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(In)Efficient Interbank Networks

We study the efficiency properties of the formation of an interbank network. Banks face a trade-off by establishing connections in the interbank market. On the one hand, banks improve the diversification of their liquidity risk and therefore can obtain a higher expected payoff. On the other hand, banks not sufficiently capitalized have risk-shifting incentives that expose them to the risk of bankruptcy. Connecting to such risky banks negatively affects expected payoff. We show that both the optimal and the decentralized networks are characterized by a core-periphery structure. The core is made of the safe banks, whereas the periphery is populated by the risky banks. Nevertheless, the two network structures coincide only if counterparty risk is sufficiently low. Otherwise, the decentralized network is underconnected as compared to the optimal one. Finally, we analyze mechanisms that can avoid the formation of inefficient interbank networks.

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It is well established that banks have incentives to form bilateral lending relationships. The most intuitive reason is to coinsure future and uncertain idiosyncratic liquidity shocks (Allen and Gale 2000, Freixas, Parigi, and Rochet 2000). There is robust evidence that documents how interbank lending plays a crucial role in providing liquidity insurance both in normal times (Furfine 2001, King

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2008, Cocco, Gomes, and Martins 2009) and in times of crisis (Furfine 2002). The overall amount of these bilateral links that banks establish forms what is generally known as the interbank network.

The 2007/2008 financial crisis witnessed a prominent role played by the interbank market in initiating and spreading the turmoil. This event prompted a surge of empirical evidence about the actual shape of the interbank network. Soromäki et al. (2007) and Bech and Atalay (2010) find that the interbank networks formed by U.S. commercial banks is quite sparse. It consists of a core of highly connected banks, while the remaining peripheral banks connect to the core banks. An almost identical feature is found in interbank networks in countries like the UK, Canada, Japan, Austria, and Germany (see, respectively, Boss et al. 2004, Inaoka et al. 2004, Embree and Roberts 2009, Craig and von Peter 2014, Langfield, Liu, and Ota 2014).

The aim of this paper is to establish under which conditions, if any, a core-periphery interbank network may be optimal. We then study the decentralized endogenous network formation game. That is, we investigate if banks have the right incentive to mimic the optimal network and under which conditions this may occur. Overall, the objective of the paper is to rationalize the stylized fact on the core-periphery shape of the interbank networks, and to understand when such a structure may entail inefficiencies.

We model an interbank network composed of several banks that anticipate how the structure of the network affects their payoff. Participating in the interbank network is beneficial because it allows banks to increase the expected payoff. We posit that such expected payoff is increasing in the number of links that a bank establishes in the network. The rationale behind this assumption is the ability of the interbank network to reduce liquidity risk by coinsuring future idiosyncratic liquidity shocks. The higher the number of connections in the network are, the higher the probability to find coinsurance is, the less resources have to be invested in liquid low-return assets, and the more resources can be invested in illiquid high-return investment projects thus increasing depositors' payoff (Castiglionesi, Feriozzi, and Lorenzoni 2019).

The benefit however has to take into account the potential cost of participating in the interbank network. Such cost is captured by assuming that banks face a standard moral hazard problem (Holmström and Tirole 1997). Each bank is financed by depositors and shareholders. The former supply their funds and expect to break even, the latter provide capital and decide the type of investment the bank chooses. Shareholders have two types of investment projects in which they can invest the bank's resources. Although one project is risk-free, that is, it guarantees a certain return, the other project is risky because it has the same payoff of the safe project if it succeeds but it delivers nothing if it fails. The risky project however gives private benefits to the bank's shareholders, therefore it represents a gambling project from the depositors' point of view.[1] Shareholders are protected by limited liability, so they find it convenient to invest in the gambling project when the bank is poorly capitalized. We assume that a

1. Throughout the paper, we use the expressions risk-free and safe bank (or project) as synonymous. Similarly for risky and gambling bank (or project).

bank, by establishing a link with banks that invest in the risky project, reduces the *ex ante* probability of serving its own depositors. In particular, we assume that the higher the ratio between the number of neighboring banks that invest in the risky project over the total neighboring banks, the lower the probability to serve the depositors is. We refer to the risk of making connections in the interbank network as counterparty (or solvency) risk.

We analyze the trade-off of participating in an interbank market in which the benefit of a reduced liquidity risk has to be weighted against the counterparty risk. First, we characterize the optimal interbank network as the solution of the planner's problem. The planner can avoid the moral hazard problem in all banks only if a sufficient amount of bank capital is available in the economy. In this case, the first-best network is characterized by a fully connected structure. Otherwise, if bank capital in the economy is scarce, the planner has to allow some banks to gamble and a constrained first-best (CFB) network is obtained.

The presence of banks investing in the risky project implies that the CFB network does not necessarily coincide with the fully connected one. Indeed, the CFB network is characterized by a core-periphery structure. The core includes all the banks that invest in the safe project and form a complete network structure among themselves. The periphery includes all the gambling banks that can be connected among themselves and/or with the core banks according to the parameters' value. With an additional assumption on the benefit of participating in the network, we are able to fully characterize the conditions under which risky banks should or should not be connected among themselves and with the core (safe) banks.

Second, we analyze the decentralized interbank network formation adopting the equilibrium notion of pairwise stability. Also in this case a core-periphery structure emerges as an equilibrium outcome. Nevertheless, the connectivity in the decentralized network does not necessarily coincide with the CFB network. We show that the structure of the decentralized interbank network is the same as the CFB one if the counterparty risk is sufficiently low. Otherwise, when the counterparty risk is not low enough, the decentralized network does not coincide with the CFB network. The reason is that the planner finds it optimal to link a safe bank with a gambling bank when the expected losses of the former (because of counterparty risk) are lower than the expected gains of the latter (represented by the higher expected payoff due to a higher liquidity coinsurance). However, these expected gains are not internalized by the safe banks that severe the link with the gambling banks even when this is not efficient. The decentralized network has an inefficiently low degree of connectivity compared to the CFB network when counterparty risk is sufficiently high.

Finally, we analyze possible mechanisms that could prevent the formation of inefficient networks. In particular, we allow for decentralized bank capital transfers before the shareholders take the investment decision. Banks investing in the safe project may find it convenient to transfer part of their bank capital to the neighboring gambling banks to change their investment decision and therefore to achieve a higher expected payoff. We show that if the probability of success of the bank's risky project is sufficiently high, then banks do not have incentive to initiate any bank capital transfer. More generally, we show that the decentralized network formation does not induce banks to form the CFB network also when bank capital transfers occur.

The type of inefficiency that our model highlights likely arose during the 2007/2008 financial crisis, where banks feared losses in their counterparts. Our model predicts that when the risk associated with the lending of funds is too high, connections become too costly relative to their benefits and safe banks inefficiently sever their interbank links. Nevertheless, our paper also stresses the fact that safer banks still have incentive to maintain their links among each other. These predictions are supported by Afonso, Kovner, and Schoar (2011) who find that interbank lending in the United States decreased substantially during the 2007/2008 crisis but it did not freeze completely. They find that riskier banks were cut off from the interbank market, whereas safer banks would be still active.[2] Moreover, consistent with our model, they show that the interbank market stress was likely coupled with inefficient provision of liquidity to the risky banks.

The reminder of the paper is organized as follows. As part of the Introduction, we discuss the related literature. Section 1 sets up the model and then it posits the benefit and cost of the interbank network and how they affect agents' expected payoff. Section 2 analyzes the planner's problem, characterizing the constrained first-best solution. Section 3 studies the decentralized network formation and its efficiency properties. Section 4 concludes. The Appendix collects the proofs.

Literature Review

Our paper is inspired by the strand of literature that models contagion as the outcome of links established by banks. In particular, banks are connected through interbank deposits that are desirable *ex ante*, but the failure of one institution can have negative effects on the institutions to which it is linked (Allen and Gale 2000, Freixas, Parigi, and Rochet 2000, Brusco and Castiglionesi 2007). The common feature of all these models is to assume an exogenous and very stylized interbank network. The present model captures the features of the banking models such as the benefits stemming from liquidity coinsurance (Allen and Gale 2000) and the gambling behavior of low capitalized banks (Morrison and White 2005, Brusco and Castiglionesi 2007), but it directly addresses the issues of the optimal design and the decentralized formation of the interbank network.

Even if the theory of network formation has been successfully applied to several economics fields, only recently there have been attempts to use such theory to understand the working of financial systems (see for a survey Allen and Babus 2009). Among the first attempts, Leitner (2005) and Babus (2016) consider models of network formation, where banks form links in order to reduce the risk of contagion.

2. Note that theoretical explanations of the liquidity dry-up based on adverse selection arguments predict that only the risky banks (i.e., the "lemons") remain in the market (Malherbe 2014, Heider, Hoerova, and Holthausen 2015).

These models rationalize the interbank network as an insurance mechanism. The idea is that banks can be surprised by unexpected liquidity shocks that can make bankrupt at least one bank in the system (like in Allen and Gale 2000). We provide an alternative rationale for the existence of interbank networks that is based on banks that fully anticipate the trade-off between the benefits and the costs of participating in the network.

Similarly to us, Farboodi (2017) provides an analysis with the aim of rationalizing the existence of a core-periphery banking network. She exogenously assumes two types of banks: those that do not have any investment opportunity and those that may have a good but risky investment opportunity. The interbank market is rationalized as a tool that channels resources from the former banks to the latter banks. In equilibrium, banks without the possibility to invest are on the periphery lending money to the core banks that instead may have the risky investment opportunity. The core banks that turn out to have the risky investment opportunity will then invest, and the core banks that do not have such investment will intermediate between the safe periphery banks and the risky core banks. In our model instead, the interbank market is viewed as a tool that provides risk sharing (that is, liquidity coinsurance) among banks. In our model, all banks invest and we endogenously determine the banks that either invest in the safe project or the risky project. Banks that invest in the safe project are part of the core because they provide the liquidity coinsurance service in the safest way. Banks that invest in the risky project will be part of the periphery because their provision of liquidity coinsurance is not safe. This is the reason of the two diametric results between her and our model.

A related fast-growing literature studies the propagation of negative shocks in financial networks. Contrary to our approach, this literature takes the structure of the network as given (Caballero and Simsek 2013, Elliott, Golub, and Jackson 2014, Acemoglu, Ozdaglar, and Tahbaz-Salehi 2015, Castiglionesi and Eboli 2018). A different approach is taken by Acemoglu, Ozdaglar, and Tahbaz-Salehi (2014) and Zawadowsky (2013) who analyze the strategic link formation among banks located however on a given network shaped as a ring. Although the former paper predicts that the equilibrium network can exhibit both under- and overconnection, the latter provides a rationale for underinsurance. Allen, Babus, and Carletti (2012) analyze the interaction between financial connections due to overlapping portfolio exposure and systemic risk. Cabrales, Gottardi, and Vega-Redondo (2016) investigate the optimal properties of a network of firms that trade-off risk sharing and risk of contagion. They find that when big shocks have low probability to occur, the complete network with uniform exposure among firms is the optimal one. When the likelihood of big shocks is large enough, it is optimal to severe links and to form disjoint components. Core-periphery structures are never optimal. However, they consider firms either with homogenous risk-return characteristics or heterogenous characteristics but only with respect to risk. We instead consider banks with heterogenous expected returns due to the presence of moral hazard. This turns out to be the main reason for the core-periphery structure to emerge both as the optimal and decentralized network.

1. THE MODEL

There are three dates $t = 0, 1, 2$ and one divisible good called "dollars" (\$). The economy is divided into n regional banks, and let $N = \{1, 2, \dots, n\}$ be the set of such banks. Each regional bank has the same (large) amount of depositors that consume at $t = 2$. Depositors have to deposit at $t = 0$ in their regional bank to access the investment opportunities of the economy. The endowment of deposits received at $t = 0$ is normalized to 1\$ in each regional bank. Besides deposits, banks are funded also through capital. Each regional bank i randomly receives an endowment $e_i \in [0, \overline{e}]$ of dollars, which represents the bank capital and it is owned by the bank's shareholders. The vector $e = (e_1, e_2, \dots, e_n)$ represents the realization of the bank capital endowments. The pair (N, e) is called an *economy*.

Let $K_i \subseteq N$ be the set of banks to whom bank i is directly linked, then the number of banks connected to bank i is $k_i \in \{0, 1, \dots, n-1\}$. The vector $K = (K_1, K_2, \dots, K_n)$ captures the interdependence among the banks, and it represents the interbank network. We restrict ourselves to undirected networks, that is bank i is related to bank j if and only if bank j is related to bank i. Let $\mathcal{K}$ denote the set of all possible interbank networks for a given economy (N, e) .

We also allow for transfers of bank capital across neighboring banks. Let $x_i = e_i + t_i$ be the bank capital for bank $i \in N$ after transfers have been made (i.e., t_i is the transfer and can be positive or negative). A vector of bank capitals $x = (x_1, x_2, \dots, x_n)$ is called feasible for a given economy (N, e) if (i) $x_i \geq 0$ for all i, and (ii) $\sum_{i \in N} x_i = \sum_{i \in N} e_i$. Let $\mathcal{X}$ denote the set of all feasible vectors of bank capital for a given economy (N, e).

After the transfers are made, each bank i has $1 + x_i$ dollars to invest. Banks may choose to invest between two types of project that mature in $t = 2$:

(i) The risk-free, or safe, project rf with an expected return of $\overline{R} > 1$ dollars per dollar invested.
(ii) The risky, or "gambling," project r that yields an expected return of $\overline{R} > 1$ dollars with probability ξ, and 0 dollars with probability $(1 - \xi)$ per dollar invested. This type of project yields also a private benefit $B > 0$ to bank's shareholders. Private benefits are realized at the moment of the investment (so they do not have dollar value, consider them as perks or investment in family business).

We refer to $s_i \in \{rf, r\}$ as the project's choice of bank i at $t = 1$. The vector s denotes the investment strategy profile, that is, $s = \{s_i\}_{i \in N}$. Let $\mathcal{S}$ denote the set of all possible investment profiles for a given economy (N, e). The sequence of events is reported in Table 1.

The timing represents the interbank network formation game. In $t = 0$, once bank capitals are realized, the banks choose the network structure. In $t = 1$, bank capital transfers are made and the shareholders choose the type of project. Accordingly, the expected payoff in $t = 2$ will depend on the structure of the financial network chosen

TABLE 1

Sequence of events

Time	Events
$t = 0$	Bank's capital is realized and interbank network is chosen.
$t = 1$	Bank's capital transfers are made and projects are chosen.
$t = 2$	Projects cash flows are realized and depositors are paid.

in $t = 0$ and on the type of project chosen by the banks in $t = 1$. The timing captures the fact that bank decisions about which interbank link to establish are long term, while the type of the investment is a decision with shorter time horizon.

1.1 Interbank Network: Benefit, Cost, and Expected Payoffs

In order to model the benefit and cost of establishing connections in the interbank market, we assume the following. On the one hand, establishing interbank lending relationships is beneficial because it allows banks to coinsure small idiosyncratic liquidity shocks that can hit the investment project before it matures. Such liquidity shocks can be smoothed out by holding a liquidity buffer and therefore do not jeopardize the survival of the bank. On the other hand, the cost of linking is given by the failure of the gambling project that represents a big shock that cannot be insured. Therefore, it may affect the survival also of the connected banks.

