Econ 714 Quarter 1: Problem set 1

Emily Case

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Consider a neoclassical growth model with preferences $\sum_{t=0}^{\infty} \beta^t U(C_t)$, production technology $F(K_t)$, and the initial capital endowment K_0. Both $U(\cdot)$ and $F(\cdot)$ are strictly increasing, strictly concave and satisfy standard Inada conditions. The capital law of motion is

$$K_{t+1} = (1-\delta)K_t + I_t - D_t$$

where D_t is a natural disaster shock that destroys a fixed amount of the accumulated capital.

1 Write down the social planner's problem and derive the intertemporal optimality condition (the Euler equation).

Note that $F(K_t) = C_t + I_t$. Then the social planner's problem is

$$\max_{C_t} \sum_{t=0}^{\infty} \beta^t U(C_t)$$
$$\text{s.t. } F(K_t) = C_t + K_{t+1} - (1-\delta)K_t + D_t$$

Now we can set up the lagrangian:

$$\sum_{t=0}^{\infty} \beta^t \left[U(C_t) + \lambda_t(-F(K_t) + C_t + K_{t+1} - (1-\delta)K_t + D_t)\right]$$

FOC with respect to C_t:

$$U'(C_t) + \lambda^t = 0$$

FOC with respect to K_{t+1}:

$$\beta^t \lambda_t = \beta^{t+1}\lambda_{t+1}[F'(K_{t+1}) + 1 - \delta]$$

Simplifying and combining FOC, we can get the Euler Equation:

$$U'(C_t) = \beta U'(C_{T+1})[F'(K_{t+1}) + 1 - \delta]$$

I worked on this Problem set with Sarah Bass, Michael Nattinger, Alex von Hafften, and Danny Edgel.

2 Given the steady-state value of $D \geq 0$, write down the system of equations that determines the values of capital $\bar{K}(D)$ and consumption $\bar{C}(D)$ in the steady state. Draw a phase diagram with capital in the horizontal axis and consumption in the vertical axis, show the steady states, draw the arrows representing the direction of change, and the saddle path.

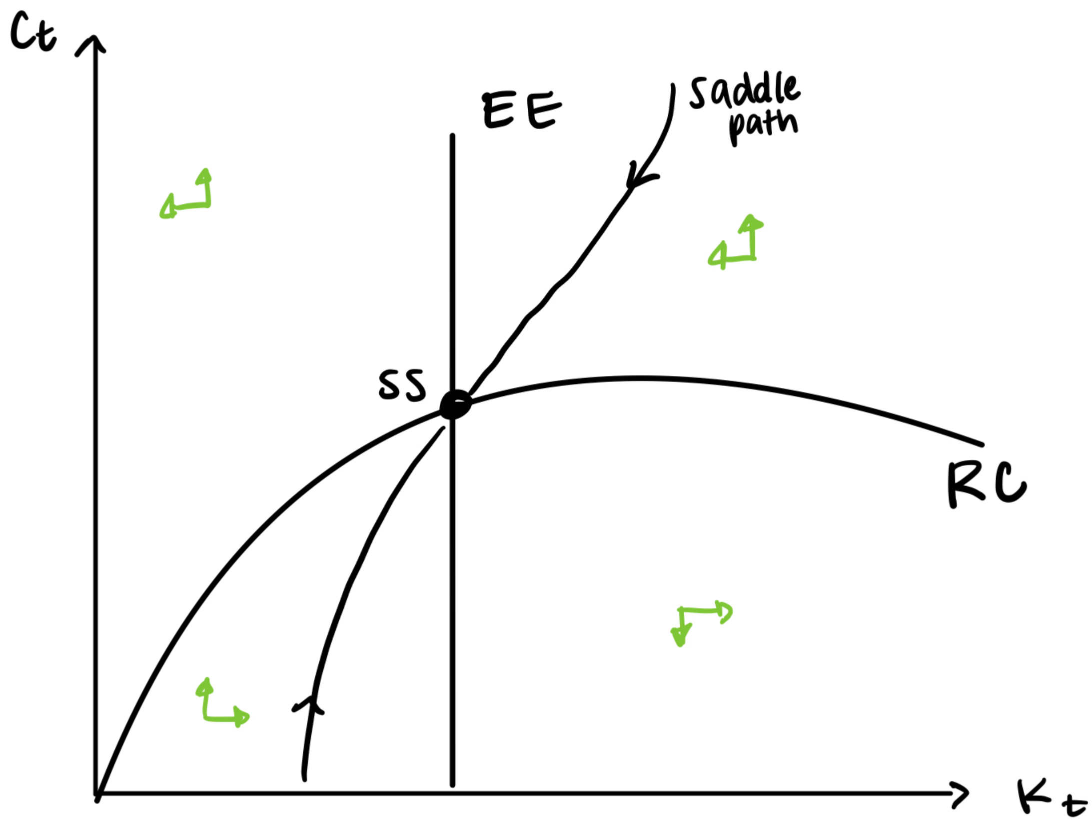
Let $K_t = K_{t+1} = \bar{K}$ and $C_t = C_{t+1} = \bar{C}$, and the Euler equation becomes:

