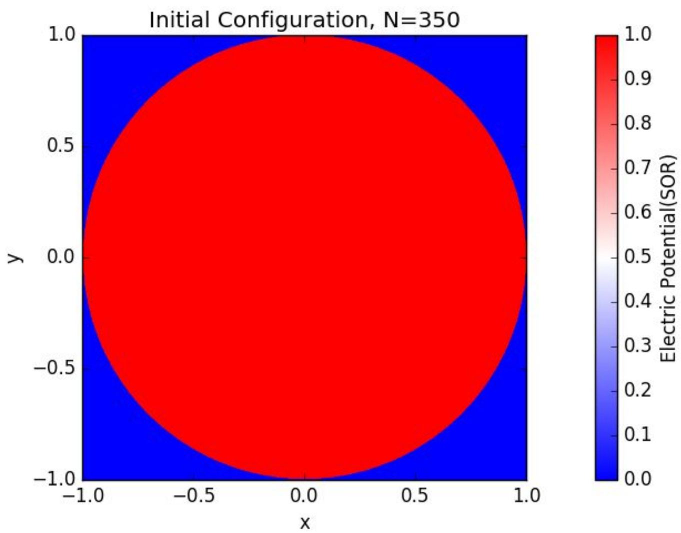
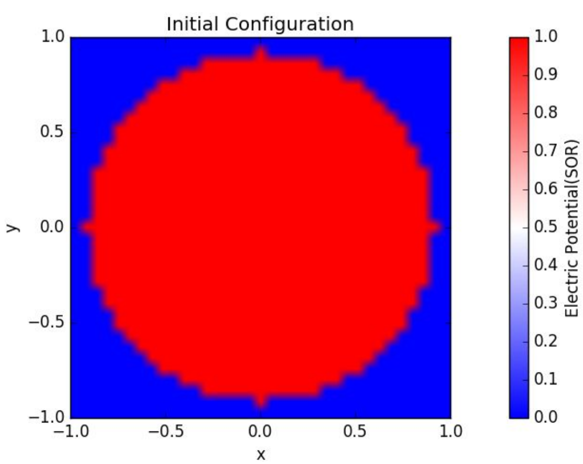
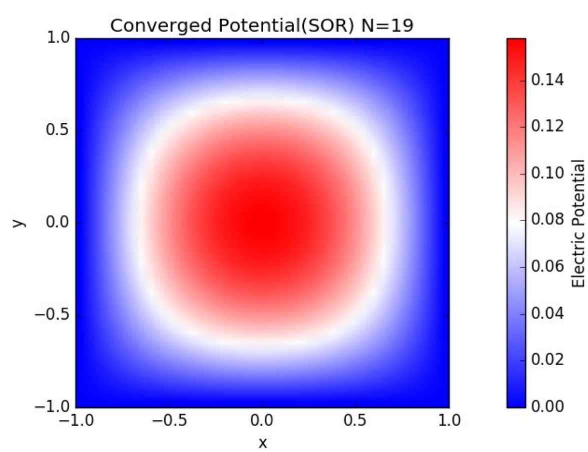
Emanuel Casiano-Díaz
PHYS-256 , Computational Physics
Prof. Adrian Del Maestro
Assignment 7 BriefReport

Problem #1:



The obtained plot represents a circle held at potential V = 1 inside a grounded plane. The cubic grid here has side lengths of N = 350 cells at it comes out to be nice and smooth. Nevertheless, when the Simultaneous Over-Relaxation Method was performed, a lower N, and thus more jagged disk, had to be used to reduce computing time and memory requirements.

