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Finding the core: Network structure in interbank markets ☆

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ABSTRACT

This paper investigates the network structure of interbank markets. Using a dataset of interbank exposures in the Netherlands, we corroborate the recent hypothesis that the core periphery model is a 'stylised fact' of interbank markets. We find a core of highly connected banks intermediating between periphery banks and pay particular attention to model selection. Our analysis can help improve systemic risk assessments, especially as more granular data is becoming available.

1. Introduction

Understanding complex interbank markets is crucial for safeguarding financial stability, as became clear during the financial crisis. Whereas the relevance of the network structure prior to crisis was mentioned only infrequently, it has now caught the attention of both academics (Tirole, 2011) and policy makers (Haldane, 2009; Yellen, 2013). Systemic risk and contagion have become keywords in finance, and debate on their precise definition and implications is ongoing.

Traditionally, the interbank market has been considered without taking the network structure into account. Bhattacharya and Gale (1987) for instance propose that the main reason for interbank trade is to co-insure against idiosyncratic liquidity shocks. For various reasons, banks might not be indifferent with whom they transact, giving rise to non-random networks. One reason might be that interbank lending functions as a peer-monitoring device (Rochet and Tirole, 1996). Recently, some have argued forcefully that relationships matter in the interbank market. Cocco et al. (2009) show that borrowing banks in Portugal pay a lower interest rate on loans from banks with whom they have a stronger relationships, and Bräuning and Fecht (2012) analyse relationship lending in Germany during the financial crisis of 2007–2008. Such relationships will be reflected in the network structure.

As the theoretical literature started to consider networks, it became clear that the actual structure of linkages between banks affects the stability of the systems and the possible contagion after large shocks. In a seminal contribution, Allen and Gale (2000) use stylised examples to show that the fragility of the system depends crucially on the structure of interbank linkages. If a network is 'complete', i.e. all banks are connected to all other banks, a shock to a single bank can easily be shared and thus the stability of the system is safeguarded. If instead

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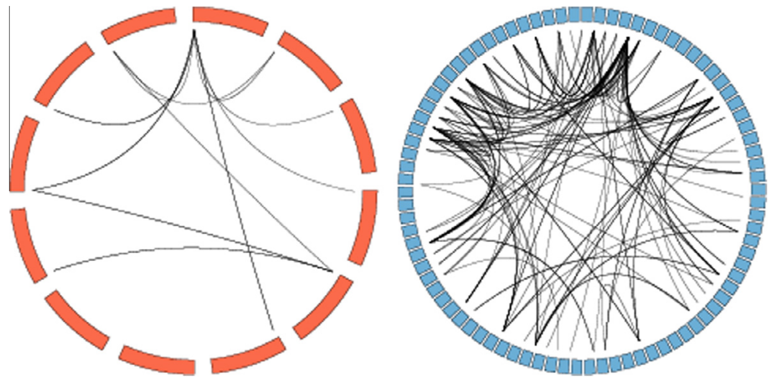

Fig. 1. Errors of the CP model for the Dutch interbank market on 2005Q1. Banks are plotted on the circle. For the core (left), the errors are the missing links between any pair of the 12 core banks: 9 out of the 132 possible links. For the periphery (right), the errors are links between any two periphery banks (87 in total): 157 out of 7482 possible links.

the network becomes clustered, spillovers to some of the banks can become substantial.[1]

Given its importance in theoretical work, the empirical analysis of financial networks is still lagging behind. Datasets are few and far between. Interbank exposures have mainly been used for stress test exercises. As noted by Upper (2011) in an overview of this literature, the estimated contagious effects are limited.[2] This is not surprising as the data generally only covers a single market and because behaviour does not change conditional on the state of the world. The limited literature that uses the tools of network theory and comes closest to our approach is discussed in Section 2.

The first goal of this paper is to find a network model that best describes the structure of actual interbank markets. This is important, as the shape of the observed network is a key determinant of the stability of the system, as discussed in Section 5. To this end we will analyse a long running panel of bank links in the Netherlands. We will focus on the estimation of a core periphery (CP) model as recently applied by Craig and von Peter (2014). We will define this model more formally below, but loosely speaking this is a structure where core banks intermediate between dependent periphery banks. Discriminating between these two types of bank is very useful for bank supervision in practice.

On a more technical note, we place more emphasis on model selection than previous studies in proposing the core periphery model for the interbank market. Another structure that recently has been put forward as a good description for interbank networks (e.g. König et al., 2014) is the so-called nested split graph (NSG). We clarify the apparent confusion between the CP and NSG models, and find a better fit of the CP model in terms of the error score. Using Monte Carlo simulations, we show that the error score is a satisfactory measure of fit to discriminate between networks close to either an CP or NSG network. The simulations also show that the good fit of the CP model is highly unlikely to be generated by random processes of other well-known, stochastic network models. Overall, by combining various existing methods to fit networks on the Dutch data, we make clear that the core periphery model gives a better fit to the interbank market than others used in the literature.

For the Netherlands we find a core of around 15 banks. This core – unsurprisingly – almost always includes all the large banks. Some small banks, however, are also part of the core (although we cannot discuss these because of confidentiality reasons). For a typical period, Fig. 1 shows that the model produces only few errors: There are few missing links in the core, and also relatively few existing links in the periphery.

The second goal is to place the network structure results in context. Are core banks different in terms of (regulatory) risk measures or business models? We will amongst other findings, show that core banks have, on average, lower capital buffers. So, even though they might be seen as more systemic, their buffers are lower. Taking the network structure into account in theoretical modelling and stress testing will be a crucial next step in developing systemic risk assessments. In the core periphery model a small set of core banks is highly connected, while periphery banks are not connected with each other but only to the core. The core banks are thus systemically more important in passing through shocks and should hence be monitored carefully to maintain financial stability.

The remainder of this paper is structured as follows. In Section 2 we discuss different network models that have been applied to the interbank market with an emphasis on the core periphery network. Section 3 describes our dataset and the market network structure. Section 4 contains our main estimation results including a discussion on testing and model selection. In Section 5, we relate our results to financial stability. Section 6 concludes.

2. Network models

Mathematical network or graph theory has been applied to many different fields including biology, technological networks, and information science (see Newman, 2010 for a comprehensive review). In this section we will discuss both *stochastic* (where link formation is determined by some random process such as the Erdös–Rényi random graph and the Barabási–Albert scale-free model) and *deterministic* models (where a structure is postulated such as nested split graphs and core periphery network).[3] After describing the Dutch interbank market data in the next section, we will then, in Section 4, compare the relevant network models and will find that the CP model best explains the data.

[1] The examples in Allen and Gale (2000) are clearly simplified and subsequent research has shown that many other aspects are relevant. For instance, Gai et al. (2011) show in a model with unsecured claims and repo activity that systemic liquidity crises as seen in 2007–2008 can arise with funding contagion spreading throughout the network. Castiglionesi (2007) argues that it is not so much the structure of the market but rather the lack of complete contracting. See Chinazzi and Fagiolo (2013) for a survey of research exploiting networks to explain contagion and systemic risk in financial markets.

[2] Studies include Furfine (2003), Upper and Worms (2004), van Lelyveld and Liedorp (2006), Degryse and Nguyen (2007), and Mistrulli (2011).

[3] We define a 'network model' as a simple representation of a structure of bilateral relations (in our case interbank loans). Thus the word 'model' does not refer here to a formal microeconomic characterisation of banks' decisions. For the four models considered, however, it will become clear that latent microeconomic models exist that capture the concepts of random interaction, preferential attachment, counterparty reliability, and intermediation.

One of the earliest theoretical models of a network was introduced by Erdõs and Rényi (1959, henceforth ER). In their random graphs, each possible link between any two nodes can occur with a certain independent and identical probability. The benchmark Erdös–Rényi model has a severe limitation for our empirical application: it is not likely to produce a network with wide ranging degrees. The *degree* of a bank in a directed interbank network is the number of banks to which it lends (*outdegree*) or from which it borrows (*indegree*) and is denoted by k_i (and k_i^{out}, k_i^{in}). The property of a fat tail in the degree distribution has been observed in many types of networks and has led to the development of scale-free models by Barabási and Albert (1999), where the probability of forming links increases proportionally with the degree (i.e. *preferential attachment*, PA). Preferential attachment for banks could result from the wish to interact with the most reliable counterparties, where trustworthiness is associated with popularity. Banks who initially have the largest number of interactions will therefore attract more linkages over time. In particular, preferential attachment distributes the probability of a connection proportionately to its links k_i.

