Quantitative Finance

ISSN: 1469-7688 (Print) 1469-7696 (Online) Journal homepage: https://www.tandfonline.com/loi/rquf20

Network topology of the interbank market

Michael Boss , Helmut Elsinger , Martin Summer & Stefan Thurner

Quantitative Finance Volume 4 (December 2004) 677–684 **Research Paper**
Taylor & Francis Ltd tandf.co.uk

Network topology of the interbank market

Michael Boss[1], Helmut Elsinger[2], Martin Summer[1] and Stefan Thurner[3,4]

[1] Oesterreichische Nationalbank, Otto-Wagner-Platz 3, A-1011 Wien, Austria

[2] Department of Finance, Universität Wien, Brünner Strasse 71, A-1210 Wien, Austria

[3] Complex Systems Research Group, HNO, Medical University Vienna, Währinger Gürtel 18–20, A-1090 Wien, Austria

E-mail: thurner@univie.ac.at

Received 19 February 2004, in final form 6 August 2004
Published 18 April 2005
Online at www.tandf.co.uk/journals/titles/14697688.asp DOI:10.1080/14697680400020325

Abstract
We provide an empirical analysis of the network structure of the Austrian interbank market based on Austrian Central Bank (OeNB) data. The interbank market is interpreted as a network where banks are nodes and the claims and liabilities between banks define the links. This allows us to apply methods from general network theory. We find that the degree distributions of the interbank network follow power laws. Given this result we discuss how the network structure affects the stability of the banking system with respect to the elimination of a node in the network, i.e. the default of a single bank. Further, the interbank liability network shows a community structure that exactly mirrors the regional and sectoral organization of the current Austrian banking system. The banking network has the typical structural features found in numerous other complex real-world networks: a low clustering coefficient and a short average path length. These empirical findings are in marked contrast to the network structures that have been assumed thus far in the theoretical economic and econo-physics literature.

1. Introduction

The problem of *systemic risk*—the large-scale breakdown of financial intermediation—has been a key concern for institutions in charge of safeguarding financial stability, mainly central banks and regulators. Systemic risk is an important issue in the banking system because banks are usually linked by a complex network of mutual credit relations originating from their activities in the interbank market. Through this network of interbank liabilities which connects individual institutions the failure of one bank might directly cause the failure of another bank in a *domino* effect. From an abstract viewpoint the system of mutual credit relations among financial institutions can be viewed as a network, where banks are the nodes and their interbank relations form financial links. An important problem is to understand how the structure of this interbank network affects the financial stability properties of the banking system as a whole. This paper takes a first step in this direction by uncovering the *empirical structure* of an interbank network using a data set provided by the Austrian Central Bank.

[4] Author to whom any correspondence should be addressed.

In our analysis we can draw on insights and concepts from ongoing and very active interdisciplinary research on networks. The physics community, in particular, has greatly contributed to the empirical analysis and to a functional understanding of the structure of complex real-world networks in general; see [1, 2] for an overview. Perhaps one of the most important contributions to recent network theory is an interpretation of network parameters with respect to the stability, robustness, and efficiency of the underlying system; see e.g. [3, 4]. Clearly, these insights are relevant for the issues of financial stability and the network structure of the mutual credit relations in the interbank market.

Our main finding is that the network structure of the interbank market is scale free, i.e. it shows power laws in the degree distributions. This means that there are very few banks with many interbank linkages, whereas there are many with only a few links. This feature of networks has repeatedly been related to the stability of networks with respect to the *random breakdown* of nodes, and, at the same time, to the risk of the *specific* removal of hubs, i.e. the very few well connected nodes in the network. In the present context, this means that—given the actually observed structure of interbank claims and liabilities—the banking system is relatively robust with respect to domino effects caused by the random breakdown of single institutes. However, the existence of power laws in the system implies the existence of hubs, the specific removal of which can have a dramatic impact on the stability of the system, and could ultimately lead to the collapse of the entire financial system. We also describe other specific features of the network such as low clustering and short average distances between institutions, which confirm the general structural characteristics of the interbank network that we find in the data. Finally, another message of this work is that it provides a direct insight into the structure of a real interbank network and its contract size distributions, which could be helpful for imposing restrictions on the large classes of potential networks for future modeling of interbank relations.

While our paper is the first to provide an empirical analysis of the structural features of a real-world interbank network using concepts from modern network theory, the topic of systemic risk and domino effects has been studied previously by various authors. In the economic literature the notion of systemic risk has been applied more broadly to refer to a fairly large variety of phenomena. It is used to describe various types of crises ranging from payment systems, bank runs, and spillover effects between financial markets, to a very broadly understood notion of financially driven macroeconomic crises; see [5–7] for a more detailed overview. In the context of banking, the problem is often considered that, due to the tight inter-dependencies between the different banks in a banking system, problems can suddenly occur at many banks simultaneously and might disrupt financial intermediation on a large scale. Clearly, from a network perspective this domino effect aspect of insolvency is the part of the systemic risk literature our paper relates to.

