CS 61A Fall 2023

Linked Lists, Efficiency

Discussion 8: October 18, 2023

Getting Started

Someone from your group should join Discord.

To get help from a TA, send a message to the `discuss-queue` channel with the @discuss tag and your discussion group number.

If you have only 1 or 2 people in your group, you can join the other group in the room with you.

Pro tip: Groups tend not to ask for help unless they've been stuck for a looooooong time. Try asking for help sooner. We're pretty helpful! You might learn something.

Q1: Ice Breaker

Everybody say your name, and then figure out who's birthday is coming up soonest. They're your group's facilitator for the day. You'll find a few instructions for facilitators, such as…

Facilitator: Ask someone to join Discord and post the first names of the members of your group to your group's Discord channel's text chat.

(If the facilitator doesn't want to be facilitator anymore or has to leave, they are always free to ask someone else to do it.)

Linked Lists

A linked list is a `Link` object or `Link.empty`. You can mutate a `Link` object `s` in two ways: - Change the first element with `s.first = ...` - Change the rest of the elements with `s.rest = ...`

```
class Link:
    """A linked list is either a Link object or Link.empty

    >>> s = Link(3, Link(4, Link(5)))
    >>> s.rest
    Link(4, Link(5))
    >>> s.rest.rest.rest is Link.empty
    True
    >>> s.rest.first * 2
    8
    >>> print(s)
    <3 4 5>
    """
    empty = ()

    def __init__(self, first, rest=empty):
        assert rest is Link.empty or isinstance(rest, Link)
        self.first = first
        self.rest = rest

    def __repr__(self):
        if self.rest:
            rest_repr = ', ' + repr(self.rest)
        else:
            rest_repr = ''
        return 'Link(' + repr(self.first) + rest_repr + ')'

    def __str__(self):
        string = '<'
        while self.rest is not Link.empty:
            string += str(self.first) + ' '
            self = self.rest
        return string + str(self.first) + '>'
```

Facilitator: Pick a way for your group to draw diagrams. Paper, a whiteboard, or a tablet, are all fine. If you don't have anything like that, ask the other group in the room if they have extra paper.

Q2: Strange Loop

In lab, there was a `Link` object with a cycle that represented an infinite repeating list of 1's.

