Autocorrelation times and dynamical critical exponents in CRBM's

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Motivated by Fig 3(b) of [1], we were interested in computing the integrated autocorrelation time of Ising model observables as a function of linear size near criticality. From this, one could then compare the dynamical critical exponent z obtained from observables sampled via CRBM/Gibbs with observables sampled with Metropolis-Hastings. Moreover, how does the autocorrelation time scale away from the critical temperature?

Dynamical critical exponent

The dynamical critical exponent in the nearest-neighbor 2D Ising model is known to be $z \approx 2.1$. Fig 3(b) of [1] seems to suggest that z differs considerably when sampling with the CRBM vs sampling with Metropolis-Hastings.

Fig. 1 shows the autocorrelation time of the magnetization near the critical temperature ($T_c \approx 2.7$). For the CRBM, the dynamical critical exponent was estimated to be $z \approx 2.10 \pm 0.03$, whereas for Metropolis, it was $z \approx 2.14 \pm 0.03$. So both estimates are the same, within error bars.

Even though the dynamical critical exponent for both methods is the same, it seems surprising that the autocorrelation time for the CRBM is larger than for Metropolis. In the [1], results are shown that indicate that as systems get larger, the CRBM should yield much less correlation between samples.

This might be due to the way that samples were measured in their case vs how I collected them. In my case, I measured Metropolis samples every single time that at least L^2 spin flips were proposed. In their case, they collect samples every time that k spin flips have been proposed, where k is the number of spin flips that can be performed for each single Gibbs step of the CRBM. This number will vary depending on the system size.

Autocorrelation times away from T_c

To try and replicate figure 3(a) from [1], and a similar other from Alcalde's thesis, I now compute the autocorrelation time away from T_c and sampling Metropolis observables only every other k steps. A table of k values for various systems sizes can be seen in an appendix of [1].

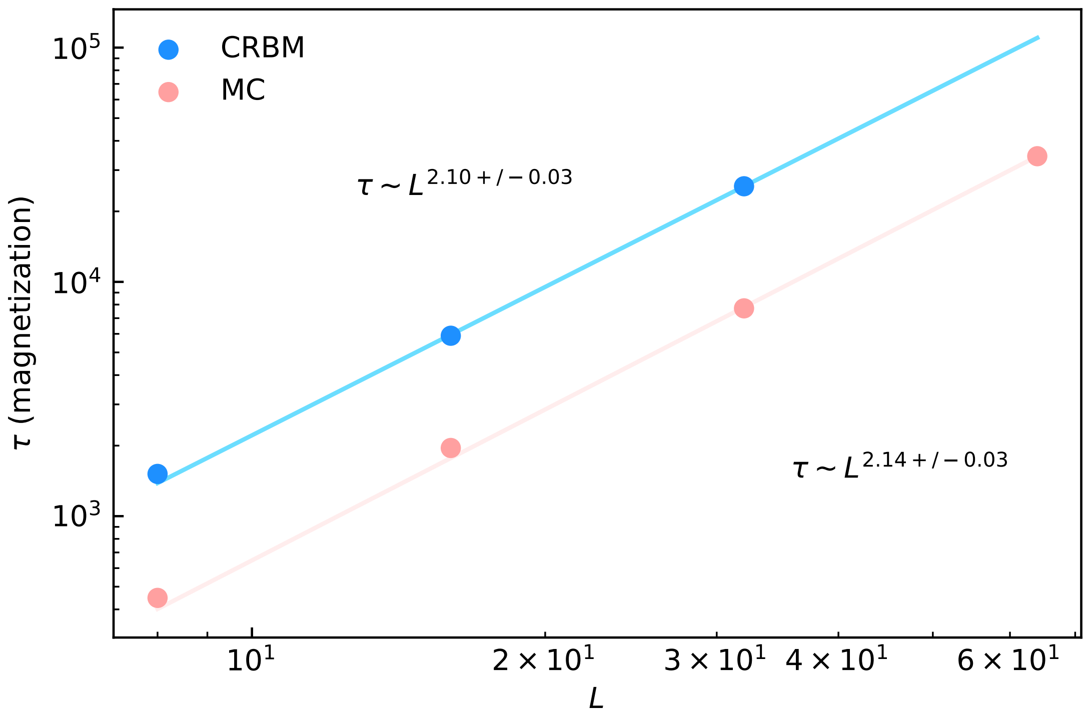

Figure 1: Autocorrelation time of magnetization at $T_c \approx 2.27$ as a function of linear size. Dynamical critical exponents are shown for CRBM and Metropolis.

Fig. 2 and Fig. 3 show the autocorrelation time of the energy as functions of L, for temperatures $T = 2.2$ and $T = 2.4$, respectively. Notice that using their sampling, we see that for $L > 100$, the CRBM outperforms Metropolis.

Should one then compare the critical exponents using this sampling?

References

[1] Daniel Alcalde Puente and Ilya M. Eremin. Convolutional restricted boltzmann machine aided monte carlo: An application to ising and kitaev models. *Physical Review B*, 102(19), nov 2020.

