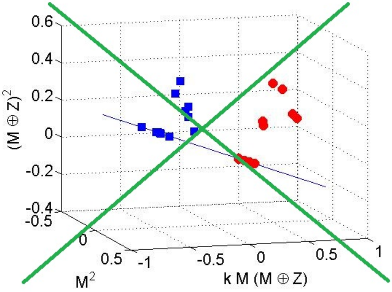

KDA
remains in $\mathbb{R}^D$

Kernel Function

$$\begin{tikzcd}
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Kernel Function

$K : \mathbb{R}^D \times \mathbb{R}^D \to \mathbb{R}$

$$K(\mathbf{x}_i, \mathbf{x}_j) = \Phi(\mathbf{x}_i) \cdot \Phi(\mathbf{x}_j) \tag{1}$$

Polynomial Kernel Function

$K(\mathbf{x}_i, \mathbf{x}_j) = (\mathbf{x}_i \cdot \mathbf{x}_j)^d \quad \leftrightarrow \quad \Phi : \mathbb{R}^D \to \mathcal{F} \subset \mathbb{R}^{\binom{D+d-1}{d}}$ all dth-degree monomials

KDA - the training

Between-class (inter-class) Covariance Matrix

LDA

- $\mathbf{S_B} = \sum_{z\in\mathcal{Z}} N_z(\vec{\mu}_s - \overline{\vec{x}})(\vec{\mu}_s - \overline{\vec{x}})^\intercal$

KDA

- $\mathbf{M} = \sum_{z\in\mathcal{Z}} N_z(\vec{M}_s - \vec{M}_T)(\vec{M}_s - \vec{M}_T)^\intercal$ [1]
-

[1] $\vec{M}_s$ and $\vec{M}_T$ are two N-sized column vectors whose entries are given by:
$\vec{M}_z[j] = \frac{1}{N_z}\sum_{i:z_i=z}^{N_z} K(\mathbf{x}_j^{z_j}, \mathbf{x}_i^{z_i})$, $\vec{M}_T[j] = \frac{1}{N}\sum_{i=1}^{N} K(\mathbf{x}_j^{z_j}, \mathbf{x}_i^{z_i})$.

[2] $\mathbf{I}$ is a $N_z \times N_z$ identity matrix, $\mathbf{I}_{N_z}$ is a $N_z \times N_z$ matrix with all entries equal to $\frac{1}{N_z}$ and $\mathbf{K}_z$ is the $N \times N_z$ sub-matrix of $\mathbf{K} = (K(\mathbf{x}_i^{z_i}, \mathbf{x}_j^{z_j}))_{\substack{i=1,\dots,N\\ j=1,\dots,N}}$ storing only columns indexed by the indices i such that $z_i = z$

KDA - the training

Within-class (intra-class) Covariance Matrix

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KDA

- $\mathbf{M} = \sum_{z \in \mathcal{Z}} N_z (\vec{M}_s - \vec{M}_T)(\vec{M}_s - \vec{M}_T)^\intercal$ [1]
- $\mathbf{N} = \sum_{z \in \mathcal{Z}} \mathbf{K}_z (\mathbf{I} - \mathbf{I}_{N_z}) \mathbf{K}_z^\intercal$ [2]
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KDA - the training

Eigenvector problem

Computational Complexity $O(D^3)$

LDA

- $\mathbf{S_B} = \sum_{z\in\mathcal{Z}} N_z(\vec{\mu}_s - \overline{\vec{x}})(\vec{\mu}_s - \overline{\vec{x}})^\intercal$
- $\mathbf{S_W} = \sum_{z\in\mathcal{Z}} \sum_{i=1}^{N_z} (\mathbf{x}_i^z - \vec{\mu}_s)(\mathbf{x}_i^z - \vec{\mu}_s)^\intercal$
- $\vec{\alpha}_i$ eigenvectors of $\mathbf{S_W}^{-1}\mathbf{S_B}$ $[D \times D]$

Computational Complexity $O(N^3)$

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$\vec{M}_z[j] = \frac{1}{N_z}\sum_{i:z_i=z}^{N_z} K(\mathbf{x}_j^{z_j}, \mathbf{x}_i^{z_i})$, $\quad \vec{M}_T[j] = \frac{1}{N}\sum_{i=1}^{N} K(\mathbf{x}_j^{z_j}, \mathbf{x}_i^{z_i})$.

