1)

$x_1(t) = Ae^{-jnw_0t}$ con $w_0 = \frac{2\pi}{T}$, $A, B > 0$, $n, m \in \mathbb{Z}$

$x_2(t) = Be^{-jmw_0t}$

$$d^2(x_1, x_2) = \lim_{T\to\infty} \frac{1}{T}\int_0^T |x_1(t) - x_2(t)|^2 dt$$

$$x_1^*(t) = Ae^{+jnw_0t}, \quad x_2^*(t) = Be^{-jmw_0t}$$

$$|x_1(t) - x_2(t)|^2 = A^2 + B^2 - 2AB\cos((n+m)w_0t)$$

$$d^2 = \lim_{T\to\infty} \frac{1}{T}\int_0^T [A^2 + B^2 - 2AB\cos((n+m)w_0t)]\, dt$$

$$d^2 = \lim_{T\to\infty}\left[\frac{1}{T}\int_0^T A^2 dt + \frac{1}{T}\int_0^T B^2 dt - 2AB \cdot \frac{1}{T}\int_0^T \cos((n+m)w_0t)\right.$$

$$\left. dt\right]$$

$$\frac{1}{T}\int_0^T A^2 dt = \frac{1}{T}\cdot A^2 T = A^2$$

$$\frac{1}{T}\int_0^T B^2 dt = B^2$$

$$d^2 = A^2 + B^2 - 2AB\left(\lim_{T\to\infty}\frac{1}{T}\int_0^T \cos((n+m)w_0t)dt\right)$$

$$\int_0^T \cos((n+m)w_0t)dt = \left.\frac{\operatorname{sen}((n+m)w_0t)}{(n+m)w_0}\right|_0^T$$

$$= \frac{\operatorname{sen}((n+m)w_0T)}{(n+m)w_0}$$

$$\frac{1}{T}\int_0^T \cos((n+m)w_0t)\,dt = \frac{\operatorname{sen}((n+m)w_0T)}{T(n+m)w_0}$$

$w_0T = 2\pi$

$\operatorname{sen}((n+m)w_0t)\,dt = \operatorname{sen}((n+m)2\pi) = 0$

$$\int_0^T \cos((n+m)w_0t)\,dt = 0$$

$n+m=0$ $\qquad$ $\operatorname{sen}(n+m) \neq 0$

$$\cos((n+m)w_0t) = \cos(0) = 1$$

$$\int_0^T \cos(0)\,dt = \int_0^T 1\,dt = T$$

sustitucion

Caso 1

$n+m \neq 0$

$$d^2 = \frac{1}{T}(A^2T + B^2T - 0) = A^2+B^2 \Rightarrow \underline{d = \sqrt{A^2+B^2}}$$

Caso 2

$n+m=0$

$$d^2 = \frac{1}{T}(A^2T + B^2T - 2ABT) = (A-B)^2 \Rightarrow d$$

$$= \underline{|A-B|}$$

RTA= $d(x_1, x_2) = \begin{cases} |A-B|, & n+m=0 \\ \sqrt{A^2+B^2}, & n+m \neq 0 \end{cases}$

$n+m=0 \rightarrow$ Componente en fase opuesta (e^{-jnw_0t} ; e^{+jnw_0t}

$A=B \rightarrow$ Diferencia promedio es 0 x el conjugado complejo de la otra

$n+m \neq 0 \rightarrow$ oscilan en diferente frecuencia

② Frecuencia (HZ)

$w_1 = 1000\pi \rightarrow f_1 = \frac{1000\pi}{2\pi} = 500 Hz$

$w_2 = 3000\pi \rightarrow f_2 = 1500 HZ$

$w_3 = 11000\pi \rightarrow f_3 = 5500 HZ$

$T_1 = 0.002 s.$

$T_2 = 0.0006666667 s$

$T_3 = 0.000181818 2 s$

Muestreo

$f_s = 5000 Hz \rightarrow T_s = \frac{1}{5000} = 0.0002 s$

Instantes: $t_n = nT_s$

Señal muestreada

$x[n] = 3\cos(1000\pi n T_s) + 10\cos(11000\pi n T_s)$

$T_s = 1/5000$:

$1000\pi n T_s = 1000\pi \frac{n}{5000} = 0.2\pi n$

$3000\pi n T_s = 0.6\pi n$

$11000\pi n T_s = 2.2\pi n$

$\cos(2.2\pi n) = \cos(2\pi n + 0.2\pi n) = \cos(0.2\pi n)$

$$\boxed{x[n] = 13\cos(0.2\pi n) + 5\sin(0.6\pi n)}$$

Chequeo Nyquist / Aliasing

$f_s/2 = 2500$ Hz.

$f_3 = 5500 > 2500$, hay aliasing

$b = 4 \Rightarrow L = 2^4 = 16$ niveles

Amplitud max aprox $\leq \sqrt{13^2+5^2} = \sqrt{194}$

≈ 13.928388

Rango $\approx [-13.928, 13.928]$

$\Delta = \frac{x_{max} - x_{min}}{L-1} = \frac{2 \cdot 13.928}{15} \approx 1.8570667$

$x_Q[n] = \Delta \cdot \text{round}\left(\frac{x[n] - x_{min}}{\Delta}\right) + x_{min}$

$SNR(dB) \approx 6.02b + 1.76 = 6.02 \cdot 4 + 1.76$

≈ 25.84 dB

(3)

$$c_n = \frac{1}{(t_i - t_f)\, n^2 \omega_0^2} \int_{t_i}^{t_f} x''(t)\, e^{-jn\omega_0 t}\, dt ; \; n \in \mathbb{Z}$$

