Define an abstract notation for the sum over all hidden configurations,

$$\sum_{\{\boldsymbol{h}\}} f[\mathbf{h}] = \left(\sum_{h_{1,1}=0}^{1} \sum_{h_{1,2}=0}^{1} \cdots \sum_{h_{j,\alpha}=0}^{1} \cdots \right) f[\mathbf{h}] = \left(\prod_{\alpha} \prod_{j} \sum_{h_{j,\alpha}=0}^{1} \right) f[\mathbf{h}].$$

More abstractly,

$$\sum_{\{\boldsymbol{h}\}} = \prod_{\alpha} \prod_{j} \sum_{h_{j,\alpha}=0}^{1} .$$

Our aim is to evaluate

$$A = \sum_{\{\boldsymbol{h}\}} e^{\sum_j \sum_\alpha W_{i-j,\alpha} h_{j,\alpha}}$$

Move sums inside exponential to products outside exponential,

$$A = \sum_{\{\boldsymbol{h}\}} \prod_{\alpha} \prod_{j} e^{W_{i-j,\alpha} h_{j,\alpha}}.$$

To simplify notation, introduce $z_{j,\alpha} = W_{i-j,\alpha} h_{j,\alpha}$, which has implicit dependence on i. Then

$$A = \sum_{\{\boldsymbol{h}\}} \prod_{\alpha} \prod_{j} e^{z_{j,\alpha}}.$$

Note the general identity,

$$\left(\prod_{x} f_x \right) \left(\prod_{x} g_x \right) = (f_1 \dots f_N)\,(g_1 \dots g_N) = \prod_{x} f_x g_x.$$

We use this to write

$$\begin{aligned} A &= \left[\prod_{\alpha} \prod_{j} \sum_{h_{j,\alpha}=0}^{1} \right] \left[\prod_{\alpha} \prod_{j} e^{z_{j,\alpha}} \right] \\ &= \prod_{\alpha} \prod_{j} \sum_{h_{j,\alpha}=0}^{1} e^{z_{j,\alpha}}. \end{aligned}$$

Using the definition of $z_{j,\alpha}$, we can explicitly evaluate the inner sum,

$$A = \prod_{\alpha} \prod_{j} \left(1 + e^{W_{i-j,\alpha}}\right).$$