STAT 534: Homework 3

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```
library(leaps)
library(gt)
library(tidyverse)
library(glmnet)
library(janitor)
library(MASS)
library(ISLR)
library(NHANES)
library(broom)
library(rsample)
library(caret)
library(GGally)
```

Exercise 1

In this problem, we will generate simulated data, and will then use this data to perform variable selection.

(a) Use the rnorm() function to generate a predictor X of length n = 100, as well as a noise vector ϵ of the same length. Then generate a response vector Y according to the model:

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 X^3 + \epsilon$$

where β_0,. . . , β_3 are constants of your choice.

```
x <- rnorm(100)
e <- rnorm(100)
```

```
y <- 9 + 1 * x + 2 * x^2 - 3 * x^3 + e
```

(b) Given the predictors X, X^2 , . . . , X^{10} , perform best subset selection in order to choose the best model. What is the best model obtained according to Cp, AIC, BIC and adjusted R^2 ? Show some plots to provide evidence for your answer, and report the coefficients of the best model obtained.

```
# Get AIC values
poly_degree <- seq(1, 10)

# Empty vector
aic <- double(length(poly_degree))

for (each_value in seq(along = poly_degree)) {
 k <- poly_degree[each_value]
```

```r
 # Polynomial Model
 aic_out <- lm(y ~ poly(x, k))

 # Assign it to vector
 aic[each_value] <- AIC(aic_out)
}

aic_values <- cbind(poly_degree,aic)
```

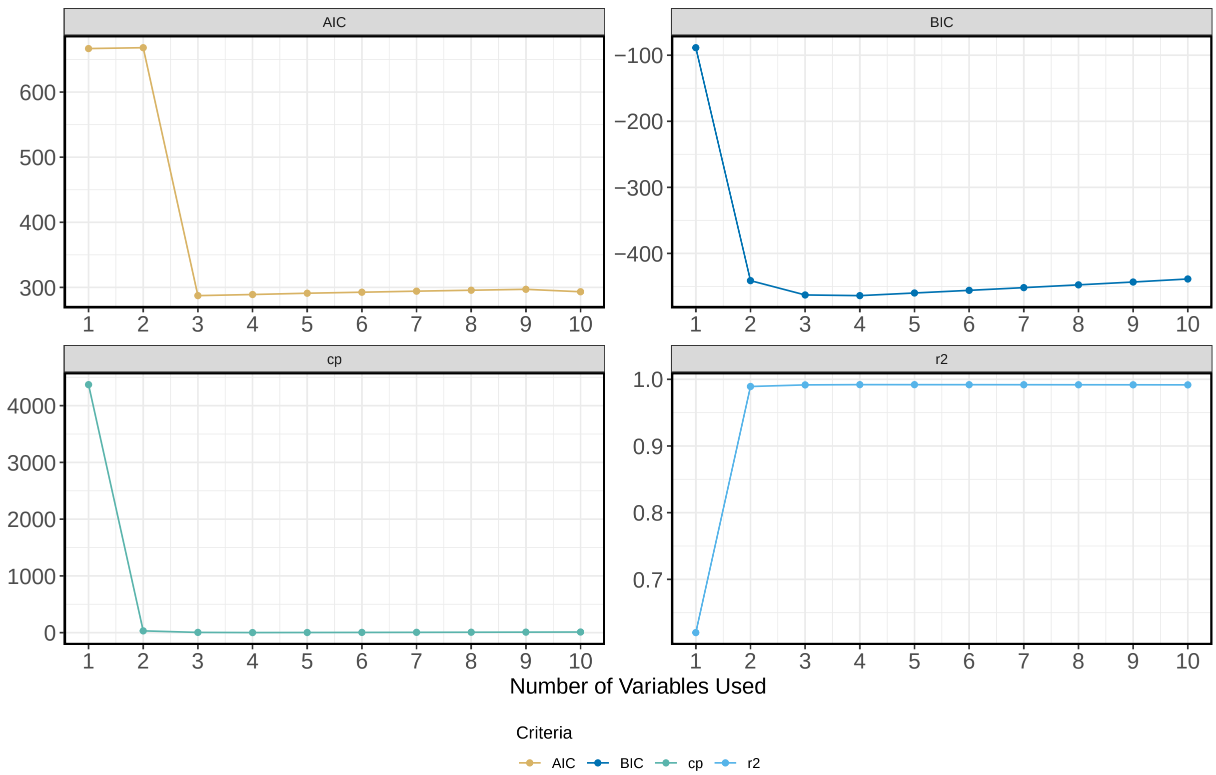
```r
data_from_linear_model <- data.frame(y,x)