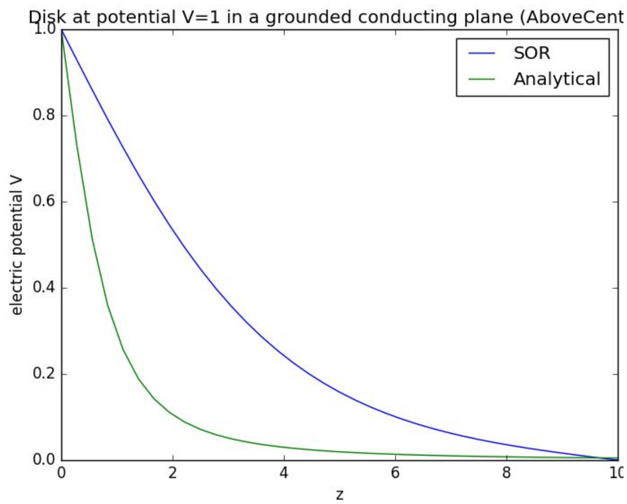


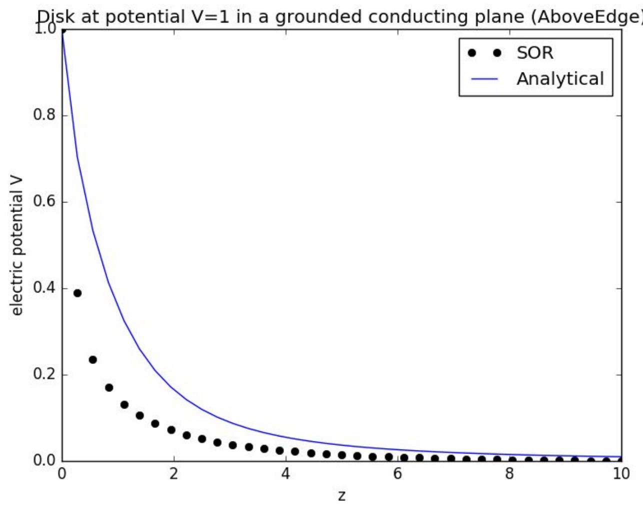


These results correspond to the same disk but with N = 19. In the converged potential image, I used the result at the layer that was half-way up of the cubical grid.

Comparison of SOR vs Analytical Results:

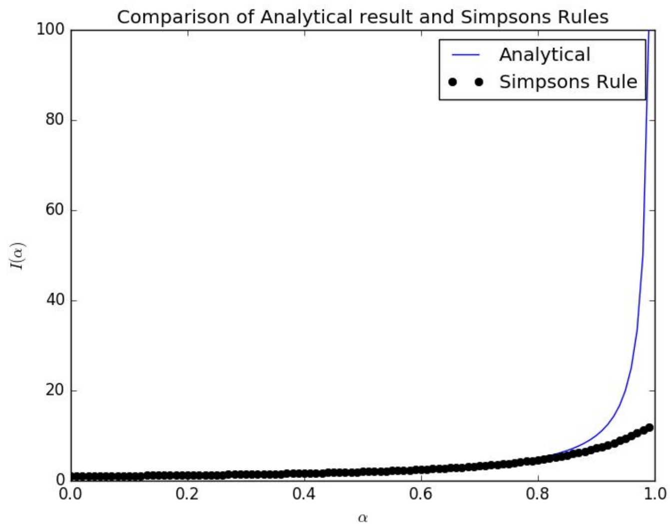
The two plots below compare the results for SOR and the exact solutions as a function of the height z above the plane. The first plot corresponds to values above the center and the second plot to values above the circumference.



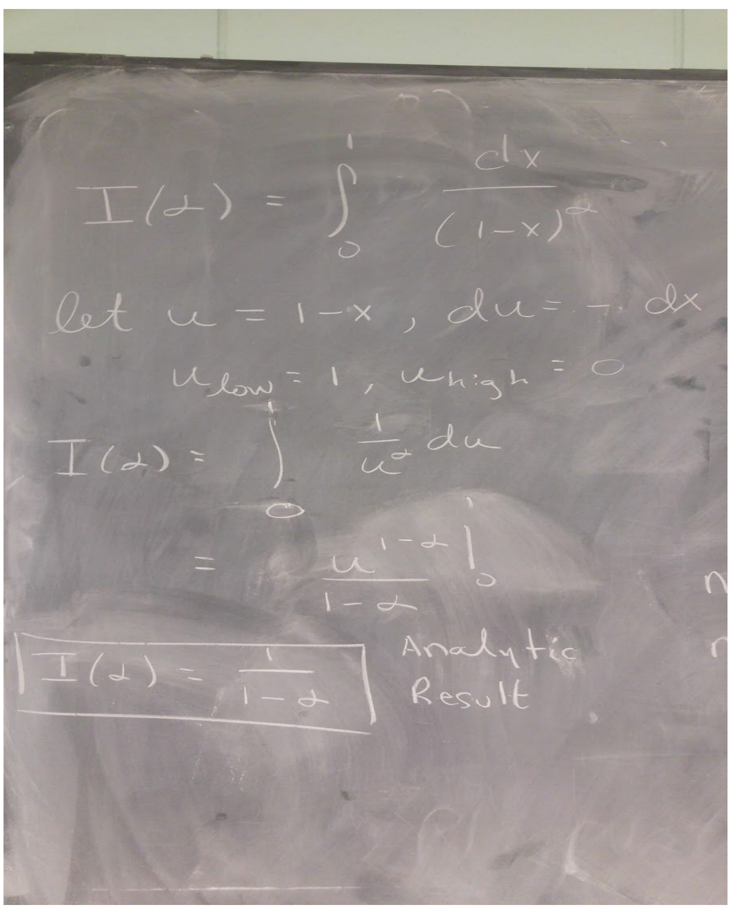


Note that there is discrepancy in the values. The tolerance was tuned all the way down to $\varepsilon = 1.0x10^{-12}$ but the values still differed. Maybe we can fix this problem by using a larger grid size, as we saw that the grid with N = 19 produced a rather jagged circle. Nevertheless, in both plots, SOR and exact results follow the same trend.