[2] $\mathbf{I}$ is a $N_z \times N_z$ identity matrix, $\mathbf{I}_{N_z}$ is a $N_z \times N_z$ matrix with all entries equal to $\frac{1}{N_z}$ and $\mathbf{K}_z$ is the $N \times N_z$ sub-matrix of $\mathbf{K} = (K(\mathbf{x}_i^{z_i}, \mathbf{x}_j^{z_j}))_{\substack{i=1,\dots,N \\ j=1,\dots,N}}$ storing only columns indexed by the indices i such that $z_i = z$

KDA - the training

New trace projection

Computational Complexity $O(D^3)$

LDA

- $\mathbf{S_B} = \sum_{z\in\mathcal{Z}} N_z(\vec{\mu}_s - \overline{\vec{x}})(\vec{\mu}_s - \overline{\vec{x}})^\intercal$
- $\mathbf{S_W} = \sum_{z\in\mathcal{Z}} \sum_{i=1}^{N_z} (\mathbf{x}_i^z - \vec{\mu}_s)(\mathbf{x}_i^z - \vec{\mu}_s)^\intercal$
- $\vec{\alpha}_i$ eigenvectors of $\mathbf{S_W}^{-1}\mathbf{S_B}$ $[D \times D]$
- $\epsilon_\ell^{LDA}(\vec{x}) = \sum_{i=1}^{D} \vec{\alpha}_\ell[i]\vec{x}[i]$

Computational Complexity $O(N^3)$

KDA

- $\mathbf{M} = \sum_{z\in\mathcal{Z}} N_z(\vec{M}_s - \vec{M}_T)(\vec{M}_s - \vec{M}_T)^\intercal$ [1]
- $\mathbf{N} = \sum_{z\in\mathcal{Z}} \mathbf{K}_z(\mathbf{I} - \mathbf{I}_{N_z})\mathbf{K}_z^\intercal$ [2]
- $\vec{\nu}_i$ eigenvectors of $\mathbf{N}^{-1}\mathbf{M}$ $[N \times N]$
- $\epsilon_\ell^{\mathrm{KDA}}(\vec{x}) = \sum_{i=1}^{N} \vec{\nu}_\ell[i] K(\mathbf{x}_i^{z_i}, \mathbf{x})$

---

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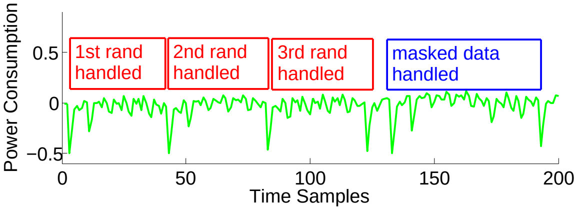
Experimental setup

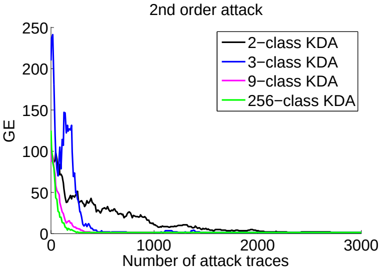
Target device and acquisitions:

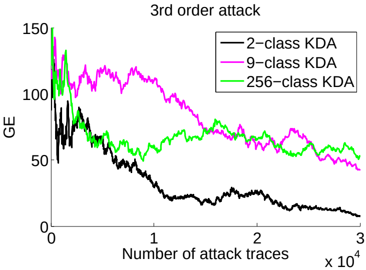
- 8-bit AVR microprocessor Atmega328P
- power-consumption acquired via the ChipWhisperer [OC14] platform
- $D = 200$, 4 clock-cycles are selected

Sensitive variable: $Z = \mathrm{Sbox}_{\mathrm{AES}}(P \oplus K^{\star})$



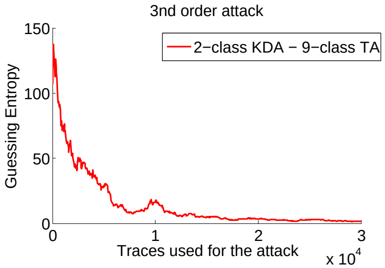
Second and third order

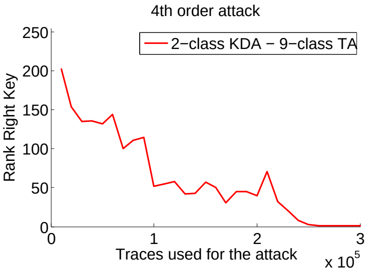




GE = Guessing Entropy (mean rank of the right key candidate)

Third and Fourth Order





- $d = 2 \quad \longrightarrow \quad \mathcal{F} = \mathbb{R}^{\binom{200+2-1}{2}} \Rightarrow$ 20.100 implicit coefficients
- $d = 3 \quad \longrightarrow \quad \mathcal{F} = \mathbb{R}^{\binom{200+3-1}{3}} \Rightarrow$ 1.353.400 implicit coefficients
- $d = 4 \quad \longrightarrow \quad \mathcal{F} = \mathbb{R}^{\binom{200+4-1}{4}} \Rightarrow$ 68.685.050 implicit coefficients

Same time of execution of the KDA algorithm!