One might classify the contributions of the present literature into two broad categories: theoretical network studies, on the one hand, and empirical papers, mostly combined with simulations, on the other. From a theoretical point of view, the economics literature on contagion [8–10] suggests various network topologies that might be interesting to look at. Allen and Gale [8] suggest the study of a fully connected graph of mutual liabilities. The properties of a banking system with this structure are then compared with the properties of systems with non-fully connected networks, where an explicit example is given for a situation where a fully connected network is more conducive for financial stability than an incomplete one. Freixas *et al* [9] contrast a circular graph with a fully connected graph, and Thurner *et al* [10] studies a much richer set of different network structures which are used in an agent-based model where banks minimize individual risk. Iori *et al* [11] study systemic risk in a simulation model without studying the impact of the network structure explicitly. There is also a large body of economics literature dealing with networks more generally. This mainly game theoretic literature is, however, mostly unconcerned with financial networks and financial stability in particular. A fairly comprehensive overview of this literature is given by Jackson [12].

An important theoretical paper from the operations research literature is that of Eisenberg and Noe [13], where the authors study a centralized static clearing mechanism for a financial system with exogenous income positions and a given structure of bilateral nominal liabilities. They provide a deterministic method to make the payment promises and the exogenous income positions consistent by an insolvency procedure which rations creditors in the system, if necessary. However, to use this analysis in the context of risk assessment, some parameters of the model have to be stochastic such as the exogenous income positions. One can think of them as random variables on a space of risk factor changes. Such an extension is suggested by Elsinger *et al* [14, 15]. Here the authors take data from the Austrian interbank market to quantitatively describe the network of bilateral liabilities and study how shocks to the exogenous income positions are propagated throughout the system. For each realization of the exogenous income position the model of Eisenberg and Noe [13] pins down a unique clearing vector of payments between banks. From this information, one can deduce which banks are insolvent. The distribution of insolvency cases can then be simulated and default probabilities for individual institutions can be deduced. Moreover, one can determine which insolvency cases occur indirectly due to contagion. The impact of network structure on contagion flow through the system by simulation has been analysed by Boss *et al* [16].

Some of the empirical, simulation-based literature on financial networks has taken the approach of measuring a liability network and studying the propagation of certain shocks through the network. The papers have mainly worked with data sets from the payment system [17, 18] or with interbank data [19–22]. All of these studies investigate contagious defaults that result from the hypothetical failure of a single institution. Elsinger *et al* [14, 15] take these studies one step further by combining the analysis of interbank connections with a simultaneous study of the banking system's overall risk exposure. Instead of performing a banking risk analysis on *ad hoc* single institution failure scenarios, they study realistic risk scenarios for the banking system which are simulated using standard risk management techniques.

Our approach provides a complementary point of view in relation to this literature. The analysis of network structure as suggested in this paper is able to explain some of the results derived in some of the simulation studies cited above. For instance, the authors of [14, 15, 19–22] unanimously find that contagion of insolvency occurs relatively rarely under realistic risk scenarios. The power laws found in the network structure of the Austrian interbank market and its consequences provide a natural explanation for these results.

The paper is organized as follows. Section 2 contains our conceptual description of the interbank data with tools from network theory. Section 3 contains our empirical results. In section 4 we discuss our findings and explore their economic and potential regulatory consequences.

2. The banking network

The interbank network is characterized by the liability (or exposure) matrix L. The entries L_{ij} are the liabilities bank i has towards bank j. We use the convention of writing liabilities in the rows of L. If the matrix is read column-wise (transposed matrix L^{T}) we see the claims or interbank assets that the banks hold with each other. Note that L is a square matrix, but not necessarily symmetric. The diagonal of L is zero, i.e. no bank self-interaction exists. In the following we are looking for the bilateral liability matrix L of all (about $N = 900$) Austrian banks, the Central Bank (OeNB) and an aggregated foreign banking sector. Our data consist of ten L matrices, each representing liabilities for quarterly single month periods between 2000 and 2003. To obtain the Austrian interbank network from Central Bank data we draw upon two major sources: we exploit structural features of the Austrian bank balance sheet data base (MAUS) and the major loan register (GKE) in combination with an estimation technique.