Problem #2:

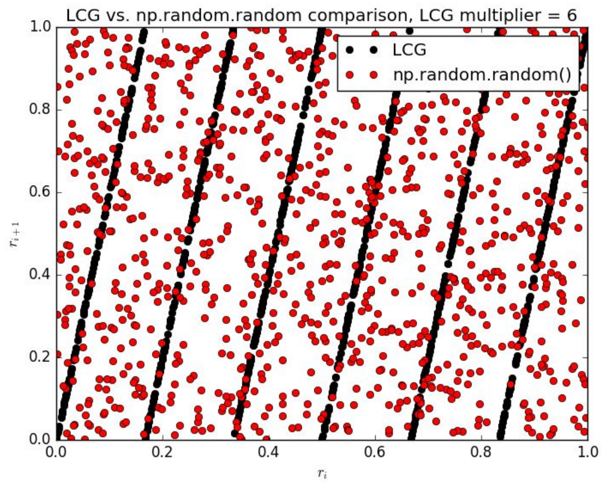


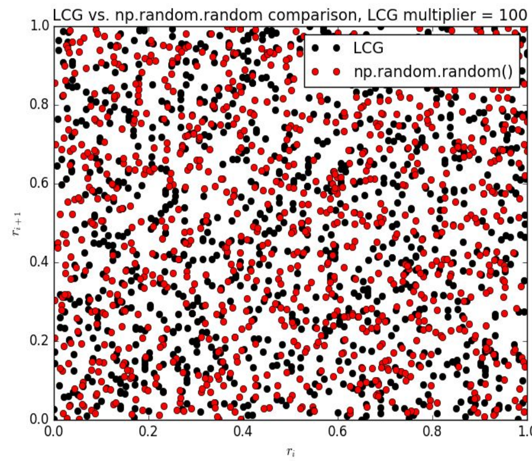
The given integral I , was calculated using Simpson's Rule for comparison with the anlaytical result, evaluated as a function of the parameter α. We see agreement in the results until around $\alpha \sim 0.9$. The exact solution to this integral was obtained easily with u-substitution:



Problem #3:

In this problem, random numbers were generated using the Linear Congruential Generator algorithm and using numpy’s np.random.random(). To test the ‘randomness’ of the results, the following plots were generated:

