Proof:

Case 1: Suppose $b = 1$. Let $\epsilon > 0$ and let N be any real number. If $n > N$, then

$$\left| b^{1/n} - 1 \right| = |1 - 1| = 0 < \epsilon.$$

Case 2: Suppose $b > 1$. Let $\epsilon > 0$ and let $N = \frac{\log(b)}{\log(1+\epsilon)}$. If $n > N$, then

$$\begin{aligned}
\frac{1}{n} < \frac{1}{N} = \frac{\log(1+\epsilon)}{\log(b)} &\implies \frac{1}{n}\log(b) < \log(1+\epsilon) \\
&\implies \log(b^{1/n}) < \log(1+\epsilon) \\
&\implies b^{1/n} < 1 + \epsilon \\
&\implies b^{1/n} - 1 = \left| b^{1/n} - 1 \right| < \epsilon.
\end{aligned}$$

Case 3.1: Suppose $0 < b < 1$. Let $0 < \epsilon < 1$ and let $N = \frac{\log(b)}{\log(1-\epsilon)}$. If $n > N$, then

$$\begin{aligned}
\frac{1}{n} < \frac{1}{N} = \frac{\log(1-\epsilon)}{\log(b)} &\implies \frac{1}{n}\log(b) > \log(1-\epsilon) \\
&\implies \log(b^{1/n}) > \log(1-\epsilon) \\
&\implies b^{1/n} > 1 - \epsilon \\
&\implies b^{1/n} - 1 > -\epsilon \\
&\implies 1 - b^{1/n} = \left| b^{1/n} - 1 \right| < \epsilon.
\end{aligned}$$

Case 3.2: Suppose $0 < b < 1$. Let $\epsilon \geq 1$ and let N be any real number. If $n > N$, then

$$\begin{aligned}
b > 0 &\implies b^{1/n} > 0 \\
&\implies -b^{1/n} < 0 \\
&\implies 1 - b^{1/n} < 1 \leq \epsilon \\
&\implies \left| b^{1/n} - 1 \right| < \epsilon.
\end{aligned}$$

Therefore we can conclude that $\lim_{n\to\infty} b^{1/n} = 1$. ■