# Generate polynomial regression up to 10
fit <- regsubsets(y ~ poly(x, 10), data = data_from_linear_model, nvmax = 10)
fit_summary <- summary(fit)
```

```r
# Choose best model
metrics <- data.frame(
  r2 = which.max(fit_summary$adjr2),
  cp = which.min(fit_summary$cp),
  BIC = which.min(fit_summary$bic),
  aic = which.min(aic_values[,2])
)
```

```r
# Generate data frame with metrics

data_model_selection <-
  data_frame(cp = fit_summary$cp,
             BIC = fit_summary$bic,
             r2 = fit_summary$adjr2) %>%

    # add aic values
    cbind(., aic_values[,2]) %>%
    rename(AIC = "aic_values[, 2]") %>%

    mutate(id = row_number())
```

```r
data_model_selection %>%

    #Transform to long format
    gather(value_type, value, -id) %>%
    ggplot(aes(id, value, col = value_type)) +
    geom_line() +
    geom_point() +
    ylab('') +
    xlab('Number of Variables Used') +
    facet_wrap(~ value_type, scales = 'free') +
    theme_bw() +
    scale_x_continuous(breaks = 1:10) +
    # Change color
    scale_colour_manual(values = c("#d8b365", "#0072B2", "#5ab4ac",
                                   "#56B4E9")) +

    # Edit the legend
    theme(axis.text.y   = element_text(size = 14),
                # Legend position and Axis size
```

```
                legend.position = "bottom",
                axis.text.x    = element_text(size = 14),
                      axis.title.y  = element_text(size = 14),
                      axis.title.x  = element_text(size = 14),
                # Add borders to the plot
                      panel.border = element_rect(colour = "black", fill= NA,size = 1.3)) +

      # Edit legend name
      labs(colour = "Criteria") +

      #Edit legend
      guides(col = guide_legend(override.aes = list(fill=NA),nrow = 1,title.position = "top",))
```



According to the plots, the model with 3 variables is the best

```
best_model <- lm(y ~ poly(x, 3), data = data_from_linear_model)
coef(best_model)
```

```
(Intercept) poly(x, 3)1 poly(x, 3)2 poly(x, 3)3
   8.906386  -84.865235   -5.308367  -64.945351
```

(c) Repeat (b), using forward selection. How does your answer compare to the results in (b)?

```
fit_forward <- regsubsets(y ~ poly(x, 10), data = data_from_linear_model,
                          nvmax = 10, method = "forward")
fit_summary_forward <- summary(fit_forward)
```

```
# Choose best model
metrics_forward <- data.frame(
```

```
  r2 = which.max(fit_summary_forward$adjr2),
  cp = which.min(fit_summary_forward$cp),
  BIC = which.min(fit_summary_forward$bic)
)
```

```
metrics %>% gt()
```

r2	cp	BIC	aic
4	4	4	3

```
metrics_forward %>% gt()
```

r2	cp	BIC
4	4	4

In this case, the forward procedure did not produced any different results

(d) Now fit a lasso model and use cross-validation to select the optimal values of λ. Create plots of the cross-validation error as a function f of λ. Report the resulting coefficient estimates, and discuss the results obtained

