Empirical Analyses of Networks in Finance

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1 INTRODUCTION

At the end of the 90s of the last century a new multidisciplinary research area took off. This research area is today called "network science". Network science is a multidisciplinary research area analyzing and modeling complex networks from the perspective of several disciplines. The major ones are computer science, economics, mathematics, sociology and psychology, and statistical physics.

The onset of the financial crisis of 2007 triggered an enormous interest in applying networks concepts and tools, originating from several different disciplines, to study the role of interlinkages in financial systems on financial stability. The vast literature covers today both theoretical and empirical aspects. It is out of the scope of the present review to cover all the research lines today present in the analysis and modeling of economic and financial systems with network concept and with tools designed specifically for this concept. We will focus instead on a selection of studies that perform empirical analyses of some crucial areas of the financial system.

Our review complements others that have been published in the last few years. Allen and Babus (2008) review theoretical work on networks with application to systemic risk, investment decisions, corporate finance, and microfinance. Bougheas and Kirman (2015) review theoretical and empirical studies on systemic risk that implement networks and complex analysis techniques. Iori and Porter (2016) review agent based models of asset markets, including heterogeneous agents and networks. Grilli et al. (2017) focus on recent theoretical work related to credit networks, discussing in particular agent based computational approaches. Benoit et al. (2017) review studies that identify sources of systemic risk both using regulatory data and market data to produce measures of systemic risk. Aymanns et al. (2018) in this volume review computational approaches to financial stability considering multiple channels of contagion such counterparty risk, funding risk, and common assets holding. While there is some overlap between these papers and the present review, our review has a more empirical focus, exploring a broad range of methodologies that have been applied to the study of financial networks, such as the investigation and modeling of the interbank market, network reconstruction techniques, multilayer characterization of interbank exposures, indirect channels of contagion, proximity based networks, association networks, and statistically validated networks.

Specifically, the empirical studies of the interbank market will be discussed in detail because this market is of major interest for studies of systemic risk of national financial systems and of the global financial system. Recent studies of the systemic risk of the banking system of different countries and of the global financial system have also shown the importance of indirect links present between financial actors. The intensity of these indirect links are for example associated with the degree of similarity of the portfolio of assets owned by the financial institutions. An action of a distressed bank acting on a specific asset and triggering a fire sale of the asset can in fact impact other financial actors even in the absence of direct financial

contracts between financial entities. Moreover, the understanding of the dynamic of contagion of distress has promoted the study of networks of influence between financial actors. For the above reasons in finance we have empirical studies covering four distinct types of networks. They are (i) the customary event or relationship networks as, for example, networks of market transactions or networks of the interbank loans exchanged between pairs of banks, (ii) proximity based network, i.e. networks obtained starting from a proximity measure often filtered with a network filtering methodology able to highlight the most significant pairwise similarity or dissimilarity present in the system, (iii) association network, i.e. networks where a link between two financial actors is set if a statistical test against a null hypothesis is rejected for a pair of financial actors (one example of association network is a network summarizing the Granger-causality relationships between all pairs of financial actors of a given financial system), and (iv) statistically validated networks, i.e. event or relationship networks where a subset of links is selected according to a statistical validation of each link performed by considering the rejection of a random null hypothesis assuming the same heterogeneity as observed in financial complex systems.

The review is organized as follows. Section 2 provides a historical perspective of the development of the new interdisciplinary research area of network science. Section 3 discusses the network approaches to the study of the stability of financial systems with a special focus on the interbank market. The section discusses the structure of national interbank markets, typical approaches in the setting and analysis of stress test scenarios, the detection of systemically important financial institutions, and the modeling of lending relationships. The methodological aspects that are considered are related to the problem of network reconstruction and to the multilayer nature of financial networks. Section 3 discusses the classes of financial networks that are different from relationship or event networks. Specifically, it discusses proximity based networks, association networks, and statistically validated networks. Contributions originating from network science, econophysics, and econometrics are illustrated and discussed. Section 4 discusses the indirect channels of contagion with an emphasis on portfolios overlap and firm-bank credit relationships. In Section 5 we conclude with a discussion on the state of the art and perspectives of empirical investigations performed in economic and financial systems.

The review also contains two appendices written to make the chapter self-contained so that readers not directly working in the field of network science could find a guide about tools and concepts that are usually defined and used in different disciplines. Appendix A briefly describes concepts and definitions used in the study of complex networks and provide a basic vocabulary needed to understand the following sections. In Appendix B we recall some econometrics systemic risk measures that have been also used and discussed in the systemic risk studies performed using network concepts.

2 A BRIEF HISTORICAL PERSPECTIVE ABOUT THE USE OF NETWORK SCIENCE IN ECONOMICS AND FINANCE

The main driver of the development of the new research area of network science was originally the invention of the World Wide Web and the rapid development and use of it that quickly involved hundreds of millions of people. Another strong input came in 2003–2004 with the creation of the first social network Myspace and the extraordinary success of Facebook. Studies about so-called complex networks benefited from the knowledge previously accumulated in several distinct disciplines. In mathematics Erdös and Rényi (1960) discovered in the 60s of the last century that a simple random graph presents an emergent phenomenon as a function of the average degree, i.e. the average number of connections each node has in the network. The emergent phenomenon concerns the setting of an infinite spanning component comprising the majority of nodes. In an infinite network this spanning component sets up exactly when the average degree is equal to one and it is absent for values lower than one. In sociology and psychology networks were used to understand social balance (Heider, 1946), social attitude towards the setting of relationships in social networks (Wasserman and Faust, 1994), and some puzzles about social networks as the so-called small world effect (Travers and Milgram, 1967). In statistical physics a pioneering paper was the one on the small world effect (Watts and Strogatz, 1998). Another seminal paper in statistical physics was the paper of Barabási and Albert that introduced the concept of scale free network and presented the preferential attachment model (Barabási and Albert, 1999). In computer science the growing importance of the physical Internet motivated the empirical analysis of it. This analysis was showing that the degree distribution of the network has a power-law behavior (Faloutsos et al., 1999). During the same years a small group of computer scientists focused on the properties of information networks with the aim of finding new solutions for the development of efficient search engines of the World Wide Web. Prominent examples of these efforts are the papers of Marchiori (1997) and Kleinberg (1998) that paved the way to the famous PageRank algorithm (Brin and Page, 1998). An early use of statistical physics concepts in the development of models for social networks can be encountered in the development of the so-called exponential random graphs (Holland and Leinhardt, 1981; Strauss, 1986).

The use of network concept in economics and finance was sporadic and ancillary during the last century. A prominent exception to this status was the model about the process of reaching a consensus in a social network introduced by the statistician Morris H. DeGroot (DeGroot, 1974). Another classic area where the role of a social network was considered instrumental to correctly interpret empirical observations was the area of the modeling of the labor market. In fact the type of a social network that it is present among job searchers and their acquaintances turned out affecting the probability of getting a job of the different social actors (Granovetter, 1973; Boorman, 1975; Calvo-Armengol and Jackson, 2004). Another pioneering empirical study used the concept of network to describe the social structure of a stock options market. The

study concluded that distinct social structural patterns of traders affected the direction and magnitude of option price volatility (Baker, 1984).

The interest about the use of network concepts in the modeling of economic and financial systems started to growth at the beginning of this century and become widespread with the onset of the 2007 financial crisis. A pioneering study was published about the stability of the financial systems (Allen and Gale, 2000). During the first years of this century the studies of economic and financial systems performed by economists with network concepts can be classified in two broad areas (Allen and Babus, 2008). The first area concerns studies primarily devoted to the theoretical and empirical investigation of network formation resulting from the rational decisions of economic actors and from their convergence to an equilibrium presenting a Paretian optimum (Goyal and Vega-Redondo, 2005; Vega-Redondo, 2007; Jackson, 2008; Goyal, 2012). Another line of research initiated by the pioneering work of DeGroot (1974) was considering the problem of learning in a distributed system. In parallel to these attempts scholars economists and financial experts in collaboration with colleagues having a statistical physics background performed a series of studies focused on the topological structure of some important financial networks. The most investigated network was the network of interbank credit relationships (Boss et al., 2004; Iori et al., 2006; Soramäki et al., 2007). Another research area focused on the development of methods able to provide proximity based networks starting from empirical financial time series (Mantegna, 1999; Onnela et al., 2004; Tumminello et al., 2005) or generated by financial models (Bonanno et al., 2003).

3 NETWORK APPROACHES TO FINANCIAL STABILITY: THE INTERBANK MARKET

Financial systems have grown increasingly complex and interlinked. A number of academics and policy-makers have pointed out the strong potential of network representation¹ and analysis to capture the intricate structure of connectedness between financial institutions and sectors of the economy. Understanding of the growth, structure, dynamics, and functioning of these networks and their mutual interrelationships is essential in order to monitor and control the build-up of systemic risk, and prevent and mitigate the impact of financial crises.

The recent global financial crisis has illustrated how financial networks can amplify shocks as they spill over through the financial system, by creating feedback loops that can turn relatively minor events into major crises. This feedback between the micro and macro states is typical of complex adaptive dynamical systems. The regulatory efforts to maintain financial stability and mitigate the impact of finan-

¹In financial networks nodes usually represent financial agents such as banks, non-bank intermediaries, firms, investors, and central banks. The edges may represent credit relationships, exposures between banks, liquidity flows, etc.

cial crises has led to a shift from micro-prudential to macro-prudential regulatory approaches. Traditional micro-prudential approaches have relied on ensuring the stability of individual financial institutions and limiting idiosyncratic risk. However, while financial market's participants have clear incentives to manage their own risk and prevent their own collapse, they have limited understanding of the potential effects of their actions on other institutions. The recent emphasis on the adoption of a macro-prudential framework for financial regulation stems from the recognition that systemic risk depends on the collective behavior of market participants acting as a negative externality that needs to be controlled by monitoring the system as a whole. The new regulatory agenda has also brought to the fore the concept that institutions may be "too interconnected to fail", in addition of being "too big to fail", and the need for methods and tools that can help to better identify and manage systemically important financial institutions.

Network analysis in finance has been used to address two fundamental questions: (i) what is the effect of the network structure on the propagation of shocks?, and (ii) what is the rational for financial institutions to form links?, with the ultimate goal of identifying the incentives, in the form of regulatory policies, that would induce the reorganization of financial system into network structures that are more resilient to systemic risk. The first question is the one that has been more extensively studied in the literature. Several authors have analyzed how the financial network structure affect the way contagion propagates through the banking system and how the fragility of the system depends on the location of distressed institutions in the network. Other authors have directed their efforts to develop algorithms for the reconstruction of bilateral exposures form aggregate balance sheet information. A number of papers have focused on detecting long lasting relationships among banks in an effort to understand the determinants of links formation. Others have looked at possible network location advantages. A smaller but growing number of papers have focused on how policy and regulations can influence the shape of the network in order to minimize the costs of systemic risk. Theoretical papers are briefly reviewed in Section 3.1. While these papers provide important insights, the connectivity structure of real financial networks departs significantly from the stylized structures assumed, or endogenously derived, by the seminal paper of Allen and Gale (2000) and subsequent work. Given the empirical emphasis of this review, the characterization of real, or reconstructed, networks of interbank exposure and empirical approaches to estimate the danger of contagion owing to exposures in the interbank bank is the main focus of Sections 3.2 to 3.6. Section 3.7 summarizes complementary approaches developed to quantify systemic risk based on econometric methods while potential benefit arising from location advantages are discussed in Section 3.8.

