1 Exercise 1

1.1 a)

A forest of size ℓ occurs when there the following patter occurs:

$$\text{EMPTY} - \underbrace{\text{TREE-TREE} \ldots \text{-TREE}}_{\ell-times} - \text{EMPTY} \quad (1)$$

The probability of a tree on site and of an empty site are:

$$p \text{ and } (1-p), \quad (2)$$

respectively.

Thus, the probability of landing in an $\ell-$sized cluster of trees (or forest) is:

$$n_\ell(p) = (1-p) * \underbrace{p * p * p * \ldots p}_{\ell-times} * (1-p) \quad (3)$$

Or

$$n_\ell(p) = (1-p)^2 p^\ell \quad (4)$$

1.2 b)

Recall that: $0 \leq p \leq 1$.

Percolation in a $1D$ lattice occurs when all sites have trees. The probability of percolation is thus p^L, where L is the lattice size. In the limit of $L \to \infty$, then it is seen that:

$$\lim_{L\to\infty} p^L = \begin{cases} 0, 0 \leq p < 1 \\ 1, p = 1 \end{cases} \quad (5)$$

Thus, for an infinitely large lattice in $1D$, percolation only occurs if $p = 1$.

$$\therefore p_c = 1 \quad (6)$$

2 Exercise 2

Via Real Space Renormalization (RSR), each triangular sublattice is replaced by a single site, or supersite. If the majority of the sites in the sublattice is open (closed), then the supersite is open (closed). The probability that a site is closed is given by p and $(1-p)$, if open. Thus, we need the count how many configurations of triangular sublattices have a majority of closed sites (i.e, 2 or 3 closed) and determine the probability that we will get one of these majority closed sites. Let the majority closed probability be defined as p'. Then, applying the discussed RSR rules to the infinitely large triangular lattice, the majority closed probability is:

$$p' = p^3 + 3p^2(1-p) \tag{7}$$

Now, setting $p' = p$, the critical probability can be determined:

$$\begin{align} p' &= p \tag{8} \\ \implies p' - p &= 0 \tag{9} \\ 2p^3 - 3p^2 + p &= 0 \tag{10} \\ p(p - \frac{1}{2})(p-1) &= 0 \tag{11} \\ & \tag{12} \end{align}$$

Thus: $p = 0, 1, \frac{1}{2}$. The first two options are just the trivial solutions discussed in the hint video. Thus, the critical probability for the infinitely large triangular lattice is:

$$p_c = \frac{1}{2} \tag{13}$$

3 Exercise 3

4 Exercise 4

5 Exercise 5