The benefit: liquidity coinsurance. We assume that the benefit of the interbank network is to increase the expected payoff that bank i can obtain by investing its resources $1 + x_i$. Let us indicate with Π_i the expected payoff that bank i obtains for each unit of investment *conditional* on its survival, and with R the return that bank i reaches when it has no interbank connections. We posit that

$$\Pi_i = f(k_i)R$$

with $f(k_i)$ increasing in k_i. We assume that $f(0) = 1$, that is, if bank i has no interbank connections ($k_i = 0$), then it gets the lowest return $R \in (1, \overline{R})$ for each unit of investment. The return R can be interpreted as the "regional" (or autarky) return that bank i obtains if it has no links with any other bank. On the other extreme, if bank i is linked to all the other regional banks ($k_i = n - 1$), we assume $f(n - 1) = \overline{R}/R$. That is, bank i gets the highest return $\overline{R}$ that can be regarded as the "global" return. For simplicity, we assume an upper bound on $\overline{R}$ given by $2R \geq \overline{R} > R$. By defining $\overline{R}/R \equiv \rho$, we have $f(k_i) \in [1, \rho]$, with $\rho \leq 2$.

The modeling choice of $f(k_i)$ captures that, all else equal, the higher the number of neighboring banks k_i is, the higher the bank i's *ex ante* probability to find a suitable counterparty. This reduces the liquidity risk that bank i is facing, allowing it to obtain, conditional on its survival, a higher return.[3] Among the many, one possible interpretation of our assumption is provided by Castiglionesi, Feriozzi, and Lorenzoni (2019). They show that a higher *ex ante* probability of finding coinsurance in the interbank market reduces the liquidity buffer that banks hold, allowing them to invest a higher amount of resources in illiquid and more profitable projects. When $k_i = 0$, bank i is therefore bearing fully the idiosyncratic liquidity risk with its own liquidity buffer. When the number of linked banks increases, bank i can coinsure the idiosyncratic liquidity risk with other regional banks and its liquidity buffer decreases. In this paper, we abstract from analyzing banks' liquidity holding, and we take as given the positive relationship between the *ex ante* liquidity insurance provided by the interbank network and the expected payoff.

The cost: counterparty (solvency) risk. The cost of belonging to an interbank network is represented by the exposure to counterparty (or solvency) risk that determines the bank's survival. A bank indeed may end up lending money to a neighboring bank that is actually investing in the risky project. Only if the risky project succeeds, the risky bank is able to pay back the borrowed money. Otherwise, the lending bank is negatively affected by the default of the gambling neighboring bank and it may not serve its depositors either. We capture this solvency risk by assuming that the higher the number of risky banks among the neighboring banks k_i is (i.e., the higher the bank i's probability ending up lending to one or more risky bank), the lower the probability bank i will serve its depositors. Although before we abstract from banks liquidity holding, here we abstract from the amount of liquidity exchanged in the interbank network.

Formally, let $p_i(K, s)$ be the probability that bank i survives. Let $g_i \in [0, k_i]$ denote the number of gambling neighbors of bank i. Then the probability $p_i(K, s)$ is defined as

$$p_i(K, s) = \begin{cases} \eta^{\frac{g_i}{k_i}} & \text{if } s_i = rf, \\ \xi \eta^{\frac{g_i}{k_i}} & \text{otherwise.} \end{cases} \tag{1}$$

When bank i and all its neighbors are investing in the risk-free project (i.e., $s_i = rf$ and $g_i = 0$), then bank i serves its depositors with $p_i(K, s) = 1$. In this case, no matter which bank (or banks) will lend money to, bank i will be paid back and then surely serve its depositors. If instead $g_i > 0$, bank i will serve its depositors with probability less than 1, namely $p_i(K, s) = \eta^{\frac{g_i}{k_i}}$. There is now a possibility that bank i will lend to one (or more) of the g_i banks that cannot repay after borrowing money. This would spill over to bank i, reducing its *ex ante* probability of being able to serve its own depositors.[4] The worse case is when all bank i's neighboring banks invest in the

3. In more abstract settings, Bloch, Genicot, and Ray (2008) and Bramoullé and Kranton (2007) offer strategic analyses of informal coinsurance in networks. They characterize bilateral agreements that provide coinsurance against income risk. We can interpret these works as microfoundation of the function $f(k)$.

4. Our assumption allows us to analyze the contagious effect of the *direct* links that connect banks, ruling out indirect contagion and systemic effects. Our results point out that the only presence of

risky project (i.e., $g_i = k_i$). In this case, counterparty risk is maximal; therefore, the probability of bank i to serve its depositors is minimal and equal to η. The parameter η then captures, all else equal, the exposure to counterparty risk: the higher the η, the lower the counterparty risk is.[5] Finally, note that counterparty risk represents a conditional probability (i.e., the bank that invests in the gambling project has to fail *and* the bank linked to it has to be a lender of the failing bank). It is then natural to assume $\eta > \xi$.

Expected payoffs and the choice of the investment project. Depositors in bank i invest one dollar in the bank and perfectly anticipate the bank's risk. Then the deposit contract that allows depositors to break even has to promise an amount equal to

$$D_i = \frac{1}{p_i(K, s)} \geq 1 \tag{2}$$

and the depositors' expected payoff $M_i(K, s)$ is

$$M_i(K, s) = p_i(K, s)D_i = 1.$$

Shareholders in bank i are residual claimant and are protected by limited liability.[6] Accordingly, they expect the following payoff

$$m_i(K, x_i, s) = \begin{cases} p_i(K, s)[(1 + x_i)f(k_i)R - D_i] & \text{if } s_i = rf, \\ p_i(K, s)[(1 + x_i)f(k_i)R - D_i] + B & \text{otherwise.} \end{cases}$$

Considering the expressions for D_i in (2) and for $p_i(K, s)$ in (1), we have

$$m_i(K, x_i, s) = \begin{cases} \eta^{\frac{g_i}{k_i}}(1 + x_i)f(k_i)Rx_i - 1 & \text{if } \ s_i = rf, \\ \xi\eta^{\frac{g_i}{k_i}}(1 + x_i)f(k_i)Rx_i - 1 + B & \text{otherwise.} \end{cases} \tag{3}$$

Shareholders decide in which type of project to invest. Therefore, for given $f(k_i)$ and s_{-i}, shareholders in bank i invest in the safe project if and only if

$$\eta^{\frac{g_i}{k_i}}(1 + x_i)f(k_i)Rx_i - 1 \geq \xi\eta^{\frac{g_i}{k_i}}(1 + x_i)f(k_i)Rx_i - 1 + B,$$

"one-step" contagion is able to generate inefficient networks. Most likely, systemic effects would magnify the inefficiencies present in our analysis.

5. The assumed monotonicity of $p_i(K, s)$ implies that solvency shocks are not perfectly correlated. That is, risky banks default in different states of the world. If solvency shocks would be perfectly correlated (i.e., risky banks fail together in the same state of the world), then we would have $p_i(K, s) = \xi$ if $s_i = r$. That is, adding a risky bank does not affect bank i's probability of default. Such alternative assumption would undermine the core-periphery result because it would make it convenient to link all gambling banks.

6. All the results of the paper are independent of how the bank's profits are shared between shareholders and depositors. This fact implies that our model could be relevant also to contexts where institutions operating in the network have different profit sharing rules and similar benefits and costs in establishing links. Indeed the core-periphery structure is not an exclusive feature of the interbank market, but it is observed also in other market as well (for example, how inter dealers are positioned in OTC markets).

which implies

$$x_i \geq \frac{B}{(1-\xi)\eta^{\frac{g_i}{k_i}} f(k_i) R} - 1 \equiv I^*(k_i, g_i, \xi, \eta).$$

Accordingly, banks with sufficiently high level of bank capital have incentive to invest in the risk-free project, whereas relatively low capitalized banks find it convenient to invest in the risky project. The threshold I^* is strictly positive whenever B is sufficiently high.

Note that two banks i and j having the same amount of capital $x_i = x_j = \bar{x}$ could invest in different projects. Assume

$$\frac{B}{(1-\xi)\eta^{\frac{g_i}{k_i}} f(k_i) R} - 1 \leq \bar{x} < \frac{B}{(1-\xi)\eta^{\frac{g_j}{k_j}} f(k_j) R} - 1,$$

then shareholders in bank i choose the safe investment, whereas shareholders in bank j choose the risky investment. The previous inequality necessarily implies $\eta^{g_j/k_j} f(k_j) < \eta^{g_i/k_i} f(k_i)$. This can occur either when bank i has more connections than bank j with the same proportion of risky counterparties, or when banks i and j have the same number of connections but bank i has fewer risky counterparties. What cannot occur is that bank j has more connections and a smaller proportion of risky counterparties than bank i.

2. CONSTRAINED FIRST-BEST INTERBANK NETWORK

Let us define an allocation as a vector (K, x, s), where $x \in \mathcal{X}$, $K \in \mathcal{K}$ and $s \in \mathcal{S}$. An allocation (K, x, s) is an *investment Nash equilibrium* (INE) for a given economy (N, e), with $x = (x_i)_{i \in N}$, if

$$m_i(K, x_i, s) \geq m_i(K, x_i, (s_{-i}, \tilde{s}_i)) \quad \text{for all } i \in N,$$

with $\tilde{s}_i \in \{rf, r\}$. That is, an allocation is an INE for a given economy if taking the financial network and capital as given there are no unilateral profitable deviations in the shareholders' choice of the investment project. Note that an allocation (K, x, s) is an INE for a given economy if and only if for all $i \in N$

$$s_i = \begin{cases} rf & \text{if } x_i \geq I^*(k_i, g_i, \xi, \eta), \\ r & \text{otherwise.} \end{cases}$$

The constrained first-best solution is characterized by the social planner problem, which objective function is to maximize the expected return of the agents in the economy (i.e., depositors and shareholders). We assume that the planner is able to (i) transfer the initial endowments of capital across banks, (ii) fix a financial network and (iii) suggest to the shareholders of each bank the type of project to invest in. Formally, the planner's problem is as follows.

Definition 1. *Given an economy* (N, e)*, an allocation* (K^*, x^*, s^*) *is a constrained first-best (CFB) allocation if it maximizes*

$$\sum_{i \in N} [M_i(K, s) + m_i(K, x_i, s)] \tag{4}$$

subject to

$$x_i \geq 0 \quad \textit{for all } i \in N, \tag{5}$$

$$\sum_{i \in N} x_i = \sum_{i \in N} e_i \equiv E, \tag{6}$$

$$s_i = \begin{cases} rf & \text{if } x_i \geq I^*(k_i, g_i, \xi, \eta) \\ r & \text{otherwise,} \end{cases} \quad \textit{for all } i \in N, \tag{7}$$

$$m_i(K, x, s) \geq \max\{R(e_i + 1) - 1, \xi R(e_i + 1) + B - 1\} \quad \textit{for all } i \in N, \tag{8}$$

The planner cannot assign a negative amount of capital to each bank (feasibility constraint 5), and she can only reallocate by means of transfering the total amount of bank capital E in the economy (feasibility constraint 6). We allow shareholders to unilaterally deviate from the planner's investment suggestion, therefore the incentive constraint (7) restricts the social planner problem in a way that moral hazard has to be taken into account. Finally, we impose the shareholders' participation constraint (8).[7]

First, note that if $B \leq (1 - \xi)\rho R$, then the solution to the planner's problem is the first best. That is, the planner successfully proposes the shareholders in each bank to connect to any other bank in the interbank network ($k = n - 1$) and to invest in the risk-free project ($g = 0$). This is achieved without any reallocation of bank capital. Choosing the risk-free project is indeed an INE given the complete and safe network as the threshold $I^*(n - 1, 0, \xi, \eta)$ is negative. Banks obtain a higher expected payoff with the first-best network proposed by the planner than in autarky. By the same reasoning, the planner achieves the first best if $\min e_i \geq I^*(n - 1, 0, \xi, \eta)$, whenever $B > (1 - \xi)\rho R$ (so that $I^*(n - 1, 0, \xi, \eta) > 0$).

Therefore, the CFB can be obtained when $B > (1 - \xi)\rho R$ and $\min e_i < I^*(n - 1, 0, \xi, \eta)$. Note that when the condition $\min e_i < I^*(n - 1, 0, \xi, \eta)$ is satisfied, then also the condition $B > (1 - \xi)\rho R$ is satisfied. Let us define the following two thresholds for the total amount of bank capital E:

$$\overline{E} = \max\left\{\frac{n-1}{\rho - 1} I^*(0, 0, \xi, \eta) - n, n I^*(n - 1, 0, \xi, \eta)\right\}$$

and

$$\underline{E} = I^*(n - 1, 0, \xi, \eta) = \frac{B}{(1 - \xi)\rho R} - 1 < \overline{E}.$$

We have the following result.

7. Recall that depositors break even in expectation, so their participation constraint is satisfied.

PROPOSITION 1. *A CFB allocation always exists. Assume* $\min e_i < \underline{E}$. *Then: (i) if* $E \geq \overline{E}$, *then any CFB yields a unique network structure* K^* *such that* $K_i^* = N \backslash \{i\}$ *for all* i *and a unique strategy profile* s^* *such that* $s_i^* = rf$ *for all* i; *(ii) if* $E < \underline{E}$, *then any CFB yields a unique strategy profile* s^* *with* $s_i^* = r$ *for all* i, *and a unique structure* K^* *such that either* $K_i^* = N \backslash \{i\}$ *for all* i, *if* $\eta \geq 1/\rho$, *or* $K_i^* = \emptyset$, *otherwise.*

The proof is in the Appendix. The existence of a CFB is guaranteed even if the constraints in the planner's problem do not define a compact set on R^n because of constraint (7). Nevertheless, by modifying the planner's problem such that the constraints define a compact set on R^n, the solution to the modified maximization problem exists and it turns out to be also the solution of the planner's problem. Proposition 1 also establishes the amount of total bank capital $\overline{E}$ that is needed to achieve the first best. That is, all the banks in the interbank network are connected and invest in the risk-free project. The intuition for this result is quite simple: when total bank capital is sufficiently high the planner can achieve the first best avoiding the moral hazard problem and satisfying shareholders participation constraint. On the other extreme case, when $E < \underline{E}$, the total bank capital is not enough to avoid the risky project even in one single bank. The planner therefore cannot induce any bank to invest in the safe project. In this case, either it is optimal to connect all the gambling banks (when the counterparty risk is sufficiently low, i.e., when $\eta \geq 1/\rho$) or disconnect all the banks getting the empty interbank network.

The following proposition establishes the shape of the CFB network when total bank capital E is such that $\underline{E} \leq E < \overline{E}$. It turns out that the CFB network is characterized by a core-periphery structure. The *core* is made of banks that invest in the safe project and are all connected to each other. The *periphery* banks choose the gambling project and they can eventually be connected to some core banks and/or some periphery.

PROPOSITION 2. *Assume* $\min e_i < \underline{E} \leq E < \overline{E}$. *Let* (K^*, x^*, s^*) *be a CFB for a given economy* (N, e). *Then, for every pair of banks* i *and* j *such that* $s_i^* = s_j^* = rf$ *we have that* $i \in K_j^*$ *and* $j \in K_i^*$.

The proof is in the Appendix. The intuition behind the optimality of the core-periphery structure is as follows. When two banks are investing in the risk-free project, it is always better to have them connected than not connected. This is true because one more neighbor increases the possibility of liquidity coinsurance and therefore the expected payoff. Such additional link does not induce any bank to switch investment decision from the risk-free to the gambling project. Indeed, if a bank has enough bank capital to choose the safe project in a given interbank network and it prefers this allocation to autarky, then the same bank capital will be sufficient to avoid the gambling behavior if the bank has one more neighbor that invests in the risk-free project.