3.1 INTERBANK NETWORKS CONNECTIVITY AND CONTAGION: THEORETICAL CONTRIBUTIONS

Theoretical work has produced important insights to better understand the role of markets interconnectivity on financial stability. Allen and Gale (2000) were the first

to show that interbank relations may create fragility in response to unanticipated shocks. In their seminal paper Allen and Gale suggested that a more equal distribution of interbank claims enhances the resilience of the system to the insolvency of any individual bank. Their argument was that in a more densely interconnected financial network, the losses of a distressed bank are divided among more creditors, reducing the impact of negative shocks to individual institutions on the rest of the system. In contrast to this view, however, others have suggested, via computational models, that dense interconnections may function as a destabilizing force (Iori et al., 2006; Nier et al., 2007; Gai and Kapadia, 2010; Battiston et al., 2012; Anand et al., 2012; Lenzu and Tedeschi, 2012; Georg, 2013; Roukny et al., 2013). Haldane (2013) has reconciled these findings by observing that highly interconnected financial networks are "robust-yet-fragile" in the sense that connections serve as shock-absorbers within a certain range but beyond it interconnections facilitate financial distress to spread through the banking system and fragility prevails.

More recent theoretical papers have confirmed these earlier computational results. Glasserman and Young (2015) show that spillover effects are most significant when node sizes are heterogeneous and the originating node is highly leveraged and has high financial connectivity. The authors also show the importance of mechanisms that magnify shocks beyond simple spillover effects. These mechanisms include bankruptcy costs, and mark-to-market losses resulting from credit quality deterioration or a loss of confidence.

Acemoglu et al. (2015) focus on the likelihood of systemic failures due to contagion of counterparty risk. The paper shows that the extent of financial contagion exhibits a phase transition: when the magnitude of negative shocks is below a certain threshold, a more diversified pattern of interbank liabilities leads to a less fragile financial system. However, as the magnitude or the number of negative shocks crosses certain thresholds dense interconnections serve as a mechanism for the propagation of shocks, leading to a more fragile financial system. While Acemoglu et al. (2015) characterize the best and worst networks, from a social planner's perspective, for moderate and very large shocks. Elliott et al. (2014) show how the probability of cascades and their extent depend, for intermediate shocks and for a variety of networks, on integration (how much banks rely on other banks) and diversification (number of banks on which a given bank's liabilities are spread over). Their results highlight that intermediate levels of diversification and integration can be the most problematic. Cabrales et al. (2017) investigate the socially optimal design of financial networks in diverse scenarios and their relationship with individual incentives. In their paper they generalize the Acemoglu et al. (2015) results by considering a richer set of possible shocks distributions and show how the optimal financial structure varies in response to the characteristics of these distributions. The overall picture that emerges from this literature is that the density of linkages has a non-monotonic impact on systemic stability and its effect varies with the nature of the shock, the heterogeneity of the players, and the state of the economy.

A large number of theoretical papers based on Agent Based simulations have investigated how the topological structure of the matrix of direct and indirect exposures

between banks affects systemic risk. For a recent review of this literature we refer the interested readers to Grilli et al. (2017).

A different branch of the theoretical literature focuses on network formation mechanisms that reproduce features of trading decisions observed empirically. Of particular relevance to interbank lending markets, are theories on the formation of core–periphery networks.²

Anufriev et al. (2016) build a model of endogenous lending/borrowing decisions, which induce a network. By extending the notion of pairwise stability of Jackson and Wolinsky (1996) they allow the banks to make binary decision to form a link, which represents a loan, jointly with the direction of the loan, its amount, and its interest rate. In equilibrium, a bipartite network is found in which borrowers and lenders form generically a unique component, which well represents interbank markets when aggregating transactions at the daily scale, van der Leij et al. (2016) provide an explanation for the emergence of core–periphery structure by using network formation theory and find that while a core-periphery network cannot be unilaterally stable when banks are homogeneous such structure can form endogenously, if allowing for heterogeneity among banks in size. Heterogeneity is indeed a common characteristics of models that generate stable core-periphery structures. In Farboodi (2015) banks are heterogeneous in their investment opportunities, and they compete for intermediation benefits. A core–periphery network is formed with investment banks taking place at the core, as they are able to offer better intermediation rates. In Bedayo et al. (2016) intermediaries bargain sequentially and bilaterally on a path between two traders. Agents are heterogeneous in their time discounting. A core-periphery network is formed with impatient agents in the core. In Castiglionesi and Navarro (2016) heterogeneity in investments arises endogenously with some banks investing in safe projects, and others in risky projects. The interbank network allows banks to coinsure future idiosyncratic liquidity needs, however, establishing a link with banks that invest in the risky project, reduces the ex-ante probability of serving its own depositors. If counterparty risk is sufficiently high, the trade off leads to a coreperiphery like structure. The core includes all the banks that invest in the safe project and form a complete network structure among themselves while the periphery includes all the gambling banks. In Chang and Zhang (2016) banks are heterogeneous in the volatility of their liquidity needs. More volatile banks trade with more stable banks, creating a multi-tier financial system with the most stable banks in the core. However, banks do not have incentives to link with other banks in the same tier. and hence, their network structure is more like a multipartite network than a coreperiphery network.

²A network core–periphery structure indicates that some nodes are part of a densely connected core, and others are part of a sparsely connected periphery. Core nodes are well-connected to peripheral nodes and peripheral nodes are not directly connected among them.

3.2 THE STRUCTURE OF NATIONAL INTERBANK NETWORKS

The mapping of interbank networks has been done for several countries, notably by Sheldon and Maurer (1998) for Switzerland; Inaoka et al. (2004) for Japan (BoJ-NET); Furfine (2003), Soramäki et al. (2007), and Bech and Atalay (2010) for the US Federal funds market (Fedwire); Boss et al. (2004), Elsinger et al. (2006), and Puhr et al. (2012) for Austria; Degryse and Nguyen (2007) for Belgium; van Lelyveld and Liedorp (2006) and Propper et al. (2013) for the Netherlands; Upper and Worms (2004) and Craig and von Peter (2014) for Germany; De Masi et al. (2006), Iori et al. (2008), and Finger et al. (2013) for the Italian based e-MID; Cont and Wagalath (2013), Tabak et al. (2010a) for Brazil; Wells (2004) and Langfield et al. (2014) for the United Kingdom; Martinez-Jaramillo et al. (2014) for Mexico; León (2015) for Colombia. In Fig. 1 we show an example of the typical shape of the interbank network. Specifically, we show the Austrian interbank market investigated in Boss et al. (2004). These studies of the interbank market have revealed a number of stylized facts and regularities. Interbank networks are sparse and display fat tailed degree distributions, with most banks attracting a few connections, and few banks concentrating most of the connections. While some papers identify a scale-free degree distribution (Boss et al., 2004; Inaoka et al., 2004; Soramäki et al., 2007; Propper et al., 2013; Bech and Atalay, 2010; Bargigli, 2014; León, 2015), others have reported heterogeneity but observe a departure from a strict power-law distribution of links (Martinez-Jaramillo et al., 2014; Craig and von Peter, 2014; Fricke and Lux, 2014; in't Veld and van Lelyveld, 2014) and propose to model the interbank market in terms of a core periphery network.

It might be worth noting that the fact that it is difficult to discriminate between a power-law scale free distribution and a core periphery network is not an accident of the empirical investigations performed in the interbank market. There are two important reasons explaining why it might be so difficult to discriminate between the two different models in empirical data. The first reason is that in empirical data a finite cut-off of a scale free distribution is unavoidable in finite systems and therefore empirically detected scale free distribution are typically observed only for a limited number of decades of the degree. The second reason is a theoretical reason that it is related to a mathematical property associated with a scale free system. In fact it has been proved by Chung and Lu (2002) that power-law random graphs with degree distribution proportional to $k^{-\beta}$ with exponent β in the interval [2, 3] (including therefore scale free networks) almost surely contain a dense subgraph (i.e. a core) that has short distance to almost all other nodes. Therefore, also random scale free networks are almost surely characterized by the presence of a core in it. This fact makes the empirical discrimination between a scale free and a core periphery model difficult to asses because, due to the presence of a "core" also in the scale free network, core periphery or scale free can be quantified closely by a fitness indicator (see, for example, the comparison provided in Craig and von Peter, 2014 by using data of the German interbank market and simulations).

Interbank networks show disassortative mixing with respect to the bank size, so small banks tend to trade with large banks and vice versa; clustering coefficients are

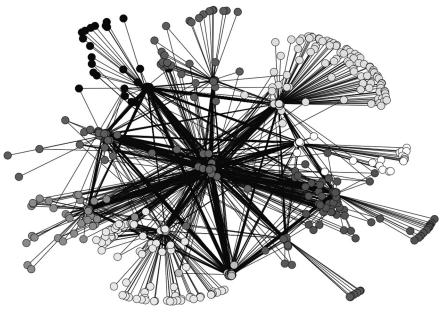


FIGURE 1

The interbank market network of Austria. Data are from September 2002. Nodes are labeled with a gray scale grouping banks according to regional and sectorial organization. Reproduced from Boss et al. (2004).

usually small and interbank networks satisfy the small-world property. Tabak et al. (2010a), Craig and von Peter (2014), Fricke and Lux (2014), and in't Veld and van Lelyveld (2014) point to a correlation between the size of financial institutions and their position in the interbank funds hierarchy in the respective Brazilian, German, Italian, and Dutch interbank markets. In these markets large banks tend to be in the core, whereas small banks are found in the periphery. The cores of the networks, composed of the most connected banks, processed a very high proportion of the total value of credit.

The dynamical evolution of interbank networks, as the subprime crisis unfolded, has been closely monitored in an attempt to identify early-warning signals of the building up of systemic risk. Fricke and Lux (2015) and Squartini et al. (2013) have analyzed respectively the e-MID market and the Dutch market. In both markets the networks only display an abrupt topological change in 2008, providing a clear, but unpredictable, signature of the crisis. Nonetheless, when controlling for banks' degree heterogeneity, Squartini et al. (2013) show that higher-order network properties, such as dyadic and triadic motifs, revealed a gradual transition into the crisis, starting

³A network is a small world network if the mean geodesic distance between pairs of nodes grows no faster than logarithmically as the number of nodes while the average clustering coefficient is not negligible.

already in 2005. Although these results provide some evidence of early warning precursors based on network properties, at least for the Dutch interbank market, a clear economic interpretation for the observed patterns is missing.

3.3 MULTILAYER NETWORKS

When institutions are interconnected trough different types of financial contracts, such as loans of different maturities, derivatives, foreign exchange and other securities, it is critical to go beyond single-layer networks to properly address systemic risk. Multilayer networks explicitly incorporate multiple channels of connectivity and constitute the natural environment to describe systems interconnected through different types of exposures.

Taking into account the multilayer nature of networks can modify the conclusions on stability reached by considering individual network layers (see Boccaletti et al., 2014 and Kivela et al., 2014 for recent reviews). Contrary to what one would expect, the literature shows that the coupling of scale free networks may yield a less robust network (Buldyrev et al., 2010). In the case of single-layer networks, scale free networks are known to be much more robust to random failures than networks with a finite second moment e.g. of Poisson networks. Indeed, scale free networks continue to have a giant component even if most of the nodes are initially damaged. A finite percolation threshold in these networks is a finite size effect, i.e. the percolation threshold disappears in the limit when the network size becomes large. The robustness of multilayer networks can be evaluated by calculating the size of their mutually connected giant component (MCGC). The MCGC of a multilayer network is the largest component that remains after the random damage propagates back and forth thought the different layers. The percolation threshold for the mutually connected component is finite also for multiplex networks formed by layers of SF networks as in the case of a multiplex in which the layers are formed by Poisson networks. Exceptions to this finding would occur when the number of links (i.e. the degree) of interdependent participants coincides across the layers. That is, scale free networks robustness is likely to be preserved if positively correlated layers exist, such that a high-degree vertex in one layer likely is high-degree in the other layers as well (Kenett et al., 2014).