```
# Generate data

data_lasso <- data.frame(cbind(y,x,x^2,x^3,x^4,x^5,x^6,x^7,x^8,
                     x^9,x^10)) %>% clean_names()

x <- model.matrix(y ~. ,data_lasso)[, -1]
y <- data_lasso[,1]
```

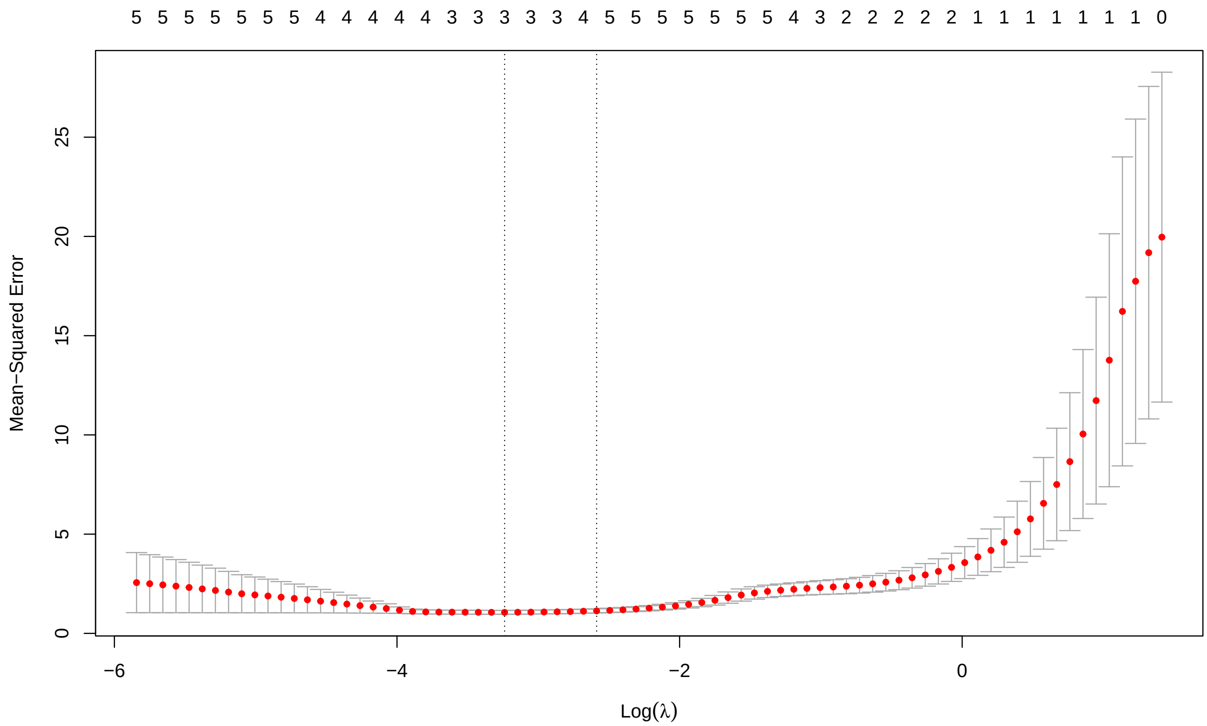
```
# split the samples into a training set and a test set

train <- sample(1:nrow(x),nrow(x) / 2)
test <- (-train)
y_test <- y[test]
```

```
#cross-validation to select the optimal values of
cv_out <- cv.glmnet(x[train,] , y[train], alpha = 1)
```

```
best_lamb <- cv_out$lambda.min
```

```
# Perform lasso regression
lasso_mod <- glmnet(x[train,] , y[train] , alpha = 1)
lasso_pred <- predict(lasso_mod , s = best_lamb ,newx = x[test,])
```

```
# Create plots of the cross-validation error as a function f of
plot(cv_out)
```




```
lasso_coef <- predict(lasso_mod, type = "coefficients",s = best_lamb)[1:11,]
lasso_coef
```

```
  (Intercept)             x            v3            v4            v5
 8.8151015699  0.0000000000  1.9114322659 -2.6892287065  0.0000000000
           v6            v7            v8            v9           v10
 0.0000000000  0.0000000000  0.0000000000  0.0000000000  0.0000000000
          v11
-0.0006294339
```

From the lasso model we can see that the variables greater than 0 have a significant effect over the y variable

Exercise 2