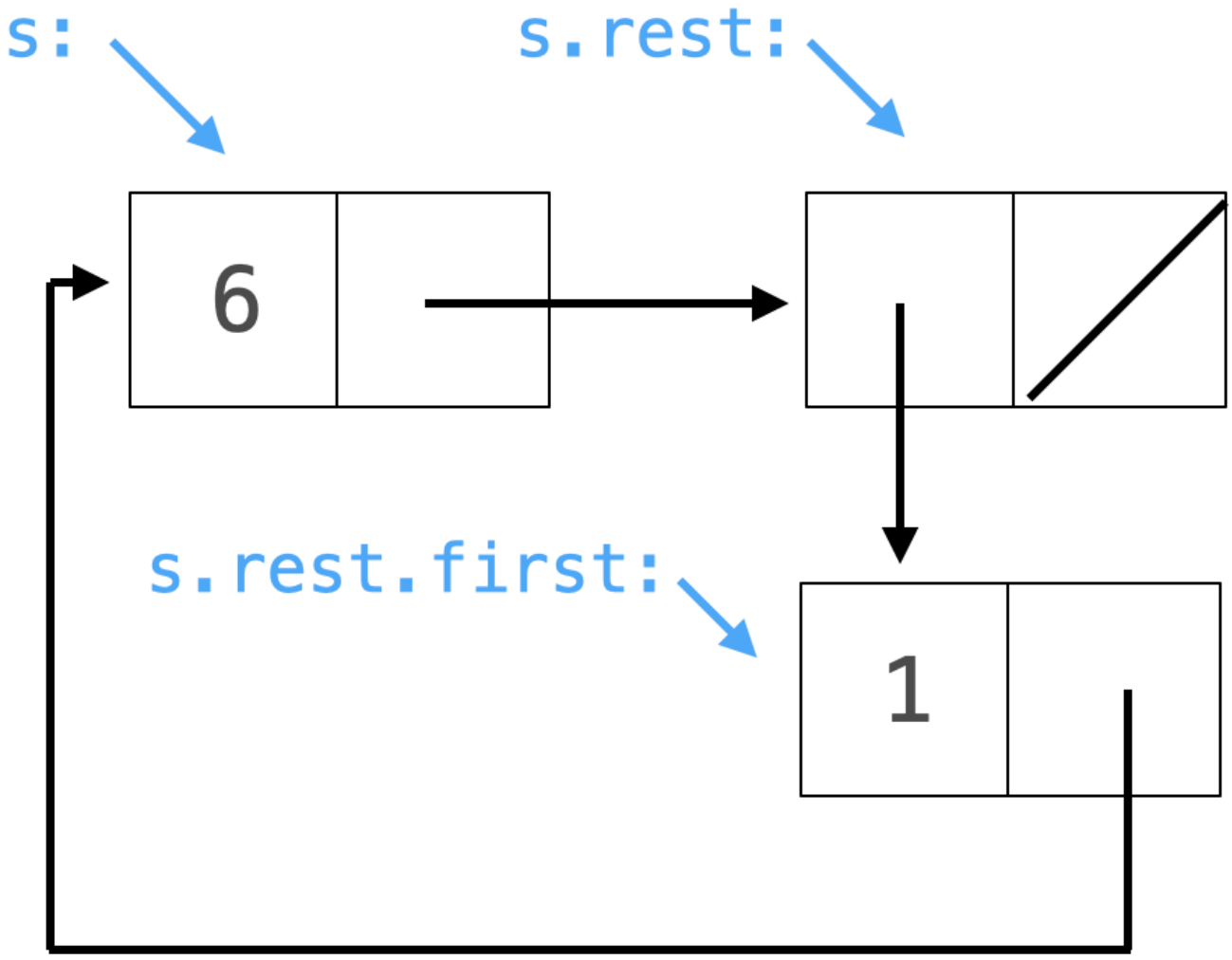
```
>>> ones = Link(1)
>>> ones.rest = ones
>>> [ones.first, ones.rest.first, ones.rest.rest.first, ones.rest.rest.rest.first]
[1, 1, 1, 1]
>>> ones.rest is ones
True
```

Implement `strange_loop`, which takes no arguments and returns a `Link` object `s` for which `s.rest.first.rest` is `s`.

Draw a picture of the linked list you want to create, then write code to create it.

Facilitator: When you think everyone has had a chance to read this far, please say: "So, what is this thing going to look like?"

For `s.rest.first.rest` to exist at all, the second element of `s`, called `s.rest.first`, must itself be a linked list.



Making a cycle requires two steps: making a linked list without a cycle, then modifying it. First create, for example, `s = Link(6, Link(Link(1)))`, then change `s.rest.first.rest` to create the cycle.

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.

```
def strange_loop():
    """Return a Link s for which s.rest.first.rest is s.

    >>> s = strange_loop()
    >>> s.rest.first.rest is s
    True
    """
    s = Link(1, Link(Link(2)))
    s.rest.first.rest = s
    return s
```

Facilitator: Ask someone to post a photo of the picture your group drew to your group's Discord channel's text chat.

Q3: Sum Two Ways

Implement both `sum_rec` and `sum_iter`. Each one takes a linked list of numbers `s` and returns the sum of its elements. Use recursion to implement `sum_rec`. Don't use recursion to implement `sum_iter`; use a `while` loop instead.

Facilitator: Tell the group which one to start with. It's your choice. You can say: "Let's start with the recursive version."

```
def sum_rec(s):
    """
    Returns the sum of the elements in s.

    >>> a = Link(1, Link(6, Link(7)))
    >>> sum_rec(a)
    14
    >>> sum_rec(Link.empty)
    0
    """
    # Use a recursive call to sum_rec
    if s == Link.empty:
        return 0
    return s.first + sum_rec(s.rest)

def sum_iter(s):
    """
    Returns the sum of the elements in s.

    >>> a = Link(1, Link(6, Link(7)))
    >>> sum_iter(a)
    14
    >>> sum_iter(Link.empty)
    0
    """
    # Don't call sum_rec or sum_iter
    total = 0
    while s != Link.empty:
        total, s = total + s.first, s.rest
    return total
```

Add `s.first` to the sum of the elements in `s.rest`. Your base case condition should be `s is Link.empty` so that you're checking whether `s` is empty before ever evaluating `s.first` or `s.rest`.