A number of financial markets have been characterized as multilayer networks. Montagna and Kok (2016) model interbank contagion in the Eurozone with a triple-layer multiplex network consisting of long-term direct bilateral exposures, short-term bilateral exposures, and common exposures to financial assets. Bargigli et al. (2015) examine a unique database of supervisory reports of Italian banks to the Banca d'Italia that includes all interbank bilateral exposures broken down by maturity and by the secured and unsecured nature of the contract. The authors found that layers have different topological properties and persistence over time.

Cont et al. (2013) use a set of different kinds of interbank exposures (i.e. fixed-income instruments, derivatives, borrowing, and lending) and study the potential contagion in the Brazilian market. Poledna et al. (2015) identify four layers of exposure among Mexican banks (unsecured interbank credit, securities, foreign exchange

and derivative markets). Aldasoro and Alves (2015) analyze the multiplex networks of exposure among 53 large European banks. Langfield et al. (2014) analyze of different layers of the UK interbank system.

León (2015) studies the interactions of financial institutions on different financial markets in Colombia (sovereign securities market, foreign exchange market, equity, derivative, interbank funds). The approximate scale free connective structure of the Colombia interbank market is preserved across the layers in the multilayer mapping with financial institutions that are "too connected to fail" that are present across many of the network layers. This positive correlated multiplexity, coupled with the ultrasmall world nature of the networks analyzed, suggest that the role of too connected financial institutions is critical for the stability of the market, not only at the single layer level, but for the market overall.

While a multilayer representation provides a more accurate characterizing the financial system, studies so far have mostly performed a comparison between the different layers. Future steps would require investigating the interconnections and interdependencies between these different layers and the implications of these interdependencies for financial stability.

3.4 FINANCIAL REGULATIONS AND NETWORK CONTROL

Of critical importance in macro-prudential policy is the identification of key players in the financial network. In September 2009, the G20 leaders requested the Financial Stability Board (FSB)⁴ to designate "Global Systemically Important Financial Institutions" (G-SIFIs). As a result, the FSB, IMF, and BIS cooperatively adopted the three valuation points – size, interconnectedness, and substitutability – as the evaluation criterion for G-SIFIs (IMF-BIS-FSB, 2009).

As we have discussed, financial networks are often characterized by skewed degree distributions, with most financial firms displaying few connections, and few financial firms concentrating many connections. In these networks the failure of a participant will have significantly different outcomes depending on which participant is selected. Those participants who are "close" (according to some measure of distance) to all other participants in the network can potentially generate widespread cascading failures if they default. A rising amount of financial literature has encouraged the usage of network metrics of centrality for identifying the institutions that are systemically important (Haldane and May, 2011; León and Murcia, 2013; Markose et al., 2012). In a broad sense, centrality refers to the importance of a node in the network. The centrality indicators typically used are constructed from measures of distance of a bank from the other banks in the network, where distance is expressed in terms of: (1) paths of length one, i.e. the number of incoming or outgoing links, for degree

⁴The Financial Stability Board is an international body representing central bankers and international financial bodies such as the Basel Committee on Banking Supervision (BCBS), and its mission is to promote financial stability.

centrality; (2) geodesics (shortest) paths (no vertex is visited more than once), for betweenness; (3) walks (vertices and edges can be visited/traversed multiple times) for eigenvector centrality, Pagerank, Sinkrank, and Katz (see Appendix A for the mathematical definition of commonly used centrality measures). Another popular iterative centrality measure is hub and authority centrality. Acemoglu et al. (2015) introduced the notion of harmonic distance over the financial network to captures the propensity of a bank to be in distress when another bank is in distress. Markose et al. (2012) proposes a measure adapted from epidemiological studies (defined as the maximum eigenvalue for the network of liabilities expressed as a ratio of Tier 1 capital) to identify the most systemic financial institutions and determine the stability of the global OTC derivatives markets. Drehmann and Tarashev (2013) explore two different approaches to measuring systemic importance: one related to banks' participation in systemic events (PA) and another related to their contribution to systemic risk (GCA). The contribution approach is rooted in the Shapley value methodology, first proposed by Shapley (1953) for the allocation of the value created in cooperative games across individual players. Shapley values are portions of system-wide risk that are attributed to individual institutions. Because Shapley values are additive, the sum of these portions across the banks in the system equals exactly the level of system-wide risk. PA assigns a higher (lower) degree of systemic importance to an interbank lender (borrower) than GCA. The reason for this is that PA attributes the risk associated with an interbank transaction entirely to the lending counterparty, i.e. the counterparty that bears this risk and can eventually transfer it onto its creditors in a systemic event. By contrast, GCA splits this risk equally between the two counterparties. GCA is computed by focusing on each subsystem or subgroup of banks that belong to the entire system, and calculating the difference between the risk of this subsystem with and without a particular bank. Averaging such marginal risk contributions across all possible subsystems delivers the systemic importance of the bank.

A novel measure of systemic importance, based on the concept of feedback centrality, is the DebtRank (DR), introduced by Battiston et al. (2012) which measures the fraction of the total economic value that is potentially lost in the case of the distress (and not necessarily default) of a particular node. This method complements traditional approaches by providing a measure of the systemic importance of a bank even when default cascades models predict no impact at all. Applications of DR based stress test analysis (see Thiago et al., 2016 for the Brazilian interbank market, and Battiston et al., 2015 for the Italian interbank market) show that systemically important FIs do not need to be large and that traditional indicators of systemic risk underestimate contagion risk.

The identification of SIFIs is crucial to direct regulatory efforts and, for example, to assess the opportunity to limit institutions' exposures, set up some form of regulatory fees or capital surcharges, or to introduce an insurance fund financed through institution-specific insurance premia (Chan-Lau, 2010). Such an approach has recently also been taken in the IMF's Interim Report for the G20 (IMF, 2010), which outlines that an ideal levy on financial institutions should be based on a network model that would take into account all possible channels of contagion. The new

Basel III rules (Basel Committee on Banking Supervision, 2013) in addition to higher capital requirements with a countercyclical component, and a framework for liquidity regulation, include limiting contagion risk as a new objective, in particular for SIFIs.

Several proposals have emerged with the purpose of creating the right incentives for institutions to control the risk they pose to the financial system. In the existing literature on prudential capital controls (Korinek, 2011), optimal policy measures are derived as tax wedges that could be implemented in a variety of equivalent ways. The opportunity cost of not receiving interest can be viewed as a Pigouvian tax. Along these lines, proposals have emerged to base capital requirements not on the risk of banks assets, but on banks' systemic importance, reflecting the impact their failure would have on the wider banking system and the likelihood of contagious losses occurring. Tarashev et al. (2010) set up a constrained optimization problem in which a policymaker equalizes banks' Shapley values subject to achieving a target level for the expected shortfall of assets to liabilities at the level of the system. Similarly Webber and Willison (2011) solve the constrained optimization problem for a policymaker who seeks to minimize the total level of capital in the UK banking system, subject to meeting its chosen systemic risk target. They show that optimal systemic capital requirements are increasing in balance sheet size (relative to other banks in the system), interconnectedness, and contagious bankruptcy costs. The paper illustrates, however, that risk-based systemic capital requirements derived in this way are procyclical and point to the need of approaches that would be explicitly countercyclical.

Gauthier et al. (2012) use different holdings-based systemic risk measures (e.g., marginal expected shortfall, ΔCoVaR , Shapley value) to reallocate capital in the banking system and to determine macroprudential capital requirements in the Canadian market using credit register data for a system of six banks. Alter et al. (2015) perform a similar exercise and compare the benchmark case, in which capital is allocated based on the risks in individual banks portfolios, and a system-based case, where capital is allocated based on some interbank network centrality metrics. Using the detailed German credit register for estimation, they find that capital rules based on eigenvectors dominate any other centrality measure, saving about 15 percent in expected bankruptcy costs.

Taxes imposed on banks in the form of contributions to a rescue fund have also been suggested. Markose et al. (2012) advocate a stabilization super-spreader fund, derived from her eigen-pair centrality measure. Like a 'bail in' escrow fund, the funds are deployed at the time of potential failure of a financial institution to mitigate tax payer bailouts of the failing bank. Similarly Zlatić et al. (2015) apply DebtRank to model cascade risk in the e-Mid market and determine a Pigouvian taxation to finance a rescue fund, which is used in the case of default of single financial players.

Poledna and Thurner (2016) and Leduc and Thurner (2017) have proposed to use a transaction-specific tax that discriminates among the possible transactions between different lending and borrowing banks. A regulator can use the systemic risk tax (SRT) to select an optimal equilibrium set of transactions that effectively 'rewire' the interbank network so as to make it more resilient to insolvency cascades. The SRT was introduced and its effect simulated using an agent-based model in Poledna

and Thurner (2016), while Leduc and Thurner (2017) prove analytically that an SRT can be applied without reducing the total credit volume and thus without making the system less efficient. Using an equilibrium concept inspired by the matching markets literature the paper shows that the SRT induces a unique equilibrium matching of lenders and borrowers that is systemic-risk efficient, while without this SRT multiple equilibrium matchings exist, which are generally inefficient.

3.5 STRESS-TEST SCENARIO ANALYSIS

A number of papers have used counterfactual simulations to test the stability of financial systems and assess the danger of contagion due to credit exposures in national interbank markets. The approach consists in simulating the breakdown of a single, or possibly more, banks and subsequently assess the scope of contagion. The baseline stress-test model runs as follows: a first bank defaults due to an exogenous shock; the credit event causes losses to other banks via direct exposures in the interbank market and one or more additional banks may default as a result; if this happens, a new round of losses is triggered. The simulation ends when no further bank defaults. Simulations are then repeated by assuming the unanticipated failure of a different bank, possibly spanning across the all system or focusing on the most important institutions.

Overall, the evidence of systemic risk from these tests is mixed. Risk of contagion has been reported, as a percentage of banking system's total assets lost, by Degryse and Nguyen (2007) for Belgium (20%), Mistrulli (2011) for Italy (16%), Upper and Worms (2004) for Germany (15%), and Wells (2004) for the UK (15%). By contrast, little possibility for contagion was found by Blavarg and Nimander (2002) for Sweden, Lubloy (2005) for Hungary, and Sheldon and Maurer (1998) for Switzerland. Furfine (2003) and Amundsen and Arnt (2005) also report only a limited possibility for contagion. Cont et al. (2013) show that in the Brazilian banking system the probability of contagion is small, however the loss resulting from contagion can be very large in some cases.

Recent studies have shown that not taking into account possible loss amplification due to indirect contagion associated with fire sales and bank exposures to the real sector can significantly underestimate the degree of fragility of financial system. We will discuss these effects in Sections 4.1 and 4.2.

3.6 NETWORK RECONSTRUCTION

A major limitation of the stress-test analysis is that often the full network of interbank liabilities is not available to regulators. For most countries bilateral linkages are unobserved and aggregate balance sheet information are used for estimating counterparty exposures. Formally the problem is that the row and column sums of a matrix describing a financial network are known but the matrix itself is not known. To estimate bank-to-bank exposures, different network reconstruction methods have been proposed. The most popular approach for deriving individual interbank liabilities from aggregates has been to minimize the Kullback–Leibler divergence between the liabilities matrix and a previously specified input matrix (see Upper, 2011 for a

review). This network reconstruction approach is essentially a constrained entropy maximization problem, where the constraints represent the available information and the maximization of the entropy ensures that the reconstructed network is maximally random, given the enforced constraints.

One drawback of the Maximum Entropy (ME) method is that the resulting interbank liabilities usually form a complete network. However, as we have seen, empirical research shows that interbank networks are sparse. The maximum entropy method might thus bias the results, in the light of the theoretical findings that better connected networks may be more resilient to the propagation of shocks. This is confirmed by Mistrulli (2011), who analyzes how contagion propagates within the Italian interbank market using actual bilateral exposures and reconstructed ME exposures. ME is found to generally underestimate the extent of contagion. Similarly Solórzano-Margain et al. (2013) showed that financial contagion arising from the failure of a financial institution in the Mexican financial system would be more widespread than from simulations based on reconstructed network based on ME algorithm.