Until recently the empirical work on the interbank exposures almost exclusively built on the scale-free model. Theoretically, a scale-free network displays a power law in the degree distribution. In one of the earliest descriptions of interbank network topology, Boss et al. (2004) fit power laws on two different regions in the degree distribution for Austria. Using data from Brazil, Cont et al. (2013) connect a systemic risk measure from a stress test exercise with local network characteristics, after calculating various properties of the network including the scale-free parameter. Martínez-Jaramillo et al. (2014) also find large degree heterogeneity in the Mexican interbank market. In sum, the empirical literature points to the apparent scale-free distributions as its main finding.[4]

However, the statistical support for many of the claims to scale-freeness of networks has been called into question (Stumpf and Porter, 2012). The problem boils down to the fact that scale-freeness is an asymptotic property, and even in the largest datasets there are few observations of extreme degrees. The limited and mostly *fixed size* of banking networks makes the scale-free network of Barabási and Albert (1999), that relies on continuing growth, inappropriate.

A different class of network models are *deterministic models*. An important advantage of such models is that they fit well within economic theories of profit optimising banks. Banks do not interact randomly with each other; interbank exposures are the complex result of agents making well-considered decisions to borrow from or lend to one another. In the following two subsections, we discuss two deterministic network models that recently have been applied to interbank markets: nested split graphs and core periphery networks. To our knowledge we are the first to compare the two models.

The two deterministic network models have some features in common, and in fact a core periphery structure is sometimes informally presented as indicative for nestedness. Indeed, we will see that both models impose the large heterogeneity in degree that is observed in the data. Moreover, they are *single-centered*: there are no distinct communities of banks that cluster together (for geographical and historical reasons). Within the small banking sector of the Netherlands, communities are not expected to play an important role. Despite their similarities, we will show below that nested split graphs and core periphery networks are actually based on different economic concepts. It is therefore useful to be able to discriminate between them in empirical interbank networks.

2.1. Nested split graphs

Nested split graphs (NSG) are networks with a simple structure based on the degrees of nodes. To see how nested split graphs can be relevant for interbank networks, assume that banks can be ranked on the basis of their reliability and that every banks takes this ranking into account when choosing counterparties. Each bank initially interacts with the most reliable counterparty; if it needs a second counterparty, it chooses the second most reliable bank, and so on. In this sense the network structure of counterparties exhibits nestedness.[5] Nested split graphs can be defined by the following strong condition (cf. König et al., 2014, p.13):

Condition NSG: For any bank, the set of its counterparties is contained in the set of counterparties of any other bank with at least as many linkages.

Fig. 2, panel (a) gives an example of a nested split graph that is perfect, i.e. it fulfills Condition NSG exactly. The shading of the nodes corresponds with the more preferable counterparties (having a higher degree). Because any set of counterparties should be contained in the set of counterparties of the bank with the largest number of linkages, any (non-empty) NSG contains at least one bank that is connected to all other banks in the system. In this case bank A has 7 links, one to every other bank. There are some banks that require more than one link, for example bank D which has an additional link to B. Condition NSG requires that all banks with more than one link have one counterparty in common: this counterparty is bank B. Two banks have three counterparties, namely C and E, and have the same set of counterparties {A,B,F}, consistent with Condition NSG.

Cohen-Cole et al. (2011) and König et al. (2014) apply nested split graphs to interbank market, and argue that they give a good fit to the datasets they consider. The idea of the nested split graph model is the strict ordering accepted by all banks with the single most preferable counterparty on top.[6] The alternative core periphery network instead suggests that there are some equally important core banks, and that periphery banks select one or more banks from this core as counterparties.

2.2. Core periphery networks

In the literature so far, there is some apparent confusion between nestedness and core periphery structures in interbank networks. We present the *core periphery* (CP) network as an alternative for nested split graphs. The idea is that core banks that intermediate between periphery banks, which in turn do not interact with each other. See Hommes et al. (2013) for a financial market model in which banks can trade indirectly through intermediation and a core periphery network formes endogenously. The formal notion of core periphery networks was introduced in the social sciences by Borgatti and Everett (1999). The concept has recently been applied to the interbank market by Craig and von Peter (2014) and subsequently by others using different datasets.[7]

[4] Fricke and Lux (2012) summarise more findings of the interbank network literature. In reviewing this literature, it is useful to contrast interbank exposures analysed in this paper with overnight interbank transactions. Whereas interbank exposures refer to stock variables on the balance sheet, interbank transactions are flow variables. Our paper is concerned with interbank exposures.

[5] Nested split graphs are related to k-cores: the sets of nodes having at least k connections. Imakubo and Soejima (2010) investigate k-cores in the Japanese interbank market.

[6] Consistent with the prominent place of league tables in trade journals such as *The Banker*.

[7] The ideas of 'money-centre banks' or a two-tiered banking systems was previously described qualitatively (and graphically) by Upper and Worms (2004), Degryse and Nguyen (2007) and Soramäki et al. (2007). The core periphery model captures these ideas quantitatively, and provides a way to test them in the data.

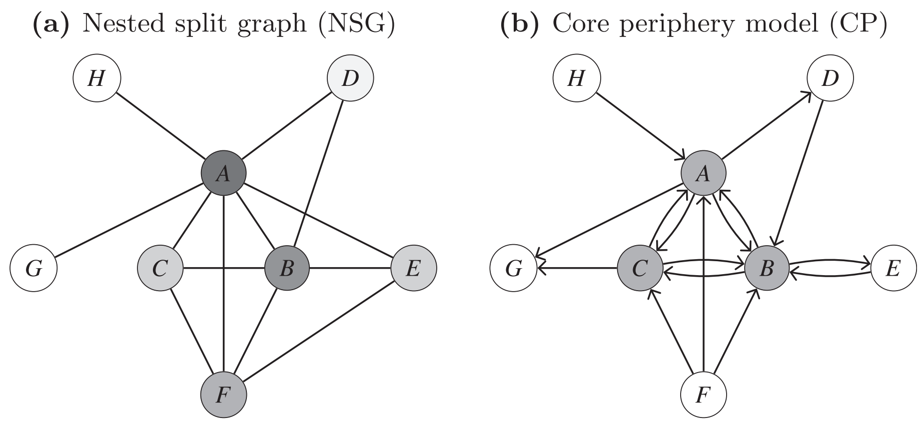

Fig. 2. Examples of perfect models.

To test the concept of an interbank core periphery network in a quantitative way, Craig and von Peter (2014) introduce a formal definition. In particular, the relation between the core and the periphery is more clearly specified than in Borgatti and Everett (1999). In a perfect core periphery network, the following three conditions are satisfied:

Condition CP1: Core banks are all bilaterally linked with each other.
Condition CP2: Periphery banks do not lend to each other.
Condition CP3: Core banks both lend to and borrow from at least one periphery bank.

This definition of a core periphery model imposes a strict distinction between core and periphery banks. In Appendix A we consider and estimate a continuous version of the model following Fricke and Lux (2012), where each bank receives a measure of its 'coreness'.

These requirements are illustrated in Fig. 2, panel (b). Banks A, B and C are core banks and all lend to each other bilaterally. These banks also intermediate between the remaining periphery banks: for example, bank A intermediates between H and D. Note that Condition CP3 is violated if the link from C to G is removed (as C would not lend to the periphery), but not by removing the link from A to G (as A lends to D). Some periphery banks, like bank D and E, may lend and borrow at the same time, as long as they are not connected to other periphery banks.

The two figures taken together show the key differences between both deterministic network models. Most importantly, the core periphery network is defined as a directed graph. The directionality in the core periphery model stems from the fact that Conditions CP1 and CP3 require links in both directions, and follows from the underlying intermediation function of the core to the periphery. This directionality gives the CP model a potential advantage over the nested split graph in explaining the data. We will show that the CP model gives a better fit to the data, suggesting that the direction of links (i.e. borrowing or lending) is indeed important in capturing the global structure of interbank relationships.

Despite the few differences in connections between the two panels in Fig. 2, the models attach a different importance to the most well-connected banks. The nested split graph orders the banks in partitions {A}, {B}, {F}, {C,E}, and so on, based on their degree. In the core periphery model, the core of systemically important banks is formed by {A,B,C} but leaves some freedom for the exact relation between the core and the periphery, provided Condition CP3 is fulfilled. There is a jump in the degree distribution from periphery banks with degree up to 3 and the least connected core bank C with degree 6. A striking difference of the CP model is that bank F is placed in the periphery while it is the third bank in the ranking of the NSG.

In sum, NSG and CP networks rely on different economic concepts (counterparty reliability and intermediation, respectively), and identify different sets of bank as most important in the system. The next section introduces our methodology to determine which of the two gives the best fit to observed networks.