Conclusions on KDA

Strong points

- KDA with d-th degree polynomial kernel function is suitable to attack $(d-1)$th-order masking
- KDA computational complexity is independent from the order d
- Tested and effective on a real case, positively compared to PP

	2nd order	3-rd order	4th order
KDA	✓	✓	✓
PP	✓	–	–

Limits and drawbacks

- Regularization hyper-parameter μ: $\mathbf{N} = \sum_{z\in\mathcal{Z}} \mathbf{K}_z(\mathbf{I} - \mathbf{I}_{N_z})\mathbf{K}_z^{\intercal} + \mu\mathbf{I}$
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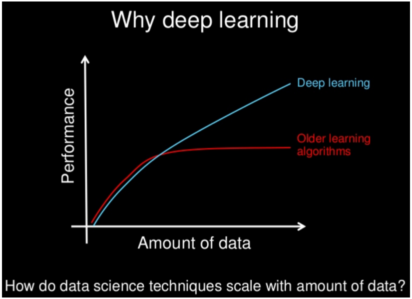
Contents

Motivations (1)



- parallelizable computation(GPU optimizations)
- not memory-based
- many hyper-parameters but faster validation

Motivations (2)

Profiling phase

- manage de-synchronization problem $[\mathcal{D}_{\text{train}} \longrightarrow \rho \colon \mathbb{R}^D \to \mathbb{R}^D]$
- mandatory dimensionality reduction $[\mathcal{D}_{\text{train}} \longrightarrow \epsilon \colon \mathbb{R}^D \to \mathbb{R}^C]$
- estimate
  - $p_{\epsilon(\rho(\vec{X})) \mid Z=z}$, $p_{\epsilon(\rho(\vec{X}))}$, p_Z (generative model)
    - Gaussian hypothesis (**Template Attack**)[CRR03]
    - Variants: *pooled* version [CK14], linear regression [SLP05]
- $p_{Z \mid \epsilon(\rho(\vec{x}))}$ (discriminative model)

Many independent preprocessing steps and assumptions

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Profiling phase

DEEP LEARNING

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Many independent preprocessing steps and assumptions
$\longleftrightarrow$ integrated and agnostic approach

Multi-Layer Perceptron

Multi-Layer Perceptron (MLP)

$$F(\vec{x}, W) = s \circ \lambda_n \circ \sigma_{n-1} \circ \lambda_{n-1} \circ \cdots \circ \lambda_1(\vec{x}) = \vec{y} \approx \Pr[Z | \vec{X} = \vec{x}]$$

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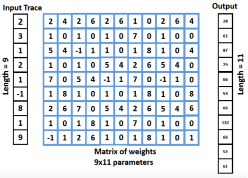
λ_i linear functions (linear combinations of time samples) depending on some **trainable weights** W



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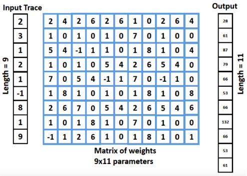

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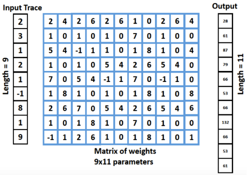

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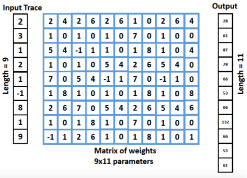

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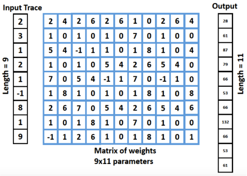
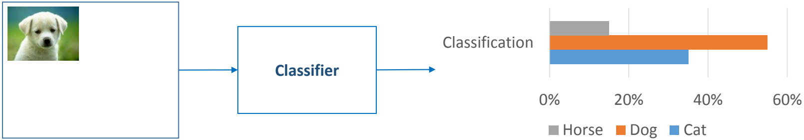


Universal approximation theorem

Convolutional Neural Networks

Translation-invariance



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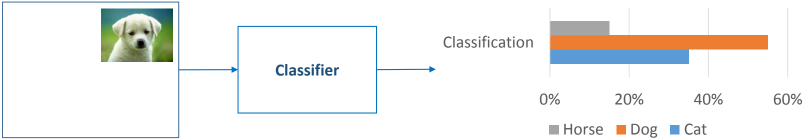

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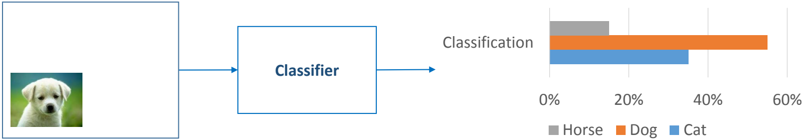