Mastromatteo et al. (2012) have proposed an extension to the ME approach using a message-passing algorithm for estimating interbank exposures. Their aim is to fix a global level of sparsity for the network and, once this is given, allow the weights on the existing links to be distributed similarly to the ME method. Anand et al. (2014) have proposed the minimum density (MD) method which consists in minimizing the total number of edges that is consistent with the aggregated interbank assets and liabilities. They argue that the MD method tends to overestimate contagion, because minimize the number of links, and therefore can together with the ME method be used to provide upper and lower bounds for stress test results. However, increasing the number of links has a non-monotonous effect on the stability of a network, thus attempts to derive bounds on systemic risk by optimizing over the degree of completeness are unlikely to be successful. Montagna and Lux (2013) construct a Monte Carlo framework for an interbank market characterized by a power law degree distribution and disassortative link formation features via a fitness algorithm (De Masi et al., 2006).

The ME approach is deterministic in the sense that the method produces a point estimate for the financial network that is treated as the true network when performing stress tests. Probabilistic approaches to stress-testing have been attempted by Lu and Zhoua (2011), Halaj and Kok (2013), Squartini et al. (2017), Bargigli (2014), Montagna and Lux (2013), and Gandy and Veraart (2017). Such approaches consist in building an ensemble of random networks, of which the empirical network can be considered a typical sample. This allows to analyze not only the vulnerability of one particular network realization retrieved from the real data, but of many plausible alternative realizations, compatible with a set of constrains. However, to generate a realistic random sample it is crucial to impose the relevant constrains on the simulated networks, reproducing not just the observed exposures but also basic network properties. Cimini et al. (2015) introduce an analytical maximum-entropy technique to reconstruct unbiased ensembles of weighted networks from the knowledge of empirical node strengths and link density. The method directly provides the expected

value of the desired reconstructed properties, in such a way that no explicit sampling of reconstructed graphs is required. Gandy and Veraart (2017) have proposed a reconstruction model, which, following a Bayesian approach, allows to condition on the observed total liabilities and assets and, if available, on observed individual liabilities. Their approach allow to construct a Gibbs sampler to generate samples from this conditional distribution that can be used in stress testing, giving probabilities for the outcomes of interest. De Masi et al. (2006) propose a fitness model, where the fitness of each bank is given by their daily trading volume. Fixing the level of heterogeneity the model reproduces remarkably well topological properties of the e-Mid interbank market such as degree distribution, clustering, and assortativity. Finally Iori et al. (2015) introduce a simple model of trading with memory that correctly reproduces features of preferential trading patterns observed in the e-Mid market.

An international study lead by several central banks has been conducted to test the goodness of network reconstruction algorithms (Anand et al., 2017). Initial results suggest that the performance of the tested methods depends strongly on the similarity measure used and the sparsity of the underlying network. This highlights that in order to avoid model risk arising from calibration algorithms, structural bilateral balance sheet, and off balance sheet, data are crucial to study systemic risk from financial interconnections.

Another common critique to stress test studies of contagion is the lack of dynamics in terms of banks' behavioral adjustments. Critics stress the importance to include indirect financial linkages, in terms of common exposures and business models, as well as fire sales or liquidity hoarding contagion driven by fear and uncertainty. To address these concerns, a few papers have explored the role of funding and liquidity risk via simulation experiments. The idea of liquidity risk as banks start fire-selling their assets, depressing overall prices in the market, was initially explored by Cifuentes et al. (2005) and has been further investigated in a simulation framework by Nier et al. (2007), Gai and Kapadia (2010), Haldane and May (2011), Tasca and Battiston (2016), Corsi et al. (2016), Cont and Wagalath (2013), Caccioli et al. (2014), and Poledna et al. (2014). The role of funding risk induced by liquidity hoarding is explored in Haldane and May (2011), Chan-Lau (2010), Espinosa-Vega and Solé (2011), Fourel et al. (2013), Roukny et al. (2013), and Gabrieli et al. (2014). Finally, simultaneous impact of market and funding liquidity risk is explored by Aikman et al. (2010) and Manna and Schiavone (2012).

While this body of work has considerably improved our understanding of the effects of the networks structure on the spreading of systemic risk unfortunately data on a range of relevant activities (securities lending, bilateral repos, and derivatives trading) remain inadequate. Moreover, the activities of non-bank market participants, such as asset managers, insurance and shadow banks, and their interconnections remain opaque. A proper assessment of systemic risk relies on a better understanding of the business models of these players and their interactions and on enhanced availability of individual transactions data.

3.7 ECONOMETRICS SYSTEMIC RISK MEASURES

Due to limited availability of supervisory data to capture systemic risk stemming from contagion via bilateral exposures, approaches have been suggested to derive systemic risk from available market data. A number of econometric measures, beyond Pearson correlation, have been proposed to measure the degree of correlation and concentration of risks among financial institutions, and their sensitivity to changes in market prices and economic conditions. Popular ones include Value-at-Risk (CoVaR) by Adrian and Brunnermeier (2016), CoRisk by Chan-Lau (2010), marginal and systemic expected shortfall by Acharya et al. (2012), distressed insurance premium by Huang et al. (2012), SRISK by Brownlees and Engle (2016), distance to distress by Castren and Kavonius (2009), and POD (probability that at least one bank becomes distressed) by Goodhart and Segoviano Basurto (2009). While these approaches are not network based, they are briefly mentioned here (see Appendix B for technical definitions) for completeness as they provide complementary methodologies to assess systemic risk and financial fragility. The underlying assumption behind these "market based" systemic risk measures is that the strength of relationships between different financial institutions, based on correlations between their asset values, is related to the materialization of systemic risk. The rationale is that common movements in underlying firms' asset values are typically driven by the business cycle, interbank linkages or shift in market sentiment that affects the valuation of bank assets simultaneously. By assuming that these are likely causes of contagion and common defaults, systemic risk should be captured by the correlations of observable equity returns.⁵ However the importance of short-term changes to market data might be overestimated, and the mechanisms that lead to the realization of systemic defaults are not well understood. In particular, when markets function poorly, market data are a poor indicator of financial environments. Correlations between distinct sectors of the financial system became higher during and after the crisis, not before, thus during non-crisis periods, correlation play little role in indicating a build-up of systemic risk using such measures. Moreover, measures based on probabilities invariably depend on market volatility, and during periods of prosperity and growth, volatility is typically lower than in periods of distress. This implies lower estimates of systemic risk until after a volatility spike occurs, which reduces the usefulness of such a measure as an early warning indicator. Overall these measures may be misleading in the build up to a crisis as they underprice risk during market booms.

3.8 LOCATION ADVANTAGES IN INTERBANK NETWORKS

In addition to having implications for financial stability, holding a central position in the interbank networks, or establishing preferential relationships, may lead to funding benefit. The exploration of potential location advantages has been the focus of recent

⁵Equity return correlation are normally used because the equity market is the most liquid financial market and thus new information on an institution's default risk can be incorporated in a timely way.

papers. Bech and Atalay (2010) analyze the topology of the Federal Funds market by looking at O/N transactions from 1997 to 2006. They show that reciprocity and centrality measures are useful predictors of interest rates, with banks gaining from their centrality. Akram and Christophersen (2013) study the Norwegian interbank market over the period 2006–2009. They observe large variations in interest rates across banks, with systemically more important banks, in terms of size and connectedness, receiving more favorable terms. Temizsoy et al. (2017) show that network measures are significant determinants of funding rates in the e-MID O/N market. Higher local measures of centrality are associated with increasing borrowing costs for borrowers and decreasing premia for lenders. However, global measures of network centrality (betweenness, Katz, PageRank, and SinkRank) benefit borrowers who receive a significant discount if they increase their intermediation activity, while lenders pay in general a premium (i.e. receive lower rates) for centrality. This effect is interpreted by the authors, as driven by the market perception that more central banks will be bailed out if in distress, because "too connected to fail". The expectation of implicit subsidies could create moral hazard and provide incentives for banks to become systemically important, exacerbating system fragility. Thus Temizsoy et al. (2017) suggest that monitoring how funding cost advantages evolve over time can act as an effective early warning indicator of systemic risk and provide a way to measure the effectiveness of regulatory policy to reduce the market perception that systemically important institutions will not be allowed to default.

During the crisis, increased uncertainty about counterpart credit risk led banks to hoard liquidity rather than making it available in the interbank market. Money markets in most developed countries almost came to a freeze and banks were forced to borrow from Central Banks. Nonetheless there is growing empirical evidence that banks that had established long term interbank relationships had better access to liquidity both before and during the crisis (Furfine, 2001; Cocco et al., 2009; Liedorp et al., 2010; Affinito, 2012; Brauning and Fecht, 2012; Temizsoy et al., 2015).

Early papers on interbank markets focus on the existence of lending relationships in the US Federal Funds markets (Furfine, 1999, 2001). Furfine (1999) shows that larger institutions tend to have a high number of counterparties while Furfine (2001) finds that banking relationships have important effects on borrowing costs and longer relationship decreases the interest rate in the funds market. Affinito (2012) uses data from Italy to analyze interbank customer relationships. His findings are that stable relationships exist and remain strong during the financial crisis. Liedorp et al. (2010) examine bank to bank relationships in the Dutch interbank market to test whether market participants affect each other riskiness through such connections. They show that larger dependence on interbank market increases risk, but banks can reduce their risk by borrowing from stable neighbors. Brauning and Fecht (2012) show that German lenders anticipated the financial crisis by charging higher interest rates in the run-up to the crisis. By contrast, when the sub-prime crisis kicked in, lenders gave a discount to their close borrowers, thus pointing to a peer monitoring role of relationship lending. Temizsoy et al. (2015) analyze the structure of the links between financial institutions participating in the e-MID interbank market. They show that, particularly after the Lehman's collapse, when liquidity became scarce, established relationships with the same bank became an important determinant of interbank spreads. Both borrowers and lenders benefited from establishing relationship throughout the crisis by receiving better rates from and trading larger volume with their preferred counterparties.

Cocco et al. (2009) suggest that size may be the main factor behind the Portuguese interbank funds connective and hierarchical architecture. Small banks acting as borrowers are more likely to rely on lending relationship than larger banks. Thus they suggest that financial institutions do not connect to each other randomly, but rather interact based on a size-related preferential attachment process, possibly driven by too-big-to-fail implicit subsidies or market power.

Hatzopoulos et al. (2015) have investigated the matching mechanism between lenders and borrowers in the e-MID market and its evolution over time. They show that, when controlling for bank heterogeneity, the matching mechanism is fairly random. Specifically, when taking a lender who makes l transactions over a given period of time and a borrower who makes b transaction over the same period, and such that they have m trades in common over that period, Hatzopoulos et al. (2015) show that m is consistent with a random matching hypothesis for more than 90% of all lender/borrower pairs. Even though matches that occur more often than those consistent with a random null model (which they call over expressed links) exist and increase in number during the crisis, neither lenders nor borrowers systematically present several over expressed links at the same time. The picture that emerges from their study is that banks are more likely to be chosen as trading partners because they are larger and trade more often and not because they are more attractive in some dimension (such as their financial healthiness).

