2) $\frac{d\Psi}{dp_i} = \frac{dF}{dp_i} + \lambda \frac{dG}{dp_i} = 0$ <u>p_i - equation</u>

$$= \frac{d}{dp_i}\left[\frac{\sum_{i=1}^{n} p_i \ln(i+a)}{-g \sum_{i=1}^{n} p_i \ln p_i} \right] + \lambda \frac{d}{dp_i}\left[-1 + \sum_{i=1}^{n} p_i \right] \quad ; \sum_{i=1}^{n} \equiv \sum_i$$

$$= \quad '' \quad + \lambda \sum \frac{dp_i}{dp_i}$$

$$= \quad '' \quad + \lambda \sum_i$$

$$= \quad '' \quad + \quad ''$$

$$= -\frac{1}{g} \frac{d}{dp_i}\left[\left(\sum_i p_i \ln(i+a)\right)\left(\sum_i p_i \ln p_i\right)^{-1} \right] + \,''$$

$$= -\frac{1}{g}\left[\left(\frac{d}{dp_i}\left(\sum_i p_i \ln(i+a)\right)\right)\left(\sum_i p_i \ln p_i\right)^{-1} + \left(\sum_i p_i \ln(i+a)\right)\frac{d}{dp_i}\left(\sum_i p_i \ln p_i\right)^{-1} \right] + \,''$$

$$= -\frac{1}{g}\left[\left(\sum_i \ln(i+a)\right)\left(\sum_i p_i \ln p_i\right)^{-1} + \left(\sum_i p_i \ln(i+a)\right)(-1)\left(\sum_i p_i \ln p_i\right)^{-2}\left(\sum_i (\ln p_i + 1)\right) \right] + \,''$$

$$= -\frac{1}{g}\left[\frac{\sum_i \ln(i+a)}{\sum_i p_i \ln p_i} - \frac{\left(\sum_i p_i \ln(i+a)\right)\left(\sum_i (\ln p_i + 1)\right)}{\left(\sum_i p_i \ln p_i\right)^2} \right] + \lambda \sum_i = 0$$

Recall: $H = -g \sum_i p_i \ln p_i$, $C = \sum_i p_i \ln(i+a)$

$H = -g \sum_i p_i \ln p_i$

$$\frac{d\Psi}{dp_i} = 0 = \frac{\sum_i \ln(i+a)}{H} - \frac{C \sum_i (\ln p_i + 1)}{H \left(\sum_i p_i \ln p_i\right)} + \lambda \sum_i \quad ; \Rightarrow \sum_i p_i \ln p_i = -\frac{H}{g}$$

$$= \frac{\sum_i \ln(i+a)}{H} + \frac{gC \sum (\ln p_i + 1)}{H^2} + \lambda \sum_i$$

$$= \sum_i \left[\frac{1}{H} \ln(i+a) + \frac{gC}{H^2} \ln p_i + \frac{gC}{H^2} + \lambda \right] = 0$$

$$\Rightarrow \frac{1}{H}\ln(i+a) + \frac{gc}{H^2}\ln p_i + \frac{gc}{H^2} + \lambda = 0$$

$$\Rightarrow \frac{gc}{H^2}\ln p_i = -\frac{1}{H}\ln(i+a) - \frac{gc}{H^2} - \lambda$$

$$\Rightarrow \ln p_i = -\frac{H}{gc}\ln(i+a) - 1 - \frac{\lambda H^2}{gc}$$

$$\ln p_i = \ln(i+a)^{-H/gc} - 1 - \frac{\lambda H^2}{gc}$$

$$\Rightarrow p_i = e^{\left[\ln(i+a)^{-H/gc} - 1 - \frac{\lambda H^2}{gc}\right]}$$

$$= e^{\ln(i+a)^{-H/gc}} e^{-1-\frac{\lambda H^2}{gc}}$$

$$\boxed{\therefore p_i = e^{-1-\frac{\lambda H^2}{gc}}(i+a)^{-H/gc}}$$ Let $\alpha \equiv \frac{H}{gc}$

$$\Rightarrow \ln p_i = \left(-1-\frac{\lambda H^2}{gc}\right) + \ln(i+a)^{-H/gc}$$

$$\ln p_i = \left(-1-\frac{\lambda H^2}{gc}\right) - \frac{H}{gc}\ln(i+a)$$

Substitute $\ln p_i$ (but no p_i) into H:

$$H = -g\sum p_i\left[-1-\frac{\lambda H^2}{gc} - \frac{H}{gc}\ln(i+a)\right]$$

$$= g\left[\sum_i p_i + \frac{\lambda H^2}{gc}\sum_i p_i + \frac{H}{gc}\underbrace{\sum p_i \ln(i+a)}_{c}\right]$$

$$= g\left[1 + \frac{\lambda H^2}{gc} + \frac{H}{g}\right]$$

$$\cancel{H} = g + \frac{\lambda H^2}{c} + \cancel{H}$$

$$\Rightarrow \frac{\lambda H^2}{c} = -g$$

$$\Rightarrow \lambda = \frac{-gC}{H^2}$$

Substitute λ into p_i:

$$p_i = e^{-1-\frac{\lambda H^2}{gC}}(i+a)^{-\frac{H}{gC}}$$

$$= e^{-1-\left(\frac{-gC}{H^2}\right)\left(\frac{H^2}{gC}\right)}(i+a)^{\frac{-H}{gC}}$$

$$= e^{-1+1}(i+a)^{\frac{-H}{gC}}$$

$$= (1)(i+a)^{\frac{-H}{gC}}$$

$$\boxed{\therefore\ p_i = (i+a)^{\frac{-H}{gC}}}$$

↳ Inverse Power Law w/ Scaling Exponent $\boxed{\alpha = \frac{H}{gC}}$