The Austrian banking system has a sectoral organization for historical reasons. Banks belong to one of seven sectors: savings banks (S), Raiffeisen (agricultural) banks (R), Volksbanken (VB), joint stock banks (JS), state mortgage banks (SM), housing construction savings and loan associations (HCL), and special purpose banks (SP). Banks have to break down their balance sheet reports on claims and liabilities with other banks according to the different banking sectors, the Central Bank and foreign banks. This practice of reporting on balance interbank positions breaks the liability matrix L down into blocks of sub-matrices for the individual sectors. The savings banks and the Volksbanken sector are organized in a two-tier structure with a sectoral head institution. The Raiffeisen sector is organized by a three-tier structure, with a head institution for every federal state of Austria. The federal state head institutions have a central institution, Raiffeisenzentralbank (RZB), which is at the top of the Raiffeisen structure. Banks with a head institution have to disclose their positions with the head institution, which gives additional information on L. Since many banks in the system hold interbank liabilities only with their head institutions, one can pin down many entries in the L matrix exactly. This information is then combined with the data from the major loans register of OeNB. This register contains all interbank loans above a threshold of 360 000 Euro. This information provides us with a set of constraints (inequalities) and zero restrictions for individual entries L_{ij}. Up to this point, one can obtain about 90% of the L matrix entries exactly.

For the rest, we employ an estimation routine based on local entropy maximization [23, 24], which has already been used to reconstruct unknown bilateral interbank exposures from aggregate information [22, 25]. The procedure finds a matrix that fulfills all the known constraints and treats all other parts (unknown entries in L) as contributing equally to the known row and column sums. These sums are known since the total claims b_i to other banks have to be reported to the Central Bank. The estimation problem can be set up as follows. Assume we have a total of K constraints. The column and row constraints take the form

$$\sum_{j=1}^{N} L_{ij} = b_i^{\mathrm{r}} \quad \forall i \text{ and } \sum_{i=1}^{N} L_{ij} = b_j^{\mathrm{c}} \quad \forall j, \tag{1}$$

with r denoting *row* and c denoting *column*. Constraints imposed by knowledge about particular entries in L_{ij} are given by

$$b^{\mathrm{l}} \leq L_{ij} \leq b^{\mathrm{u}} \quad \text{for some } i, j. \tag{2}$$

The aim is to find the matrix L (among all the matrices fulfilling the constraints) that has the least discrepancy with some *a priori* matrix U with respect to the (generalized) cross entropy measure

$$\mathcal{C}(L, U) = \sum_{i=1}^{N} \sum_{j=1}^{N} L_{ij} \ln\left(\frac{L_{ij}}{U_{ij}}\right), \tag{3}$$

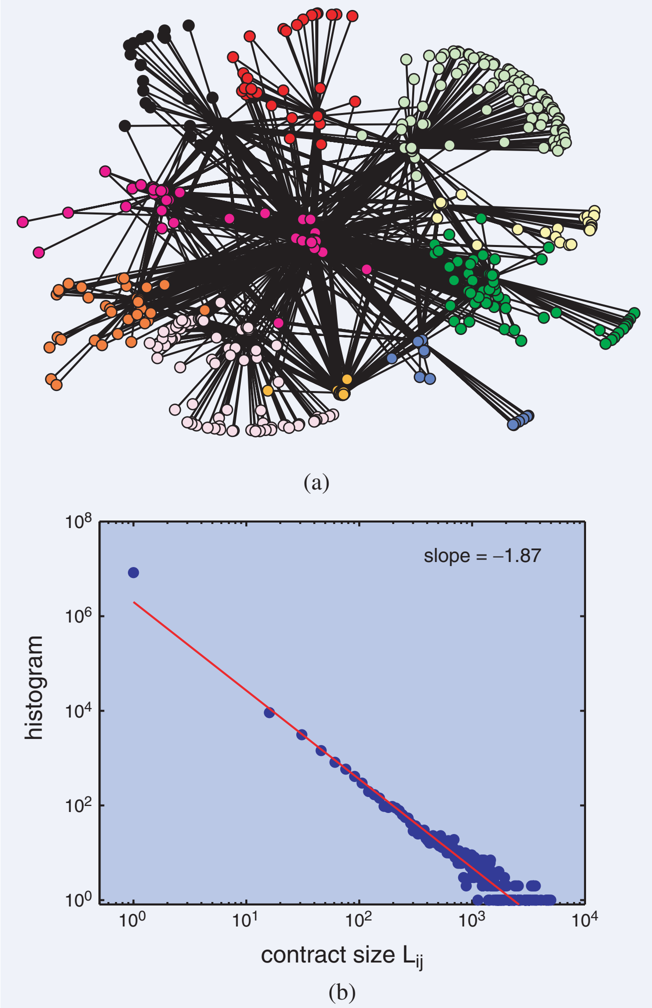

Figure 1. The banking network of Austria (*a*). Clusters are grouped (coloured) according to regional and sectorial organization. R sector with its federal state sub-structure: RB yellow, RSt orange, RK light orange, RV gray, RT dark green, RN black, RO light green, RS light yellow. VB sector: dark gray. S sector: orange-brown. Other: pink. Data from the September 2002 L matrix, which is representative for all the other matrices. (*b*) Contract size distribution within this network (histogram of all entries in L), which follows a power law with exponent -1.87. Data aggregated from all 10 matrices.