```
rm(x,y)
```

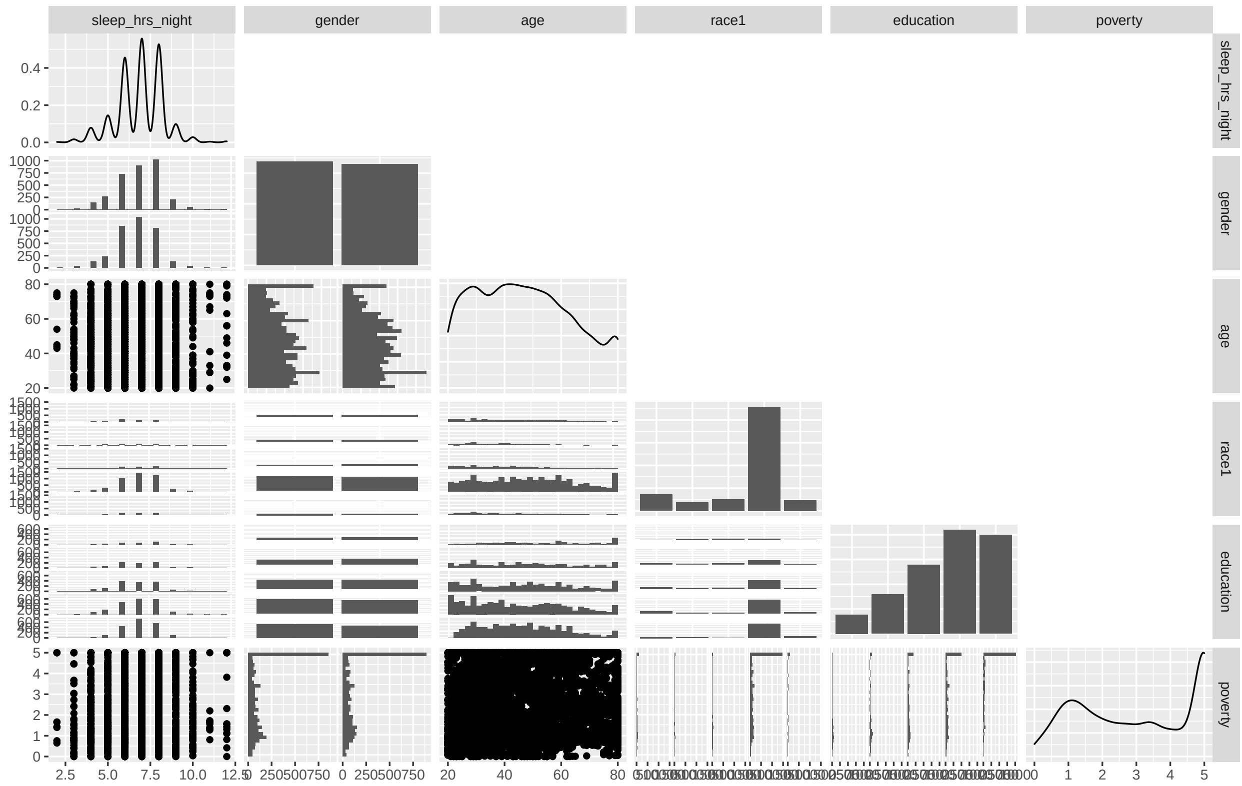
The ability to get a good night's sleep is correlated with many positive health outcomes. Use the NHANES data set from the NHANES package to predict *SleepHrsNight*. Check the R help document for detailed information about the data set.

```
# Load Data
data("NHANES")
data_p2 <-
    NHANES %>%
    clean_names()
```

(b) Select your own predictors, and create plots or summary tables to explore the variables.

```
data_selected_var <- data_p2 %>%
  dplyr::select(sleep_hrs_night,gender,age,race1,education,poverty) %>%
  na.omit()
```

```
ggpairs(data_selected_var,upper = "blank")
```



(a) First separate the data set at random into 75% training and 25% testing sets.

