ECON 714 QUARTER 1: PROBLEM SET 4

EMILY CASE

Oligopolistic competition (Atkeson and Burstein, AER 2008)

Consider a static model with a continuum of sectors $k \in [0,1]$ and $i = 1, ..., N_k$ firms in sector k, each producing a unique variety of the good. Households supply inelastically one unit of labor and have nested-CES preferences:

$$C = \left(\int C_k^{\frac{\rho-1}{\rho}} dk \right)^{\frac{\rho}{\rho-1}}, \qquad C_k = \left(\sum_{i=1}^{N_k} C_{ik}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \theta > \rho \geq 1.$$

For firm i in sector k: $Y_{ik} = A_{ik} L_{ik}$

1. Solve household cost minimization problem for the optimal demand C_{ik}, the sectoral price index P_k, and the aggregate price index P as functions of producers' prices.

$$\min_{C_{ik}} \int \sum_{i=1}^{N_k} P_{ik} C_{ik} dk$$

$$s.t. \quad C = \left(\int C_k^{\frac{\rho-1}{\rho}} dk \right)^{\frac{\rho}{\rho-1}}, \qquad C_k = \left(\sum_{i=1}^{N_k} C_{ik}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}$$

Construct a lagrangian:

$$\mathcal{L} = \int \sum_{i=1}^{N_k} P_{ik} C_{ik} dk - P \left[\left(C_k^{\frac{\rho-1}{\rho}} dk \right)^{\frac{\rho}{\rho-1}} - C \right] + \int P_k \left[C_k - \left(\sum_{i=1}^{N_k} C_{ik}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \right] dk$$

I worked on this Problem set with Sarah Bass, Michael Nattinger, Alex von Hafften, Hanna Han, and Danny Edgel.

FOC $[C_{ik}]$:

$$P_{ik} = P_k \left(\sum_{i=1}^{N_k} C_{ik}^{\frac{\theta-1}{\theta}} \right)^{\frac{1}{\theta-1}} C_{ik}^{\frac{-1}{\theta}}$$
$$\Rightarrow C_{ik} = \left(\frac{P_k}{P_{ik}} \right)^{\theta} C_k \tag{1}$$

FOC $[C_k]$:

$$P_k = P \left(C_k^{\frac{\rho-1}{\rho}} dk \right)^{\frac{1}{\rho-1}} C_k^{\frac{-1}{\rho}}$$
$$\Rightarrow C_k = \left(\frac{P}{P_k} \right)^{\rho} C \tag{2}$$

We can plug these first order conditions into the original given definitions for C and C_k:

$$C_k = \left(\sum_{i=1}^{N_k} C_{ik}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} = \left(\sum_{i=1}^{N_k} \left[\left(\frac{P_k}{P} \right)^{-\theta} C_k \right]^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}$$
$$\Rightarrow P_k = \left(\sum_{i=1}^{N_k} P_{ik}^{1-\theta} \right)^{\frac{1}{1-\theta}} \tag{*}$$

$$C = \left(\int C_k^{\frac{\rho-1}{\rho}} dk \right)^{\frac{\rho}{\rho-1}} = \left[\int \left(\left[\frac{P}{P_k} \right]^{\rho} C \right)^{\frac{\rho-1}{\rho}} dk \right]^{\frac{\rho}{\rho-1}}$$
$$= \left[C^{\frac{\rho-1}{\rho}} P^{\rho-1} \left(\int P_k^{1-\rho} dk \right) \right]^{\frac{\rho}{\rho-1}}$$
$$P = \left(\int P_k^{1-\rho} dk \right)^{\frac{1}{1-\rho}} \tag{**}$$

This gives us the sectoral price index and the aggregate price index as a function of producer's prices P_{ik}. [1]

[1] Note that we can plug (*) into (**) to get

$$P = \left(\int \left(\sum_{i=1}^{N_k} P_{ik}^{1-\theta} \right)^{\frac{1-\rho}{1-\theta}} dk \right)^{\frac{1}{1-\rho}}$$

Now to get consumption as a function of producers' prices, we use first order conditions (**??**) and (**??**):

$$C_{ik} = P_{ik}^{-\theta}\left(\sum_{i=1}^{N_k} P_{ik}^{1-\theta}\right)^{\frac{\theta-\rho}{1-\theta}} P^{\rho}C \tag{3}$$

2. Demand elasticity and the optimal price using Bertrand

I have attached in a separate pdf my handwritten algebra for this problem. Define $s_{ik} = \left(\frac{P_{ik}}{P_k}\right)^{1-\theta}$. We get:

$$\begin{aligned} P_{ik}[(1-\theta) + s_{ik}(\theta-\rho)] &= \frac{W}{A_{ik}}\left[(-\theta) + s_{ik}(\theta-\rho)\right] \\ \Rightarrow P_{ik} &= \frac{W}{A_{ik}}\left[1 - \frac{1}{(1-\theta) + s_{ik}(\theta-\rho)}\right] \end{aligned}$$

Now, to calculate the elasticity, $\eta = \frac{\partial C_{ik}}{\partial P_{ik}} \cdot \frac{P_{ik}}{C_{ik}}$:

$$\begin{aligned} \frac{\partial C_{ik}}{\partial P_{ik}} &= P^{\rho}C[(-\theta)P_{ik}^{-\theta-1}P_k^{\theta-\rho} + P_{ik}^{-\theta}(\theta-\rho)P_k^{2\theta-\rho-1}P_{ik}^{-\theta} \\ \frac{P_{ik}}{C_{ik}} &= P_{ik}P_k^{\rho-\theta}\left(\frac{P_{ik}^{\theta}}{P^{\rho}}\right)C^{-1} \\ \Rightarrow \eta &= (\theta-\rho)s_{ik} - \theta \end{aligned}$$

3. Prove that other things equal, firms with higher A_{ik} set higher markups.

Recall that the optimal price = optimal markup * marginal costs. In this model,

$$P_{ik} = \mu_{ik}\left(\frac{W}{A_{ik}}\right)$$

for markup μ_{ik}. Then also

$$\begin{aligned} \mu_{ik} &= P_{ik}\left(\frac{A_{ik}}{W}\right) \\ &= \left(1 - \frac{1}{(1-\theta) + s_{ik}(\theta-\rho)}\right) \end{aligned}$$

$$s_{ik} = \left(\frac{C_k}{A_{ik}L_{ik}}\right)^{\frac{1-\theta}{\theta}}$$

So when A_{ik} increases, s_{ik} also increases[2]. Then if s_{ik} is increasing, it is straightforward to see that μ_{ik} is also increasing.

4. Solve the model numerically

Note that to solve the model I had to implement a tuning parameter of 0.6[3], otherwise ρ poses an issue. My matlab code is attached.

5. Compute the aggregate output C of the economy and compare it to the first-best value.

Computing on matlab, I got the following results:

$$C = 5.7301$$
$$C_{SPP} = 7.238$$

The competitive equilibrium does not achieve the first best outcome because firms have some degree of market power within their sectors.

[2]Because $\theta > 1$.

[3]At the suggestion of Michael Nattinger, thanks!