where U is the matrix which contains all known exact liability entries. For those entries (bank pairs) ij where we have no knowledge from Central Bank data, we set $U_{ij} = 1$. We use the convention that $L_{ij} = 0$ whenever $U_{ij} = 0$ and define $0 \ln(0/0)$ to be 0. This is a standard convex optimization problem, and the necessary optimality conditions can be solved efficiently by the algorithm described in [23, 24]. As a result we obtain a rather precise (see below) picture of the interbank relations at a particular point in time. Given L we plot the distribution (pdf) of its entries in figure 1(*b*). The distribution of liabilities follows a power law for more than three decades with an exponent of -1.87, which is within the range well known from wealth- or firm-size distributions [26, 27]. The liability matrix is, of course, not the only way to define the relationship between financial intermediaries. For a more general view of the subject, see [28, 29] and references therein or [30] for a multi-tier approach.

To extract the network topology from our present data, there are three possible approaches to describe the structure as a graph. The first approach is to look at the liability matrix as a *directed graph*. The vertices are all Austrian banks. The Central Bank OeNB, and the aggregate foreign banking sector are represented by a single vertex each. The set of all initial (starting) vertices is the set of banks with liabilities in the interbank market; the set of end vertices is the set of all banks that are claimants in the interbank market. Therefore, each bank that has liabilities with some other bank in the network is considered an initial vertex in the directed liability graph. Each bank for which this liability constitutes a claim, i.e. the bank acting as a counterparty, is considered an end vertex in the directed liability graph. We call this representation the *liability adjacency matrix* and denote it by A^{l} (superior l indicating liability). $A^{\mathrm{l}}_{ij} = 1$ whenever a connection starts from row node i and leads to column node j, and $A^{\mathrm{l}}_{ij} = 0$ otherwise. If we take the transpose of A^{l} we get the interbank asset matrix $A^{\mathrm{a}} = (A^{\mathrm{l}})^{\mathrm{T}}$. A second way to look at the graph is to ignore directions and regard any two banks as connected if they have either a liability or a claim against each other. This representation results in an undirected graph whose corresponding adjacency matrix $A_{ij} = 1$ whenever we observe an interbank liability or claim. Our third graph representation is to define an undirected but weighted adjacency matrix $A^{\mathrm{w}}_{ij} = L_{ij} + L_{ji}$, which measures the gross interbank interaction, i.e. the total volume of liabilities and assets for each node. Which representation to use depends on the questions addressed to the network. For statistical descriptions of the network structure, matrices A, A^{a}, and A^{l} will be sufficient, and to reconstruct the community structure from a graph, the weighted adjacency matrix A^{w} will be the more useful choice.

3. Results

There are various ways to find functional clusters within a given network. Many algorithms take into account local information around a given vertex, such as the number of nearest neighbours shared with other vertices, and the number of paths to other vertices; see e.g. [31, 32]. Recently, a global algorithm was suggested which extends the concept of vertex betweenness [33] to links [34]. This elegant algorithm outperforms most traditional approaches in terms of mis-specifications of vertices to clusters. However it does not provide a measure for the differences of clusters. Zhou [35] introduces an algorithm which—while having at least the same performance rates as [34]—provides such a measure, the so-called

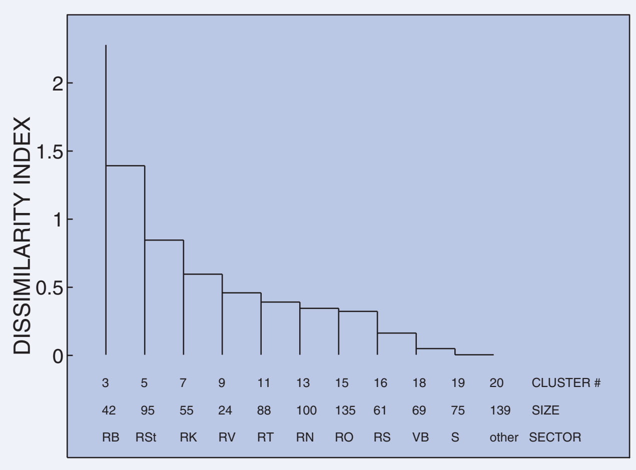

Figure 2. Community structure of the Austrian interbank market network from the September 2002 data. The dissimilarity index is a measure of the 'differentness' of the clusters.

dissimilarity index. The algorithm is based on the distance definition presented in [36].

