Assignment 05: Chaos and Curve Fitting

Due Date: 10/28/2016

1. The Dissipative Pendulum: G&N 3.6 [10 points]

Investigate the three regimes of the linear pendulum with dissipation using the Euler-Cromer (or any other suitable) method. In all three regimes, the dissipation due to friction leads to an exponential damping of the overall energy. Locate the boundary between the overdamped and underdamped regimes numerically, and compare with the analytic result (you can either derive the latter yourself, or find it in a mechanics textbook such as Marion and Thornton or Taylor).

2. Chaos and the Lyapunov Exponent: G&N 3.13 [10 points]

Write a program to calculate and compare the behavior of two, nearly identical driven non-linear pendulums. Use it to calculate the divergence of two nearby trajectories in the chaotic regime, as in Figure 3.7 of the textbook, and determine the corresponding Lyapunov exponent from the slope of a plot of $\log(|\Delta\theta|)$ as a function of t.

BONUS

Adjust your program to investigate the Lyapunov exponent as a function of the driving force F_{D}.

3. The Logisitic Map [10 points]

Recall our discussion in class of one of the simplest models that displays chaotic behavior, the logistic map, defined by the iterative equation:

$$x_{n+1} = \mu x_n(1 - x_n)$$

where the approach to chaos is controlled by tuning the parameter $0 \leq \mu < 4$.

Generate a *bifurcation diagram* for the logistic map (see Figure 3.11 in N&G). This can be accomplished by iterating the map from some random initial state x_0 until you are sure all transients have died away. Then, for a series of evenly spaced values of $1 \leq \mu < 4$, plot μ vs. x_n. Use the plot annotation methods of `pyplot` to indicate the values of μ on your figure where period doubling occurs, as well as the onset of chaos.

4. Radioactive Decay: Revisited [10 points]

Download the file `decay.dat` from BlackBoard which includes data obtained from a radioactive decay experiment. In the file, the decay rate and associated uncertainty have been measured in becquerel. Using the weighted least square fitting method, determine the half life, and its associated uncertainty by analyzing the goodness of fit parameter χ^2. Identify the radioactive nucleus and display all your results on a detailed plot showing the original data with errorbars as well as your fit.

5. Final Project **[5 points]**

Research final project topics that are of interest to you and write a short description (1-2 paragraphs) presenting the idea, the scope of the project and the numerical techniques you intend to employ. Make sure to include the main objective as well as the scientific question(s) your project will investigate. Include a minimum of two references that you have consulted or that you think will be helpful.

To get you started, some previous students in the class have studied:

- Molecular dynamics simulations of elastic constants in solids.
- Building realistic computer models of musical instruments.
- The dynamics of a wiffle ball.
- Modeling Yo-Yo tricks in python.
- Simulation of rogue waves.
- Solitons in the Korteweg-de Vries equation.
- Percolation threshold for 2d lattices.
- Variational energy calculation in the hydrogen atom.
- Chaos in a dripping faucet.
- Simulated annealing for the travelling salesman problem.
- Hartree Fock calculation of the hydrogen atom.
- Exact diagonlization study of the $S = 1/2$ Heisenberg model.
- Fractal dimension assessment for oxide thin films produced by pulsed laser deposition.
- A multi-threaded Monte Carlo method for the 2d Ising model.
- Electrical current models of cardiac cells.
- Similarity solutions for the boundary layer equations.
- The game of life (cellular automata).
- Building a star in python.
- N-body gravitational system.
- Dynamics of figure skating.
- The age of the universe.
- Synthesizers and Fourier series.
- Diffusion limited aggregation.
- 3-body satellite trajectories.
- Relativistic celestial mechanics.
- Rainbows and diffraction.
- Stadium billiards and chaos.
- Radiation around an oscillating dipole.