Maximize Your Drive

1. [15 points] A golf ball has a mass of $m = 45.93\,\text{g}$, a diameter of $D = 42.67\,\text{mm}$ and a professional golfer can impart an initial velocity of $v_0 = 72\,\text{m/s}$ at a launch angle of $\theta_0 = 9^\circ$ with a perfectly swung one-wood. Due to its uniform dimpled exterior, the drag coefficient of a golf ball can be parametrized as follows:

$$C = \begin{cases} 0.5 & ; \quad v < 14\,\text{m/s} \\ \frac{7.0}{v} & ; \quad v \geq 14\,\text{m/s} \end{cases} \tag{1}$$

with the total drag force given as $\vec{F}_{\text{drag}} = -C\rho A v^2 \hat{v}$. Assuming an adiabatic approximation for the atmosphere, the density of air at a distance y above our perfectly flat driving range is

$$\rho(y) = \rho_0 \left(1 - \frac{ay}{T_0}\right)^{5/2} \tag{2}$$

where $\rho_0 = 1.225\,\text{kg/m}^3$ is the density of air at sea level at $T_0 = 288\,\text{K}$ and $a = 6.5 \times 10^{-3}\,\text{K/m}$.

As the slanted face of the driver makes contact, its rough surface imparts some amount of backspin on the ball which controls the Magnus force $\vec{F}_{\text{magnus}} = S_0 \vec{\omega} \times \vec{v}$ where $S_0 = 3.828 \times 10^{-5}\,\text{kg}$. In a right handed coordinate system where $\hat{y}$ points upwards and $\hat{x}$ outward along the driving range, this backspin is in the $\hat{z}$ direction and it can be varied from $\omega = 0$ to $\omega = 500\,\text{rad/s}$ by varying the material and design of the club face. We make the physically reasonable simplifying assumption that the angular velocity is constant during the trajectory of the golf ball.

If our golfer imparts no motion in the z-direction to the ball when it is hit off a tee at the point $(x, y) = (0, 2.5\,\text{cm})$, find the optimal value of the backspin velocity ω (to an accuracy of $0.5\,\text{rad/s}$) which maximize the range and produce a fully labeled plot of the ideal trajectory including the final range.

Instructions:
A code scaffold has been provided for you on BlackBoard named `max_drive_scaffold.py`. Rename this file `max_drive.py`, complete the required code and upload it to the appropriate place on BlackBoard in a compressed file which expands to `Lastname_Initial/Midterm/max_drive.py`. For full marks, your code needs to run without errors and all plots must be properly labeled.