To analyse our interbank network we apply this latter algorithm to the weighted adjacency matrix A^{w}. As the only preprocessing step we clip all entries in A^{w} above a level of 300m Euro for numerical reasons, i.e. $A^{\mathrm{w}}_{\mathrm{clip}} = \min(A^{\mathrm{w}}, 300\mathrm{m})$. The computed community structure, i.e. the clusters [34, 35] found in the liability network L, is shown in figure 1(*a*) and can now be compared with the actual community structure in the real world. The result for the community structure obtained from one representative data set is shown in figure 2. The results from other data sets are practically identical. The algorithm identifies communities of banks which are coupled by a two- or three-tier structure, i.e. the R, VB, and S sectors. For banks that, in reality, are not structured in a hierarchical way, such as banks in the SP, JS, SM, HCL sectors, no strong community structure is expected. Using the algorithm these banks are grouped together in a cluster called 'other'. The Raiffeisen sector, with its substructure in federal states, is further grouped into clusters which are clearly identified as R banks within one of the eight federal states (B, St, K, V, T, N, O, S). In figure 2 these clusters are marked as, for example, 'RS', with 'R' indicating the Raiffeisen sector and 'S' the state of Salzburg. Overall, there were 31 mis-specifications into wrong clusters within the total $N = 883$ banks, which is a mis-specification rate of 3.5%, demonstrating the quality of the dissimilarity algorithm and, more importantly, the quality of the entropy approach to reconstruct matrix L.

Like many real-world networks, the degree distribution of the interbank market follows power laws for all three graphs A^{l}, A^{a}, and A. Figures 3(*a*) and (*b*) show the out-degree (liabilities) and in-degree (assets) distribution of the vertices in the interbank liability network. Figure 3(*c*) shows the degree distribution of the interbank connection graph A. In all three cases we find two regions

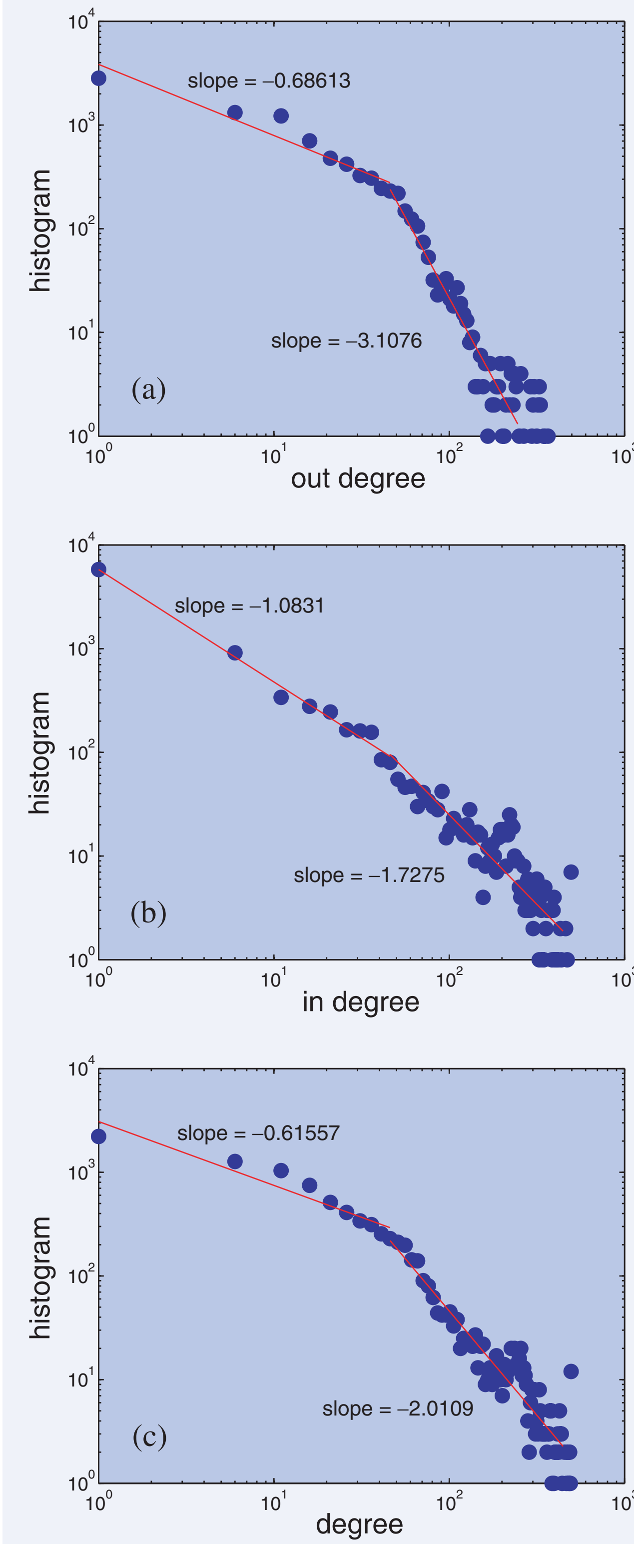

Figure 3. Empirical out-degree (*a*), in-degree and (*b*) distribution of the interbank liability network. (*c*) Degree distribution of the interbank connection network. All plots are histograms of aggregated data from all 10 datasets.

