Econ 714 Quarter 1: Problem set 1

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Consider a neoclassical growth model with preferences $\sum_{t=0}^{\infty} \beta^t U(C_t)$, production technology $F(K_t)$, and the initial capital endowment K_0. Both $U(\cdot)$ and $F(\cdot)$ are strictly increasing, strictly concave and satisfy standard Inada conditions. The capital law of motion is

$$K_{t+1} = (1-\delta)K_t + I_t - D_t$$

where D_t is a natural disaster shock that destroys a fixed amount of the accumulated capital.

1 Write down the social planner's problem and derive the intertemporal optimality condition (the Euler equation).

Note that $F(K_t) = C_t + I_t$. Then the social planner's problem is

$$\max_{C_t} \sum_{t=0}^{\infty} \beta^t U(C_t)$$
$$\text{s.t. } F(K_t) = C_t + K_{t+1} - (1-\delta)K_t + D_t$$

Now we can set up the lagrangian:

$$\sum_{t=0}^{\infty} \beta^t \left[U(C_t) + \lambda_t(-F(K_t) + C_t + K_{t+1} - (1-\delta)K_t + D_t)\right]$$

FOC with respect to C_t:

$$U'(C_t) + \lambda^t = 0$$

FOC with respect to K_t:

$$\beta^{t+1}\lambda_{t+1} = \beta^t \lambda_t(1-\delta+F'(K_t))$$

Simplifying and combining FOC, we can get the Euler Equation:

$$\beta\lambda_{t+1} = \lambda_t(1-\delta+F'(K_t))$$

$$\beta U'(C_{t+1}) = U'(C_t)(1-\delta+F'(K_t))$$

$$\beta\frac{U'(C_{t+1})}{U'(C_t)} = 1-\delta+F'(K_t)$$

I worked on this Problem set with Sarah Bass, Michael Nattinger, Alex von Hafften, and Danny Edgel.

2 Given the steady-state value of $D \geq 0$, write down the system of equations that determines the values of capital $\bar{K}(D)$ and consumption $\bar{C}(D)$ in the steady state. Draw a phase diagram with capital in the horizontal axis and consumption in the vertical axis, show the steady states, draw the arrows representing the direction of change, and the saddle path.

Let $K_t = K_{t+1} = \bar{K}$ and $C_t = C_{t+1} = \bar{C}$, and the Euler equation becomes:

$$\beta = 1 - \delta + F'(\bar{K})$$

Also, the resource constraint becomes:

$$F(\bar{K}) = \bar{C} + \delta\bar{K} + D$$

this absolutely does not seem right

3 The scientists forecast an earthquake T periods from now that will destroy $D > 0$ units of capital. Assuming that economy starts from a steady state with $D = 0$, draw a phase diagram that shows the optimal transition path. Make two separate graphs showing the evolution of capital and consumption in time.

drawn on ipad but want to fix it so that shock puts us on the saddle path below new SS, so that they increase again. However the way I have it drawn maybe indicates that if you don't drop consumption low enough and save enough, you won't have great insurance for earthquake? ask about this...

could also plot on matlab using question 4's values? if i have time

4 Assume that $U(C) = \frac{C^{1-\sigma}-1}{1-\sigma}$ and $F(K) = K^{\alpha}$ and the values of parameters are $\sigma = 1$, $\alpha = 1/3$, $\beta = 0.99^{1/12}$ (monthly model), $\delta = 0.01$, $T = 12$, $D = 1$. Using a shooting algorithm, solve numerically for the optimal transition path and plot dynamics of consumption and capital.

Updated Euler Equation:

$$\beta \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} = 1 - \delta + \alpha K_t^{1-\alpha}$$
$$\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} = \frac{1}{\beta}\left(1 - \delta + \alpha K_t^{1-\alpha}\right)$$
$$C_{t+1} = C_t \beta^{1/\sigma}(1 - \delta + \alpha K_t^{1-\alpha})^{-1/\sigma}$$

Updated resource constraint:

$$K_t^{\alpha} = C_t + K_{t+1} - (1-\delta)K_t + D_t$$

Notes:

- SS before shock: K = 170.57 and C = 3.84

General idea for code:

Use a consumption grid instead of a capital grid. initialize that and define the parameters. Find the steady state values, without the shock value first, because that steady state is where it will start at.

Now use the shooting method to find the trajectory for this SS (no shock yet). Given a K, what would the optimal consumption be? do this for all K in order to find the saddle path trajectory.

in general very stuck here.... have half tried my own method half tried to learn Danny's...