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, e^{-jn\omega_0 t}\, dt$$

integramos partes

$u = x(t)$ $\quad du = x'(t)\,dt$

$dv = e^{-jn\omega_0 t}\, dt$ $\quad v = \frac{j e^{-j\omega_0 t}}{n\omega_0}$

$$\int u\,dv = uv - \int v\,du$$

sustituyendo

$$c_n = \frac{1}{T}\left[\left(x(t) \frac{j e^{-jn\omega_0 t}}{n\omega_0} \right) \Bigg|_{-T/2}^{T/2} - \underbrace{\frac{j}{n\omega_0} \int_{-T/2}^{T/2} x'(t)}_{\text{nuevamente}} \right.$$

$$\left. \underbrace{e^{-j\omega_0 t}\, dt}_{\text{por partes}} \right]$$

$u = x'(t) \rightarrow du = x''t\,dt$

$dv = e^{-jn\omega_0 t}\, dt \rightarrow v = \frac{j e^{-jn\omega_0 t}}{n\omega_0}$

$$\int_{-T/2}^{T/2} x'(t)\, e^{-jn\omega_0 t}\, dt = \left[x'(t) \frac{j e^{-jn\omega_0 t}}{n\omega_0} \right] \Bigg|_{-T/2}^{T/2}$$

$$-\frac{j}{n\omega_0}\int_{-T/2}^{T/2} x''(t)\, e^{-j\omega_0 t}\, dt$$

$$c_n = \frac{1}{T}\left(\left[x(t)\,\frac{j e^{-jn\omega_0 t}}{n\omega_0}\right|_{-T/2}^{T/2} - \frac{j}{n\omega_0}\left(\left[x'(t)\cdot \frac{j e^{-jn\omega_0 t}}{n\omega_0}\right]\right|_{-T/2}^{T/2} - \frac{j}{n\omega_0}\int_{-T/2}^{T/2} x''(t)\, e^{-jn\omega_0 t}\, dt\right]$$

$$= \frac{1}{T}\left(j\, x(t)\,\frac{e^{-jn\omega_0 t}}{n\omega_0}\right)\Bigg|_{-T/2}^{T/2} + \frac{1}{T}\left[x(t)\,\frac{e^{-jn\omega_0 t}}{n^2\omega_0^2}\right]\Bigg|_{-T/2}^{T/2}$$

$$- \frac{1}{Tn^2\omega_0^2}\int_{-T/2}^{T/2} x''(t)\, e^{-jn\omega_0 t}\, dt$$

$$\omega_0 = \frac{2\pi}{T}$$

$$\frac{1}{T}\left(j\, x(t)\,\frac{e^{-jn\omega_0 t}}{n\omega_0}\right)\Bigg|_{-T/2}^{T/2} = \frac{1}{T}\left(j\, x(T/2)\,\frac{T e^{-jn\pi}}{2n\pi}\right.$$

$$- j\, x(-T/2)\,\frac{T e^{-jn\pi}}{2n\pi}$$

$$= \frac{1}{T}\left(\frac{jT}{2n\pi}\left[x(T/2)\,e^{-jn\pi} - x(-T/2)\,e^{jn\pi}\right]\right)$$

$$= \frac{1}{n\pi}\left[x(T/2) + x(-T/2)\,sen(n\pi)\right]$$

$$= 0$$

Se evalua la 2 integral paso en vez de $x(t) \rightarrow x'(t)$ lo cual sera igual a lo anterior entonces sera 0

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\,e^{-jnw_0 t}\,dt = \frac{1}{Tn^2w_0^2}\int_{-T/2}^{T/2} x''(t)\,e^{-jnw_0 t}\,dt$$

Tomamos rango $T = t_f - t_i$

$$= \frac{1}{(t_f - t_i)n^2w_0^2}\int_{t_i}^{t_f} x''(t)\,e^{-jnw_0 t}\,dt$$

$$c_n = \frac{1}{(t_i - t_f)n^2w_0^2}\int_{t_i}^{t_f} x''(t)\,e^{-jnw_0 t}\,dt$$

$$c_n = \frac{1}{(t_i - t_f)n^2\omega_0^2} \int_{t_i}^{t_f} x''(t)\, e^{-jn\omega_0 t}\, dt$$

$$c_n = \frac{1}{(t_i - t_f)n^2\omega_0^2} \int_{t_i}^{t_f} x''(t) [\cos(n\omega_0 t) - j\sin(n\omega_0 t)] dt$$

$$c_n = \frac{1}{(t_i - t_f)n^2\omega_0^2} \int_{t_i}^{t_f} x'' \cos(n\omega_0 t) - \frac{j}{(t_i - t_f)n^2\omega_0^2} \int_{t_i}^{t_f} x''(t)$$

$\sin(n\omega_0 t)\, dt$

utilizando la iguardda

$a_n = 2\,\mathrm{Re}\{c_n\}$ si tiene

$$\boxed{a_n = \frac{2}{(t_i - t_f)n^2\omega_0^2} \int_{t_i}^{t_f} x''(t) \cos(n\omega_0 t)\, dt}$$

Para la otra parte imaginaria

$b_n = -2\,\mathrm{Im}\{c_n\}$

$$\boxed{b_n = \frac{2}{(t_i - t_f)n^2\omega_0^2} \int_{t_i}^{t_f} x''(t) \sin(n\omega_0 t)\, dt}$$