2.3. Fitting deterministic network models

Deterministic network models impose strong restrictions on networks: either a network satisfies the conditions or it does not. Of course, in practice the data will rarely coincide exactly with a nested split graph. Similarly, the core periphery model will generally have both missing links within the core as well as links existing within the periphery. To evaluate the fit of the models, we count the number of *errors*, i.e. the number of links that have to be added or deleted to end in a perfect deterministic model. An alternative measure of fit would be the Pearson correlation between the observed network and a perfect CP network, as discussed by Boyd et al. (2006). We follow previous studies in choosing the number of errors as the fitness measure, sometimes known as the Hamming distance, and motivate it below.

For the core periphery model, minimising the number of errors determines the estimated set of core banks, and thus the fit of the model to network data. For any chosen set of core banks, the errors with respect to the (perfect) CP model can be counted by checking the Conditions CP1, CP2 and CP3. The *optimal core* is defined by the core producing the smallest number of errors, and finding the optimal core is similar to running a regression.[8] The domain of the minimisation problem contains all possible cores, so that both the size and the composition of the set of core banks can be determined optimally from an observed network. For our dataset the found solutions intuitively correspond to core periphery networks, as can easily be checked by the high density of the core.

Fitting a nested split graph can be done in a similar way by counting errors for different orderings of banks and choosing the ordering with the lowest number of errors. However, because nested split graphs are defined in terms of undirected links, the

[8] Following Craig and von Peter (2014), violating Condition CP3 is punished strongly, producing errors for every periphery bank which it could have lent to or borrow from. This way of counting errors is convenient, as it ensures that the optimal core satisfies Condition CP3. Alternative versions of the loss function for the model selection procedure – such as counting all violations as one error – are possible, but will not change the results qualitatively. As the theoretical work on network structures and financial stability further develops, it will be possible and even desirable to adjust the loss function for the relative costs of erroneously choosing one network model over another. We thank an anonymous referee for pointing this out.

directionality in the network data first has to be transformed. This is done by defining an undirected link between a pair of banks if it has a lending relationship in at least one direction. Information contained in the network is thus reduced, but this is necessary for fitting a nested split graph.

Fig. 3 provides an example of a network with 8 banks. As in the interbank exposure data, links are directed. The two bottom plots of Fig. 3 show how this network can lead to the perfect structures as shown in the panels in Fig. 2 by making some small changes in the connections. For the fit of the NSG in the bottom right plot, the directionality of the network is first removed as discussed above. To have a perfect core periphery structure, the missing link between core banks A and B should be added (error 1) and the link between periphery banks E and F should be removed (error 2). To have a perfect nested split graph, bank A with the largest degree should link to all other banks including to E (error 1), while bank G is not allowed to link to C, as C is not the second bank in the ordering (error 2). Fitting the network to other orderings of banks for the NSG model or other sets of core banks for the CP model requires more changes in the structure. So for this example network, the distance to both the closest CP structure and the closest NSG is two errors.

It is easy to see how slight changes in the network in Fig. 3 could lead to a further distance from one deterministic model without changing the fit to the other. For the CP network, for example, additional errors by removing a link between A and C or by adding a reciprocal link from E to F leaves the undirected network unchanged, and therefore the fit of the NSG is unchanged as well. For the NSG, removing the link from A to D or adding a link from C to H further limits the extent of nestedness, but does not conflict with Condition CP3 about the relation between core and periphery.

These modifications to the example network show that having a lower number of errors gives a good indication about which structure gives a better fit to the data.

Nevertheless the number of errors is a raw measure of distance, as for example the maximum number of errors that can occur depends on the number of banks. Moreover, undirected networks generally consist of fewer links than directed networks, driving down the number of errors when fitting undirected networks. A improved measure of distance is therefore the *error score* (Craig and von Peter, 2014):

$$e = \frac{E}{L}. \tag{1}$$

The error score expresses the number of errors E as a proportion of the number of actual links L. Continuing with the example in Fig. 3, we find an error score for the CP model equal to $e^{CP} = \frac{2}{16}$ versus an error score for the NSG model of $e^{NSG} = \frac{2}{13}$, that is, a better fit for the CP model using the error score.

In Section 4, the error score will be used to compare the fit of different network structures to one particular observed network, as well as between the fit to different observed networks over time. Using Monte Carlo simulations, we argue in Section 4.2 that the error score is a satisfactory measure of fit in the statistical sense: when the real network is close to an CP, the error score of fitting an CP will be shown to be much lower than the error score of fitting an NSG, and vice versa.

A convenient property of the error score is that it is bounded by 1. To see that this holds for the CP network and NSG, note that these network models include an empty network without links as a possible perfect structure. The empty network satisfies Condition NSG, as the set of counterparties is empty for every bank; and

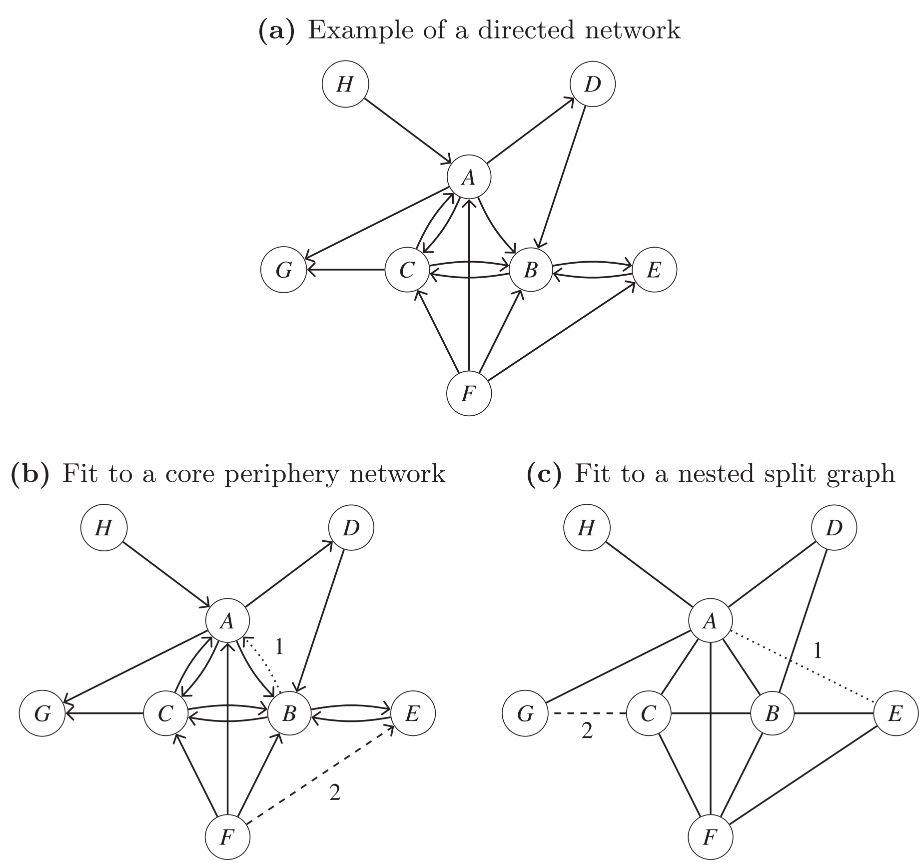

Fig. 3. Example of a directed network with 8 banks (a), its fit to a core periphery network (b) and the fit of the (projected and hence) undirected network to a nested split graph (c). The numbers indicate the two errors in both fits: one dotted link that should be added (1) and one dashed link that should be removed (2) to fulfill the conditions of each model.

it also satisfies Conditions CP1, CP2 and CP3 if the set of core banks is empty. Compared to an empty network model, any observed network has as many errors as links, so $e = \frac{L}{L} = 1$. The fitted CP network and the fitted NSG are the result of optimising over all possible structures and by definition cannot have higher error score than the empty network. Having an error score considerably below 1 is therefore a first requirement in evaluating the fit of a deterministic model.

For small networks, fitting deterministic models can easily be done by an exhaustive search. However, because the size of this domain rises exponentially with the number of banks, there exist 'greedy' algorithms for both the NSG and the CP model with a reduced, linear running time. The algorithm iteratively improves the outcome by shuffling banks that generate most errors. Craig and von Peter (2014) developed such an algorithm for the CP model and show its robustness to the initial partition of core and periphery. For nested split graphs, a similar greedy algorithm is provided by Mannila and Terzi (2007).

3. Data description

Financial institutions interact on many levels. Sometimes these interactions are very short-lived with contractual obligations expiring before the end of the trading day. Other contracts, such as for instance swaps, can be long-running and can last up to 30 years. Our data combines all of these interactions on a quarterly reporting frequency, where we are limiting the sample to exposures up to one year. We use prudential reporting of balance sheet positions of Dutch banks. Each quarter banks have to report the interbank assets and liabilities to the market as a whole. In addition, banks have to report exposures to their largest counter parties. Assuming that the distribution of interbank exposures is equal to the distribution of claims in general, given by the large exposure reporting, we construct a matrix of interbank exposures for the Dutch banking market.