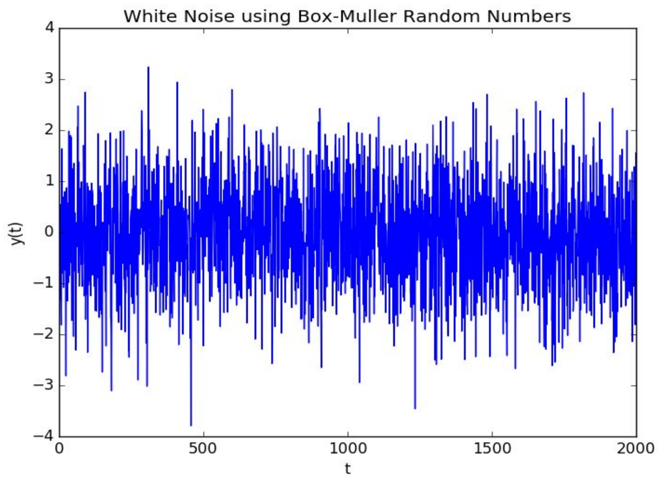




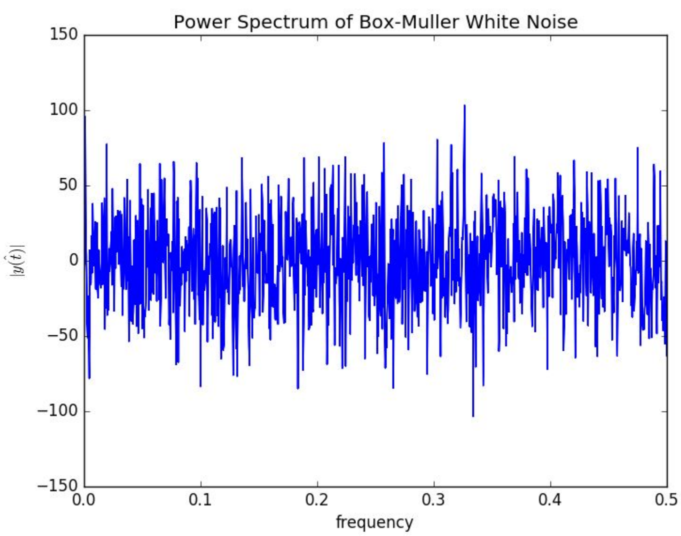
As we can see, there is some pattern in the LCG random numbers in the first plot, which uses an LCG multiplier of 6. By increasing the multiplier, we get more randomness in the results, as seen from the second plot, which uses a multiplier of 100.

Problem #4:

White noise was generated using the Box-Muller method:



And a power spectrum plot of the white noise was computed using a Fast Fourier Transform:



The power spectrum plot is almost as ‘messy’ as original plot. This comes as no surprise due to how random, instead of periodic the white noise is.

EXTRA CREDIT:

The following is the derivation of the potential in part 1(a) using Green's functions and Cylindrical Coordinates.:

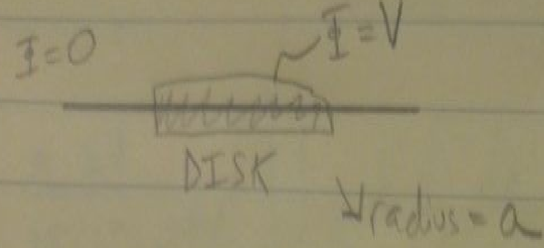
$$G_D(\vec{x},\vec{x}') = \frac{+1}{|\vec{x}-\vec{x}'|} + \frac{-1}{|\vec{x}'-\vec{x}''|}$$

where $\vec{x}' = (x', y', z')$ and $\vec{x}'' = (x', y', -z')$

$$\boxed{G_D(\vec{x},\vec{x}') = \frac{1}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}} + \frac{-1}{\sqrt{(x-x')^2+(y-y')^2+(z+z')^2}}}$$

Let $x' = s'\cos\phi'$ and $y' = s'\cos\phi'$ (Cylindrical Coords)

$$\Rightarrow \boxed{G_D(\vec{x},\vec{x}') = \frac{1}{\sqrt{s^2+s'^2-2ss'\cos\phi'+(z-z')^2}} + \frac{-1}{\sqrt{s^2+s'^2-2ss'\cos\phi+(z+z')^2}}}$$



$$\left.\frac{\partial G}{\partial z'}\right|_{z'=0} = \frac{-2z}{(s^2+s'^2-2ss'\cos\phi'+z^2)^{3/2}}$$

The potential in terms of Green's Function is given by:

$$\rho(\vec{x}')=0 \qquad 0$$

$$\Phi(\vec{x}) = \cancel{\int_V \rho(\vec{x}')\, G_D(\vec{x},\vec{x}')\, d^3x'} - \oint_S \Phi(\vec{x}')\frac{\partial G_D}{\partial n'}\, da'$$

$$\Phi(\vec{x}) = \frac{V}{\cancel{4}^{2}\pi}\int_0^a\int_0^{2\pi}\frac{\cancel{2}z\, s'ds'd\phi'}{(s^2+s'^2-2ss'\cos\phi'+z^2)^{3/2}}$$

$$\boxed{\Phi(s,\phi,z) = \frac{zV}{2\pi}\int_0^a\int_0^{2\pi}\frac{s'ds'd\phi'}{(s^2+s'^2-2ss'\cos\phi'+z^2)^{3/2}}}$$

$$\text{For } s=0,\ \Phi(\vec{x}) = \frac{zV}{\cancel{2\pi}} \int_0^a \frac{s'\,ds'}{(s'^2+z^2)^{3/2}} \cancel{\int_0^{2\pi} d\phi}$$

$$\text{Let } u = s'^2 + z^2 \Rightarrow du = 2s'\,ds' \Rightarrow s'\,ds' = \frac{du}{2}$$

$$\Phi(\vec{x}) = \frac{zV}{2} \int_{z^2}^{a^2+z^2} u^{-3/2}\,du = -zV\, u^{-1/2} \Big|_{z^2}^{a^2+z^2}$$

$$= -zV \left[\frac{1}{\sqrt{a^2+z^2}} - \frac{1}{z} \right]$$

$$\Rightarrow \boxed{\Phi(z) = V\left[1 - \frac{z}{\sqrt{a^2+z^2}}\right]}$$