The next proposition determines the size of the core in a CFB network. Given an allocation (K, x, s), we denote by $C(K, x, s)$ the set of banks that choose the risk-free project. Note that if (K, x) is equal to $(\emptyset, e)$, then all the banks are in autarky. It is easy to verify that there is a unique INE in autarky, denoted $(\emptyset, e, s^A)$, such that for any bank i

$$s_i^A = \begin{cases} rf & \text{if } e_i \geq \frac{B}{(1-\xi)R} - 1, \\ r & \text{otherwise.} \end{cases} \tag{9}$$

PROPOSITION 3. *Assume* $\min e_i < \underline{E} \leq E < \overline{E}$. *Let* (K^*, x^*, s^*) *be a CFB for a given economy* (N, e). *Then,* $C(\emptyset, e, s^A) \subseteq C(K^*, x^*, s^*)$. *That is, the size of the core in a CFB network can only increase as compared to the number of safe banks in autarky. Moreover, the core in a CFB network includes the banks that invest in the risk-free project in autarky.*

The proof is in the Appendix. The intuition is as follows. A bank that invests in the safe project in autarky is a bank with a relatively high initial endowment of bank capital. But the higher the initial endowment of bank capital, the higher the capital the planner has to allocate to satisfy the shareholders' participation constraint of that bank. Precisely, the minimum bank capital the planner has to offer to make shareholders participate in the optimal network is sufficiently high to induce them to choose the safe project in an INE for that network. Because the optimal allocation must be an INE, Proposition 3 follows.

We determine now the number of optimal links that each bank should have in the core-periphery CFB network. This number will depend on the relationship between the benefit, captured by the function $f(k)$, and the cost, represented by the counterparty risk η, of participating in the interbank network. The following assumption guarantees that the function $f(k)$ satisfies the increasing power ratios (IPR) property.

ASSUMPTION (IPR property). *The function* $f(k)$ *satisfies the IPR property if the expression* $(\frac{f(k_i)}{f(k_i+1)})^{k_i+1}$ *is increasing in* k_i.

Under the assumption of the IPR property,[8] the relationship between $f(k)$ and η can be parameterized in terms of a number $k(\eta)$ that is defined as follows

$$k(\eta) = \begin{cases} 0 & \text{if } \eta < \frac{1}{f(1)}, \\ k^* & \text{if } \left(\frac{f(k^*-1)}{f(k^*)}\right)^{k^*} \leq \eta < \left(\frac{f(k^*)}{f(k^*+1)}\right)^{k^*+1} \\ & \quad \text{for some } k^* \in (0, n-1), \\ n-1 & \text{if } \eta \geq \left(\frac{f(n-2)}{f(n-1)}\right)^{n-1}. \end{cases} \tag{10}$$

To understand the meaning of the number $k(\eta)$, suppose that shareholders in bank i know that there are $k - 1$ safe banks. Establishing links to those safe banks is beneficial because there is no counterparty risk. However, when is it beneficial to connect to a kth risky bank? The answer is positive, all else equal, if and only if the expected payoff obtained from connecting to $k_i = k$ banks (with $g_i = 1$) is greater

8. An increasing function like $f(k)$ satisfies the IPR property whenever it is also concave (i.e., $f''(k) < 0$), $f'''(k) > 0$, and $f'(0)$ is sufficiently high (result upon request). It is interesting to note that simple concave functions, like $f(k) = 1 + \ln(k+1)$ or $f(k) = 1 + \exp\{A - \frac{1}{k}\}$ with $A > 0$, satisfy the IPR property.

than (or equal to) the expected payoff of connecting to $k_i = k - 1$ safe banks (i.e., $g_i = 0$). This is equivalent to

$$\eta^{\frac{1}{k}} f(k)R \geq \eta^{\frac{0}{k-1}} f(k-1)R,$$

which implies

$$\eta \geq \left(\frac{f(k-1)}{f(k)}\right)^k.$$

When $f(k)$ satisfies the IPR property and $\eta \geq (\frac{f(k-1)}{f(k)})^k$, then $\eta \geq (\frac{f(k_i)}{f(k_i+1)})^{k_i+1}$ for any $k_i \leq k-1$. That is, the shareholders are willing to connect to an additional kth risky bank if they are connected to at most $k(\eta) - 1$ safe banks. Instead, when $f(k)$ satisfies the IPR property and $\eta < (\frac{f(k)}{f(k+1)})^{k+1}$, then $\eta < (\frac{f(k_i)}{f(k_i+1)})^{k_i+1}$ for any $k_i \geq k$. That is, shareholders are not willing to connect to an additional kth risky bank if they are connected to at least $k(\eta)$ safe banks.[9]

The next proposition characterizes the number of optimal links in the CFB network.

Proposition 4. *Assume that* $\min e_i < \underline{E} \leq E < \overline{E}$ *and that* $f(k)$ *satisfies the IPR property. Let* (K^*, x^*, s^*) *be a CFB allocation for a given economy* (N, e)*, and* $k(\eta)$ *be as defined in (10). Then*

(i) If $k(\eta) = n - 1$*, then* $K_i^* = N \backslash \{i\}$ *for all* $i \in N$.
(ii) If $k(\eta) = 0$*, then* $g_i^* = 0$ *for all* i *such that* $s_i^* = r$.

In the Appendix, we provide the proof of a longer version of Proposition 4 where results regarding the intermediate values of $k(\eta)$ are also included. Statement 1 establishes that the CFB network is the complete network structure in which all banks are connected if $\eta \geq (\frac{f(n-2)}{f(n-1)})^{n-1}$. The intuition is that when counterparty risk is small with respect to the benefits provided by the interbank network, connecting until the last gambling bank in the network increases the total expected payoff. Statement 2 determines that when $\eta < \frac{1}{f(1)}$ the CFB network is characterized by a sparse periphery in which the gambling banks are disconnected among them. In this case, counterparty risk is sufficiently high that it is not optimal to link two gambling banks.

For intermediate values of $k(\eta)$, we show, among other results, that at most one bank can hold less than $k(\eta)$ connections in a CFB network. Indeed, if there were at least two banks holding less than $k(\eta)$ links, the total expected payoff is increased by connecting two such banks. Moreover, periphery banks that hold more than $k(\eta)$ links cannot be directly connected. That is, holding more than $k(\eta)$ connections reduces the expected payoff if the banks who hold more than $k(\eta)$ connections are investing in the risky project. Severing their connections then increases the expected payoff.

9. The threshold $k(\eta)$ captures the nonmonotonic relationship between interconnectedness and individual and systemic risk found in recent theoretical literature. Establishing new connections is beneficial at the individual and global level when the number of connections in the network is low, while it is harming otherwise (Wagner 2011, Acemoglu, Ozdaglar, and Tahbaz-Salehi 2015).

Note that according to Statement 2 of Proposition 4 the planner may still find it optimal to link a gambling bank to a safe bank. The following Proposition establishes a sufficient condition under which the CFB network has an empty periphery, that is gambling banks are disconnected also from safe banks.

PROPOSITION 5. *Assume that* $\min e_i < \underline{E} \leq E < \overline{E}$ *and that* $f(k)$ *satisfies the IPR property. Let* (K^*, x^*, s^*) *be a CFB for a given economy* (N, e)*. Then, if* $\eta < (\frac{1-\xi}{\rho[1+n\rho]})^{n-1}$*, then* $K_i^* = \emptyset$ *for all i with* $s_i^* = r$.

The proof is in the Appendix. The intuition is as follows. The planner finds it optimal to connect a gambling bank with a safe bank if the expected benefit for the former are high enough to outweigh the counterparty risk faced by the latter. At some point, counterparty risk becomes so high that the expected benefit for the gambling bank does not outweigh the risk taken by the safe bank.[10]

3. DECENTRALIZED INTERBANK NETWORK

In this section, we study the decentralized interbank network formation. We assume first that bank capital transfers are not allowed at $t = 1$. Characterizing the efficiency properties without transfers makes it easier to analyze the effects of bank capital transfers.

3.1 Decentralized Interbank Networks without Transfers

We define an *economy without transfers* (N, e) in which the sequence of events in Table 1 does not include bank capital transfers across banks in $t = 1$. We solve the model backward. Banks choose the network structure given their initial endowment of bank capital, anticipating the INE played in that network. For a given economy (N, e), an allocation without transfers (K, e, s) is an INE if there are no unilateral profitable deviations in the type of project in which the shareholders invest.

Without bank capital transfers the network formation game is basically a static one, that is banks choose simultaneously to whom they want to connect. Accordingly, we adopt the equilibrium notion of *pairwise stability* introduced by Jackson and Wolinsky (1996).[11] Given a network K, let $K \cup ij$ denote the network resulting from adding a link joining banks i and j to the existing network K. On the contrary, for any two

10. The condition in Proposition 5 is more stringent than the condition $\eta < \frac{1}{f(1)}$ in Proposition 4 (statement 2). We have:

$$\left(\frac{1-\xi}{\rho[1+n\rho]}\right)^{n-1} < \left(\frac{1-\xi}{\rho[1+n\rho]}\right) < \frac{1}{\rho[1+n\rho]} < \frac{1}{f(1)} < 1,$$

given that $1 > \xi > 0$, $n > 2$ and $1 < f(1) < \rho$.

11. Using the stronger equilibrium notion of pairwise-Nash stability (Block and Jackson 2006, Calvo and Ilkilic 2009), the results, available upon request, remain qualitatively the same.

banks i and j connected in K, let $K\backslash ij$ denote the resulting network from severing the link joining banks i and j from K.

DEFINITION 2. *An allocation without transfers* (K, e, s) *is* pairwise stable *(PSWT) if the following holds:*

(i) For all i and j directly connected in K, $m_i(K, e, s) \geq m_i(K\backslash ij, e, \tilde{s})$ and $m_j(K, e, s) \geq m_j(K\backslash ij, e, \tilde{s})$ for all allocations $(K\backslash ij, e, \tilde{s})$ that are INE.
(ii) For all i and j not directly connected in K, if there is an INE $(K \cup ij, e, \tilde{s})$ such that $m_i(K, e, s) < m_i(K \cup ij, e, \tilde{s})$, then $m_j(K, e, s) > m_j(K \cup ij, e, \tilde{s})$.

The definition of PSWT captures two ideas. The first refers to the network's internal stability: no pair of banks directly connected in the current interbank network individually gain from severing their link. This implies that any of the two banks could sever the link unilaterally. The second establishes the network external stability: if one bank could gain from creating a link with another bank, it has to be that the other bank cannot gain from that link. This implies that both banks have to agree in order to create a link. Note that if one bank strictly gains with the creation of one link and the other bank is indifferent, it is assumed that the link is formed.

DEFINITION 3. *An allocation without transfers* (K, e, s) *is a decentralized equilibrium (DEWT) if it is INE and PSWT.*

We assess now the relationship between the CFB and the DEWT. We first establish the conditions for a DEWT to be shaped as the complete network both when banks choose the risk-free project (i.e., the first best) and when they choose the risky project.

PROPOSITION 6. *Assume that (N, e) define an economy without transfers and $f(k)$ satisfies the IPR property. Then: (i) the allocation (K^e, e, s^e) with $K_i^e = N\backslash\{i\}$ and $s_i^e = rf$ for all i is a DEWT if and only if $\min_{i\in N} e_i \geq \underline{E}$; (ii) if $\max_{i\in N} e_i < \underline{E}$, then (K^e, e, s^e) with $K_i^e = N\backslash\{i\}$ and $s_i^e = r$ for all i is a DEWT. Furthermore, if $\eta > \frac{1}{f(1)}$, then (K^e, e, s^e) with $K_i^e = N\backslash\{i\}$ and $s_i^e = r$ for all i is the unique DEWT, whereas if $\eta \leq \frac{1}{f(1)}$, then (K^e, e, s^e) with $K_i^e = \emptyset$ and $s_i^e = r$ for all i is also a DEWT (and there are no other DEWT).*

The proof is in the Appendix. Let us compare the conditions in Proposition 6 with those in Proposition 1. The condition in Proposition 1 that guarantees the first best is not enough to achieve it as a DEWT. The planner can sustain the first best as far as the *total* capital is high enough ($E \geq \overline{E}$), because she can transfer capital among banks. In the DEWT, no transfers are possible and hence the first best is a DEWT if and only if each *individual* bank capital is high enough ($\min e_i \geq \underline{E}$). Because the latter condition is more stringent than the former condition, whenever the first best is a DEWT, it is also achieved by the planner, whereas the opposite is not always true.

On the opposite case, all banks are connected and choose the risky project in the DEWT if each *individual* bank capital is low enough ($\max e_i < \underline{E}$). According to Proposition 1, the fully connected network with gambling banks is a CFB only if *total* capital is low enough ($E < \underline{E}$). Because the latter condition is more stringent than the former condition, whenever the complete network with gambling banks is a CFB, it is also DEWT, whereas the opposite is not always true. Again, the planner can pool the capital of several banks to induce at least one bank to choose the safe project that is not possible in a DEWT. Finally, note that in the DEWT the complete network with gambling banks is unique if $\eta > 1/f(1)$. According to Proposition 1, the condition $\eta \geq 1/\rho$ guarantees that the CFB complete network with gambling banks is unique. Because $1/f(1) > 1/\rho$, whenever $\eta > 1/f(1)$, gambling banks are all connected both in the CFB and in the DEWT.[12]

The remaining propositions characterizes the decentralized network for the remaining possible values of bank capital e_i. That is, whenever $\min e_i < \underline{E} < \max e_i$ (note this condition, as before, implies $B > (1 - \xi)\rho R$). The following proposition states that any DEWT is also characterized by a core-periphery structure.

PROPOSITION 7. *Assume that* $\min e_i < \underline{E} < \max e_i$ *and* (N, e) *define an economy without transfers. Then, a DEWT is a core-periphery structure, that is, if* (K^e, e, s^e) *is a DEWT, then, for every pair of banks* i *and* j *such that* $s_i^e = s_j^e = rf$, *we have that* $i \in K_j^e$ *and* $j \in K_i^e$.

The proof is in the Appendix, whereas the intuition is as follows. On the one hand, a bank agrees to be connected to any neighbor that is choosing the risk-free project because this decision entails no counterparty risk. On the other hand, if a bank invests in the risk-free project, any other bank would like to be connected to it for the same reason. Because a link in this case is beneficial, two banks choosing the safe project have the right incentive to be connected. A core-periphery structure emerges in which the safe banks are connected among themselves. Similar to the analysis of the CFB network, the connectivity in the decentralized network of the banks choosing the gambling project depends on the relationship between $f(k)$ and η. We have the following.

PROPOSITION 8. *Assume that* $\min e_i < \underline{E} < \max e_i$ *and* $f(k)$ *satisfies the IPR property. Let* $k(\eta)$ *be as defined in (10). Then*

(i) If $k(\eta) = n - 1$, *then* $K_i^e = N\backslash\{i\}$ *for all* $i \in N$.
(ii) If $k(\eta) = 0$, *then* $g_i^e = 0$ *for all* i.

Similarly to Proposition 4, in the Appendix, we provide the proof of a more general version of Proposition 8 that takes into account also intermediary values of $k(\eta)$. Statement 1 shows that the decentralized interbank network is characterized by the complete structure when $\eta \geq (\frac{f(n-2)}{f(n-1)})^{n-1}$. This is the same condition of the CFB network (recall Statement 1 in Proposition 4). When counterparty risk is very low, the incentives in the formation of the decentralized network are aligned with those of the planner. It is important to note that Proposition 8 does not imply that investment decisions are also the same in both networks. In the DEWT, the investment decisions

12. The discrepancy between these two conditions is due to the equilibrium notion of pairwise stability. With the stronger notion of pairwise-Nash stability, the condition on uniqueness for the DEWT coincides with the one for the CFB (i.e., $\eta \geq 1/\rho$).

might be suboptimal (i.e., too many banks investing in the risky project). However, when η tends to 1, the risk-free and the risky projects yield (almost) the same expected payoff. As counterparty risk vanishes, the only factor that determines the expected payoff in the economy *is* the network structure.