Data are available from 1998Q1 to 2008Q4 with the number of reporting banks varying between 91 and 103. This number includes Dutch banks, foreign subsidiaries, branches of foreign banks and investment firms. To get a feel for the data we show the overall domestic market volume in Fig. 4.[9] In the long run the volume has been growing, dropping markedly in the financial crisis. The two vertical lines indicate (1) the beginning of the crisis in August 2007 once the subprime mortgages made the headlines, and (2) the bankruptcy of Lehman Brothers on 15 September 2008. Computed network measures such as path length tell a similar story: the network became more connected over time with a marked reversal due to the crisis.

In order to apply unweighted or binary network models discussed in Section 2 to our dataset, we reduce our weighted links to binary links. We use the same €1.5 million threshold as Craig and von Peter (2014) to determine the most important links. For every period a sparse network remains of about 8% of the possible number of links. This makes the Dutch interbank network much denser than the German network (where less than 1% of possible links materialise). It seems that the main difference of the German market compared to the Netherlands is the large number of small banks, with relatively less links above the threshold.

Imposing a threshold necessarily involves some arbitrariness. In the present context, a higher threshold will result in fewer qualifying links and consequently a smaller core. However, this dependency does not change the key feature of the interbank market consisting of two groups. A more relevant question is how much information is lost by using the threshold. In Appendix A we estimate a different version of the CP model using the weighted network of exposures and show that the estimation results, at least for the CP model, do not depend on the threshold.

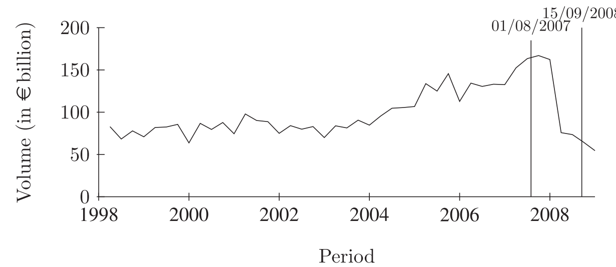

Fig. 4. Domestic market volume over time.

As the first property of our binary network we first check the distribution of the degrees $\{k_i^{out}\}$ and $\{k_i^{in}\}$. The distribution of the number of links per bank is an important feature of different network models and was also analysed by Boss et al. (2004) and later interbank studies. We show selected distributions of in- and out-degree in Fig. 5 (i.e. the first period, the last period and the overall aggregate over all periods). These distributions are plotted as the inverse cumulative distribution (showing the proportion of banks with degree higher than k) on a log–log scale. Scale-free networks have in the limit infinitely many banks displaying a power law in the degree distribution, corresponding to a straight line in such log–log plots.

The plots already give a first feel for the relevance of the different network models. There clearly exist some banks which are highly connected, with the number of counterparties reaching up to more than 90 counterparties. The ER model seems therefore inappropriate as it typically produces a much lower maximum degree. As the log–log plots show no linear relations, we did not try to fit a power law on these degree distributions. The distribution of indegree looks like a concave decreasing function that could be present in both the PA, NSG and CP models. However, for many periods such as the 1998Q1 shown in Fig. 5, there is a jump in the outdegree. This discontinuity in degree is a first indication of a clear distinction between core and periphery. For the last period 2008Q4, there is no jump in the degree distribution.

4. Estimation results

Given the empirical interbank network in the Netherlands, we will evaluate the fit of the CP model in relation to the alternative models. The models are either stochastic (ER, PA) or deterministic (NSG, CP), but all four are in principle suitable candidates for the single-centered, fixed-size Dutch interbank market. As we will see, the core periphery structure is the best way to describe the data. In SubSection 4.1 we estimate the core periphery model, finding the number of estimated core members and the error score (as defined in SubSection 2.3) for every period. We compare the error score of the CP model with the error score of the NSG model, and also relate our results with previous analysis of interbank markets in other countries. In SubSection 4.2, we pay further attention to selecting the CP model using Monte Carlo simulations.

4.1. Estimating the core

To estimate the CP model, we use the sequential optimisation algorithm developed by Craig and von Peter (2014). In our dataset

[9] The dataset has been used before in Liedorp et al. (2010). Unfortunately, our information about relations with foreign counterparties is restricted to the total exposures of each Dutch bank to foreign counterparties. We do not know the identities of the foreign banks nor the exposure to any specific foreign counterparty. This limitation is arguably quite restrictive, at least in terms of volume: the domestic market volume reported in Fig. 4 amounts to around 15% of the total exposures including foreign counterparties.

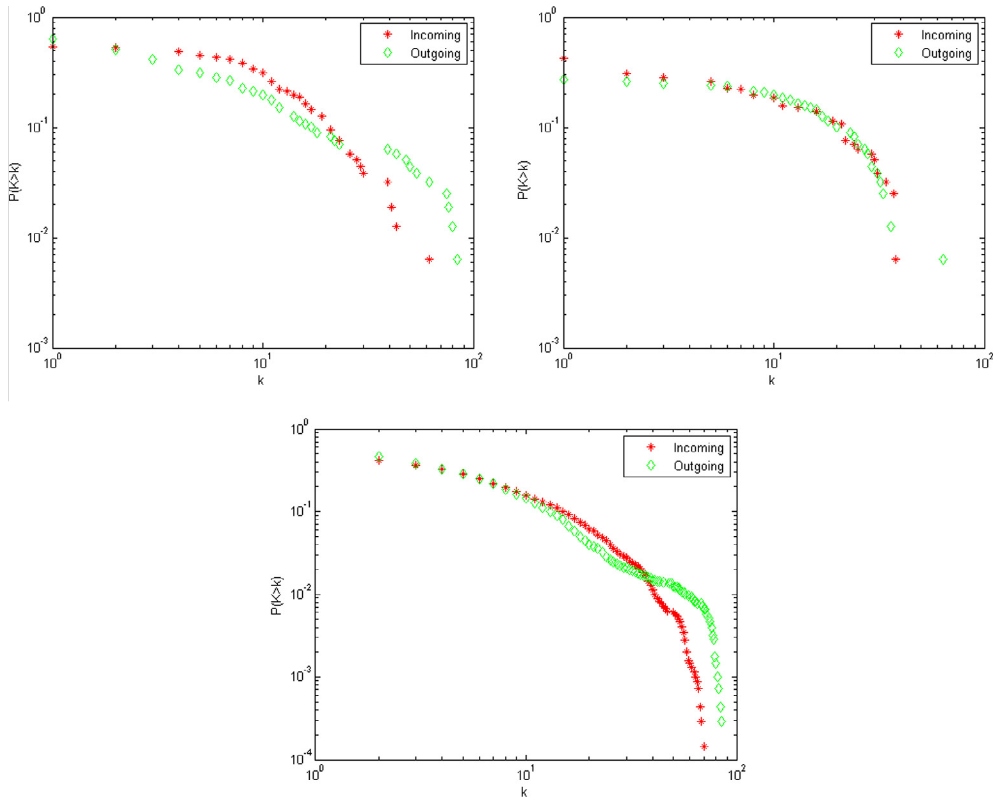

Fig. 5. In- and out-degree distributions for 1998Q1 (left), 2008Q4 (right) and over all periods (down).

with around 100 banks, the optimal core varies between 10 and 19 banks. Fig. 6 plots the core size per period. Although in the first two periods a relatively large core is found of respectively 18 and 19 banks, the core size thereafter stays stable and close to the average of around 13 banks.

There are two important points with respect to the core size. First, the relative core size is closely related to the density of the network. As the density of the network clearly depends heavily on the data construction, one should therefore be careful not to take the core size too literarily. Second, there is no correlation with the total volume in the interbank market: although volumes almost doubled, the higher number of links above the threshold stayed relatively stable which leads to a stable core size.

Fig. 7 shows the error scores of the CP and NSG models. The fit of the CP model is between 0.21 and 0.38, with an average of 0.29. We also calculate the error scores of NSG model using the algorithm of Mannila and Terzi (2007). Because the NSG model is fitted on the undirected version of the empirical network, the number of actual links by which the number of errors is divided is different for both models. The error scores of the NSG model turn out to be much higher for most periods, reaching up to 0.64, more than twice as high as the error score of the CP model for some periods.[10]

Interestingly, the fit of the CP model is clearly worse in the last four quarters, after the subprime crisis had started to spread. In fact, the fit of the NSG model is better in these periods. This suggests that the CP structure, as far as it a good description of the Dutch market, dissolved during the crisis. Further indications for the deterioration of the core periphery structure are the more regular degree distribution in the last period (Fig. 5) and the large number of errors in the core, particularly in period 2008Q4. In Appendix A we estimate a different version of the core periphery model on the weighted network of exposures, and find a similar deteriorated fit in 2008Q1–Q4.