Introduce a new name, such as `total`, then repeatedly (in a `while` loop) add `s.first` to `total` and set `s = s.rest` to advance through the linked list, as long as `s is not Link.empty`.

Discussion time: When adding up numbers, the intermediate sums depend on the order.

$(1 + 3) + 5$ and $1 + (3 + 5)$ both equal 9, but the first one makes 4 along the way while the second makes 8 along the way. For the same linked list, will `sum_rec` and `sum_iter` both make the same intermediate sums along the way? Answer in your group's Discord channel's text chat. If yes, post "Same way all day." If no, post "Sum thing is

different."

Facilitator: After that discussion, please ask: "Does anyone have questions about how these functions work before we move on?" (If there are questions, try to answer them as a group, but call a TA if you want some more guidance.)

Q4: Overlap

Implement `overlap`, which takes two linked lists of numbers called `s` and `t` that are sorted in increasing order and have no repeated elements within each list. It returns the count of how many numbers appear in both lists.

This can be done in *linear* time in the combined length of `s` and `t` by always advancing forward in the linked list whose first element is smallest until both first elements are equal (add one to the count and advance both) or one list is empty (time to return). Here's a lecture video clip about this (but the video uses Python lists instead of linked lists).

Facilitator: Take a vote to decide whether to use recursion or iteration. Either way works (and the solutions are about the same complexity/difficulty).

```
def overlap(s, t):
    """For increasing s and t, count the numbers that appear in both.

    >>> a = Link(3, Link(4, Link(6, Link(7, Link(9, Link(10))))))
    >>> b = Link(1, Link(3, Link(5, Link(7, Link(8)))))
    >>> overlap(a, b)  # 3 and 7
    2
    >>> overlap(a.rest, b)  # just 7
    1
    >>> overlap(Link(0, a), Link(0, b))
    3
    """
    if s is Link.empty or t is Link.empty:
        return 0
    if s.first == t.first:
        return 1 + overlap(s.rest, t.rest)
    elif s.first < t.first:
        return overlap(s.rest, t)
    elif s.first > t.first:
        return overlap(s, t.rest)
```

```
    if s is Link.empty or t is Link.empty:
        return 0
    if s.first == t.first:
        return ________________
    elif s.first < t.first:
        return ________________
    elif s.first > t.first:
        return ________________
```

```
    k = 0
    while s is not Link.empty and t is not Link.empty:
        if s.first == t.first:
            ------------------
        elif s.first < t.first:
            ------------------
        elif s.first > t.first:
            ------------------
    return k
```

Q5: Overlap Growth

The alternative implementation of `overlap` below does not assume that `s` and `t` are sorted in increasing order. What is the order of growth of its run time in terms of the length of `s` and `t`, assuming they have the same length? Choose among: *constant*, *logarithmic*, *linear*, *quadratic*, or *exponential*.

```
def length(s):
    if s is Link.empty:
        return 0
    else:
        return 1 + length(s.rest)

def filter_link(f, s):
    if s is Link.empty:
        return s
    else:
        frest = filter_link(f, s.rest)
        if f(s.first):
            return Link(s.first, frest)
        else:
            return frest

def contained_in(s):
    def f(s, x):
        if s is Link.empty:
            return False
        else:
            return s.first == x or f(s.rest, x)
    return lambda x: f(s, x)

def overlap(s, t):
    """For s and t with no repeats, count the numbers that appear in both.

    >>> a = Link(3, Link(4, Link(6, Link(7, Link(9, Link(10))))))
    >>> b = Link(1, Link(3, Link(5, Link(7, Link(8, Link(12))))))
    >>> overlap(a, b)  # 3 and 7
    2
    >>> overlap(a.rest, b.rest)  # just 7
    1
    >>> overlap(Link(0, a), Link(0, b))
    3
    """
    return length(filter_link(contained_in(t), s))
```

Presentation time: Once your group agrees on an answer (or wants help), call a TA to explain your answer. Send a message to the `#discuss-queue` channel with the `@discuss` tag, your discussion group number, and the message "Over here!" and a member of the course staff will join your voice channel to hear your explanation and give feedback.