which can be fitted by a power. Accordingly, we fit one regression line to the small degree distribution and one to the obvious power tails of the data using an iteratively re-weighted least square algorithm. The power decay exponents γ_{tail} to the tails of the degree distributions are $\gamma_{\text{tail}}(A^{\text{l}}) = 3.11$, $\gamma_{\text{tail}}(A^{\text{a}}) = 1.73$, and $\gamma_{\text{tail}}(A) = 2.01$. The size of the out-degree exponent is within the range of several other complex networks, such as, for example, collaboration networks of actors (3.1) [3], and sexual contacts (3.4) [37]. Exponents in the range of 2 are, for example, the Web (2.1) [38] or mathematicians' collaboration networks (2.1) [39], and examples for exponents of about 1.5 are email networks (1.5) [40] and co-authorships (1.2) [41]. For the left part of the distribution (small degrees) we find $\gamma_{\text{small}}(A^{\text{l}}) = 0.69$, $\gamma_{\text{small}}(A^{\text{a}}) = 1.01$, and $\gamma_{\text{small}}(A) = 0.62$. These exponents are small compared with other real-world networks. Compare, for example, with food-webs (1) [42]. We have checked that the distribution for the low degrees is almost entirely dominated by banks of the R sector. Typically, in the R sector, most small agricultural banks have links to their federal state head institution and very few contacts with other banks, leading to a strong hierarchical structure, which is also visible in figure 1(*a*). This hierarchical structure is perfectly reflected by the small scaling exponents [43].

To quantify clustering phenomena within the banking network we use the so-called clustering coefficient C, defined as

$$C = \frac{3 \times (\text{number of triangles on the graph})}{\text{number of connected triples of vertices}}. \quad (4)$$

It provides the probability that two vertices that are connected to any given vertex are also connected with one another. A high clustering coefficient means that two banks that have interbank connections with a third bank have a greater probability of having interbank connections with one another than will any two banks randomly chosen on the network. The clustering coefficient is only well defined in undirected graphs. We find the clustering coefficient of the connection network (A) to be $C = 0.12 \pm 0.01$ (mean and standard deviation over the 10 data sets), which is relatively small compared with other networks. In the context of the interbank market, a small C is a reasonable result. While banks might be interested in some diversification of interbank links, keeping a link is also costly. Therefore, if, for instance, two small banks have a link with their head institution, there is no reason for them to additionally open a link between themselves.

We calculate the average path length for the three networks A^{l}, A^{a}, and A with the Dijkstra algorithm [44] and find an average path length of $\bar{\ell}(A^{\text{l}}) = \bar{\ell}(A^{\text{a}}) = 2.59 \pm 0.02$. Note the possibility that, in a directed graph, not all nodes can be reached and we restrict our statistics to the giant components of the directed graphs. The average path length in the (undirected) interbank connection network A is $\bar{\ell}(A) = 2.26 \pm 0.03$. From these results the Austrian interbank network looks like a very small world with about three degrees of separation. This result appears natural in light of the community structure described earlier. The two- and three-tier organization with head institutions and sub-institutions apparently leads to short interbank distances via the upper tier of the banking system and thus to a low degree of separation.

4. Discussion

Our analysis provides a first picture of an interbank network topology by studying a data set for the Austrian interbank market. Even though the Austrian interbank market is small it is structurally very similar to the interbank systems in many European countries, including the large economies of Germany, France, and Italy. We show that the liability (contract) size distribution follows a power law, which can be understood as being driven by the underlying size and wealth distributions of the banks, which show similar power exponents. We find that the interbank network shows, like many other realistic networks, power law dependencies in the degree distributions. We could show that different scaling exponents relate to different network structures in different banking sectors within the total network. The scaling exponents by the agricultural banks (R) are very low, due to the hierarchical structure of this sector, while the other banks lead to scaling exponents also found in other complex real-world networks. The interbank network shows a low clustering coefficient, a result that mirrors the analysis of community structure, which shows a clear network pattern, where banks first have links with their head institution, whereas the few head institutions have links with each other. A consequence of this structure is that the interbank network is a small world with a very low *degree of separation* [2] between any two nodes in the system. A further important message of this work is that it suggests the use of a more realistic class of scale-free networks for the future modeling of interbank relations, rather than the networks that have been studied in the literature thus far.