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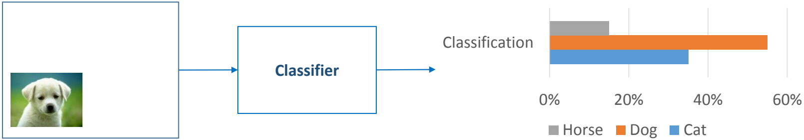

It is important to explicit the data translation-invariance

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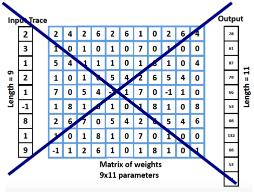

Figure: Linear layer in an MLP (*Fully Connected Layer*)

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Convolutional Neural Networks: share weights across space

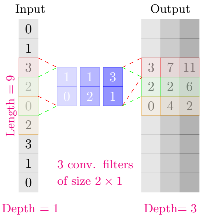
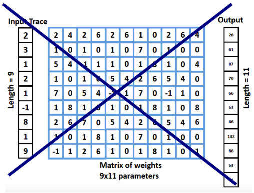


Figure: Linear layer in a ConvNet (*Convolutional Layer*)



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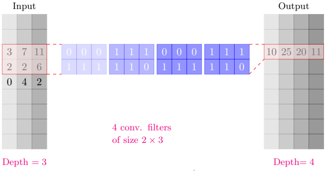

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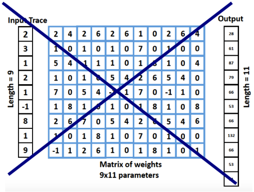

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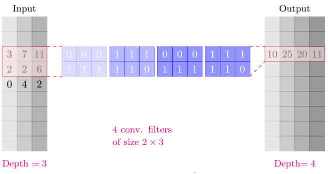

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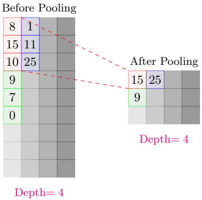
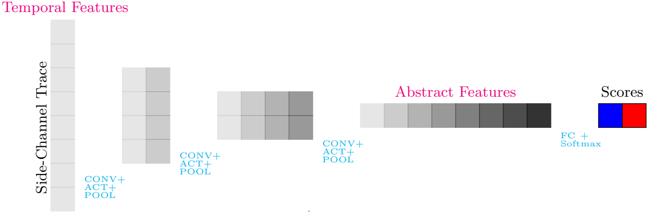


Figure: Max Pooling Layer

A kind of CNN architecture



VGG-like [SZ14]:

- Reduce temporal features to only one
- Maintain time complexity of each layer (one-half pooling when number of feature maps are doubled)
- Small filters

Model used in our experiments

- 4 Conv + Pool layers
- tanh activations
- batch normalisation [**batch_norm**]
- 1 *fully connected layer* + softmax

Training and overfitting

Training

Profiling set $\rightarrow \begin{cases} \text{Training set} \\ \text{Validation set} \end{cases}$

Randomly partition training set into batches

Iterative optimization algorithm over batches (cost function, stochastic gradient descent)

Epoch:= one pass over the entire training set

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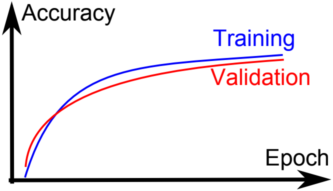
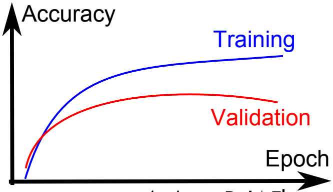
Evaluate and compare training and validation accuracy

Understand significant features



Learn by heart (**OVERFITTING**)



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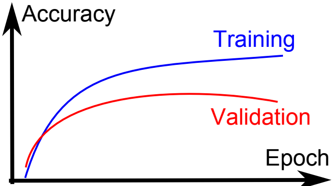
Why?

- Too complex model
- Not enough training data

Solution?

- Data augmentation

Learn by heart (**OVERFITTING**)



Data Augmentation

Data Augmentation

Artificially generate new training data by deforming those previously acquired,
Applying transformations that preserve the label Z

Countermeasure Emulation Idea

Emulate the effects of misaligning countermeasures to generate new traces

SHIFTING

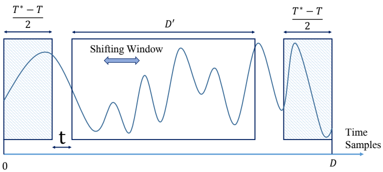


Figure: SH_T

ADD-REMOVE

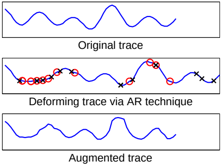


Figure: AR_R

Parameter T: ♯ of possible positions
Parameter R: ♯ of added and removed points
Data Augmentation techniques are applied online during training phase.