Statement 2 in Proposition 8 establishes that the condition $\eta < 1/f(1)$ is sufficient to observe an empty periphery in the decentralized network. That is, banks that invest in the risky project are not linked with any other bank *including* the safe ones. However, statement 2 in Proposition 4 shows that the same condition is not sufficient for the empty periphery to be optimal. In this case, the planner allows safe banks to have more risky links compared to the decentralized network. The condition to have an empty periphery in the CFB network is given in Proposition 5. As a consequence, banks that invest in the risky project can be inefficiently underconnected in the decentralized interbank network whenever[13]

$$\eta \in \left[\left(\frac{1-\xi}{\rho[1+n\rho]} \right)^{n-1}, \frac{1}{f(1)} \right].$$

For intermediary values of $k(\eta)$, like in the CFB network (recall the long version of Proposition 4), also the general version of Proposition 8 establishes that in the decentralized network there is at most one bank holding less than $k(\eta)$ connections. The reason is the same as in the CFB. More importantly, in the decentralized network, a bank with more than $k(\eta)$ links will be connected only to banks that invest in the risk-free project. This does not coincide with the condition of the CFB network, where $k(\eta)$ is the highest number of links that two connected periphery risky banks can have. Although it is not individually optimal to hold more than $k(\eta)$ links when there is at least one risky neighbor, the planner could find it optimal to have a bank holding more than $k(\eta)$ links if the expected benefits of the risky banks compensate for the expected loss of the safe banks.

The decentralized interbank network may show fewer connections than the CFB network when banks that invest in the risk-free project consider engaging in bilateral insurance too risky. The benefit provided by the interbank network is neglected when counterparty risk is particularly high.[14] The emergence of inefficient networks may be due to the absence of a mechanism that allows banks to internalize the network externalities. We are going to analyze this possibility in the next section.

3.2 Allowing for Transfers

In our model, the inefficiency is rooted in two sources of externalities: those related to the decision of establishing the connections in the network (i.e., due to the network formation process) and those related to the investment decision (i.e., risk-free versus

13. The interval is nonempty (see footnote 10).

14. Clearly, because counterparty risk η is not the only parameter in determining the shape of a CFB network (bank capital endowments are important as well), the decentralized network is "underconnected" whenever the more connected network is feasible given the initial capital endowments.

risky project). The two sources of externalities are intertwined in our model and they cannot be separated. Let us explain why this is the case. When bank i invests in the risky project, then it reduces also the probability of success of its directly connected neighbors (respect to the case in which bank i invests in the risk-free project). This is the externality that concerns the type of project chosen: the expected payoff of bank i's neighbors depends on the investment decision of bank i. Furthermore, whether bank i decides to invest in the risk-free or the risky project depends on its network position (i.e., whether other banks want to be connected to it). In turn, bank i's network position depends on bank i's investment decision. Note that when bank i invests in the risk-free project any bank would like to connect to it, otherwise, when bank i invests in the risky project, other banks may connect depending on the severity of counterparty risk.

Externalities due to the network formation process have been analyzed by Bloch and Jackson (2007), whereas externalities due to the investment decision are peculiar to our model. Bloch and Jackson (2007) show that efficient networks are supported, although not uniquely, by pairwise stable equilibria when agents can make transfers *contingent* on the network formation process. As it will become clearer later on in this Section, allowing *only* for transfers contingent on the network formation process is not enough to obtain the CFB network (because, as said, the two sources of inefficiency cannot be separated in our model). Therefore, the interesting question is whether allowing only for transfers affecting the investment decisions in general suffice to sustain the CFB network.

We consider then a sequential-move game in which banks can transfer bank capital to their neighboring banks. Following the sequence of events in Table 1, we consider transfers that are made *before* the investment decisions take place.[15] The goal of a bank making such transfer is to induce the desired choice of the investment project of its neighbors in the attempt to reduce the exposure to counterparty risk and to increase the expected payoff.[16]

We again solve the model backward. We analyze the INE given the transfers and the network. Then, given the network, we solve for the transfers anticipating the INE played in the last stage. Finally, we characterize the decentralized network anticipating the transfers and the INE played in the following stages. Recall that our INE is determined by a simultaneous-move game in which it might not be unique.[17] This can be problematic in a decentralized context where there is no coordination device. Therefore, to avoid inefficiencies due to coordination failure in the INE, we consider a sequential-move game with perfect information in the transfer stage assuming a well-defined rule of order.

15. Although transfers conditional on network formation necessarily have to occur in $t = 0$, transfers conditional on the investment decision can also occur in $t = 1$ but before the investment decisions take place.

16. We can interpret such capital transfers as capital-rich banks acquiring shares in capital-poor banks.

17. Note that the definition of PSWT does not preclude the possibility of coordination failure, that is the possibility of encountering more than one INE given network K and the initial endowment e.

We rank banks according to their bank capital endowment, and we start the order of transfers from the highest to the lowest capital endowed bank. Once the bank endowed with the lowest capital has taken his transfer decision, we move on to the investment stage. In this stage, the bank with the highest capital (which now might be different from the bank with the highest capital endowment because of the transfers) decides the type of investment, and the other banks follow according to their level of bank capital. The backward solution allows us to select one INE and one profile of transfers for a given network, that will be a Subgame Perfect Equilibrium in the transfer and investment game. Once the transfer and the investment profile are uniquely determined in equilibrium, we can apply pairwise stability to the network formation process.

We have the following.

PROPOSITION 9. *Let K be a given network and x be a vector of bank capital reallocation such that there are multiple INE following (K, x). Let $Q(s, s') \subseteq N$ be the group of banks for which investment decisions change from the risky to the risk-free project in two INE s, s' following (K, x). Then the backward induction argument in the sequential-move investment game does not select the INE where agents in Q choose the risky project, independently of the rule of order.*

The proof is in the Appendix. Proposition 9 states that with sequential investment decisions there is no coordination problem among banks. This means that if banks arrive at the investment stage after choosing the optimal network and having done the optimal transfers, the decentralized interbank network would coincide with the CFB network. We can then explore whether bank capital transfers in a sequential move game lead to a CFB allocation, assuming that INE decisions in the last stage are optimal.

We first provide a condition under which bank capital transfers are never initiated by the banks.

PROPOSITION 10. *There exists a $\overline{\xi} \in [0, 1]$ such that if $\xi > \overline{\xi}$, then there are no bank capital transfers in the sequential-move transfer game.*

The proof is in the Appendix. The intuition is the following. When the probability of success of the gambling project is sufficiently high, then no bank capital transfers occur. The high probability of success of the gambling project implies that counterparty risk is relatively low. Therefore, the amount of bank capital that has to be transferred becomes too costly. Whenever ξ is sufficiently high, safe banks prefer to face the (low) counterparty risk. It follows that if the decentralized network is not a CFB it remains an inefficient network even when transfers affecting the investment decisions are allowed.

We now analyze whether allowing for transfers when the probability of success of the gambling project becomes lower (and counterparty risk becomes higher) is enough to sustain the CFB network. It turns out that, even when transfers are correct (i.e., they give the right investment incentives to the neighboring banks), the decentralized allocation with bank capital transfers is not able to mimic the CFB network. We illustrate this result with a couple of examples.[18] In particular, we show that allowing for transfers affecting the investment decisions, not only does not avoid the formation of underconnected decentralized network (like it was the case without such transfers), but it may actually induce the formation of overconnected decentralized networks.

Overconnected decentralized network. Consider four banks. Bank 1 has a large amount of capital e_1, whereas the other three banks have an amount of capital close to zero: $e_2 = e_3 = e_4 = \varepsilon \approx 0$. Assume the total capital E is such that $2I^*(1, 0, \xi, \eta) \leq E < 3I^*(2, 0, \xi, \eta)$, and couterparty risk is such that $\eta < 1/f(1)$. Under these assumptions, the CFB interbank network is made of Bank 1 connected to one of the three banks. Let us indicate as Bank 2 the bank connected to Bank 1. The two connected banks invest in the risk-free project after a capital transfer is made from Bank 1 to Bank 2. In the decentralized network, Bank 1 would transfer to Bank 2 exactly the amount of capital necessary to switch from the risky to the risk-free project. Such transfer t is equal to

$$x_2 = e_2 + t = \frac{B}{(1-\xi)f(1)R} - 1,$$

and Bank 2's expected payoff is equal to $\frac{B}{(1-\xi)} - 1$.

What happens if Bank 2 connects to one of the two risky banks that are disconnected in the CFB? For some values of the parameters, Bank 1 still transfers to Bank 2 the amount of capital necessary to switch investment decision (so that transfers are correct). However, the transfer now needs to make Bank 2 change its investment decision when Bank 2 has two connections (one of them risky). The transfer $\tilde{t}$ is now equal to

$$x_2 = e_2 + \tilde{t} = \frac{B}{(1-\xi)\eta^{1/2} f(2)R} - 1,$$

and Bank 2's expected payoff is as before equal to $\frac{B}{(1-\xi)} - 1$. Given that $\eta < \frac{1}{f(1)}$ implies that $\eta < (\frac{f(1)}{f(2)})^2$ we have $\tilde{t} > t$. Therefore Bank 1's expected payoff is equal to $f(1)R(e_1 + 1 - t) - 1$ when it is linked only to Bank 2, and equal to $f(1)R(e_1 + 1 - \tilde{t}) - 1$ when it is linked to Bank 2, which is linked to another bank. Clearly, Bank 1 strictly prefers the first network over the second network. The latter network therefore is overconnected, in the sense that the link between Banks 2 and another risky bank imposes a negative externality on Bank 1. In other terms, the transfer made by Bank 1 to Bank 2 represents a positive externality for the gambling banks because it induces a positive switch, from the risky to the safe project, in the investment decision of Bank 2. The other banks have an incentive to over connect.

Finally, note that both the disconnected risky banks strictly gain from connecting to Bank 2. The disconnected banks have an expected payoff equal to $\xi Rf(1)(\varepsilon + 1) + B - 1$ if they connect to Bank 2 as opposed to the lower expected payoff $\xi R(\varepsilon +$

18. More elaborated numerical examples are available upon request.

$1) + B - 1$ if disconnected. Overall, the CFB cannot be an equilibrium because two banks that should not be connected would gain from linking by free-riding on Bank 1's transfer.

Let us illustrate with this example why transfers *only* contingent on the network formation process (*a lá* Bloch and Jackson 2007) are not enough to restore the CFB network. Assume risky banks can propose a transfer to the safe Bank 1 conditional on being connected to Bank 1. This transfer can be interpreted as a repayment with high interest in case the risky project succeed. Likely the safe bank may find it convenient to link to only one of the risky banks *but* the investment behavior of the linked risky bank will not change, because its bank capital does not increase. The CFB (including the proper investment decision) cannot be reached. It is true that the CFB network structure, that is, two banks linked, can be obtained but the two banks will not invest *both* in the risk-free project. It could be tempting to conclude that at least the externality due to the network formation process is solved, as the network structure coincides with the CFB one, although not the investment decision. However, this is *not* true in general.

Let us slightly modify the example by assuming that the total capital E is such that $3I^*(2, 0, \xi, \eta) \leq E < 4I^*(3, 0, \xi, \eta)$. In this case, the CFB consists of three safe banks in the core and one risky bank in the periphery disconnected from the core. Assume again only transfers conditional on the network formation process are possible. The three risky banks can then propose a transfer to the safe bank conditional on being connected to Bank 1. Again Bank 1 will accept to connect to the risky banks but the risky banks will not change investment behavior. Because η is small enough that it is not convenient to connect a risky bank to a safe bank, *a fortiori* a risky bank is not willing to connect to another risky bank and their connection will not be created. In this case, the equilibrium network is a star where the safe Bank 1 is the center and the three gambling banks are connected to it. However, the CFB network is made of a core with three safe banks and a disconnected risky periphery bank. Hence, in general, the CFB is not supported when *only* transfers conditional on the network formation are possible. In our model, transfers that are conditional on the investment decision are necessarily needed to have hope to restore efficiency. However, this example has shown that also such transfers alone cannot guarantee the CFB network.

Underconnected decentralized network. Consider the same four banks, and assume that $3I^*(2, 0, \xi, \eta) \leq E < 4I^*(3, 0, \xi, \eta)$ so that the CFB interbank network has three banks in the core. In the CFB, Bank 1 makes a transfer to two neighbors that is sufficient to make them switch the investment decision from the risky to the risk-free project. What happens if Bank 1 deletes one of his connections in the CFB? For some values of the parameters, Bank 1 still transfers the exact amount of capital that induces to switch the investment decision of the remaining connected bank (so transfers are correct) but it severs the connection and the relative transfer with the other bank. The reason is that in the CFB the planner guarantees that the expected gains of the two core banks receiving the transfers compensate for the expected loss that Bank 1 suffers, such that the three banks are better off than in autarky. This is not incompatible with the fact that Bank 1 individually prefers being connected to one bank (making only one transfer) than being linked to two banks (making two transfers), even though the latter situation is better than autarky. The CFB cannot be an equilibrium, as Bank 1 has an incentive to delete one of its links.

Overall assessment. Our examples show that, even allowing for transfers conditional on the investment decisions, the equilibrium networks feature either too many or too few connections with respect to the CFB network. Therefore, such transfers do not guarantee in general that the CFB network is sustained in equilibrium. We also show that transfers conditional only on the network formation are not effective in our model. The overall conclusion is that we need transfers that affect *both* the network formation process *and* the investment decision in order to sustain CFB network structures in equilibrium. How realistic is such possibility? Bloch and Jackson (2007) must consider a wide array of transfers contingent on the network formation process (transfers can be proposed to all the agents in order to subsidize the formation of a link and to incentivate the severance of a link). We consider such transfers contingent on the network formation *coupled* with the capital transfers affecting investment decision to be too demanding from a practical point of view. It clearly requires a high level of coordination among all the banks in the system even before the network is formed and investment decisions are taken. If however such coordination is considered to be feasible, then inefficiencies both in the investment decision and in the network formation process are less of a concern.

4. CONCLUSIONS

We present a model of interbank network formation and characterize the set of optimal networks as core-periphery structures. The decentralized interbank networks also show a core-periphery structure. However, when the counterparty risk is sufficiently high the decentralized core-periphery interbank network may be underconnected with respect to the optimal one. The root of the inefficiency is that a bank's investment choice has a direct effect on the expected payoff of its neighbors in the interbank network. This network externality in general is not internalized even if bank capital transfers conditional on the investment decision are allowed. A more comprehensive set of transfers, including those conditional on the network formation process, should be feasible for banks to avoid inefficiency.

Some of our modeling choices need some qualification. First, our exogenous assumptions on the benefits and costs of connecting to the interbank network allow us to sharp our analysis on the optimal network structure and the decentralized network formation. Even if we provide, in our opinion, a reasonable justification for these assumptions, a better understanding of the microfoundations that are able to rationalize such benefits and costs is clearly needed.

Second, we assume that all banks have access to the same coinsurance independently of the connections of their counterparts. Consider, for example, a network in which bank 1 and bank 2 are disconnected but both connected to bank 3, and another network in which the three banks are directly connected. In our setting, bank 3 has the same access to coinsurance in both networks. However, there could be reasons to believe that bank 3 coinsurance becomes weaker when also bank 1 and bank 2 are directly connected.[19] Including this possibility in our model represents a challenge for future research on interbank network formation.

Finally, it is consistently observed across countries that large banks are in the core of interbank networks. Our model, like many in this literature, features homogeneous bank size so it cannot be directly related to this evidence. However, as long as size can be considered a proxy for safety our model seems to be supported by this evidence. Indeed, a larger size is related to better diversification and therefore lower risk. Also bigger banks exploit the implicit guarantee due to the too-big-to-fail policy. Clearly, both more sophisticated models and, because of the evident endogeneity issues between size and risk, more detailed evidence are needed to shed light on this challenging research agenda.