Overall the empirical evidence suggests that, particularly at a time of deteriorating trust towards credit rating agencies, private information acquired through repeated transactions plays an important role in mitigating asymmetric information about a borrower's creditworthiness and can ease liquidity redistribution among banks. These results show that interbank exposures are used as a peer-monitoring device. Private information acquired through frequent transactions, supported liquidity reallocation in the e-MID market during the crisis by improving the ability of banks to assess the creditworthiness of their counterparties. Relationship lending thus play a positive role for financial stability and can help policymakers to assess market discipline. Furthermore, the analysis of patterns of preferential relationships may help identifying systemically important financial institutions. If a bank, who is the preferential lender to several borrowers defaults, or stop lending, this may pose a serious funding risk for its borrowers who may find it difficult to satisfy their liquidity needs from other lenders and may be forced to accept deals at higher rates. This may eventually put them under distress and increase systemic risk in the system. Similarly if preferential borrowers exit the interbank market, such lenders may find it difficult to reallocate their liquidity surplus if they fail to find trusted counterparties. The resulting inefficient reallocation of liquidity, may in turn increase funding costs of other borrowers and again contribute to the spread of systemic risk. In this sense relationship lending provides a measure of the financial substitutability of a bank in the interbank market. Thus when establishing if a bank is too connected to fail, regulators should not only look at how connected a bank is, but also at how preferentially connected it is to other players. Finally, reliance on relationship lending is an indicator of trust evaporation in the banking system and monitoring the effect of stable relations on spreads and traded volume may help regulators to identify early warning indicator of a financial turmoil.

4 NETWORKS AND INFORMATION FILTERING

So far we have primarily considered what we have called relationship or event networks. Event or relationship networks are the most common types of networks that are investigated in finance. In these networks a link between two nodes is set when a relationship or an event is occurring between them in a given period of time. For example two banks are linked when a credit relationship is present between them. In addition to this type of networks other classes of networks have been investigated in finance. We call these types of networks (i) proximity based networks, (ii) association networks, and (iii) statistically validated networks.

4.1 PROXIMITY BASED NETWORKS

In proximity based networks a similarity or a dissimilarity measure (i.e. technically speaking a proximity measure, see a reference text of Data Mining for a formal definition as, for example, Han et al., 2011) computed between all pairs of elements of a system is used to determine a network. The network obtained highlights the most relevant proximities observed in the system. Let us make an example to clarify the concept. Let us consider two banks having no credit relationship in the interbank market connecting them in a given time period. Is this meaning that the two banks are isolated the one to the other unless a chain of bankruptcy of banks occurs? The general answer is no. We can see this by considering the fact that the two banks have both a portfolio of assets and that the two portfolios can present a given degree of similarity. Two banks characterized by a high degree of similarity of their portfolios will be impacted by market dynamics and by exogenous news in a similar way in spite of the fact that they do not present direct credit relationship. For example, the fire sale of a distressed bank of a given asset will impact all those banks having large weights of that asset in their portfolios. The interlinkages associated with different degree of similarity of the set of elements of a given system can therefore be summarized and visualized in a network. In the literature these networks are sometime called with different names. For example, in neuroscience they are called functional networks because they are obtained starting from the functional activity detected by functional magnetic resonance signals detected in different regions of the brain. The proximity can be estimated by considering features of the considered elements that can be numeric, binary or even categorical.

4.1.1 The Minimum Spanning Tree

The first example of proximity based network was introduced in Mantegna (1999). This study proposed to obtain a network starting from a dissimilarity measure estimated between pairs of stocks traded in a financial market. A dissimilarity measure between two financial stocks can be obtained by first estimating the Pearson correlation coefficient ρ between the two time series of return of stocks and then obtaining a dissimilarity d (more precisely a distance) through the relation $d = \sqrt{2(1-\rho)}$. By following this approach, one can always associate a similarity or dissimilarity matrix to any multivariate time series and use this matrix to define a weighted complete network. The weight of each link being the value of the proximity measure.

For this type of weighted complete networks all information which is present in the proximity matrix is retained and it has associated the same degree of statistical reliability. However, in real cases this ideal condition is hardly verified. In fact, not all the information present in a proximity matrix has the same statistical reliability. The specific level of reliability depends on the way the proximity measure is obtained and limitations are always present when the estimation of the proximity measure is done with a finite number of experimental records or features. The unavoidable presence of this type of limitation is quantified by random matrix theory (Metha, 1991) in a very elegant way. For example, the eigenvalue spectrum of the correlation matrix for a random Gaussian multivariate set is given by the Marčenko–Pastur distribution (Marčenko and Pastur, 1967). Several studies have shown that the Marčenko–Pastur distribution computed for a correlation matrix of *n* stocks with a finite number of records widely overlaps with the ones empirically observed in financial markets although also clear deviations from this null hypothesis are observed (Laloux et al., 1999; Plerou et al., 1999).

In empirical analyses, it is therefore important and informative to perform a meaningful filtering of any proximity matrix obtained from a multivariate system described by a finite number of records or features. One of the most effective filtering procedures of a proximity matrix is the extraction from it of its minimum (for dissimilarity) or maximum (for similarity) spanning tree (Mantegna, 1999). The minimum spanning tree is a concept of graph theory and it is the minimum (or maximum) tree connecting all the nodes of a system through a path of minimal (maximal) length. The minimum spanning tree can be determined by using Prim's or Kruskal's algorithms. The information it selects from the original proximity measure is related to the hierarchical tree that can be obtained from the same proximity matrix by using the hierarchical clustering procedures known as single linkage. For more details about the extraction of minimum spanning trees from a proximity matrix one can consult (Tumminello et al., 2010).

In Fig. 2 we show an example of minimum spanning tree obtained for a set of 100 highly capitalized stocks traded in the US equity markets during the time period January 1995–December 1998. In the MST several stocks, such as, for example, BAC (Bank of America Corp.), INTC (Intel Corp.), or AEP (America Express), are linked with several stocks belonging to the same economic sector (financial, technology, and utility sector respectively) whereas others (the most notable case is General

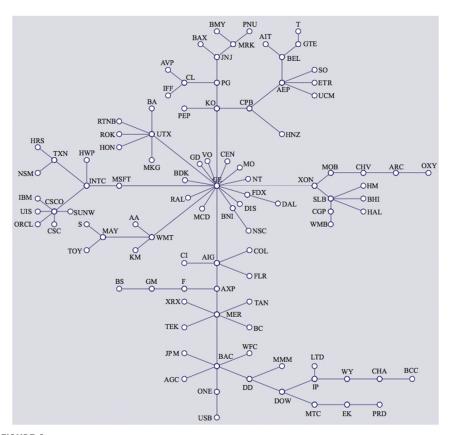


FIGURE 2

Minimum spanning tree of a set of 100 highly capitalized stocks traded in the US equity markets during the time period January 1995–December 1998. The similarity measure used to obtain the tree was the Pearson linear correlation measured between each pair of 1 day stock returns. Each circle represents a stock labeled by its tick symbol. The clustering of stocks according to their economic sector and subsector is evident. Reproduced from Bonanno et al. (2001).

Electric Company (GE)) are linked to stocks of different sectors. In general the clustering in terms of economic sectors of the considered companies is rather evident. Since the original proposal of Mantegna (1999), minimum spanning trees have been investigated in a large number of empirical studies involving different financial markets operating worldwide or in diverse geographical areas. The classes of assets and geographically located markets investigated through the proximity based network methodology comprises⁶: stocks traded in equity markets geographically located in

⁶The following references are not exhaustive. For an attempt to cover a wider number of reference see Bonanno et al. (2004), Tumminello et al. (2010), Marti et al. (2017).

US (Mantegna, 1999; Bonanno et al., 2001; Onnela et al., 2002; Miccichè et al., 2003; Bonanno et al., 2003; Onnela et al., 2003; Precup and Iori, 2007; Eom et al., 2007; Tumminello et al., 2007; Brida and Risso, 2008; Zhang et al., 2011), in Europe (Coronnello et al., 2005), in Asia (Jung et al., 2006; Eom et al., 2007; Zhuang et al., 2008), market indices of major stock exchanges (Bonanno et al., 2000; Coelho et al., 2007; Gilmore et al., 2008; Song et al., 2011), bonds and interest rates (Di Matteo and Aste, 2002; Dias, 2012, 2013), currencies (McDonald et al., 2005; Mizuno et al., 2006; Górski et al., 2008; Jang et al., 2011; Wang et al., 2012, 2013), commodities (Sieczka and Holyst, 2009; Barigozzi et al., 2010; Tabak et al., 2010b; Kristoufek et al., 2012; Zheng et al., 2013; Kazemilari et al., 2017), interbank market (Iori et al., 2008), housing market (Wang and Xie, 2015), credit default swaps market (León et al., 2014).

As for other data mining approaches, there are multiple approaches to perform information filtering. The choice of specific approach depends on the posed scientific question and on the ability of the filtering process to highlight the information of interest. This is similar to the case of hierarchical clustering where there is no a priori recipe to select the most appropriate algorithm performing the clustering. In addition to the approach of the minimum spanning tree several other approaches have been proposed in the literature to perform information filtering on networks. We will discuss some examples in the following subsection.

4.1.2 Other Types of Proximity Based Networks and the Planar Maximally Filtered Graph

One of the first alternative approaches to the minimum spanning tree was the one proposed in Onnela et al. (2004) where a network is built starting from a correlation matrix by inserting links between two nodes when their correlation coefficient is above a given threshold. This approach is retaining a large amount of information but suffers by the arbitrariness in choosing the threshold. Moreover, the network can cover only part of the system when the correlation threshold is relatively high. When the threshold is selected by considering an appropriate null model as, for example, an uncorrelated multivariate time series characterized by the same return distributions as in real data, most of the estimated correlation coefficient are rejecting the null hypothesis of uncorrelated returns ending up with an almost complete correlation based graph.

To highlight information present in the system by selecting links in a proximity based network richer than the minimum spanning tree without inserting an arbitrarily selected threshold the use of a network topological constraint was proposed in Tumminello et al. (2005). Specifically, Tumminello et al. (2005) introduce a method to obtain a planar graph starting from a similarity or a dissimilarity matrix. The method of the network construction requires that the network remains always planar, i.e. can be embedded in a surface of genus 0, until all the nodes of the system are included in the network. The resulting network has the property of including the minimum (or maximum in case of similarity) spanning tree and of presenting also loops and cliques of 3 and 4 nodes. The planar maximally filtered graph is therefore extracting

an amount of information larger than the minimum spanning tree but still linear in the number of nodes of the system. In fact the minimum spanning tree selects n-1 links among the possible n(n-1)/2 links and the planar maximally filtered graph selects 3(n-2) links. The approach of filtering the network under topological constraints can be generalized (Aste et al., 2005) by considering the embedding of the network into surfaces with genus larger than 0. The genus is a topological property of a surface. Roughly it is given by the integer number of handles observed in the connected, orientable surface. Unfortunately, this general approach which is very well defined from a mathematical point of view, is pretty difficult to implement computationally already for values of the genus equal to 1.

The study of proximity based networks is strongly interlinked with methodological approaches devoted to detect hierarchical structure and clustering of the considered nodes. Examples of this type of interlinkages are the clustering procedure achieved by considering Potts super-paramagnetic transitions (Kullmann et al., 2000). Within this approach, in the presence of anti-correlation, the methodology associates anti-correlation to a physical repulsion between the stocks which is reflected in the obtained clustering structure. Another approach to hierarchical clustering is using maximum likelihood procedure (Giada and Marsili, 2001, 2002), where authors define the likelihood by using a one-factor model, then varied to detect a clustering with high likelihood. In Tumminello et al. (2007) authors introduce the so-called average linkage minimum spanning tree, i.e. a tree associated with the hierarchical clustering procedure of the average linkage. An unsupervised and deterministic clustering procedure, labeled as directed bubble hierarchical tree, often finding high quality partitions based on the planar maximally filtered graph, is proposed in Song et al. (2012). Other clustering approaches have relied more on concepts originating from network science as it is the case for the approaches (i) using the concept of modularity maximization for cluster detection obtained from a correlation matrix (MacMahon and Garlaschelli, 2013), and (ii) using the concept of p-median to construct every cluster as a star network at a given level (Kocheturov et al., 2014).

