The goal is to sample (τ_1, τ_2) from the joint probability density

$$W(\tau_1, \tau_2) \propto e^{c(\tau_2 - \tau_1)}, \tag{1}$$

subject to constraints

$$a < \tau_1 < \tau_2 < b. \tag{2}$$

If we "marginalize out" τ_2 we get a probability density for τ_1 alone,

$$\begin{aligned} P(\tau_1) &= \int_{\tau_1}^{b} d\tau_2 W(\tau_1, \tau_2) \\ &\propto \int_{\tau_1}^{b} d\tau_2 e^{c(\tau_2 - \tau_1)} \\ &= e^{-c\tau_1} \int_{\tau_1}^{b} d\tau_2 e^{c\tau_2} \\ &= e^{-c\tau_1} \frac{1}{c} \left(e^{cb} - e^{c\tau_1} \right) \\ &\propto e^{c(b-\tau_1)} - 1. \end{aligned} \tag{3}$$

We want to sample $\tau_1 \sim P(\tau_1)$ under the constraint $a < \tau_1 < b$. Once accomplished, we can fix the τ_1 sample, and then sample τ_2 from Eq. (1).

Sampling $\tau_1 \sim P(\tau_1)$ is the hard part, so we focus on that. With explicit normalization, we wish to sample from the density

$$P(\tau_1) = \frac{e^{c(b-\tau_1)} - 1}{Z}$$

where

$$Z = \frac{1}{c} \left[e^{c(b-a)} - 1 \right] - (b - a).$$

The cumulative probability is

$$\begin{aligned} F(\tau_1) &\equiv \int_{a}^{\tau_1} P(\tau_1) d\tau_1 \\ &= \frac{1}{Z} \left\{ \frac{1}{c} \left[e^{c(b-a)} - e^{c(b-\tau_1)} \right] - (\tau_1 - a) \right\}. \end{aligned} \tag{4}$$

As required, the net probability is 1, i.e. $F(b) = 1$.

In principle, to sample $\tau_1 \sim P(\tau_1)$ we can solve for the value τ_1 that satisfies

$$x = F(\tau_1),$$

where x is a random variable uniformly sampled between 0 and 1. Inverting the function $F(\tau_1)$ is where it gets hard. Rearrange,

$$Zx - \frac{1}{c} e^{c(b-a)} - a = -\frac{1}{c} e^{c(b-\tau_1)} - \tau_1.$$

Introduce

$$\begin{aligned} y &= Zx - \frac{1}{c}e^{c(b-a)} - a \\ A &= -\frac{1}{c}e^{cb}, \end{aligned}$$

such that we must solve

$$y = Ae^{-c\tau_1} - \tau_1.$$

According to Wolfram Alpha `https://www.wolframalpha.com/input/?i=Solve+%28y+%3D%3D+A+Exp%5B-c+t%5D+-+t%29+for+t`, the solution is given in terms of the Lambert W_k function

$$\tau_1 = \frac{W_k(Ace^{cy})}{c} - y.$$

My guess is that we want the principle branch ($k = 0$). Apparently numerical evaluation requires about 4 iterations using Halley's method, each one requiring an evalution of the exponential function, `https://math.stackexchange.com/a/464668/660903`.