BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI

Work Integrated Learning Programmes Division

Cluster Programme - M. Tech in DSE

I Semester , 2023 – 24(APRIL,2024)

Comprehensive Examination **(MAKEUP)**

Q.1.Consider the following joint probability density function. [**7 Marks]**

f (x + y) = c(x + y)/2 , 0 < x < 2,0 < y < 3

Then find

i) c value
ii) Marginal probability distributions of X,Y
iii) P(X < 1, Y< 2)
iv) Are X and Y are independent? Validate.

ISM MAKE UP ANSWER KEY

April - 2023

Q1. Consider the following joint probability density function

$$f(x,y) = \frac{c(x+y)}{2}, \quad 0<x<2, \quad 0<y<3$$

Then find

(i) c value
(ii) Marginal probability distributions of X, Y
(iii) P(X<1, Y<2)
(iv) Are X and Y are independent? Validate [7 M]

Sol: (i) To find c value:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$$

$$\int_0^3\int_0^2 c\frac{(x+y)}{2}\,dx\,dy = 1$$

$$\Rightarrow \frac{c}{2}\int_0^3 \left(\frac{x^2}{2}+xy\right)_0^2 dy = 1$$

$$\Rightarrow \frac{c}{2}\int_0^3 (2+2y)\,dy = 1 \Rightarrow \frac{c}{2}\left(2y+y^2\right)_0^3 = 1$$

$$\Rightarrow \frac{c}{2}(6+9) = 1 \Rightarrow \frac{15c}{2} = 1 \Rightarrow c = \frac{2}{15}$$ 2M

(ii) Marginal probability distribution of x

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_0^3 \frac{2}{15}\frac{(x+y)}{2}\,dy$$

$$= \frac{1}{15}\left(xy+\frac{y^2}{2}\right)_0^3 = \frac{1}{15}\left(3x+\frac{9}{2}\right) = \frac{1}{10}(2x+3)$$ 1M

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_0^2 \frac{2}{15}\frac{(x+y)}{2}\,dx$$

$$= \frac{1}{15}\left(\frac{x^2}{2} + xy\right)_0^2 = \frac{1}{15}(2+2y) = \frac{2}{15}(1+y)$$ 1M

(iii) $P(X<1, Y<2) = \int_{y=0}^{2}\int_{x=0}^{1} \frac{2}{15}\frac{(x+y)}{2}\,dx\,dy$

$$= \frac{1}{15}\int_{y=0}^{2}\int_{x=0}^{1}(x+y)\,dx\,dy = \frac{1}{15}\int_{y=0}^{2}\left(\frac{x^2}{2}+xy\right)_0^1 dy$$

$$= \frac{1}{15}\int_0^2\left(\frac{1}{2}+y\right)dy = \frac{1}{15}\left(\frac{y}{2}+\frac{y^2}{2}\right)_0^2$$

$$= \frac{1}{15}(1+2) = \frac{3}{15} = 0.2$$ 2M

(iv) Consider,

$$f_X(x)\cdot f_Y(y) = \left(\frac{1}{10}(2x+3)\right)\left(\frac{2}{15}(1+y)\right) \neq \frac{1}{15}(x+y)$$

$$\neq f(x,y)$$

$\therefore$ X and Y are NOT independent. 1M

Q.2. Validate the hypothesis that product A is superior to product B in terms of performance. A sample of 20 items of product A is having mean life of 12 months with standard deviation 15 days where as product B is having mean life of 10 months with standard deviation 10 days. Use p – value and validate the hypothesis. [**7 Marks]**

H0: product A is not significantly superior to product B in terms of performance

(1 mark)

H1: product A is significantly superior to product B in terms of performance

(one tailed)

(1mark)

Pooled sd = s = $\sqrt{\frac{(n1-1)S1^2+(n2-1)S2^\wedge 2}{n1+n2-2}}$ = $\sqrt{\frac{(20-1)15^2+(20-1)10^2}{20+20-2}}$ =12.74

(1.5 mark)

t = $\frac{\overline{x1}-\overline{x2}}{S\sqrt{(\frac{1}{n1}+\frac{1}{n2})}}$ = $\frac{12-10}{12.74\sqrt{(\frac{1}{20}+\frac{1}{20})}}$ = 0.496

(1.5 mark)

P = P(t38df > 0.496) = 0.3113

(1 mark)

Since P(=0.3113) > the level of significance (= 5% =0.05) then we fail to reject H0 and we conclude that product A is not significantly superior to product B in terms of performance (1 mark)

Q.3.a).If the true proportion of voters who support Proposition A is P =0.4, what is the probability that a sample of size 200 yields a sample proportion between 0.40 and 0.45?

[3 Marks]

Solution

If P = 0.4 and n = 200, what is $P(0.40 \le \hat{p} \le 0.45)$?