The finding that the core periphery structure dissolved during the financial crisis seems to be confirmed by analysis of other datasets. Using a dataset up to 2010 for the Italian market, Fricke and Lux (2012) established a significant structural break in the crisis, and a deteriorated fit of the model afterwards. Analysing different network metrics (i.e. "motifs"), Squartini et al. (2013) find early signs for a changing network as well.

We also calculate the transition matrix between the states of being in the core and in the periphery, and being inactive in the interbank market ('Exit'). Most importantly, the transition from core to core indicates that on average 83% of the core banks stay in the core the next period. As we found that the number of core banks is quite stable, the flow from and to the core is in absolute terms almost equal. The higher persistence in the periphery merely reflects that it consists of many more banks.

	Core	Periphery	Exit
Core	83%	16%	1%
Periphery	2%	96%	2%
Exit	0%	2%	98%

In Table 1, the empirical results of the CP model for the Netherlands are summarised and compared to Germany, Italy, the UK and Mexico. The fit of 0.29 in the Dutch market is worse than found in the German and Mexican markets (0.12 and 0.25),[11] but better than in the Italian and UK equivalent markets (0.42 and 0.47). Plots of all

[10] When the CP model is estimated on the undirected network, we find even lower error scores. The conclusion that the CP model gives a better fit does therefore not depend on the directionality of the model, but rather on the exact conditions.

[11] The better fit in the German market can partly be explained by the lower density. We tried a higher threshold of € 1 billion, in which case the density of the Dutch interbank market is 0.5%, very close to the density in the German market. The average error score decreases to 0.15.

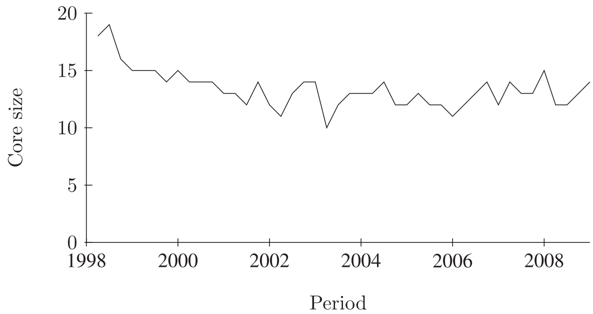

Fig. 6. Number of core banks over time.

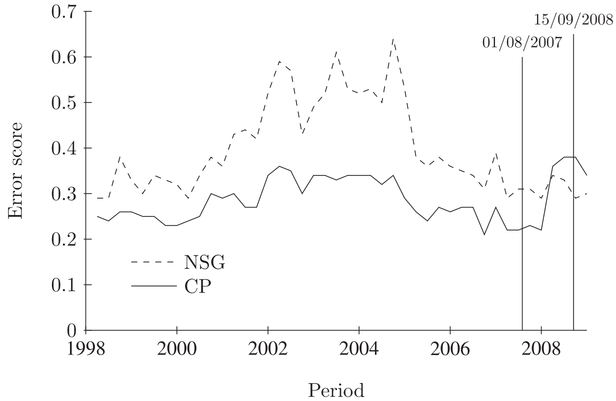

Fig. 7. Error score of the CP and NSG models over time, 1998Q1–2008Q4.

errors in both the core and the periphery for all 44 periods show that most of the errors occur in the periphery, where a perfect CP structure would have no links (i.e. an empty subgraph).

4.2. Monte Carlo simulations

We have shown that the error score of CP model was lower than that of the NSG model in the sample from 1998Q1 until 2008Q1. Now we will assess the fit of the core periphery structure using Monte Carlo simulations. First, we simulate ER and PA networks using a data generating process that takes into account the size and density of the actual network. For each of these simulated networks, we calculate the errors of the core periphery model. In this way we can use the error score as a test statistic to evaluate whether the actual number of errors is likely to be drawn from the simulated distribution functions. Second, we make similar Monte Carlo simulations for data generating processes that are close to the fitted CP and NSG networks. This way, we check whether the error score is a well-behaved measure of fit to discriminate between the two models. Finally, we also investigate the relation between the fit of the CP model and the observed degree distributions $\{k_i^{out}\}$ and $\{k_i^{in}\}$.

Starting with the explanatory power of the CP model over stochastic network models, we plot the distribution function of ER and PA networks for a typical period (1998Q1) in Fig. 8. We generate 1000 replications of ER random graphs and PA networks with 103 banks and 1135 links as in 1998Q1, and fit the CP model on every replication. We find that the CP model is a much better description of the Dutch interbank data than the ER random graphs. If the probability of forming a link is completely random, i.e. not dependent on the number of already existing links, the generated networks have a barely skewed degree distribution. Fitting such random networks to a CP model will show a core that is too small. Note that although it is theoretically possible to end up in a perfect CP network (zero errors) this is extremely unlikely for a finite number of replications.

We reject the hypothesis that the actual network is drawn from the distribution of PA networks in favour of the CP mechanism as the actual error score is much lower than the entire distribution of generated PA networks. Note that the precise distribution of errors as shown in Fig. 8 depends on our choice of the Xie et al. (2008)-procedure which is appropriate given the fixed size of around 100 banks.[12]

Using similar Monte Carlo simulations, we now argue that the error score is a satisfactory measure of fit to discriminate between CP and NSG networks. In particular, we start from perfect CP and NSG structures and simulate randomised networks using a small probability that each link is rewired. To mimic the empirical problem where we do not know a priori which model is better, we fit both the CP and NSG model on every simulated network, irrespective of whether it was generated from a perfect CP or NSG structure. For example, we generate randomised networks from a perfect CP network with a low probability of rewiring, and fit both models. Of course we expect to find much fewer errors from fitting a CP model than from fitting a NSG model, but depending on the relations between the core and periphery and on the random outcomes of rewiring, it is possible that the NSG model gives a better fit.

This Monte Carlo procedure has three steps. First, to have reasonable choices of perfect CP and NSG structures, we start from the fits of these models to the first quarter in our dataset, 1998Q1. The fit of the CP model gives a directed network satisfying Conditions CP1, CP2 and CP3, while the fit of the NSG model is an undirected network satisfying Condition NSG. Second, we randomise these networks using the procedure of Watts and Strogatz (1998). This procedure rewires each link from bank i to bank j with a probability p to a different bank $k \neq i, j$. We generate $R = 1000$ replications of simulated networks per underlying generating process. Third, we estimate both the NSG and the CP model on the simulated network.[13]

Table 2 presents the simulation results. In the first column the true type of simulated network is indicated, with either the fitted CP or the fitted NSG and the rewiring probability p. The top row indicates which network model has been fitted to these simulated networks. The results clearly show that if the underlying network is an NSG the average error score of fitting an NSG is much lower than for fitting an CP, and vice versa a much lower error score is achieved when fitting an CP structure when the actual underlying network is a rewired CP. We conclude that the error score rightfully discriminates between CP and NSG networks and works well as a measure of distance.

Finally, as typical in the complex network literature, we have also tested whether the network outcomes are significant under

[12] Xie et al. (2008) suggest the following procedure to generate a PA network: starting from any (random) initial network, iteratively (1) remove a random link, and (2) add a new link between two banks that are both selected with probabilities $\frac{k_i}{\sum_j k_j}$. As noted by Zhong and Liu (2012), the model implicitly allows self-loops, multiple links and many isolated nodes. We exclude self-loops and multiple links, and let the procedure run until we attain a maximum of 10% isolated links. The resulting degree distribution no longer follows a power law, but does generate a large asymmetry in degrees. Fricke and Lux (2012) follow a different procedure by Goh et al. (2001), but their procedure imposes the power law degree distribution, rather than deriving it from preferential attachment. Finally, Craig and von Peter (2014) use another procedure that does not generate stochastic scale-free networks, so they do not find an approximately normal Monte Carlo distribution.

[13] As discussed in Section 2.3, we have to make sure that the directionality of the network and the model coincide. To fit the CP model on the undirected NSG simulations, we add directionality to undirected links before the rewiring in step two. We choose probabilities to transform an undirected link in either two reciprocal directed links or one directed link that ensure that the number of directed links is close to the observed value of 1135 links in 1998Q1. To fit the NSG model, we remove the directionality of the generated networks after the rewiring.

Table 1
Comparing the CP model for the Dutch interbank market to Germany (Craig and von Peter, 2014), Italy (Fricke and Lux, 2012), the UK (Langfield et al., 2012) and Mexico (Solis-Montes, 2013).

	Netherlands	Germany	Italy	UK	Mexico
Description					
Total number of banks	100	1800	±120	176	46
Network density	8%	0.4%	±15%	3.2%	26%
Average number of core banks	±15	±45	±30	16	±16
Average core size	±15%	±2.5%	±25%	9.1%	±35%
Fit					
Error frequency, as % of links	29%	12%	42%	47%	25%
Transition prob. core → core	83%	94%	83%	NA[a]	94%

[a] Langfield et al. (2012) only have one period and thus cannot compute a transition probability.

