CSE 211: Discrete Mathematics **(Due: 03/01/22)**

Homework #3

Instructor: Dr. Zafeirakis Zafeirakopoulos
Assistant: Gizem Süngü

Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- The homeworks (both latex, pdf and/or source code files in a zip file) will be submitted into the course page of Teams.
- The latex, pdf or source code and zip files of the homeworks should be saved as "StudentId".{tex, pdf, c, py, cpp, java, zip}.
- If the answers of the homeworks have only calculations without any formula or any explanation -when needed- will get zero.

Problem 1: Relations (20 points)

Let $\mathbb{Z}$ be the set of all integers. Define relation R on $\mathbb{N}$ as follows.

$$\forall a, b \in \mathbb{N}, (a, b) \in R \text{ iff } \exists i \in \mathbb{Z}, \frac{a}{b} = 2^i$$

Prove that R is an equivalence relation. Show your work step by step.

Problem 2: Relations (15 points)

Define he relation R on $\mathbb{N}$ as

$$\forall c, d \in \mathbb{N}, (c, d) \in R \text{ iff } c + d \text{ is even.}$$

(a) Prove that R is an equivalence relation.
(b) How many equivalence classes does R have?

Problem 3: Relations (10 points)

Let R and S be two relations both on A. Show that
(a) If R and S are both reflexive, is R $\cap$ S also reflexive?
(b) If R and S are both symmetric, is R $\cap$ S also symmetric?
(c) If R and S are both transitive, is R $\cap$ S also transitive?
Note: Of course, yes or no answers are not acceptable. Show your work briefly.

Problem 4: Relations (15 points)

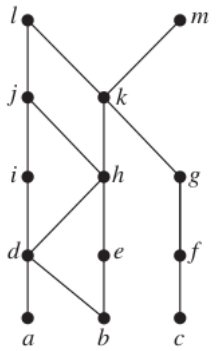
Give a poset that has
(a) a minimal element but no maximal element.
(b) a maximal element but no minimal element.
(c) neither a maximal nor a minimal element.

Problem 5: Relations (15 points)

Suppose that (S, $\preccurlyeq_1$) and (T , $\preccurlyeq_2$) are posets. Show that (S $\times$ T , $\preccurlyeq$) is a poset where (s, t) $\preccurlyeq$ (u, v) if and only if s $\preccurlyeq_1$ u and t $\preccurlyeq_2$ v. Show your work in details.

Problem 6: Relations (10 points)

Answer these questions for the partial order represented by this Hasse diagram.

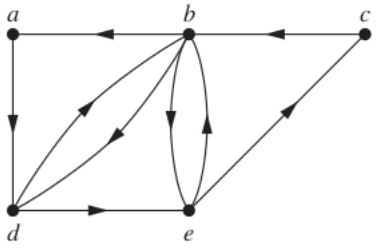


(a) Find the maximal elements.
(b) Find the minimal elements.
(c) Is there a greatest element?
(d) Is there a least element?
(e) Find all upper bounds of $\{a, b, c\}$.
(f) Find the least upper bound of $\{a, b, c\}$, if it exists.
(g) Find all lower bounds of $\{f, g, h\}$.
(h) Find the greatest lower bound of $\{f, g, h\}$, if it exists.

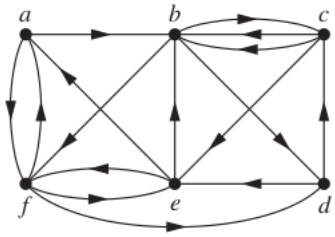
Problem 7: Graphs (10 points)

Determine whether the directed graph shown has an Euler circuit. Construct an Euler circuit if one exists. If no Euler circuit exists, determine whether the directed graph has an Euler path. Construct an Euler path if one exists.

(a)



(b)



Problem 8: Graphs (25 points)

Remember graph coloring problem in the problem session.
(a) Draw the graph of the following code block.
(b) Explain how you represent the components of the code in graph coloring problem.
(c) What is the minimum number of registers that you need to compile the code? Show which variables can be in the same register.

Line 1: a = 3

Line 2: b = 6

Line 3: c = 4

Line 4: d = b + c

Line 5: e = d + a

Line 6: if e $<$ 4 then f = 4 $\times$ b

Line 7: else then f = 6 $\times$ a

Line 8: d = f - c

Problem 9: Graphs (10 points)

Draw graphs which are given in adjaceny matrices as follows. Explain if these graphs are isomorphic?

$$G_1 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \qquad G_2 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$