APPENDIX: PROOFS

Proof of Proposition 1. The existence of a CFB can be proven in two steps. First, for given network K and vector of investments s, we maximize the objective function with respect to the vector of capital x subject to

$$x_i \geq 0 \quad \text{for all } i \in N, \tag{A1}$$

$$\sum_{i \in N} x_i = \sum_{i \in N} e_i \equiv E, \tag{A2}$$

$$x_i \geq I^*(k_i, g_i, \xi, \eta), \quad \text{if } s_i = rf, \tag{A3}$$

$$x_i < I^*(k_i, g_i, \xi, \eta) \quad \text{if } s_i = r. \tag{A4}$$

Second, once the maximum total expected payoff is obtained given K and s , it suffices to choose the combination of K and s that delivers the highest expected total payoff. Note that the second step does not create problems of existence as we are choosing the highest number on a discrete set of numbers. However, to guarantee that the set of constraints define a closed set in $\Re^n$ in the first step, the problematic restriction is (A4). Let us modify the planner problem by rewriting such constraint

19. Castiglionesi and Wagner (2013) provide a three-bank model where this issue is analyzed.

as

$$x_i \le I^*(k_i, g_i, \xi, \eta) \quad \text{if } s_i = r.$$

In the modified problem, a maximum always exists. Furthermore, the solution of the modified problem is also a maximum of the social planner's problem. If this were not true, then there is an economy for which the solution (K^*, x^*, s^*) of the modified problem is such that $x_i^* = I^*(k_i, g_i, \xi, \eta)$ for at least one bank i with $s_i = r$. But if $x_i^* = I^*(k_i, g_i, \xi, \eta)$ for at least one bank i, then the triple (K^*, x^*, s'), where s' and s^* differ only on the choice of investment by bank i, is also feasible and yields a higher expected payoff to at least bank i and its neighbors (and no bank gets lower expected payoff) than in s^*. Therefore, (K^*, x^*, s^*) could not have been a maximum of the modified problem, which is a contradiction.

Let us now consider the second statement of the proposition. We have to check that both the participation and incentive constraints have to be satisfied. First, note that if $\sum_{i\in N} e_i = E \ge \overline{E}$ there is enough bank capital to satisfy the incentive constraint in all banks. This is because $\overline{E} \ge nI^*(n-1, 0, \xi, \eta)$.

Regarding the participation constraints, note that the higher the initial capital endowment of a bank, the harder it is for the planner to satisfy the participation constraint. The most difficult case is when one bank is endowed with all the bank capital in the economy E and the remaining banks have 0 bank capital. The banks with 0 bank capital choose the risky project given that $B > (1-\xi)R$, or, equivalently, $I^*(0, 0, \xi, \eta) > 0$. We separate two cases depending on the type investment project chosen in autarky by the all-endowed bank.

If the "all-endowed" bank chooses the risk-free project, then the planner needs to give $\max\{\frac{B}{(1-\xi)\rho R} - 1, \frac{E+1}{\rho} - 1\}$ dollars of bank capital to the all-endowed bank and $\frac{B}{(1-\xi)\rho R} - 1$ to each of the other banks. Given that $B > (1-\xi)\rho R > (1-\xi)R$, this redistribution of capital gives an expected payoff equal to $\frac{B}{1-\xi} - 1$ to each gambling bank, which is higher than the expected payoff in autarky equal to $\xi R + B - 1$. Note that if the all-endowed bank chooses the risk-free project in autarky, we have $\frac{E+1}{\rho} \ge \frac{B}{(1-\xi)\rho R}$. Therefore, the planner needs E to be at least equal to $\frac{E+1}{\rho} - 1 + (n-1)(\frac{B}{(1-\xi)\rho R} - 1)$, or, rearranging

$$E \ge \frac{(n-1)}{(\rho-1)}\frac{\rho B}{(1-\xi)R} - \frac{n\rho - 1}{\rho - 1} = \frac{n-1}{\rho-1} I^*(0, 0, \xi, \eta) - n.$$

If the all-endowed bank chooses the risky project, then $E < \frac{B}{(1-\xi)R} - 1$, which implies $\frac{B}{(1-\xi)R}(n-\rho) < n\rho - \rho$. The planner now needs to give $\max\{\frac{B}{(1-\xi)\rho R} - 1, \xi\frac{E+1}{\rho} + \frac{B}{\rho R} - 1\}$ of bank capital to the all-endowed bank and $\frac{B}{(1-\xi)\rho R} - 1$ to each of the remaining banks. As before, given that $B > (1-\xi)R$, this reallocation of capital guarantees an expected payoff of $\frac{B}{1-\xi} - 1$ to each gambling bank, which is higher than the expected payoff in autarky $\xi R + B - 1$. Note that if the all-endowed bank chooses the risky project in autarky, it is $\xi \frac{E+1}{\rho} + \frac{B}{\rho R} < \frac{B}{(1-\xi)\rho R}$. Therefore, the planner needs E to be at least equal to $n(\frac{B}{(1-\xi)\rho R} - 1) = nI^*(n-1, 0, \xi, \eta)$.

If the banks had the same bank capital endowment, it is easy to check that the condition for INE is sufficient to induce shareholders of all banks to participate in the first best network with $x_i = e_i$.

Finally, assume $E < I^*(n-1, 0, \xi, \eta) = \frac{B}{(1-\xi)\rho R} - 1$, then $E < \frac{B}{(1-\xi)\eta^{g/k} f(k)} - 1$ for any $k \in [1, n-1)$ and $g \leq k$. This means that at any INE no bank that is connected can choose the safe project, even if the planner pools all capital into one bank. Given that f is increasing, *if* it is optimal to have some banks connected, then it is optimal to have all of them connected as $\eta f(k) \leq \eta\rho$. It is optimal to have all gambling banks connected (so $k = g = n-1$) when $\eta\rho \geq 1$. Nevertheless, it is optimal to keep banks disconnected if $1 > \eta\rho$ because the empty network yields a higher expected payoff than the complete network. □

To prove the following propositions, it is useful to make use of reallocations of bank capital that are payoff equivalent to the shareholders' expected payoff in autarky. We indicate the expected payoff in autarky for bank i as $m_i^A = \max\{R(e_i+1) - 1, \xi R(e_i+1) + B - 1\}$. For given K and s, let $x_i^A(K, s)$ be such that $m_i(K, x_i^A(K, s), s) = m_i^A$. That is, $x_i^A(K, s)$ is a reallocation of bank capital that makes bank i indifferent between participating or not in the network K with strategy profile s. Note that the reallocation of bank capital $x_i^A(K, s)$ is unique given (K, s).

Proof of Proposition 2. The proof is made by contradiction. Assume that (K^*, x^*, s^*) is a CFB for (N, e) but there are two disconnected banks i and j such that $s_i^* = s_j^* = rf$. Take a new allocation $(\hat{K}, x^*, s^*)$ for the same economy (N, e), where the allocation of bank capital and strategies are the same but the new network structure adds the link between banks i and j. Formally, $\hat{K}_i = K_i^* \cup \{j\}$, $\hat{K}_j = K_j^* \cup \{i\}$, and $\hat{K}_b = K_b^*$ for all banks $b \neq i, j$. We show now that the allocation $(\hat{K}, x^*, s^*)$ is an INE: it satisfies the participation constraints in any bank and it yields a higher expected total payoff. Therefore, the initial allocation (K^*, x^*, s^*) cannot be a solution to the planner's problem and Proposition 2 follows.

Note that, by definition of $\hat{K}$, $p_b(\hat{K}, s^*) = p_b(K^*, s^*)$ for all $b \in N$, and

$$\hat{k}_b = \begin{cases} k_b^* + 1 & \text{if } b = i \text{ or } b = j \\ k_b^* & \text{otherwise.} \end{cases}$$

Note that (K^*, x^*, s^*) is an INE given that it is a CFB. This means, because both i and j are choosing the risk-free project, that $x_i^* \geq I^*(k_i^*, g_i, \xi, \eta)$ and $x_j^* \geq I^*(k_j^*, g_j, \xi, \eta)$. By definition of the functions I^*, it is true then that $x_i^* \geq I^*(k_i^* + 1, g_i, \xi, \eta)$ and $x_j^* \geq I^*(k_j^* + 1, g_j, \xi, \eta)$. Therefore, the allocation $(\hat{K}, x^*, s^*)$ is an INE.

Given that (K^*, x^*, s^*) satisfies the participation constraints for any bank, it is true that $x_i^* \geq x_i^A(K^*, s^*)$ and $x_j^* \geq x_j^A(K^*, s^*)$. Recall that: (i) $x_i^A(K^*, s^*) =$

$\frac{m_i^A+1}{\eta^{g_i/k_i} f(k_i^*)R} - 1$ and $x_j^A(K^*, s^*) = \frac{m_j^A+1}{\eta^{g_j/k_j} f(k_j^*)R} - 1$; (ii) $x_i^A(\hat{K}, s^*) = \frac{m_i^A+1}{\eta^{g_i/(k_i+1)} f(k_i^*+1)R} - 1$ and $x_j^A(\hat{K}, s^*) = \frac{m_j^A+1}{\eta^{g_j/(k_j+1)} f(k_j^*+1)R} - 1$. Given that $f(k)$ is increasing in k and η is a probability, it is true that $x_i^A(K^*, s^*) > x_i^A(\hat{K}, s^*)$ and $x_j^A(K^*, s^*) > x_j^A(\hat{K}, s^*)$. Therefore, $x_i^* \geq x_i^A(\hat{K}, s^*)$ and $x_j^* \geq x_j^A(\hat{K}, s^*)$. This means that the allocation $(\hat{K}, x^*, s^*)$ satisfies the shareholders' participation constraints in any bank.

Finally,

$$\sum_{b\in N}\left[m_b\left(\hat{K}, x_b^*, s^*\right) + M_b\left(\hat{K}, s^*\right)\right] - \sum_{b\in N}\left[m_b(K^*, x^*, s^*) + M_b\left(K^*, s^*\right)\right]$$
$$= R\left(x_i^*+1\right)\left[\eta^{\frac{g_i}{(k_i^*+1)}} f\left(k_i^*+1\right) - \eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right)\right] + R\left(x_j^*+1\right)\left[\eta^{\frac{g_j}{(k_j^*+1)}} f(k_j^*+1)\right.$$
$$\left. - \eta^{\frac{g_j}{k_j^*}} f\left(k_j^*\right)\right] > 0$$

because $f(k)$ is increasing in k and η is a probability. Given the last inequality, the allocation $(\hat{K}, x^*, s^*)$ yields a higher expected payoff and therefore (K^*, x^*, s^*) was not a CFB. □

Proof of Proposition 3. We prove that if an allocation is an INE and it satisfies the shareholders' participation constraints in any bank, then it has to be that any bank choosing the risk-free project in autarky chooses the same project in the optimal allocation as well. Take any bank i such that $\max\{R(e_i+1)-1, \xi R(e_i+1)+B-1\} = R(e_i+1)-1$, that is, it chooses the safe project in autarky. Assume by contradiction that there exists an allocation (K, x, s) that is an INE, it satisfies the shareholders' participation constraints for any bank and $s_i = r$. This implies

(i) $x_i < I^*(k_i, g_i, \xi, \eta)$ because (K, x, s) is an INE, and
(ii) $\xi\eta^{g_i/k_i} f(k_i)R(x_i+1) + B - 1 \geq R(e_i+1) - 1$ because (K, x, s) satisfies the participation constraint.

Given that bank i chooses the risk-free project in autarky, we have $R(e_i+1) \geq \frac{B}{(1-\xi)}$. The last condition together with the participation constraint for bank i (item 2 above) implies

$$\xi\eta^{g_i/k_i} f(k_i)R(x_i+1) + B \geq \frac{B}{(1-\xi)},$$

and, after rearranging, we have

$$x_i \geq \frac{B}{(1-\xi)\eta^{g_i/k_i} f(k_i)R} - 1 = I^*(k_i, g_i, \xi, \eta),$$

a contradiction with (K, x, s) being an INE (item 1 above). Therefore, any bank i choosing the risk-free project in autarky chooses the same type of project in any INE satisfying the shareholders' participation constraint (for at least that bank i). □

PROPOSITION 4 (Long version). *Assume that* $\min e_i < \underline{E} \leq E < \overline{E}$ *and* $f(k)$ *satisfies the IPR property. Let* (K^*, x^*, s^*) *be a CFB allocation for a given economy* (N, e). *Let* $k(\eta)$ *be as defined in* (10). *Then*:

(1) *If* $k(\eta) = n - 1$, *then* $K_i^* = N\backslash\{i\}$ *for all* $i \in N$.
(2) *If* $k(\eta) = 0$, *then* $g_i^* = 0$ *for all* i *such that* $s_i^* = r$.
(3) *If* $k(\eta) \in (0, n-1)$, *then*:

(a) $k_i^* < k(\eta)$ *for some* i *implies that* $s_i^* = r$ *and* $k_j^* \geq k(\eta)$ *for all* $j \notin K_i^*$.
(b) $k_i^* > k(\eta)$ *and* $s_i^* = r$ *implies that for all* $j \in G_i$:

(bi) $k_j \leq k(\eta)$;
(bii) *there is no other bank* b *with* $k_b^* < k(\eta)$ *and* $b \notin K_j$;
(biii) for any other bank b *with* $k_b^* > k(\eta)$: $s_b^* = r$ *and there is no bank* $z \in G_b$ *such that* $z \notin K_j$.

In order to prove Proposition 4 (Long version), it is useful to prove the following lemma.

LEMMA A1. *Let* $\underline{k}(\eta)$ *be the highest* $k \in \{1, \ldots, n-1\}$ *such that* $\eta \geq (\frac{f(k-1)}{f(k)})^k$ *and let* $\overline{k}(\eta)$ *be the lowest* $k \in \{0, 1, \ldots, n-2\}$ *such that* $\eta < (\frac{f(k)}{f(k+1)})^{k+1}$. *Then:*

(1) If (K^*, x^*, s^*) *is a CFB allocation, then,* $k_i^* < \underline{k}(\eta)$ *for some* i *implies that* $s_i^* = r$ *and* $k_j^* \geq \underline{k}(\eta)$ *for all* $j \notin K_i^*$.
(2) If (K^*, x^*, s^*) *is a CFB allocation, then,* $k_i^* > \overline{k}(\eta)$ *and* $s_i^* = r$ *implies that for all* $j \in G_i$:
(a) $k_j \leq \overline{k}(\eta)$;
(b) there is no other bank b *with* $k_b^* < \overline{k}(\eta)$ *and* $b \notin K_j$;
(c) for any other bank b *with* $k_b^* > \overline{k}(\eta)$, $s_b^* = g$ *there is no bank* $z \in G_b$ *such that* $z \notin K_j$.

PROOF OF LEMMA A1. We prove all statements by contradiction.

Proof of Statement 1. Assume that (K^*, x^*, s^*) is a CFB allocation where there are two banks i and j not directly connected and such that $k_i < \underline{k}(\eta)$ and $k_j < \underline{k}(\eta)$. Take a new allocation $(\hat{K}, x^*, s^*)$ for the same economy (N, e), where the allocation of capital and strategies are the same but the network structure adds the link between banks i and j. Formally, $\hat{K}_i = K_i^* \cup \{j\}$, $\hat{K}_j = K_j^* \cup \{i\}$, and $\hat{K}_b = K_b^*$ for all $b \neq i, j$.

We show first that the participation constraint is satisfied by $(\hat{K}, x^*, s^*)$ for all banks. Later, we show that $(\hat{K}, x^*, s^*)$ yields a higher total expected payoff than (K^*, x^*, s^*). Therefore, the initial allocation (K^*, x^*, s^*) cannot be a solution to the planner's problem and Statement 1 follows. We separate cases depending whether bank i chooses the risk-free or the risky project in the investment profile s^*.

Note that if (K^*, x^*, s^*) is a CFB, then it has to be an INE and it has to satisfy the shareholders participation constraints in any bank. If at least one of them, for example bank i, is choosing the risk-free project, then $x_i^* \geq I^*(k_i^*, g_i, \xi, \eta)$. The other bank, say j, is choosing the risky project. We know from Proposition 2 that both banks cannot invest in the risk-free project because in such a case (K^*, x^*, s^*) would not be a CFB allocation.