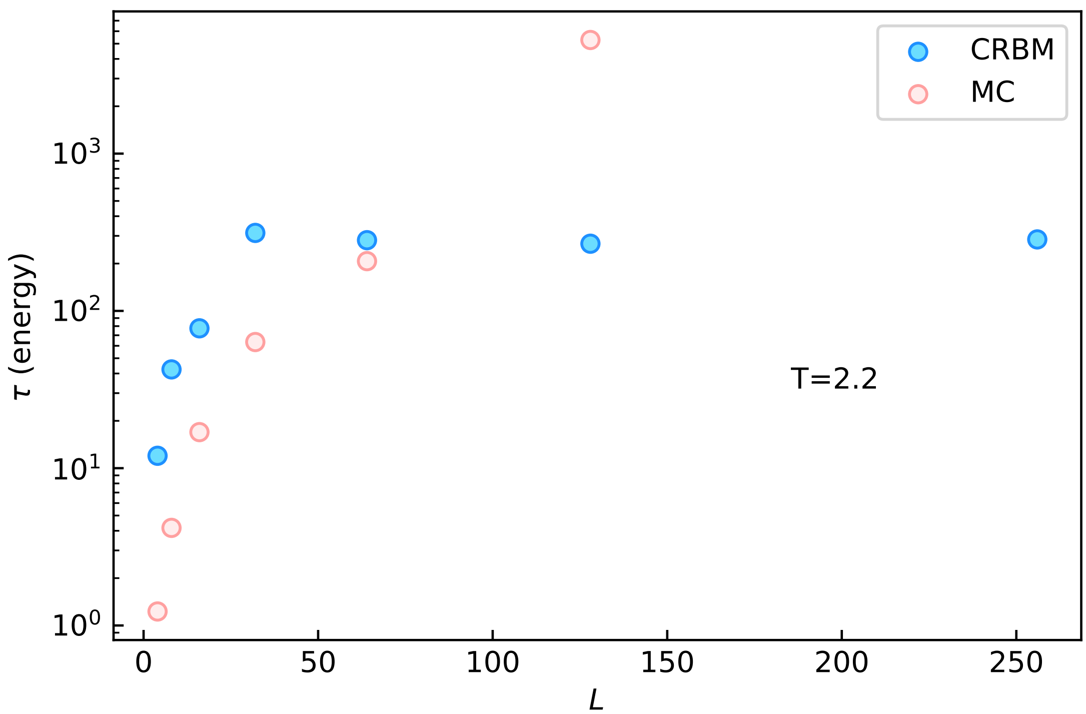

Figure 2: Autocorrelation time of the energy at $T = 2.2$ as a function of L.

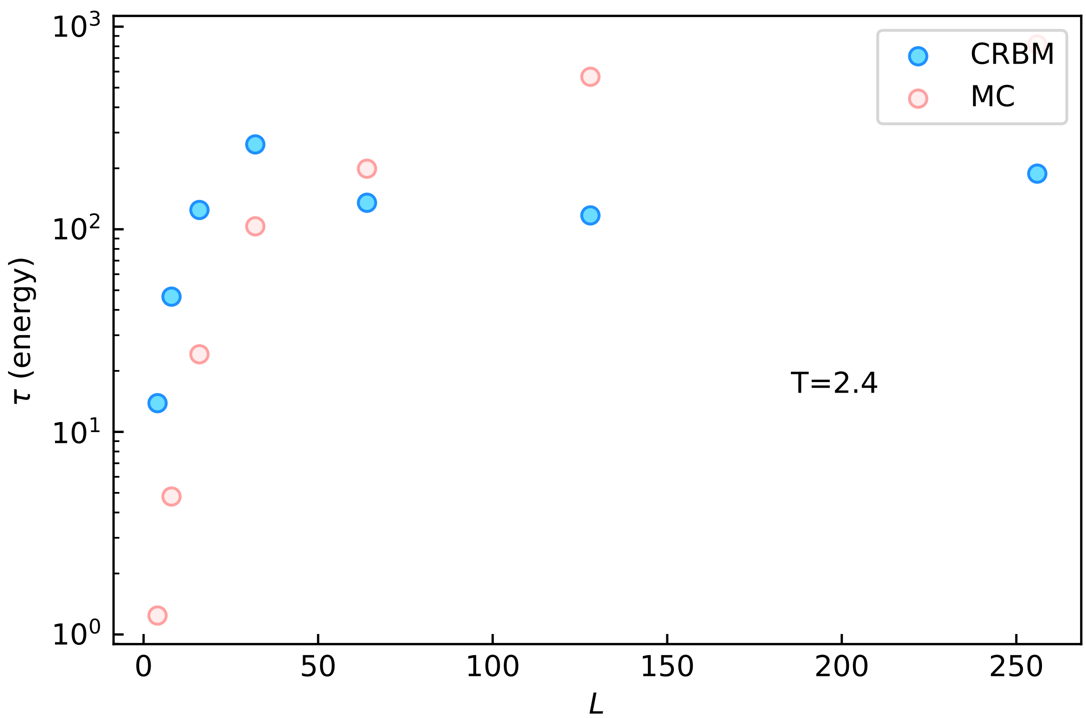

Figure 3: Autocorrelation time of the energy at $T = 2.4$ as a function of L.