Econ 714 Quarter 1: Problem set 1

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Consider a neoclassical growth model with preferences $\sum_{t=0}^{\infty} \beta^t U(C_t)$, production technology $F(K_t)$, and the initial capital endowment K_0. Both $U(\cdot)$ and $F(\cdot)$ are strictly increasing, strictly concave and satisfy standard Inada conditions. The capital law of motion is

$$K_t + 1 = (1 - \delta)K_t + I_t - D_t$$

where D_t is a natural disaster shock that destroys a fixed amount of the accumulated capital.

1 Write down the social planner's problem and derive the intertemporal optimality condition (the Euler equation).

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I worked on this Problem set with Sarah Bass, Michael Nattinger, Alex von Hafften, and Danny Edgel.

2 **Given the steady-state value of $D \geq 0$, write down the system of equations that determines the values of capital $\bar{K}(D)$ and consumption $\bar{C}(D)$ in the steady state. Draw a phase diagram with capital in the horizontal axis and consumption in the vertical axis, show the steady states, draw the arrows representing the direction of change, and the saddle path.**

3 **The scientists forecast an earthquake T periods from now that will destroy $D > 0$ units of capital. Assuming that economy starts from a steady state with $D = 0$, draw a phase diagram that shows the optimal transition path. Make two separate graphs showing the evolution of capital and consumption in time.**

4 **Assume that $U(C) = \frac{C^{1-\sigma}-1}{1-\sigma}$ and $F(K) = K^{\alpha}$ and the values of parameters are $\sigma = 1,\ \alpha = 1/3,\ \beta = 0.99^{1/12}$ (monthly model), $\delta = 0.01,\ T = 12,\ D = 1$. Using a shooting algorithm, solve numerically for the optimal transition path and plot dynamics of consumption and capital.**