We note that $I^*(k_i^*, g_i, \xi, \eta) \geq I^*(k_i^* + 1, g_i + 1, \xi, \eta)$ if and only if $\eta^{\frac{g_i+1}{k_i^*+1}} f(k_i^* + 1) \geq \eta^{\frac{g_i}{k_i^*}} f(k_i^*)$, or, equivalently, $\eta^{\frac{k_i^*-g_i}{k_i^*(k_i^*+1)}} \geq \frac{f(k_i^*)}{f(k_i^*+1)}$. Given that the ratio $(\dfrac{f(k_i)}{f(k_i+1)})^{k_i+1}$ is increasing in k_i we have that

$$\left(\frac{f\left(\underline{k}(\eta) - 1\right)}{f\left(\underline{k}(\eta)\right)}\right)^{\underline{k}(\eta)} \geq \left(\frac{f(k_i)}{f(k_i + 1)}\right)^{k_i+1} \text{ for any } k_i \leq \underline{k}(\eta) - 1,$$

$\eta < 1$ and $\frac{k_i^*-g_i}{k_i^*(k_i^*+1)} < \frac{1}{(k_i^*+1)}$. Hence, $\eta > (\frac{f(\underline{k}(\eta)-1)}{f(\underline{k}(\eta))})^{\underline{k}(\eta)}$ implies that $\eta > (\dfrac{f(k_i^*)}{f(k_i^*+1)})^{k_i^*+1}$ because $k_i^* \leq \underline{k}(\eta) - 1$. The latter inequality implies in turn that $\eta^{\frac{k_i^*-g_i}{k_i^*(k_i^*+1)}} \geq \frac{f(k_i^*)}{f(k_i^*+1)}$. Therefore, the allocation $(\hat{K}, x^*, s^*)$ is an INE as far as $x_j < I^*(k_i + 1, g_i, \xi, \eta)$. If it were not, there is an allocation $(\hat{K}, x^*, \hat{s})$ in which bank j chooses the risk-free project instead of the risky one. This would be an INE for bank i because $I^*(k_i^* + 1, g_i + 1, \xi, \eta) > I^*(k_i^* + 1, g_i, \xi, \eta)$.

Participation constraints. Assume that bank i chooses the risk-free project in (K^*, x^*, s^*). If $\eta \geq (\frac{f(\underline{k}(\eta)-1)}{f(\underline{k}(\eta))})^{\underline{k}(\eta)}$, then $\eta^{\frac{1}{k_i^*+1}} f(k_i^* + 1) > f(k_i^*)$ for $k_i < \underline{k}(\eta)$. As $\frac{g_i+1}{k_i^*+1} - \frac{g_i}{k_i^*} = \frac{k_i^*-g_i}{k_i^*(k_i^*+1)} < \frac{1}{k_i^*+1}$, the latter inequality implies

$$\eta^{\frac{g_i+1}{k_i^*+1}} f\left(k_i^* + 1\right) R\left(x_i^* + 1\right) - 1 > \eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right) R\left(x_i^* + 1\right) - 1.$$

Given that (K^*, x^*, s^*) satisfies the participation constraints for any bank, it has to be that $\eta^{\frac{g_i}{k_i^*}} f(k_i^*)R(x_i^* + 1) - 1 \geq m_i^A$, and therefore, $(\hat{K}, x^*, s^*)$ satisfies the participation constraints for bank i.

If bank i chooses the risky project in (K^*, x^*, s^*), the argument is equivalent. Note that if $\eta \geq (\frac{f(\underline{k}(\eta)-1)}{f(\underline{k}(\eta))})^{\underline{k}(\eta)}$, then $\eta^{\frac{1}{k_i^*+1}} f(k_i^* + 1) > f(k_i^*)$ for $k_i < \underline{k}(\eta)$. As $\frac{g_i+1}{k_i^*+1} - \frac{g_i}{k_i^*} = \frac{k_i^*-g_i}{k_i^*(k_i^*+1)} < \frac{1}{k_i^*+1}$, the latter inequality implies

$$\xi\eta^{\frac{g_i+1}{k_i^*+1}} f\left(k_i^* + 1\right) R\left(x_i^* + 1\right) + B - 1 \geq \xi\eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right) R\left(x_i^* + 1\right) + B - 1.$$

Given that (K^*, x^*, s^*) satisfies the participation constraints for any bank, it has to be that $\xi \eta^{\frac{g_i}{k_i^*}} f(k_i^*)R(x_i^*+1) + B - 1 \geq m_i^A$, and therefore, $(\hat{K}, x^*, s^*)$ satisfies the participation constraint for bank i.

Checking that the participation constraint for bank j, which chooses the gambling project in (K^*, x^*, s^*), is satisfied in $(\hat{K}, x^*, s^*)$ works the same way as in the previous case and it is therefore omitted.

Total expected payoff. If bank i chooses the risk-free project in (K^*, x^*, s^*) we have

$$\sum_{b\in N}\left[m_b\big(\hat{K}, x_i^*, s^*\big) + M_b\big(\hat{K}, s^*\big)\right] - \left[m_b\big(K^*, x_i^*, s^*\big) - M_b\big(K^*, s^*\big)\right] =$$

$$= \eta^{\frac{g_i}{k_i^*}} R\big(x_i^*+1\big)\left[\eta^{\frac{k_i^*-g_i}{k_i^*+1}} f\left(k_i^*+1\right) - f\left(k_i^*\right)\right]$$

$$+ \xi\eta^{g_j} R\big(x_j^*+1\big)\left[\eta^{\frac{1}{k_j^*+1}} f\left(k_j^*+1\right) - \eta^{\frac{1}{k_j^*}} f\left(k_j^*\right)\right]. \tag{A5}$$

If banks i and j both choose the risky project in (K^*, x^*, s^*), we have

$$\sum_{b\in N}\left[m_b\big(\hat{K}, x_i^*, s^*\big) + M_b\big(\hat{K}, s^*\big)\right] - \left[m_b\big(K^*, x_i^*, s^*\big) + M_b\big(K^*, s^*\big)\right] =$$

$$= \xi\eta^{\frac{g_i}{k_i^*}} R\big(x_i^*+1\big)\left[\eta^{\frac{k_i^*-g_i}{k_i^*+1}} f\left(k_i^*+1\right) - f\left(k_i^*\right)\right]$$

$$+ \xi\eta^{\frac{g_j}{k_j^*}} R\big(x_j^*+1\big)\left[\eta^{\frac{k_j^*-g_j}{k_j^*+1}} f\left(k_j^*+1\right) - f\left(k_j^*\right)\right]. \tag{A6}$$

Expressions (A5) and (A6) are greater than 0 because $1 > \eta \geq (\frac{f(k_i^*)}{f(k_i^*+1)})^{k_i^*+1}$ and $1 > \eta \geq (\frac{f(k_j^*)}{f(k_j^*+1)})^{k_j^*+1}$ for $k_i^* < \underline{k}(\eta)$ and $k_j^* < \underline{k}(\eta)$. Therefore, the allocation $(\hat{K}, x^*, s^*)$ yields a higher total expected payoff and therefore (K^*, x^*, s^*) is not a CFB.

Furthermore, equation (A5) implies that if $s_i^* = rf$ any bank $j \notin K_i^*$ would be better off connecting to bank i. Hence, there is no bank j such that $j \notin K_i^*$ if $s_i^* = rf$ and $k_i^* < k(\eta)$. But then $k_i^* = n-1$, a contradiction. Therefore, if $k_i^* < \underline{k}(\eta)$, then $s_i^* = r$ and $k_j^* > \underline{k}(\eta)$ for any bank $j \notin K_i^*$. Finally, if $(\hat{K}, x^*, s^*)$ is not an INE it is because bank j would choose the safe project as well, once $\hat{K}$ is given, or, when both banks i and j choose the risky project in (K^*, x^*, s^*), an INE would select at least one of the two banks to choose the safe project. In all these cases, the new INE will yield a higher expected payoff for both banks i and j. This implies that the participation constraint would be satisfied, and therefore a higher total expected payoff than in $(\hat{K}, x^*, s^*)$.

Proof of Statements 2(a), 2(b), and 2(c). Assume that (K^*, x^*, s^*) is a CFB allocation where there is at least one bank i with $s_i^* = r$ and $g_i^* \neq 0$. As before, we show that for all the three cases there is an allocation $(\hat{K}, x^*, s^*)$ that satisfies the participation constraint for any bank and yields an higher total expected payoff. In particular, the allocation $(\hat{K}, x^*, s^*)$ yields both individual and total expected payoffs higher than the allocation (K^*, x^*, s^*), which then cannot be a solution to the planner's problem.

Assume that statement 2(a) is not true and $k_j > \overline{k}(\eta)$ for some $j \in G_i^*$. Take a new allocation $(\hat{K}, x^*, s^*)$ where the bank capital and strategies are the same but the network structure removes the link between banks i and j. Formally, $\hat{K}_i = K_i^* \backslash \{j\}$, $\hat{K}_j = K_j^* \backslash \{i\}$, and $\hat{K}_b = K_b^*$ for all $b \neq i, j$.

Assume that statement 2(b) is not true and there is a bank $b \neq j$ with $k_b < \overline{k}(\eta)$ and b and j are not directly connected. Take a new allocation $(\hat{K}, x^*, s^*)$ where the bank capital and strategies are the same but the network structure removes the link between banks i and j and creates a link between banks j and b. Formally, $\hat{K}_i = K_i^* \backslash \{j\}$, $\hat{K}_j = K_j^* \backslash \{i\} \cup \{b\}$, and $\hat{K}_b = K_b^* \cup \{j\}$ and $\hat{K}_l = K_l^*$ for all $l \neq i, j, b$.

Finally, assume that statement 2(c) is not true and there is a bank $b \neq j$ with $k_b^* > \overline{k}(\eta)$ and $s_b^* = r$ such that one of its direct gambling neighbors z is not directly connected to j. Take a new allocation $(\hat{K}, x^*, s^*)$ where the bank capital and strategies are the same but the network structure severs the links between banks i and j, and between b and z, and it creates a link between banks j and z. Formally, $\hat{K}_i = K_i^* \backslash \{j\}$, $\hat{K}_j = K_j^* \backslash \{i\} \cup \{z\}$, $\hat{K}_b = K_b^* \backslash \{z\}$, and $\hat{K}_z = K_z^* \backslash \{b\} \cup \{j\}$ and $\hat{K}_l = K_l^*$ for all $l \neq i, j, b, z$.

Participation constraints. If $\eta < (\frac{f(\overline{k}(\eta))}{f(\overline{k}(\eta)+1)})^{\overline{k}(\eta)+1}$, then $\eta^{\frac{1}{k_i^*}} f(k_i^*) < f(k_i^* - 1)$ because $k_i^* > \overline{k}(\eta)$. This implies $\eta^{\frac{g_i-1}{k_i}} \times \eta^{\frac{1}{k_i^*}} f(k_i^*) < \eta^{\frac{g_i-1}{k_i}} \times f(k_i^* - 1)$, and hence

$$\xi \eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right) R\left(x_i^*+1\right)+B-1<\xi \eta^{\frac{g_i-1}{k_i^*-1}} f\left(k_i^*-1\right) R\left(x_i^*+1\right)+B-1,$$

or

$$\eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right)<\eta^{\frac{g_i-1}{k_i^*-1}} f\left(k_i^*-1\right). \tag{A7}$$

Given that (K^*, x^*, s^*) satisfies the participation constraints for any bank, it has to be that $\xi \eta^{\frac{g_i}{k_i^*}} f(k_i^*) R(x_i^* + 1) + B - 1 \geq m_i^A$, and therefore $(\hat{K}, x^*, s^*)$ satisfies the participation constraints for bank i. The proof is equivalent for bank j if statement 2(a) is considered and for bank b if statement 2(c) is considered. Note that bank j in statements 2(b) and 2(c) and bank z in statement 2(c) are indifferent between the allocations (K^*, x^*, s^*) and $(\hat{K}, x^*, s^*)$. Consider now the participation constraint for bank b in statement 2(b). Note that given the definition of $\overline{k}(\eta)$, it has to be that $\eta^{\frac{1}{k_b^*+1}} f(k_b^* + 1) > f(k_b^*)$ for $k_b^* < \overline{k}(\eta)$. This implies

$$\xi \eta^{\frac{g_b+1}{k_b^*+1}} f\left(k_b^*+1\right) R\left(x_b^*+1\right)+B-1>\xi \eta^{\frac{g_b}{k_b^*}} f\left(k_b^*\right) R\left(x_b^*+1\right)+B-1,$$

or

$$\eta^{\frac{g_b+1}{k_b^*+1}} f\left(k_b^*+1\right) > \eta^{\frac{g_b}{k_b^*}} f\left(k_b^*\right),$$

if b chooses the risky project. If bank b chooses the risk-free project, we have

$$\eta^{\frac{g_b+1}{k_b^*+1}} f\left(k_b^*+1\right) R\left(x_b^*+1\right) - 1 > \eta^{\frac{g_b}{k_b^*}} f\left(k_b^*\right) R\left(x_b^*+1\right) - 1,$$

or

$$\eta^{\frac{g_b+1}{k_b^*+1}} f\left(k_b^*+1\right) > \eta^{\frac{g_b}{k_b^*}} f\left(k_b^*\right).$$

Given that (K^*, x^*, s^*) satisfies the participation constraints for any bank, it has to be that $\xi \eta^{\frac{g_b}{k_b^*}} f(k_b^*) R(x_b^*+1) + B - 1 \geq m_b^A$ if bank b is gambling, or $\eta^{\frac{g_b}{k_b^*}} f(k_b^*) R(x_b^*+1) \geq m_b^A$, otherwise. Therefore, the allocation $(\hat{K}, x^*, s^*)$ satisfies the participation constraints for bank b as well.

Total expected payoff. Let

$$\sum_{l \in N} m_l\left(\hat{K}, x_l^*, s^*\right) - \sum_{l \in N} m_l\left(K^*, x_l^*, s^*\right) \equiv \Delta,$$

and, depending on which of the three cases is considered, we have:

Statement 2(a)

$$\Delta = \xi R\left(x_i^*+1\right)\left[\eta^{\frac{g_i-1}{k_i^*-1}} f\left(k_i^*-1\right) - \eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right)\right] + \xi R\left(x_j^*+1\right)\left[\eta^{\frac{g_j-1}{k_j^*-1}} f\left(k_j^*-1\right) - \eta^{\frac{g_j}{k_j^*}} f\left(k_j^*\right)\right],$$

where both $k_i^* > \overline{k}(\eta)$ and $k_j^* > \overline{k}(\eta)$;

Statement 2(b)

$$\Delta = \xi R\left(x_i^*+1\right)\left[\eta^{\frac{g_i-1}{k_i^*-1}} f\left(k_i^*-1\right) - \eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right)\right] + \xi R\left(x_b^*+1\right)\left[\eta^{\frac{g_b+1}{k_b^*+1}} f\left(k_b^*+1\right) - \eta^{\frac{g_b}{k_b^*}} f\left(k_b^*\right)\right],$$

where $k_i^* > \overline{k}(\eta)$ and $k_b^* < \overline{k}(\eta)$;

Statement 2(c)

$$\Delta = \xi R\left(x_i^*+1\right)\left[\eta^{\frac{g_i-1}{k_i^*-1}} f\left(k_i^*-1\right) - \eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right)\right] + \xi R\left(x_b^*+1\right)\left[\eta^{\frac{g_b-1}{k_b^*-1}} f\left(k_b^*-1\right) - \eta^{\frac{g_b}{k_b^*}} f\left(k_b^*\right)\right],$$

where both $k_i^* > \overline{k}(\eta)$ and $k_b^* > \overline{k}(\eta)$.