4.2 ASSOCIATION NETWORKS

Another class of networks is the class we name association networks. In this type of networks two nodes of a complex system are connected in a network by computing a quantity that is putting in relation node i with node j of a given system. The quantity computed can be a complex indicator as, for example, the partial correlation between the time evolution of node i with node j or an indicator of the rejection of a statistical test against a given null hypothesis.

A prominent example of association network is the partial correlation network investigated in Kenett et al. (2010). Partial correlation is a measure of how the correlation between two variables, e.g., stock returns, is affected by a third variable. Kenett et al. (2010) define a proxy of stock influence, which is then used to construct a partial correlation network. The empirical study performed on stocks traded at the US equity markets concluded that stocks of the financial sector and, specifically, of investment services sub-sector, are taking a special position in the association network

based on partial correlation. Partial correlation networks have also been investigated by Anufriev and Panchenko (2015) for publicly traded Australian banks and for their connections to domestic economic sectors and international markets.

In their paper, Billio et al. (2012) propose a method to construct a directed network starting from a statistical test performed between each pair of time series of a multivariate system. Specifically they investigate Granger-causality between monthly return time series, and starting from the time series they infer a network of dependencies of hedge funds, publicly traded banks, broker/dealers, and insurance companies. In other words an association network of Granger-causal relations among these institutions is identified via the pairwise Granger-causality test. Granger causality is a statistical notion of causality based on the forecast power of two time series. Time series j is said to "Granger-cause" time series i if past values of j contain information that helps predict i above and beyond the information contained in past values of i alone. In their investigation authors conclude that their results show a pronounced asymmetry in the degree of connectedness among hedge funds, publicly traded banks, broker/dealers, and insurance companies sectors, with banks playing a much more important role in transmitting shocks than other financial institutions.

Another example of association networks concerns the networks obtained when performing pairwise co-integration tests. First attempts to produce association networks based on pairwise co-integration test are reported in Yang et al. (2014) and Tu (2014).

The need of many statistical tests to be used for the construction of an association network raises some problems directly originating form the pairwise nature of the performed tests. For example, in the case of Granger-causality test, as pointed out by Basu et al. (2017), direct pairwise estimation does not take into account indirect effects on the pair under consideration. For example in a system with 3 nodes and 2 "true" Granger causal dependencies: $B \to C, C \to A$, a Granger causal test would detect an additional (spurious) pairwise Granger causal effect $B \to A$, due to indirect effect through C. In fact a similar concern arises with the use of CoVaR, since these models estimate the covariance of an institution with the rest of the system without conditioning it on all other participants. Such inconsistency may impose large economic costs; for example, a number of institutions that are not highly interconnected may end up being wrongly classified as inter-connected under such approaches.

To address this problem Basu et al. (2017) develop and estimate a measure of network connectivity that explicitly recognizes the possibility of connectivity of all the firms in the network, in contrast with extant measures of systemic risk that, either explicitly or implicitly, estimate such connections using pair-wise relationships between institutions. The system-wide approach, is based on the Lasso penalized Vector Auto-Regression (LVAR) model that only focuses on estimating the strongest interconnections, while forcing weaker relationships to zero. Basu et al. (2017) also consider explicitly the problem of the control of the family-wise error rate. In fact when a large number of statistical tests are performed to obtain an association network the absence of the control of the family-wise error rate can produce a large number of false positive. This problem was first recognized in Tumminello et al.

(2011) where the concept of the so-called statistically validated network was introduced (see next section). In Tumminello et al. (2011) it was shown that the so-called Bonferroni correction is highly precise but often characterized by low power and therefore associated with a relatively low accuracy. For this reason, as it is common in statistical approaches needing a control of the family-wise error rate the approach based on the control of the false discovery rate (Hochberg and Tamhane, 1987) was proposed.

A similar system-wide approach was developed in Diebold and Yimaz (2014). However the Diebold and Yimaz (2014) approach requires fitting full vector autoregressive models, simultaneously for all firms, and use variance decomposition of the forecast error of the fitted model to define the network topology and extract connectivity measures. Since the number of parameters to be estimated, even for the simplest lag-1 VAR model in this approach is quadratic in the number of institutions under consideration, the model can only be estimated for a limited number of firms (15 in Diebold and Yimaz, 2014). The advantage of the LVAR model lays on the fact that it needs significantly lower number of time points to estimate as long as the underlying network is approximately sparse, and can be applied to more realistic setting involving all important banks of the economy.

Barfuss et al. (2016) have recently introduced an information filtering methodology (LoGo) to produce parsimonious models associated with a meaningful structure of dependency between variables of interest. The strength of this methodology is that the global sparse inverse covariance matrix is produced from a simple sum of local inversions. This makes the method computationally very efficient and statistically robust. Applied to financial data the method results computationally more efficient than state-of-the-art methodologies such as Glasso producing, in a fraction of the computation time, models that can have equivalent or better performances but with a sparser inference structure.

4.3 STATISTICALLY VALIDATED NETWORKS

In the previous section we have seen that a statistical test is performed to asses the existence of an undirected or directed link between each pair of nodes of the system in the study of association networks. Moreover, a statistical validation procedure can also be performed in ordinary networks to detect those links that are presenting marked differences from a given null hypothesis. According to Tumminello et al. (2011), when statistical tests are performed to highlight specific links (or the absence of links) between pairs of nodes of the investigated systems, we say we are in the presence of statistically validated networks detected against a given null hypothesis. The value of this statistical approach primarily relies on the fact that most systems of interest are highly heterogeneous with respect to economic or financial actions taken in a given period of time (e.g. number of loans, number of syndicated loans, amount of loans provided to firms, etc.) and therefore detecting over-expression or under-expression of interaction with respect to a null hypothesis taking into account for the heterogeneity of the action of each node can highlight important properties of the analyzed system.

The first example of statistical validation in weighted network is the work of Serrano et al. (2009). In this paper the statistical validation is performed on the outgoing weight of each node of the network statistically compared with a null hypothesis of uniform distribution of the total node strength in the outgoing arcs. In their paper authors statistically validate each link by deciding, which of the links carry disproportionate fraction of the weights of a given node. Specifically they compare the empirical value of each link of a node with a null model that considers all links of each node as equally probable. By performing the statistical validation node-by-node they select only those links that reject the null hypothesis therefore obtaining what they call the "multiscale backbone" of the system. It is worth noting that the method is not affected by the heterogeneity of the strength of nodes allowing for a selection of nodes working at all scales of the weight of links. Their approach was then generalized to the case of non-uniform distribution of weights by Radicchi et al. (2011).

Heterogeneity is a typical fingerprint of almost all complex systems. Economic and financial systems do not deviate from this general property and in fact economics was one of the first disciplines where power law distributions were documented and modeled. Vilfredo Pareto discovered a power law distribution in the wealth distribution of individuals in many countries and historical time periods as early as in 1897 (Pareto, 1897). An indication about the essential heterogeneity of economic and financial systems can be seen in the observation that the majority of economic and financial networks are presenting a degree distribution with a pronounced level of leptokurtosis. This observation has motivated network scientists to investigate complex network by estimating the deviation observed from the basic heterogeneity observed in the system.

The first approach designed to perform statistical tests of the over-expression of a given node action (e.g. participating to a credit relationship, performing a market transaction, etc.) against a null hypothesis taking into account the heterogeneity of the nodes was first presented for undirected interaction in Tumminello et al. (2011) and then extended to directed interaction and to the investigation of under-expression of interactions in Hatzopoulos et al. (2015). The method is designed for complex systems where two sets of agents repeats an action involving two actors a number of times in a given time interval. For example the two sets of agents can be (i) calling mobile phone subscribers and (ii) receiving mobile phone subscribers. Another example is the one where the two sets of agents are (i) lending banks and (ii) borrowing banks in the interbank market. Usually these complex systems are highly heterogeneous systems and therefore detecting links that are rejecting a null hypothesis taking into account the heterogeneity of the activity of each node in a statistical test can highlight informative interlinkages of the investigated system. In Fig. 3 we show the statistically validated network of the Italian segment of the e-MID market obtained in Hatzopoulos et al. (2015) from data of the maintenance period from 10 September 2008 to 9 December 2008.

It should be noted that the methodology used to obtain a statistically validated network needs performing a large number of statistical tests, that are usually done on all links of a large network. These means that this statistical procedure needs the

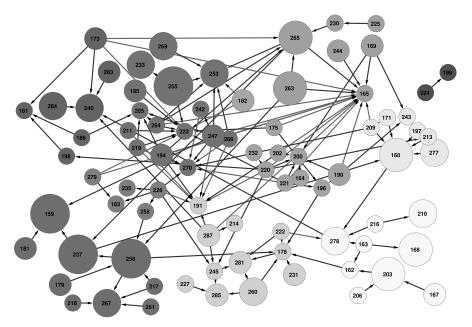


FIGURE 3

Statistically validated network (with the Bonferroni multiple hypothesis test correction) of the Italian segment of the e-MID market during the maintenance period from 10 September 2008 to 9 December 2008. Arrows originate from the lender aggressor. The different gray levels indicate the node membership obtained by applying a community detection algorithm. Light gray links are under-expressed links, while black links are over-expressed ones. Reproduced from Hatzopoulos et al. (2015).

so-called control of family-wise error rate. Tumminello et al. (2011) were the first pointing out the need of multiple hypothesis test correction when statistical tests are used to select or construct networks. Family-wise errors arise from the fact that when a large number of statistical tests are simultaneously performed a certain number of false positive are unavoidably observed due to the parallel repetition of the statistical test many times.

There are different types of procedures to correct multiple hypothesis testing procedures to avoid the presence of false positive. The most restrictive multiple hypothesis test correction is the so-called Bonferroni correction (Hochberg and Tamhane, 1987) that is performed by setting the statistical threshold as $\alpha_B = \alpha/N_t = 0.01/N_t$, where α is the chosen univariate statistical threshold and N_t is the number of tests to be performed. The Bonferroni correction controls the number of false positive therefore ensuring a high degree of statistical precision to the multiple hypothesis test performed. Unfortunately, in some cases it does not guarantee sufficient statistical accuracy because it may provide a large number of false negative. The procedure controlling the false discovery rate introduced in Benjamini and Hochberg (1995) reduces the number of false negative by controlling the expected proportion of re-

jected null hypothesis without significantly expanding the number of false positive. For this reason the multiple hypothesis test correction controlling the false discovery rate is today widely used also when investigating statistically validated or association networks.

Statistically validated networks originating from bipartite networks have been investigated in a variety of systems. The methodology was first illustrated by investigating the bipartite relationship between genomes and proteins in simple organisms, the price dynamics of financial stocks described in terms of categorical variables and the Internet Movie Database (Tumminello et al., 2011). It has also been used to detect and classify investment strategies of single investors (Tumminello et al., 2012). The methodology is rather flexible and easily adapted to directional relationships such as the ones of any form of communication from a caller to a receiver, any form of loan provided from a lender to a borrower or any form of market transaction from a seller to a buyer, etc. With this generalization of the methodology statistically validated networks have been detected and analyzed in mobile phone communication networks (Li et al., 2014a, 2014b) and in the empirical investigation of the e-MID interbank market (Hatzopoulos et al., 2015).

5 INDIRECT CHANNELS OF CONTAGION

The evaluation of the fragility and resilience of the financial system is a research area where different types of networks need to be taken into account. In fact, finance is probably one of the research areas where both relationships networks and proximity networks play a role in the correct assessment of systemic risk of the system. In the following subsections we will discuss the role of two major indirect channels of contagion that can be detected by using information about similarity of banks' characteristics. The first subsection discusses the role of overlapping portfolios of assets owned by banks and the second concerns the bank–firm credit relations and the indirect channel of contagion that can originate from them.

