ECON 714 QUARTER 1: PROBLEM SET 4

EMILY CASE

Oligopolistic competition (Atkeson and Burstein, AER 2008)

Consider a static model with a continuum of sectors $k \in [0,1]$ and $i = 1, ..., N_k$ firms in sector k, each producing a unique variety of the good. Households supply inelastically one unit of labor and have nested-CES preferences:

$$C = \left(\int C_k^{\frac{\rho-1}{\rho}} dk \right)^{\frac{\rho}{\rho-1}}, \qquad C_k = \left(\sum_{i=1}^{N_k} C_{ik}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \theta > \rho \geq 1.$$

For firm i in sector k: $Y_{ik} = A_{ik} L_{ik}$

1. Solve household cost minimization problem for the optimal demand C_{ik}, the sectoral price index P_k, and the aggregate price index P as functions of producers' prices.

$$\min_{C_{ik}} \int \sum_{i=1}^{N_k} P_{ik} C_{ik} dk$$

$$s.t. \quad C = \left(\int C_k^{\frac{\rho-1}{\rho}} dk \right)^{\frac{\rho}{\rho-1}}, \qquad C_k = \left(\sum_{i=1}^{N_k} C_{ik}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}$$

Construct a lagrangian:

$$\mathcal{L} = \int \sum_{i=1}^{N_k} P_{ik} C_{ik} dk - P \left[\left(C_k^{\frac{\rho-1\rho}{d}} k \right)^{\frac{\rho}{\rho-1}} - C \right] + \int P_k \left[C_k - \left(\sum_{i=1}^{N_k} C_{ik}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \right] dk$$

I worked on this Problem set with Sarah Bass, Michael Nattinger, Alex von Hafften, Hanna Han, and Danny Edgel.

FOC $[C_{ik}]$:

$$\begin{aligned} P_{ik} &= P_k \left(\sum_{i=1}^{N_k} C_{ik}^{\frac{\theta-1}{\theta}} \right)^{\frac{1}{\theta-1}} C_{ik}^{\frac{-1}{\theta}} \\ \Rightarrow C_{ik} &= \left(\frac{P_k}{P_{ik}} \right)^{\theta} C_k \end{aligned} \tag{1}$$

FOC $[C_k]$:

$$\begin{aligned} P_k &= P \left(C_k^{\frac{\rho-1}{\rho}} dk \right)^{\frac{1}{\rho-1}} C_k^{\frac{-1}{\rho}} \\ \Rightarrow C_k &= \left(\frac{P}{P_k} \right)^{\rho} C \end{aligned} \tag{2}$$

We can plug these first order conditions into the original given definitions for C and C_k:

$$\begin{aligned} C_k &= \left(\sum_{i=1}^{N_k} C_{ik}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \\ &= \mathbf{XXXXX} \\ P_k &= \left(\sum_{i=1}^{N_k} P_{ik}^{1-\theta} \right)^{\frac{1}{\theta-1}} \end{aligned} \tag{*}$$

$$\begin{aligned} C = \left(\int C_k^{\frac{\rho-1}{\rho}} dk \right)^{\frac{\rho}{\rho-1}} &= \left[\int \left(\left[\frac{P}{P_k} \right]^{\rho} C \right)^{\frac{\rho-1}{\rho}} dk \right]^{\frac{\rho}{\rho-1}} \\ &= \left[C^{\frac{\rho-1}{\rho}} P^{\rho-1} \left(\int P_k^{1-\rho} dk \right) \right]^{\frac{\rho}{\rho-1}} \\ P &= \left(\int P_k^{1-\rho} dk \right)^{\frac{1}{\rho-1}} \end{aligned} \tag{**}$$

This gives us the sectoral price index and the aggregate price index as a function of producer's prices P_{ik}. [1]

Now to get consumption as a function of producers' prices, we use first order conditions (1) and (2)

[1]Note that we can plug (*) into (**) to get

$$P = \left(\int \left(\sum_{i=1}^{N_k} P_{ik}^{1-\theta} \right)^{\frac{1-\rho}{\theta-1}} dk \right)^{\frac{1}{\rho-1}}$$

check this