Applying the same reasoning used above, namely condition (A7) and the IPR property, it follows that $\Delta > 0$ in each of the three cases. □

PROOF OF PROPOSITION 4 (Long version). Assume first that $\underline{k}(\eta) = n - 1$. According to Statement 1 of Lemma A1, if there is any bank i with $k_i^* < n - 1$ then, for all bank j not directly connected to bank i , we have $k_j^* = n - 1$. But this is a contradiction, because any bank j with $n - 1$ connections has to be directly connected to bank i. Assume now that $\overline{k}(\eta) = 0$. According to Statement 2(a) of Lemma A1, if there is any bank i that invest in the risky project and it is directly connected to another gambling bank j, it has to be that $k_j^* = 0$. But again this is a contradiction with the fact that bank j is directly connected to bank i. Finally, assume $\underline{k}(\eta) \neq n - 1$ and $\overline{k}(\eta) \neq 0$. By definition, $\overline{k}(\eta)$ is the minimum number k such that $\eta^{\frac{1}{k+1}} f(k + 1) \leq f(k)$. Then it has to be that $\eta^{\frac{1}{k+1}} f(k + 1) > f(k)$ for $k \leq \overline{k}(\eta) - 1$. This follows because f satisfies the IPR property. □

PROOF OF PROPOSITION 5. Assume that bank i is choosing the risk-free project in the CFB allocation (K^*, x^*, s^*), with $g_i^* > 0$. As shown, if $\eta < \dfrac{1}{f(1)}$, we have $g_j = 0$ for any $j \in G_i$. Take a new allocation $(\hat{K}, \hat{x}, s^*)$ where the investment strategies are the same but (i) the bank capital given to every $j \in G_i$ is the autarky-equivalent payoff $x_j^A(\hat{K}, s^*)$, and (ii) the network structure severs all the risky links of bank i. Formally, (i) $\hat{x}_j = x_j^A(\hat{K}, s^*)$ for any $j \in G_i$, $\hat{x}_i = x_i^* + \sum_{j \in G_i}(x_j^* - \hat{x}_j)$, and $\hat{x}_b = x_b^*$ for all $b \notin G_i$, $b \neq i$, and (ii) $\hat{K}_i = K_i^* \backslash \{G_i\}$, $\hat{K}_j = K_j^* \backslash \{i\}$, for all $j \in G_i$, and $\hat{K}_b = K_b^*$ for all $b \notin G_i$, $b \neq i$. We show that the new allocation $(\hat{K}, \hat{x}, s^*)$ satisfies the participation constraints for any bank, it is an INE, and it yields a higher total expected payoff. Therefore, the initial allocation (K^*, x^*, s^*) cannot be a solution to the planner's problem.

Participation constraints. We consider only bank i, because for any bank $j \in G_i$ the investor participation constraints are satisfied by definition. For any other bank, the participation constraints are satisfied because the allocation (K^*, x^*, s^*) satisfies the participation constraints. For shareholders in bank i, we need to show that

$$x_i^* + \sum_{j \in G_i} \left(x_j^* - \hat{x}_j\right) \geq x_i^A\left(\hat{K}, s^*\right) \quad \text{for } \eta < \left(\frac{1 - \xi}{1 + n\rho}\right)^{n-1}.$$

Recall that, given that (K^*, x^*, s^*) satisfies the participation constraint, it has to be that $x_i^* \geq x_i^A(K^*, s^*)$ and $x_j^* \geq x_j^A(K^*, s^*)$ for any $j \in G_i$. Therefore,

$$x_i^* + \sum_{j \in G_i} \left(x_j^* - \hat{x}_j\right) \geq x_i^A\left(K^*, s^*\right) + \sum_{j \in G_i} \left(x_j^A\left(K^*, s^*\right) - \hat{x}_j\right).$$

Note that (K^*, x^*, s^*) is an INE satisfying the participation constraint. By Proposition 3, any $j \in G_i$ chooses the risky project in autarky. This means that

$x_j^A(K^*, s^*) = \frac{e_j+1}{f(k_j^*)} - 1$ and $x_j^A(\hat{K}, s^*) = \frac{e_j+1}{f(k_j^*-1)} - 1$ for any $j \in G_i$. Because bank i chooses the risk-free project, we have $x_i^A(K^*, s^*) = \frac{m_i^A+1}{\eta^{\frac{g_i}{k_i^*}} f(k_i^*)R} - 1$ and $x_i^A(\hat{K}, s^*) = \frac{m_i^A+1}{\eta^{\frac{g_i}{k_i^*-g_i}} f(k_i^*-g_i)R} - 1$, with $m_i^A = \max\{R\,(e_i+1) - 1, \xi R\,(e_i+1) + B - 1\}$.

Consider first $x_j^A(K^*, s^*)$ for any $j \in G_i$. Because (K^*, x^*, s^*) is an optimal allocation it has to be a core-periphery structure. Furthermore, $g_j = 0$ for any $j \in G_i$ given that $\eta < \frac{1}{f(1)}$. Then, $k_j^* \leq k_i^*$ for any $j \in G_i$. This is so given that (i) bank i is connected to all other banks choosing the safe project and to all banks in G_i, that is $k_i^* = c^* - 1 + g_i$ (with c^* being the number of banks in the core); (ii) any bank j choosing the gambling project can be connected at most to all the banks choosing the safe project (and no bank choosing the gambling project), that is $k_j^* \leq c^*$. Then, because $g_i \geq 1$, we have $k_j^* \leq k_i^*$ and therefore

$$x_j^A\left(K^*, s^*\right) = \frac{e_j+1}{f(k_j^*)} - 1 \geq \frac{e_j+1}{f(k_i^*)} - 1 \quad \text{for any } j \in G_i. \tag{A8}$$

Consider now $x_j^A(\hat{K}, s^*)$ for any $j \in G_i$. Note that

$$x_j^A\left(\hat{K}, s^*\right) = \frac{e_j+1}{f\left(k_j^*-1\right)} - 1 \leq e_j \quad \text{for any } j \in G_i,$$

given that $f(k_j^*-1) \geq 1$ for any $k_j^* \geq 1$. Then we have

$$x_i^A\left(K^*, s^*\right) + \sum_{j\in G_i}\left(x_j^A\left(K^*, s^*\right) - \hat{x}_j\right) \geq \frac{m_i^A+1}{\eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right)R}$$
$$-1 + \sum_{j\in G_i}\left(\frac{e_j+1}{f\left(k_i^*\right)} - 1 - e_j\right). \tag{A9}$$

Note that

$$\frac{m_i^A+1}{\eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right)R} - 1 + \sum_{j\in G_i}\left(\frac{e_j+1}{f\left(k_i^*\right)} - 1 - e_j\right) \geq \frac{m_i^A+1}{f\left(k_i^*-g_i\right)R} - 1$$
$$= x_i^A\left(\hat{K}, s^*\right),$$

or

$$\frac{m_i^A+1}{R}\left[\frac{1}{\eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right)} - \frac{1}{f\left(k_i^*-g_i\right)}\right] \geq \frac{f\left(k_i^*\right)-1}{f\left(k_i^*\right)}\sum_{j\in G_i}(e_j+1),$$

given that (i) $\frac{m_i^A+1}{R}[\frac{1}{\eta^{\frac{g_i}{k_i^*}} f(k_i^*)} - \frac{1}{f(k_i^*-g_i)}] \geq \frac{B}{R} \times \frac{1-\eta^{\frac{1}{n-1}}\rho}{\eta^{\frac{1}{n-1}}\rho}$, because $m_i^A + 1 \geq B$, (ii) $1 \leq f(.) \leq \rho$, (iii) $\frac{f(k_i^*)-1}{f(k_i^*)} \sum_{j \in G_i} (e_j + 1) \leq \frac{B}{R} \times \frac{n-1}{1-\xi}$, given that by Proposition 3 we know that any $j \in G_i$ chooses the risky project in autarky and $f(k_i^*) - 1 \leq f(k_i^*)$, and (iv) $\eta^{\frac{1}{n-1}}\rho < \frac{1-\xi}{n-\xi}$ if $\eta^{\frac{1}{n-1}}\rho < \frac{1}{(1+n\rho)}$ because $\frac{1-\xi}{(1+n\rho)} < \frac{1-\xi}{n-\xi}$.

The allocation $(\hat{K}, \hat{x}, s^*)$ *is an INE*. If (K^*, x^*, s^*) is a CFB, then it has to be an INE. Consider any player $j \in G_i$. Given that (K^*, x^*, s^*) is an INE, it has to be that

$$x_j^* < \frac{B}{(1-\xi) f(k_j^*) R} - 1,$$

for any $j \in G_i$, because $s_j^* \in G_i$ and $g_j = 0$. From equation (A8), $x_j^* \geq \frac{e_j+1}{f(k_j^*)} - 1$. The two last inequalities imply

$$e_j < \frac{B}{(1-\xi)R} - 1. \tag{A10}$$

By definition of $\hat{x}$, we have $\hat{x}_j = x_j^A(\hat{K}, s^*) = \frac{e_j+1}{f(k_j^*-1)} - 1$. From (A10), $\frac{e_j+1}{f(k_j^*-1)} - 1 < \frac{B}{(1-\xi) f(k_j^*-1) R} - 1$, and therefore $\hat{x}_j < I^*(k_j^* - 1, 0, \xi, \eta)$. Consider now bank i and recall that $\hat{x}_i = x_i^* + \sum_{j \in G_i} (x_j^* - \hat{x}_j)$. From equation (A9), we know that

$$x_i^* + \sum_{j \in G_i} \left(x_j^* - \hat{x}_j\right) \geq \frac{m_i^A + 1}{\eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right) R} - 1 + \sum_{j \in G_i} \left(\frac{e_j + 1}{f\left(k_i^*\right)} - 1 - e_j\right).$$

We prove that

$$\frac{m_i^A + 1}{\eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right) R} - 1 + \sum_{j \in G_i} \left(\frac{e_j + 1}{f\left(k_i^*\right)} - 1 - e_j\right) \geq \frac{B}{(1-\xi)R} - 1 \tag{A11}$$

because $\frac{B}{(1-\xi)R} - 1 \geq \frac{B}{(1-\xi) f(k_i^*-g_i) R} - 1 = I^*(k_i^* - g_i, 0, \xi, \eta)$. Rearranging terms, equation (A11) is equivalent to

$$\frac{m_i^A + 1}{\eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right) R} \geq \frac{B}{(1-\xi)R} + \frac{f\left(k_i^*\right) - 1}{f\left(k_i^*\right)} \sum_{j \in G_i} (e_j + 1).$$

Note that $m_i^A + 1 \geq B$ and $\eta^{\frac{g_i}{k_i^*}} f(k_i^*) R \leq \eta^{\frac{1}{n-1}} \rho R$. Thus,

$$\frac{m_i^A + 1}{\eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right) R} \geq \frac{B}{\eta^{\frac{1}{n-1}} \rho R} \geq n \frac{B}{(1-\xi)R},$$

the last inequality being true for $\eta^{\frac{1}{n-1}}\rho < \frac{1-\xi}{n}$. By assumption, $\eta^{\frac{1}{n-1}}\rho < \frac{1-\xi}{1+n\rho}$, with $\frac{1-\xi}{1+n\rho} < \frac{1-\xi}{n}$. Finally, we have

$$n\frac{B}{(1-\xi)R} \geq \frac{B}{(1-\xi)R} + \frac{f\left(k_i^*\right)-1}{f\left(k_i^*\right)}\sum_{j\in G_i}(e_j+1),$$

given that $\frac{f(k_i^*)-1}{f(k_i^*)}\sum_{j\in G_i}(e_j+1) < g_i\frac{B}{(n-1)R}$, because $e_j < \frac{B}{(1-\xi)R} - 1$ for any $j \in G_i$, and that $g_i \leq (n-1)$.

Expected total payoff. We need to prove that

$$\begin{aligned}
&\sum_{b\in N}\left[m_b(\hat{K},\hat{x},s^*) + M_b(\hat{K},s^*)\right] - \sum_{b\in N}\left[m_b(K^*,x^*,s^*) + M_b(K^*,s^*)\right] = \\
&= Rf\left(k_i^* - g_i\right)(\hat{x}_i+1) - \eta^{\frac{g_i}{k_i^*}} f(k_i^*)R\left(x_i^*+1\right) + \\
&+ \xi\sum_{j\in G_i} f(k_j^*-1)R\left(\hat{x}_j+1\right) - \xi\sum_{j\in G_i} f(k_j^*)R\left(x_j^*+1\right) > 0,
\end{aligned}$$

for $\eta < (\frac{1-\xi}{\rho(1+n\rho)})^{n-1}$. Recall that $\hat{x}_i = x_i^* + \sum_{j\in G_i}(x_j^* - \hat{x}_j)$, where $\hat{x}_j = \frac{e_j+1}{f(k_j^*-1)} - 1$. Rearranging terms, we have to prove that

$$\begin{aligned}
&\left[f\left(k_i^*-g_i\right) - \eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right)\right]\left(x_i^*+1\right) + \sum_{j\in G_i}\left[f\left(k_i^*-g_i\right) - \xi f\left(k_j^*\right)\right]x_j^* - \\
&- \sum_{j\in G_i}\left[f\left(k_i^*-g_i\right) - \xi f\left(k_j^*-1\right)\right]\hat{x}_j > \xi\sum_{j\in G_i}\left[f\left(k_j^*\right) - f\left(k_j^*-1\right)\right].
\end{aligned}$$

First note that $k_i^* = c^* - 1 + g_i \geq c^* \geq k_j^*$, where c^* is the number of core banks. Given that $\xi < \eta < \frac{1}{f(1)}$, and that the ratio $\frac{f(k)}{f(k+1)}$ is increasing in k—f is concave—we have

$$\sum_{j\in G_i}\left[f\left(k_i^*-g_i\right) - \xi f\left(k_j^*\right)\right]x_j^* \geq \sum_{j\in G_i}\left[f\left(k_j^*-1\right) - \xi f\left(k_j^*\right)\right]x_j^* \geq 0.$$

Furthermore, $f(k_i^*-g_i) - \eta^{\frac{g_i}{k_i^*}} f(k_i^*) > \xi\sum_{j\in G_i}[f(k_j^*) - f(k_j^*-1)]$ because $\xi < \eta^{\frac{1}{n-1}} < \frac{1-\xi}{\rho(1+n\rho)}$. So, it suffices to show that

$$\left[f\left(k_i^*-g_i\right) - \eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right)\right]x_i^* > \sum_{j\in G_i}\left[f\left(k_i^*-g_i\right) - \xi f\left(k_j^*-1\right)\right]\hat{x}_j$$

to prove our claim.

We have (i) $[f(k_i^* - g_i)] - \eta^{\frac{g_i}{k_i^*}} f(k_i^*) \geq (1 - \eta^{\frac{1}{n-1}}\rho)$; (ii) $x_i^* \geq \frac{B}{(1-\xi)\eta^{\frac{1}{n-1}}\rho R} - 1$, because x^* satisfies the incentive constraint; (iii) $f(k_i^* - g_i) - \xi f(k_j^* - 1) < \rho$ for any $j \in G_i$; (iv) $\hat{x}_j = \frac{e_j+1}{f(k_j^*-1)} - 1 \leq e_j < \frac{B}{(1-\xi)R} - 1$, given that by Proposition 3, bank j chooses the risky project in autarky. Then the following inequality holds

$$\left[f\left(k_i^* - g_i\right) - \eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right)\right] x_i^* \geq \left(1 - \eta^{\frac{1}{n-1}}\rho\right)\left\{\frac{B}{(1-\xi)\eta^{\frac{1}{n-1}}\rho R} - 1\right\} >$$

$$> (n-1)\rho\left\{\frac{B}{(1-\xi)R} - 1\right\}$$

$$> \sum_{j \in G_i}\left[f\left(k_i^* - g_i\right) - \xi f\left(k_j^* - 1\right)\right]\hat{x}_j,$$

where the second inequality follows from the assumption $\eta^{\frac{1}{n-1}} < \frac{1-\xi}{\rho(1+n\rho)}$. □

Proof of Proposition 6. Assume that (N, e) and η are given and that f is increasing in k and satisfies the IPR property.