Experimental Results

- Random delays
- Artificial Jitter
- Real Jitter

Keras 1.2.1 library with Tensorflow backend [Cho+15] (open source, today 2.2.4)

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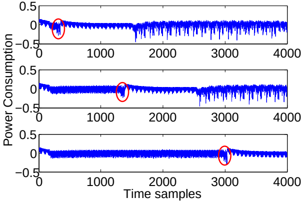


(a) One leaking operation

Setup

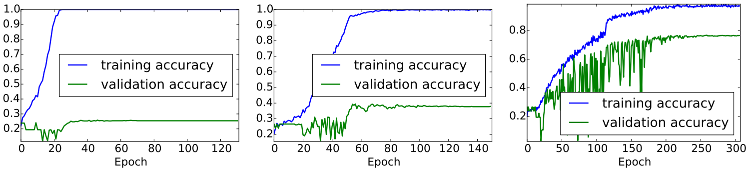
- Target Chip: Atmega328P
- Target Variable: $Z = \mathrm{HW}(\mathrm{Sbox}(P \oplus K))$
- Acquisition: through *ChipWhisperer*® platform, $\approx 4,000$ time samples
- Countermeasure: Random Delays - insertion of r *nop* operations, $r \in [0, 127]$ uniform random
- $1,000$ training traces

Random delays

Data augmentation vs overfitting

Metrics

- Test accuracy: classification accuracy over the attack traces
- $N^{\star}$: minimum number of attack traces to make *guessing entropy* of the right key permanently equal to one ($N^{\star}$ estimated over 10 independent attacks)



SH_0 SH_{100} SH_{500}

Random delays

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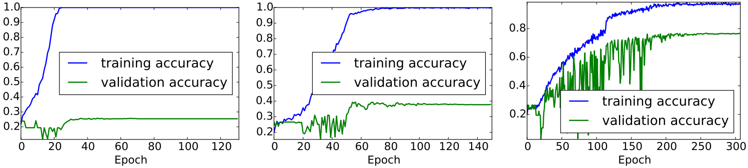


SH_0 SH_{100} SH_{500}

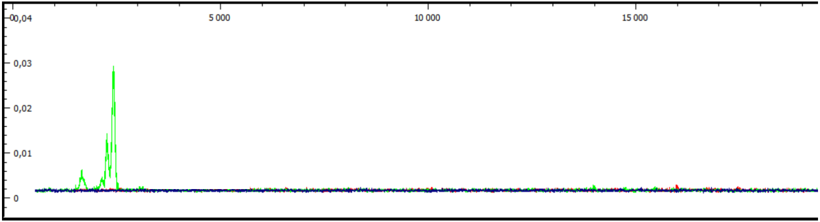
		SH_0		SH_{100}		SH_{500}	
Acc	$N^\star$	27.0%	$> 1,000$	31.8%	101	**78%**	**7**

Real Jitter (1)

Target

- AES hardware implementation
- strong jitter effect
- Target Variable: $Z = \mathrm{Sbox}(P \oplus K)$
- $2,500$ selected time samples
- $99,000$ training traces

SNR first Sbox



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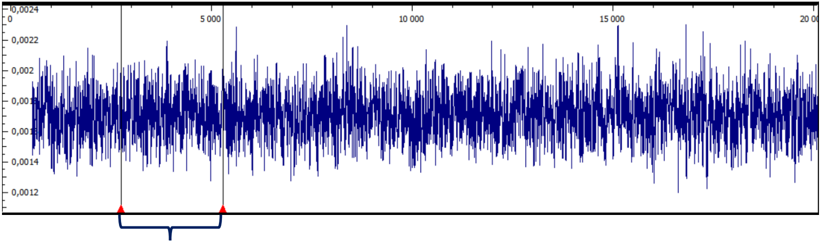
SNR second Sbox without realignment



Entry region for CNN (2,500 pts)

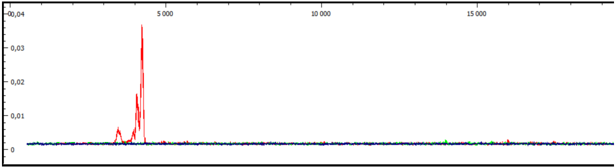
Real Jitter (2)

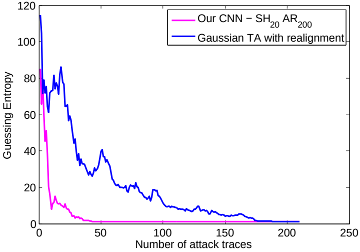
		SH_0AR_0		$SH_{10}AR_{100}$		$SH_{20}AR_{200}$	
Acc	$N^\star$	1.2%	137	1.3%	89	1.8%	54

Real Jitter (2)