5.1 OVERLAPPING PORTFOLIOS AND FEEDBACK EFFECTS

Systemic risk is the research area where the use of event or relationship networks and proximity based networks is jointly required for a realistic model of the financial system. In fact, both direct and market mediated indirect channels of contagion need to be taken into account for the estimation of systemic risk of the financial system. One indirect channel can be highlighted by estimating the amount of overlapping of portfolios of different institutions. Market impact of portfolio deleveraging in stress scenarios can in fact induces contagion paths that are indirect (Cont and Wagalath, 2013; Huang et al., 2013; Caccioli et al., 2014). The magnitude of these additional market mediated contagion paths depends on the portfolios overlap of economic actors acting in the financial system (Cifuentes et al., 2005; Braverman and Minca, 2014; Cont and Schaanning, 2017). The most prominent aspect of this approach con-

cerns fire sales occurring during market downturns. It is documented that financial institutions in the presence of financial distress can be forced to fire sales of assets (Kyle and Xiong, 2001; Manconi et al., 2012; Cont and Wagalath, 2013). Due to market impact fire sales can generate a positive feedback loop that will increase endogenous risk (Shin, 2010) creating a channel of contagion for those financial institutions with high similarity in the holding of given assets.

Recent papers have proposed possible methods to include fire sales induced contagion channels into macro-stress tests (Duarte and Eisenbach, 2015; Greenwood et al., 2015; Cont and Schaanning, 2017). In Duarte and Eisenbach (2015) authors construct a systemic risk measure quantifying vulnerability to fire-sale spillovers using detailed regulatory balance sheet data for US commercial banks and repo market data for broker-dealers. They find that fire sale externalities can be substantial, even for moderate shocks in normal times. For example, they estimated that for commercial banks a 1 percent exogenous shock to assets in 2013 in the first quarter can produce fire sale externalities of the order of 21 percent of system capital. Greenwood et al. (2015) introduced a model where fire sales propagate shocks across bank balance sheets. Authors showed how this contagion spreads across the banking sector, and how it can be estimated empirically using balance sheet data. In particular, authors computed bank exposures to system-wide deleveraging, as well as the spillovers induced by individual banks. The study presents an application to European banks. Cont and Schaanning (2017) suggest an operational framework able to quantify the impact of deleveraging in stress scenarios when the financial institutions are subjected to portfolio constraints. The authors show that this price mediated loss contagion may be quantified through a liquidity-weighted overlaps across portfolios. The paper also performs a case study using data on European banks. By performing the case study authors show that indirect contagion effects affect the outcome of bank stress tests and lead to heterogeneous losses of banks which cannot be replicated in a stress test without deleveraging effects. The study highlights the difference between insolvency and illiquidity clarifying that the presence of different levels of illiquidity and insolvency can lead to substantially different loss estimates compared to standard models just based on leverage targeting.

Risk metrics based on the mechanism of portfolio rebalancing through fire sales require the full knowledge of the portfolio holdings of each institution in the economy. However, such detailed information may not be available, especially at frequency higher than a quarter. Di Gangi et al. (2015) propose to apply the maximum entropy approach to the inference of the network of portfolio weights in order to estimate metrics of systemic risk due to fire sales spillovers, starting form a reduced information set. The authors show that this approach, while underestimating systemic risk in interbank networks (Mistrulli, 2011), works well when applied to reconstruct the exposures of US commercial banks available via FFIEC Call Reports. Gualdi et al. (2016) propose a method to assess the statistical significance of the overlap between heterogeneously diversified portfolios, in order to investigate patterns of portfolio overlap. When applied to a historical database of SEC 13-F filings, the authors find that the proposed proxy of fire sale risk, after reaching a peak in 2008 had

subsequently decreased. However fire sale risk has been increasing again from 2009 to the end 2013 and the number of securities that can be involved in a potential fire sale has been steadily growing in time, with an even stronger proliferation of contagion channels.

5.2 FINANCIAL SECTOR AND THE REAL ECONOMY

Another important channel of indirect contagion of banks due to external shocks is the indirect channel due to common exposure to firms. When a large firm get distressed it will present difficulties in paying back the loans obtained from a number of banks. In other words the links existing from the real economy to the financial sector can act as channels of contagion from a firm or a set of firms to the banking system especially in a period of economic crisis when a state of financial distress may impact simultaneously many firms or firms of an entire economic sub-sector. The impact of the bank-firm network on systemic risk and its role as channel of contagion is investigated theoretically in Lux (2016). In this study, the author proposes a stochastic model of a bipartite credit network observed between banks and the nonbank corporate sector. The model is based on stylized facts observed empirically in large data sets of bank-firm loans for some countries. The model is investigated computationally and it shows a pronounced non-linear behavior under shocks. In fact, the default of a single unit will typically have no immediate avalanche effect, but might lead to a wide collapse of the entire system in a certain number of cases. The model and its numerical investigations suggest that when one distinguishes between contagion due to interbank credit and due to joint exposures to counterparty risk via loans to firms, the later channel appears more important for contagious spread of defaults. Other recent papers that assess how the interbank network structure affects lending to the wider economy via macro-finance ABM simulations are Gabbi et al. (2015) and Gurgone et al. (2017). In the first paper, firms behavior is exogenous and stochastic, and lending to firms short term. In the second firm dynamics are endogenized, lending to firms is long term, and banks implement precautionary liquidity hoarding strategies. Both papers show that constrains to interbank lending affect rates and volumes on the bank-firm credit markets. Instability arising from this channel dominates direct knock-on effects. However, the connectivity of the interbank market has a non-monotonous effect on macro-financial stability.

Starting from 2010 a series of empirical studies have considered the structure of the bank–firm network observed by considering the credit network provided by the banking system of to the firms operating in a given country. The first studies were conducted by analyzing the credit relations of Japanese banks with large Japanese firms (De Masi et al., 2011), and a subset of the Italian banking and corporate system (De Masi and Gallegati, 2012). More recently, the bank interlinkages originating from joint exposure to firms have been investigated also in Brazil (Miranda and Tabak, 2013) and China (Wang and Yang, 2017). The bank–firm network has been investigated by focusing on the topology of the projected network of banks (De Masi et al., 2011), by considering the time evolution of the credit relationships arising from the

clustering of the bank–firm relationships investigated with tools of community detection directly applied to a bipartite network (Marotta et al., 2015), and by highlighting the backbone of the credit relationships (Marotta et al., 2016).

The analysis of the Japanese credit market of 2004 performed in De Masi et al. (2011) pointed out that big Japanese banks privileged in that period of time long-term contracts. An analysis performed by using the minimum spanning tree disclosed a highly hierarchical backbone, where the central positions of MSTs were occupied by the largest banks. A strong geographical characterization of the various branches of the trees was also observed, while the clusters of firms did not show apparently specific common properties. Moreover, authors observed that while larger firms presented multiple lending in large, the demand for credit of firms with similar sizes was very heterogeneous.

In the analysis performed directly on the bank-firm bipartite network (Marotta et al., 2015), the analysis was repeated for each calendar year of the 32-year-long time period ranging from 1980 to 2011. In this study, authors investigated the time evolution of the networked structure of the credit market by tracking the time evolution of detected groups of banks and firms. The different groups were then characterized by analyzing the composition of the groups with respect to the over-expression of attributes of firms and banks. Specifically, authors considered as attributes the economic sector and the geographical location of firms and the type of banks. With their analysis authors were able to detect a long term persistence of the over-expression of attributes of communities of banks and firms together with a slow dynamic of changes from some specific attributes to new ones. In Marotta et al. (2016) authors detected the backbone of the weighted bipartite network of the Japanese credit market relationships. The backbone was detected by adapting the method introduced in Serrano et al. (2009). The analysis was done with a yearly resolution and covered the time period from 1980 to 2011. The study showed the time evolution of the backbone by detecting changes occurring in network size, fraction of credit explained, and attributes characterizing the banks and the firms present in the backbone.

6 CONCLUDING REMARKS

Starting from the end of the last century networks have been used to analyze and model financial systems. In the present review we primarily focused on empirical analyses of financial networks. In particular we have extensively discussed (i) analyses and modeling of the interbank market and (ii) different types of networks that have been empirically investigated in finance. In fact, finance is one of the disciplines where in addition to the customary study of event or relationship networks there is also a large number of investigations of association networks and statistically validated networks.

Empirical analysis needs access to accurate and cured data. Finance is one of the research areas with an extremely high rate of production of business data. Most of economic and market activities are today performed or assisted with tools of in-

formation technology and therefore market activities are producing a gargantuan amount of data. Some of these data are transparent and easily accessible other are distributed, confidential, and extremely hard to obtain also for monitoring authorities. For this reason many methods have been proposed to reconstruct interlinkages which are present between financial actors starting from aggregate information publicly released by them. In this review we discuss some of the attempts performed in this direction during the last years.

Our review has shown that many efforts have been done and are continuously done to properly detect and model the interlinkages that are of relevance for the evaluation of the fragility and resilience of the financial system. Several approaches have considered interbank credits in the presence or absence of collaterals and by considering different maturities of the loans. Some of these networks have been investigated to detect systemic important financial institutions defined with a variety of criteria ranging from their size to their interconnectedness with respect to a specific market or to groups of markets. In addition to the direct channels of contagion, a series of recent studies have also considered indirect channels of contagion that can originate from similarity of the banking activities. The wide variety of the methods and approaches proposed shows the importance that interlinkages certainly play in systemic risk assessment and it indicates that currently there is no widespread consensus on the way the fragility of the financial system can be detected and monitored efficiently. The research community is therefore considering multiple approaches partly overlapping that are covering different channels of contagion and different feedback mechanisms. The scientific falsification of the different approaches proposed will provide information that will allow the research community to gradually converge towards a successful consensus methodology. Some recent work in this direction has started. For example Anand et al. (2017) perform a systematic comparison of network reconstruction methodologies from 24 financial networks. These authors show that while some approaches work better than others, performance depends on the network characteristics, such as, for example, its sparsity. Thus, for the moment, we believe that the methods proposed and presented throughout this review need to be considered in parallel until a consensus methodology will eventually emerge.

The network approach to the analysis and modeling of complex financial systems is a truly interdisciplinary activity involving scholar with background in disciplines as different as economics, finance, mathematics, statistics, computer science, physics, econometrics, and social sciences. Typically the cultural background of the researcher connotes the specific approach to the study and some of the different disciplines seem not to communicate among them. In this review we have made efforts to cite empirical investigations of financial systems performed in finance, financial mathematics, econophysics, and econometrics. These different research communities present overlaps of themes, results, and fruitful interactions but also cases of isolation within each specific community of research are frequent. We hope that our review might provide a wider perspective and might promote the diffusion of knowledge obtained by the different research communities bridging the efforts done and cross fertilizing future results.

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APPENDIX A BASIC CONCEPTS IN NETWORK SCIENCE

Among its definitions, the Oxford Dictionary states that a network is "A group or system of interconnected people or things". More generally, a network is a collection of elements connected by a number of links or interlinkages. This is a rather generic definition. The lack of a more precise definition is probably related to the fact that network science is a truly multidisciplinary research area. In fact network studies have been performed within the research communities of social sciences, mathematicians, statistical physicists, computer scientists, biologists, chemists, medical scientists, and economists. Therefore in network science is not unusual to encounter approaches, methodologies, and technical languages that are reflecting the characteristic of the research community that has originated the studies and the results. One prominent example is graph theory. Graph theory is a well defined research are of mathematics where the object of interest are graphs (i.e. networks) primarily described in terms of a collection of points joined together in pairs by lines. In this last case the definition of a network (here a graph) is much more precise but appropriate just for the purposes of graph theory and not always usable for wider use of the concept of networks. For these reasons we will consider ourselves satisfied with a generic definition saying that a network is a collection of elements connected by a number of links or interlinkages.