```
# Select X-Y for models
x_sleep <- model.matrix(sleep_hrs_night ~ . ,data_selected_var)[, -1]

y_sleep <- data_selected_var$sleep_hrs_night

# Sample data
index <-  sample(1:nrow(x_sleep), 0.75 *nrow(x_sleep))

# Create the training data 75%
train_data = x_sleep[index,]
(nrow(train_data)/6671)*100
```

```
[1] 74.99625
```

```
# Create the test data 25%
test_data = x_sleep[-index,]
(nrow(test_data)/6671)*100
```

```
[1] 25.00375
```

(c) Build the following models using the training set with your predictors of choice:

- Multiple linear regression

```
m1 <- lm(y_sleep[train] ~ x_sleep[train,])
```

- Ridge regression

```
# Model
m2_ridge <- glmnet(x_sleep[train, ], y_sleep[train] ,alpha = 0)

# Get best lambda
lambda_sleep_ridge <- cv.glmnet(x_sleep[train,], y_sleep[train], alpha = 0)

best_lamb_ridge <- lambda_sleep_ridge$lambda.min
```

- LASSO regression

```
# Model
m3_lasso <- glmnet(x_sleep[train , ], y_sleep[train] ,alpha = 1)


# Get best lambda
lambda_sleep_lasso <- cv.glmnet(x_sleep[train,], y_sleep[train], alpha = 1)

best_lamb_lasso <- lambda_sleep_lasso$lambda.min
```

(d) Compare the effectiveness of each model on training vs. testing data.

I found that the MSE were generally low, indicating the effectiveness of each model on training vs testing data

- Linear regression MSE

```
linear_prediction <- predict(m1,newx = x_sleep[test_data, ])

y_sleep_test <- y_sleep[test_data]
mean((linear_prediction - y_sleep_test)^2)
```

```
[1] 5.611041
```

- Ridge MSE

```
ridge_pred <- predict(m2_ridge , s = best_lamb_ridge, newx = x_sleep[test_data,])
mean((ridge_pred - y_test)^2)
```

```
[1] 217.6218
```

- Lasso MSE

```
lasso_pred <- predict(m3_lasso , s = best_lamb_lasso, newx = x_sleep[test_data,])
mean((lasso_pred - y_test)^2)
```

```
[1] 217.9877
```

(e) Choose one best model and interpret the results. What have you learned about people's sleeping quality?

Based on the MSE I chose the Multiple linear regression model because it has the lowest MSE(~5)