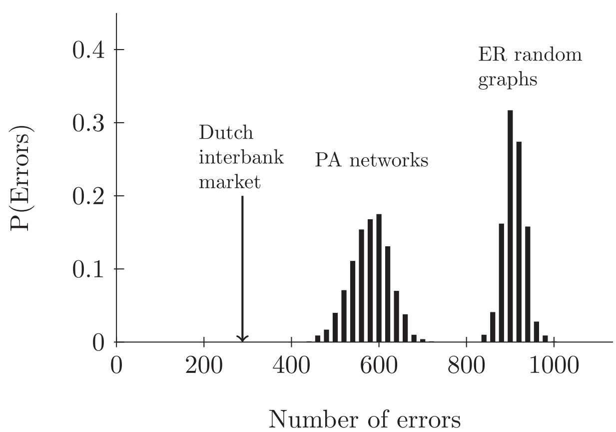

Fig. 8. Simulated distribution function of number of errors for 1998Q1, 1000 PA networks and 1000 ER random graphs.

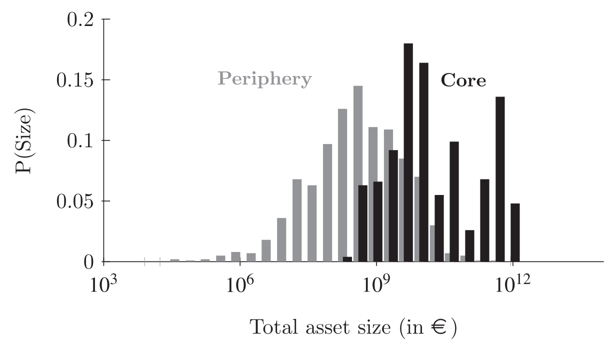

Fig. 9. Distribution of total asset size for core and periphery banks over all periods.

Table 2
Mean of the error score (standard deviation in brackets) from fitting CP and NSG models to $R = 1000$ simulated networks based on fits of the Dutch interbank market in 1998Q1.

Simulated network	Error scores			
	e^{CP}		e^{NSG}	
CP fit, $p = 1\%$	0.5%	(0.4%)	16.2%	(0.5%)
CP fit, $p = 5\%$	2.6%	(0.7%)	20.4%	(1.0%)
NSG fit, $p = 1\%$	12.7%	(0.7%)	1.6%	(0.5%)
NSG fit, $p = 5\%$	14.6%	(0.8%)	7.5%	(1.1%)

the so-called *configuration model*. Such a configuration model uses the full degree distributions and is thus a very different approach from the one presented so far (see Appendix B for further elaboration). This alternative approach shows that, given the degree distribution, the CP model does not add anything. Note that this has no bearing on whether the CP model is plausible in itself.

In summary of the estimation results, we favour the core periphery structure over random graphs (ER), preferential attachment networks (PA), or nested split graphs (NSG), because the error score in the data is much lower for the CP model than for ER, PA or NSG models. The better fit seems to be fully attributed to the degree distribution, with a small subset of banks having many links, removed sharply from the majority of banks having only a few.

5. Network structure in perspective

In this final section, we place our discussion of the interbank network structure in a wider context. We will first describe how core/periphery membership relates to various measures of bank health and business models. This is followed by a discussion of how these results might be relevant for financial stability assessment.

Core banks tend to be the larger banks (cf. Craig and von Peter, 2014) and authorities might have already identified these as systemic. More interestingly, however, some smaller, less obvious banks also enter the core (although we cannot discuss these for confidentiality reasons). Fig. 9 plots the distribution of total asset size (in log scale) for all core and periphery banks, over all periods. While the core has a higher average size than the periphery, we observe that the group of core banks can be divided in the small set of the largest banks, and an additional group of medium-sized banks of a size similar to many periphery banks.

Core and periphery banks might not only differ in size but also on other dimensions. An often used regulatory classification of bank soundness are the CAMELS indicators. These indicators rate a bank on Capital, Asset quality, Management capability, Earnings, Liquidity and Sensitivity to market risks. As discussed in detail in Liedorp et al. (2010), there are various well known proxies and we have investigated them all. For almost all of these variables there does not seem to be a difference between the two groups of banks. In Fig. 10 we show a selection of these variables. The first panel a) plots the average buffer of the core and periphery members over time. The difference is 10 percentage points on average and is statistically significant. Core banks thus tend to have a *lower* buffer, defined as the Tier1 capital over the total assets.[14] Panel b) shows the returns on assets, which are largely similar except for the crisis period where core banks (which we showed to be larger and

[14] Alternative measures condition the buffer on risk sensitivity, e.g. using Basel Risk Weighted Assets (RWA), ratings or CDS spreads. Unfortunately, both the regulatory (RWAs) and the public measures are not available for the vast majority of the periphery banks. However, simple measures, such as the leverage ratio shown, have recently been demonstrated to outperform more complex, risk sensitive measures as predictors of financial distress (cf. Hilscher and Wilson, 2011; Haldane and Madouros, 2012).

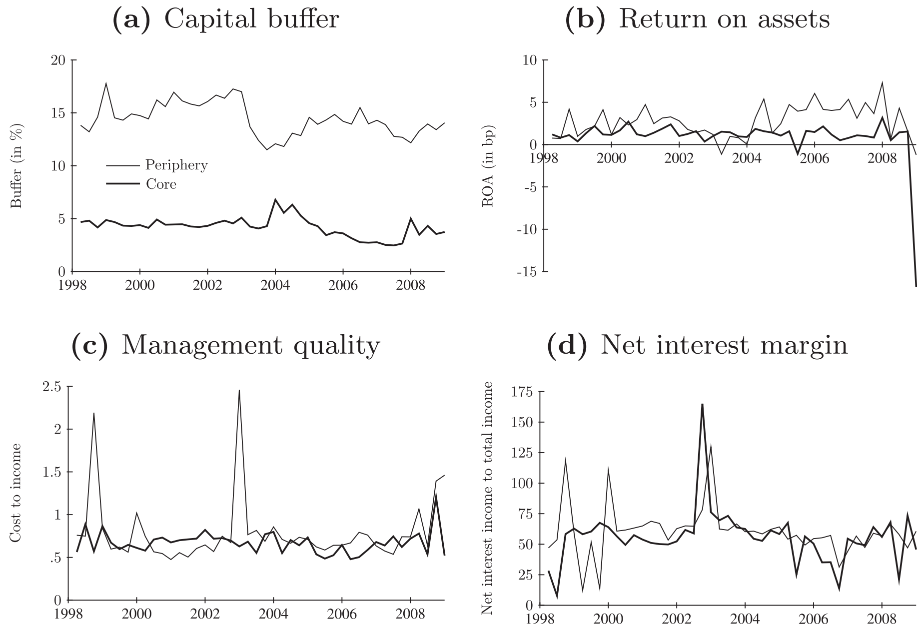

Fig. 10. Selected CAMELS and business model indicators.

more thinly capitalised) faced significant losses. The third panel (c) plots the cost to income ratio, an often used proxy for management quality, which is typical for many of the other indicators: both the core and the periphery banks are largely similar. The final panel (d) shows that the net interest margin, another measure of banks' business models, gives also no difference between the two groups.

We have shown that the core periphery model gives the best fit out of the network models examined, although the structure deteriorated during the financial crisis. In any case, the way interbank linkages are formed is far from random. In all periods, the degree distribution shows that there are certain banks with many more counterparties than expected based on the benchmark ER random graph model. This persistent asymmetry in degrees has not yet been incorporated in the theoretical literature on financial contagion and bankruptcy cascades, which follows Allen and Gale (2000) in assuming homogeneous banks. Freixas et al. (2000) do have – limited – heterogeneity as they allow for one bank to be the clearing bank.

Taking this heterogeneity seriously could start with an analysis as in Nier et al. (2007) who investigate how connectivity affects the diffusion of shocks within an ER random graph. If such simulation studies would be performed in a core periphery network, the failure of a core bank would lead to much more severe contagion throughout the system, while a shock to a periphery bank can more easily be absorbed by its counterparties in the core. The fact that these core banks have lower buffers would exacerbate this. Georg (2013) takes the results of Nier et al. (2007) further by comparing different interbank network structures and shows that money-centre networks are more stable than random networks.[15]

6. Conclusions

The interbank market plays an important role for Dutch banks in the management of their liquidity positions. While foreign counterparties are easily found in the small and open banking environment of the Netherlands, there exists a core of domestic banks that are connected to many periphery banks. Following studies in other countries, we described the network that arises in the interbank market. We identify the most important banks based on the network structure. We also made efforts to visualise the network, which aims at helping authorities to supervise complex market structures.