		SH_0AR_0		$SH_{10}AR_{100}$		$SH_{20}AR_{200}$	
Acc	$N^\star$	1.2%	137	1.3%	89	1.8%	54

SNR second Sbox with realignment





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Contents

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  - Classification-oriented techniques for dimensionality reduction
  - LDA and KDA generalization to tackle masking countermeasure
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Today and in the future

- ASCAD database and SCA/DL community
- From CNN to PoIs, visualizing techniques
- Advanced-attack-oriented machine learning task (instead of multiple classification)
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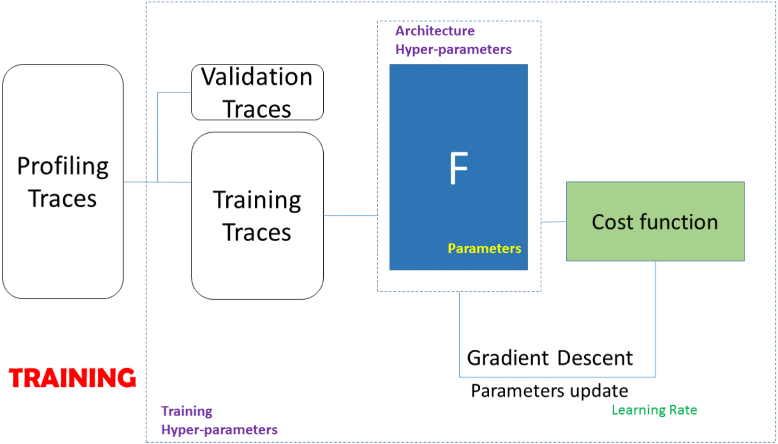
Training-Validation-Test



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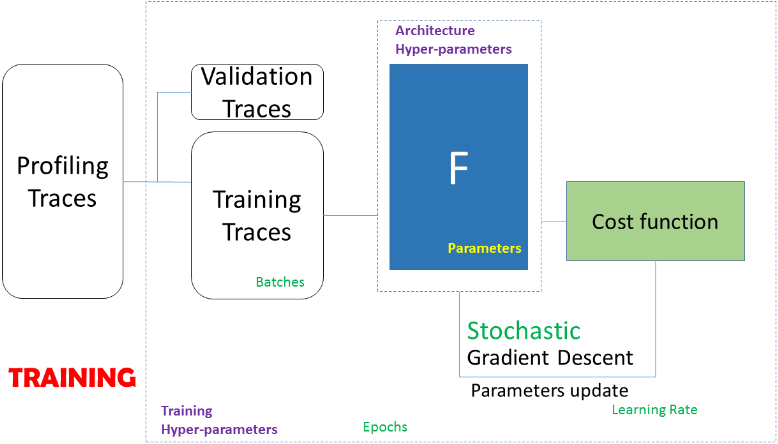
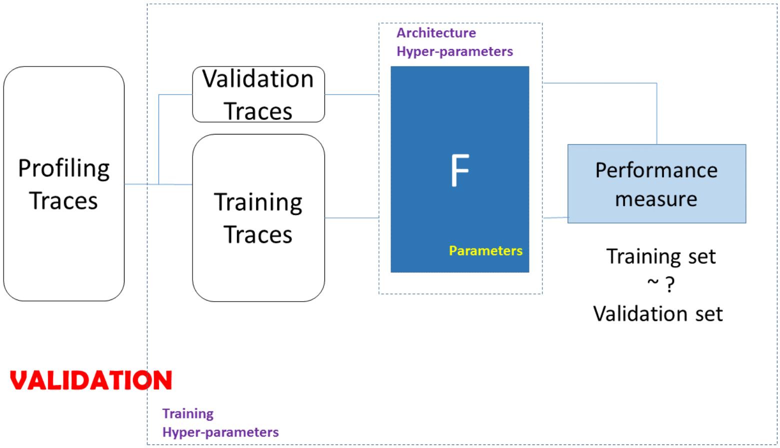


TRAINING

Training-Validation-Test



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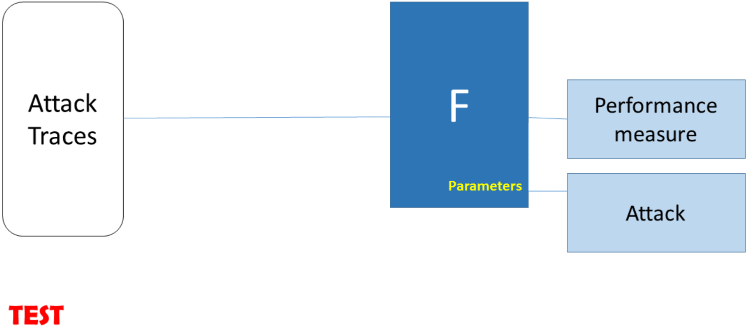