```
model <- lm(sleep_hrs_night ~ .,  data = data_selected_var )
```

```
summary(model)
```

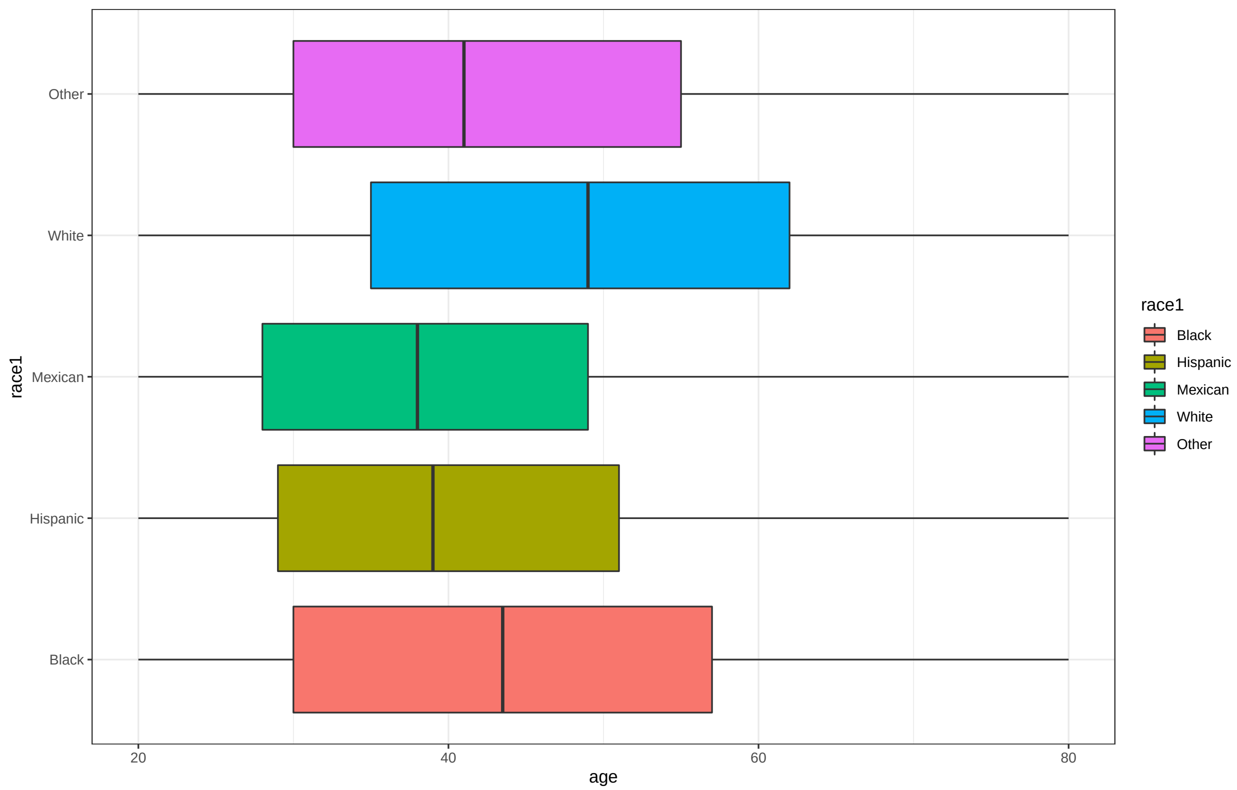
```
Call:
lm(formula = sleep_hrs_night ~ ., data = data_selected_var)

Residuals:
    Min      1Q  Median      3Q     Max 
-5.1653 -0.8528  0.0466  0.9925  5.2933 

Coefficients:
                         Estimate Std. Error t value Pr(>|t|)    
(Intercept)              6.459462   0.105150  61.431  < 2e-16 ***
gendermale              -0.172464   0.032651  -5.282 1.32e-07 ***
age                      0.002964   0.001010   2.934  0.00336 ** 
race1Hispanic            0.211124   0.085156   2.479  0.01319 *  
race1Mexican             0.413867   0.078782   5.253 1.54e-07 ***
race1White               0.331176   0.054131   6.118 1.00e-09 ***
race1Other               0.214679   0.078972   2.718  0.00658 ** 
education9 - 11th Grade -0.102837   0.084828  -1.212  0.22544    
educationHigh School    -0.085695   0.080851  -1.060  0.28922    
educationSome College    0.007952   0.080063   0.099  0.92089    
educationCollege Grad    0.108830   0.083642   1.301  0.19326    
poverty                  0.028880   0.011520   2.507  0.01220 *  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.326 on 6659 degrees of freedom
Multiple R-squared:  0.02086,	Adjusted R-squared:  0.01924 
F-statistic:  12.9 on 11 and 6659 DF,  p-value: < 2.2e-16
```

```r
anova(model)
```

```
Analysis of Variance Table

Response: sleep_hrs_night
            Df  Sum Sq Mean Sq F value    Pr(>F)    
gender       1    48.4  48.402 27.5318 1.593e-07 ***
age          1    25.0  24.988 14.2135 0.0001646 ***
race1        4   103.0  25.746 14.6450 6.497e-12 ***
education    4    62.0  15.493  8.8127 4.301e-07 ***
poverty      1    11.0  11.048  6.2843 0.0122046 *  
Residuals 6659 11706.7   1.758                      
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

From the variables selected, all had an effect over the amount of sleeping hours. For example it seems non-white people sleeps less when compared to white people.

```r
ggplot(data = data_selected_var, aes(x = age, y = race1, fill = race1))+
  geom_boxplot() + theme_bw()
```


Other
White
Mexican
Hispanic
Black
race1
20
40
60
80
age
race1
Black
Hispanic
Mexican
White
Other