$$F'(\bar{K}(D)) = \beta - (1 - \delta)$$

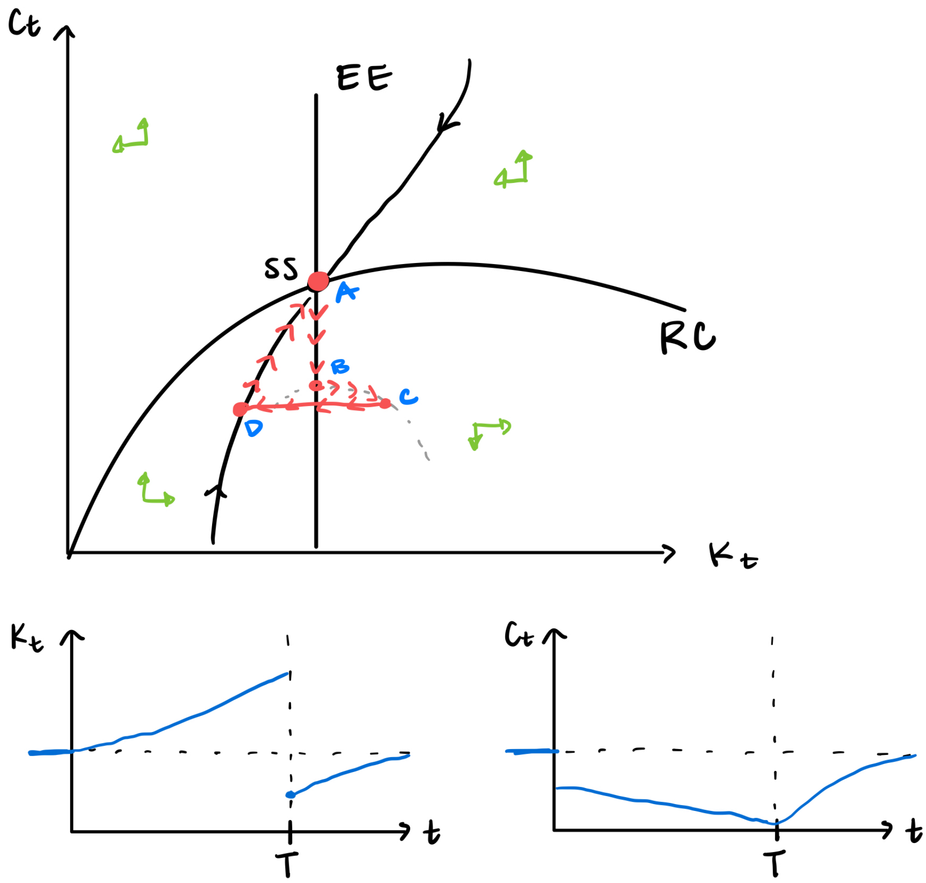
$$1 = \beta[F'(\bar{K}(D)) + 1 - \delta]$$

Also, the resource constraint becomes:

$$\bar{C}(D) = F(\bar{K}(D)) - \delta\bar{K}(D) - D$$



3 **The scientists forecast an earthquake T periods from now that will destroy $D > 0$ units of capital. Assuming that economy starts from a steady state with $D = 0$, draw a phase diagram that shows the optimal transition path. Make two separate graphs showing the evolution of capital and consumption in time.**