Cost function - Cross-entropy

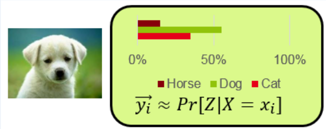
- batch of training data $(\vec{x}_i, z_i)_{i\in I}$, outputs of the current model $(\vec{y}_i)_{i\in I}$
- labels $z_i = s_j$ are *one-hot encoded*: $\vec{z}_i = \vec{s}_j = (0,\ldots,0,\underbrace{1}_{j},0,\ldots,0)$

Loss function

$$\mathcal{L} = -\frac{1}{|I|}\sum_{i\in I}\sum_{t=1}^{|\mathcal{Z}|}\vec{z}_i[t]\log\vec{y}_i[t] \tag{2}$$

Maximum-*a-posteriori* or Cross-entropy

- $\vec{y}_i \approx \Pr[Z \mid \vec{X} = \vec{x}_i]$



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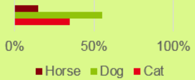
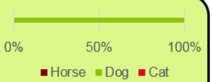
- $\vec{y}_i \approx \Pr[Z \mid \vec{X} = \vec{x}_i]$
- $\vec{z}_i \approx \Pr[Z \mid Z = \vec{s}_j]$
- $\mathbb{H}(\vec{z}_i, \vec{y}_i) = \mathbb{H}(\vec{z}_i) + D_{KL}(\vec{z}_i||\vec{y}_i) = \mathbb{E}_{\vec{z}_i}[-\log\vec{y}_i] = -\sum_{t=1}^{|\mathcal{Z}|}\vec{z}_i[t]\log\vec{y}_i[t]$



$\vec{y}_i \approx Pr[Z|X = x_i]$



$\vec{z}_i = Pr[Z|Z = s_j]$

Capacity-Overfitting-Regularization

Regression example

Performance metric: Mean Square Error (MSE)

MSE_train=44.228280, MSE_test=330.984916

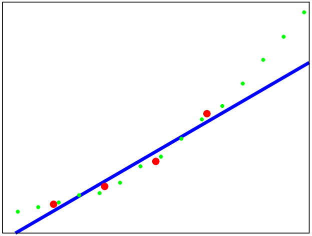


Figure: Linear regression $\rightarrow$ underfitting

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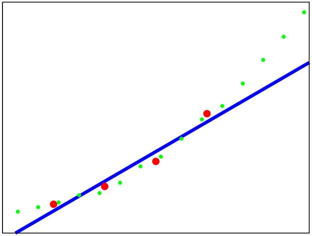

Figure: Linear regression → underfitting

MSE_train=2.243097, MSE_test=61.891672

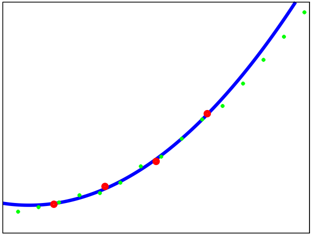

Figure: Quadratic regression → fits

Capacity-Overfitting-Regularization

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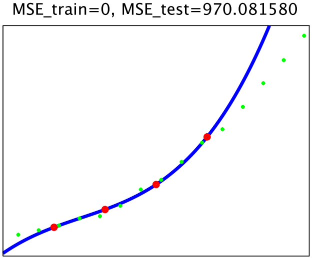
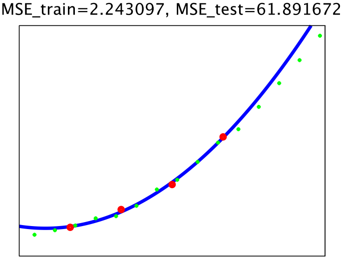


Figure: Cubic regression → overfitting



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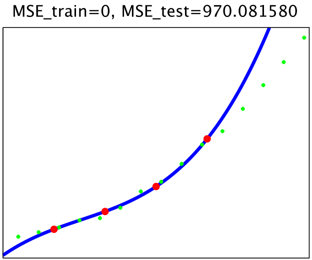

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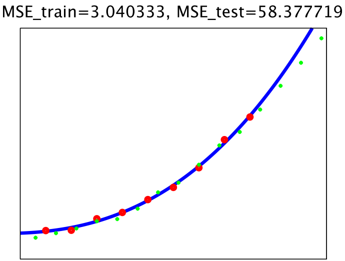


Figure: Cubic regression with more training data

Capacity-Overfitting-Regularization

Regression example

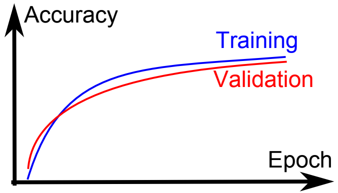
Performance metric: Mean Square Error (MSE)