Document the Occasion

Please all fill out the attendance form (one submission per person per week).

Extra Challenge

This last question is meant to be similar to an A+ question on an exam. Feel free to skip it.

Q6: Decimal Expansion

Definition. The *decimal expansion* of a fraction `n/d` with `n < d` is an infinite sequence of digits starting with the 0 before the decimal point and followed by digits that represent the tenths, hundredths, and thousands place (and so on) of the number `n/d`. E.g., the decimal expansion of 2/3 is a zero followed by an infinite sequence of 6's: 0.6666666….

Implement `divide`, which takes positive integers `n` and `d` with `n < d`. It returns a linked list with a cycle containing the digits of the infinite decimal expansion of `n/d`. The provided `display` function prints the first `k` digits after the decimal point.

For example, 1/22 would be represented as `x` below:

```
>>> 1/22
0.045454545454545456
>>> x = Link(0, Link(0, Link(4, Link(5))))
>>> x.rest.rest.rest.rest = x.rest.rest
>>> display(x, 20)
0.04545454545454545454...
```

```
def divide(n, d):
    """Return a linked list with a cycle containing the digits of n/d.

    >>> display(divide(5, 6))
    0.8333333333...
    >>> display(divide(2, 7))
    0.2857142857...
    >>> display(divide(1, 2500))
    0.0004000000...
    >>> display(divide(3, 11))
    0.2727272727...
    >>> display(divide(3, 99))
    0.0303030303...
    >>> display(divide(2, 31), 50)
    0.06451612903225806451612903225806451612903225806451...
    """
    assert n > 0 and n < d
    result = Link(0)  # The zero before the decimal point
    cache = {}
    tail = result
    while n not in cache:
        q, r = 10 * n // d, 10 * n % d
        tail.rest = Link(q)
        tail = tail.rest
        cache[n] = tail
        n = r
    tail.rest = cache[n]
    return result

def display(s, k=10):
    """Print the first k digits of infinite linked list s as a decimal.

    >>> s = Link(0, Link(8, Link(3)))
    >>> s.rest.rest.rest = s.rest.rest
    >>> display(s)
    0.8333333333...
    """
    assert s.first == 0, f'{s.first} is not 0'
    digits = f'{s.first}.'
    s = s.rest
    for _ in range(k):
        assert s.first >= 0 and s.first < 10, f'{s.first} is not a digit'
        digits += str(s.first)
        s = s.rest
    print(digits + '...')
```

The decimal expansion of 1/22 could be constructed as follows:

```
>>> n, d = 1, 22
>>> n/d
0.045454545454545456
>>> result = Link(0)
>>> tail = result
>>> q, r = 10 * n // d, 10 * n % d
>>> tail.rest = Link(q)  # Adds the 0: 0.0
>>> tail = tail.rest
>>> n = r
>>> n
10
>>> q, r = 10 * n // d, 10 * n % d
>>> tail.rest = Link(q)  # Adds the 4: 0.04
>>> tail = tail.rest
>>> n = r
>>> n
12
>>> q, r = 10 * n // d, 10 * n % d
>>> tail.rest = Link(q)  # Adds the 5: 0.045
>>> tail = tail.rest
>>> n = r
>>> n
10
>>> result
Link(0, Link(0, Link(4, Link(5))))
>>> tail.rest = result.rest.rest
>>> display(result, 20)
0.04545454545454545454...
```

Place the division pattern from the example above in a `while` statement:

```
>>> q, r = 10 * n // d, 10 * n % d
>>> tail.rest = Link(q)
>>> tail = tail.rest
>>> n = r
```

While constructing the decimal expansion, store the `tail` for each `n` in a dictionary keyed by `n`. When some `n` appears a second time, instead of constructing a new `Link`, set its original link as the rest of the previous link. That will form a cycle of the appropriate length.