Studies about networks have been performed by many scholars in different disciplines since the solution of the so-called Königsberg bridge problem from Euler in 1735. However a large scale interest towards this type of problems triggered at the beginning of the nineties of the last century when the fast development of the World Wide Web manifested the huge importance information networks have in all human activities. Starting from that period several communities of researcher started to work on network problems. Basic concepts about networks can today be easily found in several authoritative books (Newman, 2010; Barabási, 2016) and reviews (Boccaletti et al., 2006). Hereafter we briefly recall a few basic concepts just to set

up the terminology we are using in the present review in a way that the reader will find the basic information discussed in this review self-contained.

In a mathematical representation networks are described in terms of graphs. A graph G = (V, E) is a mathematical object consisting of a set V of vertices (also called nodes) and a set E of edges (also called links). Edges are defined in terms of pairs $\{i, j\}$ of distinct vertices i, j belonging to the set V. In undirected networks the order of vertices is not informative. The number of vertices N_v is called the order of the graph. The number of edges N_E is called the size of the graph. A graph $G_S = (V_{G_S}, E_{G_S})$ is a subgraph of $G = (V_G, E_G)$ if $V_{G_S} \subseteq V_G$ and $E_{G_S} \subseteq E_G$. A graph $G_I = (V_{G_I}, E_{G_I})$ is an induced subgraph of $G = (V_G, E_G)$ if $V_{G_I} \subseteq V_G$ is a pre-specified set of vertices and $E_{G_I} \subseteq E_I$ are the edges observed among them. In some cases vertices have self-links (i.e. both edges connected to the same vertex, sometimes also called self-loops). A graph with either self-links and/or multi-edges between two vertices is called a multi-graph or a multiplex. A graph without selflinks and without multi-edges is addressed as a simple graph. A graph where the relationship between vertex i and vertex j is directional is called a directed graph. Directed graphs have directed edges also called arcs. In a directed graph the direct edge $\{i, j\}$ is different from $\{i, j\}$. Conventionally, the formalism $\{i, j\}$ states that the directionality is from the tail i to the head j. Note that in a simple directed graph we might have up to 2 directed edges between two vertices. When both are present one says that the two arcs are mutual.

Two vertices $i, j \in V$ are adjacent if joined by an edge belonging to set E. Two edges $e_i, e_j \in E$ are adjacent if connected by a vertex belonging to set V. The degree of a vertex is the number of incident edges on it. A degree sequence is obtained by arranging the degree of vertices in non-decreasing order and associating to each vertex its degree. A degree distribution is the probability distribution of the degree observed in a specific network. In a directed graph one can consider both in-degree and out-degree sequence and/or distribution. A walk on a graph G from an initial vertex i to a final vertex j is the path associated with a sequence of vertices and edges connecting the two vertices in the graph. A walk with n edges (where n is counting edges each time they are encountered including multiple counts) has length n. A walk without repeated vertices and repeated edges is called a path. A walk, of length at least three, beginning and ending at the same vertex but with all vertices distinct from each other is called a cycle.

A vertex i is reachable from a vertex j if there exists at least a walk connecting i to j. A graph G is said to be connected if every vertex is reachable for every other one. A component of a graph is a maximally connected subgraph. The component with the largest number of vertices is called the largest connected component. In the case of directed graphs the concept of connectedness is specialized in two cases. A directed graph is weakly connected if the underlying undirected graph is connected. A directed graph is strongly connected if every vertex i is reachable by every other vertex j through a directed walk. A widely used notion of distance between two vertices of a graph is defined as the length of the shortest path(s) between the two vertices. Another indicator is the diameter of a network defined as the maximum length of shortest

paths observed in the network. When an edge of a graph has associated a numerical weight the graph is called a weighted graph. The notion of degree is generalized to take into account the weights of the edges. The generalization is called the strength of the vertex and it is the sum of the weights of all incident edges. When edges are weighted the length of a walk (or of a path) is defined as the sum of the values of the edges composing the walk (path).

Some basic network structures have gained special attention due to their specificity. We are briefly recalling some of these specific network structures hereafter. A complete graph is a graph where every vertex is linked to every other vertex. A complete subgraph is called a clique. A *d*-regular graph is a graph where all vertices have degree *d*. A connected graph with no cycles is called a tree. The disjoint union of trees is called a forest. A directed graph whose underlying graph is a tree is called a directed tree. Directed tree may show a root, i.e. a unique vertex from which there is a directed path to any other vertex of the graph. When a root is present in a directed tree the tree is called a rooted tree. A bipartite graph is a graph where the set of vertices can be classified in two disjoint sets and edges are present only between a pair of vertices of different type (for example, actors and movies, authors and papers, students and courses, etc.). From bipartite networks it is quite common to extract one or two projected, i.e. networks containing only vertices of the same type. A graph is planar if its edges can be embedded on a surface like a plane or a sphere, without crossings of the edges.

The information about the edges present in a network are summarized in the adjacency matrix or in the edge list. The adjacency matrix is a square matrix $n \times n$ of elements A_{ij} different from zero when an edge is present between node i and node j. In the presence of an edge, the value of the element is 1 for unweighted networks and equal to the weight of the link w_{ij} for weighted ones.

Networks are investigated under several respects. For example they are investigated both by considering properties of single vertices and edges and by considering a series of ensemble properties of vertices or of groups (also called communities or partitions) of them. Other investigations concerns the relative abundance of structures of a few vertices called triads in social sciences and motifs in natural sciences. Among the measures of single vertex there is the degree (with the corresponding indicator of the strength) and other centrality measures as, for example, the betweenness. The betweenness is a centrality measure obtained by considering which fraction of all shortest paths connecting all pairs of vertices are passing through a specific vertex or edge. Other measures as the clustering coefficient are measuring the degree of local compactness of a network. The local clustering coefficient of vertex *i* is defined as the ratio between the number of edges observed among the first neighbors of node *i* and the total number of edges possible among them. The global clustering coefficient computes the fraction of paths of length 2 that are closed.

The most investigated ensemble property of vertices is the degree (and/or strength) distribution. The degree distribution can present a characteristic scale or can be a broad distribution. The characteristics of the degree distribution have been often put in relation with widespread models of networks. A generic model charac-

terized by the same degree distribution of the one of the system of interest and with no additional information is addressed as a configuration model of the considered network. In real networks configuration models are often used as statistical null models to detected structures and regularities that cannot be associated with the type of heterogeneity of the considered system.

The most widespread models of networks are the Erdös–Rényi model (Erdös and Rényi, 1960), the small world model (Watts and Strogatz, 1998), the preferential attachment model (Barabási and Albert, 1999), and the core-periphery model (Borgatti and Everett, 2000). The classic Erdös-Rényi model (Erdös and Rényi, 1960) is a model of random connection between all pair of vertices belonging to a network. The surprising result obtained by Erdös and Rényi was to conclude that in an infinite system the presence of a spanning cluster, i.e. a connected cluster covering large part of the vertices, occurs abruptly ad a function of the average degree in a way that is reminiscent of a percolation phenomenon. The small world model was proposed to solve the apparent puzzle of the observation of many networks that present a high degree of clustering simultaneously with the presence of a short average path length. By studying a transition from a d-regular network to a random network, Watts and Strogatz were able to show that the presence of a very limited number of links connecting remote regions of the network are able to decrease significantly the average path length without equally affecting the degree of local clustering thus providing a rational for the frequent observation of "small world" phenomena. The Barabási-Albert scale free network is a type of network presenting a high heterogeneity of the degree of vertices. One way to generate this wide class of network is by hypothesizing a network growth process based on preferential attachment. The core-periphery model originated in the field of social sciences was originally motivated by a series of studies on national elites, interlocking directorates, and scientific citation networks. These previous studies triggered the developing of a formal model proposed by Borgatti and Everett (2000) where vertices belong to a "core" or to a "periphery". The vertices that belongs to the core are strongly interconnected between them and have also links with vertices of the periphery. The vertices of the periphery have links with vertices of the core but do not have links between them.

The investigation of triads, i.e. subnetworks involving three actors and their links, and their interpretations was originally introduced in studies of social networks performed during the seventies of the last century (Wasserman and Faust, 1994). The same structures have also been investigated in other types of networks such as biological networks (Milo et al., 2002) where they have been labeled as 3-motifs and generalized as k-motifs. Real networks present local structures where vertices have a degree of interconnectedness towards internal vertices much more pronounced that the one observed towards external vertices. When this type of structures are detected in a network it is said that the network is presenting a community (i.e. a cluster) structure. Several methods and algorithms have been proposed to properly detect this type of structures. Comprehensive reviews about the problem of community detection can be found in Fortunato (2010) and Fortunato and Hric (2016).

APPENDIX B ECONOMETRICS SYSTEMIC RISK MEASURE

The first step in market based approaches to quantify systemic risk is to compute the system-wide loss distribution and define a set of systemic events, which are states of the world that occur with a small probability but in which aggregate losses exceed a critical threshold. The systemic importance of a particular bank is then set equal to the expected losses it generates, conditional on systemic events. In other words, systemic importance is measured as the expected participation of individual institutions in systemic events. CoVaR measures the value-at-risk (VaR) of financial institutions conditional on other institutions experiencing financial distress. It provides a boundary on a large loss for some institution(s), given that a particular institution is stressed to a certain degree. While the q% VaR gives the minimum large loss that is not exceeded (1-q%) of the time, $\hat{CoVaR_q^{j|i}}$ gives the q-percent VaR value for institution j when institution i is at its q-percent VaR value. If one of the institution is replaced by the overall financial system, then CoVaR estimates the size of the losses to the system caused by financial distress of one institution. CoVaR examine the spillover effect from one bank's failure to the whole system, but underplays the importance of institutional size by design. Another disadvantage of CoVaR is that it can be used only to identify systemically important institutions but cannot appropriately aggregate the systemic risk contributions of individual institutions.

The expected shortfall (ES) is the expected loss conditional on the loss being greater than a given limit (usually the 95% VaR). The marginal expected shortfall (MES) of a financial firm is the short-run expected equity loss conditional on the banking sector stock index decreases to its lower fifth percentile (note that the MES can be expressed as a function of the COVAR). Systemic Expected Shortfall is defined as the amount of a bank's equity drop below its target level (which is a fraction of its assets) in case of a systemic crisis when the aggregate banking capital is less than a fraction of aggregate assets. Acharya et al. (2012) show that the SES is related to the MES measured during bad markets outcomes, scaled up by a factor to account for the worse performance in a true crisis. Benoit et al. (2017) show that systemic risk rankings of financial institutions based on their MES tend to mirror rankings obtained by sorting firms on market betas capturing systematic risk rather then systemic risk.

The DIP indicator of systemic risk, is defined as the insurance premium that protects against the distressed losses of banks portfolio. Technically, it is the risk neutral price of a hypothetical insurance contract calculated issued to protect against loses that equal or exceed a fixed level (for example, 10% of the gross liability of all the sampled financial institutions). DIP is similar to the MES measure in that both focus on each bank's potential loss conditional on the system being in distress exceeding a threshold level, and both are coherent risk measures. The threshold definition however is different. The extreme condition is defined by the percentile distribution in the MES setting but by a given threshold loss of the underlying portfolio in the case of DIP. The risk premium comprises two components: the default risk premium, which reflects uncertainty about a financial institution's creditworthiness; and the liquidity risk premium, which reflects market liquidity. The DIP is designed to capture the risk

emanating from both deteriorating creditworthiness among financial institutions but also increased risk aversion or price volatility. Unlike the CoVaR and the MES, the DIP uses the incidence of default instead of the plunge of stock prices. The variable utilized by this measure is the default probability of the financial institutions based on their CDS spreads.

Goodhart and Segoviano Basurto (2009) estimate individual banks default probabilities from structural approaches or securities prices and aggregate them at the system level using copulas. From the obtained multivariate distribution of defaults, they derive several banking distress measures.

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