Our main methodological contribution lies in combining new methods for comparing different network models. Our comparison shows that the core periphery model fits better empirically than random graphs, preferential attachment networks, and nested split graphs. This implies that the simple and economically intuitive division between two tiers of banks is justified. In our dataset of Dutch interbank connections from 1998Q1 until 2008Q4, we find a core of roughly 15 out of the 100 active banks. The distinction between core and periphery banks is found to be stable over the entire sample. Thus the hierarchical structure seems to be a 'new stylised fact' for interbank markets (cf. Fricke and Lux, 2012).

Our findings raise the question why individual banks' decisions on loan contracts align with a structure of strongly connected core banks and dependent periphery banks. In line with Craig and von Peter (2014) and Hommes et al. (2013), we have discussed the core periphery model in terms of loan intermediation. This seems like a fruitful idea in developing an improved understanding of the mechanisms generating interbank networks. Fricke and Lux (2012) discuss different explanations of a core periphery structure and emphasise the comparative advantage of core banks in gathering information (e.g. about counterparty risk), an advantage mostly driven by bank size. We have shown that although core membership is indeed correlated with total asset size, there are several medium-sized and unexpected banks in the core of the Dutch interbank market.

Irrespective of what determines the structure, we have shown that there exists a core of systemically important banks. Moreover, core banks' difficulties will arguably have larger effects to the system as a whole than the failure of periphery banks. Authorities might therefore consider to require the core members to hold higher buffers. In contrast, however, the core banks in our sample have lower buffers.

The core periphery structure deteriorated considerably during the financial crisis, as was also found by Fricke and Lux (2012)

[15] Georg (2013) also provides evidence that the central bank only stabilises interbank markets in the short run. See van Lelyveld et al. (2011) for an analysis of how market structure affects stability in reinsurance markets.

for the interbank market of Italy. More research is required to explain the deteriorated fit by individual banks' behaviour and to provide further guidelines for supervisory authorities during financial distress. The present paper opens up new opportunities for systemic risk assessments of the interbank market, especially as more granular data is becoming available.

Appendix A. Robustness analysis using the weighted network

In this appendix we make use of the size of the exposure data and check the robustness of the results on the CP model. We denote by $\mathbf{D}$ the exposure matrix at some particular point in time, where element $\mathbf{D}_{i,j} > 0$ represents the size of an exposure of bank i on bank j. First of all, we consider the loan size distribution over all periods. Fig. A.11 plots this inverse cumulative distribution (of loans in €) in log–log scale. The plot shows that a quite high number of the links (around 30%) is estimated below € 1000. As these small loans are due to the data construction (see below) rather than actual banking agreements, it is important to use some sort of threshold value in our analysis of unweighted links.

Fig. A.11 shows a wide dispersion of loans within the sample. We could estimate a power law distribution on the right tail to see whether loan sizes in the Netherlands display scale free behaviour. In an early analysis of interbank network structure, Boss et al. (2004) fit power laws to the Austrian loans.

In the analysis of Section 4 we have used a threshold of € 1.5 million to construct the unweighted network $\mathbf{A}$ of binary links (right of the dotted line in Fig. A.11):

$$\mathbf{A}_{i,j} = \begin{cases} 1 & \text{if } \mathbf{D}_{i,j} \geqslant \text{€ 1.5 million} \\ 0 & \text{if } \mathbf{D}_{i,j} < \text{€ 1.5 million.} \end{cases}$$

In Fig. A.12, both the total number of links and the 'significant' links above the threshold are presented over time. Interestingly, the total number of links shows a sharp drop in 2008Q1 when almost half of the links disappear permanently. This might be driven by the way the data is constructed. For the smaller loans, it is assumed that the totals of the interbank assets and liabilities for a given bank (observable from the balance sheet) follow the same distribution over other banks as the payment system. After the beginning of the financial crisis in August 2007, the payment system between banks was already under stress, and this might have caused a direct effect on the estimation of the small loans. See Liedorp et al. (2010) for a full description of the construction of the dataset.

While the small loans seem to drop in number for reasons unrelated to actual balance sheet management of the banks, the number of links above the threshold shows a much milder decrease during of the crisis. The steady number of 'significant' links is therefore a more reliable source of information than considering all links with nonzero weight. Also note that while only around 1000 links remain, the remaining links cover practically all market volume (more than 99.5%). This observation is in accordance with the wide dispersion of link size as noted above. Given that a threshold is necessary for the (discrete) network models in Section 2, the choice of € 1.5 million seems not too restrictive.

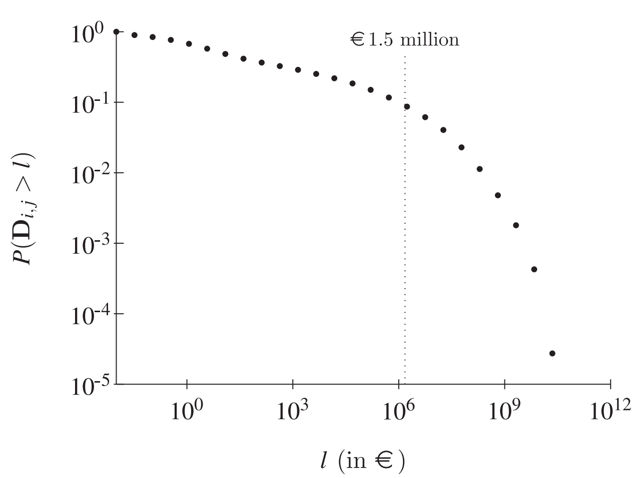

Fig. A.11. Loan size distribution over all periods.

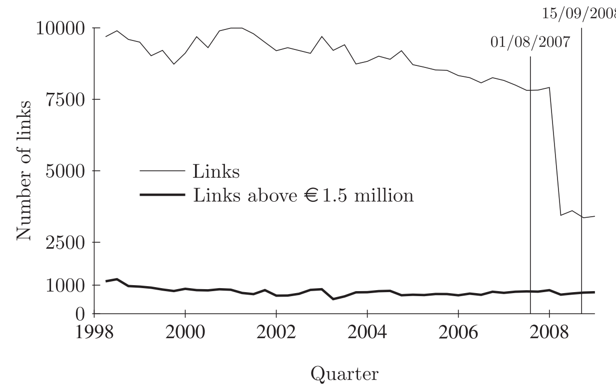

Fig. A.12. Number of links over time.

To investigate whether the choice of the threshold has an effect of our results for the core periphery structure, we estimate the model using the different methodology of Fricke and Lux (2012). They consider a continuous version of a core periphery model where is each bank is assigned a level of 'coreness'. The optimisation procedure chooses the 'coreness' of each bank such that the implied 'pattern matrix' $\mathbf{P}$ is closest to the observed matrix of interbank exposures $\mathbf{D}$, as will be specified below. Apart from the advantage that no threshold has to be imposed on the exposures $\mathbf{D}_{i,j}$, this method can be used to detect differences between bank in terms of their coreness, both within the core and between core and periphery.

Fricke and Lux (2012) distinguish between two versions of the continuous model depending on the specification of the pattern matrix $\mathbf{P}$. In the symmetric version, there is one vector c where $0 \leqslant c_i \leqslant 1$ is a measure of coreness for bank i and $\mathbf{P} = cc'$. The discrete model can now be seen as a special case with $c_i \in \{0, 1\}$, which leads to Conditions CP1 and CP2.[16] In the asymmetric version, there are two vectors u and v representing the 'out-coreness' and 'in-coreness' of each bank, and $\mathbf{P} = uv'$. The asymmetric version can be useful if there is lot of difference between lending and borrowing behaviour of core banks.

For the continuous models we use the *proportional reduction of error* (*PRE*) as the measure of fit, defined as:

$$PRE = 1 - \frac{SS(\mathbf{D} - \mathbf{P})}{SS(\mathbf{D} - \langle D \rangle)},$$

with $\langle D \rangle$ the global average of $\mathbf{D}$ (across all elements, excluding the diagonal), and $SS(.)$ the sum of squared deviations of the off-diagonal elements of the input matrix. Thus, maximising the *PRE* is equivalent to minimising the distance between $\mathbf{P}$ and $\mathbf{D}$. Note that the *PRE* is similar in spirit as the error score for the discrete model. Both measures scale the amount of errors of the optimal structure to that

[16] Condition CP3 would not follow from this example, because $c_i c_j = 0$ if i is in the core and j in the periphery. Instead, Condition CP3 leaves the structure of links between the core and periphery for the most part unspecified, indicating just a minimum of one link to and one from the periphery. Fricke and Lux (2012) estimate the discrete model with and without Condition CP3, and conclude that the differences are negligible in their dataset.

of a trivial structure of just a periphery, constructed by only using the information about the number of banks and the number of links.