Consider $\sigma_{\hat{p}} = \sqrt{\frac{P(1-P)}{n}} = \sqrt{\frac{.4(1-.4)}{200}} = .03464$ (1Mark)

$P(.40 \le \hat{p} \le .45) \quad = P(\frac{.40-.40}{.03464} \le Z \le \frac{.45-.40}{.03464})$ (1Mark)

$= P(0 \le Z \le 1.44) = 0.4251$ (1Mark)

b). A sample of 11 circuits from a population has a mean resistance of 2.00 ohms. We know from past testing that the population standard deviation is 0.35 ohms. Determine a 99% confidence interval for the true mean resistance of the population. **[3 Marks]**

$$\bar{x} \pm z\frac{\sigma}{\sqrt{n}}$$ (1Mark)

$$= 2 \pm 2.58\left(\frac{0.35}{\sqrt{11}}\right)$$ (1Mark)

$$= 2 \pm .2723$$

$$1.7277 < \mu < 2.2723$$ (1Mark)

Q.4. Find the exponential smoothing for α=0.4 and α=0.6 for the following data and also find out which weighting factor gives better smoothing. **[7 Marks]**

Month/ Year	1.200 1	2.200 1	3.200 1	4.200 1	5.200 1	6.200 1	7.200 1	8.200 1	9.200 1	10.200 1	11.200 1	12.200 1
Deflecti on	-213	-564	-35	-15	141	115	-420	-360	203	-338	-431	194

Month/ Year	Deflection	$F_{t+1} = 0.4Y_t + 0.6F_t$	$F_{t+1} = 0.6Y_t + 0.4F_t$	$\Delta_t = (Y_t - F_t)^2$	$\Delta_t = (Y_t - F_t)^2$
	α	0.4	0.6	0.4	0.6
1.2001	-213	-213	-213		
2.2001	-564	-213	-213	123201	123201
3.2001	-35	-353.40	-423.60	101378.56	151009.96
4.2001	-15	-226.04	-190.44	44537.88	30779.19
5.2001	141	-141.62	-85.18	79876.33	51155.58
6.2001	115	-28.57	50.53	20613.61	4156.43
7.2001	-420	28.86	89.21	201471.13	259296.70
8.2001	-360	-150.69	-216.32	43812.02	20645.30
9.2001	203	-234.41	-302.53	191329.32	255556.64
10.2001	-338	-59.45	0.79	77591.64	114778.36
11.2001	-431	-170.87	-202.48	67668.48	52219.48
12.2001	194	-274.92	-339.59	219886.91	284722.21
			Sum	**1171366.88**	**1347520.87**
			MSE	**106487.90**	**122501.90**

The MSE at α = 0.4 is lower than at α = 0.6 hence the forecasting is better at α = 0.4.

Q.5.The following data gives the frequency of Cargo exports in years Y_t in a state of a country.

Find the autocorrelation at the lag 1. **[6 Marks]**

Year (t)	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Cargo exports (Y_t)	1230	1345	1382	1416	1593	1802	1817	1995	2212	2607

Key answer:

Year (t)	Cargo exports(Y_t)	Lag 1 (Y_{t-1})	y_t=(Y_t-mean)	y_{t-1}=(Y_{t-1}-mean)	y_t*y_{t-1}
2011	1230				
2012	1345	1230	-394.9	-509.9	201359.51
2013	1382	1345	-357.9	-394.9	141334.71
2014	1416	1382	-323.9	-357.9	115923.81
2015	1593	1416	-146.9	-323.9	47580.91
2016	1802	1593	62.1	-146.9	-9122.49
2017	1817	1802	77.1	62.1	4787.91
2018	1995	1817	255.1	77.1	19668.21
2019	2212	1995	472.1	255.1	120432.71
2020	2607	2212	867.1	472.1	409357.91
Mean	1739.9			Sum(y_t*y_{t-1})=	1051323.19
Variance	1720145				
Autocovariance	116813.6878				
Autocorrelation	**0.61118**				

Q.6.a).For the following data on sales, fit a linear trend by the method of least squares. Forecast the sales for the year 2025. **[7 Marks]**

Year	1995	2000	2005	2010	2015	2020
Sales ('000)	16	20	18	15	18	21

Date | Page No.

Let $X = x - 2010$ ——— (1)

$Y = y - 18$ ——— (2)

Now let the straight line to be fitted in terms of variables X & Y be given by

$Y = a + bX$ ——— (3)

Then the normal equations are

$\Sigma Y = na + b\Sigma X$ ——— (4)

& $\Sigma XY = a\Sigma X + b\Sigma X^2$ ——— (5)

Here, $n = 6$

x	X	y	Y	XY	X^2
1995	−15	16	−2	30	225
2000	−10	20	2	−20	100
2005	−5	18	0	0	25
2010	0	15	−3	0	0
2015	5	18	0	0	25
2020	10	21	3	30	100
	−15		0	40	475

Substituting these values into (4) & (5)

$0 = 6a + b(-15)$

$6a = 15b$

$2a = 5b$ ——— (6)

$40 = a(-15) + b(475)$

$8 = -3a + 95b$

$8 = -3a + 19(5b)$

$8 = -3a + 19(2a)$ From eqn. (6)

$8 = -3a + 38a$

$8 = 35a$

$a = \frac{8}{35}$

f $b = \frac{2a}{5}$ From eqn. 6

$= \frac{2}{5} \times \frac{8}{35}$

$= \frac{16}{175}$

From eqn. 3,

$$Y = \frac{8}{35} + \frac{16}{175} X$$

$$y - 18 = \frac{8}{35} + \frac{16}{175}(x - 2010)$$

$$y = \frac{8}{35} + 18 + \frac{16}{175} x - \frac{16}{175}(2010)$$

$$y = \frac{638}{35} + \frac{16x}{175} - \frac{\boxed{\cancel{402}}}{35}$$ 16x402

$$\boxed{\cancel{y = \frac{236}{35} + \frac{16x}{175}}}$$

Ans

y=-165.54+0.0914x