4.1. Economic consequences

Only recently has the network literature begun to study the dependence between the structure and the stability properties of networks. The most important results in this direction have been the notion of robustness of scale-free networks, i.e. networks with a power law degree distribution, with respect to the *random* breakdown of nodes, and the danger of *intentional attack* against the hubs in such networks [3, 4]. This insight can be understood theoretically, see e.g. Newman [45].

The power laws detected in the Austrian interbank network suggest that this interbank network should have a strong resilience towards random shocks. Indeed, Elsinger *et al* [14, 15], using a contagion flow model, show that the system is very stable in terms of individual default probability, but, more interestingly, also with respect to the domino effects of insolvencies following economic shocks to bank incomes. One of the few papers on contagion in the theoretical economic literature [8] suggests that a fully connected interbank liability graph might be the best safeguard against financial fragility. However, Elsinger *et al* [15] show that this claim is not true in general. By comparing a fully connected network with the actual scale-free network structure of the Austrian data, these authors show that, in the fully connected network, domino effect risk actually increases. Concerning the risk of the specific removal of a hub, e.g. as a consequence of an intentional attack, it was demonstrated by Boss *et al* [16] that, within the same Austrian banking network, the removal of a few network hubs will have a huge impact on the stability of the entire banking system. We conclude that both the stability of the network with respect to the random removal of nodes, and the disastrous consequences of the removal of specific hubs, are a direct consequence of the power laws of the degree distribution revealed in this paper.

4.2. Regulatory consequences

As far as banking regulation is concerned, our analysis is useful for reforming traditional approaches to banking supervision. These traditional approaches are strongly focused on the supervision of individual institutions. For the analysis of systemic risk it is, however, crucial to assess risk exposure at the level of the system as a whole because only then will the sources of systemic risk, such as correlated exposure and risk of domino effects through inter-linkages, become visible. Our methods provide a building block towards this direction. It reveals the hubs of the network, it shows the critical links and it helps to specify potentially systemically important banks. For instance, we show [16] that there is a linear dependence between the betweenness coefficient [33] of a node and its contagion impact. Thus a local parameter retrieved by network analysis can provide global information about a systemically relevant risk parameter of banking systems.

Clearly, to draw more far reaching and practical conclusions concerning contagion dynamics and financial stability we would need to couple the network topology to a dynamical model including external risk factors, other (non-interbank) income sources, and perhaps even behavioural assumptions about financial institutions. Only such a combination of topology and a dynamical model would allow for a rigorous analysis of some of the conclusions and conjectures we have formulated previously. The ultimate goal of our research on financial networks is to contribute to a deeper understanding of contagion dynamics. The work presented here is a first step in this direction, where an empirical analysis of network topology can serve as a guideline for modeling systemic financial stability problems such as contagion of bank insolvency or systemic risk in payment systems.

Acknowledgements

This work was supported by the Austrian Science Foundation project FWF P17621-G05. We thank J D Farmer for valuable comments to improve the paper and Haijun Zhou for making his dissimilarity index algorithm available. S T would like to thank the SFI and, in particular, J D Farmer for their great hospitality in the summer of 2003.

References

[1] Dorogovtsev S N and Mendes J F F 2003 *Evolution of Networks: From Biological Nets to the Internet and WWW* (Oxford: Oxford University Press)
[2] Watts D J 1999 *Small Worlds: The Dynamics of Networks Between Order and Randomness* (Princeton University Press)
[3] Albert R and Barabasi A-L 2000 Topology of evolving networks: local events and universality *Phys. Rev. Lett.* **85** 5234–7
[4] Albert R, Jeong H and Barabasi A-L 2000 Error and attack tolerance of complex networks *Nature* **406** 378–82
[5] Dow J 2000 What is systemic risk? Moral hazard, initial shocks and propagation *IMES Discussion Paper No. 2000-E-17*
[6] De Bandt O and Hartmann P 2000 Systemic risk: a survey *CEPR Dicussion Paper No. 2634*
[7] Summer M 2003 Banking regulation and systemic risk *Open Economies Rev.* **1** 43
[8] Allen F and Gale D 2000 Financial contagion *J. Political Economy* **108** 1
[9] Freixas X, Parigi L and Rochet J C 2000 Systemic risk, interbank relations and liquidity provision by the Central Bank *J. Money, Credit Banking* **32**
[10] Thurner S, Hanel R and Pichler S 2003 Risk trading, network topology, and banking regulation *Quant. Finance* **3** 306–19
[11] Iori G, Jafaray S and Padilla F 2001 Interbank lending and systemic risk *e-print*: arXiv:cond-mat/0104080
[12] Jackson M 2003 A survey of models of network formation: stability and efficiency http://www.hss.caltech.edu/jacksonm/netsurv.pdf
[13] Eisenberg L and Noe T H 2001 Systemic risk in financial networks *Manage. Sci.* **47** 236–49
[14] Elsinger H, Lehar A and Summer M 2002 Risk assessment for banking systems *Austrian National Bank Working Paper 57*
[15] Elsinger H, Lehar A and Summer M 2004 Risk assessment for banking systems http://www.bwl.univie.ac.at/bwl/fiwi3/members/lehar/interbankv25.pdf
[16] Boss M, Summer M and Thurner S 2004 Contagion flow through banking networks *Lecture Notes in Computer Science* 3038 eds M Bubak, G D v Albada, P M A Sloot and J J Dongarra (Berlin: Springer) pp. 1070–7
[17] Humphrey D B 1986 Payments finality and risk of settlement failure *Technology and Regulation of Financial Markets: Securities, Futures and Banking* eds S Anthony, A S Saunders, J Lawrence and L J White (Lexington Books)