Figs. A.13 and A.14 presents the main results of the estimation of the continuous core periphery models. Following Fricke and Lux (2012), we have log-transformed the data in the form $\widetilde{\mathbf{D}} = \log(1+\mathbf{D})$ in order to adjust for the skewness of the data. As also found by these authors, fitting the continuous models on the raw network matrix $\mathbf{D}$ leads to a very poor fit that is not comparable to the fit of the discrete model. The fit in terms of the *PRE* (Fig. A.13) shows that the symmetric continuous model has a deteriorated fit during the four quarters of 2008, similar to the discrete model.

Surprisingly, the asymmetric version of the model performs better during the financial crisis than before 2008. The fit of the asymmetric version should always be better than the fit of the symmetric model, as it has twice as many parameters. In the period 1999Q1–2007Q4 the ratio of the *PRE*s is around 1.6, but in 2008Q1–Q4 it almost doubles to around 2.6. As it turns out, the reduced fit of a model with one set of systemically important banks for the Dutch interbank market during the crisis can be improved by making the distinction between important lending banks and important borrowing banks. In contrast, Fricke and Lux (2012) found that for their data both model versions gave a worse fit during the crisis.

Fig. A.14 shows how the fit is achieved for the asymmetric model and relates the results back to those of the discrete model. It plots the level of in-coreness v_i and out-coreness u_i for all banks i and over all periods. The colour of the dots indicates whether the bank was assigned to the core or to the periphery in the discrete model. The asymmetric continuous model confirms that these binary assignments to core or periphery are sensible: core banks have higher in-coreness and out-coreness. While there is not a very complete separation between the two groups, many core banks have distinctly higher coreness measures. We find that core banks have relatively balanced positions (in the sense of similar in- and out-coreness) and are thus important for both lending and borrowing in the interbank market.

In general, we find that different models of the core periphery structures give comparable results, confirming the results of the discrete model presented in Section 4. To make the relation between the different model versions more explicit, we follow Fricke and Lux (2012) in calculating the correlations between the (stacked) coreness vectors over all periods, i.e. the binary core vectors for the discrete model, and the vectors c, u and v for the continuous models. The lowest correlation found is 30% between in-coreness and out-coreness, indicating that there is some heterogeneity within the core of the interbank market. The overall core of the discrete model is somewhat higher correlated with out-coreness than with in-coreness.[17] Nevertheless, we can safely say that the discrete approach gives satisfactory results for our dataset.

$$\left(\begin{array}{lccc} & \text{Symmetric} & \text{Out-coreness} & \text{In-coreness} \\ \text{Discrete} & 74\% & 60\% & 53\% \\ \text{Symmetric} & & 81\% & 77\% \\ \text{Out-coreness} & & & 30\% \end{array}\right)$$

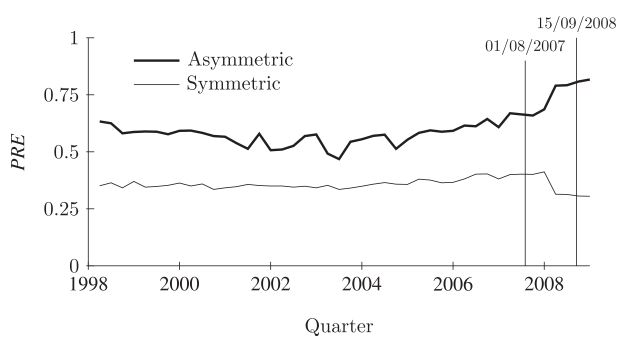

Fig. A.13. Fit of the continuous models in *PRE*.

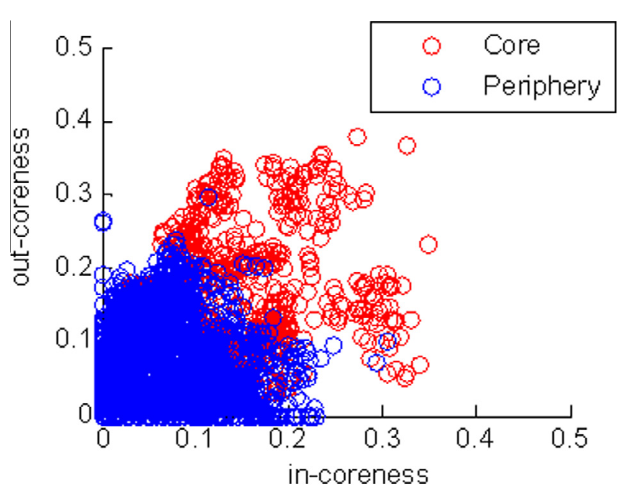

Fig. A.14. In-coreness vs. out-coreness over all periods, labeled by the discrete model.

Appendix B. Testing the CP against the configuration model

As typical in the complex network literature, we also test whether the network outcomes are significant given the same degree distribution, under a so-called *configuration model*. Such a model is a very different from the models discussed so far, as it makes use of the full degree distributions $\{k_i^{out}\}$ and $\{k_i^{in}\}$ instead of just the density of the network. A useful way to think of this model is by considering a rewiring algorithm, where two distinct links in the actual network are broken and the nodes are reconnected reversely.[18] The distribution of the error score under the configuration model indicates whether or not, given the degree distribution, there is additional evidence for the CP structure in the 'ordering' of the links.

Instead of a rewiring algorithm, we use the analytical maximum likelihood method of Squartini and Garlaschelli (2011) which is computationally much less demanding, so that we can test the configuration model in all periods. As a first easy exercise, we use the method on the random graph model, so that only the density is included in the information set. The best estimator for the probability of a link p is then simply equal to the density of the network. For the configuration model, the linking probabilities p_{ij} are such that the degrees match the observations in expectation:

$$k_i^{out} = \sum_{j \neq i} p_{ij}$$

$$k_i^{in} = \sum_{j \neq i} p_{ji}$$

Squartini and Garlaschelli (2011) show that the linking probabilities should be of the form:

[17] Interestingly, Fricke and Lux (2012) found that overall coreness is almost completely driven by the outgoing links of core banks, as shown by correlations of 73% between discrete and out-coreness, and of 26% between discrete and in-coreness. In their dataset the asymmetric version of the continuous CP model leads to a considerably improved fit.

[18] This procedure is sometimes called "Maslov–Sneppen" rewiring (cf. Squartini and Garlaschelli, 2011).

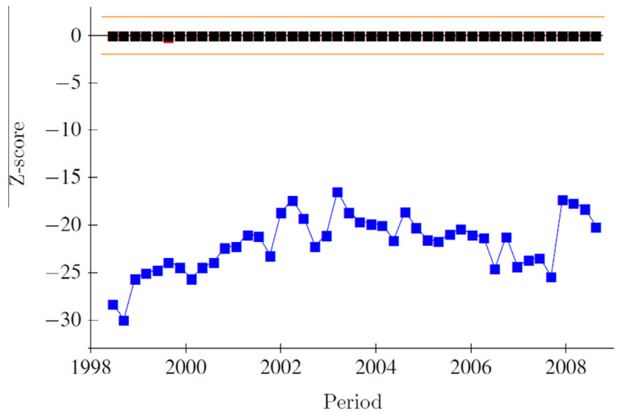

Fig. B.15. *z*-Scores for the number of errors under the ER model (blue, down) and the configuration model (black, up). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$$p_{ij} = \frac{x_i y_j}{1 + x_i y_j}.$$

The analytical method amounts to solving the equation for the parameters $\{x_i\}$ and $\{y_i\}$.

Given the maximum likelihood estimations on the probability of linkages between banks, the *z*-score is defined as the observed number of errors expressed in standard deviations from the expected value under the null model. Fig. B.15 shows the *z*-score over all periods for both the random graph and the configuration models. As the *z*-scores of the random graphs are very negative and large, and those of the configuration model indistinguishable from 0, the observed CP structure can be completely explained by the in- and out-degree sequences.

The *z*-scores very close to 0 might seem conspicuous, but they do have a meaning. Note that the errors in the CP model equal the number of existing links in the periphery plus the number of missing links in the core up to a constant (assuming Condition CP3 holds). The observed error score can in expectation be reproduced exactly by the degree distributions $\{k_i^{out}\}$ and $\{k_i^{in}\}$ and the core membership for all banks *i*. The variance of this number is small yet non-zero, as the number of links within the core or the periphery can change according to the probabilities p_{ij}. This results in a 0 *z*-score.

From this test we can conclude that there is no additional evidence for the CP structure apart from the degree distribution, nothing more and nothing less. It is important to note that in a perfect CP structure, there are no rewirings possible that would generate errors from this network. The simulated distribution function would therefore consist of a single point, and *z*-scores would be undefined. Testing the CP structure against the configuration model can therefore not give any indication of whether the CP structure is plausible in itself.

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