Assume first that $\min_{i \in N} e_i \geq \frac{B}{(1-\xi)\rho R} - 1$. We show that (K^e, e, s^e) with $K_i^e = N \backslash \{i\}$ and $s_i^e = rf$ for all i is a DEWT. The necessary condition it is straightforward to check. Note that $m_i(K^e, e, s^e) = \rho R(e_i + 1) - 1$ if $K_i^e = N\backslash\{i\}$ and $s_i^e = rf$ for all i . First, we check that (K^e, e, s^e) with $K_i^e = N\backslash\{i\}$ and $s_i^e = rf$ for all i is an INE. Indeed, note that

$$\rho R(e_i + 1) - 1 \geq \xi\rho R(e_i + 1) + B - 1$$

because $e_i \geq \frac{B}{(1-\xi)\rho R} - 1$ for all i and hence (K^e, e, s^e) is an INE. We prove now that there is no other INE $(\hat{K}, e, \hat{s})$ that can yield a higher payoff $m_i(\hat{K}, e, \hat{s})$ to any $i \in N$. We consider two cases: (i) If bank $\hat{s}_i = rf$, then

$$m_i(K^e, e, s^e) = \rho R(e_i + 1) - 1 \geq \eta^{\frac{\hat{g}_i}{\hat{k}_i}} f(\hat{k}_i)R(e_i + 1) - 1 = m_i(\hat{K}, e, \hat{s}),$$

because $\eta < 1$ and $f(\hat{k}_i) \leq \rho$. (ii) If bank $\hat{s}_i = r$, then

$$m_i(K^e, e, s^e) = \rho R(e_i + 1) - 1 \geq \xi\rho R(e_i + 1) + B - 1$$

$$\geq \xi\eta^{\frac{\hat{g}_i}{\hat{k}_i}} f(\hat{k}_i)R(e_i + 1) + B - 1 = m_i(\hat{K}, e, \hat{s}),$$

because $e_i \geq \frac{B}{(1-\xi)\rho R} - 1$, $\eta < 1$, and $f(\hat{k}_i) \leq \rho$.

Hence, no bank i can strictly gain from deleting any link, either if the investment decision is risky or risk-free after the deletion of the link. As K^e is already complete, no new links can be established. Hence, (K^e, e, s^e) is pairwise stable without transfers. Given that (K^e, e, s^e) is INE and pairwise stable without transfers, it is a DEWT.

Assume now that $\max_{i \in N} e_i < \frac{B}{(1-\xi)\rho R} - 1$. First note that if (K, e, s) is an INE, then $s_i = r$ for all i because

$$I^*(k_i, g_i, \xi, \eta) = \frac{B}{(1-\xi)\eta^{\frac{g_i}{k_i}} f(k_i)R} - 1 \geq \frac{B}{(1-\xi)\rho R} - 1$$

given that $\eta < 1$ and $f(k_i) \leq \rho$. Hence, (K^e, e, s^e) with $s_i^e = r$ for all i is an INE. We now check that if $\eta f(1) \geq 1$, then (K^e, e, s^e) with $s_i^e = r$ and $K_i^e = N \backslash \{i\}$ for all i is the unique pairwise stable network without transfers, and hence the unique DEWT. We prove that if two banks i and j are disconnected in an INE (K, e, s), they always benefit from creating their connection. Indeed, given that (K, e, s) is an INE, it has to be that $s = r$ for all i. Take two banks i and j such that $i \notin K_j$ (therefore, $j \notin K_i$). We have that, if $k_i > 0$, then

$$\begin{aligned} m_i(K, e, s) &= \xi \eta f(k_i) R(e_i + 1) + B - 1 < \xi \eta f(k_i + 1) R(e_i + 1) + B - 1 \\ &= m_i(K \cup \{i, j\}, e, s), \end{aligned}$$

because in any INE $s_b = r$ for all b and f is increasing, and if $k_j > 0$,

$$\begin{aligned} m_i(K, e, s) &= \xi \eta f(k_i) R(e_i + 1) + B - 1 < \xi \eta f(k_i + 1) R(e_i + 1) + B - 1 \\ &= m_i(K \cup \{i, j\}, e, s), \end{aligned}$$

because in any INE $s_b = r$ for all b and f is increasing. This means that when banks hold $n - 2$ connections they always benefit from creating the last one, and, turning the argument around, they never benefit from breaking the last connection (i.e., they prefer holding $n - 1$ connections than $n - 2$). In particular, no pair of banks has an incentive to delete their link, and hence (K^e, e, s^e) is pairwise stable without transfers and therefore a DEWT. If $k_i = 0$, then

$$\begin{aligned} m_i(K, e, s) &= \xi R(e_i + 1) + B - 1 \leq \xi \eta f(1) R(e_i + 1) + B - 1 \\ &= m_i(K \cup \{i, j\}, e, s), \end{aligned}$$

because $\eta f(1) \geq 1$. Hence, bank i always gains from building new links and the same argument can be applied to bank j. Given that two disconnected banks always benefit from building their connection, the only pairwise stable allocation without transfers, and hence the only DEWT, is (K^e, e, s^e) with $s_i^e = r$ and $K_i^e = N \backslash \{i\}$ for all i.

We finally check that if $\eta f(1) < 1$, then there are two DEWT: (i) (K^e, e, s^e) with $s_i^e = r$ and $K_i^e = \emptyset$ for all i (the empty network), or (ii) (K^e, e, s^e) with $s_i^e = r$ and $K_i^e = N \backslash \{i\}$ (the complete network). The complete network is pairwise stable without transfers because, as argued just above, any bank i gains from creating a new link as far as $k_i > 0$, in particular whenever $k_i = n - 2$, because f is increasing and all banks are choosing the risky project in any INE.

In order to check that the empty network is also DEWT, we check that it is pairwise stable without transfers if $\eta f(1) \leq 1$. Indeed, if $K_i = \emptyset$, then

$$\begin{aligned} m_i(K, e, s) &= \xi R(e_i + 1) + B - 1 \geq \xi \eta f(1) R(e_i + 1) + B - 1 \\ &= m_i(K \cup \{i, j\}, e, s), \end{aligned}$$

and no bank has a strict incentive to create the first link. Hence, (K^e, e, s^e) with $s_i^e = r$ and $K_i^e = \emptyset$ for all i is pairwise stable without transfers, and given that it is also an INE, it is also a DEWT. □

Proof of Proposition 7. Assume by contradiction that (K^e, e, s^e) is a DEWT, but there are two banks i and j such that $s_i^e = s_j^e = rf$ with $i \notin K_j^e$, and therefore $j \notin K_i^e$. We prove that $(K^e \cup ij, e, s^e)$ is an INE and that both $m_i(K^e \cup ij, e, s^e) > m_i(K^e, e, s^e)$ and $m_j(K^e \cup ij, e, s^e) > m_j(K^e, e, s^e)$, contradicting the fact that (K^e, e, s^e) is a PSWT, and therefore it cannot be a DEWT.

Note that, because (K^e, e, s^e) is a DEWT, it has to be an INE. This means that $e_i \geq I^*(k_i, g_i, \xi, \eta)$ and $e_j \geq I^*(k_j, g_j, \xi, \eta)$. Furthermore, it has to be $I^*(k_i, g_i, \xi, \eta) \geq I^*(k_i + 1, g_i, \xi, \eta)$ and $I^*(k_j, g_j, \xi, \eta) \geq I^*(k_j + 1, g_j, \xi, \eta)$ given that $f(k)$ and $\eta^{\frac{g}{k}}$ are increasing in k. This implies $e_i \geq I^*(k_i + 1, g_i, \xi, \eta)$ and $e_j \geq I^*(k_j + 1, g_j, \xi, \eta)$. Therefore, $(K^e \cup ij, e, s^e)$ is also an INE. Finally, note that

$$\begin{aligned} m_i(K^e \cup ij, e, s^e) &= \eta^{\frac{g_i}{k_i+1}} f(k_i + 1) R(e_i + 1) - 1 > \eta^{\frac{g_i}{k_i}} f(k_i) R(e_i + 1) - 1 \\ &= m_i(K^e, e, s^e) \end{aligned}$$

and

$$\begin{aligned} m_j(K^e \cup ij, e, s^e) &= \eta^{\frac{g_j}{k_j+1}} f(k_j + 1) R(e_j + 1) - 1 > \eta^{\frac{g_j}{k_j}} f(k_j) R(e_j + 1) - 1 \\ &= m_j(K^e, e, s^e) \end{aligned}$$

because the functions $f(k)$ and $\eta^{\frac{g}{k}}$ are increasing in k. Therefore, (K^e, e, s^e) cannot be a DEWT. □

Proposition 8 (General version). *Assume that*$\min e_i < \underline{E} < \max e_i$ *and* $f(k)$ *satisfies the IPR property. Let* $k(\eta)$ *be as defined in* (10). *Then*:

(i) *If* (K^e, e, s^e) *is a DEWT allocation, then* $k_i^e < k(\eta)$ *for some* i*implies that* $s_i^e = r$ *and* $k_j^e \geq k(\eta)$ *for all* $j \notin K_i^e$.
(ii) *If* (K^e, e, s^e) *is a DEWT allocation, then* $k_i^e > k(\eta)$ *only if* $G_i = \emptyset$.

Corollary of Proposition 8 (General version) (Proposition 8 in the text). *Assume that* $\min e_i < \underline{E} < \max e_i$ *and* $f(k)$ *satisfies the IPR property. Let* $k(\eta)$ *be as defined in* (10). *Then*

(i) *If* $k(\eta) = n - 1$, *then* $K_i^e = N \backslash \{i\}$ *for all* $i \in N$.
(ii) *If* $k(\eta) = 0$, *then* $g_i^e = 0$ *for all* i.

PROOF OF PROPOSITION 8 (General version). The proof is based on Lemma A1. Assume $k_i^e < \underline{k}(\eta)$. This means that $k_i^e \leq \underline{k}(\eta) - 1$, and by definition of $\underline{k}(\eta)$ and the fact that the ratio $(\frac{f(k)}{f(k+1)})^{k+1}$ is increasing in k, we have $\eta \geq (\frac{f(k_i^e)}{f(k_i^e+1)})^{k_i^e+1}$. Hence, bank i will gain if connecting to any other bank $j \notin K_i^e$, no matter j's strategy. Therefore, K^e can be a pairwise stable structure only if there is no other bank j willing to connect to bank i. If bank i is choosing the risk-free project, any bank not yet connected to bank i would be better off by connecting to it. Therefore, if $s_i^e = rf$, then $K_i^e = N \backslash \{i\}$. This is a contradiction with $k_i < k(\eta) \leq n - 1$. If $s_i^e = r$, it has to be that no other bank j not connected to bank i could be better off from connecting to bank i. This could only happen if $k_j^e \geq \underline{k}(\eta)$ for any bank $j \notin K_i^e$, given that any bank j would like to connect with bank i if $k_j^e < \underline{k}(\eta)$.

Assume that $k_i^e > \overline{k}(\eta)$ for some bank i. This means that $k_i^e \geq \overline{k}(\eta) + 1$, and by definition of $\overline{k}(\eta)$ and the fact that the ratio $(\frac{f(k)}{f(k+1)})^{k+1}$ is increasing in k, we know that $\eta < (\frac{f(k_i^e-1)}{f(k_i^e)})^{k_i^e}$. This means that bank i is better off if it unilaterally disconnects any of its links with banks investing in the risky project. So, the allocation (K^e, e, s^e) can only be PSWT if $G_i = \emptyset$. □

PROOF OF COROLLARY OF PROPOSITION 8 (General version). Substituting $k(\eta)$ by its extreme values in the general version of Proposition 8, we obtain the statements. The arguments used are similar to the ones used in the proof of Proposition 4 and can therefore be omitted. □

PROOF OF PROPOSITION 9. Given the definition of Q, we have

$$Q = \left\{ i \text{ such that } I^*(k_i, \widehat{g_i}, \xi, \eta) \leq x_i < I^*(k_i, \widehat{g_i} + q_i, \xi, \eta) \right\},$$

where $\widehat{g_i}$ is the number of banks in $K_i \backslash Q$ that choose the gambling project, and q_i is the minimum number of banks in Q connected to bank i that, by choosing the gambling project, would make bank i switch from the risk-free project to the gambling project. Note that $1 \leq q_i \leq |K_i \cap Q|$. Namely, q_i has to be at least one, otherwise bank i will always choose the gambling project and would not belong to $Q(s, s')$. Furthermore, q_i has to be at most $Q \cup K_i$, that is, the number of banks in Q that are connected to bank i. Otherwise, bank i will always choose the risk-free project and would not belong to $Q(s, s')$. Assume that the sequential-move investment game calls bank i to make the investment decision (given the choices of banks Q.) If history in the game is such that q_i banks in Q connected to bank i choose the gambling project, bank i chooses the gambling project as well. If history in the game is such that there are less than q_i banks in Q connected to bank i choosing the gambling project, bank i chooses the safe project. But if history is such that if by choosing the safe project there are less than q_i, say, c_1 banks choosing the gambling project in Q and if by choosing the gambling project there are more than q_i, say, c_2 banks choosing the gambling project, bank i chooses the gambling asset only if

$$\eta^{\frac{\widehat{g_i}+c_1}{k_i}} f(k_i) R(x_i + 1) - 1 < \xi \eta^{\frac{\widehat{g_i}+c_2}{k_i}} f(k_i) R(x_i + 1) + B - 1, \tag{A12}$$

where $c_1 < q_i \leq c_2$. The previous inequality implies

$$x_i < \frac{B}{\left(1 - \xi\eta^{\frac{c_2 - c_1}{k_i}}\right)\eta^{\frac{\widehat{g}_i + c_1}{k_i}} f(k_i)R} - 1, \tag{A13}$$

a contradiction with the fact that

$$x_i \geq \frac{B}{(1-\xi)\eta^{\frac{\widehat{g}_i + c_1}{k_i}} f(k_i)R} - 1 = I^*(k_i, \widehat{g}_i, \xi, \eta),$$

given that c_1 banks in $|Q \cap K_i|$ are not enough to make bank i switching from the safe project to the gambling one.

This implies that every time a bank is critical in the rule of order to decide among different continuation paths, it will choose the safe path. Then we can conclude that for any rule of order, the sequential-move investment game selects the INE profiles where the highest number of banks choose the risk-free project. □

PROOF OF PROPOSITION 10. Recall that $x_i = e_i + t_i \geq 0$. Assume bank i is considering to make a transfer t_i. Denote by j one bank (there might be many) that receives a transfer t_j to be induced to choose the risk-free project. Therefore, $x_j = e_j + t_j \geq I^*(k_j, g_j, \xi, \eta)$. Given that $x_i \geq 0$, it has to be that $x_i + x_j \geq I^*(k_j, g_j, \xi, \eta) > \frac{B}{(1-\xi)\rho R} - 1$. Clearly, $x_i + x_j \leq E = \sum_{i \in N} e_i$. Thus, for $E < \frac{B}{(1-\xi)\rho R} - 1$, there is no transfer t_i that can induce bank j to invest in the risk-free project, no matter the values of x_i or x_j. This happens when $\xi > 1 - \frac{B}{\rho R(E+1)}$. We have $\overline{\xi} = \max\{1 - \frac{B}{\rho R(E+1)}, 0\}$. □

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