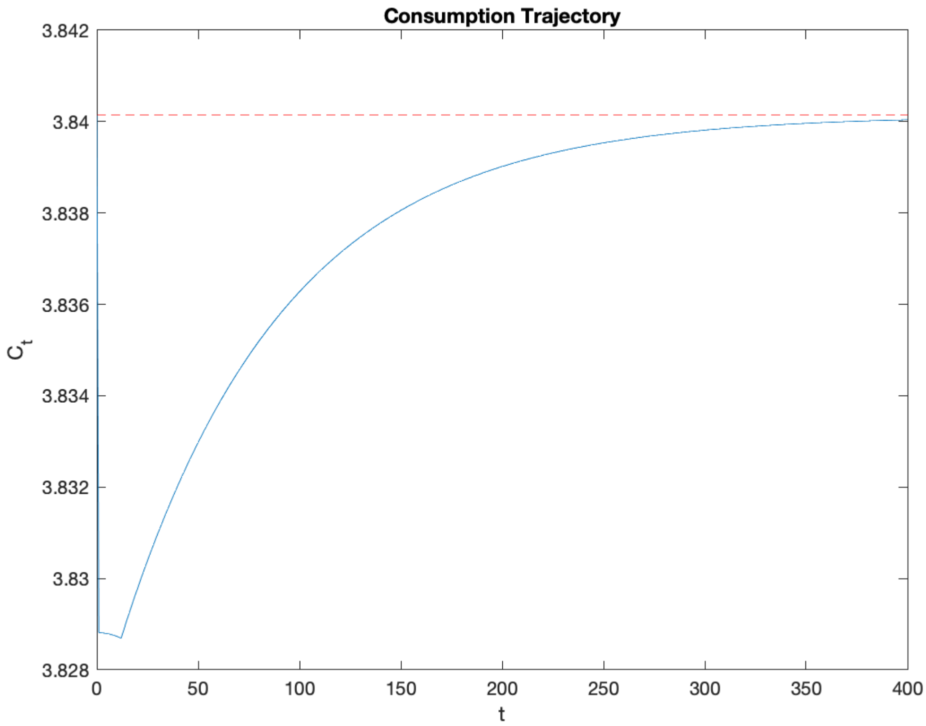
4 Assume that $U(C) = \frac{C^{1-\sigma}-1}{1-\sigma}$ and $F(K) = K^\alpha$ and the values of parameters are $\sigma = 1,\ \alpha = 1/3,\ \beta = 0.99^{1/12}$ (monthly model), $\delta = 0.01,\ T = 12,\ D = 1$. Using a shooting algorithm, solve numerically for the optimal transition path and plot dynamics of consumption and capital.

Updated Euler Equation:

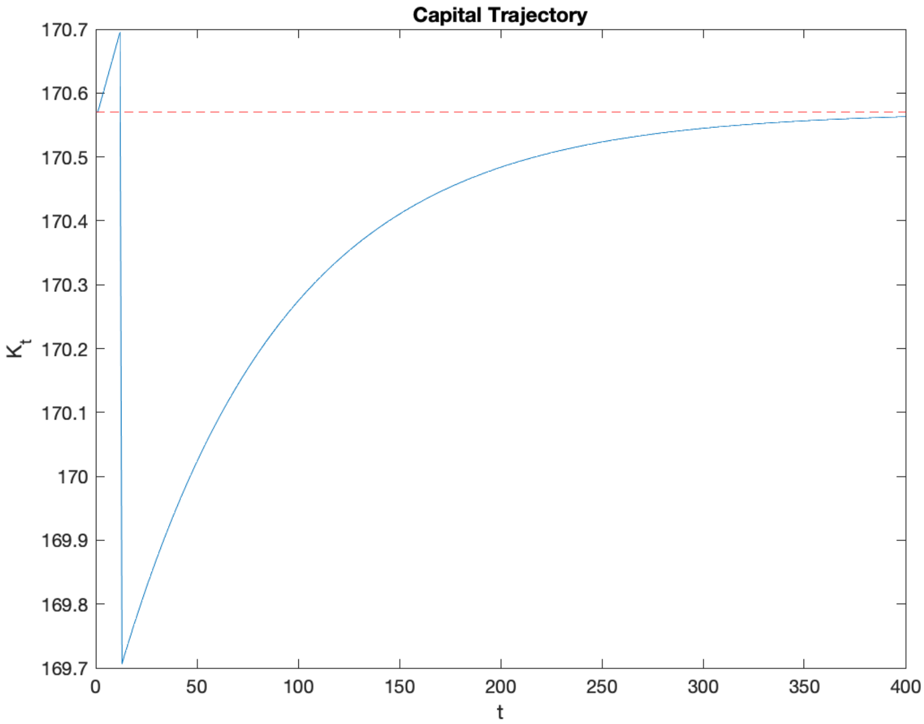
$$C_{t+1} = \beta^{1/\sigma} C_t [\alpha K_{t+1}^{\alpha-1} + 1 - \delta]^{1/\sigma}$$

Updated resource constraint:[1]

$$K_t^\alpha = C_t + K_{t+1} - (1-\delta)K_t + D_t$$



[1] *Note:* SS before shock: K = 170.57 and C = 3.84.


Capital Trajectory
170.7
170.6
170.5
170.4
170.3
170.2
170.1
170
169.9
169.8
169.7
K_t
0
50
100
150
200
250
300
350
400
t