[18] Angelini P, Maresca G and Russo D 1996 Systemic risk in the netting system *J. Banking Finance* **20** 853–68
[19] Wells S 2002 *Bank of England*, *Financial Stability Review*, 175–82
[20] Degryse H and Nguyen G 2004 Interbank exposures: an empirical examination of systemic risk in the Belgian banking system *Belgian National Bank*, *Working Paper*
[21] Furfine C 2003 Interbank exposures: quantifying the risk of contagion *J. Money, Credit Banking* **35** 11–128
[22] Upper C and Worms A 2002 Estimating bilateral exposures in the German Interbank Market: is there a danger of contagion? *Deutsche Bundesbank*, *Dicussion paper 09*
[23] Fang S C, Rajasekra J R and Tsao J 1997 *Entropy Optimization and Mathematical Programming* (Dordrecht: Kluwer Academic)
[24] Blien U and Graef F 1997 Entropy optimizing methods for the estimation of tables *Classification, Data Analysis, and Data Highways* eds I Balderjahn, R Mathar and M Schader (Berlin: Springer)
[25] Sheldon G and Maurer M 1998 Interbank lending and systemic risk *Swiss J. Economics Statistics* **134** 685
[26] Solomon S and Levy M 2000 *e-print*: arXiv: cond-mat/0005416
[27] Axtell R L 2001 *Science* **293** 1818
[28] Nagurney A and Siokos S 1997 *Financial Networks: Statics and Dynamics* (Berlin: Springer)
[29] Nagurney A and Hughes M 1992 Financial flow of funds networks *Networks* **22** 145
[30] Nagurney A and Ke K 2001 Financial networks with intermediation *Quant. Finance* **1** 441–51
[31] Wasserman S and Faust K 1994 *Social Network Analysis: Methods and Applications* (Cambridge: Cambridge University Press)
[32] Ravasz E, Somera A L, Mongru D A, Oltvai Z N and Barabasi A-L 2001 *Science* **297** 1551
[33] Freeman L C 1977 *Sociometry* **40** 35
[34] Girvan M and Newman M E J 2002 *Proc. Natl Acad. Sci.* **99** 7831
[35] Zhou H 2003 *Phys. Rev. E* (in press) *e-print*: arXiv: cond-mat/0302030
[36] Zhou H 2003 Distance, dissimilarity index, and network community structure *e-print*: arXiv:physics/0302032
[37] Liljeros F, Edling C F, Amaral L A N, Stanley H E and Aberg Y 2001 The web of human sexual contacts *Nature* **411** 907
[38] Albert R, Jeong H and Barabasi A-L 1999 Diameter of the world wide web *Nature* **401** 130
[39] Barabasi A-L, Jeong H, Neda Z, Ravasz E, Schubert A and Vicsek T 2002 Evolution of social network of scientific collaborations *Physica* A **311** 590
[40] Ebel H, Mielsch L I and Bornholdt S 2002 Scale free tolopogy of e-mail networks *Phys. Rev.* E **66** 036103
[41] Newman M E J 2001 Who is the best connected scientist? A study of scientific coauthorship networks, scientific collaboration networks. I. Network construction and fundamental results *Phys. Rev.* E **64** 016131, 016132
[42] Montoya J M and Sole R V 2000 Toplogical properties of food webs. From real data to community assembly models *Santa Fe Institute Working Paper* 00-10-059
[43] Trusina A, Maslov S, Minnhagen P and Sneppen K 2003 *e-print*: arXiv:cond-mat/0308339
[44] Gibbons A 1985 *Algorithmic Graph Theory* (Cambridge: Cambridge University Press)
[45] Newman M E J 2002 Random graphs as models of networks *Santa Fe Institute Working Paper* 02-02-05