Birla Institute of Technology & Science, Pilani
Work Integrated Learning Programmes Division
First / Second Semester 2024-2025

Comprehensive Examination
(EC-3 Make Up)

Course No. : AIMLCZC418
Course Title : Introduction to Statistical Methods
Nature of Exam : Open Book
Weightage : 40%
Duration : 150 Minutes
Date of Exam : 05-04-2025_FN

No. of Pages = 4
No. of Questions = 5

Note to Students:
1. Please follow all the *Instructions to Candidates* given on the cover page of the answer book.
2. All parts of a question should be answered consecutively. Each answer should start from a fresh page.
3. Assumptions made if any, should be stated clearly at the beginning of your answer.

1 a) A mould is mounted on a moulding machine by using two different techniques and the mounting time is a crucial period in production management. Hence the manager needs to determine whether there is any significant difference in the two moulding procedures. A sample is collected for both processes and is listed in the following table. State the hypothesis and test with 0.05 level of significance. [4 marks]

Process-1 (in Hrs)	Process-2 (in Hrs)
2	3
4	7
9	5
3	8
2	4
---	3

b) A company wants to study the relationship between **daily working hours (X) and employee productivity (Y) in terms of completed tasks.** The data is as follows: [4 Marks]

X	5	6	4	7	8	5.5
Y	15	18	12	20	23	16

Using this data, answer the following:
i) Compute the **Pearson correlation coefficient (r)**.
ii) Does increasing working hours significantly improve employee productivity?

2 a) For the following data: production of wheat in tons, calculate the 4-year centered moving average. Also find the mean square error (MSE) and mean absolute deviation (MAD). [4 Marks]

Year	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Production (tons)	76	83	89	96	103	117	126	137	141	150

2 b) A factory produces electronic components, and each component is independently defective with probability p. In a random sample of 10 components, 3 were found to be defective. [4 Marks]

(i) Find the maximum likelihood estimate (MLE) of p.
(ii) Using the estimated p, determine the probability that in the next batch of 5 components, exactly 2 will be defective.

3 a) Find the 98% confidence interval for the average time students spend on a deep learning project. A sample of 15 students was taken, and their recorded project completion times (in hours) were: 76, 85, 90, 78, 83, 88, 91, 75, 84, 79, 87, 92, 80, 86, 89

Assume the time spent follows a normal distribution, but the population standard deviation is unknown. [4 Marks]

3 b) A WILP faculty believes that the distribution of students across different extracurricular activities (Sports, Music, Art, and Drama) is equal. A random sample of 200 students is surveyed, and the observed number of students in each category is recorded as follows: [4 Marks]

Activity	Sports	Music	Art	Drama	Total
Observed (O)	55	45	60	40	200

Test the principal's claim at a 5% significance level using the Chi-square goodness-of-fit test.

4 a) Consider the following time series data representing the **monthly sales** (in thousands) for a company over 5 months: [4 Marks]

Month	**Sales (in '000s)**
1	50
2	55
3	53
4	60
5	58

Compute the **autocorrelation** at **lag 2** for the given time series data. Interpret your conclusion.

4 b) A professor wants to develop a model to predict the **weight of students (y)** based on their **height (x)**. The professor has collected the following data on the height and weight of 10 students: [4 Marks]

i) Fit a **linear regression line** of weight (y) on height (x).
ii) Write your **observations** and **comments** about the fit of the model. Suggest the best model that fits the data.

x	85	94	101	50	88	88	61	68	94	74
y	61	75	97	85	68	44	79	59	98	73

5) Scientists developed a vaccine. At an initial phase of trial, the vaccine is tested on people of three different regions and four different age groups. The effectiveness of vaccine is measured at a level from 1(non-effective) to 10 (satisfactorily effective) which is tabulated as below:

Age group / Region	Children	Teenagers	Young	Adult
Region 1	5	3	7	5
Region 2	6	5	5	8
Region 3	7	4	9	8

Test at 5% level of significance, if there is any significant difference in effectiveness of vaccine due to age and due to region. [8 Marks]

Solution:

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Chi-Square (χ^2) Distribution

Degrees of Freedom	Area to the Right of Critical Value									
	0.995	**0.99**	**0.975**	**0.95**	**0.90**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

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