Classification via Neural Network

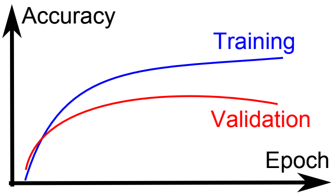
Performance measure: Accuracy (Classification rate)

Evaluate and compare training and validation accuracy

Understand significant features



Learn by heart (**OVERFITTING**)



Capacity-Overfitting-Regularization

Regression example

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Classification via Neural Network

Performance measure: Accuracy (Classification rate)
Evaluate and compare training and validation accuracy

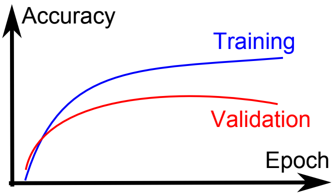
Learn by heart (**OVERFITTING**)

Why?

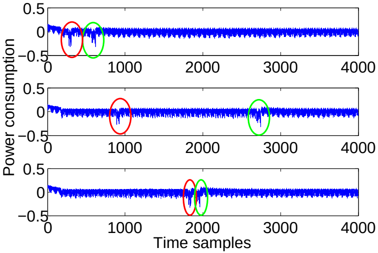
- Too complex model
- Not enough training data

Solution?

- Data augmentation



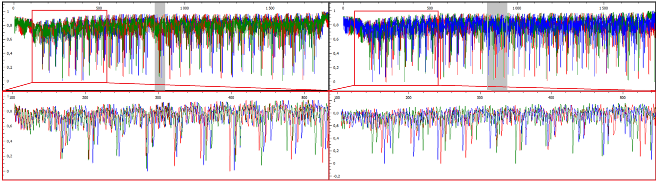
Random Delays - Two Leaking Operations



Two leaking operations

First operation - Test acc: 76.8%, $N^\star = 7$
Second operation - Test acc: 82.5%, $N^\star = 6$

Artificial Jitter



(a) Low artificial jitter (b) High artificial jitter

Target

- Target Variable: $Z = \mathrm{HW}(\mathrm{Sbox}(P \oplus K))$
- $\approx$ 2000 time samples
- Countermeasure: artificial signal treatment simulating clock jitter
- 10000 training traces

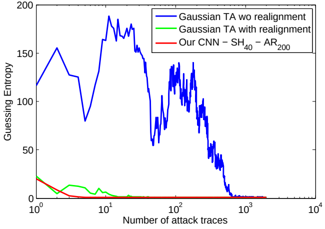
Artificial Jitter (2)

Low_jitter

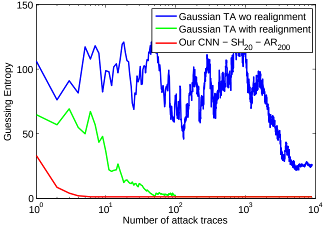
Acc / $N^\star$	SH_{0}		SH_{20}		SH_{40}	
AR_{0}	57.4%	14	82.5%	6	83.6%	6
AR_{100}	86.0%	6	87.0%	5	87.5%	6
AR_{200}	86.6%	6	85.7%	6	**87.7%**	5

High_jitter

Acc / $N^\star$	SH_{0}		SH_{20}		SH_{40}	
AR_{0}	40.6%	35	51.1%	9	62.4%	11
AR_{100}	50.2%	15	72.4%	11	73.5%	9
AR_{200}	64.0%	11	**75.5%**	8	74.4%	8



(c) Low Jitter



(d) High Jitter

Artificial Jitter

DS_low_jitter

a	b								
c	d								
		SH_0		SH_{20}		SH_{40}		SH_{200}	
AR_0		100.0%	68.7%	99.8%	86.1%	98.9%	84.1%		
		57.4%	14	82.5%	6	83.6%	6		
AR_{100}		87.7%	88.2%	82.4%	88.4%	81.9%	89.6%		
		86.0%	6	87.0%	5	87.5%	6		
AR_{200}		83.2%	88.6%	81.4%	86.9%	80.6%	88.9%		
		86.6%	6	85.7%	6	87.7%	5		
AR_{500}								85.0%	88.6%
								86.2%	5

DS_high_jitter

a	b								
c	d								
		SH_0		SH_{20}		SH_{40}		SH_{200}	
AR_0		100%	45.0%	100%	60.0%	98.5%	67.6%		
		40.6%	35	51.1%	9	62.4%	11		
AR_{100}		90.4%	57.3%	76.6%	73.6%	78.5%	76.4%		
		50.2%	15	72.4%	11	73.5%	9		
AR_{200}		83.1%	67.7%	82.0%	77.1%	82.6%	77.0%		
		64.0%	11	75.5%	8	74.4%	8		
AR_{500}								83.6%	73.4%
								68.2%	11